Title: TBA

Date: Nov 26, 2018 11:00 AM

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Abstract: <p>Abstract TBD.</p>


Mirror symmetry & representation theory
Work in progress

Mirror symmetry & representation theory

Work in progress
Jt. w/ M McBreen?

G reductive grp, $\mathfrak{g} = \mathrm{Lie}(G)$, $G^\vee =$ Langlands dual

$\mathfrak{g}, [-,-] \rightsquigarrow \mathfrak{g}^*$ is a Poisson manifold

GS resolution

$$\begin{tikzcd} \tilde{\mathfrak{g}}^* \arrow[r] \arrow[dr, "\tilde{\chi}"'] & \mathfrak{g}^* \arrow[d, "\chi"] \\ & \mathfrak{h}^*/W \end{tikzcd}$$

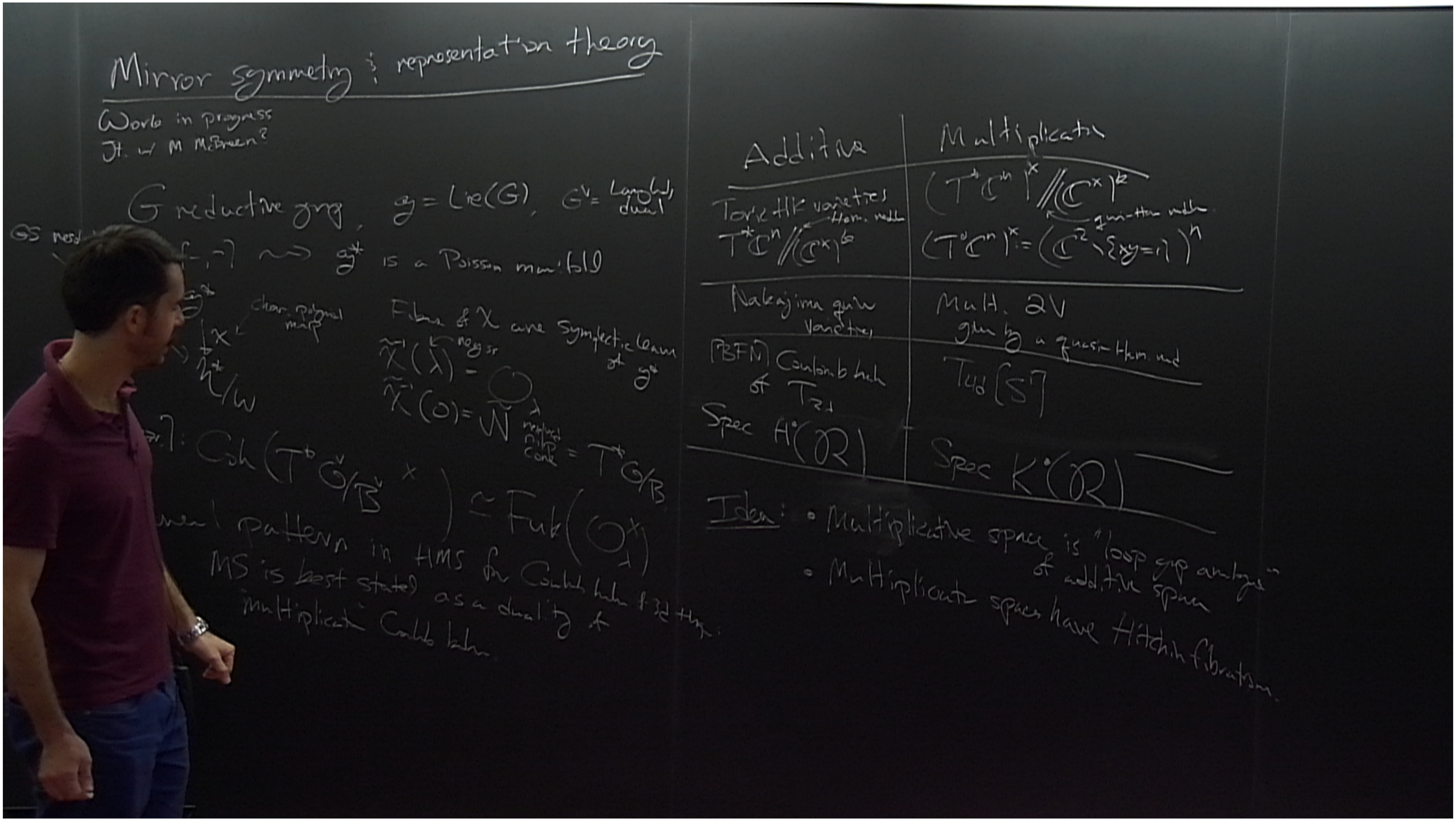
char. polynomial map

Fibers of χ are symplectic leaves of $\mathfrak{g}^*$

$\tilde{\chi}^{-1}(\lambda) = \mathcal{O}_\lambda$ reg sr

$\tilde{\chi}^{-1}(0) = \tilde{\mathcal{N}}$ resolved nilp cone $= T^*G/B$

[Gij/Bezr.]: $\mathrm{Coh}(T^*G^\vee/B^\vee \times) \simeq \mathrm{Fuk}(\mathcal{O}_\lambda)$


Mirror symmetry & representation theory
Work in progress
Jt. w/ M McBreen?
G reductive grp
is a Poisson manifold
Fibres of χ are symplectic leaves
Additive
Multiplicative
Toric HK varieties
Nakajima quiver varieties
Coulomb branch
Spec
Idea:
• Multiplicative space is "loop grp analogue" of additive space
• Multiplicative spaces have Hitchin fibrations
MS is best stated as a duality of multiplicative Coulomb branches

Want: Group analogue of previous construction

Replace $\mathfrak{g}^*$ by G^* ← dual Poisson-Lie group

$$G^* \xrightarrow{\sim} B_- B_+ \hookrightarrow G$$

(B_-: lower-Δ matrices, B_+: upper-Δ matrices)

Fact: $B_- B_+ = \left\{ g \in G \;\middle|\; \text{upper-left minors of } g \text{ are all nonzero} \right\}$

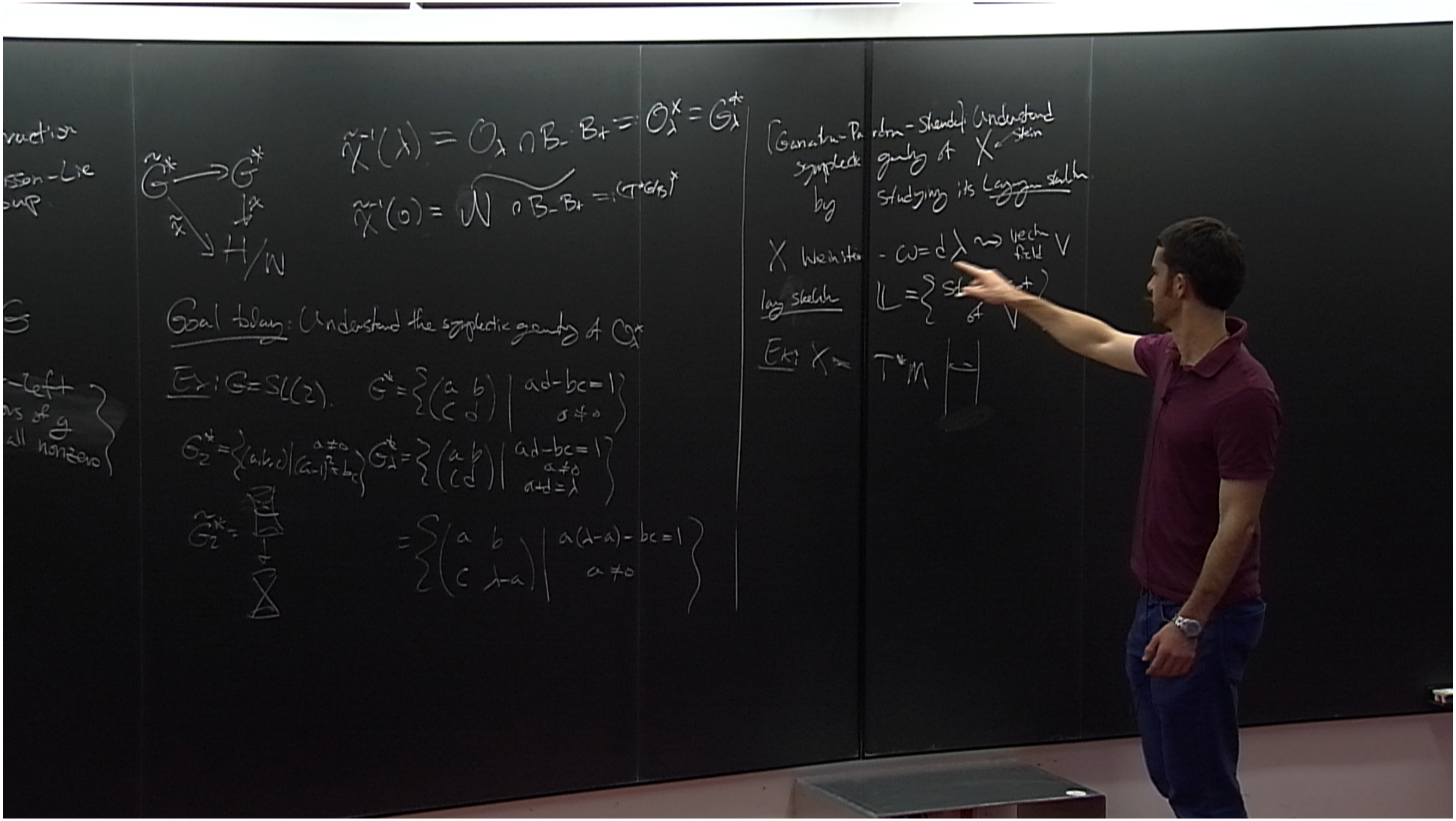
Ex: $\left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \;\middle|\; \begin{matrix} a \neq 0 \\ ae - bd \neq 0 \end{matrix} \right\}$

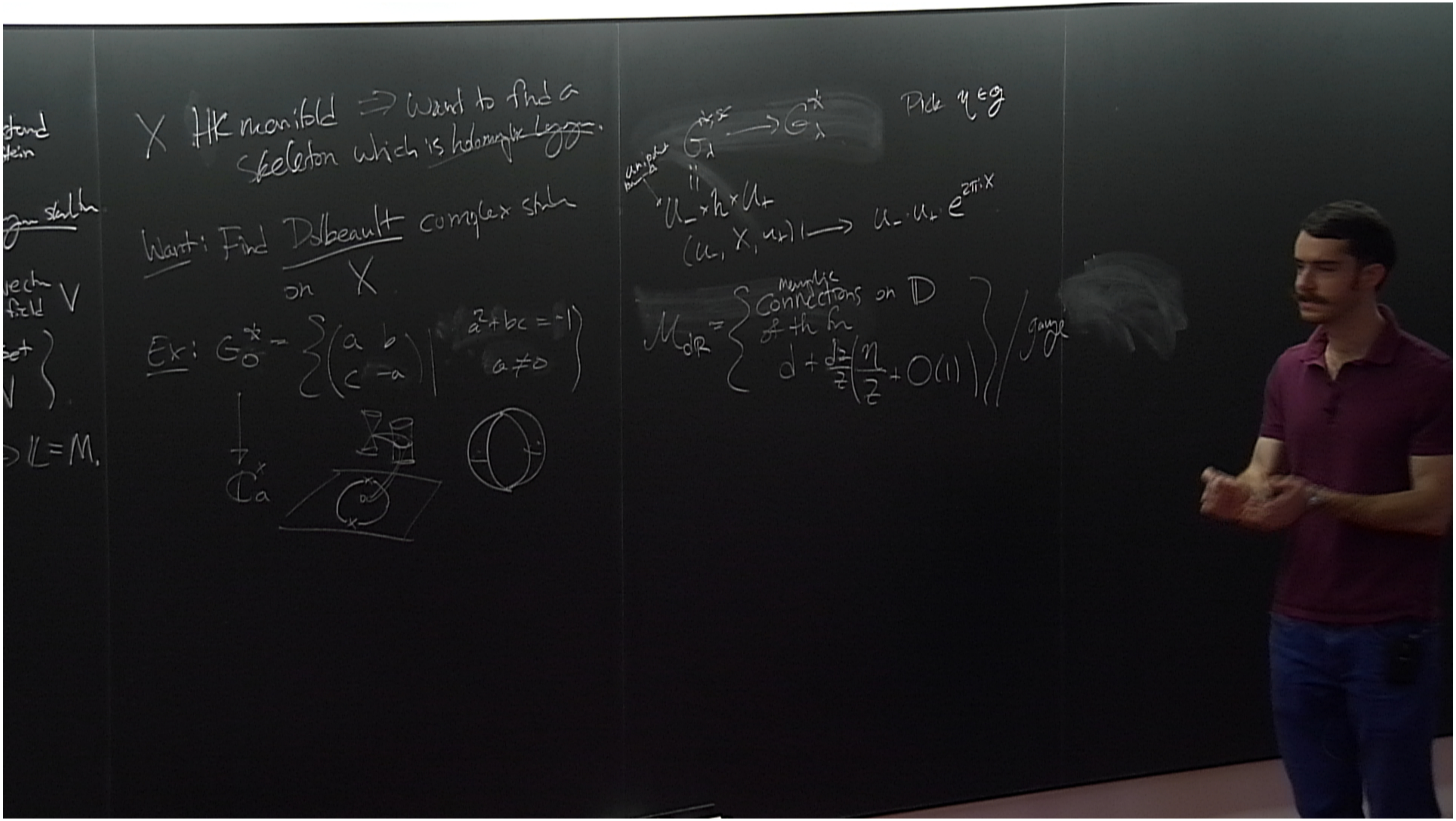
$$\begin{tikzcd} \tilde{G}^* \arrow[r] \arrow[dr, "\tilde{\chi}"'] & G^* \arrow[d, "\chi"] \\ & H/W \end{tikzcd}$$

$$\tilde{\chi}^{-1}(\lambda) = \mathcal{O}_\lambda \cap B_- \cdot B_+ =: \mathcal{O}_\lambda^\times$$

$$\tilde{\chi}^{-1}(0) = \mathcal{N} \cap B_- B_+ =: (T^*G/B)^\times$$

analogue ... fibration


Goal today: Understand the symplectic geometry of O
Ex: G=SL(2)
Understand
symplectic geometry of X
by studying its
Weinstein
Lag skeleton


X HK manifold ⇒ Want to find a skeleton which is holomorphic Lagrangian.
Want: Find Dolbeault complex str on X
Ex: $G_0^\times = \left\{ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \middle| \begin{array}{c} a^2+bc=-1 \\ a \neq 0 \end{array} \right\}$
$\mathbb{C}_a^\times$
$G_\lambda^{\times, \mathrm{ss}} \longrightarrow G_\lambda^\times$
unipotent
$U_- \times \mathfrak{h} \times U_+$
$(u_-, X, u_+) \longmapsto u_- \cdot u_+ \cdot e^{2\pi i X}$
Pick $\eta \in \mathfrak{g}$
$\mathcal{M}_{dR} = \left\{ \begin{array}{l} \text{Meromorphic connections on } \mathbb{D} \\ \text{of the form} \\ d + \frac{dz}{z}\left(\frac{\eta}{z} + O(1)\right) \end{array} \right\} / \text{gauge}$

...stend
...tein

...gn stalk

vector field V

...set ... $V \}$

$\Rightarrow \mathbb{L} = M.$

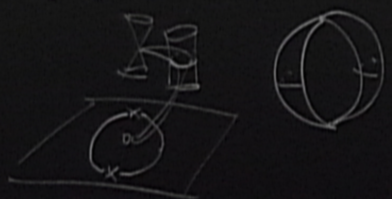
X HK manifold $\Rightarrow$ Want to find a skeleton which is holomorphic Lagrangian.

Want: Find Dolbeault complex structure on X

Ex: $G_0^* = \left\{ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \middle| \begin{array}{c} a^2+bc=-1 \\ a \neq 0 \end{array} \right\}$

$\downarrow$

$\mathbb{C}_a^\times$



$G_\lambda^{*,sc} \longrightarrow G_\lambda^*$ Pick $\eta \in \mathfrak{g}$

unipotent

$\| \quad U_- \times \mathfrak{h} \times U_+$

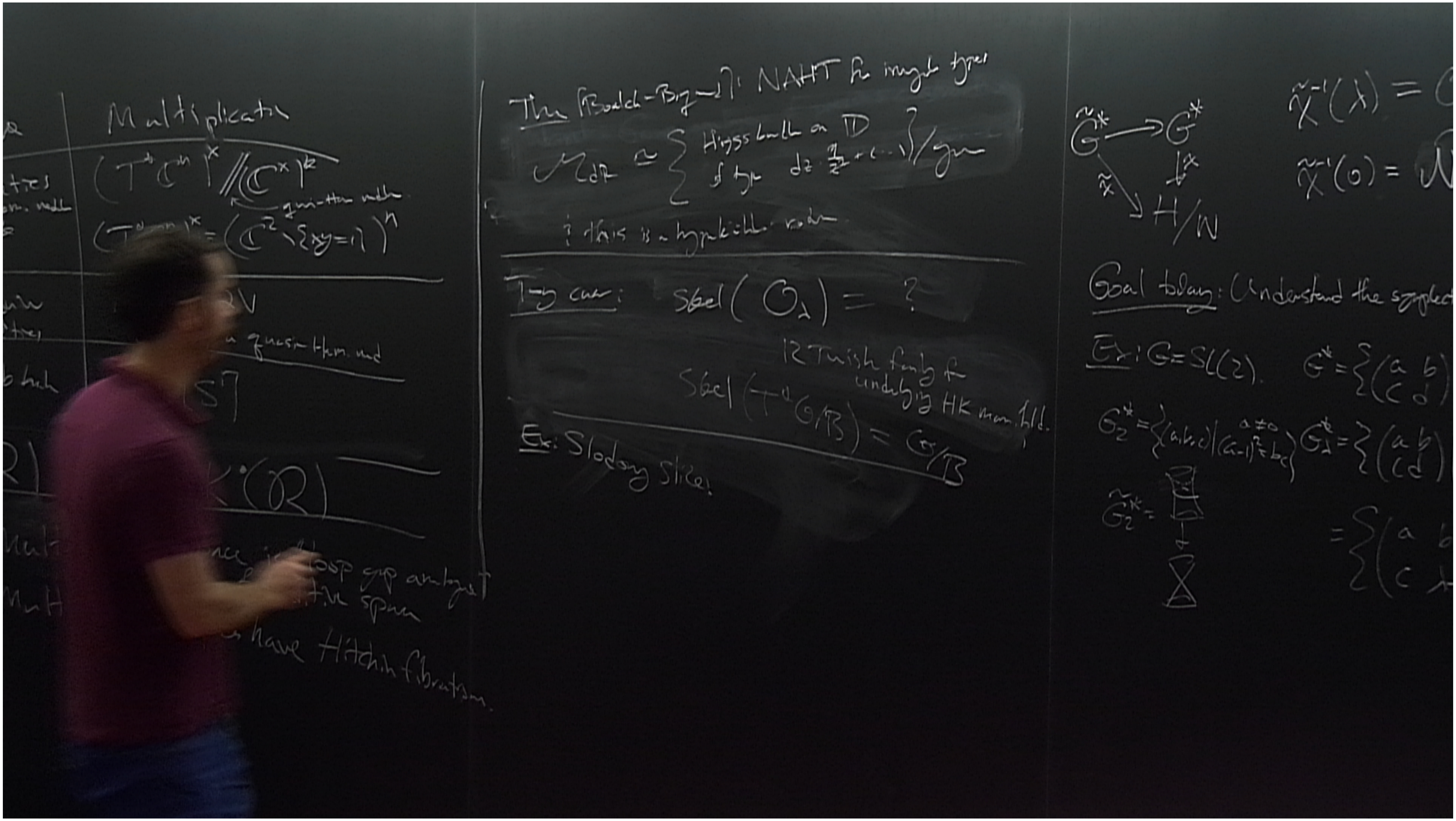
$(u_-, X, u_+) \longmapsto u_- \cdot u_+ \cdot e^{2\pi i X}$

$\mathcal{M}_{dR} = \left\{ \begin{array}{l} \text{meromorphic connections on } \mathbb{D} \\ \text{of the form} \\ \quad d + \frac{dz}{z}\left(\frac{\eta}{z} + O(1)\right) \end{array} \right\} \Big/ \text{gauge}$

IRH/Z

$\mathcal{M}_B = \left\{ \begin{array}{l} \text{Stokes} \\ \text{data fn} \end{array} \right\} \Big/ \dots$

Thm [Boalch]: $G^{*,sc} = \mathcal{M}_B$.


Multiplicative
NAHT for irregular types
Higgs bundles on D
this is a hyperkähler mfd
Sobel$(\mathcal{O}_\lambda)$ = ?
Ex: Slodowy slice
Goal today: Understand the symplectic
Ex: G = SL(2)
have Hitchin fibration.

Moral of this talk:

$\mathcal{M}_{\mathrm{Dol}}$ = affine version of Slodowy slice

Skeleton = affine Springer fib.

$\mathcal{M}_{\mathrm{Dol}} = \mathcal{N} /\!\!/ T^*\mathcal{F}\ell$

$\Longrightarrow \mathrm{Fuk}(\mathcal{O}_\lambda^{\kappa}) \hookrightarrow D\text{-mod}^{(N,\psi)}(\mathcal{F}\ell)$.

Thm (Boalch–Biquard): NAHT for irregular types

$\mathcal{M}_{dR,\lambda} \cong \left\{ \begin{array}{l} \text{Higgs bundles on } \mathbb{D} \\ \text{of type } dz \cdot \frac{\lambda}{z^2} + (\ldots) \end{array} \right\} / \text{gauge} =: \mathcal{M}_{\mathrm{Dol}}$

; this is a hyperkähler rotation

Toy case: $\mathrm{Skel}(\mathcal{O}_\lambda) = \;?$

1 2 Twistor family of underlying HK manifold.

$\mathrm{Skel}(T^*(G/B)) = G/B$

Ex: Slodowy slice $S = \mathcal{N} /\!\!/_{\psi} \mathfrak{g}^*$. Symplectic leaves are $\mathcal{O}_\lambda \cap S^{*}$ (Slodowy slice)

As above, $\mathcal{O}_\lambda \cap S \underset{\text{family}}{\overset{HK}{\sim}} \widetilde{\mathcal{N} \cap S} \sim \mathcal{N} /\!\!/_{\psi} T^* G/B$

$\mathrm{Skel}(\widetilde{\mathcal{N} \cap S}) = $ Springer fib.

$\widetilde{G}^* \longrightarrow G^*$

$\widetilde{\chi} \searrow \quad \downarrow \chi$

H/W

Goal today:

Ex: $G = SL($

$G_2^* = \{(a,b,c) \mid$

$\widetilde{G}_2^* =$