Title: K-theoretic Donaldson-Thomas theory and the Hilbert scheme of points on a surface

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Abstract: <p>Integrals of characteristic classes of tautological sheaves on the Hilbert scheme of points on a surface often arise in enumerative problems. We explain how to use the K-theoretic Donaldson-Thomas theory of toric Calabi-Yau threefolds to study K-theoretic versions of such expressions.</p>

<p>Namely, we explicate a precise relationship between K-theoretic Donaldson-Thomas theory and the refined topological vertex of Iqbal, KosÃ§az and Vafa. Applying such results to specific toric threefolds, we deduce dualities satisfied by certain generating series that control integrals over the Hilbert scheme of points on a surface. We then explain how to use these dualities to evaluate certain Euler characteristics of tautological bundles on the Hilbert scheme of points on a general surface.</p>

K-theoretic DT invariants

+

Hilb(S)

K-theoretic DT invariants
+
Hilb(S)

(quasi)

S - sm proj surface / $\mathbb{C}$

$$S^{[n]} = \left\{ Z \subset S \;\middle|\; \begin{array}{l} \dim Z = 0 \\ \dim_{\mathbb{C}} H^0(\mathcal{O}_Z) = n \end{array} \right\}$$

(quasi)

Smooth, projective, of dimension 2n

$\mathcal{L}$ on $S \rightsquigarrow \mathcal{L}^{[n]}$ on $S^{[n]}$

line bundle — rk n bundle

over $Z \in S^{[n]}$

fiber of $\mathcal{L}^{[n]}$ is $H^0(\mathcal{O}_Z \otimes \mathcal{L})$

$$\int_{S^{[n]}} S_{2n}(\mathcal{L}^{[n]})$$

→ Conj of Lehn

→ proved by Marian-Oprea-Pandharipande, Voisin

$S^{[n]}$

$\Theta_{?}\otimes\mathcal{L}$)

of Lehn
by
-Oprea-Pandharipande
...ish

Ellingsrud-Göttsche-Lehn

Given a series

$$\sum q^n \int_{S^{[n]}} c_{2n}(\mathcal{L}^{[n]})$$

or more generally

$$(\star)\sum q^n \int_{S^{[n]}} \Phi_1(\mathcal{L}^{[n]})\, \Phi_2(TS^{[n]}),$$

mult. char. class

Such integrals are given by universal series

There exist $A_{\Phi}(q), B_{\Phi}, C_{\Phi}, D_{\Phi}$
(that do not depend on $S, \mathcal{L}$)
(Φ_1,Φ_2)
such that

$$(\star) = A_{\Phi}(q)^{c_2(S)}\, B_{\Phi}(q)^{c_1(S)^2}\, C_{\Phi}(q)^{c_1(S)c_1(\mathcal{L})}\, D_{\Phi}(q)^{c_1(\mathcal{L})^2}$$

S

$S^{[n]}$

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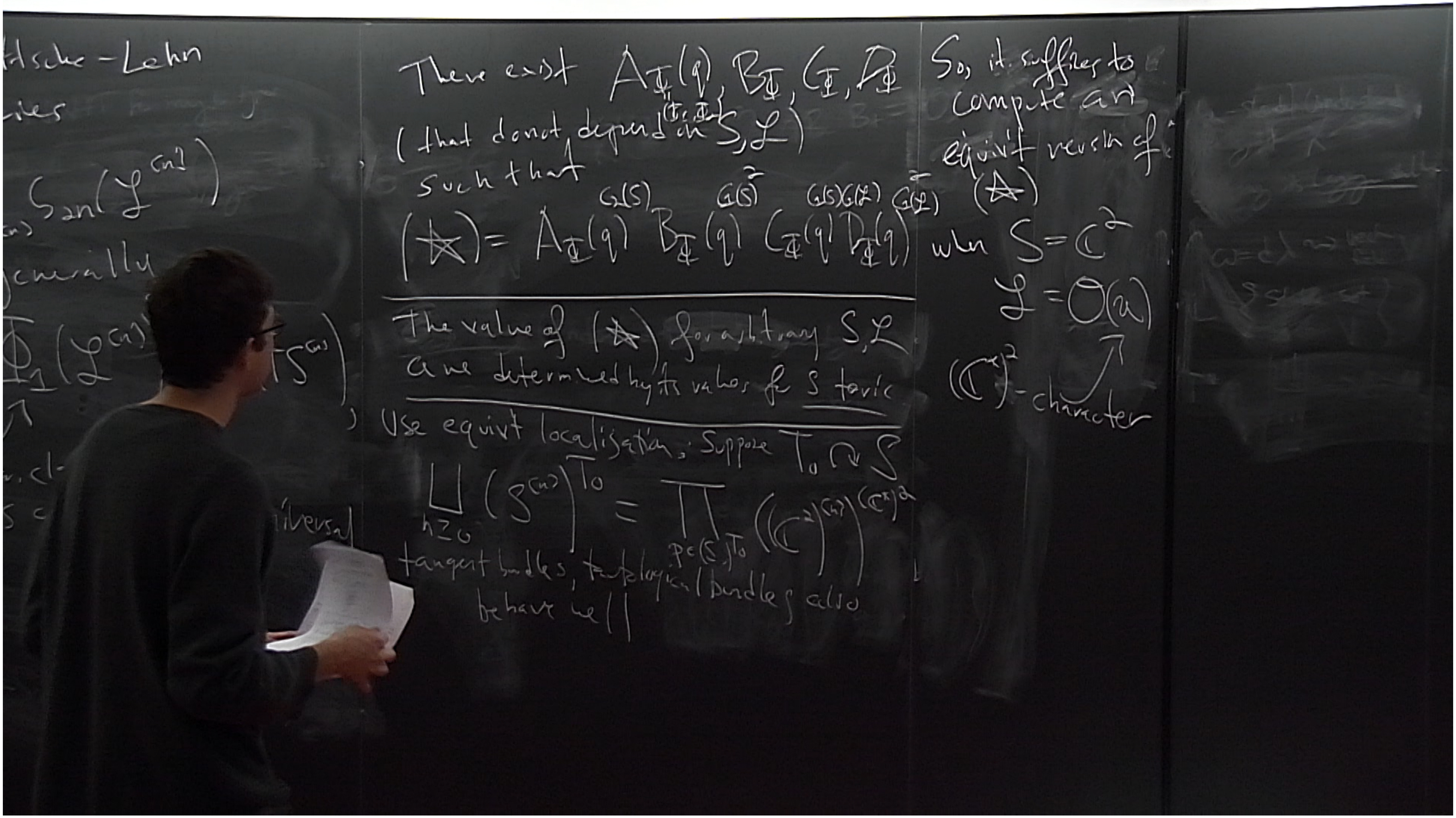
The value of $(\star)$ for arbitrary $S, \mathcal{L}$ are determined by its values for S toric

Use equiv't localisation: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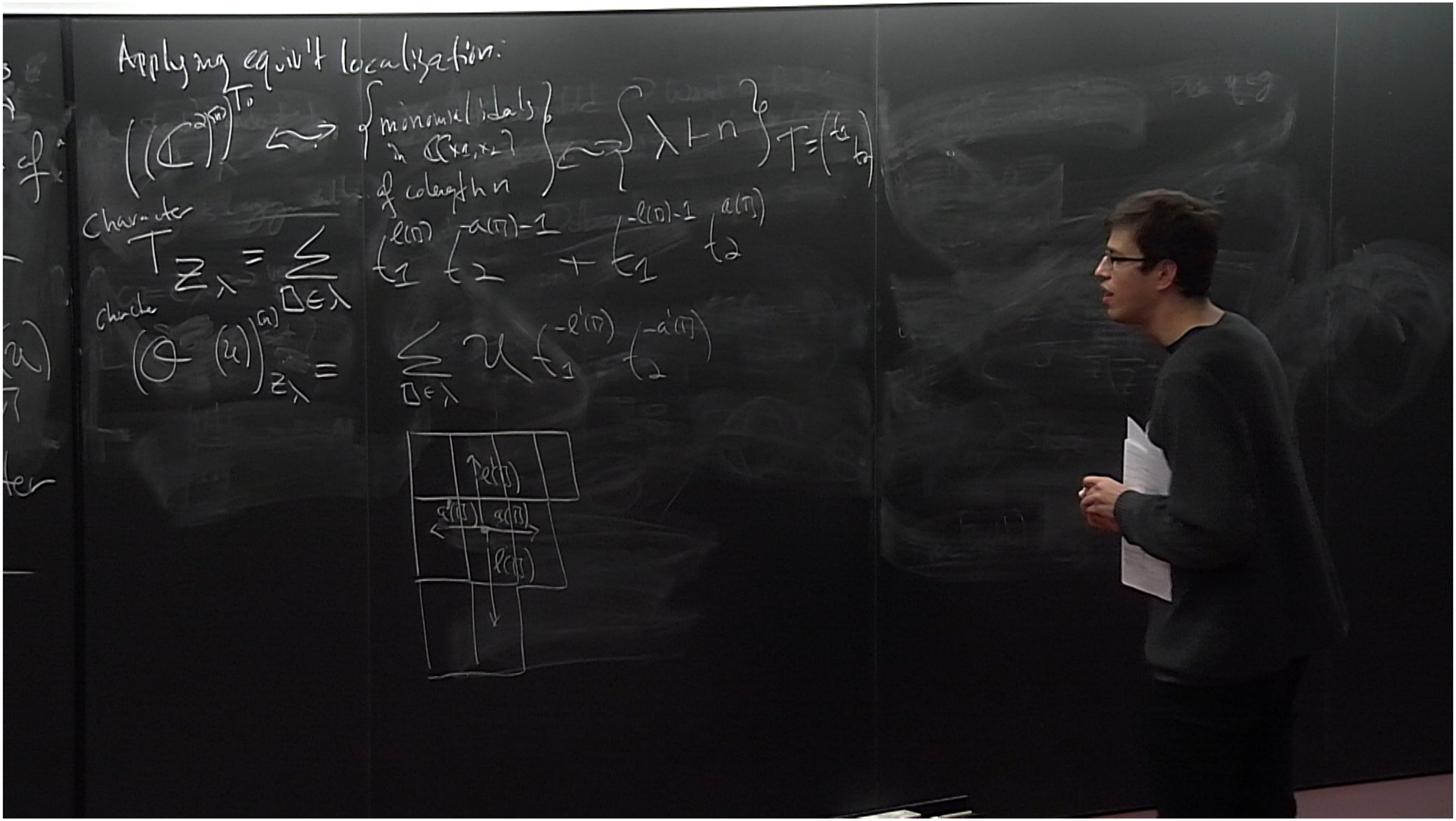
The value of $(\star)$ for arbitrary $S, \mathcal{L}$ are determined by its values for S toric
Use equiv't localisation: Suppose $T_0 \curvearrowright S$
$\bigsqcup_{n \geq 0} (S^{[n]})^{T_0} = \prod_{p \in (S)^{T_0}} ((\mathbb{C}^2)^{[n]})^{(\mathbb{C}^*)^2}$
tangent bundles, tautological bundles also behave well
So, it suffices to compute an equiv't version of $(\star)$ when $S = \mathbb{C}^2$, $\mathcal{L} = \mathcal{O}(u)$
$(\mathbb{C}^*)^2$-character


Applying equiv't localization:
$((\mathbb{C}^2)^{[n]})^{T_0} \leftrightsquigarrow$ {monomial ideals in $\mathbb{C}[x_1,x_2]$ of colength n} $\leftrightsquigarrow$ {$\lambda \vdash n$}
$T=\begin{pmatrix} t_1 & \\ & t_2 \end{pmatrix}$
Character
$T_{Z_\lambda} = \sum_{\square\in\lambda} t_1^{\ell(\square)} t_2^{-a(\square)-1} + t_1^{-\ell(\square)-1} t_2^{a(\square)}$
Character
$(\mathcal{O}(u))^{[n]}_{Z_\lambda} = \sum_{\square\in\lambda} u\, t_1^{-\ell'(\square)} t_2^{-a'(\square)}$
$\ell'(\square)$
$a'(\square)$
$a(\square)$
$\ell(\square)$

Can write series ☆ purely in terms of combinatorics:

eg: $$\sum_{i,j,n\geq 0} q^n m^i y^j \chi\left((\mathbb{C}^2)^{[n]}, \Lambda^i\left((\Theta(\omega))^{[n]}\right) \otimes \mathrm{Sym}^j\left((\Theta(\omega))^{[n]}\right)\right)$$

$$= \sum_{\lambda} q^{|\lambda|} \frac{1}{\prod_{w \in T_{Z_\lambda}} (1-w^{-1})} \prod_{v \in (\Theta(\omega))^{[n]}_{Z_\lambda}} \frac{1-mv}{1-yv}$$

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eg: $\sum_{i,j,n\geq 0} q^n m^i y^j \chi\left((\mathbb{C}^2)^{[n]}, \Lambda^i\left((\Theta(\omega))^{[n]}\right) \otimes S^j_{sym}\left((\Theta(\omega))^{[n]}\right)\right)$

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For arbitrary Φ this could be intractable.

For particular choices of Φ some nice structure emerges.

Set

$F(q, m_1, m_2, y, \underbrace{t_1, t_2}_{\text{suppress from notation}})$

$$\sum_{n, i_1, i_2, j} (-q)^n m_1^{i_1} (m_2 y)^{i_2} y^j \chi\left((\mathbb{C}^2)^{[n]}, \Lambda^{i_1}(\mathcal{O}^{[n]})^* \otimes \Lambda^{i_2}(\mathcal{O}^{[n]}) \otimes \mathrm{Sym}^j \mathcal{O}^{[n]} \otimes \det(\mathcal{O}^{[n]})\right)$$

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Ellip…

Giv…

$\sum$

$(\star) \sum q^{\dots}$

Such …

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$$= \sum_\lambda \frac{q^{|\lambda|}}{\prod_{w \in T_\lambda}(1-w^{-1})} \prod_{v \in \mathcal{O}^{[n]}|_\lambda} \frac{(1-m_1 v^{-1})(1-m_2 y v)}{y - v^{-1}}$$

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Ellip[...]

Giv[...]

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$(\star) \sum q^{[...]}$

Such [...]

is

$\otimes \Lambda^{\bullet}(\mathcal{O}^{[n]})$

$\otimes \det(\mathcal{O}^{[n]})$

$v^{-1})(1-m_2 y v)$

$y - v^{-1}$

Thm:

$$\frac{F(q, m_1, m_2, y)}{F(q, m_1, m_2, 0)} = \frac{F(q, m_2, m_1, y)}{F(q, m_2, m_1, 0)} = \frac{F(y, m_1, m_2, q)}{F(y, m_1, m_2, 0)}$$

Prop:

$$F(q, m_1, m_2, 0) = \exp\left(\sum_{n>0} \frac{q^n}{n} \frac{(1-m_1^n)}{(1-t_1^n)(1-t_2^n)} \right) = S^{\bullet}\left(\frac{q\,(1-m_1)}{(1-t_1^{-1})(1-t_2^{-1})} \right)$$

There exist

(that do not

such that

$(\star) = A$

The value of

are determ

Use equivt loca

$\coprod_{n \geq 0} (S^{(n)})$

tangent bundles

behave

Proof

$$F(q, m_1, m_2, t) = \exp\left(\sum_{n>0} \frac{q^n}{n} \frac{(1-m_1^n)}{(1-t_1^n)(1-t_2^n)}\right)$$

$$= S^{\bullet}\left(\frac{q\,(1-m_1)}{(1-t_1^{-1})(1-t_2^{-1})}\right)$$

Application:

$$\sum_{n \geq 0} q^n \chi(S^{(n)}, \mathrm{Sym}^2 \mathcal{L}^{(n)})$$

want to understand terms of y-deg ≤ 2

$\rightsquigarrow$ of

$$\sum_{\lambda} \frac{q^{|\lambda|}}{\prod_{w \in \mathbb{Z}_\lambda} (1 - w^{-1})} \prod_{v \in \mathcal{O}_\lambda^{(n)}} \frac{1}{(1 - y v)}$$

every partition λ could contribute terms of y-deg ≤ 2

So, it suffices to compute an equiv't version of $(\star)$ when $S = \mathbb{C}^2$

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$(\mathbb{C}^*)^2$-character

$$\sum_{n>0} \frac{q^n}{n} \frac{(1-m_1^n)}{(1-t_1^n)(1-t_2^n)}$$

$$\frac{q\,(1-m_1)}{(1-t_1^{-1})(1-t_2^{-1})}$$

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Thm lets me

prefactor

$$\sum_\lambda \frac{y^{|\lambda|}}{\prod(1-w^{-1})} \prod \frac{1}{1-qv}$$

only have to work with $|\lambda| \leq 2$

$\longleftrightarrow$ of monomial in $\mathbb{C}[x_1, \ldots$ of colength

$$\sum_{\square \in \lambda}$$

$t_1^{\ell(\square)}$

Then lets use

prefactor

$$\sum_\lambda \frac{q^{|\lambda|}}{\prod(1-w^{-1})} \prod \frac{1}{1-qv}$$

only have to work with $|\lambda| \le 2$

eg.

$$\chi(S^{(n)}, \mathrm{Sym}_m^2 \mathcal{L}^{(n)}) = \binom{\chi(\mathcal{O}_S)+n-3}{n-1}\chi(\mathcal{L}^{\otimes 2}) + \binom{\chi(\mathcal{O}_S)+n-3}{n-2}\binom{\chi(\mathcal{L})+1}{2}$$

For arbitrary n,k $\chi(S^{(n)}, \mathrm{Sym}_m^k \mathcal{L}^{(n)})$ seem to have no concise expression

But: When $\chi(\mathcal{O}_S)=1$, and $n \ge k$,

$$\chi(S^{(n)}, \mathrm{Sym}_m^k \mathcal{L}^{(n)}) = \binom{\chi(\mathcal{L})+k-1}{k}$$

Can write series
Combinatorics

eg: $\sum_{i,j,n\ge 0} q^n m^i y^j \chi$

$= \sum_\lambda q^{|\lambda|}$

For arbitrary

For particular
emerges.

Strategy:

F, integrals over $(\mathbb{C}^2)^{(n)}$, instanton moduli space $=$ Nekrasov partition functions $\sim$ partition functions of SUSY gauge theories $\overset{\text{geometric engineering}}{=\!=}$ refined topological string partition function

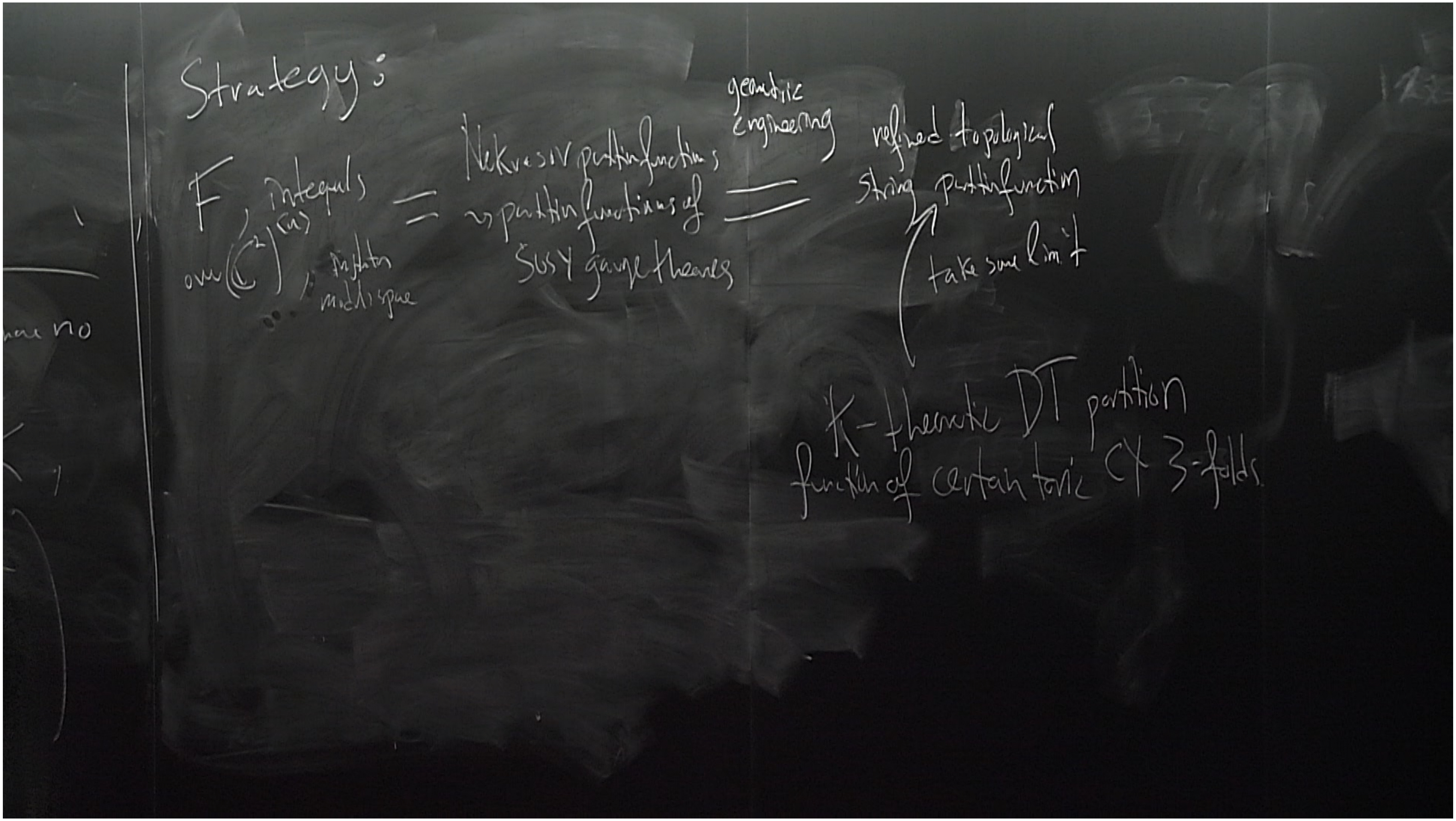
take some limit

K-theoretic DT partition

em to have no

$\geq K$,

$t+k-1$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= Nekrasov partition functions ~ partition functions of SUSY gauge theories
geometric engineering
= refined topological string partition function
take some limit
K-theoretic DT partition function of certain toric CY 3-folds

X - toric CY 3-fold, $T \curvearrowright X$

such that $\quad T = \begin{pmatrix} t_1 & & \\ & t_2 & \\ & & t_3 \end{pmatrix}$

weight of Ω_X is $t_1 t_2 t_3$

$$DT(X) = \mathrm{Hilb}(X, \text{curves})$$

$$= \bigsqcup_{\beta, n} \mathrm{Hilb}(X, \beta, n)$$

$$\mathrm{Hilb}(X, \beta, n) = \left\{ Y \subset X \;\middle|\; \begin{array}{l} \dim Y \leq 1 \\ [Y] = \beta \\ \chi(\mathcal{O}_Y) = n \end{array} \right\}$$

$1 = F$

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symmetric

These spaces carry a perfect obstruction theory

$T^{vir}_Y = \chi(\mathcal{O}_Y) + \chi(\mathcal{O}_Y, \mathcal{O}_X) - \chi(\mathcal{O}_Y, \mathcal{O}_Y)$

$= \mathrm{Def} - \mathrm{Obs}$

Symmetry $\Rightarrow$ $\mathrm{Def} = (\mathrm{Obs})^* \otimes t_1 t_2 t_3$

X - toric CY 3-fold, $T \circlearrowright X$ with $T = (t_1, t_2, t_3)$

such that

weight of Ω_X is $t_1 t_2 t_3$

$DT(X)$ ... (X, curves)

$= \bigsqcup_{\beta}$ ... (X, β, n)

Hil ... $) = \{ Y \subset X \mid \dim Y \leq 1,\ [Y] = \beta,\ \chi(\mathcal{O}_Y) = n \}$

These spaces carry a perfect symmetric obstruction theory

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Symmetry $\Rightarrow$ $\mathrm{Def} = (\mathrm{Obs})^* \otimes t_1 t_2 t_3$

$K^{\mathrm{vir}} = \dfrac{\det \mathrm{Obs}}{\det \mathrm{Def}}$

Prop. [Nekrasov–Okounkov]. The bundle K^{vir} admits an equiv't square root $(K^{\mathrm{vir}})^{1/2}$ in the $\tilde{T}$-equiv't Picard group where $\tilde{T}$ is a double cover of T.

Prop

$F(q$

symmetric

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Def:

$Z_{DT}(X)$

$= \sum_{\beta, n} Q^n u^\beta \chi(\mathrm{Hilb}(X, \beta, n), \mathcal{O}^{vir} \otimes (K^{vir})^{1/2})$

understood via equiv't localization:

If $Y \in (DT(X))^T$

can write $T^{vir}_Y = \sum_i a_i^Y - b_i^Y$

where $a_i^Y b_i^Y = t_1 t_2 t_3$

$n \geq 0$

want to understand

$\rightsquigarrow$ ... of

$\sum_\lambda \dfrac{q^{|\lambda|}}{\prod_{w \in Z_\lambda} (1 - w^{-1})}$

every partition λ ...

of q-deg ≤ 2 ...

$\mathrm{DT}(X)$

$= \sum_{\beta,n} Q^n u^\beta \chi(\mathrm{Hilb}(X,\beta,n), \mathcal{O}^{vir} \otimes (K^{vir})^{1/2})$

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$$Z_{DT}(X) = \sum_{Y \in \mathrm{DT}(X)^T} Q^{\chi(\mathcal{O}_Y)} u^{[Y]} \prod_i \frac{(b_i^Y)^{1/2} - (b_i^Y)^{-1/2}}{(a_i^Y)^{1/2} - (a_i^Y)^{-1/2}}$$

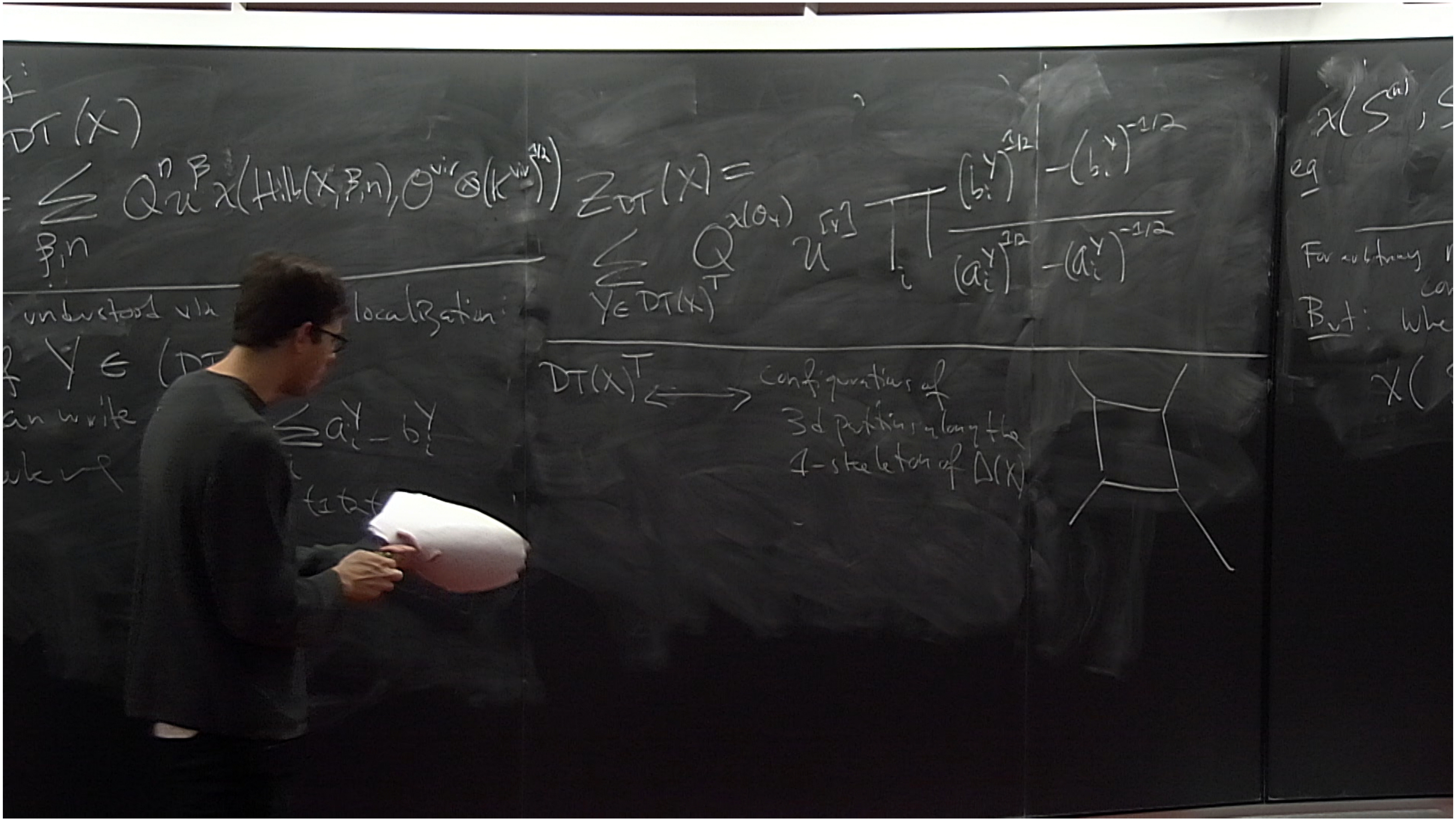
$\chi(S^{(n)}, S$

eg

For arbitrary

But: Whe

$\chi($


$DT(X)$
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understood via localization:
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$\sum a_i^Y - b_i^Y$
$DT(X)^T \longleftrightarrow$ configurations of 3d partitions along the 1-skeleton of $\Delta(X)$
eg
For arbitrary
But: Whe

$Z_{DT}(X)$

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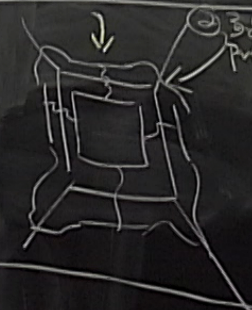
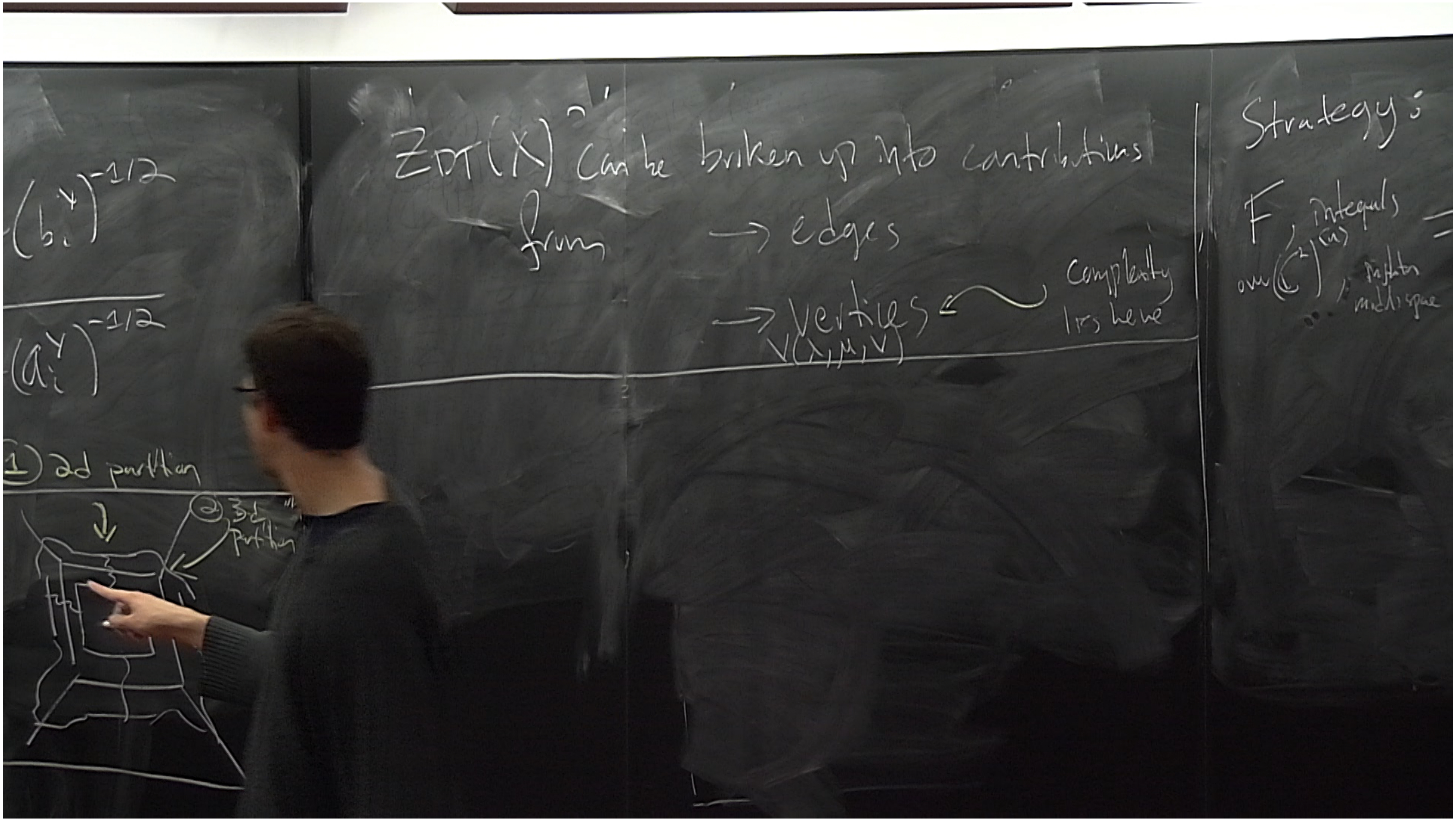
$DT(X)^T \longleftrightarrow$ configurations of 3d partitions along the 1-skeleton of $\Delta(X)$

① 2d partition

② 3d partition



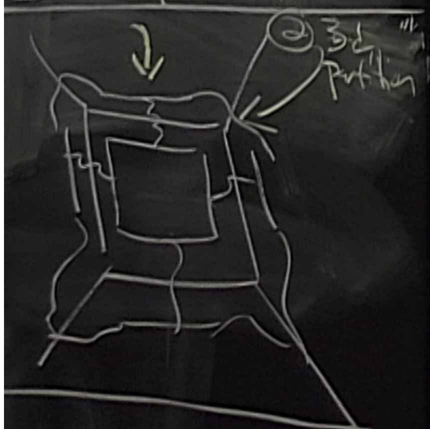

ZDT(X) can be broken up into contributions
from
→ edges
→ vertices
V(λ, μ, ν)
complexity lies here
Strategy:
integrals
over
moduli space
1) 2d partition
2) 3d partition

$(*)_2$

$$\frac{(b_i^Y)^{-1/2}}{(a_i^Y)^{-1/2}}$$

1) 2d partition

2) 3d partition



$Z_{DT}(X)$ can be broken up into contributions

from $\longrightarrow$ edges

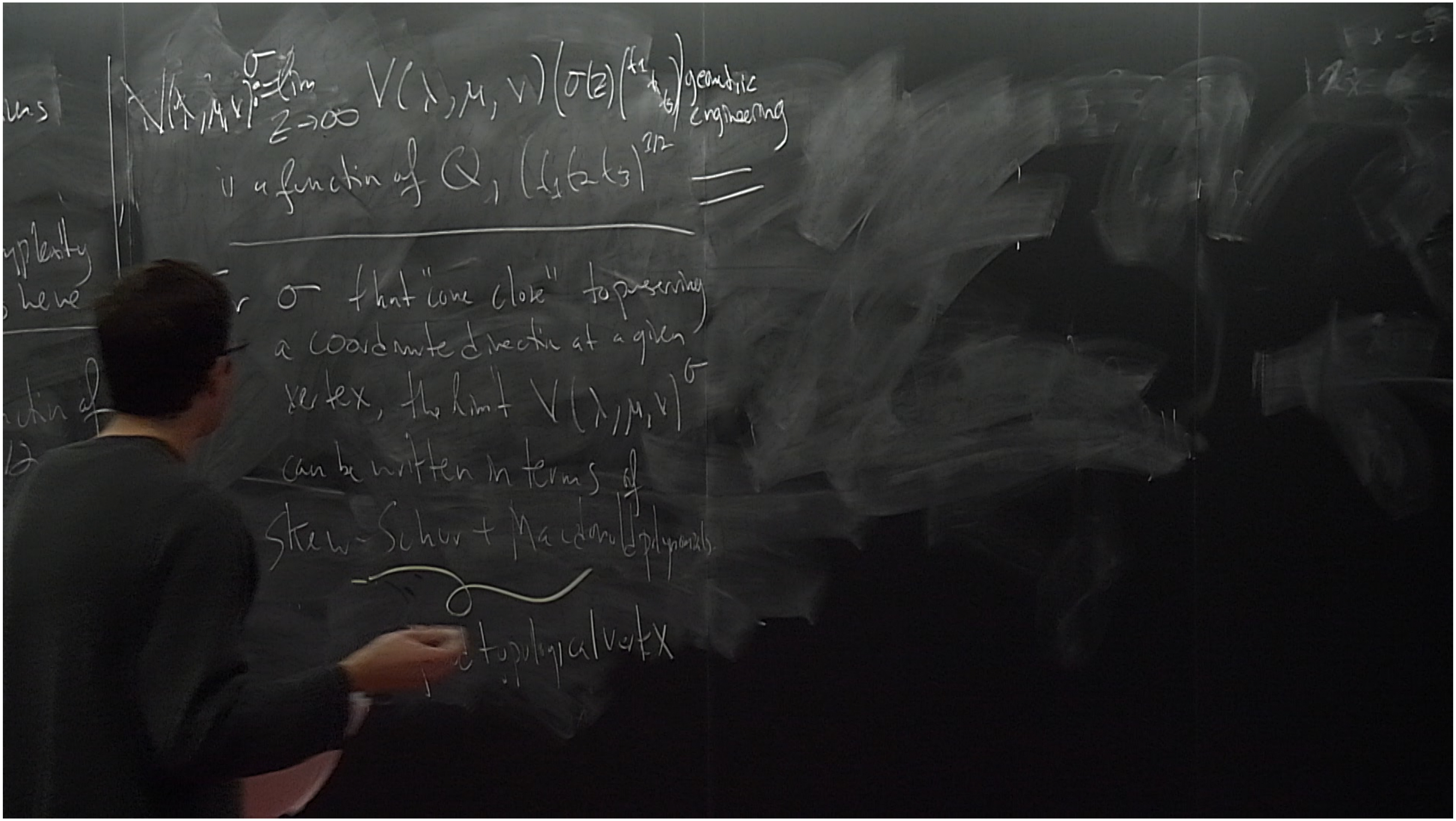
$\longrightarrow$ vertices $V(\lambda,\mu,\nu)$ — complexity lies here

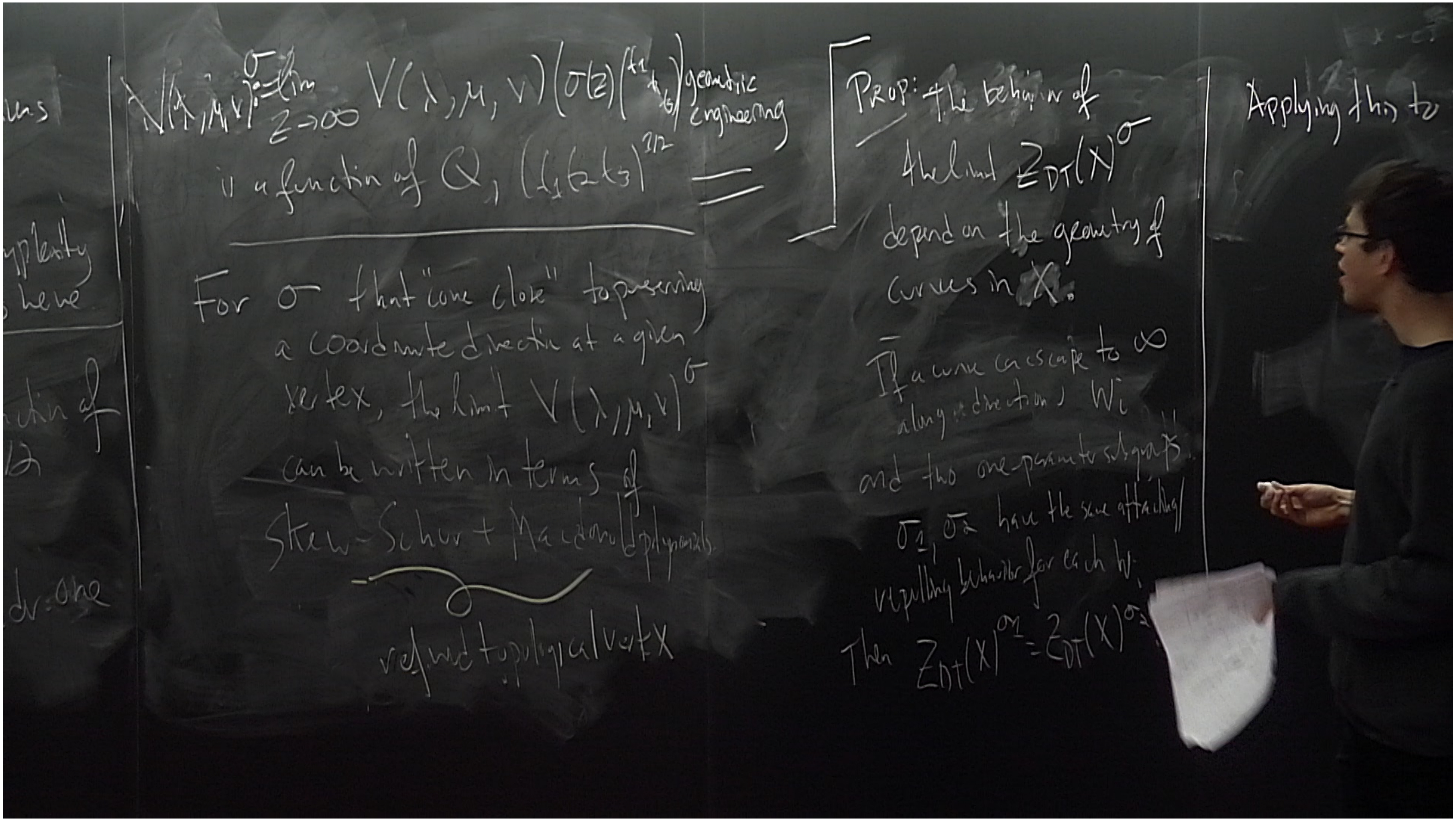
In general $V(\lambda,\mu,\nu)$ is a complicated function of

$$Q,\ t_1,\ t_2,\ t_3,\ (t_1t_2t_3)^{1/2}$$

However, the symmetry of $(*)$ implies that under one parameter subgroups $\sigma: \mathbb{C}^* \to (T_{CY}) \subset T$

$$T_{CY} = \mathrm{Ker}(t_1t_2t_3)$$


V(λ,μ,ν)^σ := lim z→∞ V(λ,μ,ν)(σ(z))(t1,t2,t3) geometric engineering
is a function of Q, (t1t2t3)^1/2
For σ that "come close" to preserving a coordinate direction at a given vertex, the limit V(λ,μ,ν)^σ
can be written in terms of
skew-Schur + Macdonald polynomials
topological vertex


$N(\lambda,\mu,\nu)^{\sigma} := \lim_{z\to\infty} V(\lambda,\mu,\nu)(\sigma(z))(t_1, t_2, t_3)$ geometric engineering
is a function of $Q, (t_1 t_2 t_3)^{1/2}$
For σ that "come close" to preserving a coordinate direction at a given vertex, the limit $V(\lambda,\mu,\nu)^{\sigma}$ can be written in terms of skew-Schur + Macdonald polynomials.
refined topological vertex
Prop: the behavior of the limit $Z_{DT}(X)^{\sigma}$ depend on the geometry of curves in X.
If a curve can escape to ∞ along directions W_i and two one-parameter subgroups σ_1, σ_2 have the same attracting/repelling behavior for each W_i
Then $Z_{DT}(X)^{\sigma_1} = Z_{DT}(X)^{\sigma_2}$
Applying this to

$V(\lambda,\mu,\nu)^{\sigma} := \lim_{z\to\infty} V(\lambda,\mu,\nu)\left(\sigma(z)\begin{pmatrix} t_1 \\ t_2 \\ t_3\end{pmatrix}\right)$ geometric engineering

is a function of Q, $(t_1 t_2 t_3)^{1/2}$ $=$

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refined topological vertex

Prop: The behavior of the limit $Z_{DT}(X)^{\sigma}$ depend on the geometry of curves in X.

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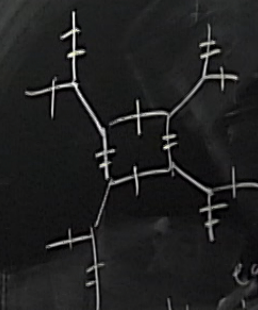
Applying this to



, we recover the earlier identity.

$\ldots V(\lambda, \mu, \nu)(\sigma(z))^{(t_1, t_2, t_3)}$ geometric engineering

...tion of Q, $(t_1 t_2 t_3)^{1/2}$ =

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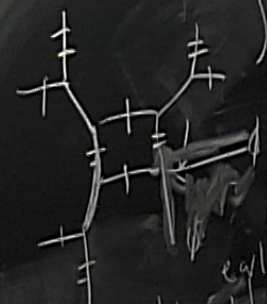
Then $Z_{DT}(X)^{\sigma_1} = Z_{DT}(X)^{\sigma_2}$.

Applying this to



$\int_{M(z)}$

$(\lambda, \mu, \nu, \phi)$

earlier

we recover the identity.