Title: Geometry from Entanglement in Double Trace States

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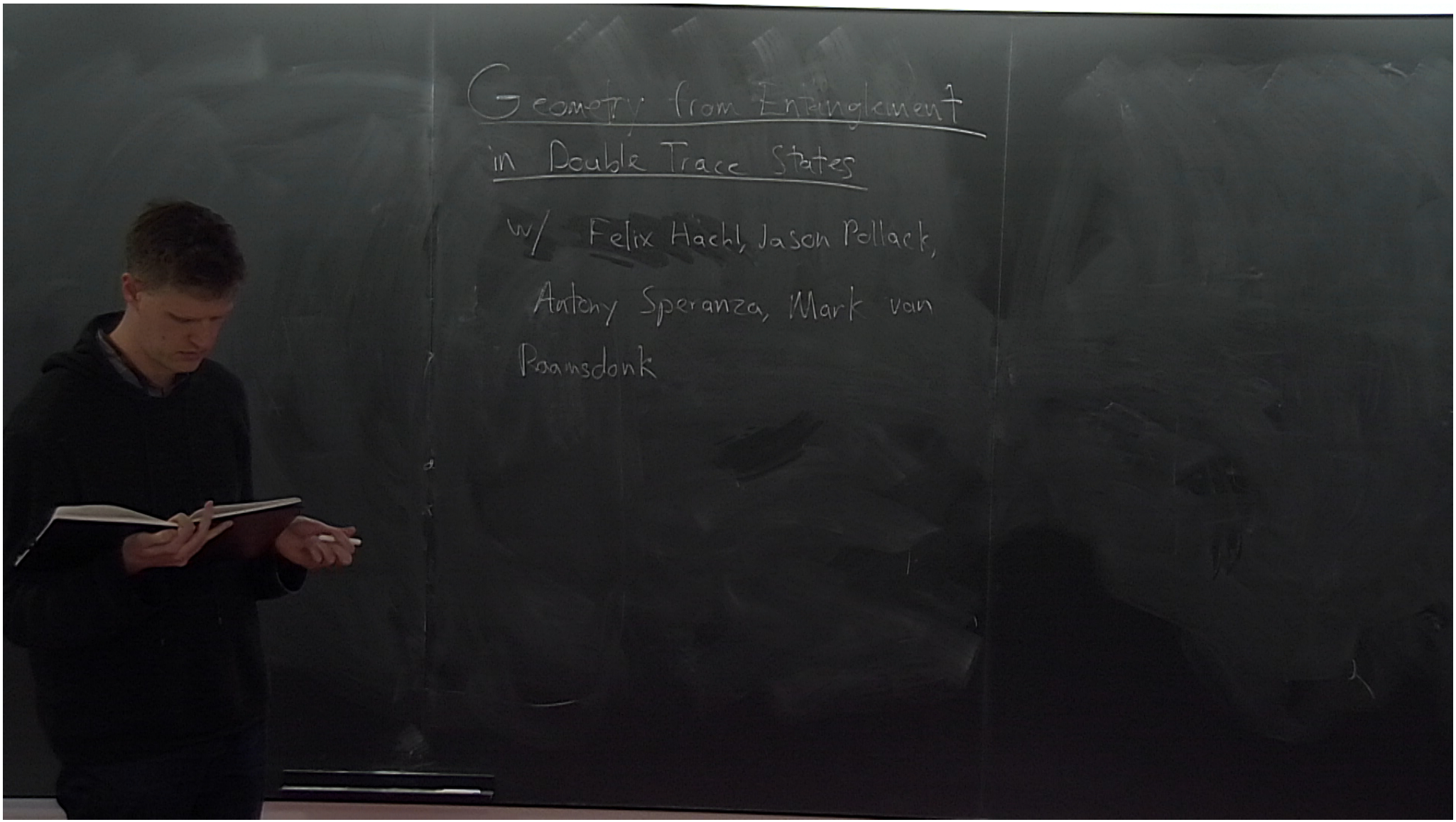
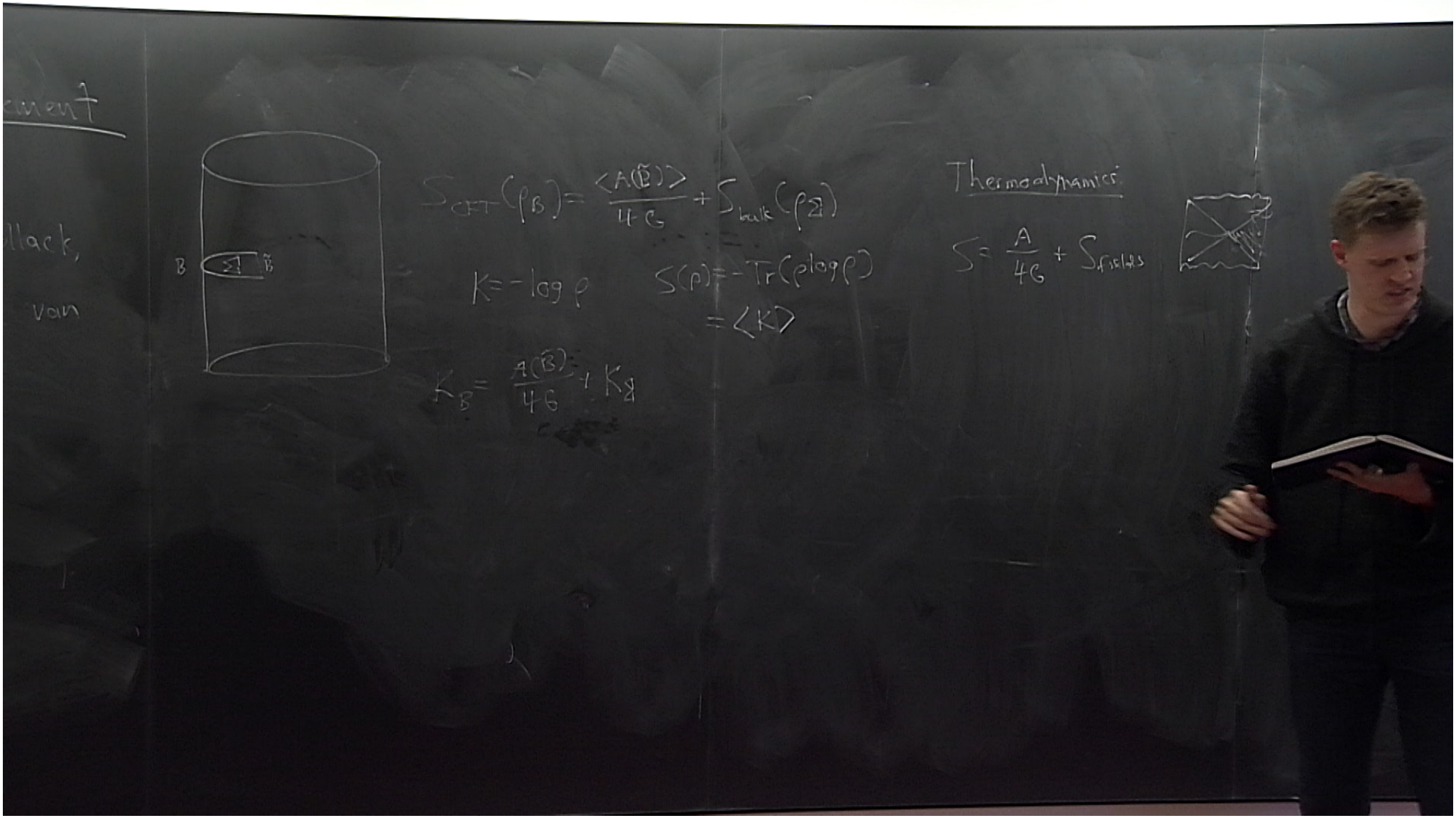
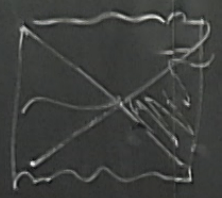
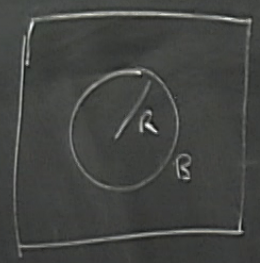
Abstract: Bulk coherent states produced by Euclidean CFT sources can be used to study how geometry arises from entanglement: when these states obey the Ryu-Takayanagi formula, the linearized Einstein equations hold in the bulk without assuming the complete AdS/CFT correspondence. However, the Faulkner-Lewkowycz-Maldacena bulk entanglement term that corrects the RT formula does not play a role in these previous studies, since to leading order coherent states have the same entanglement entropy as the vacuum. We study states CFT states created by Euclidean, bilocal, double trace sources, which produce bulk squeezed states. We consider possible modifications to RT and study divergences in the perturbative expansion that occur for certain choices of source. We work towards determining constraints on the bulk geometry and state that arise from requiring FLM, akin to the linearized Einstein equations that arose for coherent states.


Geometry from Entanglement
in Double Trace States
w/ Felix Hacl, Jason Pollack,
Antony Speranza, Mark van
Raamsdonk


ement
llack,
van
B
Σ
B̃
$S_{CFT}(\rho_B) = \frac{\langle A(\tilde{B}) \rangle}{4G} + S_{bulk}(\rho_\Sigma)$
$K = -\log \rho$
$S(\rho) = -\mathrm{Tr}(\rho \log \rho)$
$= \langle K \rangle$
$K_B = \frac{A(\tilde{B})}{4G} + K_\Sigma$
Thermodynamics
$S = \frac{A}{4G} + S_{fields}$

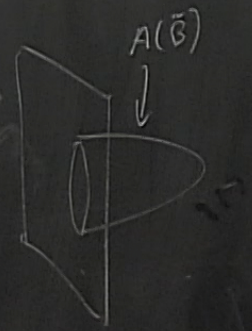


$t = t_0$ CFT



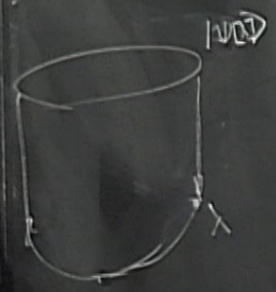
$$\rho_B = \mathrm{Tr}_{\bar{B}}(|0\rangle\langle 0|)$$

$$S_{CFT}(\rho_B) \sim \begin{cases} a_d^* \log(\xi/R) & d \text{ odd} \\ a_d^* & d \text{ even} \end{cases}$$

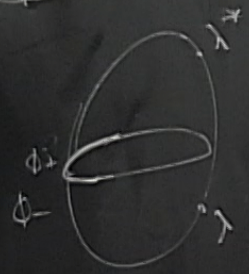


$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

$$\ell = 1/a^{d-1}$$



$$\langle \phi_0 | \psi[\lambda] \rangle = \int_{-\infty}^{\phi(\tau=0)=\phi_0} D\phi\, e^{-S+\int \lambda \mathcal{O}}$$



$$\langle \phi_+ | \psi[\lambda] | \phi_- \rangle = \int_{\phi(\tau=0)=\phi^-}^{\phi(\tau=0)=\phi^+} D\phi\, e^{-S+\int \lambda \mathcal{O}}$$

$$\rho = \rho_0 + \int \rho_0 \lambda \mathcal{O} \equiv \rho_0 + \delta\rho$$

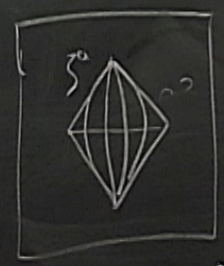
$$\delta S(\rho) = S(\rho) - S(\rho_0)$$

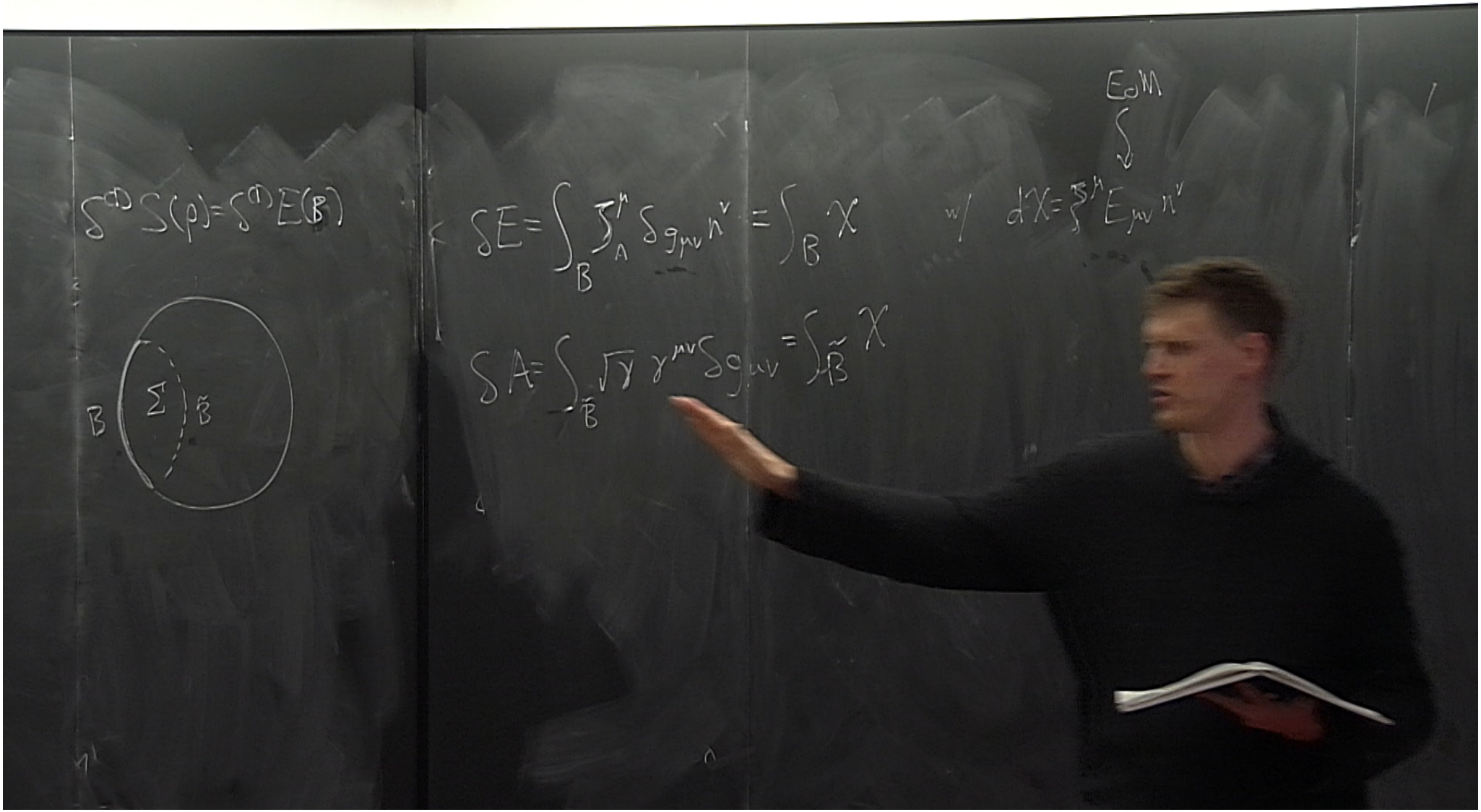
$$= \mathrm{Tr}[\delta\rho\, K_0] - S^{rel}(\rho|\rho_0)$$

$$S^{rel}(\rho|\rho_0) = \mathrm{Tr}[\rho \log \rho] - \mathrm{Tr}[\rho \log \rho_0]$$

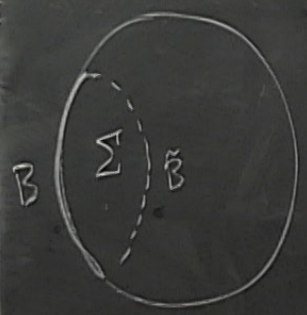
$$\delta^{(1)} S^{rel}(\rho|\rho_0) = 0$$

$$K_0(B) = \int \xi^a T_{ab} n^b$$




$\delta^{(1)} S(\rho) = \delta^{(1)} E(B)$
B
Σ
$\tilde{B}$
$\delta E = \int_B \xi^\mu_A \, \delta g_{\mu\nu} n^\nu = \int_B X$
w/ $dX = \xi^\mu E_{\mu\nu} n^\nu$
EoM
$\delta A = \int_{\tilde{B}} \sqrt{\gamma} \, \gamma^{\mu\nu} \delta g_{\mu\nu} = \int_{\tilde{B}} X$

$$\delta^{(1)} S(\rho) = \delta^{(1)} E(B)$$

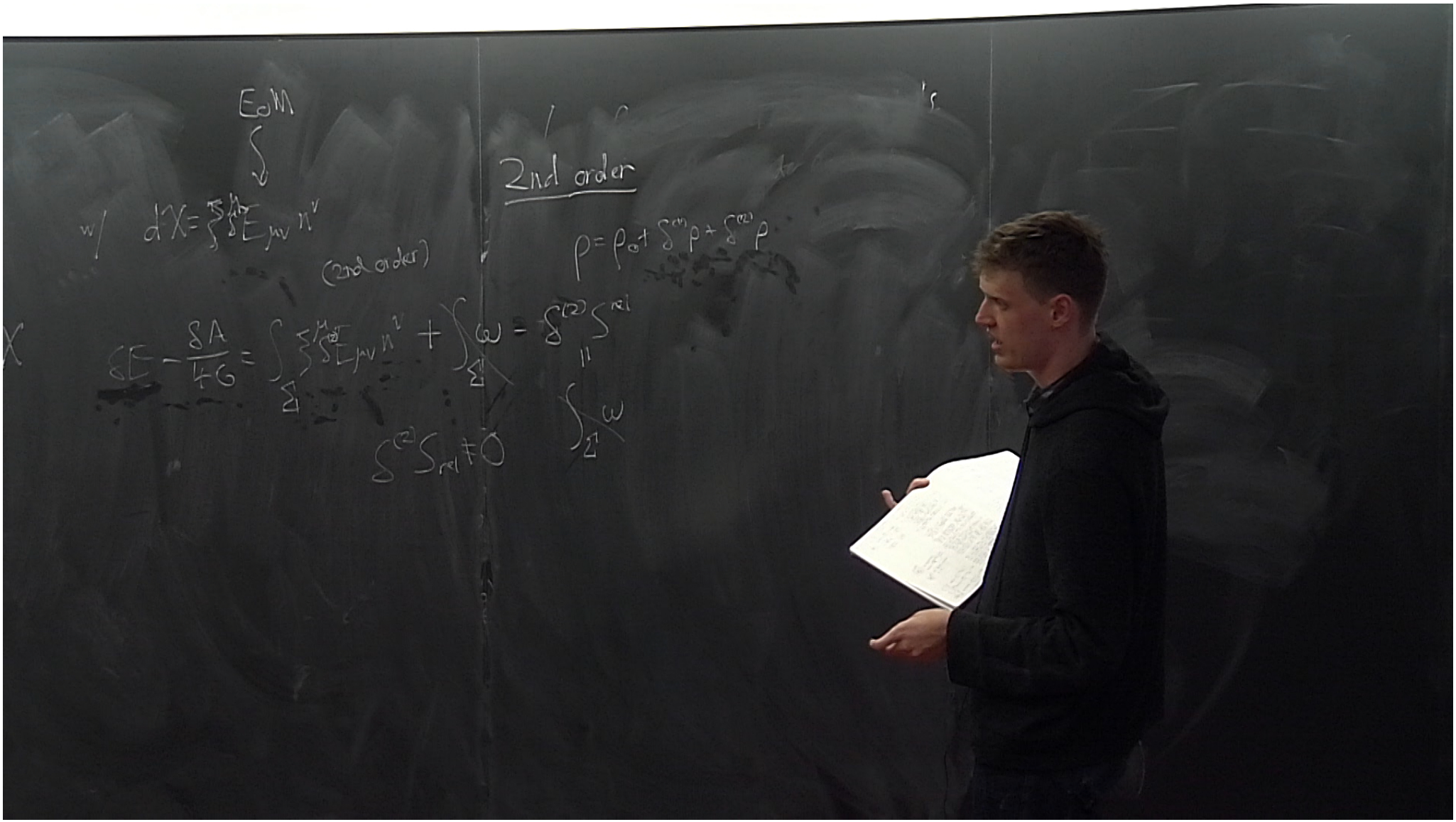


$$\delta E = \int_B \xi^\mu_A \, \delta g_{\mu\nu} n^\nu = \int_B X \qquad \text{w/} \quad dX = \xi^\mu E_{\mu\nu} n^\nu$$

$$\delta A = \int_{\tilde{B}} \sqrt{\gamma}\, \gamma^{\mu\nu} \delta g_{\mu\nu} = \int_{\tilde{B}} X \qquad \delta E - \frac{\delta A}{4G} = \int_\Sigma \xi^\mu E_{\mu\nu} n^\nu$$

EoM

EoM
2nd order
w/ $dX = \xi^\mu \delta E_{\mu\nu} n^\nu$
(2nd order)
$\rho = \rho_0 + \delta^{(1)}\rho + \delta^{(2)}\rho$
$\delta E - \frac{\delta A}{4G} = \int_\Sigma \xi^\mu \delta^{(2)} E_{\mu\nu} n^\nu + \int_\Sigma \omega$
$\delta^{(2)}$


EoM
w/ $dX = \xi^\mu \delta E_{\mu\nu} n^\nu$
(2nd order)
2nd order
$\rho = \rho_0 + \delta^{(1)}\rho + \delta^{(2)}\rho$
$\delta E - \frac{\delta A}{4G} = \int_\Sigma \xi^\mu \delta E_{\mu\nu} n^\nu + \int_\Sigma \omega = \delta^{(2)} S^{rel}$
$\int_\Sigma \omega$
$\delta^{(2)} S_{rel} \neq 0$

EoM

w/ $d^2X = \xi^\mu \delta E_{\mu\nu} n^\nu$

(2nd order)

$$\delta E - \frac{\delta A}{4G} = \int_\Sigma \xi^\mu \delta E_{\mu\nu} n^\nu + \int_\Sigma \omega = \delta^{(2)} S^{rel}$$

$$\delta^{(2)} S_{rel} \neq 0$$

2nd order

$$\rho = \rho_0 + \delta^{(1)}\rho + \delta^{(2)}\rho$$

$$\int_\Sigma \omega$$

$$S_{CFT}(\rho_D) = \frac{\langle A \rangle}{4G} + S_{bulk}(\rho_\Sigma)$$

$$\delta S_{bulk} = 0$$

$$\langle K_{0,bulk} \rangle = S^{rel}_{bulk}$$

JLMS

$$S^{rel}_{CFT}(\rho|\sigma) = S^{rel}_{bulk}(\rho|\sigma)$$

1. bulk interactions

2. non-perturb.

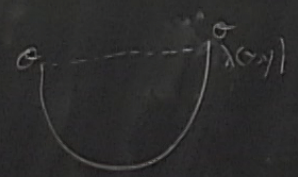
bulk

simple single-trace

$$\mathcal{O}|0\rangle\langle 0|\mathcal{O} \to a^\dagger|0\rangle\langle 0|a$$

$$\int \lambda(x,y)\,\mathcal{O}(x)\,\mathcal{O}(y)$$

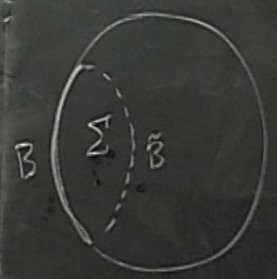
3. $\rho \sim \int D\phi\, e^{-S+\int \lambda(x,y)\,\mathcal{O}(x)\,\mathcal{O}(y)}$



$$\lambda(x,y) = \lambda\,\delta(x-y)$$

$$\int \lambda\, T\bar{T}$$

$$\delta^{(1)} S(\rho) = \delta^{(1)} E(B)$$



States

Jason Pollack,

, Mark van

$$S_{CFT}(\rho_D) = \frac{\langle A \rangle}{4G} + S_{bulk}(\rho_\Sigma)$$

$$\delta S_{bulk} = 0$$

$$\langle K_{0,bulk} \rangle = S^{rel}_{bulk}$$

JLMS

$$S^{rel}_{CFT}(\rho|\sigma) = S^{rel}_{bulk}(\rho|\sigma$$

$$e^{-\int \lambda a} \qquad e^{a - a^\dagger}$$

$$e^{-\int \lambda a a} \qquad e^{a^\dagger a^\dagger}$$

lement

Pollack,

k van

$\mathcal{O} \sim 1/N \, \mathrm{Tr}[\cdot]$ $\quad \lambda \sim N^2$

$\langle \mathcal{O}\mathcal{O} \rangle_c \sim 1/N^2$

$\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle_c \sim 1/N^4$

S

bu

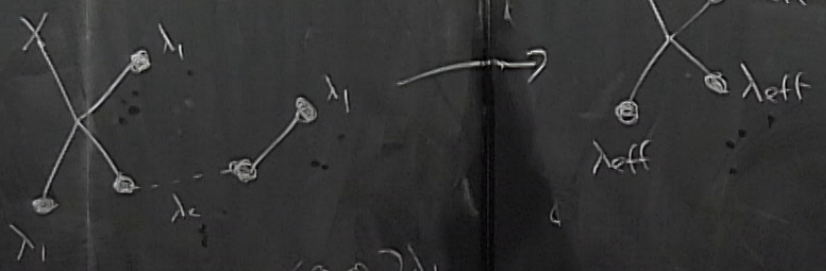
$$-S \to -S + \int \lambda_1(x)\, \mathcal{O}(x) + \int \lambda_2(x_1, x_2)\, \mathcal{O}(x_1)\, \mathcal{O}(x_2)$$

$$\left[S_{CFT}\left(\rho_{\{\lambda_1, \lambda_2\}}\right) = S_{CFT}\left(\rho_{\lambda_{eff}}\right) \right]_{O(N^2)}$$

$$\lambda_{eff}(x) = \lambda_1(x) + \int dx'\, \lambda_2(x, x')\, \langle \mathcal{O}(x') \rangle_{\lambda_{eff}}$$

1) $\langle \mathcal{O} \rangle_{\{\lambda_1, \lambda_2\}} = \langle \mathcal{O}(x) \rangle_{\lambda_{eff}}$

$\sum_{\text{trees}}$



$\lambda_1 \lambda_1 \langle \mathcal{O}(x) \mathcal{O} \mathcal{O} \mathcal{O} \rangle \lambda_2 \langle \mathcal{O} \mathcal{O} \rangle \lambda_1$

$$\ln Z(\{\lambda_1, \lambda_2\}) = \ln Z(\lambda_{eff}) - \int dx_1\, dx_2\, \lambda_{12}(x_1, x_2) \langle \mathcal{O}(x_1) \rangle_{\lambda_{eff}} \langle \mathcal{O}(x_2) \rangle_{\lambda_{eff}}$$

$$\frac{\delta \ln Z(\{\lambda_1, \lambda_2\})}{\delta \lambda_1} = \langle \mathcal{O} \rangle_{\lambda_1, \lambda_2} = \langle \mathcal{O} \rangle_{\lambda_{eff}} = \frac{\delta \ln Z(\lambda_{eff})}{\delta \lambda_{eff}}$$

3) $S(\{\lambda_1, \lambda_2\}) = -\frac{d}{dn}\left(\ln Z_n(\{\lambda_1, \lambda_2\}) - n \ln Z(\{\lambda_1, \lambda_2\})\right)\Big|_{n=1}$

$\| $

$S(\lambda_{eff})$

$$-\int dx_1 dx_2\, \lambda_2(x_1, x_2) \frac{d\langle \mathcal{O}\mathcal{O}\rangle_{\lambda_{eff}}}{dn} \langle \mathcal{O}(x_2)\rangle_{\lambda_{eff}}$$

$$\underbrace{\frac{\delta \ln Z(\lambda_{eff})}{\delta \lambda_{eff}}}_{\langle \mathcal{O}\rangle_{\lambda_{eff}}} \frac{d\lambda_{eff}}{dn}$$

$$\phi(x,z) \sim z^{d-\Delta}\left(\alpha(x) + O(z^2)\right) + z^{\Delta}\left(\beta(x) + O(z^2)\right)$$

$$= S + \int dx\, W[\Phi]$$

$$\alpha(x) = \frac{\delta W[\beta]}{\delta \beta}$$

$$z) \sim z^{d-\Delta}\left(\alpha(x) + O(z^2)\right) + z^{\Delta}\left(\overset{\langle \mathcal{O} \rangle}{\beta(x)} + O(z^2)\right)$$

$$= S + \int dx\, W[\mathcal{O}]$$

$$\alpha(x) = \frac{\delta W[\beta]}{\delta \beta}$$

rel entropy

$$\delta^{(2)} S = \iint \lambda(\tau_1,\tau_2)\lambda(\tau_3,\tau_4) \int_{-\infty+ib}^{\infty+ib} \frac{ds}{\sinh(s/2)} \langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\mathcal{O}(\tau_3+is)\,\mathcal{O}(\tau_4+is)\rangle$$

$$|\tau_1-\tau_2| + |\tau_3-\tau_4| > 2\pi$$

order Λ

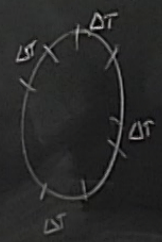
$\Delta\tau$

$$\tau_1, \tau_2)\,\lambda(\tau_3,\tau_4)\int_{-\infty+ib}^{\infty+ib}\frac{ds}{\sinh^4(s/2)}\,\langle T\{O(\tau_1)\,O(\tau_2)\}\,T\{O(\tau_3+is)\,O(\tau_4+is)\}\rangle$$

$$|\tau_1-\tau_2|+|\tau_3-\tau_4|>2\pi$$

order n

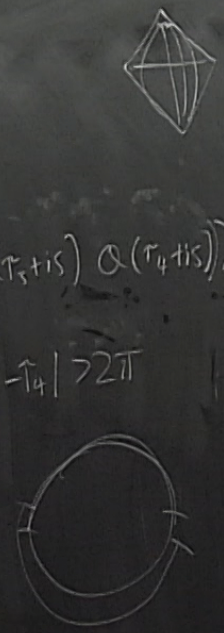
$\Delta\tau$ $\quad \frac{\Delta\tau}{n} < 2\pi$



ntropy

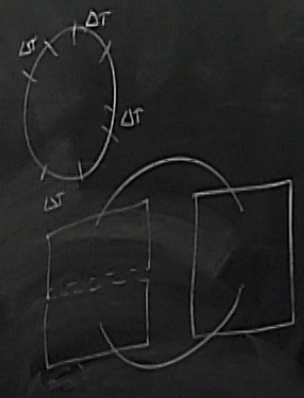
$$\iint \lambda(\tau_1,\tau_2)\lambda(\tau_3,\tau_4)\int_{-\infty+ib}^{\infty+ib}\frac{ds}{\sinh^2(s/2)}\langle T\{\mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\}\mathcal{O}(\tau_3+is)\,\mathcal{O}(\tau_4+is)\rangle$$

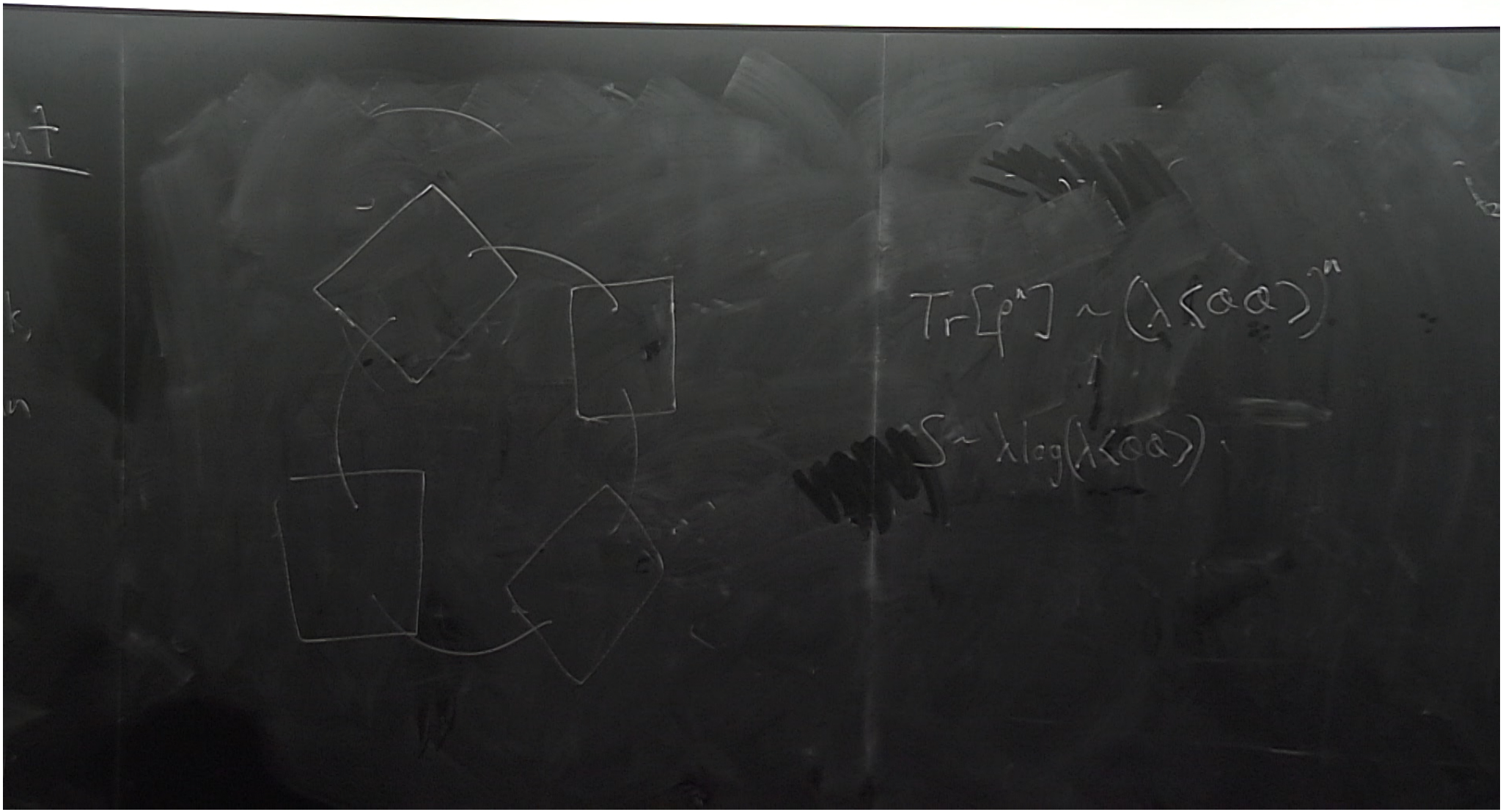
$$|\tau_1-\tau_2|+|\tau_3-\tau_4|>2\pi$$

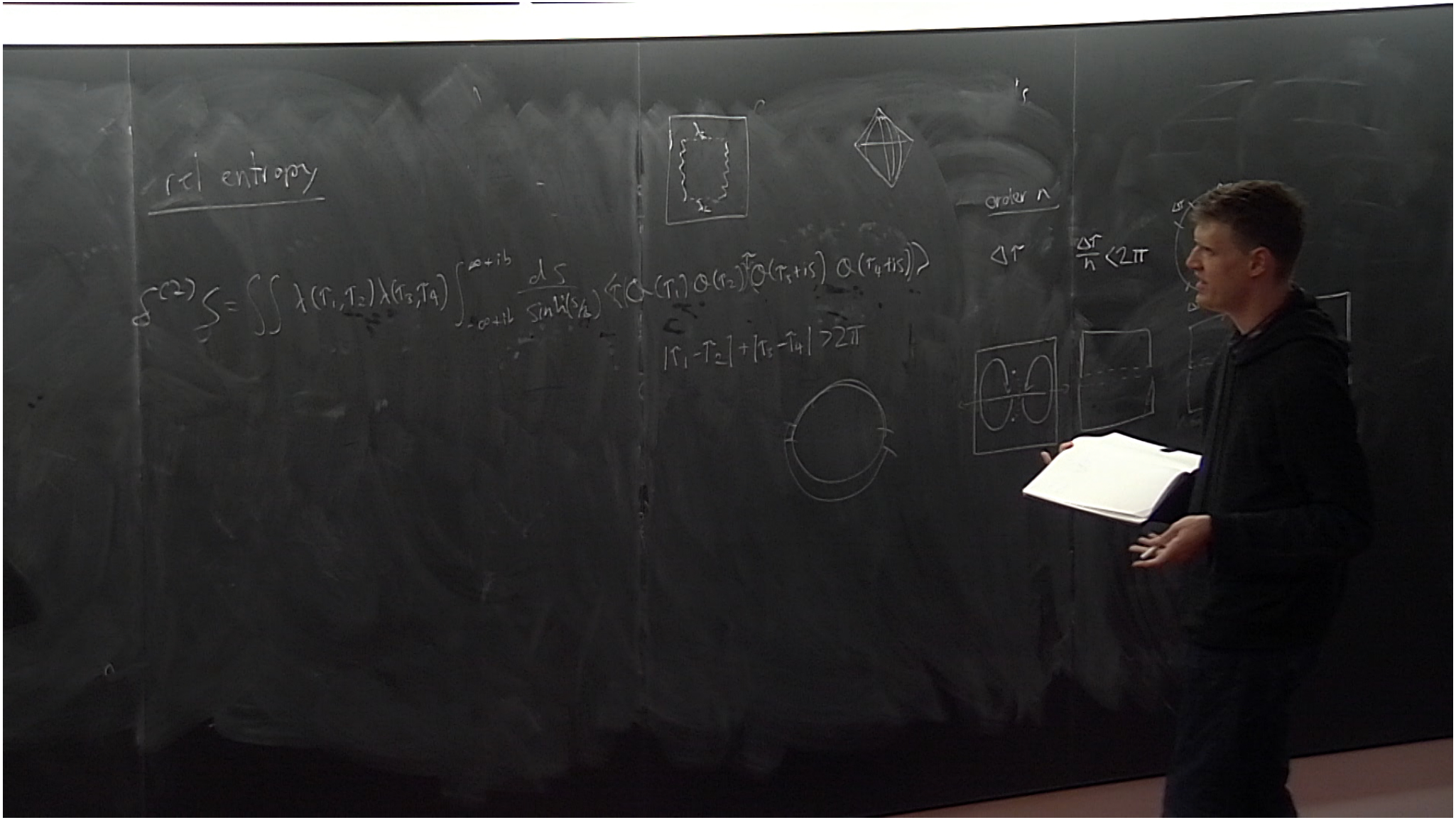


order n

$\Delta\tau$ $\quad \frac{\Delta\tau}{n}<2\pi$




$Tr[\rho^n] \sim (\lambda \langle QQ \rangle)^n$
$S \sim \lambda \log(\lambda \langle QQ \rangle)$


rel entropy
order n
$\Delta\tau$
$\frac{\Delta\tau}{n} < 2\pi$
$|\tau_1 - \tau_2| + |\tau_3 - \tau_4| > 2\pi$

nent

ack,

von

Have:

classical geo

$S_{CFT}^{rel} \sim N^0$

Want:

$|\Psi\rangle = |0\rangle + c_1 a^\dagger a^\dagger |0\rangle + c_2 a_n^\dagger |0\rangle$

s.t.

$$\delta S_{ent}(B) = \frac{\delta\langle A\rangle}{4G} + \delta S_{bulk}(|\Psi\rangle, \tilde{B}) + \int \xi^a \langle E_{ab}\rangle n^b$$

$$\mathrm{Tr}[\rho^n] \sim (\lambda \langle QQ\rangle)^n$$

$$S \sim \lambda \log(\lambda \langle QQ\rangle)$$

Prob

S