Title: Superselection Sectors of Gravitational Subregions

Date: Nov 22, 2018 11:00 AM

URL: http://pirsa.org/18110090

Abstract: <p>Motivated by the problem of defining the entanglement entropy of the graviton, we study the division of the phase space of general relativity across subregions. Our key requirement is demanding that the separation into subregions is imaginary---i.e., that entangling surfaces are not physical. This translates into a certain condition on the symplectic form. We find that gravitational subregions that satisfy this condition are bounded by surfaces of extremal area. We characterise the 'centre variables' of the phase space of the graviton in such subsystems, which can be taken to be the conformal class of the induced metric in the boundary, subject to a constraint involving the traceless part of the extrinsic curvature. We argue that this condition works to discard local deformations of the boundary surface to infinitesimally nearby extremal surfaces, that are otherwise available for generic codimension-2 extremal surfaces of dimension â‰¥ 2.</p>

Superselection Sectors of Gravitational Subregions

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1810.01802

indebted to
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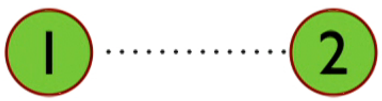
Why subregions

- In many situations it is useful to think of a system as the union of its constituents
- Natural constituents for fields: subregions?
- But, continuum subtleties (test functions)

Reminder of entanglement

- Entanglement in quantum mechanics: more information in the whole than in the parts



$$|\psi\rangle = \sum_i \lambda_i \, |\psi_1\rangle_i \otimes |\psi_2\rangle_i$$

- Quantified with, eg, entanglement entropy

$$S_1 = S_2 = -\sum_i |\lambda_i|^2 \, \log |\lambda_i|^2$$

- UV divergences

Entanglement and GR

Bombelli, Koul, Lee, Sorkin
Srednicki

- In quantum field theory, entanglement entropy satisfies an area law

$$S \propto \frac{A}{\epsilon^2} + \# \log \epsilon + \cdots$$

- Black hole entropy, non-perturbatively in G?

- Perturbatively: Geometry + quantum fluctuations

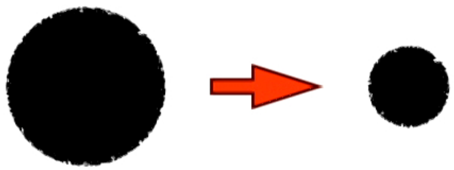
$$S_{\rm BH} = \frac{A}{4G} + S_{\rm out}$$

- In AdS/CFT, entanglement considered a crucial ingredient for emergence of space

Black Hole entropy

Bekenstein
Wall

- Black holes evaporate, but entropy increases $\delta S \geq 0$



- Solution: add entropy of fields to entropy of geometry

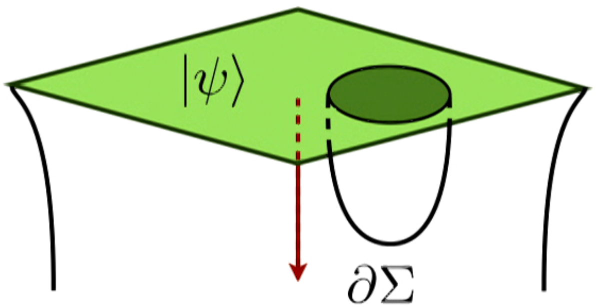
$$S = \frac{A}{4G} + S_{\rm out}$$

- Total entropy increases with time

Entanglement, AdS/CFT

Ryu, Takayanagi
Faulkner, Lewkowycz, Maldacena

- Duality between quantum field theory and quantum gravity



$$S_{\text{EE, bdry}} = \frac{A}{4G} + S_{\text{EE, bulk}}$$

Extremal surface

- Quantum error correction, bulk reconstruction

Almheiri, Dong, Harlow
Dong, Harlow, Wall

Quantum effects on gravitational entropy

- Quantum corrections to gravitational entropy = entanglement of the fields across subregions.

$$S = \frac{A}{4G} + S_{\text{out}}$$

- One of the fields is the graviton

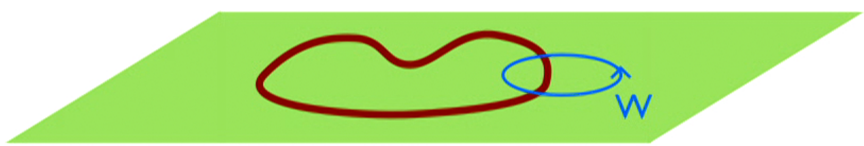
Gauge Subregions

Casini, Huerta, Rosabal
Donnelly, Freidel

- In a gauge theory, observables are gauge invariant
- Because $A_i(x)$ is not gauge invariant, instead of $A_i(x)$ and $E^i(x)$, we have Wilson loops and $E^i(x)$

$$\oint_W A_i(x)dx^i \qquad\qquad E^i(x)$$

- Wilson loops are extended: some do not factorise across regions.



Algebraic definition

Casini, Huerta, Rosabal

- Regions for gauge = Boundary conditions for fields
- Position/momentum space:

$$\psi\left[E^i(x)\right] \qquad\qquad \psi\left[\oint_W A_i dx^i\right]$$

- Fix gauge potential / electric field on $\partial\Sigma$
- State decomposes into selection sectors

$$\rho = \bigoplus_{\{E_\partial\}} p_{E_\partial}\, \rho_{E_\partial}$$

- Entropy

$$S = \sum_{\{E_\partial\}} p_{E_\partial} S_{E_\partial} + H_{E_\partial} \qquad H_{E_\partial} = -\sum_{\{E_\partial\}} p_{E_\partial} \log p_{E_\partial}$$

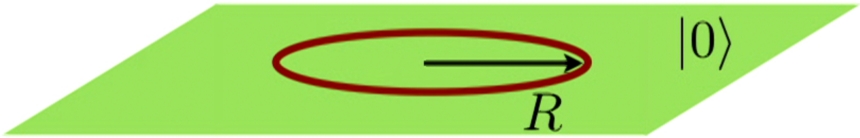
Comments

- Nothing is fixed at the boundary

$$\rho = \bigoplus_{\{E_\partial\}} p_{E_\partial} \, \rho_{E_\partial}$$

- We drop A_∂ from the algebra, then E_∂ does not change by action of remaining operators (or vv)
- UV feature, at $\partial\Sigma$. Mutual Info and Relative Entropy are UV finite and blind to these choices

Photon ambiguities



- Choice of algebra affects what you get
- EE of the photon, in the vacuum, across a sphere, with electric bcs

$$S_{\rm el} = \#\frac{A}{\epsilon^2} - \frac{31}{45}\log\frac{R}{\epsilon} + \ldots$$

Donnelly, Wall

- With another prescription

$$S_{\rm M.I.} = \#'\frac{A}{\epsilon^2} - \frac{16}{45}\log\frac{R}{\epsilon} + \ldots$$

Casini, Huerta

Recap Gauge

- Gauge theories describe extended dofs
- What is a subregion?
- Several definitions available
- EE, including 'universal contributions', depends on the definition

Gravity: preliminaries

- Gravity has no local observables: a point x^μ is not diff-invariant
- Worse than for spin-1
- Generic surfaces $X^\mu(\sigma^i)$ are not gauge invariant
- Does 'gravitational region' make sense in general?

Symplectic form

- The relevant object is the symplectic form W: two-form in phase space

$$W(\delta_1\phi, \delta_2\phi) = -W(\delta_2\phi, \delta_1\phi)$$

- This informs us about the structure of phase space: space of solutions to the eoms
- For a 1D particle: $W = dq \wedge dp$

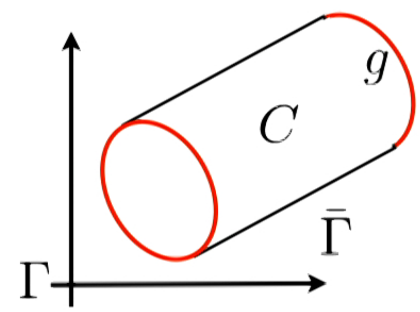
Symplectic reduction

Lee, Wald

- It is common to embed the physical phase of a gauge thy $\bar{\Gamma}$ in a larger space Γ , eg $\{A_i(x), E^j(x)\}$
- On a constraint surface C in Γ, eg $\nabla_i E^i = 0$, the symplectic form has null directions g

$$I_g W|_C = 0$$

- Can be dispensed via 'symplectic reduction' if this is also a symmetry: $L_g \, W|_C = 0$
- C : Fiber bundle over the physical $\bar{\Gamma}$



Comments

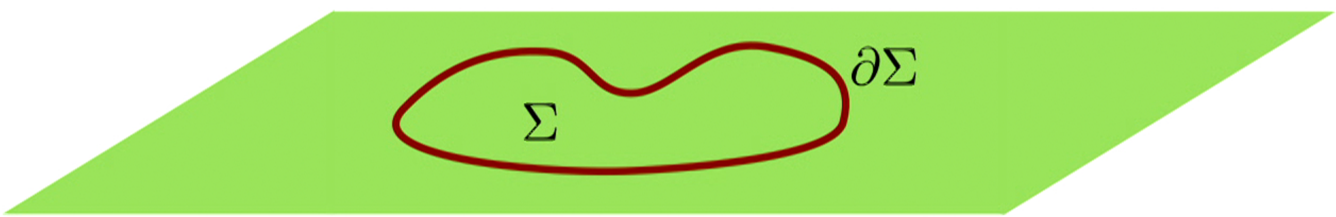
- Since closedness is preserved under restriction, one normally expects trivially

$$L_g \, W|_C = \delta I_g W + I_g \delta W = 0$$

- Failure of g to be hamiltonian on C , $I_g W \neq \delta H_g$ would indicate that restriction $W|_C$ has not been done properly

Reduction & Subregions

- Summary: a gauge, non-degree of freedom is such that it is a null direction and a symmetry of W
- Imaginary subregions do not introduce dofs
- If they had dofs, they would be physical



$$W\left(g|_{\partial\Sigma}, \delta\right) = 0$$

Symplectic photon

- Symplectic form

$$W(\delta_1, \delta_2) = \int_\Sigma d^3x\, \delta_1 A_i\, \delta_2 E^i - (1 \leftrightarrow 2)$$

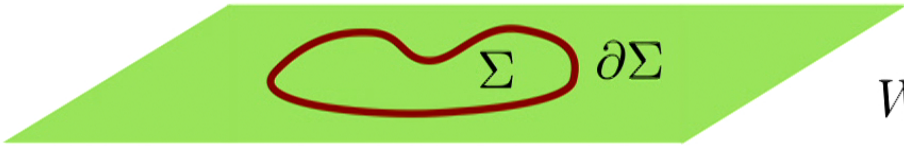
- For a gauge transformation $\delta A = d\varepsilon$

$$W(\varepsilon, \delta) = \int_{\partial\Sigma} d^2x\, \varepsilon\, \delta E^\perp$$

- Choices

$\varepsilon = 0$ $\delta E^\perp = 0$

Comments



$$W(\varepsilon,\delta) = \int_{\partial\Sigma} d^2x\, \varepsilon\, \delta E^{\perp}$$

- Magnetic choice:

$$\delta A_i|_{\partial\Sigma} = 0 \qquad\qquad \varepsilon|_{\partial\Sigma} = 0$$

- Electric choice:

$$\delta E^{\perp}\big|_{\partial\Sigma} = 0$$

- ε is always a symmetry of W (because $\delta^2 = 0$)

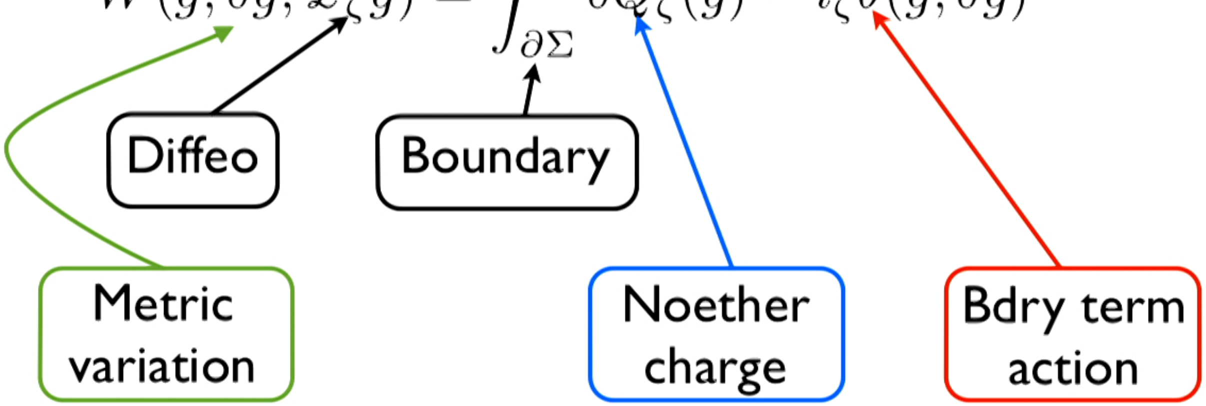
$$H_\varepsilon = \int_{\partial\Sigma} d^2x\, \varepsilon\, E^{\perp}$$

Symplectic graviton

Hollands, Wald
Jafferis, Lewkowycz, Maldacena, Suh

- Symplectic form on a diffeomorphism

$$W(g;\delta g,\pounds_\zeta g)=\int_{\partial\Sigma}\delta Q_\zeta(g)-i_\zeta\theta(g;\delta g)$$



Notation

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (h_{ij} + 2K_{ija}x^a)\,dy^i dy^j + \cdots$$

Induced metric (h_{ij}) — Extrinsic curvature (K_{ija})

$$h_{ij} = e^{2\Omega}\bar{h}_{ij} \qquad \det \bar{h} = 1$$

$$K_{ija} = \frac{1}{D-2}K_a h_{ij} + e^{2\Omega}\bar{K}_{ija} \qquad h^{ij}\bar{K}_{ija} = 0$$

Types of non-trivial diffs

- Boundary diffs 
- Boosts 
- Translations 

Ham	Null
Y	$\delta \bar{h}_{ij} = 0$
Y	$\delta A = 0$
???	???

$$W(g;\delta g, \pounds_{\zeta^a} g) = \frac{1}{16\pi}\int_{\partial\Sigma}\left[\frac{2}{D-2}\zeta^b {\epsilon^a}_b\, \delta(K_a\, \epsilon_h)\right.$$
$$\left. +2\left(\frac{D-3}{D-2}\delta K_a + \frac{1}{2}{\bar{K}^{ij}}_a\, \delta\bar{h}_{ij}\right)\zeta^b {\epsilon^a}_b\, \epsilon_h\right]$$

Graviton bdry conds.

$$W(g;\delta g,\pounds_{\zeta^a} g) = \frac{1}{16\pi}\int_{\partial\Sigma}\left[\frac{2}{D-2}\zeta^b \epsilon^a{}_b\,\delta(K_a\,\epsilon_h)\right.$$

$$\left.+2\left(\frac{D-3}{D-2}\delta K_a + \frac{1}{2}\bar{K}^{ij}{}_a\,\delta\bar{h}_{ij}\right)\zeta^b\epsilon^a{}_b\,\epsilon_h\right]$$

- Most natural: fix $K^a = 0$ and $\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$
- Superselection sectors available when $K^a = 0$
- Superselection sectors labelled by $\delta\bar{h}_{ij}$, such that

$$\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$$

- Examples: Biff surfaces, Ryu-Takayanagi surfaces

Intepretation of BCs

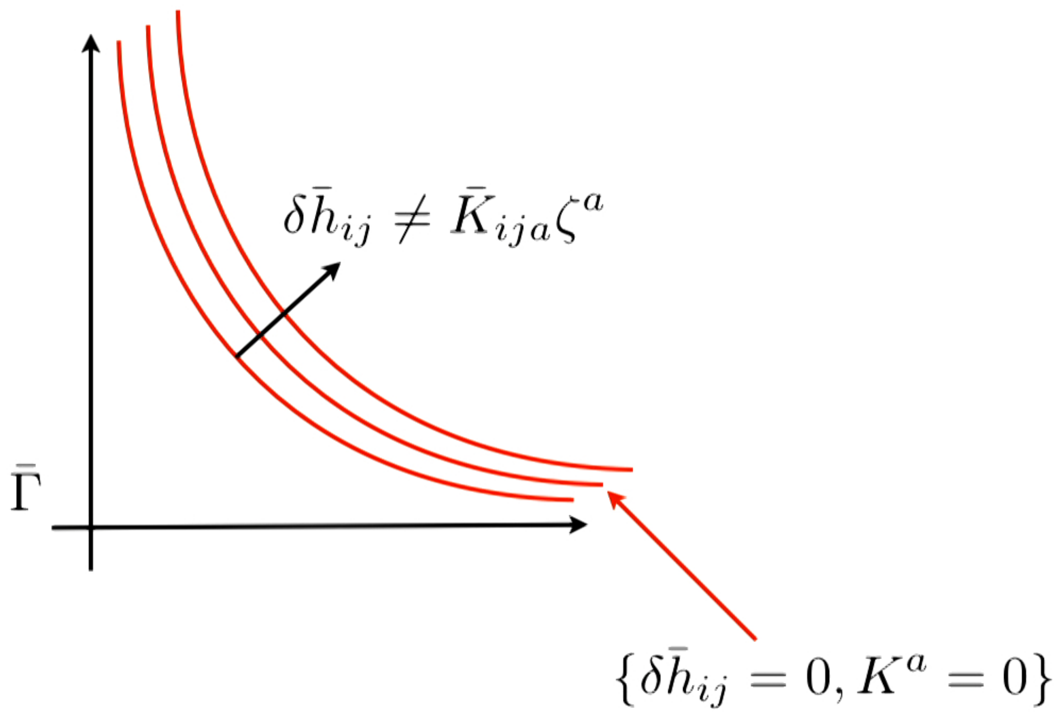
$$\rho_\Sigma = \bigoplus_{\{\delta\bar{h}_{ij} | \bar{K}^{ija}\,\delta\bar{h}_{ij}=0\}} p_{\delta\bar{h}_{ij}}\,\rho_{\delta\bar{h}_{ij}}$$

- $K^a = 0$ is a good subregion: It is diff-invariant
- $\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$ discards $\delta\bar{h}_{ij}$ achievable by a displacement (gauge transformation):

$$\delta\bar{h}_{ij} = 2\bar{K}_{ija}\zeta^a$$

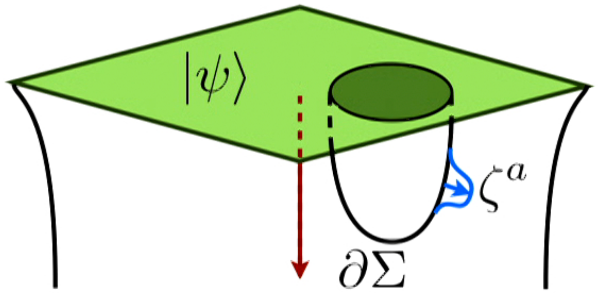
- Locally deformable extremal surfaces?
- Jacobi fields?

Cartoon



Deformability of $\partial\Sigma$.

- Claim: Generically, there may be nearby extremal surfaces reachable with ***local*** translations



- Jacobi equation

$$-D^2\zeta^a + V^a{}_b\zeta^b = 0$$

- When

$$V^a{}_b = -\bar{K}_{ij}{}^a\bar{K}^{ij}{}_b \approx -\bar{K}^2\delta^a{}_b \qquad \boxed{\zeta^a = c^a\, J_0\left(\sqrt{\bar{K}^2}\,|y|\right)}$$

Recap: Graviton

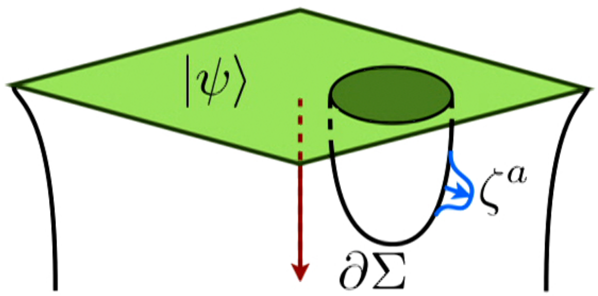
- Principle: Subregions are imaginary separations: Do not introduce degrees of freedom to the system.
- Symplectic reduction: diffs of subregions should annihilate and be symmetries of W . Non-trivial.
- Boundaries of subregions are extremal surfaces.
- Superselection sectors labeled by $\bar{h}_{ij}$ on $\partial\Sigma$, such that $\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$ .This discards $\delta\bar{h}_{ij} = \bar{K}_{ija}\zeta^a$, that take us to nearby extremal surfaces.

Outlook: (i) AdS/CFT

- Quantitative quantum corrections to RT:

$$S_{\text{EE, bdry}} = \frac{A}{4G} + S_{\text{EE, bulk}}$$

- Consequences of deformability of RT surfaces?



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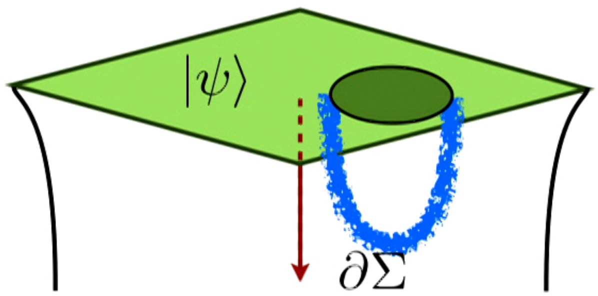

- Fuzzy bulk reconstruction?

(ii) Black Hole entropy

- With Euclidean methods, quantum corrections to BH entropy can be accounted for precisely for extremal BHs in string theory.
- Mismatch for Schwarzschild black hole:

$$S_{\text{Sen}} = \frac{A}{4G} + \frac{77}{90}\log\frac{A}{4G} + \cdots$$

- And LQG:

$$S_{\text{LQG}} = \frac{A}{4G} - \log\frac{A}{4G} + \cdots$$

- Real-time entanglement-across-horizon picture?

Summary

- Regions are subtle with gauge symm
- Important for, eg, Q corrections to BH entropy
- The phase space of a gravitational subregion is gauge invariant if the boundary is extremal surface
- Extremal surfaces are locally deformable?