Title: Unruh effect without spacetime

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Abstract: <p>I show that Unruh effect can be associated with the partitioning of the real line, and derived from the basic representation theory of the group of affine transformations in one dimension. This result shows that thermal distributions naturally emerge in connecting quantum
states belonging to representations related to distinct notions of translational symmetry.</p>

Unruh effect without spacetime

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SHK Reebok

BH thermodynamics

- Bekenstein* suggested that BHs have entropy

$$S \sim \frac{A}{l_{Pl}^2}$$

- Spectacularly confirmed by Hawking** BH radiate at temperature

$$T_H = \frac{1}{8\pi GM}$$

* Phys. Rev. D7, 2333 (1973); ** Nature 248, 30 (1974):

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
  - ➢ The enigmatic nature of degrees of freedom that BH entropy is counting;
  - ➢ The fate of unitarity in BH quantum evaporation: do BHs evolve pure states into mixed states?

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
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Crucial to the BH thermodynamics puzzles is the quantum temperature perceived by accelerated observer, but not by inertial one.

Horizon temperature

- Unruh* motivated by Hawking discovery that black holes radiate thermally at T_H, associates temperature to the Rindler horizon

$$T_U = \frac{a}{2\pi}$$

* Phys. Rev. D 14, 870 (1976), 2003))

Horizon temperature

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- **What is the universal, basic structure behind Unruh temperature?**

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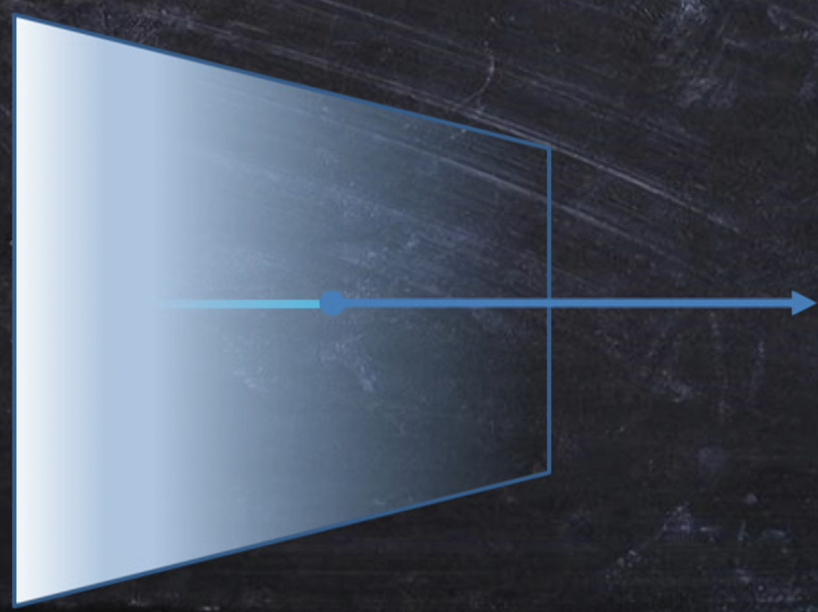
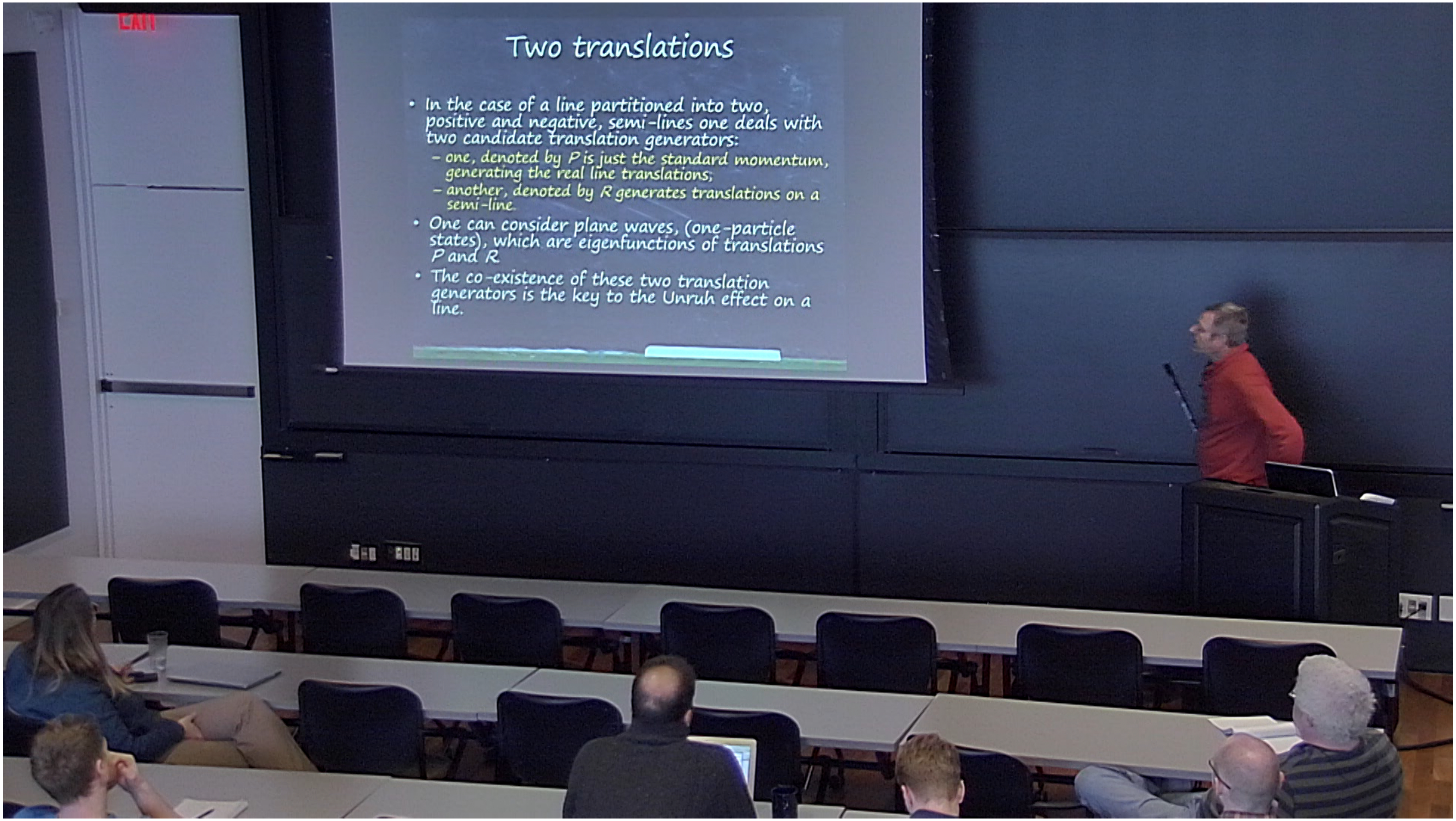
- In this talk I will describe **connection between the geometric notion of a null boundary and thermal quantum states** in the setup stripped down to the bone.
- Minimal setting: only **group theoretic ingredients** associated with symmetries of space-time
  - No space-time
  - No metric
  - No quantum fields (almost)

Null surface

- Consider a generic null surface (codimension 1) in Minkowski space in t-x plane. This surface is the basic ingredient of Unruh effect setup.




EXIT
Two translations
• In the case of a line partitioned into two, positive and negative, semi-lines one deals with two candidate translation generators:
– one, denoted by P is just the standard momentum, generating the real line translations;
– another, denoted by R generates translations on a semi-line.
• One can consider plane waves, (one-particle states), which are eigenfunctions of translations P and R
• The co-existence of these two translation generators is the key to the Unruh effect on a line.

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Poincaré–Weyl algebra

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$$[P_t, P_x] = 0,\ [D, N] = 0$$

$$[N, P_t] = iP_x,\ [N, P_x] = iP_t$$

$$[D, P_t] = iP_t,\ [D, P_x] = iP_x$$

ax+b algebra

- We can form light-cone generators

$$P \equiv P_t - P_x\,, \quad R \equiv \frac{1}{2}(N - D)$$

- Which satisfy a simple algebra (actually, the simplest nontrivial)

$$\boxed{[P, R] = iP}$$

- Unruh effect can be directly derived from representation theory of this algebra.

ax+b group

- … is a group of transformations g=(a,b)

$$x \to ax + b, \quad a \in \mathbb{R}^+, b \in \mathbb{R}$$

- consisting of two transformations, translations and dilations

$$T(\alpha) = (1, \alpha), \quad S(\lambda) = (e^{-\lambda}, 0)$$

- with representation on a Hilbert space (positive energy states)

$$T(\alpha)|k\rangle_+ = e^{-i\alpha P}|k\rangle_+ = e^{-i\alpha k}|k\rangle_+, \quad S(\lambda)|k\rangle_+ = e^{-i\lambda R}|k\rangle_+ = |e^{-\lambda}k\rangle_+$$

$$P|k\rangle_+ = k|k\rangle_+, \quad R|k\rangle_+ = -ik\frac{d}{dk}|k\rangle_+, \quad k \in \mathbb{R}^+$$

Fourier transform

- For "functions on momentum space"

$$T(\alpha)\psi(k) = e^{i\alpha k}\psi(k), \quad S(\lambda)\psi(k) = \psi(e^{\lambda}k), \quad \psi(k) = {}_{+}\langle k|\psi\rangle$$

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- One can take Fourier transform to get the position space picture

$$\psi(x) \equiv \langle x|\psi\rangle = \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dk}{k}\left(e^{ikx}\,{}_{+}\langle k|\psi\rangle + e^{-ikx}\left({}_{+}\langle k|\psi\rangle\right)^*\right)$$
$$= \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dk}{k}\left(e^{-ikx}\,a(k) + e^{ikx}\,a^*(k)\right)$$

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$$T(\alpha)\psi(x) = \psi(x+\alpha), \quad S(\lambda)\psi(x) = \psi(e^{-\lambda}x)$$

$$P\psi(x) = -i\frac{d}{dx}\psi(x), \quad R\psi(x) = ix\frac{d}{dx}\psi(x)$$

P-momenta vs. R-momenta

$$[P,R]=iP$$

- So far we discussed the representation in which P was diagonal. Let us now take another representation, in which R is diagonal.

$$|k\rangle_+ = \int_{-\infty}^{\infty} d\omega \langle \omega | k\rangle_+ |\omega\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega k^{-i\omega} |\omega\rangle, \quad k \in \mathbb{R}^+,$$

$$|\omega\rangle = \int_0^{\infty} \frac{dk}{k} k^{i\omega} |k\rangle_+$$

$$P|k\rangle_+ = k|k\rangle_+, \quad R|k\rangle_+ = -ik\frac{d}{dk}|k\rangle_+,$$

$$P|\omega\rangle = |\omega - i\rangle, \quad R|\omega\rangle = -\omega|\omega\rangle$$

ω-representation

- The "field" is now

$$\psi(x)=\frac{1}{2\pi}\int_0^\infty \frac{dk}{k}\int_{-\infty}^\infty d\omega\left(e^{ikx}k^{i\omega}\langle\omega|\psi\rangle+e^{-ikx}k^{-i\omega}\langle\omega|\psi\rangle^*\right)$$

- And can be rewritten as

$$\psi(x)=\frac{1}{2\pi}\int_{-\infty}^\infty d\omega\left(x^{-i\omega}e^{-\frac{\pi\omega}{2}}\Gamma(i\omega)\langle\omega|\psi\rangle+x^{i\omega}e^{-\frac{\pi\omega}{2}}\Gamma(-i\omega)\langle\omega|\psi\rangle^*\right)$$

$$=\frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{d\omega}{\omega}\left(x^{-i\omega}b(\omega)+x^{i\omega}b^*(\omega)\right)$$

k- vs. ω-representation

- ψ(x) is the same function in k- and ω-representation, so that

$$b(\omega)=\frac{\omega}{\sqrt{2\pi}}\Gamma(i\omega)\int_0^\infty \frac{dk}{k}k^{-i\omega}\left[a(k)e^{\pi\omega/2}+a^*(k)e^{-\pi\omega/2}\right]$$

- From which we can read-off the Bogolyubov coefficients.

Quantum (field) theory

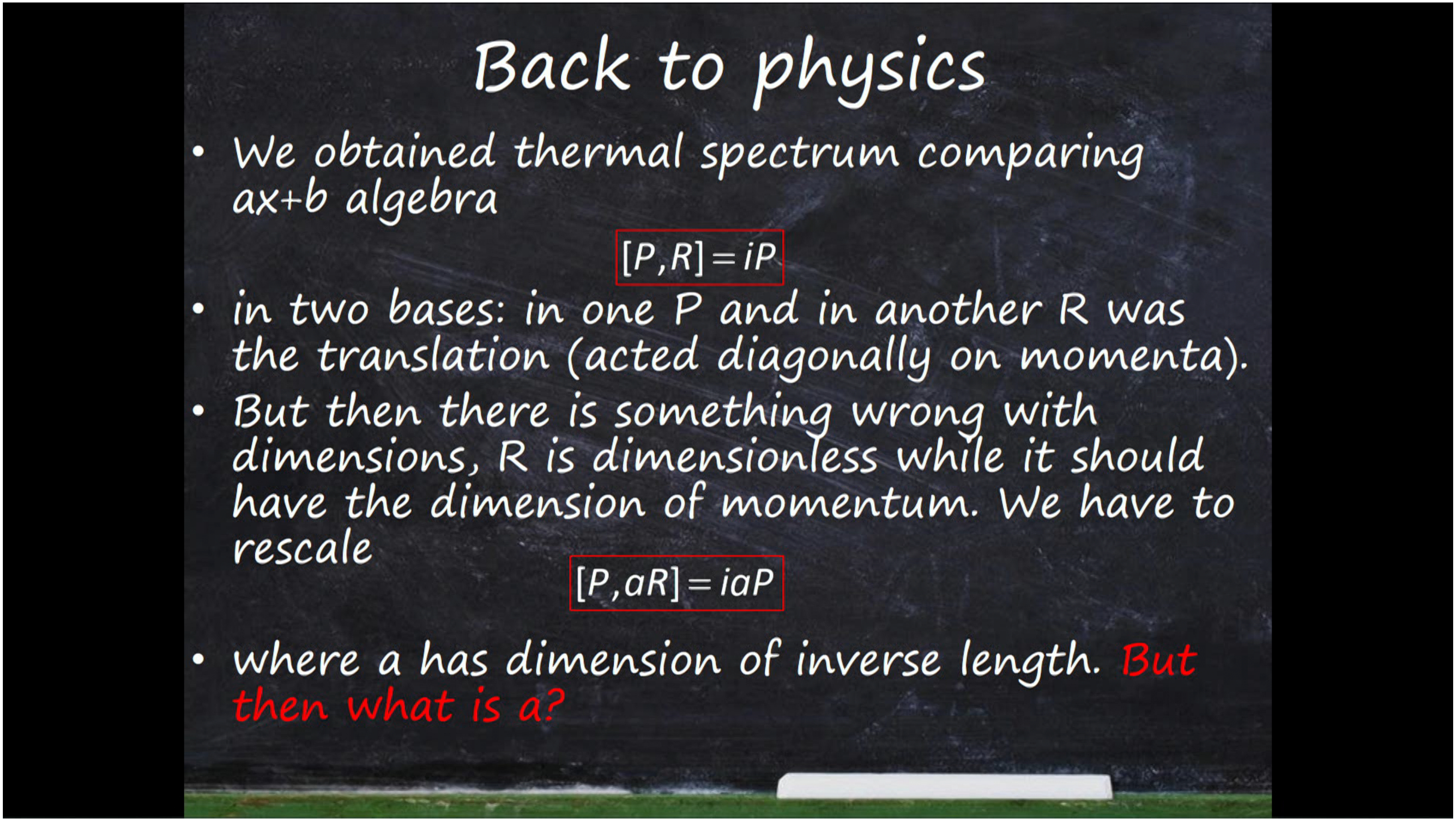
- We upgrade the functions a and b to quantum operators:

$$\left[\hat{a}(k),\hat{a}^{\dagger}(k')\right]=k\,\delta(k-k')\,,\quad \left[\hat{b}(\omega),\hat{b}^{\dagger}(\omega')\right]=2\pi\omega\,\delta(\omega-\omega')$$

$$\hat{a}(k)\,|0\rangle_a=0,\qquad \hat{b}(\omega)\,|0\rangle_b=0$$

- And then we find that $N_b(\omega)=\frac{1}{2\pi\omega}\hat{b}^{\dagger}(\omega)\hat{b}(\omega)$

$$n_b=\frac{N_b}{\delta(0)}=\frac{1}{\omega}\frac{1}{\delta(0)}\;{}_a\!<0\,|\,\hat{b}^{\dagger}(\omega)\hat{b}(\omega)\,|\,0>_a=\frac{1}{e^{2\pi\omega}-1}$$


Back to physics
We obtained thermal spectrum comparing ax+b algebra
$[P,R]=iP$
in two bases: in one P and in another R was the translation (acted diagonally on momenta).
But then there is something wrong with dimensions, R is dimensionless while it should have the dimension of momentum. We have to rescale
$[P,aR]=iaP$
where a has dimension of inverse length. But then what is a?

Accelerated observers

- Accelerated worldline is the Lorentz orbit of the vector (0; 1/a)

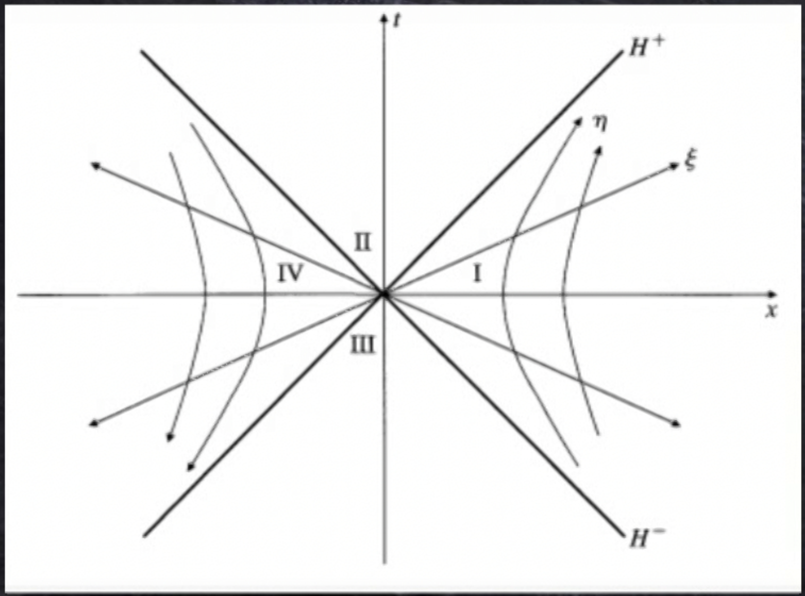
$$t^2(\tau) - x^2(\tau) = \frac{1}{a^2}$$

- Lorentz transformations move points along the orbit of constant acceleration; but what is the transformation that changes the acceleration?
- It is the dilation that does the job.

$$(t, x) \rightarrow (t', x') = e^{\delta}(t, x)$$

Rindler space

- We define the Rindler coordinates with the help of the boost and dilation parameters.



$$t = \frac{1}{a} e^{a\xi} \sinh a\eta$$

$$x = \frac{1}{a} e^{a\xi} \cosh a\eta$$

Poincaré-Weyl algebra

- We have two translation operators P_t and P_x generating translation in Minkowski space;
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k- vs. ω-representation

- So when we compare k and ω bases we are comparing theories as seen by Minkowski and Rindler observers.
- The thermal distribution we found is the number of quanta Rindler observer sees in Minkowski vacuum.

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Unruh effect

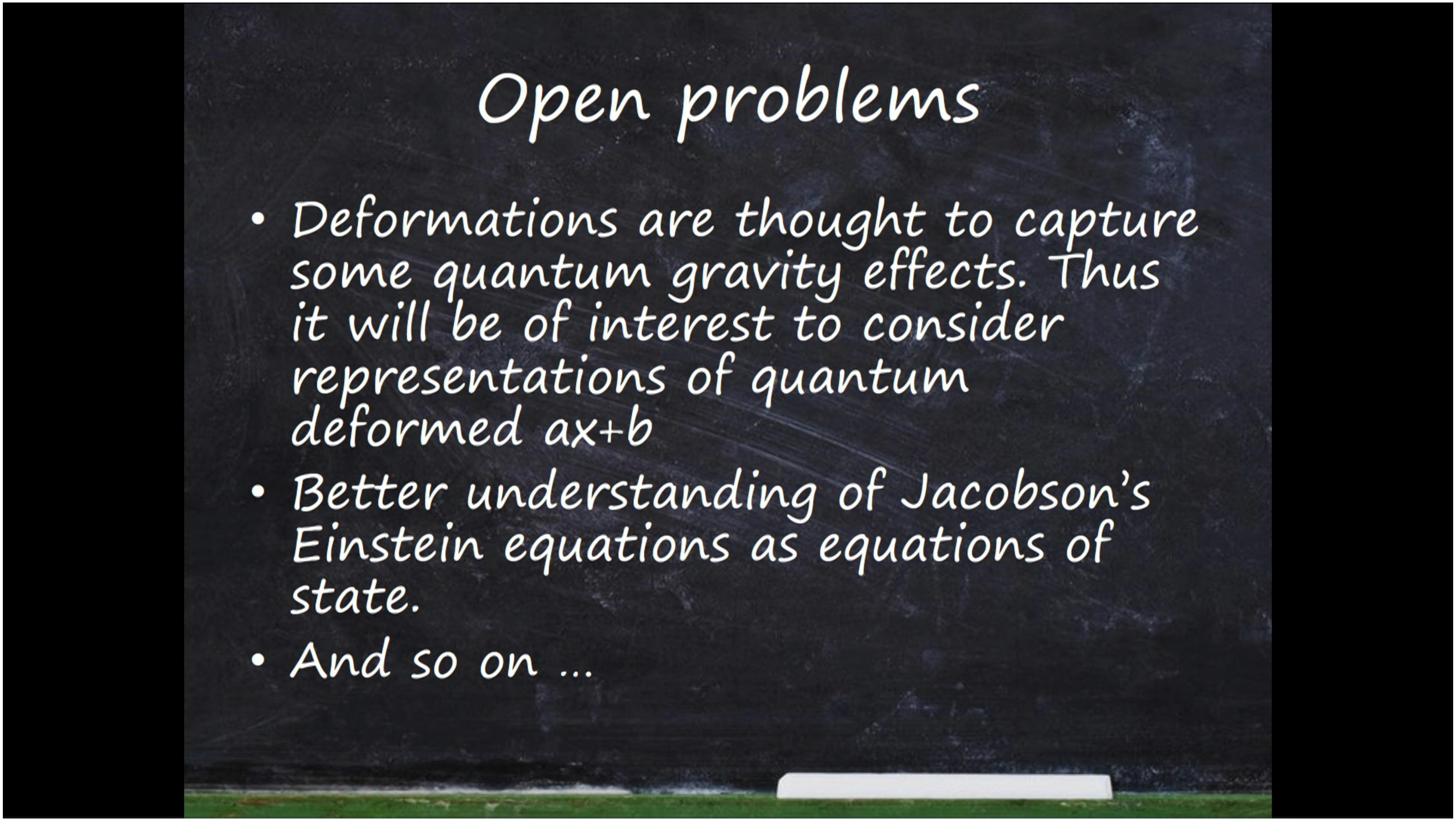
- Then the thermal spectrum becomes

$$n_b = \frac{1}{e^{2\pi\omega/a} - 1}$$

- which is thermal distribution at temperature T=2π/a, Unruh temperature, physically interpreted as the temperature of the thermal bath the accelerating observer is immersed in.

Big picture

- The Unruh effect is just a particular example of the construction based on representations of ax+b group.
- There might be (should be) other instances where representation theory of ax+b group might be of use.


Open problems
• Deformations are thought to capture some quantum gravity effects. Thus it will be of interest to consider representations of quantum deformed ax+b
• Better understanding of Jacobson's Einstein equations as equations of state.
• And so on ...