Title: Symmetry-protected self-correcting quantum memories

Date: Nov 21, 2018 04:00 PM

URL: http://pirsa.org/18110089

Abstract: <p>A self-correcting quantum memory can store and protect quantum information for a time that increases without bound with the system size, without the need for active error correction. We demonstrate that symmetry can lead to self-correction in 3D spin lattice models. In particular, we investigate codes given by 2D symmetry-enriched topological (SET) phases that appear naturally on the boundary of 3D symmetry-protected topological (SPT) phases. We find that while conventional onsite symmetries are not sufficient to allow for self-correction in commuting Hamiltonian models of this form, a generalized type of symmetry known as a 1-form symmetry is enough to guarantee self-correction. We illustrate this fact with the 3D `cluster state' model from the theory of quantum computing. This model is a self-correcting memory, where information is encoded in a 2D SET ordered phase on the boundary that is protected by the thermally stable SPT ordering of the bulk. We also investigate the gauge color code in this context. Finally, noting that a 1-form symmetry is a very strong constraint, we argue that topologically ordered systems can possess emergent 1-form symmetries, i.e., models where the symmetry appears naturally, without needing to be enforced externally. Joint work with Stephen Bartlett.</p>

Symmetry-protected self-correcting quantum memories

Sam Roberts, Stephen Bartlett

based on arXiv:1805.01474

THE UNIVERSITY OF SYDNEY

EQUS

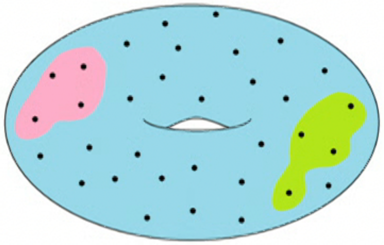
Motivation

How can we protect quantum information in a noisy environment?

Motivation

How can we protect quantum information in a noisy environment?

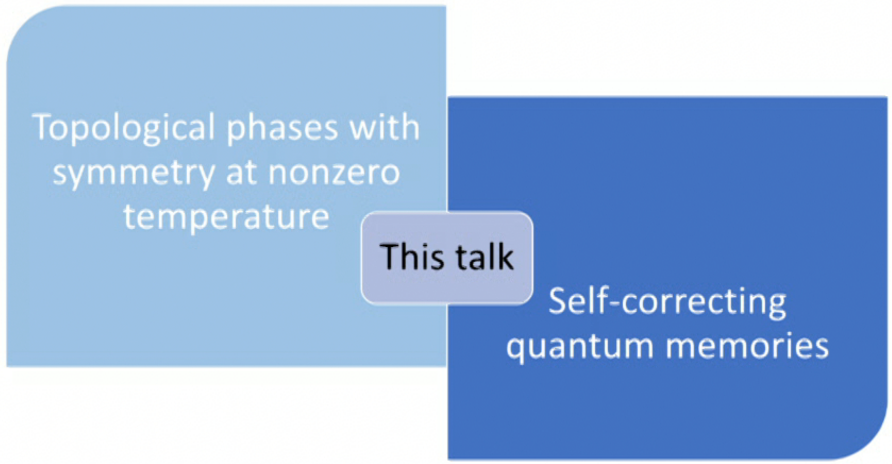
Quantum error-correcting codes...
using topological phases of matter.



Error-correction is expensive
How long can we store quantum information without intervention or active error correction?

Self-correcting quantum memories

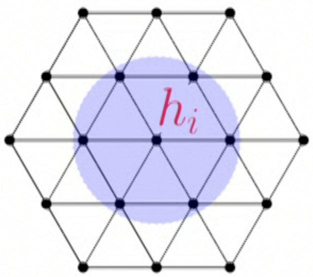
Self correction, symmetry, and topological phases at $T \neq 0$



1. Self-correcting quantum memories
2. Symmetry as a mechanism for self correction
   - Symmetry-protected topological phases for quantum memories
   - Simple examples demonstrating self-correction
3. Relationship between self-correction and topological order at nonzero temperature
4. Avenues to symmetry

The (Caltech) rules for self correction

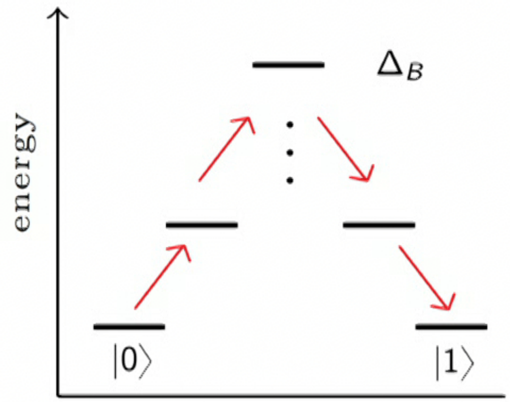
1. Finite density of spins in $\mathbb{R}^3$
2. Local Hamiltonian $H = \sum_i h_i$ with bounded strength terms $\|h_i\| \leqslant 1$
3. Degenerate ground space (in thermodynamic limit), perturbatively stable
4. Coupled to a thermal bath, the lifetime τ of encoded information diverges (exponentially) with the system size
5. Efficient classical decoder



$$\rho_0 \quad \longrightarrow \quad \rho(t) = \mathrm{tr}_B \left(e^{-i \sum_a S_a \otimes B_a t} \rho_0 e^{i \sum_a S_a \otimes B_a t} \right) \quad \longrightarrow \quad \Phi_{\text{e.c.}}(\rho(t)) =_\epsilon \rho_0$$

The energy barrier

- Energy barrier: The minimal energy cost accrued to implement a logical operator through local operations



Arrhenius Law
for memory time
$\tau \sim \exp(\frac{\Delta_B}{T})$

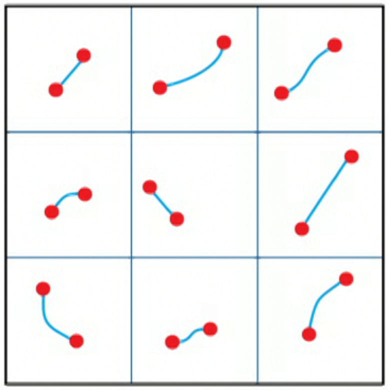
- If Δ_B grows with N: the bigger the system, the more quantum it becomes
- Necessary for stabilizer Hamiltonians (Temme 14, Temme & Kastoryano 15), 2D quantum doubles (Komar *et al.* 16)
- Can a macroscopic energy barrier exist in a 3D model?

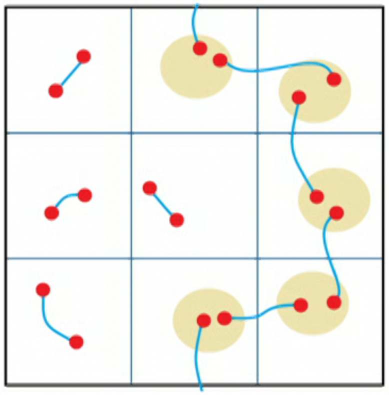
Dimension game and the energy barrier

	Energy barrier	Memory time
2D toric code	O(1)	$O(1) = e^{c\beta}$
3D toric code	O(1)	$O(1) = e^{c\beta}$
4D toric code	$O(L)$	$\exp(\beta L)$

The 2D toric code

- Excitations appear at the end of error strings





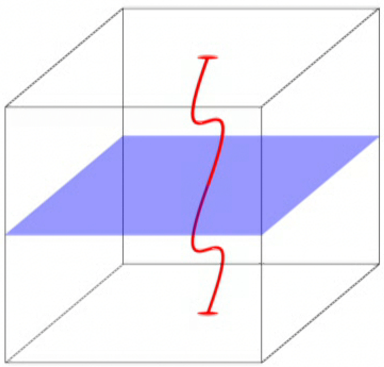
- Rapidly approaches a uniform excitation density $\sim e^{\beta\Delta}$
- Excitations need only walk distance $O(1)$ to cause an error
- With active error correction can achieve $\tau \sim \exp(L)$

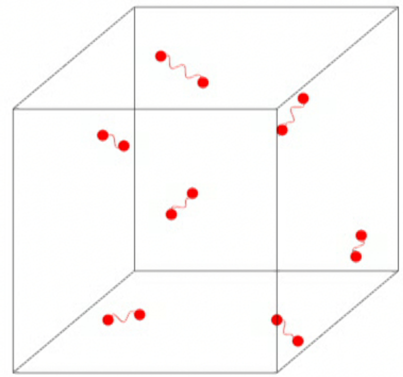
Dimension game and the energy barrier

	Energy barrier	Memory time
2D toric code	O(1)	$O(1) = e^{c\beta}$
3D toric code	O(1)	$O(1) = e^{c\beta}$
4D toric code	$O(L)$	$\exp(\beta L)$

The 3D toric code

- Loop and membrane logical operators
- Point-like and loop like excitations





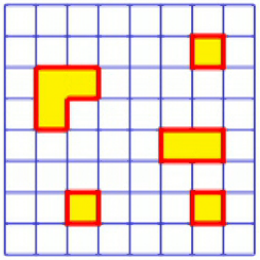
- Point-like sector (loop logical) thermalizes quickly
- Loop excitations are suppressed $\rightarrow$ can store a classical bit

Dimension game and the energy barrier

	Energy barrier	Memory time
2D toric code	O(1)	$O(1) = e^{c\beta}$
3D toric code	O(1)	$O(1) = e^{c\beta}$
4D toric code	$O(L)$	$\exp(\beta L)$

The 4D toric code

- Both logical operators are two-dimensional
- Errors cost energy according to their boundary size



- Large loop excitations are suppressed by the Boltzmann factor
- A self-correcting **quantum** memory! (Dennis *et al.* 02, Alicki *et al.* 10)

Dimension game and the energy barrier

	Energy barrier	Memory time
2D toric code	O(1)	$O(1) = e^{c\beta}$
3D toric code	O(1)	$O(1) = e^{c\beta}$
4D toric code	$O(L)$	$\exp(\beta L)$

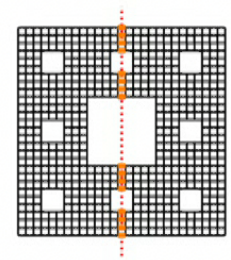
Can self-correcting quantum memories exist in three spatial dimensions or less?

Interesting proposals in 3D

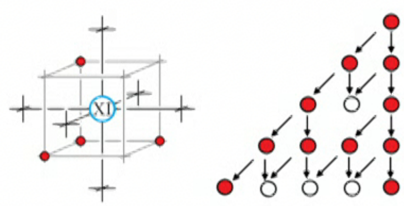
Long range interacting anyons
Hamma *et al.* 09, Chesi *et al.* 10, Pedrocchi *et al.* 13

$$H = \overbrace{\sum_{p,p'} V_{p,p'} h_p^{t.c.} h_{p'}^{t.c.}}^{\text{interacting toric code}} + \overbrace{\sum_q \epsilon_q \tilde{a}_a^\dagger \tilde{a}_q}^{\text{scalar field}}$$

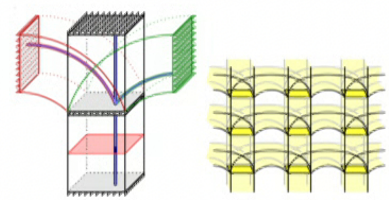
Fractal product codes
Brell 2016



Fractal models
Haah 11



Welded codes
Michnicki, 2014



No-go theorems

Necessity of energy barrier

- Stabilizer Hamiltonians (Temme 14, Temme & Kastoryano 15)
- 2D quantum doubles (Komar *et al.* 16). No entropic protection.

2D models

- 2D stabilizer codes $\Delta_B = O(1)$ (Bravyi & Terhal 09, Kay & Kolbeck 08, Haah & Preskill 12)
- 2D commuting projectors $\Delta_B = O(1)$ (Landon-Cardinal & Poulin 12)

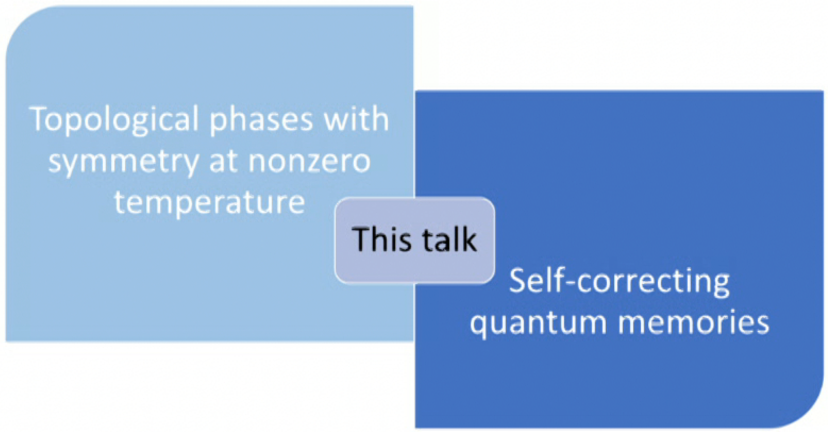
Instability of topological order

- Decay of topological entanglement entropy in 2D and 3D toric code (Castelnovo & Chamon 07,11)
- 2D commuting projectors Gibbs triviality (Hastings 11)

3D models

- Stabilizer models with translational and scale invariance $\Delta_B = O(1)$ (Yoshida 11)
- Stabilizer models with non-Clifford gate $\Delta_B = O(1)$ (Pastawski & Yoshida 15)
- Translationally invariant stabilizers $\Delta_B \leqslant O(\log(L))$ (Haah 13)

New direction: symmetries



- Two related insights:
  1. The presence of symmetry can lead to new phases of matter
     (*i*) Symmetry-enriched topological phases
     (*ii*) Symmetry-protected topological phases

     some are stable at nonzero temperature in 3D
  2. Symmetry gives constraints on local noise (in definition of energy barrier)

New direction: topological phases with symmetry

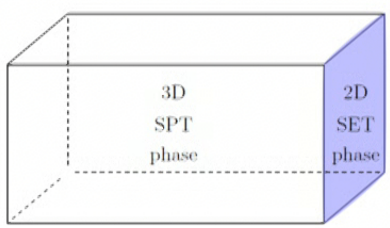
Two types of topological phases with symmetry $S(g), g \in G$.

Symmetry-protected

- Bulk is short-range entangled, non-degenerate
- No anyonic excitations in the bulk
- Bulk boundary correspondence: boundaries support 'protected modes'
- New exotic phases obtained by gauging the symmetry (later)

Symmetry enriched

- Long range entanglement
- Anyonic excitations that may transform under the symmetry
- Symmetries lead to interesting defects
- Some SETs only realisable on the boundary of higher dim SPTs



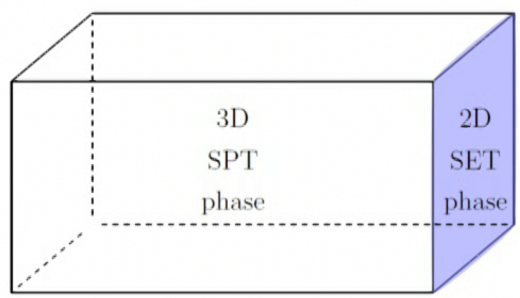
New direction: Topological phases with symmetry

- Natural candidates for the following reasons:
  1. Bulk is confining (absence of anyons)
  2. The boundary theory must either
     - (*i*) break the symmetry
     - (*ii*) be gapless
     - (*iii*) **be topological ordered**

> Goal: Characterize feasibility of symmetry-protected topological phases as self-correcting quantum memories.

Thermal topological order $\Longleftrightarrow$ self-correction

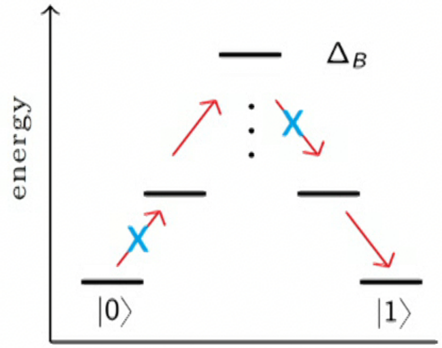
Weakest possible symmetry

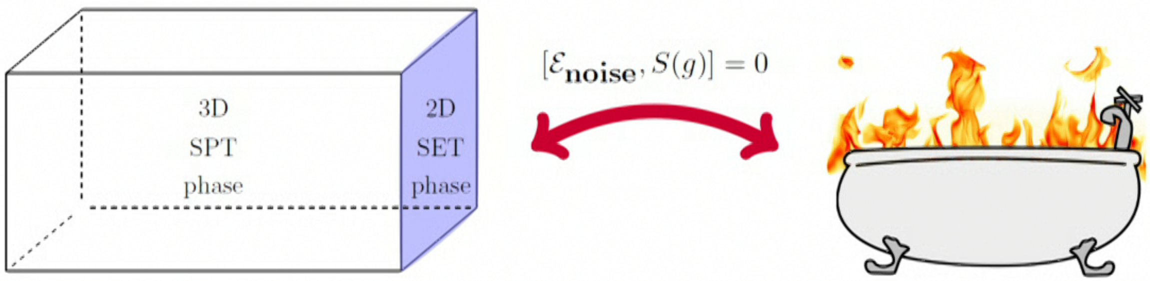


A bulk boundary correspondence at $T \geqslant 0$

The framework

- Hamiltonian: Topological ordered models on the boundary of 3D SPT phases
- Symmetry: $S(g), g \in G$ (given)
- Noise model: local noise respecting symmetry
- Lifetime: Determined by the *symmetric* energy barrier

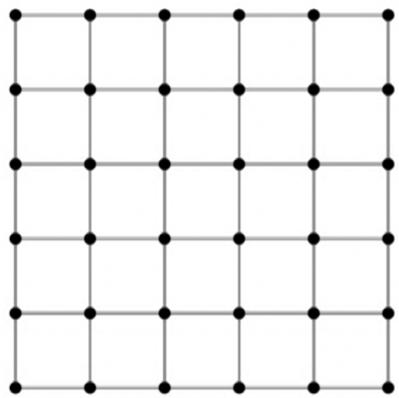




First observations: the onsite symmetry case

- Onsite symmetries are most commonly considered

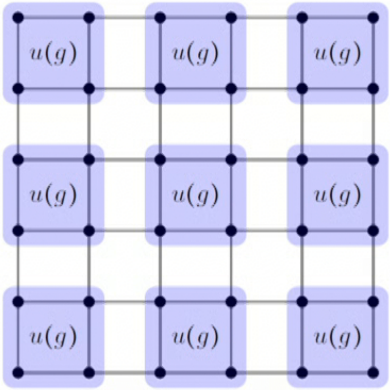
$$S(g) = \otimes_i u_i(g), \qquad u_i(g)\,|g_i\rangle \mapsto |gg_i\rangle$$



First observations: the onsite symmetry case

- Onsite symmetries are most commonly considered

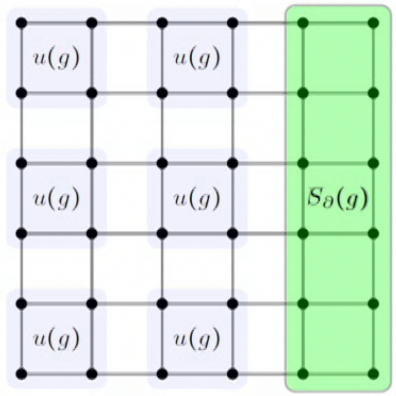
$$S(g) = \otimes_i u_i(g), \qquad u_i(g)\,|g_i\rangle \mapsto |gg_i\rangle$$



First observations: the onsite symmetry case

- Onsite symmetries are most commonly considered

$$S(g) = \otimes_i u_i(g), \qquad u_i(g)\,|g_i\rangle \mapsto |gg_i\rangle$$



- Boundary degrees of freedom transform 'anomalously'

$$S_\partial(g) = N(g)R(g)$$

- $R(g)$ onsite, $N(g)$ local circuit diagonal in $|g_i\rangle$ basis

- Dimension reduction: look at 2D boundary separately with anomalous symmetry
- 2D boundary unstable: string-like operators free to propagate
- Symmetric energy barrier $= O(1)$

Memory time $\tau = O(1)$ for onsite symmetry-protected memories.

First observations: the onsite symmetry case

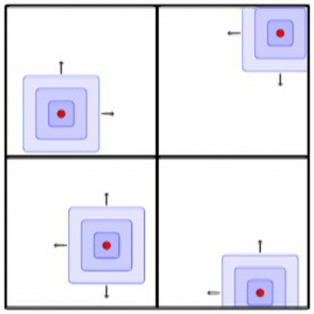
- Can SPT order survive at nonzero temperature?
- Thermal nontriviality: The Gibbs state is "not a classical state":
  ρ is trivial if a spin K_i can be added at each site i, and a symmetric local unitary U can be found such that

$$\rho = \mathrm{tr}_K(U\rho_{\mathrm{cl}}U^\dagger), \qquad \rho_{\mathrm{cl}} \sim \exp(-\beta H_{\mathrm{cl}})$$

No thermal SPT with on-site global symmetry

Proof Sketch:

1. Construct approximate thermal state by removing terms from Hamiltonian
2. Missing terms act as sinks, where point-like excitations can be created/destroyed
3. Dividing the lattice into a grid, the sinks can be used to construct a symmetric disentangler



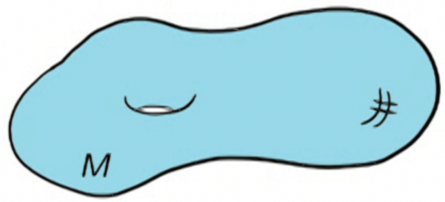
SR, Yoshida, Kubica, Bartlett 17

Beyond onsite symmetries

- Onsite symmetric models:
  1. Not topologically ordered (circuit depth) at any $T > 0$
  2. Not self-correcting

- Look toward subsystem symmetries, which generalize the onsite case
  - ~~line-like (rigid)~~
  - ~~fractal (rigid)~~
  - **higher-form (deformable)**

$S_M(g) =$



$$= \prod_{i \in M} u_i(g)$$

k-form: M closed codimension-k submanifold

Kapustin and Thorngren (2013)

Kapustin and Seiberg (2014)

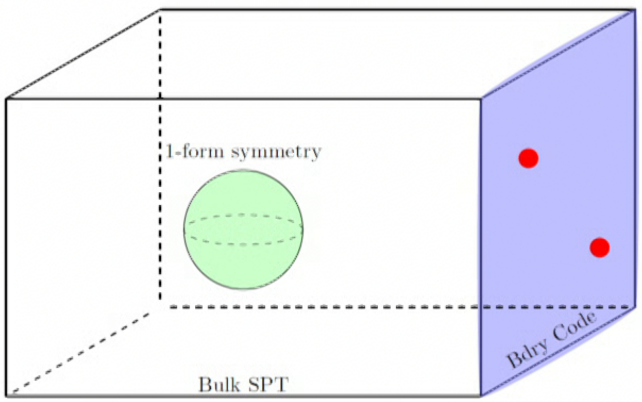
Giaotto, Kapustin, Seiberg, and Willett (2015)

Yoshida (2015)

- Note: 0-form = onsite case

A first example of a symmetry-protected self-correcting quantum memory

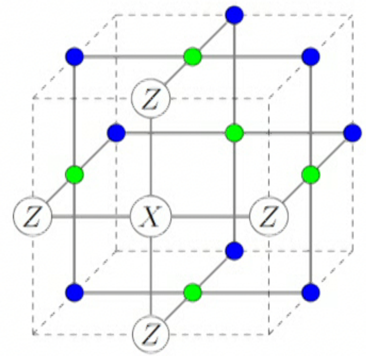
1. Bulk Hamiltonian: Raussendorf, Bravyi, Harrington (RBH) cluster state
2. Boundary Hamiltonian: dressed toric code
3. Symmetry: $\mathbb{Z}_2^2$ 1-form symmetry



$$H = H_{\text{bulk}} + H_{\text{boundary}}$$

Constructing the model (bulk)

- Bulk cubic lattice
- Qubits on edges and faces
- Hamiltonian $H_{\text{bulk}} = \sum_i K_i$
- Ground state = cluster state



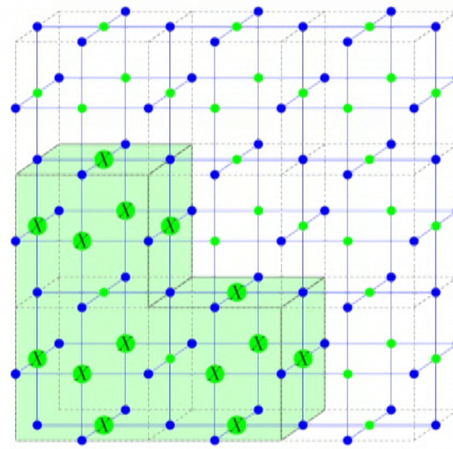
$$K = \begin{matrix} & Z & \\ & | & \\ Z - & X & - Z \\ & | & \\ & Z & \end{matrix}$$

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ 1-form symmetry operators

$$S_M(g) = \prod_{i \in M} X_i \quad \text{M a 2-dim closed surface}$$

Constructing the model (bulk)

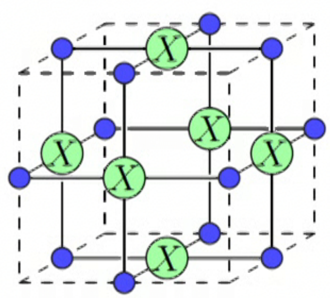
- Bulk cubic lattice
- Qubits on edges and faces
- Hamiltonian $H_{\text{bulk}} = \sum_i K_i$
- Ground state = cluster state

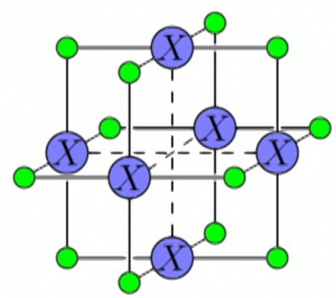


$$K = \begin{array}{c} Z \\ | \\ Z\text{-}X\text{-}Z \\ | \\ Z \end{array}$$

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ 1-form symmetry operators

$$S_M(g) = \prod_{i \in M} X_i \quad \text{M a 2-dim closed surface}$$

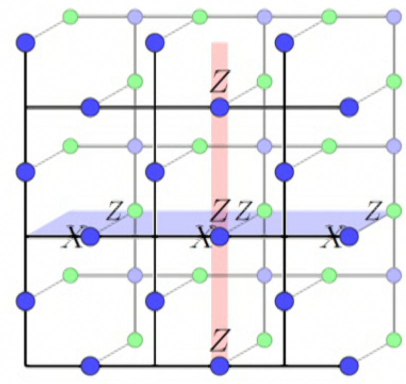




Constructing the model (boundary)

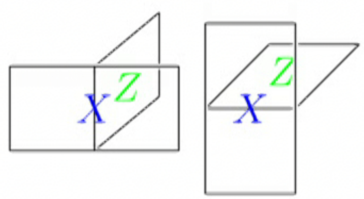
- Construct a toric code out of boundary operators

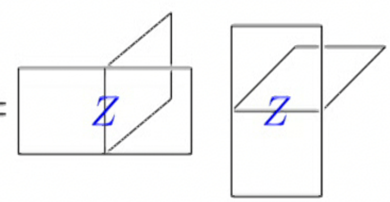
$$H_b = -\sum_v \tilde{A}_v - \sum_f \tilde{B}_f$$



- String logical operators (single logical qubit with planar boundary conditions)

- Boundary modes

$\tilde{X}_b =$ 

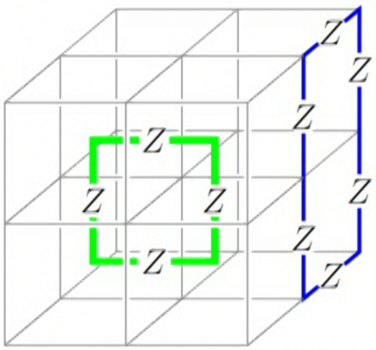
$\tilde{Z}_b =$ 

What excitations can be introduced?

- Bulk excitations: flipped cluster terms from chains of Z operators

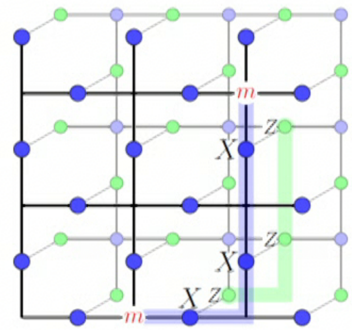
$$\{K_i, Z_i\} = 0, \qquad [K_i, Z_j] = 0 \qquad \forall i \neq j$$

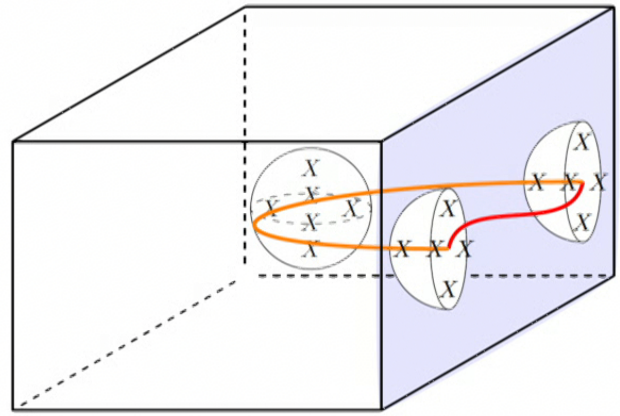
- Symmetric $\Longleftrightarrow$ closed loop



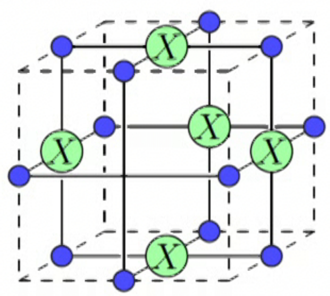
What excitations can be introduced?

Boundary anyons can exist only at the end of bulk string excitations



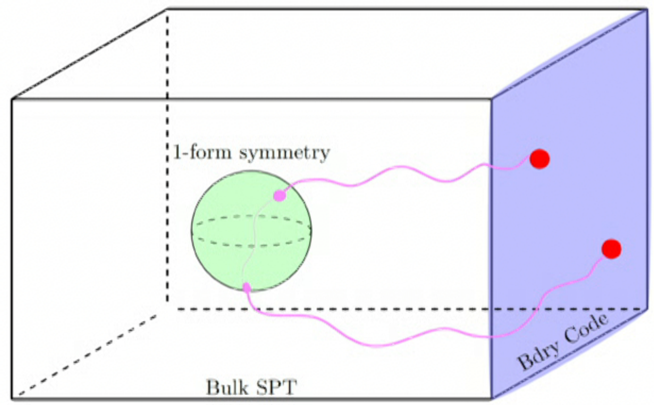


- Due to bulk boundary coupling



$$S_c = \tilde{B}_p \prod K_f$$

Energy barrier



- Tension between anyons introduced by bulk string excitations.
- Local symmetric error channels that drag anyons apart have an energy that grows with their separation
- Energy barrier grows with the code distance. $\Delta_B = O(L)$

> Lifetime of symmetry-protected quantum memory $\tau = O(\exp(L))$ below a critical temperature T_c.

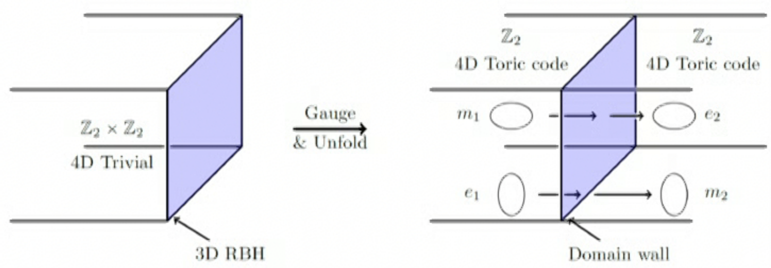
Thermal nontriviality

- Bulk retains SPT order up to a critical temperature

$$\rho \not\sim U\rho_{\text{cl}}U^\dagger, \quad \text{for any} \quad \rho_{\text{cl}} \sim \exp(-\beta H_{\text{cl}}), \quad U \text{ sym local unitary}$$

- Many ways to prove this
  1. Error corrected non-local order parameters
  2. Gauging to produce a nontrivial domain wall in the 4D toric code
  3. Symmetric energy barrier is a topological invariant

$$\text{Non-constant energy barrier} \implies \text{nontrivial SPT order}$$

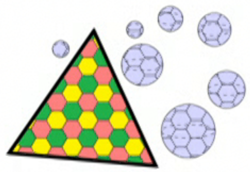


SR, Yoshida, Kubica, Bartlett 17

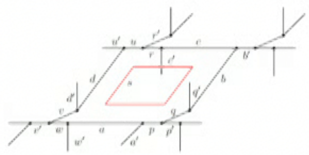
- Can we obtain the same result using a 1-form symmetric **trivial** phase ($H = -\sum_i X_i$) for the bulk?
  - No! Bulk SPT order is required to couple bulk and boundary systems
  - Bulk boundary correspondence at nonzero temperature

Other Symmetry Protected SCQMs

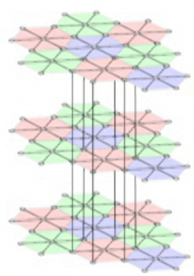
- Gauge fixed version of the gauge color code $H_{fix} = -\sum_{f \in \mathbf{B}}(X_f + Z_f)$



- Modular Walker Wang enriched by symmetries



- Foliated stabilizer codes (a simpler subclass of WW models)

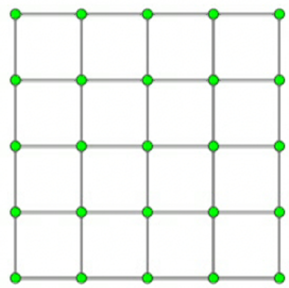


Emergent symmetries

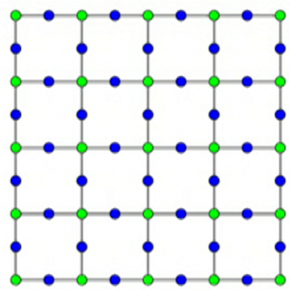
Can the $\mathbb{Z}_2 \times \mathbb{Z}_2$ 1-form symmetry emerge from underlying physics

- Hint 1: Emergent local $\mathbb{Z}_2$ symmetry in the toric code
- What is the symmetry operator that detects this?
- Kitaev: we can find a symmetry if we introduce redundant qubits

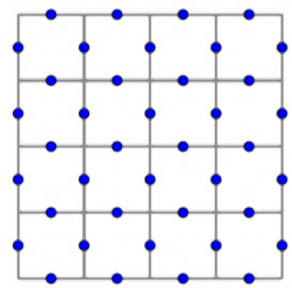
$$\mathbb{Z}_2 \text{ paramagnet} \underset{\text{ungauge}}{\overset{\text{gauge}}{\rightleftarrows}} \mathbb{Z}_2 \text{ gauge theory} \underset{LU}{\overset{LU}{\rightleftarrows}} \text{toric code}$$



$\mathbb{Z}_2$ symmetry $\prod_i X_v$



Valid states $X_v A_v = +1$



Emergent $\mathbb{Z}_2$

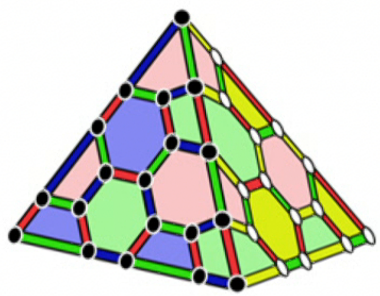
- Models with enforced symmetries can be dual to models with emergent symmetries: artificial symmetry becomes real

Emergent symmetries

- Gauging can be performed on subsystem and higher-form symmetries too (e.g. Vijay, Haah, & Fu 16, Williamson 16, Yoshida 16)
- One can gauge (several copies) of RBH with 1-form symmetries to a particular gauge color code Hamiltonian (Kubica & Yoshida 18)

Emergent symmetries

- Gauging can be performed on subsystem and higher-form symmetries too (e.g. Vijay, Haah, & Fu 16, Williamson 16, Yoshida 16)
- One can gauge (several copies) of RBH with 1-form symmetries to a particular gauge color code Hamiltonian (Kubica & Yoshida 18)
- Evidence in favour of gauge color code stability?



Bombin 2016

$$H = -\sum_f (X_f + Z_f)$$

- Product constraints $\Longrightarrow$ loop excitations, bulk and boundary coupled
- Does the gauge color code enforce a 1-form symmetry on its own

Emergent symmetries (a second hint)

- Prethermalisation and approximate conservation laws in closed systems
- Some closed quantum systems have an exponentially slow approach to thermal equilibrium
  - Obstructions to the equilibration of degrees of freedom encoded in edges or defects (Else *et al.* 17)
- What is the underlying symmetry?

> Theorem (Abanin, De Roeck, Huveneers and Ho 15)
>
> $$H = \sum_X P_X + Y \qquad \text{(commuting proj. + small pert.)}$$
>
> there exists local unitary U such that
>
> $$UHU^\dagger = \sum_X P_X + D + E$$
>
> where $\left[\sum_X P_X, D\right] = 0$ and $\|E\| = O(e^{-cn_*})$

- $\sum_X P_X$ is conserved for a time e^{-cn_*} $\implies$ approximate $U(1)$ symmetry
- Can we find higher-form analogues?

Summary

	Topological order $T>0$	Self-correction
SPT with onsite	×	×
Trivial with 1-form	×	×
SPT with 1-form	✓	✓
2D commuting proj.	× Hastings 11	× Landon-Cardinal & Poulin 12
3D fractal	× Siva & Yoshida 17	× Bravyi & Haah 13
4D toric	✓ Hastings 11	✓ Alicki *et al.* 10

Find dualities relating RBH to other models

Questions

- Higher-form symmetries arppear necessary for stability in 3D commuting models. Can they emerge more naturally in e.g. subsystem codes?
  - Find more solvable models of the gauge color code (ongoing work with Tomas Jochym-O'Connor - ask me later!)
- 2D SET boundaries of 3D SPT correspond to topological defects in 4D (thermally stable?)
- Understand symmetry-first principles for single-shot error correction and connection to thermal SPT order
- Dualities with subsystem codes (gauging/ungauging relates emergent and enforced symmetries) e.g. Kubica and Yoshida 18
- Understanding closed system and open system thermalisation results (prethermalisation, localisation, approximate $U(1)$ conservation laws...)
- Other notions of thermal topological order