Title: Holographic Thermodynamics of Accelerating Black Holes

Date: Nov 22, 2018 01:00 PM

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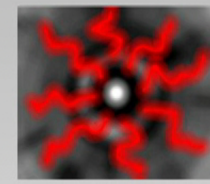
Abstract: <p>I will address a long-standing problem of describing the thermodynamics of an accelerating black hole, concentrating on the special case of slowly accelerating black holes in AdS. The key
ingredient of consistent thermodynamics is to ensure that the system is not over-constrained by including the possibility of varying the string tensions that are responsible for the acceleration of the black hole. The first law assumes the standard form, with the entropy given by one quarter of the horizon area and other quantities identified by standard methods. The dual
energy-momentum tensor can be written as a three-dimensional perfect fluid plus a non-hydrodynamic contribution. Some novel thermodynamic phase behavior related to the black hole acceleration will also be discussed.</p>

Black hole thermodynamics



- First law of black hole thermodynamics:

$$\delta M = T\delta S + \sum_i \Omega_i \delta J_i + \Phi \delta Q + V\delta P \qquad P = -\frac{1}{8\pi}\Lambda$$

- Smarr-Gibbs-Duhem relation:

$$\frac{d-3}{d-2}M = TS + \sum_i \Omega_i J_i + \frac{d-3}{d-2}\Phi Q - \frac{2}{d-2}VP$$

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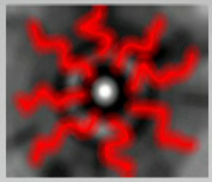

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Works for black holes of various asymptotic properties, horizon topologies, charges, and rotation parameters. With a remarkable exception: **C-metric**

Plan of the talk

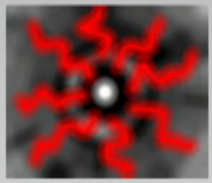
I. What are accelerated black holes?

II. Thermodynamics of accelerated black holes
   a) Capture of a cosmic string by a black hole
   b) Intermezzo 1: Calculation of mass
   c) Thermodynamics of AdS C-metric

III. Adding spin and charge
   a) Intermezzo 2: Meaning of conjugate quantities
   b) Thermodynamics of rotating & charged C-metric
   c) What about other C-metric settings?

IV. Summary & Future directions

Black hole thermodynamics



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Based on

Mostly wrong:

Paper 1: M. Appels, R. Gregory, DK, *Thermodynamics of Accelerating Black Holes*, Phys. Rev Lett. 117 (2016) 131303, ArXiv: 1604.08812.

Paper 2: M. Appels, R. Gregory, DK, *Black Hole Thermodynamics with Conical Defects*, J. High Energy Phys. 1705 (2017) 116, arXiv:1702.00490.

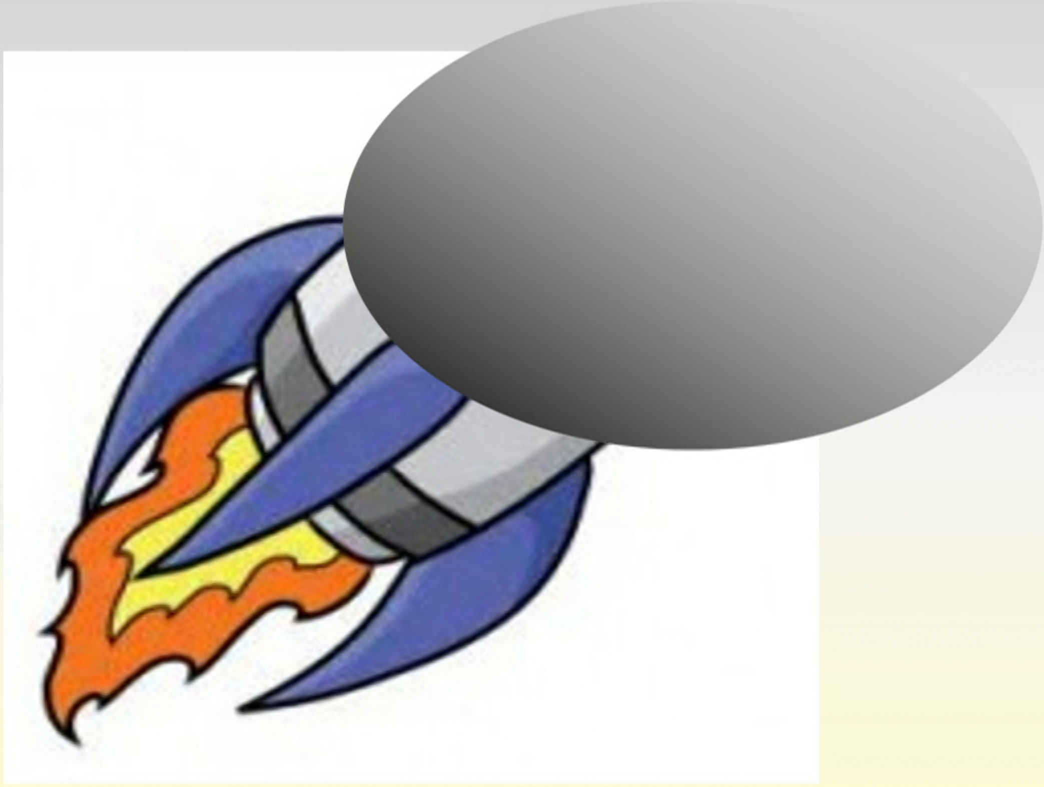
Almost correct?

Paper 3: A. Anabalon, M. Appels, R. Gregory, DK, R.B. Mann, A. Ovgun, *Holographic Thermodynamics of Accelerating Black Holes*, ArXiv:1805.02687.

Paper 4: A. Anabalon, F. Gray, R. Gregory DK, R.B. Mann, *Thermodynamics of Charged, Rotating, and Accelerating Black Holes*, arXiv:1811.04936.

Paper 5: N. Abbasvandi, W. Cong, DK, R.B. Mann, *Snapping swallowtails in accelerating black hole thermodynamics*, in preparation.

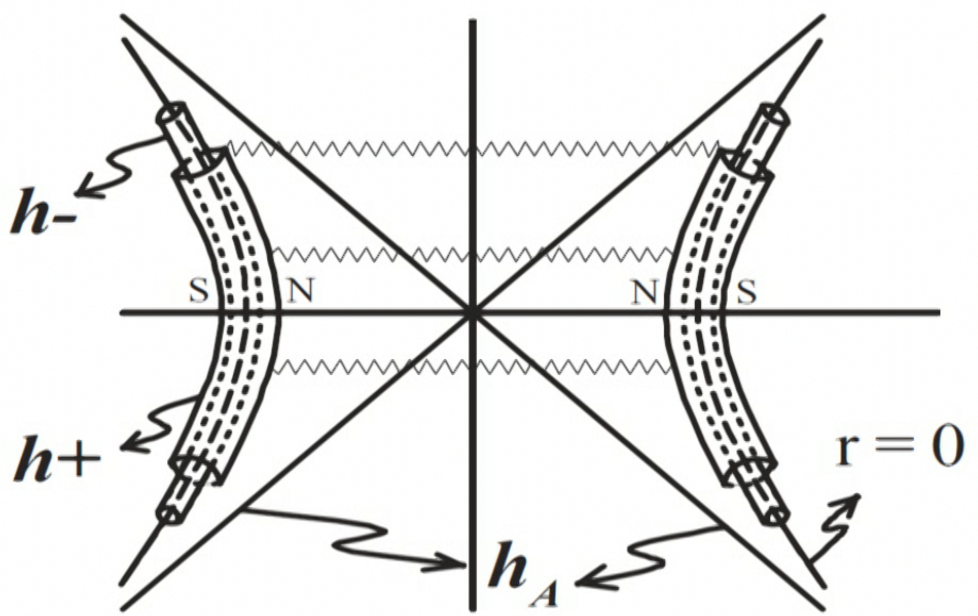
I) What are accelerated black holes?



Introducing the C-metric

- Exact **stationary**, **axisymmetric** black hole solution of Einstein equations (with EM field and cosmological constant)

Weyl (1917), Levi-Civita (1918), Ehlers & Kundt (1963), Kinnersley & Walker (1970),…



O. Dias and J. Lemos, Phys.Rev. D67 (2003) 064001.

- Typically describes a pair of **accelerated black holes,** pushed away by a strut in between them (represented by a conical singularity)

What is it good for?

- Exact **radiative spacetime –** used to study **radiative patterns** in spacetimes with various asymptotics (Bicak, Krtous, Ortaggio, Podolsky, Pravda, Pravdova,…)

- Used to study **black hole nucleation**: for example in electric field, de Sitter space (see Ross 03 for review)

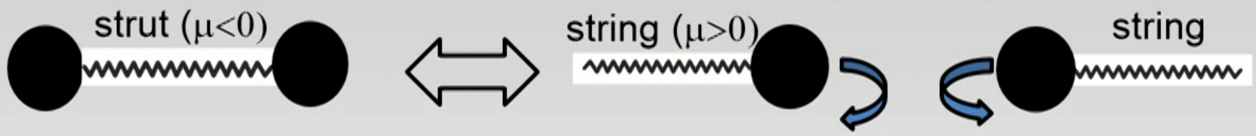
- Used to construct **black ring** of Emparan and Reall (Wick rotated C-metric)

- AdS/CFT correspondence: **black funnels and droplets** (Hubeny, Marolf, Rangamani 10)

- Provides means fro **splitting cosmic strings** (Gregory, Hindmarsh; Eardley, Horowitz, Kastor, Traschen 95)

Various setups

- **Strut** can be replaced by two **cosmic strings** stretching to infinity (represented by a conical deficit and positive tension)



- **Regular C-metrics (Ernst 1976):**

  The conical deficits are removed by placing the charged black hole in an **external** electric or magnetic field

- **C-metrics with various asymptotics** (including scalar fields).

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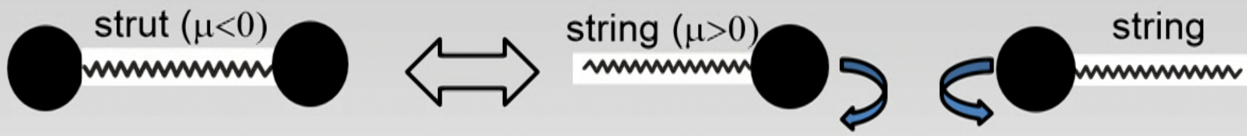

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- **C-metrics with various asymptotics** (including scalar fields).

> - No known generalization to higher dimensions
> - Not well established **thermodynamics**

Simplest TD setup: AdS C-metric with small acceleration

$$ds^2 = \frac{1}{\Omega^2}\left[-f\,dt^2 + \frac{dr^2}{f} + r^2\left(\frac{d\theta^2}{h} + h\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

where:

$$\Omega = 1 + Ar\cos\theta\,, \qquad h = 1 + 2mA\cos\theta$$

$$f(r) = (1 - A^2r^2)\left(1 - \frac{2m}{r}\right) + \frac{r^2}{\ell^2}\,.$$

acceleration parameter

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A ← acceleration parameter

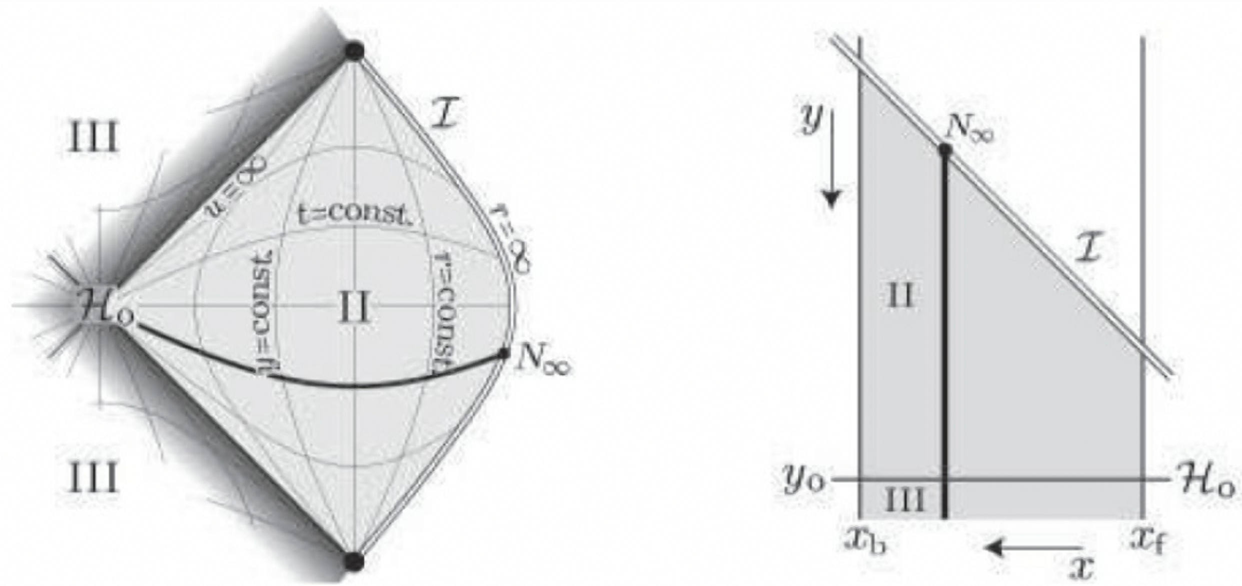
- **Conformal factor**: $\Omega = 1 + Ar\cos\theta$

(determines conformal infinity)

Slow acceleration case

$A \lesssim 1/l$

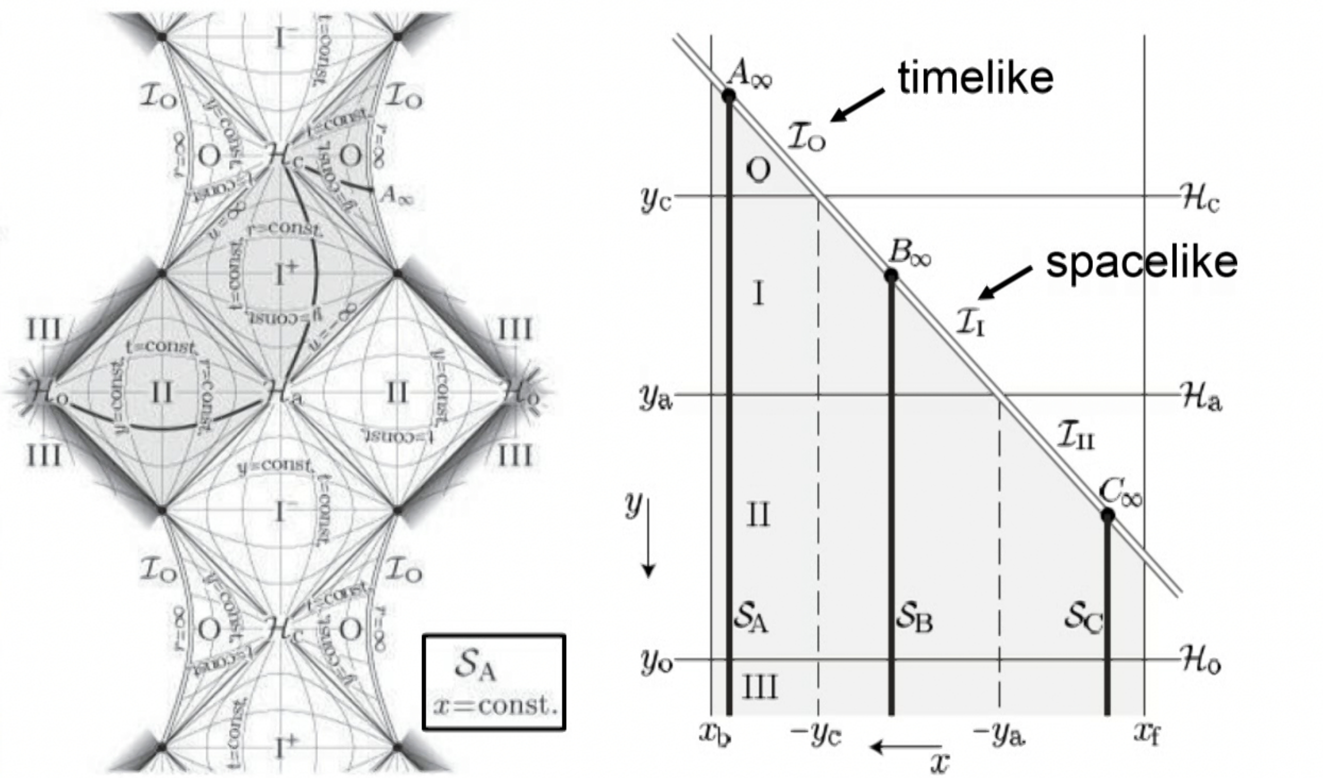
(no acceleration horizon is present)



J. Podolsky, M. Ortaggio, P. Krtous, Phys.Rev. D68 (2003) 124004.

J. Podolsky, *Accelerating black holes in anti-de Sitter universe*, Czech J. Phys. 52 (2002) 1; gr-qc/0202033.

Large acceleration case



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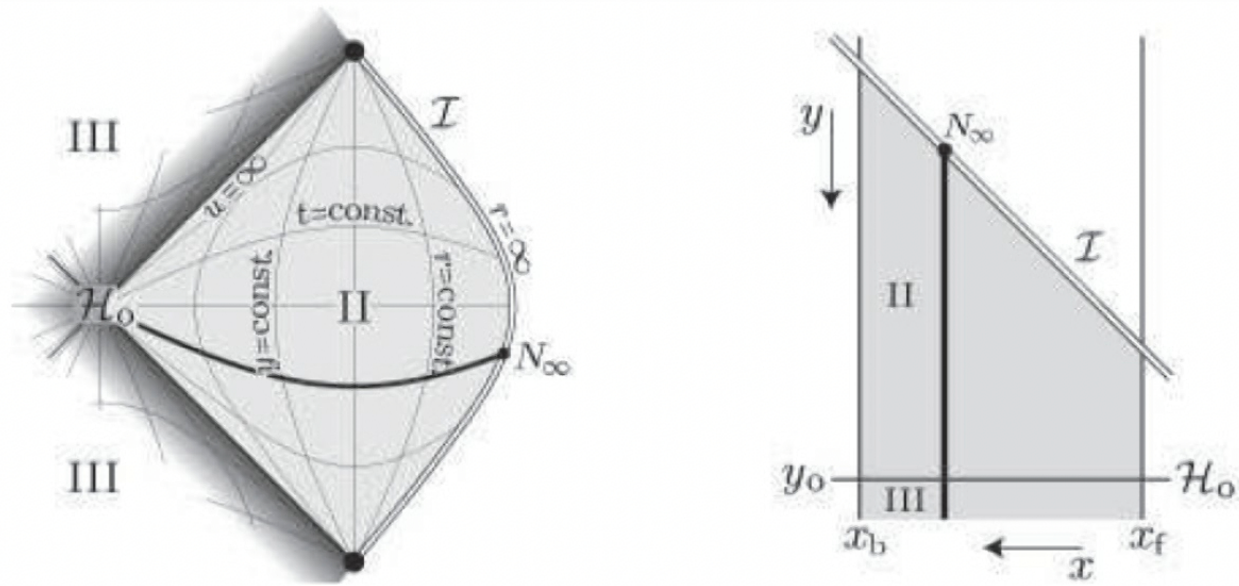

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We focus on slow acceleration

- Only **one black hole** exists in the spacetime, there are no acceleration or cosmological horizons (no black holes on the boundary)

- Since only one (black hole) horizon is present, the system has unique temperature and **thermodynamics** should be well defined.

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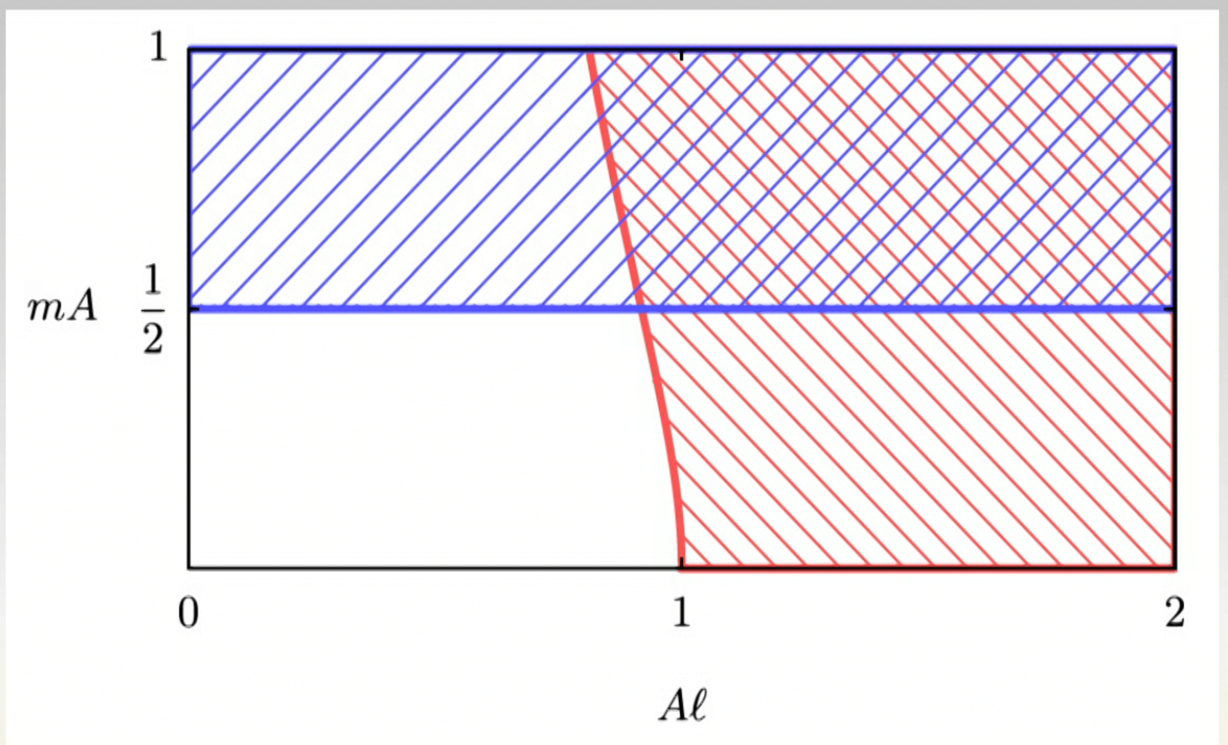
Warning:

- We still have a **cosmic string**. Does it influence thermodynamics? Can the string be dynamical?

- Since even in this case the black hole is uniformly accelerated (though the metric is static) what about **radiation**? (Is there some kind of "weird balance" between retarded and advanced potentials?)

Parameter space



- Metric has physical signature and natural azimuthal coordinate
- There is a bulk black hole (not naked singularity)
- Slow acceleration regime

Conical deficits and cosmic strings

- determined from the behavior of function h at the poles

  North pole: $\theta_+ = 0$ South pole: $\theta_- = \pi$

- Surfaces with constant t and r:

$$ds^2_{\mathrm{II}} = \frac{r^2}{h(\theta_\pm)}\left[d\vartheta^2 + \vartheta^2 \frac{h(\theta_\pm)^2}{K^2} d\phi^2\right], \quad \vartheta = \pm(\theta - \theta_\pm)$$

"distance to poles"

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"distance to poles"

- **Conical deficits** ($\phi \sim \phi + 2\pi$)

$$\delta_\pm = 2\pi\left(1 - \frac{h(\theta_\pm)}{K}\right) = 2\pi\left(1 - \frac{1 \pm 2mA}{K}\right)$$

- Correspond to **cosmic strings** with the following **tensions**:

$$\mu_\pm = \frac{\delta_\pm}{8\pi}$$

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A standard choice: North pole is regular $(\mu_+ = 0)$

$$\Rightarrow \quad K = h(\theta_+) = 1 + 2mA \qquad \mu_- = \frac{mA}{K}$$

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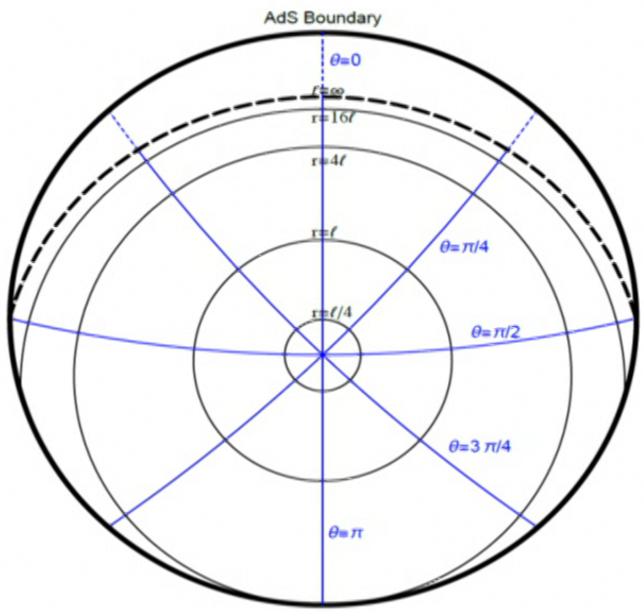
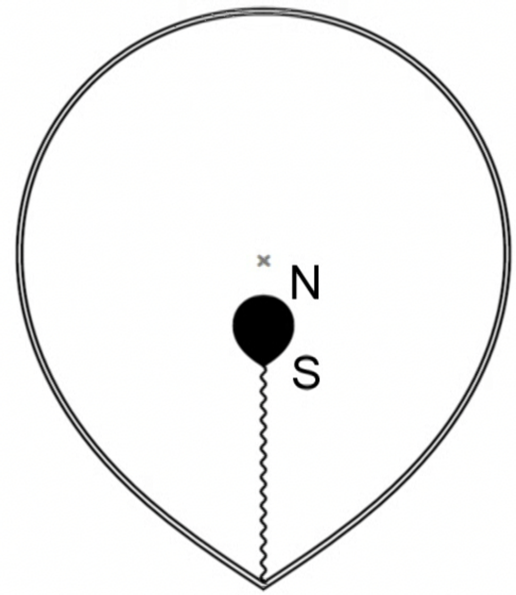




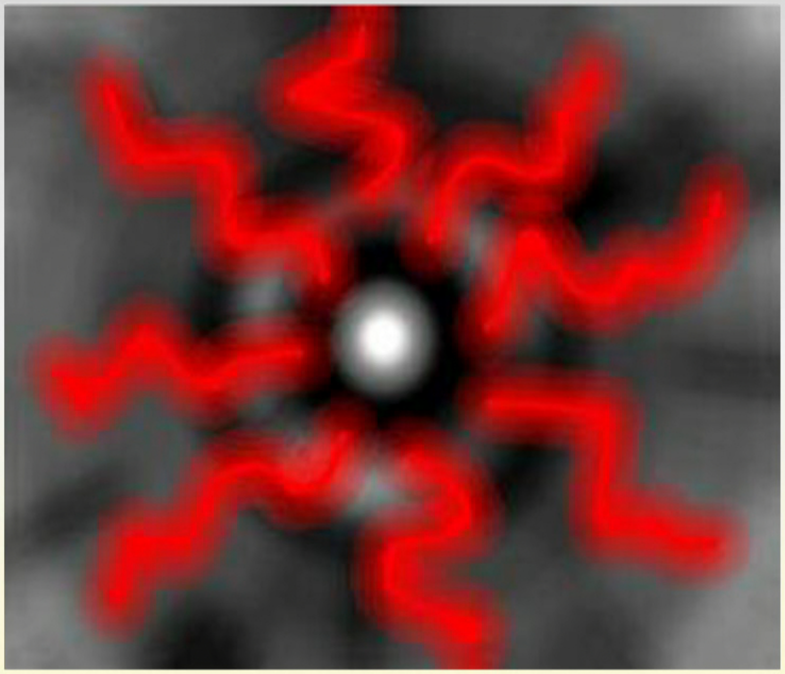
- Left: **Slowly accelerating Rindler** (m=0) **-** origin is displaced from the center of the Poincare disc.
- Right: **C-metric -** Black hole distorts the Poincare discs with a conical deficit which extends all the way to the boundary.

Are the string tensions fixed?

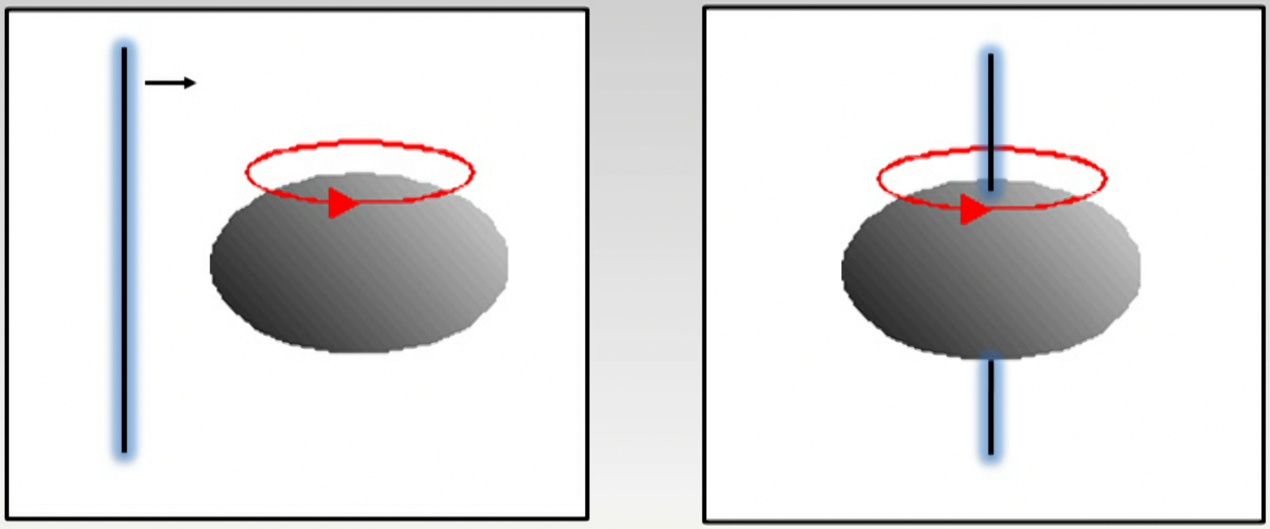
- Tension of cosmic strings, which are **topological defects** is typically quantized. Paper 1 (2016) employs this choice.

- In what follows we want to describe more general physical situations where **string tensions may vary**: keep the tensions unspecified. Paper 2 (2017).

II) Thermodynamics of accelerated black holes



Motivation: capture of a cosmic string by a black hole



- There is known solution for a black hole sporting **cosmic string hair** (Achucarro, Gregory & Kijken 1995)

- Rotating black hole – R. Gregory, DK & D. Wills 2013

Provides an example where tensions vary

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\theta^2 + r^2 \sin^2\theta \left(\frac{d\phi}{K}\right)^2 , \quad f = 1 - \frac{2m}{r}$$

Corresponds to a cosmic string with tension

$$\mu_+ = \mu_- = \mu = \frac{\delta}{8\pi} = \frac{1}{4}\left[1 - \frac{1}{K}\right]$$

(K describes the strength of the overall defect running through the black hole)

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Using the Komar integration, Euclidean trick, and the area law we have

$$M = \frac{m}{K}, \quad T = \frac{|f'(r_+)|}{4\pi} = \frac{1}{4\pi r_+}, \quad S = \frac{\mathcal{A}}{4} = \frac{\pi r_+^2}{K}$$

Can verify the Smarr and first laws:

$$M = 2TS$$

$$\delta M = T\delta S - 2\lambda\delta\mu$$

where $$\lambda = r_+ - KM = \frac{1}{2}r_+$$

is the **thermodynamic length** (string “captured” by the black hole)

- This first law provides a reasonable description of cosmic string capture (see Paper 2 for more details)

More generally

We seek the following laws for the **accelerated black holes**:

$$\delta M = T\delta S - \lambda_+ \delta\mu_+ - \lambda_- \delta\mu_- + V\delta P + \ldots$$
$$M = 2TS - 2VP + \ldots$$

Process example (due to Rob Myers)

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We seek the following laws for the **accelerated black holes**:

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$$M = 2TS - 2VP + \ldots$$

Process example (due to Rob Myers)

- Such first law is of "full cohomogeneity"
- It reduces to standard 1st law upon fixing the tensions:

$$\delta\mu_+ = 0 = \delta\mu_-$$

Intermezzo 1: Calculation of mass

- Komar integration does not work

$$M = -\frac{1}{8\pi}\int_{S_\infty} *dk\,, \quad k^a = (\partial_t)^a$$

- Nor does the background substracted Komar

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Conformal method

(Ashtekar & Das, Clas. Qauntum Gravity 17 (2000) L17)

$$Q(\xi) = \frac{\ell}{8\pi}\lim_{\bar{\Omega}\to 0}\oint \frac{\ell^2}{\bar{\Omega}} N^\alpha N^\beta \bar{C}^\nu{}_{\alpha\mu\beta}\xi_\nu d\bar{S}^\mu$$

where $\bar{g}_{\mu\nu} = \bar{\Omega}^2 g_{\mu\nu}$ is the conformal completion of g (divergencies removed)

$N_\mu = \partial_\mu \bar{\Omega}$ is the normal to the boundary

Euclidean action calculation: AdS counterterms

$$I[g] = \frac{1}{16\pi}\int_M d^4x\sqrt{-g}\left[R + \frac{6}{\ell^2}\right] + \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{-h}\mathcal{K}$$
$$- \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{-h}\left[\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right],$$

Then we have:

$$F = I/\beta = M - TS$$

where $\beta = 1/T$ is the periodicity of Euclidean time

Holographic stress tensor

$$8\pi\mathcal{T}_{ab}=\ell\mathcal{G}_{ab}\left(h\right)-\frac{2}{\ell}h_{ab}-\mathcal{K}_{ab}+h_{ab}\mathcal{K}$$

- **AdS boundary** parametrized by **Fefferman-Graham** coordinates:

$$ds^2=\frac{\ell^2}{\rho^2}d\rho^2+\frac{\rho^2}{\ell^2}\left(\gamma_{ab}^{(0)}+\frac{1}{\rho^2}\gamma_{ab}^{(2)}+\ldots\right)dx^a dx^b$$

- Expectation value of the CFT3 energy momentum:

$$\left\langle\mathcal{T}_a^b\right\rangle=\lim_{\rho\longrightarrow\infty}\frac{\rho}{\ell}\mathcal{T}_a^b$$

- Mass is an integral over the energy density

$$M=\int\rho_E\sqrt{-\gamma^{(0)}}$$

Which one is the correct mass?

- **In general**, the three methods yield **different results** (especially when the AdS asymptotics is not approached sufficiently quickly (e.g. in the presence of scalar fields)

  e.g. Lu, Pope, Wen, JHEP 1503 (2015) 165.

- Provided some conditions, they yield the same mass

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- **The correct mass** is the **thermodynamic mass** (a mass that satisfies the first law of BH thermodynamics)

  Problem: To find it one needs to identify the correct (non-rotating and properly normalized) **timelike Killing vector**. This is often a complicated task.

Thermodynamics of the C-metric

- **Key is to identify the correct timelike Killing vector**

Let the correct time is $\tau = \alpha t$

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Method 1: set m=0 and do a coordinate transf. to AdS space

$$1 + \frac{R^2}{\ell^2} = \frac{1 + (1 - A^2\ell^2)r^2/\ell^2}{(1 - A^2\ell^2)\Omega^2}, \quad R\sin\vartheta = \frac{r\sin\theta}{\Omega}$$

….recovers AdS in global coordinates provided we set

$$\alpha = \sqrt{1 - A^2\ell^2}$$

Method 2: boundary metric (m=0) must be AdS with a round sphere

Method 3: the action variation has to vanish

$$\delta I = \int_{\partial M} \sqrt{-\gamma}\tau_{ab}\delta\gamma^{ab} d^3x = 0$$

Thermodynamic mass

All 3 methods give the same mass:

$$M = \frac{m}{K}\frac{1 - A^2 l^2}{\alpha} = \frac{m\alpha}{K}$$

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Other thermodynamic quantities

$$T = \frac{f'(r_+)}{4\pi\alpha}$$ (Wick)

$$S = \frac{\mathcal{A}}{4} = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)}$$ (area law)

…are consistent with the **action calculation** (Paper 3)

$$I = \frac{\beta}{2\alpha K}\left(m - 2mA^2\ell^2 - \frac{r_+^3}{\ell^2(1 - A^2 r_+^2)^2}\right)$$

we also have $$\mu_\pm = \frac{1}{4}\left(1 - \frac{1 \pm 2mA}{K}\right)$$

The first law and the Smarr are satisfied

$$\delta M = T\delta S + V\delta P - \lambda_+ \delta\mu_+ - \lambda_- \delta\mu_-$$
$$M = 2TS - 2PV\,,$$

Provided we set:

$$V = \frac{4}{3}\frac{\pi}{K\alpha}\left[\frac{r_+^3}{(1-A^2r_+^2)^2} + mA^2\ell^4\right],$$
$$\lambda_\pm = \frac{1}{\alpha}\left[\frac{r_+}{1-A^2r_+^2} - m\left(1 \pm \frac{2A\ell^2}{r_+}\right)\right]$$

This is a “**full cohomogeneity**” first law (all variations independent: r+, l, A, K)

"Isolated black hole": string tensions are fixed

(Strings do not instantaneously change their tension)

$$\delta\mu_+ = 0 = \delta\mu_-$$

Recalling that $\mu_\pm = \frac{1}{4}\Big(1 - \frac{1 \pm 2mA}{K}\Big)$ yields

2 conditions: $$F = mA = const$$

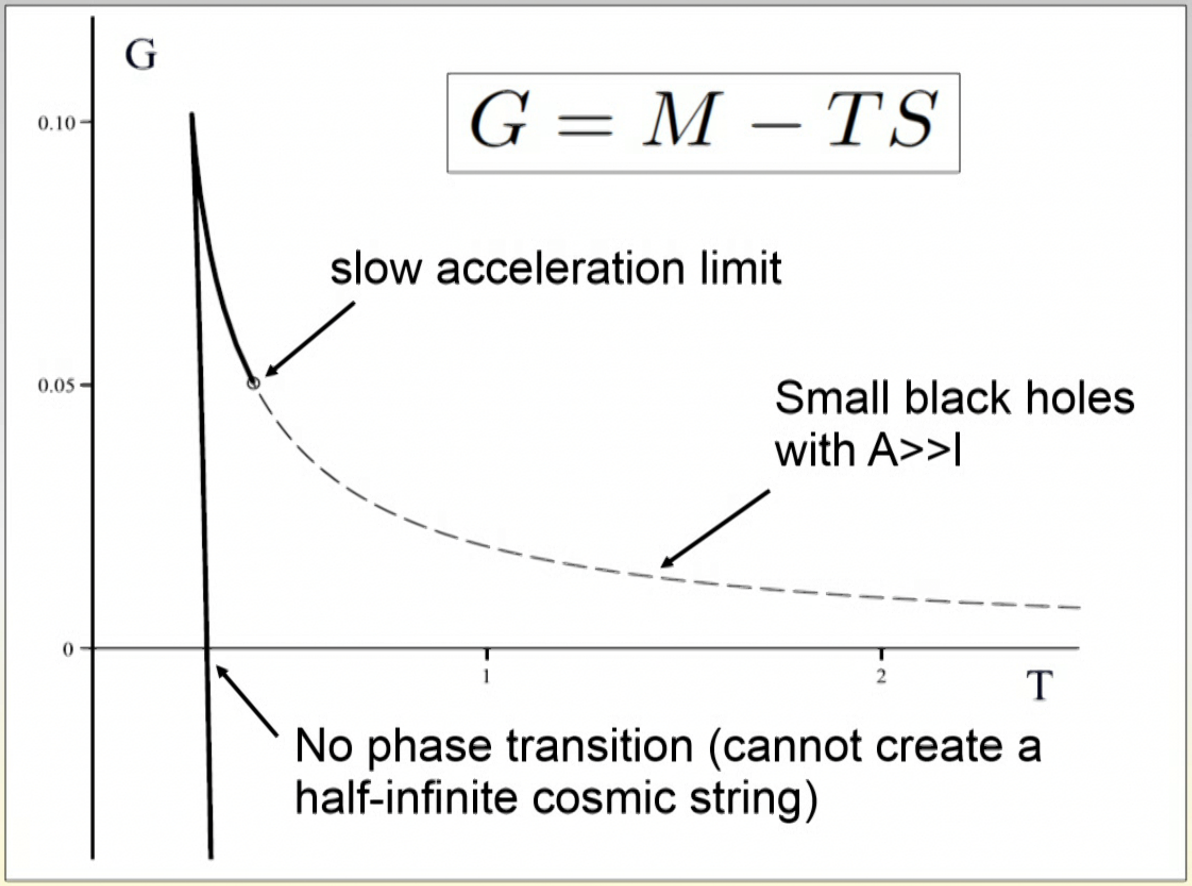
$$\delta K = 0$$

"more massive object accelerates more slowly"

Standard first law holds:

$$\delta M = T\delta S + V\delta P$$

- **<u>Free energy - fixed tensions:</u>** Superficially analogous to Hawking-Page transition



A few remarks on holography

$$ds^2_{(0)} = -\omega^2 d\tau^2 + \frac{\omega^2\alpha^2\ell^2 dx^2}{X(1 - A^2\ell^2 X)^2} + \frac{X\omega^2\alpha^2\ell^2 d\phi^2}{K^2(1 - A^2\ell^2 X)}$$

$$X = (1 - x^2)\,(1 + 2mAx)$$

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Energy momentum tensor is not that of perfect fluid

$$\langle \mathcal{T}_a^b \rangle = \lim_{\rho \longrightarrow \infty} \frac{\rho}{\ell} \mathcal{T}_a^b = \begin{pmatrix} -\rho_E & 0 & 0 \\ 0 & \frac{\rho_E}{2} + \Pi & 0 \\ 0 & 0 & \frac{\rho_E}{2} - \Pi \end{pmatrix}$$

A few remarks on holography

$$ds^2_{(0)} = -\omega^2 d\tau^2 + \frac{\omega^2\alpha^2\ell^2 dx^2}{X(1-A^2\ell^2 X)^2} + \frac{X\omega^2\alpha^2\ell^2 d\phi^2}{K^2(1-A^2\ell^2 X)}$$

$$X = (1-x^2)\,(1+2mAx)$$

Energy momentum tensor is not that of perfect fluid

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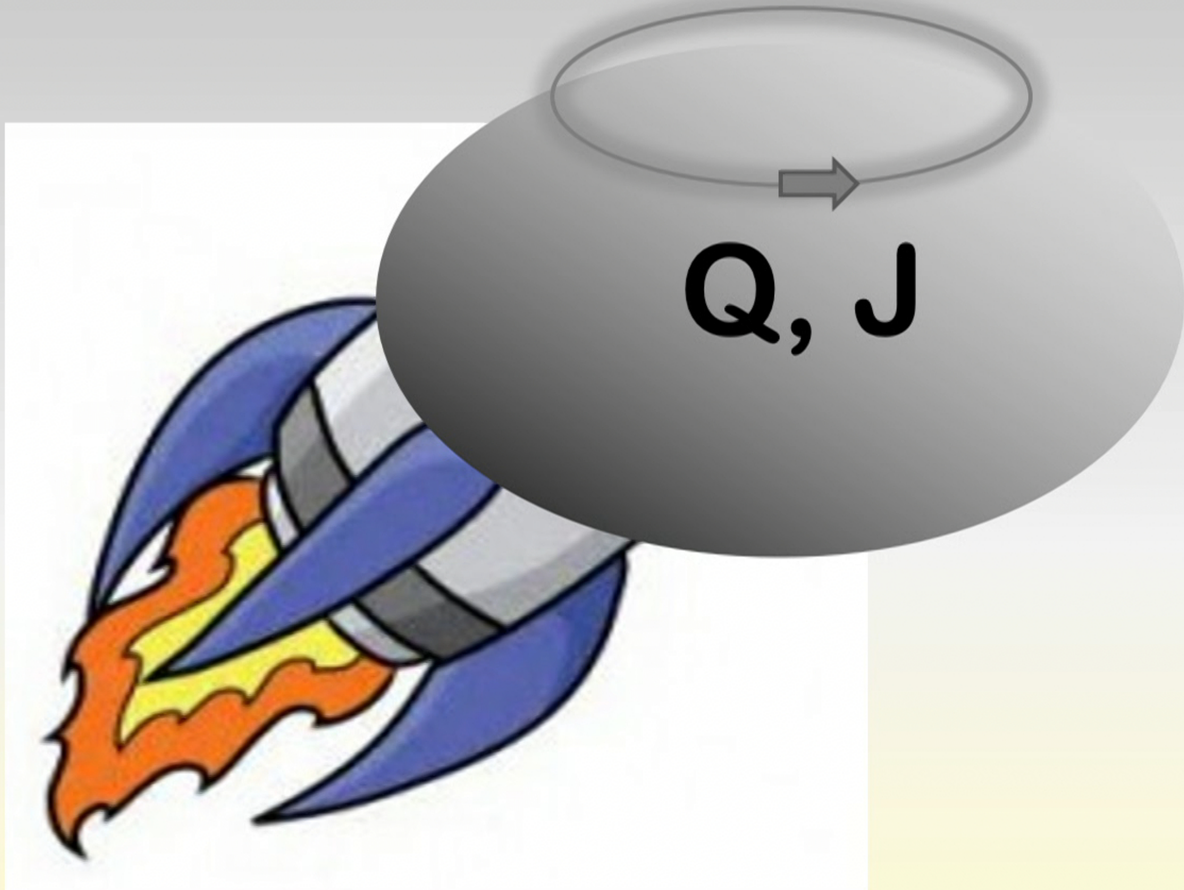
It can be written as [c.f. de Freitas & Reall, JHEP 1406, 148 (2014)]

$$\langle \mathcal{T}_{ab} \rangle = \frac{3}{2}d(x)U_a U_b + \frac{d(x)}{2}\gamma^{(0)}_{ab} + \xi\Theta_{ab}$$

$$\Theta_{ab} = C_{abd}U^d + C_{bad}U^d \qquad \xi = \frac{\ell^2}{8\pi\sqrt{3}} = \sqrt{\frac{2}{3}}\frac{1}{12\pi}k^{1/2}N^{3/2}$$

III) Adding charge and spin



Rotating and charged AdS C-metric

$$ds^2 = \frac{1}{\Omega^2}\Bigg\{ -\frac{f(r)}{\Sigma}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right]^2 + \frac{\Sigma}{f(r)}dr^2 + \frac{\Sigma r^2}{h(\theta)}d\theta^2 + \frac{h(\theta)\sin^2\theta}{\Sigma r^2}\left[\frac{adt}{\alpha} - (r^2+a^2)\frac{d\varphi}{K}\right]^2\Bigg\}$$

$$F = dB\,, \qquad B = -\frac{e}{\Sigma r}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right] + \Phi_t dt\,,$$

where

$$f(r) = (1-A^2r^2)\left[1 - \frac{2m}{r} + \frac{a^2+e^2}{r^2}\right] + \frac{r^2+a^2}{\ell^2}$$

$$h(\theta) = 1 + 2mA\cos\theta + \left[A^2(a^2+e^2) - \frac{a^2}{\ell^2}\right]\cos^2\theta$$

$$\Sigma = 1 + \frac{a^2}{r^2}\cos^2\theta\,, \qquad \boxed{\Omega = 1 + Ar\cos\theta}$$

Intermezzo 2: meaning of conjugate quantities

$$\delta M = T\delta S + V\delta P + \Omega\delta J + \Phi\delta Q$$

What are V, Φ, and Ω?

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What are V, Φ, and Ω? **They are TD conjugate quantities…**

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What are V, Φ, and Ω? **They are TD conjugate quantities…**

- **Electrostatic potential**

$$\Phi_H = -k\cdot B\big|_H$$

However, such electrostatic potential is not gauge invariant!

Physical quantity is $\Phi = \Phi_H - \Phi_\infty$

…may not be well defined

Hawking-Ross prescription
[Phys Rev D52, 5865 (1995)]
Counterterm that changes ensembles

$$\Phi = \frac{1}{4\pi Q\beta}\int_{\partial M}\sqrt{h}n_a F^{ab}B_b$$

- **Angular velocity?**

  = angular velocity of a ZAMO observer on the horizon

$$L = (\partial_\phi)^a u_a = u_\phi = g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi} = 0$$

$$\omega = \frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}} \Rightarrow \Omega_H = -\frac{g_{t\phi}}{g_{\phi\phi}}\bigg|_H$$

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This does not work: Hawking, Hunter, Taylor-Robinson 1999

Considered Kerr-AdS in standard “BL coordinates”

$$\tilde{M} = \frac{m}{\Xi} \qquad J = \frac{ma}{\Xi^2} \qquad \Omega_H = \frac{a\Xi}{r_+^2 + a^2}$$

- Checked $F \equiv TI = \tilde{M} - TS - \Omega_H J$
- Have not checked the first law $\delta\tilde{M} \neq T\delta S + \Omega_H \delta J + \tilde{V}\delta P$

- As noted by Gibbons, Perry, and Pope 2004, the **frame rotates at infinity** $\Omega_\infty = -\frac{a}{l^2}$

  suggested to use $\Omega = \Omega_H - \Omega_\infty$

  However $\tilde{M} = \frac{m}{\Xi}$ still does not work!

- To get correct mass, transform to **non-rotating frame** & use conformal method $\varphi = \phi + \frac{a}{l^2}t$

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<u>In general</u>, the proposal $\Omega = \Omega_H - \Omega_\infty$

- May not work if the latter is azimuthal dependent. Sometimes helps to switch off the mass and/or consider special theta.
- Absence of Hawking-Ross like prescription!

Thermodynamics of charged and rotating C-metric

$$M=\frac{m(\Xi+a^2/\ell^2)(1-A^2\ell^2\Xi)}{K\Xi\alpha(1+a^2A^2)}, \qquad T=\frac{f'_+r_+^2}{4\pi\alpha(r_+^2+a^2)}, \qquad S=\frac{\pi(r_+^2+a^2)}{K(1-A^2r_+^2)},$$

$$Q=\frac{e}{K}, \quad \Phi=\Phi_t=\frac{er_+}{(r_+^2+a^2)\alpha}, \quad J=\frac{ma}{K^2}, \quad \Omega=\Omega_H-\Omega_\infty, \quad \Omega_H=\frac{Ka}{\alpha(r_+^2+a^2)}$$

$$P=\frac{3}{8\pi\ell^2}, \quad V=\frac{4\pi}{3K\alpha}\left[\frac{r_+(r_+^2+a^2)}{(1-A^2r_+^2)^2}+\frac{m[a^2(1-A^2\ell^2\Xi)+A^2\ell^4\Xi(\Xi+a^2/\ell^2)]}{(1+a^2A^2)\Xi}\right]$$

$$\lambda_\pm=\frac{r_+}{\alpha(1\pm Ar_+)}-\frac{m}{\alpha}\frac{[\Xi+a^2/\ell^2+\frac{a^2}{\ell^2}(1-A^2\ell^2\Xi)]}{(1+a^2A^2)\Xi^2}\mp\frac{A\ell^2(\Xi+a^2/\ell^2)}{\alpha(1+a^2A^2)},$$

$$\mu_\pm=\frac{1}{4}\left[1-\frac{\Xi\pm 2mA}{K}\right]=\frac{1}{4}\left[1-\frac{K_\pm}{K}\right] \qquad \Xi=1-\frac{a^2}{l^2}+A^2(e^2+a^2)$$

$$\Omega_\infty=-\frac{aK(1-A^2\ell^2\Xi)}{\ell^2\Xi\alpha(1+a^2A^2)} \qquad \alpha=\frac{\sqrt{(\Xi+a^2/\ell^2)(1-A^2\ell^2\Xi)}}{1+a^2A^2}$$

Satisfy both the first law and Smarr

$$\delta M=T\delta S+\Phi\delta Q+\Omega\delta J-\lambda_+\delta\mu_+-\lambda_-\delta\mu_-+V\delta P$$

$$M=2(TS+\Omega J-PV)+\Phi Q$$

- **Adding rotation** is interestingly simpler than charge.

$$\Omega = \Omega_H - \Omega_\infty \qquad \Omega_\infty = \omega(m=0)|_\infty$$

  - This defines a non-rotating frame and calculations are "straightforward".
  - Moreover α can be derived by coordinate trans. (method 1)

- **Adding charge** is much bigger challenge
  - One needs to use Hawking-Ross prescription to find ϕ
  - There is non-trivial electric flux from pole to pole at infinity

$$F = eA\sin\theta\, dt\wedge d\theta$$

"Voltage" is angle-dependent !

  - Only method 3 works for finding α:

$$\delta I = \int_{\partial M} \sqrt{-\gamma}\tau_{ab}\delta\gamma^{ab} d^3x = 0$$

Parameter-dependent diff. changes the value!

What about other C-metric settings?

- **Asymptotically flat case**
  - Dutta, Ray, Traschen, *Boost mass and the mechanics of accelerated black holes*, CQG 23, 335 (2006).
  - Astorino, *CFT duals for accelerating black holes*, Phys.Lett. B760 (2016) 393, ArXiv: 1605.06131.
  - Paper 4.
- **Regular case**
  - Astorino, Thermodynamics of regular accelerating black holes, Phys. Rev. D95 (2017) 064007, ArXiv: 1612.04387.

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Marco's procedure

- Employ **Iyer-Wald construction** for asymptotic charges.
- Used **substraction of reference values (**Astorino, Compere, Oliveri, Vandervoorde 2016)

- To take into account non-trivial asymptotics, horizon generator as well as gauge fields were multiplied by **constant** "**integration factor**" α.

$$\delta\mathcal{M} = \alpha\Big(T_H\delta S + (\Omega_H - \Omega_{int})\delta J + (\Phi_H - \Phi_{int})\delta Q\Big)$$

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$$\delta\mathcal{M} = \alpha\Big(T_H\delta S + (\Omega_H - \Omega_{int})\delta J + (\Phi_H - \Phi_{int})\delta Q\Big)$$

Problems?

- Thermodynamics is employed to fix thermodynamic quantities (there is no independent check & big freedom)
- The first law is of "higher cohomogenity" than it seems (more parameters, including tensions, are varied than there are number of terms in the first law).
- TD quantities are relatively complicated. In paper 4 we have a "simpler proposal" for TDs of AF C-metric, using the variable tension trick (also no independent check).

Summary

1) **C-metric** is one of the most unusual and surprising exact solutions of classical general relativity that has many potential applications and interesting properties.

2) We showed how to construct **consistent thermodynamics** for **slowly accelerating** AdS black holes, with entropy given by one quarter of the horizon area and other thermodynamic quantities identified by "standard methods" (Eucl. Action calculation, conformal method, holography).

3) The key ingredient is to include the **variations of cosmic string tensions,** restoring the full cohomogeneity of the first law:

$$\delta M = T\delta S - \lambda_+ \delta\mu_+ - \lambda_- \delta\mu_- + V\delta P + \dots$$

$$M = 2TS - 2VP + \dots$$

The standard first law is restored upon fixing the string tensions.

Future directions

1) Understand why at all is the **thermodynamic description** possible (presence of a string & radiation)

2) Expand the analysis to **more general settings** of AF and regular C-metrics. What happens at the acceleration horizon? Do we need to consider a dipole mass?

3) Understand better the **conjugate quantities** (what is the meaning of λ's?). Construct a "Hawking-Ross" prescription for angular velocity.

4) Discuss the **holographic interpretation** of the energy momentum tensor in the rotating and charged case.