Title: Dai-Freed Anomalies in Particle Physics

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Abstract: <p>Anomaly cancellation conditions place strong constraints on many physical theories. In the traditional framework, local and global anomalies are detected by computing an eta invariant of a certain Dirac operator on a mapping torus. Recent research has uncovered the existence of finer anomaly cancellation conditions, not visible in these traditional settings. I will review the traditional and refined anomalies, and apply them to various symmetries of interest in particle physics. Examples include the (various global forms of) the Standard Model, GUT's, and discrete gauge symmetries of physical interest such as proton triality and a certain Z4 symmetry in the (MS)SM.</p>

Our motivation:

Q: Do gauge anomalies cancel in the Standard Model?

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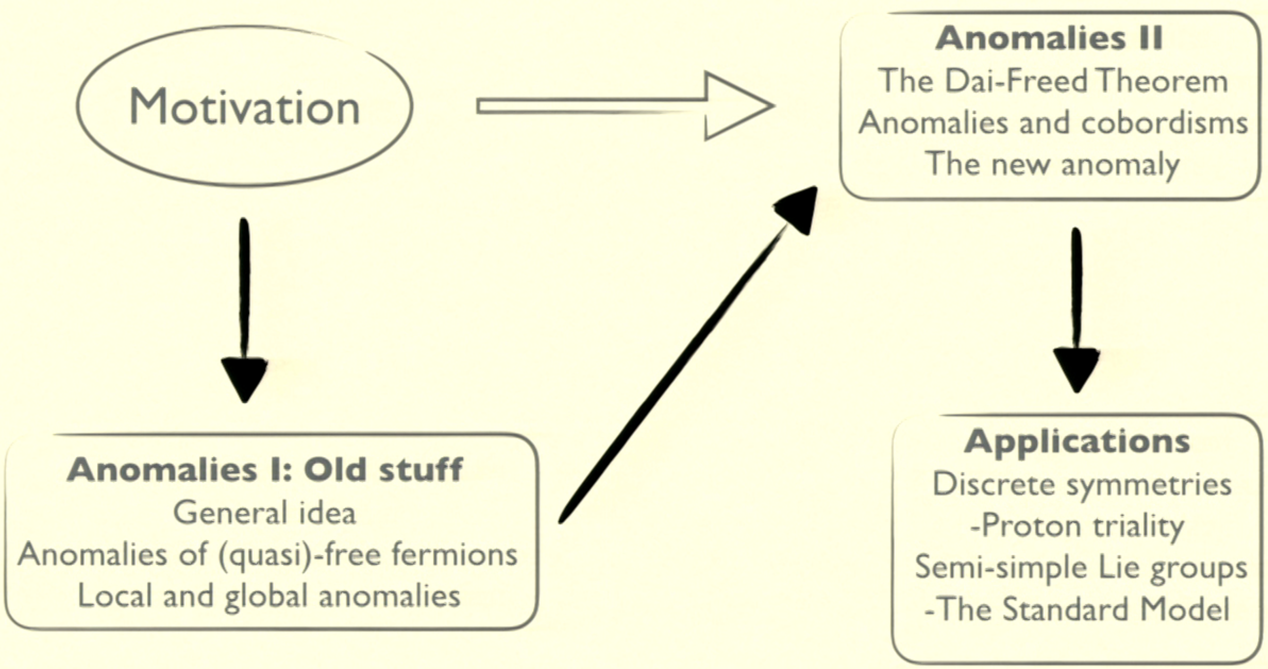
A: Maybe, but the answer is not in the literature!

Recent discovery of finer anomaly conditions.
[Kapustin-Thorngren-Turzillo-Wang '14, Hsieh-Cho-Ryu '15, Witten '15…]

We study these anomalies in pheno interesting QFT's.

PLAN OF THE TALK



ANOMALIES I

What is an anomaly?

- We will work in **Euclidean signature** and will only consider **lagrangian theories**.
- Basic object: Partition function with sources. If there is a symmetry G, we can couple to a background gauge field:

$$Z = \int \mathcal{D}\Psi \exp(-S[\Psi, A_G])$$

- There is an **anomaly** if the partition function is not gauge-invariant.

- We can **gauge the symmetry** by summing over physically inequivalent gauge fields.
- Two classes of anomalies:
  - **Local:** Anomaly in infinitesimal gauge transformation
  - **Global:** Anomaly in transformation not continuously connected to the identity.
- Rest of the talk: **Fermion anomalies**, where

$$S = \frac{i}{2} \int \bar{\Psi} \not{D} \Psi$$

- Local fermion anomalies are well understood in terms of the **Wess-Zumino** descent procedure:

$$\delta_g S = \delta_g I_{d+1}$$

$$I_{d+1} = dI_{d+2}$$

- Local anomaly cancellation in d dimensions is equivalent to vanishing of (d+2) anomaly polynomial:

$$\sum_{\text{fermions}} \left[\hat{A}(R)ch(F)\right]_{d+2} = 0$$

- Global anomalies are subtler. Let us discuss the prototype [Witten '82] : A 4d Weyl fermion in the fundamental of SU(2). Since

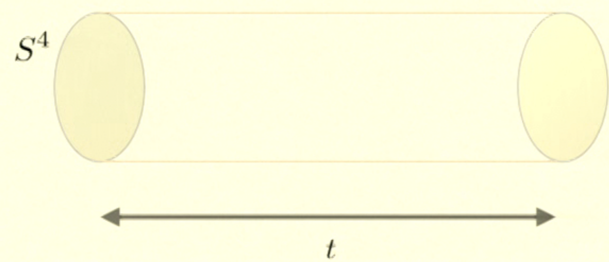
$$\pi_4(SU(2)) = \mathbb{Z}_2$$

on S^4 there is a gauge transformation not connected to the identity.

- Consider 1-parameter family of connections

$$A_t = (1-t)A_0 + tA_g$$

These can be regarded a gauge field on $S^4 \times S^1$(the **mapping torus)**



- Witten shows that there is an anomaly:

$$Z[A_g] = Z[A_0]\exp(i\pi\alpha)$$

with α (which can only be 0 or 1) given by a quantity called the **mod 2 index** of a different five-dimensional real Dirac operator defined on the mapping torus

- Anomaly computed via a **higher dimensional auxiliary theory**.

ANOMALIES II

Back to the future

- We want a **general prescription** to compute fermion anomalies. Let's just evaluate the fermion path integral (easy for free fermion; interactions don't change topology)

$$\int [D\psi] e^{-S} = \prod_{\lambda} \int d\psi_\lambda d\bar{\psi}_\lambda e^{-\lambda \bar{\psi}_\lambda \psi_\lambda} = \mathrm{Pf}(i\not{D})$$

- If the Pfaffian is not a well-defined function of the connection, there is an anomaly.

- However, it is **always** a section of the **Pfaffian line bundle**.

THE DAI-FREED THEOREM

- The **Dai-Freed** theorem [Dai-Freed '94] tells us that there is **another quantity** $e^{-2\pi i\eta}$ which is also a section of the Pfaffian line bundle!
- That means that

$$Pf(i\not{D})e^{-2\pi i\eta}$$

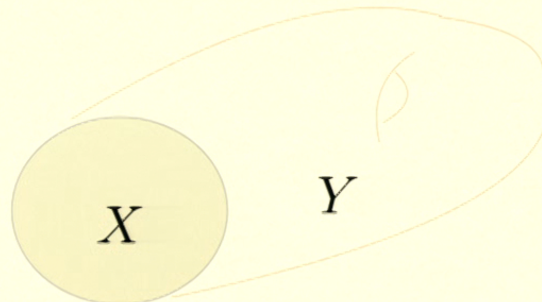
is a complex number, or equivalently, that we can write

$$Pf(i\not{D}) = |Pf(i\not{D})|e^{2\pi i\eta}$$

- The **point** is we know how to compute η .

- The **definition** of η is convoluted: If the theory is defined on manifold X, pick Y such that boundary(Y)=X

- On Y, define a **new** Dirac operator D_X, related to the one on X in a particular way [Witten '85, Witten '15, Yonekura '16]



- Put (generalized) **APS boundary conditions** so that D_X is **self-adjoint** [Atiyah-Patodi-Singer '75, Yonekura '16]

- Then η is the **APS-η invariant of** D_X

$$\eta \equiv \frac{1}{2}\left(\sum_{\lambda\neq 0} \mathrm{sign}(\lambda) + \mathrm{Dim}\ \mathrm{Ker}(i\not{D}_X)\right)_{\mathrm{reg.}}$$

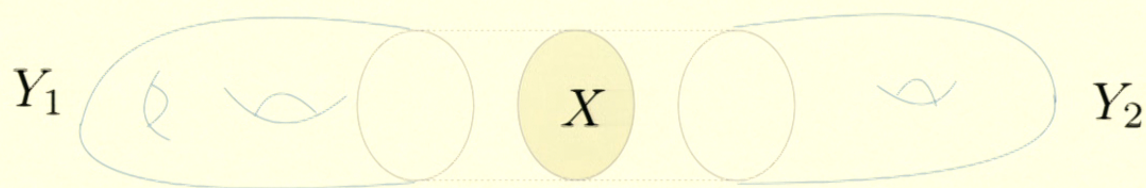
- In this language: **Anomaly = dependence on the choice of Y**

- Familiar story: Defining a CS interaction on X using an auxiliary manifold Y.

$$\int_{X=\partial Y} F \wedge A = \int_Y F \wedge F$$

The theory is well-defined **if it is trivial on a closed 4-manifold**

$$\int_{Y_1} F \wedge F - \int_{Y_2} F \wedge F = \int_{Y_1 \cup Y_2} F \wedge F \in \mathbb{Z}$$



- Similarly, anomaly free means that

$$\exp(2\pi i \eta_{Y'}) = 1$$

  on any "allowed" closed (d+1)-manifold Y (η behaves nicely under gluing)

- Two classes of Y's are particularly interesting:
  - Y's that are themselves boundaries of (d+2)-dimensional Z
  - Mapping tori X x S^1

- If Y is the boundary of some Z, we can use the APS index theorem

$$\mathrm{Ind}(\not{D}_Z) = \eta_Y + \int_Z \hat{A}(R)ch(F)$$

which means that

$$\exp(2\pi i \eta_Y) = \exp\left(2\pi i \int_Z I_{d+2}\right)$$

In other words, requiring no anomalies for these Y is equivalent to **local anomaly cancellation.**

- Once local anomalies cancel, η is a topological invariant.

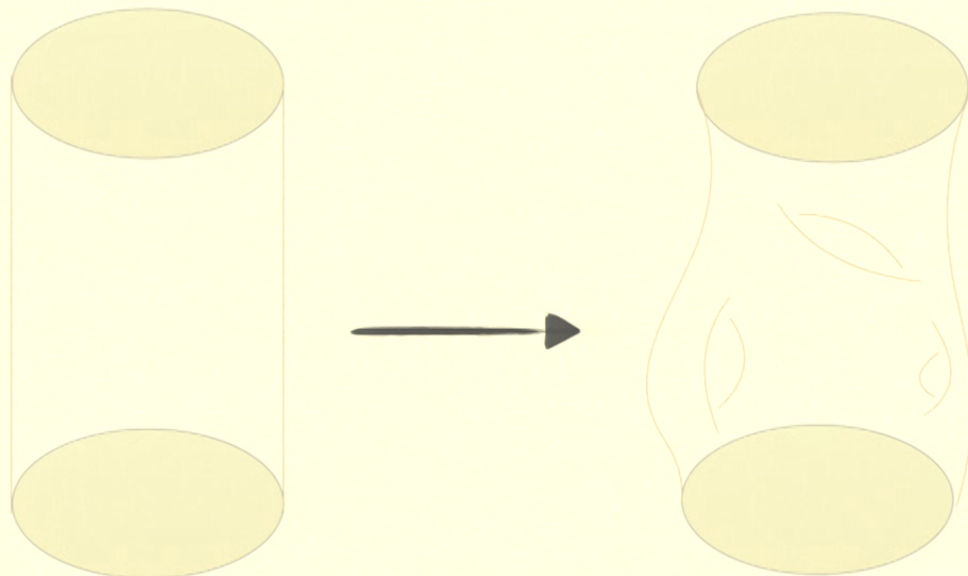
- If Y is a mapping torus for a gauge transformation not connected to the identity, then $\exp(2\pi i\, \eta_Y)=1$ means absence of the associated global anomaly.
- What happens if we demand $\exp(2\pi i\, \eta_Y)=1$ for **any Y**?

We get additional constraints.

- Original example: 3d Majorana fermions on non-orientable manifolds (top. superconductor). Mapping torus sees mod 8 anomaly; η of RP^4 sees a **mod 16.**

- Why should this "anomaly" cancel?
    - Consistency with gluing and pasting & reflection positivity [Witten '15+…]

- Another reason: In gravity, topology can change!



- Allowing topology change produces new nontrivial "closed paths" in configuration space.

- Just one caveat to keep in mind: **Green-Schwarz mechanism.**
- Local anomalies: Add extra light degrees of freedom, and anomalous spectrum is OK
- Global anomalies [Garcia-Etxebarria-Hayashi-Ohmori-Tachikawa-Yonekura '17]: Couple to tQFT which either
    - Has the same anomaly as the anomalous fermions.
    - Forbids the anomalous background
    - **No** extra local degrees of freedom

APPLICATIONS

Dai-Freed anomalies have only been studied in a few systems.

A priori, **any** gauge theory could be Dai-Freed anomalous!

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A priori, **any** gauge theory could be Dai-Freed anomalous!

Rest of talk: Apply **Dai-Freed** to symmetries of interest in **particle physics**.

Is the Standard Model Dai-Freed anomalous?

- Once local anomalies cancel, η is a **bordism invariant:**

$$\exp(2\pi i \eta_{Y_1}) = \exp(2\pi i \eta_{Y_2})$$

- **Bordism** is an equivalence relation, which defines **bordism groups**

$$\Omega^{Spin}_{d+1}(BG)$$

These classify (d+1)-dimensional manifolds, with a principal G-bundle, modulo bordism (bundle extends over bordism too)

- Computed using **AHSS**.

- η is a **group homomorphism** from the relevant bord. group to U(1).

GENERAL STRATEGY

- Compute relevant bordism group
    - If it vanishes, there is no new anomaly.
- Find a nontrivial manifold Y, compute η.
    - If it vanishes, there is no new anomaly.
- If η is nonvanishing, there is an anomaly.

SEMISIMPLE LIE GROUPS

$\mathbf{G}$	$\mathbf{\Omega_d^{\text{Spin}}(BG)}$								
	0	1	2	3	4	5	6	7	8
$SU(2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$4\mathbb{Z}$
$SU(n>2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
$USp(2k>2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$5\mathbb{Z}$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}$	0	$2\mathbb{Z}$	0	–	–	–
$PSU(2^k)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{2^k}$	0	–	–	–	–	–
$PSU(p^k, p \text{ odd})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{p^k}$	0	$2\mathbb{Z}$	0	–	–	–
$Spin(n \geq 8)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
$SO(3)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$e(\mathbb{Z}_2, \mathbb{Z}_2)$	0	$2\mathbb{Z}$	0	–	–	–
$SO(n>3)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$e(\mathbb{Z}_2, \mathbb{Z}_2)$	0	$e(\mathbb{Z}, \mathbb{Z} \oplus \mathbb{Z}_2)$	0	–	–	–
E_6, E_7, E_8	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
G_2	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
F_4	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0	–

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$USp(2k>2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$5\mathbb{Z}$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}$	0	$2\mathbb{Z}$	0	–	–	–
$PSU(2^k)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{2^k}$	0	–	–	–	–	–
$PSU(p^k, p \text{ odd})$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{p^k}$	0	$2\mathbb{Z}$	0	–	–	–
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G_2	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
F_4	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0	–

THE STANDARD MODEL

- Experiments only probe the gauge **algebra** of the SM. There are four possibilities [Tong '17…]

$$\frac{SU(3) \times SU(2) \times U(1)}{\Gamma}, \quad \Gamma \in \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$$

- The discrete group Γ acts trivially on the SM fermions.
- $\Gamma = Z_6$ is "maximal": Includes bundles for any other choice of Γ.
    - This is also the group that embeds in SU(5).

- The SM fermion spectrum falls into **SU(5) representations.**
- Any (**SU(3) x SU(2) x U(1))/Z**$_6$ bundle is a **SU(5) bundle** too!
- As far as anomalies are concerned, the SM is **equivalent** to the SU(5) GUT. But since

$$\Omega_5^{Spin}(BSU(5)) = 0$$

The SM is free of Dai-Freed anomalies

- Similar situation for Spin(10).

- To get an anomaly, we need to look at **more general** spaces.
- What about the SM in **non-orientable spaces?**
  - Only makes sense if one assumes CP breaking in SM is **spontaneous.**
  - Need a Pin structure to define fermions, which can change cob. groups, e.g. $\Omega_6^{Pin-}=Z_{16}$, but $\Omega_6^{Spin}=0$.
  - Majorana masses require a Pin^+ structure [Berg et al '00].
- We have again

$$\Omega_5^{Pin^+}(BSU(5)) = \Omega_5^{Pin^-}(BSU(5)) = 0$$

- Last try: SM + right-handed neutrinos + gauged (B-L).
- Since all fermion charges under (B-L) are odd, we can now consider the SM on **Spin^c manifolds** or even **Pin^c manifolds**

DISCRETE CYCLIC GROUPS

- Lucky! Bordism groups & η invariants already computed by mathematicians [Bahri-Gilkey ' 87, Gilkey '89, Gilkey-Botvinnik '94] both for Spin and Spin^c cases. They are **nontrivial.**

- We can compare with **known anomalies** of discrete symmetries. [Ibañez-Ross' 91]. These were originally obtained by demanding that the Z_n embeds in a U(1).

$$2\sum s_i \equiv 0 \bmod N, \quad \sum s_i^3 \equiv 0 \bmod N$$

- Only **linear** constraints are "UV-independent" [Banks-Dine' 91]

- For Z_3,

$$\sum s_i \equiv 0 \mod 3 \qquad \text{(Linear IR)}$$
$$\sum s_i \equiv 0 \mod 9 \qquad \text{(Dai-Freed)}$$

This one has phenomenological consequences: Proton triality (and also hexality) in the MSSM is a IR-anomaly free Z_3 symmetry, but it has a mod 9 anomaly

Q	u	d	l	e	H	$\bar{H}$
0	-1	1	-1	-1	1	-1
0	-2	-5	-5	1	5	5

$$\sum_{MSSM} s_i = 3 \mod 9$$

- Dai-Freed anomaly cancellation requires 3k generations.

- Consistent with previous results [Dreiner et al. '04]: U(1) embedding of proton triality only with gen. dependent charges.

SM = TOP. SUPERCONDUCTOR

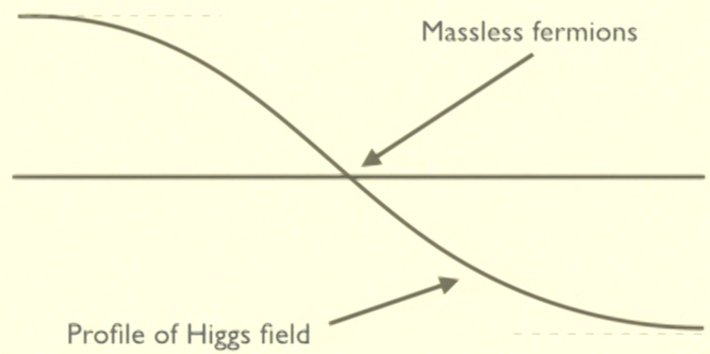
- **Topological superconductor**: 1st example of Dai-Freed anomaly [Kapustin-Thorgren-Turzillo-Wang '14, Witten 015, Hsieh-Cho-Ryu '16]
  - T-invariant 3d fermions. Global grav. anomaly requires multiple of 8.
  - Dai-Freed enhances to a multiple of 16, because

$$\Omega_4^{Pin^+} = \mathbb{Z}_{16}$$

- # of fermions/ generation in SM + rh neutrinos = 16. **Not a coincidence!**

- The SM + rh neutrinos has a Z_4 symmetry (center of Spin(10)) that acts on every fermion by multiplication by i.
- We can use this to put the SM on manifolds with a $Spin^{\mathbb{Z}_4}$ structure [Tachikawa-Yonekura '18]. Transition functions of the spinors in (Spin x Z_4)/Z_2
- The **Smith homomorphism** maps

$$\Omega_{d+1}^{Spin^{\mathbb{Z}_4}} \to \Omega_d^{Pin^+}$$



- Physical interpretation: Higgsing the Z_4 w. a nontrivial bundle, there is a 3d locus with massless Pin^+ fermions.

CONCLUSIONS

- We've explored a new kind of anomaly in four dimensional gauge theories of phenomenological interest.
- SM and GUT's are anomaly free. Can put SM on non-Spin manifolds (related to topological superconductor).
- New anomalies for discrete symmetries e.g. proton triality.
- Top. GS means these anomalies can be cancelled; still good for 't Hooft anomaly matching
- Outlook
    - Dai-Freed anomalies with twisted spin structures [Wang-Wen-Witten '18]

Thank you!