Title: Exact Strong-ETH Violating Eigenstates in the Rydberg-blockaded Atom Chain

Date: Nov 19, 2018 03:30 PM

URL: http://pirsa.org/18110072

Abstract: A recent experiment in the Rydberg atom chain observed unusual oscillatory quench dynamics with a charge density wave initial state, and theoretical works identified a set of many-body ``scar states" in the Hamiltonian as potentially responsible for the atypical dynamics. In the same nonintegrable Hamiltonian, we discover several eigenstates at infinite temperature that can be represented exactly as matrix product states with finite bond dimension, for both periodic boundary conditions (two exact E = 0 states) and open boundary conditions (two E = 0 states and one each E =Â± âˆš2). This discovery explicitly demonstrates violation of strong eigenstate thermalization hypothesis in this model. These states show signatures of translational symmetry breaking with period-2 bond-centered pattern, despite being in 1D at infinite temperature. We show that the nearby many-body scar states with energies E ~Â± 1.33 andE ~ Â± 2.66 can be well approximated as ``quasiparticle excitations" on top of our exact E = 0 states, and propose a quasiparticle explanation of the strong oscillations observed in experiments.

Exact Strong-ETH Violating Eigenstates in the Rydberg-blockaded Atom Chain

Cheng-Ju Lin, Olexei I. Motrunich

Nov. 19. 2018
arXiv: 1810.00888

Caltech IQIM

How can an isolated quantum system become thermal?

- **Prepare a pure initial state** $|\Psi_0\rangle$ **Unitary evolution** $U = e^{-iHt}$
- **Quantum dynamics** $\hat{O}$: Local observable

$$\langle\Psi_0|e^{iHt}\hat{O}e^{-iHt}|\Psi_0\rangle = \sum_E \underbrace{|c_E|^2\langle E|\hat{O}|E\rangle}_{\text{Equilibration}} + \sum_{E_1\neq E_2} \underbrace{c_{E_1}^* c_{E_2}\langle E_1|\hat{O}|E_2\rangle e^{i(E_1-E_2)t}}_{\text{Dynamics}}$$

$c_E \equiv \langle E|\Psi_0\rangle$ $|E\rangle$: **eigenstate of H**

- **Assuming in the long-time, the off-diagonal part dephases, and system equilibrates** (or take long-time average)

Thermalization: $$\sum_E |c_E|^2\langle E|\hat{O}|E\rangle \overset{?}{=} \frac{1}{Z}\mathbf{Tr}[\rho\hat{O}]$$

$Z = \mathbf{Tr}[\rho]$

ρ : Thermal ensemble

$$\rho_{MC} = \frac{1}{N}\sum_{E\in I}|E\rangle\langle E|,$$

$$\rho_C = \frac{1}{Z}\sum_E e^{-\beta E}|E\rangle\langle E|$$

Eigenstate Thermalization Hypothesis

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$Z = \mathbf{Tr}[\rho]$

ρ : Thermal ensemble

- **(Diagonal) eigenstate thermalization hypothesis (ETH):**

  $\langle E|\hat{O}|E\rangle \to O(E)$ **converges to a smooth function of E in thermodynamic limit, therefore giving the same expectation value for all ensembles at the same energy density.**

  In particular, $\rho_0 = \sum_E |C_E|^2 |E\rangle\langle E|, \quad \rho_{MC} = \frac{1}{N}\sum_{E\in I} |E\rangle\langle E|, \quad \rho_C = \frac{1}{Z}\sum_E e^{-\beta E}|E\rangle\langle E|$

  all give the same expectation value. (Deutsch '91, Srednicki '93)

- **Strong-ETH:** $\longleftrightarrow$ ***All*** **initial states thermalize**

  Take the extreme: $|C_E|^2 = \delta_{E,E^*}$ **(Initial state is an eigenstate** $|E^*\rangle$**.)**

  $\langle E^*|\hat{O}|E^*\rangle = \frac{1}{Z}\mathbf{Tr}[\rho\hat{O}]$ **(*every* eigenstate satisfy ETH)**

- **Some consequences: eigenstates at nearby energies are locally indistinguishable; volume-law entanglement scaling in subsystem size.**

Hyungwon Kim, Tatsuhiko N. Ikeda, and David A. Huse, PRE **90**, 052105 (2014)
James R. Garrison and Tarun Grover, PRX **8**, 021026 (2018)

A Little More About the Dynamics

- **If we only probe the eigenstates locally, they behave as drawing from some random matrix ensemble.**

$$\langle \Psi_0 | e^{iHt} \hat{O} e^{-iHt} | \Psi_0 \rangle = \sum_E |c_E|^2 \langle E | \hat{O} | E \rangle + \sum_{E_1 \neq E_2} \underbrace{c^*_{E_1} c_{E_2} \langle E_1 | \hat{O} | E_2 \rangle e^{i(E_1 - E_2)t}}_{\text{Dynamics}}$$

$\langle E_1 | \hat{O} | E_2 \rangle$ **behave as some "random number"**

No frequency (E_1-E_2) is special.

- **Off-diagonal ETH** $\langle E_\alpha | \hat{O} | E_\beta \rangle = O(E)\delta_{\alpha\beta} + e^{-S(E)/2} f(E, \omega) R_{\alpha\beta}$

$S(E)$: **Thermodynamic entropy at** $E = (E_\alpha + E_\beta)/2$

$f(E, \omega)$: **smooth function** $\omega = E_\alpha - E_\beta$

$R_{\alpha\beta}$: **random number with zero mean and unit variance**

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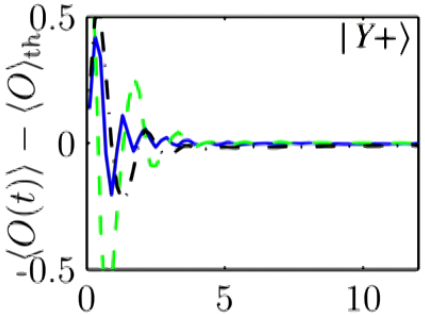
Examples of atypical quench dynamics

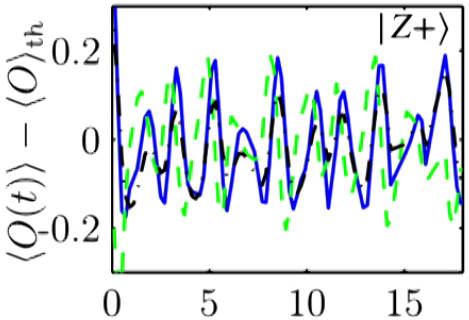
$$H = -J\sum_j Z_j Z_{j+1} - h_x \sum_j X_j - h_z \sum_j Z_j$$

- "Weak thermalization" (J=1, h_x=-1.05, h_z=0.5)

M. C. Bañuls, J. I. Cirac, and M. B. Hastings, PRL 106, 050405 (2011)



Strong thermalization: thermalizes quickly



Weak thermalization: Strong oscillations

$\langle\sigma_x\rangle$ (blue solid line)

$\langle\sigma_y\rangle$ (dashed green line)

$\langle\sigma_z\rangle$ (dash-dotted black line)

Due to quasiparticles in the low-energy part of spectrum

(C.-J. Lin and O. I, Motrunich, Phys. Rev. A **95**, 023621 (2017))

- Oscillatory dynamics due to "mesons" (confined domain walls), h_x<1, small h_z.

M. Kormos, M. Collura, G. Takács and P. Calabrese, Nature Physics **13** 246–249 (2017)

A. J. A. James, R. M. Konik and N. J. Robinson arXiv: 1804.09990

N. J. Robinson, A. J. A. James and R. M. Konik, arXiv: 1808.10782

“Thermometer” of systems

Strong-ETH: all eigenstate are thermal

In between? Many proposals, asymptotic many-body localization, quantum disentangled liquid ……

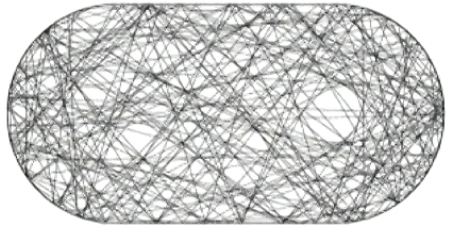
- Recent discoveries:
  Weak ergodicity breaking/ Quantum many-body scar states

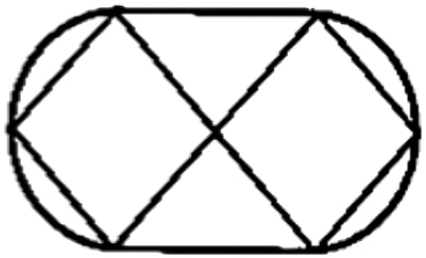
- For now:
  many-body scar states = eigenstates in the middle of many-body spectrum that are nonthermal; while other eigenstates are thermal

Many-body localization, integrable models: all eigenstates are nonthermal

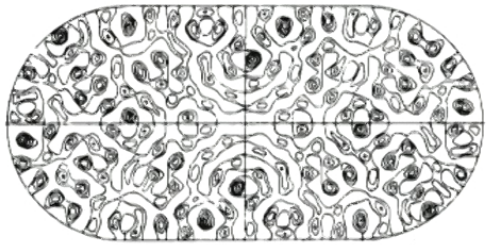
Single particle scar states

Classical particle:

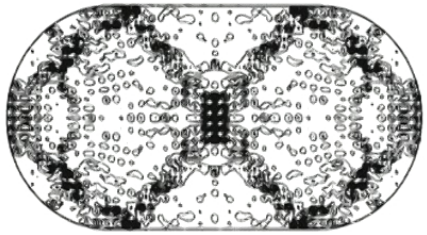




Quantum particle: $\nabla^2\psi(x,y) = \lambda\psi(x,y)$



Most of the eigen wavefunctions are "ergodic".



Some wavefunctions are concentrated around the closed orbit - scar wavefunction

What about in the many-body systems?

Analytical Results of Strong-ETH Violation

- Exact eigenstates in AKLT model (Nonintegrable):
  S. Moudgalya, S. Rachel, B. A. Bernevig, and N. Regnault, arXiv:1708.05021
  S. Moudgalya, N. Regnault, and B. A. Bernevig, arXiv:1806.09624

  - Constructed a tower of exact eigenstates

  - Showed logarithmic entanglement scaling at finite energy density - violating strong ETH

Nonthermal eigenstates embedded in the many-body spectrum - quantum many-body scar states.

ARTICLE

doi:10.1038/nature24622

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien[1], Sylvain Schwartz[1,2], Alexander Keesling[1], Harry Levine[1], Ahmed Omran[1], Hannes Pichler[1,3], Soonwon Choi[1], Alexander S. Zibrov[1], Manuel Endres[4], Markus Greiner[1], Vladan Vuletić[2] & Mikhail D. Lukin[1]

Controllable, coherent many-body systems can provide insights into the fundamental properties of quantum matter, enable the realization of new quantum phases and could ultimately lead to computational systems that outperform existing computers based on classical approaches. Here we demonstrate a method for creating controlled many-body quantum matter that combines deterministically prepared, reconfigurable arrays of individually trapped cold atoms with strong, coherent interactions enabled by excitation to Rydberg states. We realize a programmable Ising-type quantum spin model with tunable interactions and system sizes of up to 51 qubits. Within this model, we observe phase transitions into spatially ordered states that break various discrete symmetries, verify the high-fidelity preparation of these states and investigate the dynamics across the phase transition in large arrays of atoms. In particular, we observe robust many-body dynamics corresponding to persistent oscillations of the order after a rapid quantum quench that results from a sudden transition across the phase boundary. Our method provides a way of exploring many-body phenomena on a programmable quantum simulator and could enable realizations of new quantum algorithms.

Rydberg Atom Chain

● ○ ● ● ○ ● ● ●

$$H_R = \frac{1}{2}\Omega \sum_j X_j - \Delta \sum_j n_j + \sum_{i<j} V_{ij} n_i n_j$$

$$V_{ij} = \frac{C}{|i-j|^6}$$

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$|0\rangle$:Atomic ground state

$|1\rangle$:Rydberg excitation

Nearest-Neighbor Rydberg Blockade

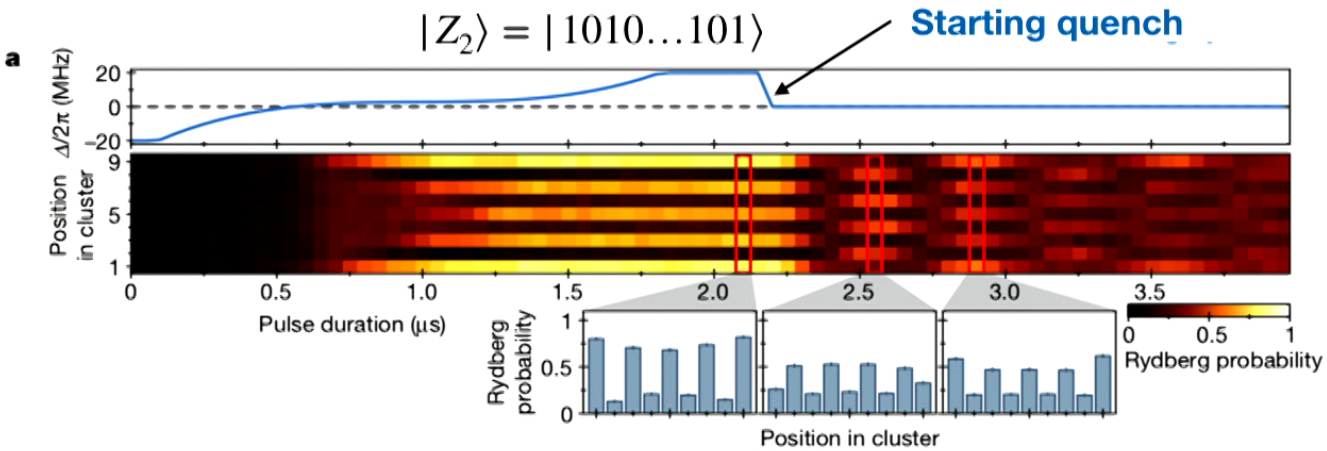
Constraint Hilbert space $\mathscr{H}$: **No** $|\ldots 11 \ldots\rangle$

$$H = \frac{1}{2}\Omega \sum_j X_j - \Delta \sum_j n_j$$

Fun facts: **dim**$(\mathscr{H}) \sim \phi^L$ $\phi = \frac{1+\sqrt{5}}{2}$ **golden ratio**

1-1 correspondence to the fusion tree of Fibonacci anyons.

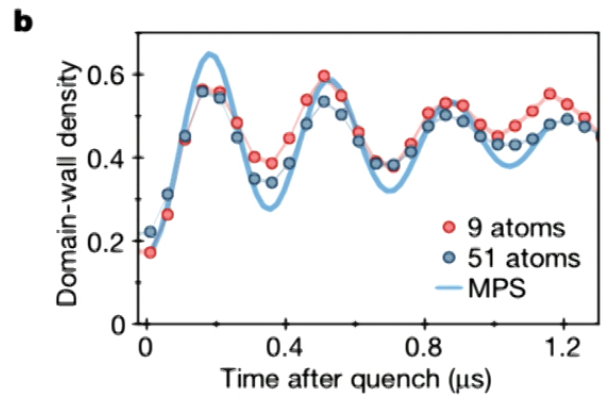
Quench Dynamics



$$H_c = \frac{1}{2}\Omega \sum_j X_j$$

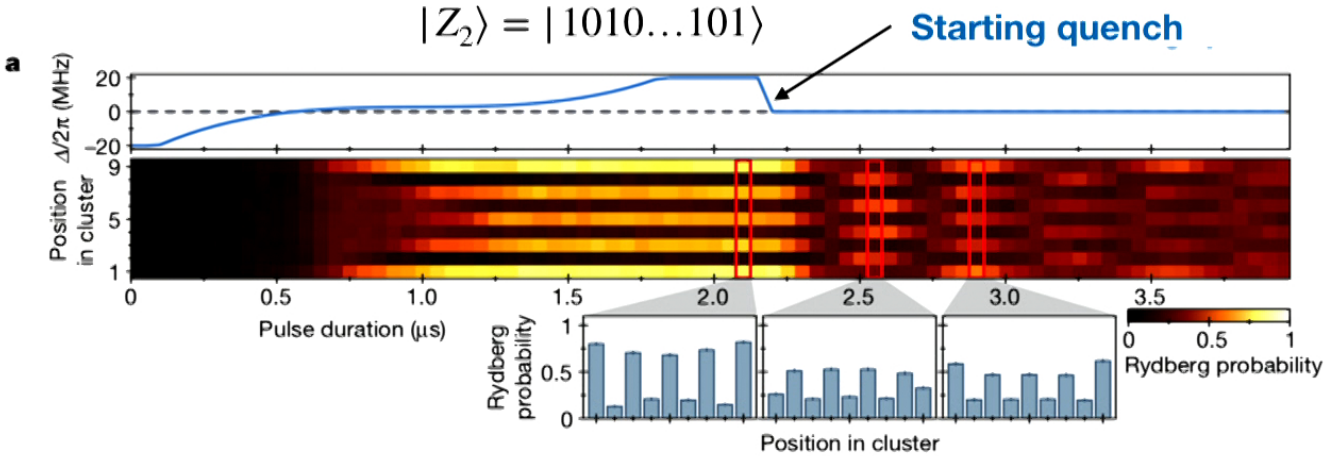
Domain-wall number

$$D_{j,j+1} = P_j P_{j+1} \qquad P = |0\rangle\langle 0|$$



Quench Dynamics

$$|Z_2\rangle = |1010\ldots101\rangle$$



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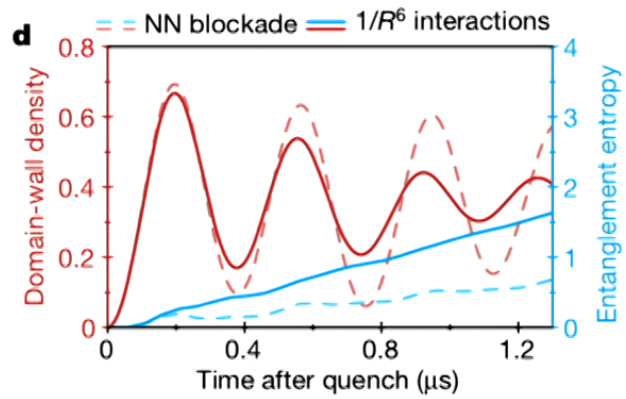
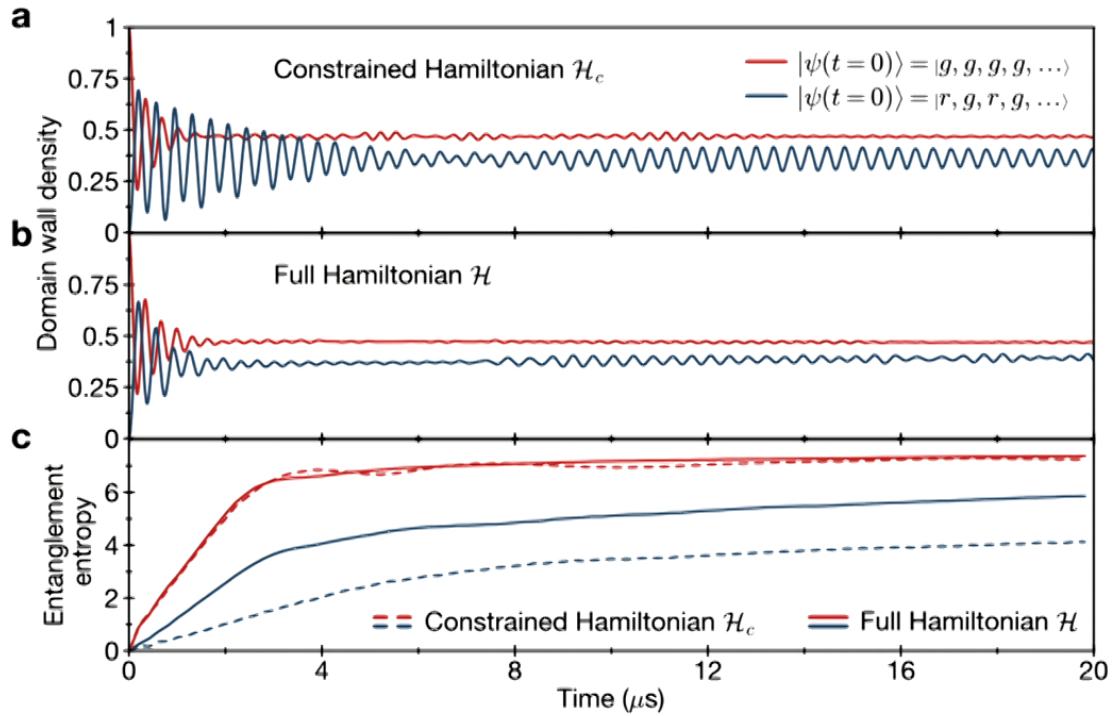


Comparing Two Initial States

$|Z_2\rangle = |1010\ldots101\rangle \qquad |\mathbf{O}\rangle = |000\ldots0\rangle$

These two states are at the same energy density, but different thermalization behavior!



MPS calculations

Some Properties of the PXP model

$$H = \sum_j P_{j-1} X_j P_{j+1}$$

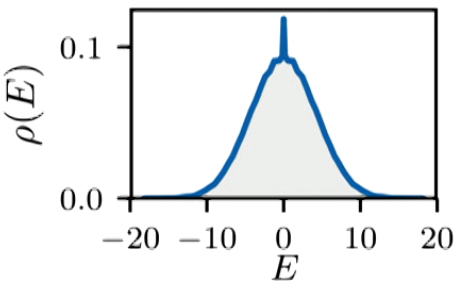
In the constraint Hilbert space $P = |0\rangle\langle 0|$

- Inversion symmetry: $I : j \to L - j + 1$

- Particle-hole transformation:

$$\mathscr{C} = \prod_j Z_j \qquad \mathscr{C} H \mathscr{C}^{-1} = -H \qquad Z = |1\rangle\langle 1| - |0\rangle\langle 0|$$

- Exponentially-many zero-energy states
  (See also Schecter and Iadecola,
  Phys. Rev. B **98** (2018) 035139)



- Level-spacing statistics indicates nonintegrable;
  Khemani *et. al. suggests could be a deformation from some integrable model (arXiv:1807.02108)*

Quantum Dynamics

- **Prepare a pure initial state** $|\Psi_0\rangle$ **Unitary evolution** $U = e^{-iHt}$

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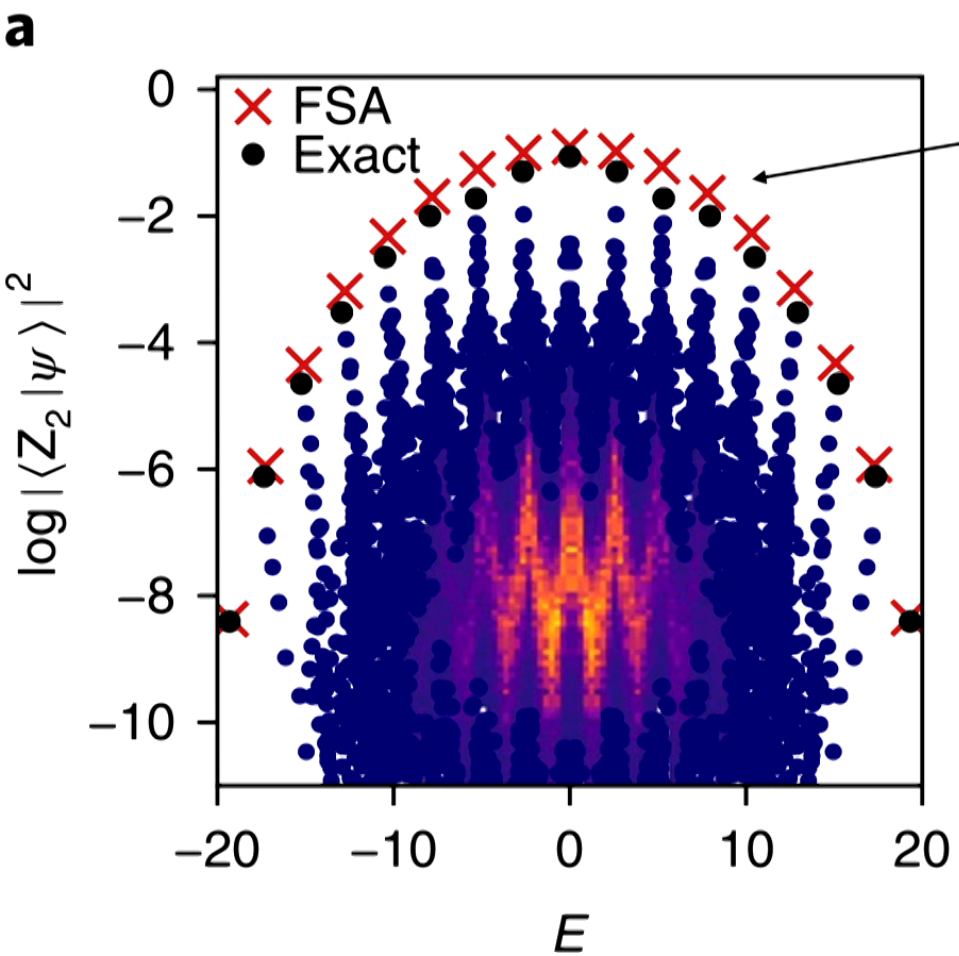
$c_E \equiv \langle E|\Psi_0\rangle$ $|E\rangle$: **eigenstate of H**

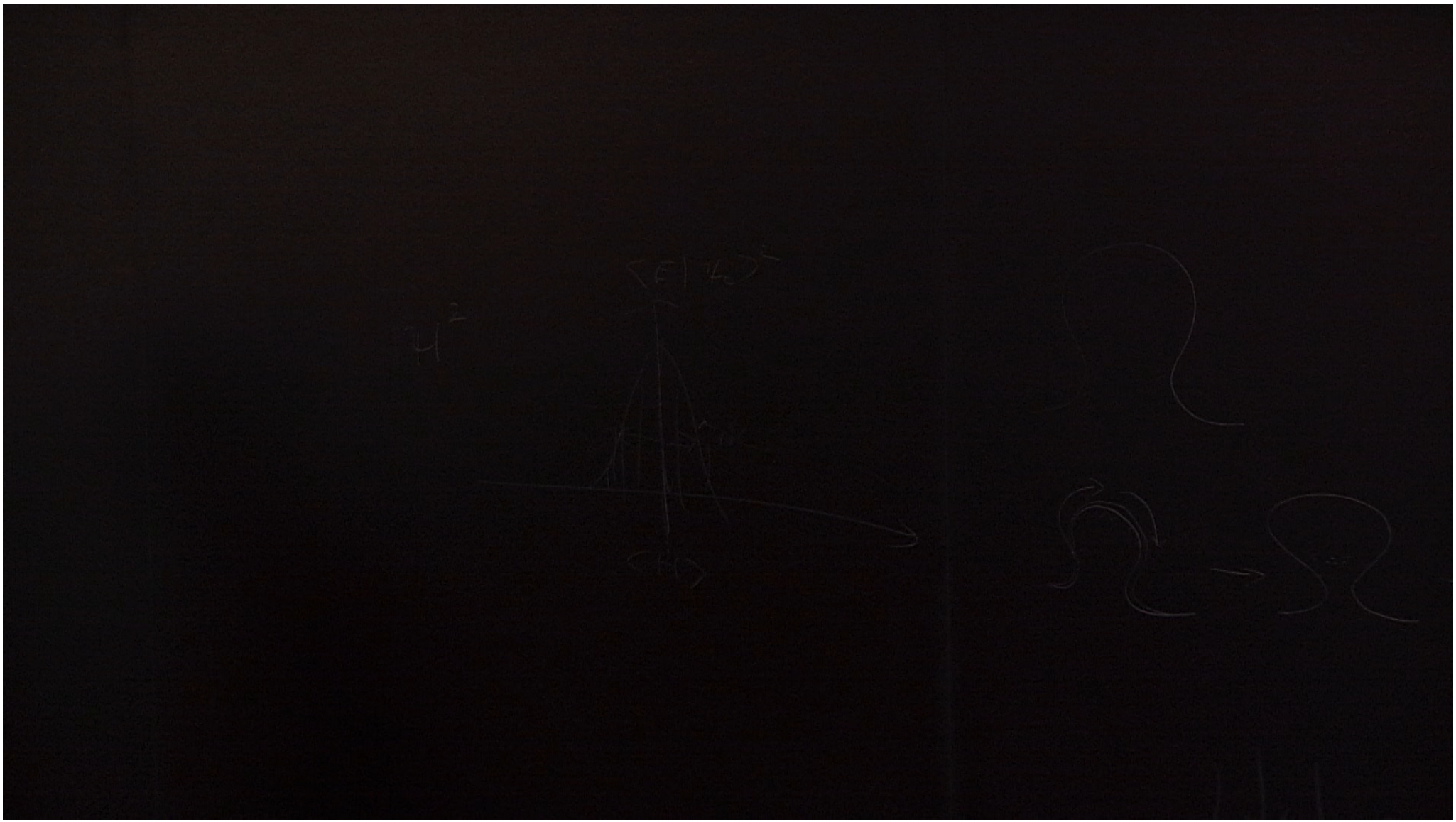
Expanded in eigenstates of $H = \sum_j P_{j-1}X_jP_{j+1}$

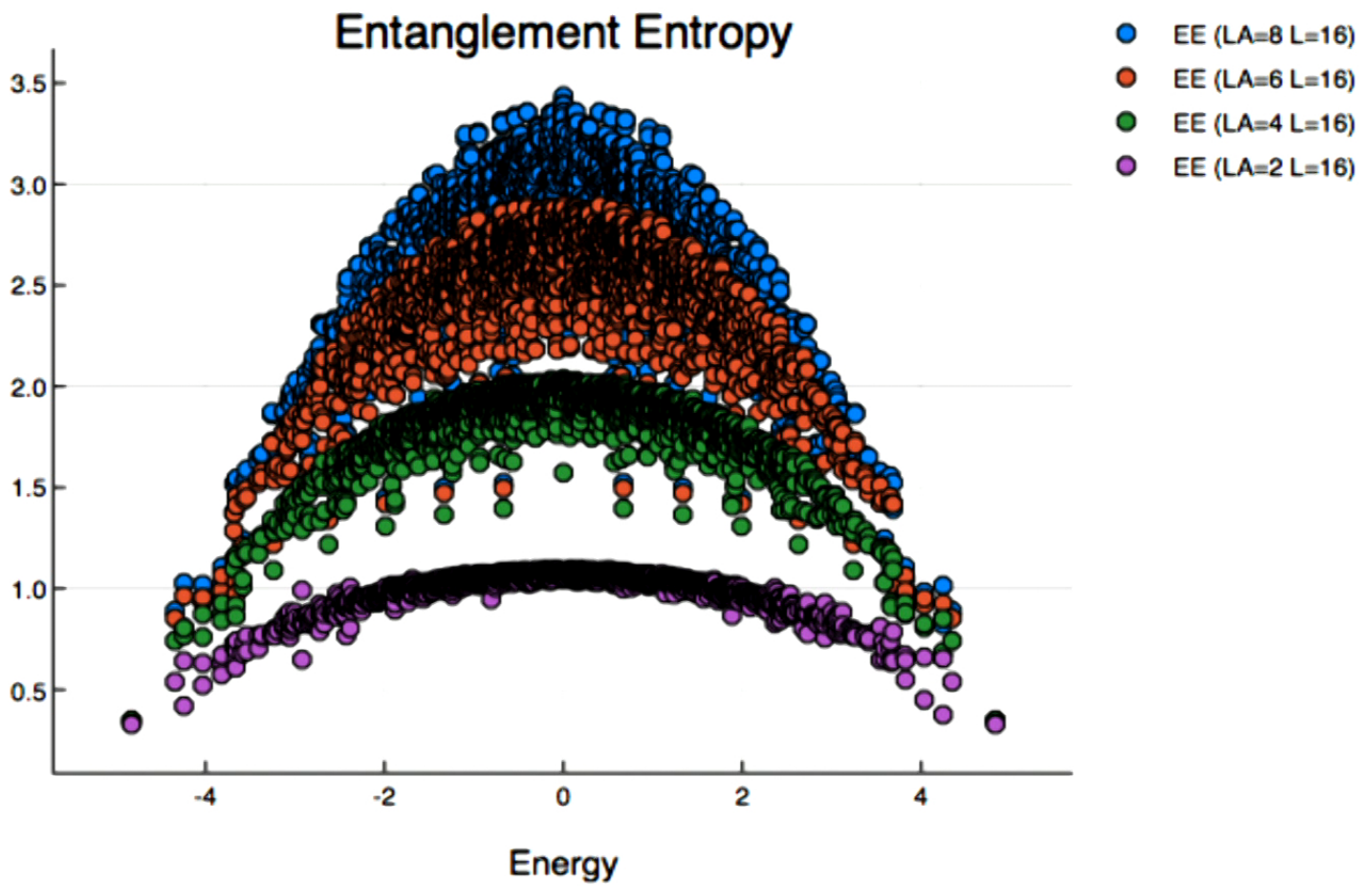
Z_2 scar states



- Unusual high overlaps with towers of special states.
- Towers have almost equal energy spacing: observed oscillation frequency in the experiment.

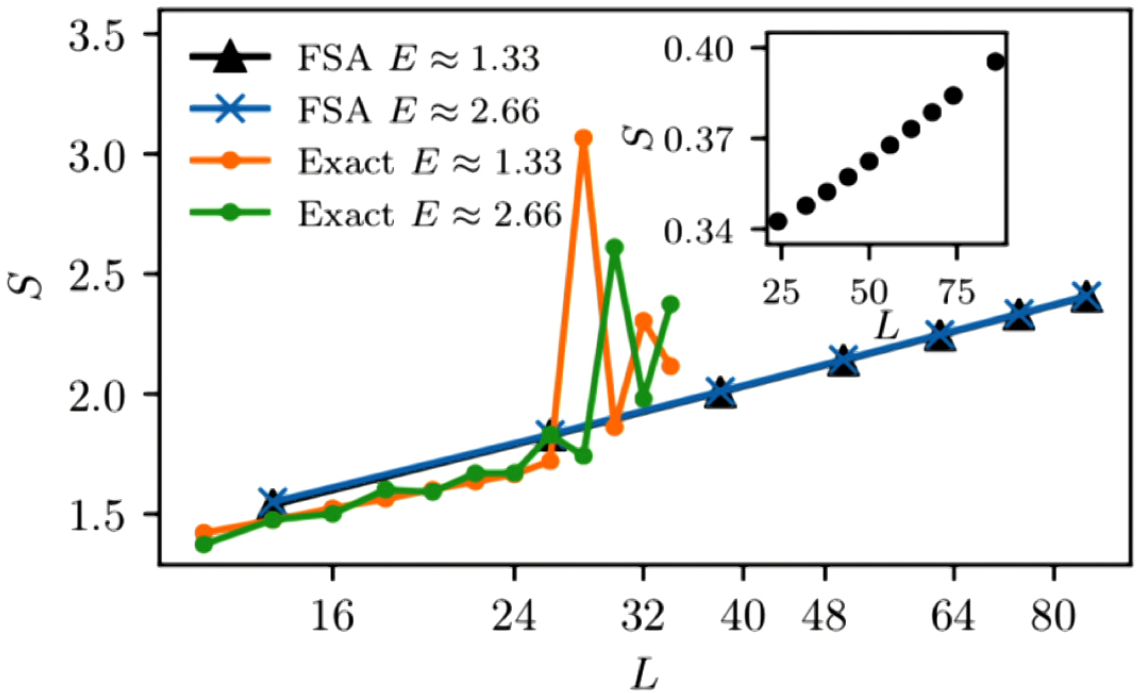
C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Nature Physics **14** 745–749 (2018)





See also Turner *et. al., arXiv: 1806.10933*

Non-ETH Behavior



Turner *et. al.* (arXiv: 1806.10933) argued logarithmic entanglement scaling numerically.

Our work:
analytical discovery of exact nonthermal eigenstate!

Periodic Boundary Condition

$$H = \sum_j P_{j-1} X_j P_{j+1}$$ **Only exist in L even:**

$$|\Phi_1\rangle = \sum_{\{\sigma\}} \mathrm{Tr}[B^{\sigma_1} C^{\sigma_2} \dots B^{\sigma_{L-1}} C^{\sigma_L}] |\sigma_1 \dots \sigma_L\rangle$$

E=0
(unnormalized)

$$|\Phi_2\rangle = \sum_{\{\sigma\}} \mathrm{Tr}[C^{\sigma_1} B^{\sigma_2} \dots C^{\sigma_{L-1}} B^{\sigma_L}] |\sigma_1 \dots \sigma_L\rangle$$

$$B^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad B^1 = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$C^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad C^1 = \sqrt{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix}$$

constraints are satisfied

Not orthogonal but linearly independent for L>6.

L_b=L/2

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E=0
(unnormalized)

```
2.0      [0, 0, 0, 0, 0, 0, 0, 0]
-2.0     [1, 0, 1, 0, 0, 0, 0, 0]
-2.0     [0, 1, 0, 1, 0, 0, 0, 0]
2.0      [0, 1, 0, 0, 1, 0, 0, 0]
-2.0     [0, 0, 1, 0, 1, 0, 0, 0]
2.0      [1, 0, 0, 0, 0, 1, 0, 0]
-2.0     [0, 0, 0, 1, 0, 1, 0, 0]
-2.0     [1, 0, 0, 0, 0, 0, 1, 0]
2.0      [0, 0, 0, 1, 0, 0, 1, 0]
-2.0     [0, 0, 0, 0, 1, 0, 1, 0]
4.0      [1, 0, 1, 0, 1, 0, 1, 0]
-2.0     [0, 1, 0, 0, 0, 0, 0, 1]
2.0      [0, 0, 1, 0, 0, 0, 0, 1]
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L_b=L/2

Sketch of the proof:

Block two-sites **R=(01), O=(00), L=(10)**

$$H = \sum_{b=1}^{L_b} h_{b,b+1} \ , \quad h_{b,b+1} = (|R\rangle\langle O| + |O\rangle\langle R|)_b \otimes (I - |L\rangle\langle L|)_{b+1} + (I - |R\rangle\langle R|)_b \otimes (|L\rangle\langle O| + |O\rangle\langle L|)_{b+1}$$

$$|\Phi_1\rangle = \sum_{\{s\}} \mathrm{Tr}[A^{s_1} \dots A^{s_{L_b}}]|s_1 \dots s_{L_b}\rangle \qquad A^O = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ , \ A^R = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} \ , \ A^L = \begin{pmatrix} 0 & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$$

Two-body terms annihilate the state.

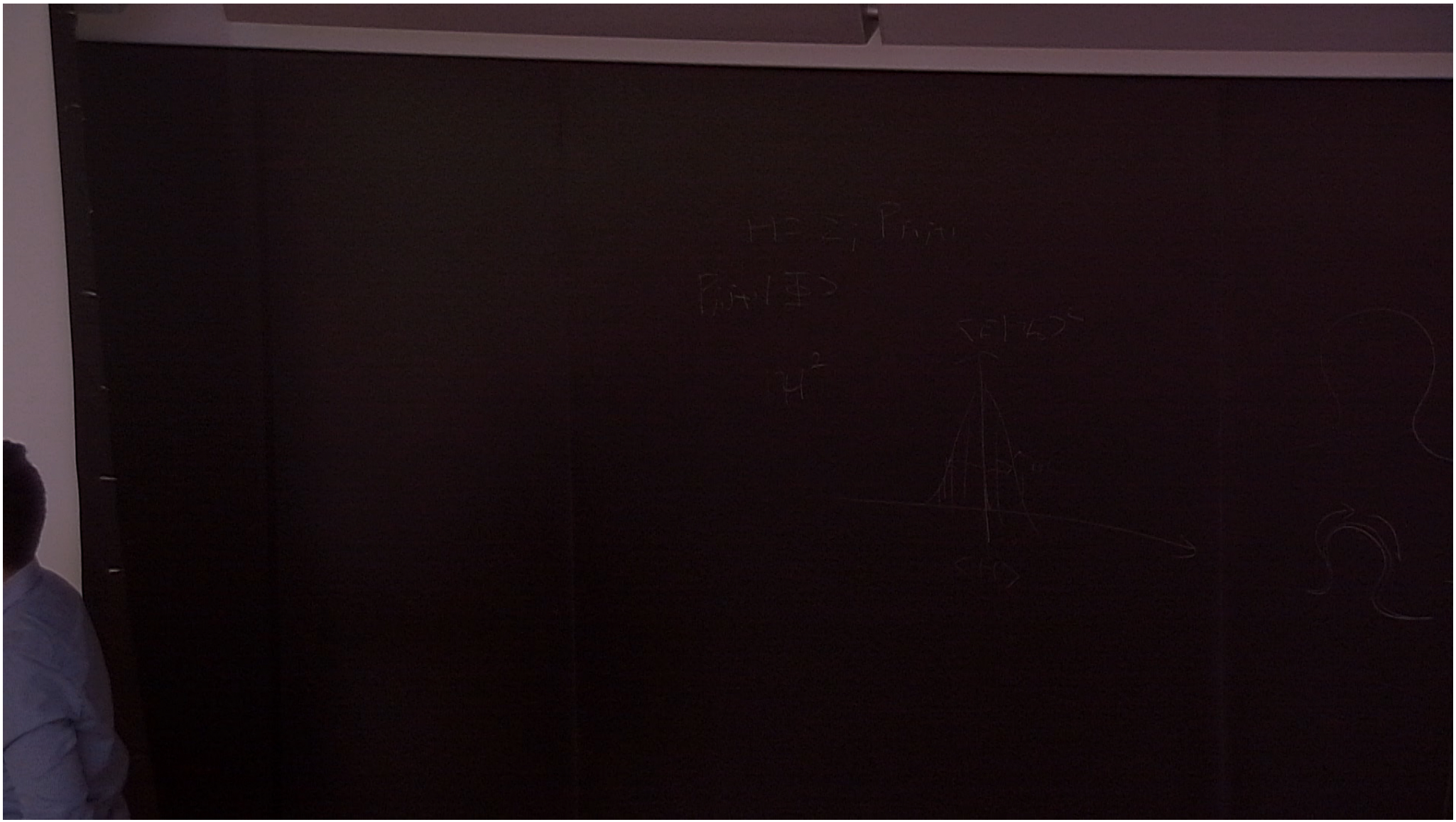
One-body term:

$$\Big(|R\rangle\langle O| + |O\rangle\langle R| + |L\rangle\langle O| + |O\rangle\langle L|\Big)_b |\Phi_1\rangle = \sum_{\{s\}} \mathrm{Tr}[A^{s_1} \dots F^{s_b} \dots A^{s_{L_b}}]|s_1 \dots s_{L_b}\rangle$$

$$H|\Phi_1\rangle = \sum_{b=1}^{L_b} \sum_{\{s\}} \mathrm{Tr}[A^{s_1} \dots F^{s_b} \dots A^{s_{L_b}}]|s_1 \dots s_{L_b}\rangle$$

$$F^O = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix} \ , \quad F^R = F^L = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$F^s = XA^s - A^sX, \text{ where } X = \frac{1}{\sqrt{2}}\sigma_x \qquad H|\Phi_1\rangle = 0.$$



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2.0      [1, 0, 0, 0, 0, 1, 0, 0]
-2.0     [0, 0, 0, 1, 0, 1, 0, 0]
-2.0     [1, 0, 0, 0, 0, 0, 1, 0]
2.0      [0, 0, 0, 1, 0, 0, 1, 0]
-2.0     [0, 0, 0, 0, 1, 0, 1, 0]
4.0      [1, 0, 1, 0, 1, 0, 1, 0]
-2.0     [0, 1, 0, 0, 0, 0, 0, 1]
2.0      [0, 0, 1, 0, 0, 0, 0, 1]
-2.0     [0, 0, 0, 0, 0, 1, 0, 1]
4.0      [0, 1, 0, 1, 0, 1, 0, 1]
```

$$B^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad B^1 = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$C^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad C^1 = \sqrt{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix}$$

L_b=L/2

Translation Symmetry Breaking

- **Cannot be detect by one-site observables:**

$$\rho^{\text{one-site}} = \frac{2}{3}|0\rangle\langle 0| + \frac{1}{3}|1\rangle\langle 1|$$

Fun fact: average Rydberg excitation number in Gibbs ensemble

$$\langle n_j \rangle_{T=\infty} = (1+\phi^2)^{-1} \approx 0.2764$$ **-> does not satisfy ETH!**

- **Can be detect by** $|01\rangle\langle 10| + |10\rangle\langle 01|$ **: 0 on [1,2]; -2/9 on [2,3]**

$$\rho^{\text{two-site}}_{[1,2]} = \frac{1}{3}(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10|)$$

$$\rho^{\text{two-site}}_{[2,3]} = \frac{1}{3}(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10|) - \frac{1}{9}(|01\rangle\langle 10| + |10\rangle\langle 01|) \ .$$

Fun fact: Domain-wall density $\langle P_j P_{j+1} \rangle_{T=\infty} = \phi/(\phi+2) \approx 0.4472$

Inversion Symmetry and Particle-hole

	I	$\mathscr{C}$
$\lvert\Phi_1\rangle$	$(-1)^{L_b}$	$(-1)^{L_b}$
$\lvert\Phi_2\rangle$	$(-1)^{L_b}$	$(-1)^{L_b}$

$$I|\Phi_1\rangle = \sum_{\{\sigma\}} \mathrm{Tr}[B_I^{\sigma_1} C_I^{\sigma_2} \ldots B_I^{\sigma_{L-1}} C_I^{\sigma_L}]|\sigma_1 \ldots \sigma_L\rangle$$

$$B_I^\sigma \equiv [C^\sigma]^T \text{ and } C_I^\sigma \equiv [B^\sigma]^T$$

Gauge transformation $X_I \equiv i\sigma_y \qquad Y_I \equiv \mathrm{diag}(-1,-1,1)$

$$X_I B_I^\sigma Y_I^{-1} = B^\sigma \text{ and } Y_I C_I^\sigma X_I^{-1} = -C^\sigma$$

For $|\Phi_2\rangle$, note that since $IT_x = T_x^{-1} I$ and $T_x^2|\Phi_i\rangle = |\Phi_i\rangle$

Inversion Symmetry and Particle-hole

	I	$\mathscr{C}$
$\lvert\Phi_1\rangle$	$(-1)^{L_b}$	$(-1)^{L_b}$
$\lvert\Phi_2\rangle$	$(-1)^{L_b}$	$(-1)^{L_b}$

$$\mathcal{C}_{\rm ph}|\Phi_1\rangle = \sum_{\{\sigma\}} {\rm Tr}[B_c^{\sigma_1} C_c^{\sigma_2} \ldots B_c^{\sigma_{L-1}} C_c^{\sigma_L}]|\sigma_1 \ldots \sigma_L\rangle$$

$$B_c^0 = -B^0,\ B_c^1 = B^1,\ C_c^0 = -C^0, \text{ and } C_c^1 = C^1$$

Gauge transformation $X_c \equiv \sigma_z \quad Y_c \equiv {\rm diag}(-1, 1, -1)$

$$X_c B_c^\sigma Y_c^{-1} = B^\sigma,\ Y_c C_c^\sigma X_c^{-1} = -C^\sigma$$

For $|\Phi_2\rangle$, noting that $\mathcal{C}_{\rm ph} T_x = T_x \mathcal{C}_{\rm ph}$

Q: Yeah, right, you are just lucky to get low-entanglement eigenstates out of the exponentially-degenerate E=0 space……

A: But we have other exact eigenstates at $E = \pm\sqrt{2}$ in OBC!

Open Boundary Condition

L even $|\Gamma_{\alpha,\beta}\rangle = \sum_{\{\sigma\}} v_\alpha^T B^{\sigma_1} C^{\sigma_2} \dots B^{\sigma_{L-1}} C^{\sigma_L} v_\beta |\sigma_1 \dots \sigma_L\rangle$ **(unnormalized)**

Boundary vectors: $v_1 = (1,1)^T \quad v_2 = (1,-1)^T$

$$B^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad B^1 = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$C^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad C^1 = \sqrt{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix}$$

	$\|\Gamma_{1,2}\rangle$	$\|\Gamma_{1,1}\rangle$	$\|\Gamma_{1,2}\rangle$	$\|\Gamma_{2,1}\rangle$
E	√2	0	0	-√2

Same bulk as in PBC, expectation values followed as in PBC.

$$\langle X_j \rangle_{\alpha,\beta} \equiv \langle \Gamma_{\alpha,\beta} | X_j | \Gamma_{\alpha,\beta} \rangle / \langle \Gamma_{\alpha,\beta} | \Gamma_{\alpha,\beta} \rangle \qquad \text{decay length } 2\ln(3)$$

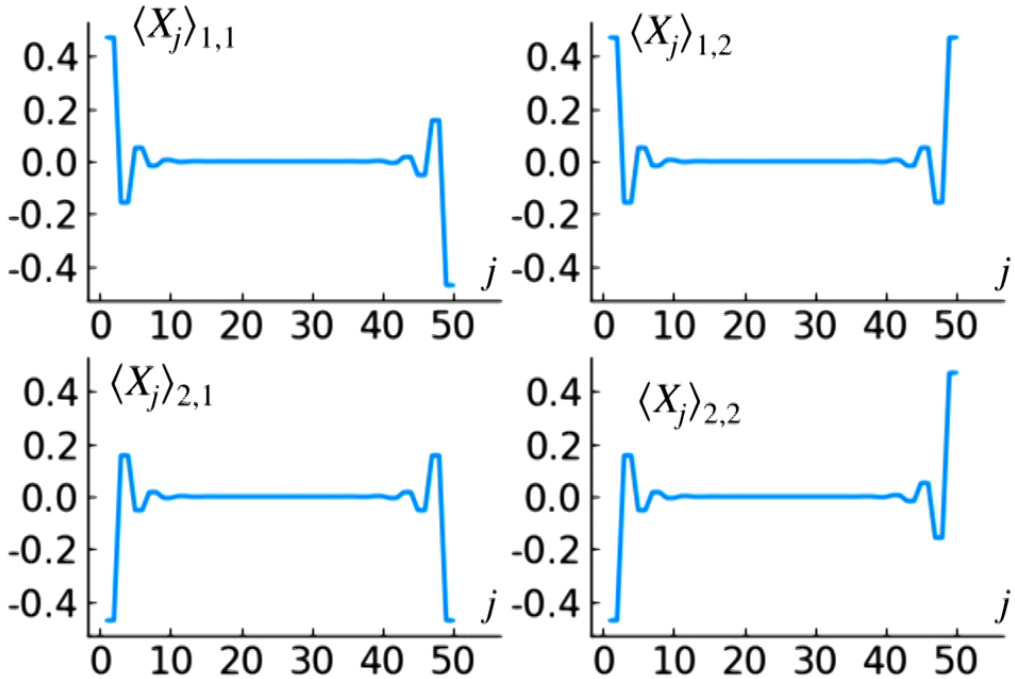


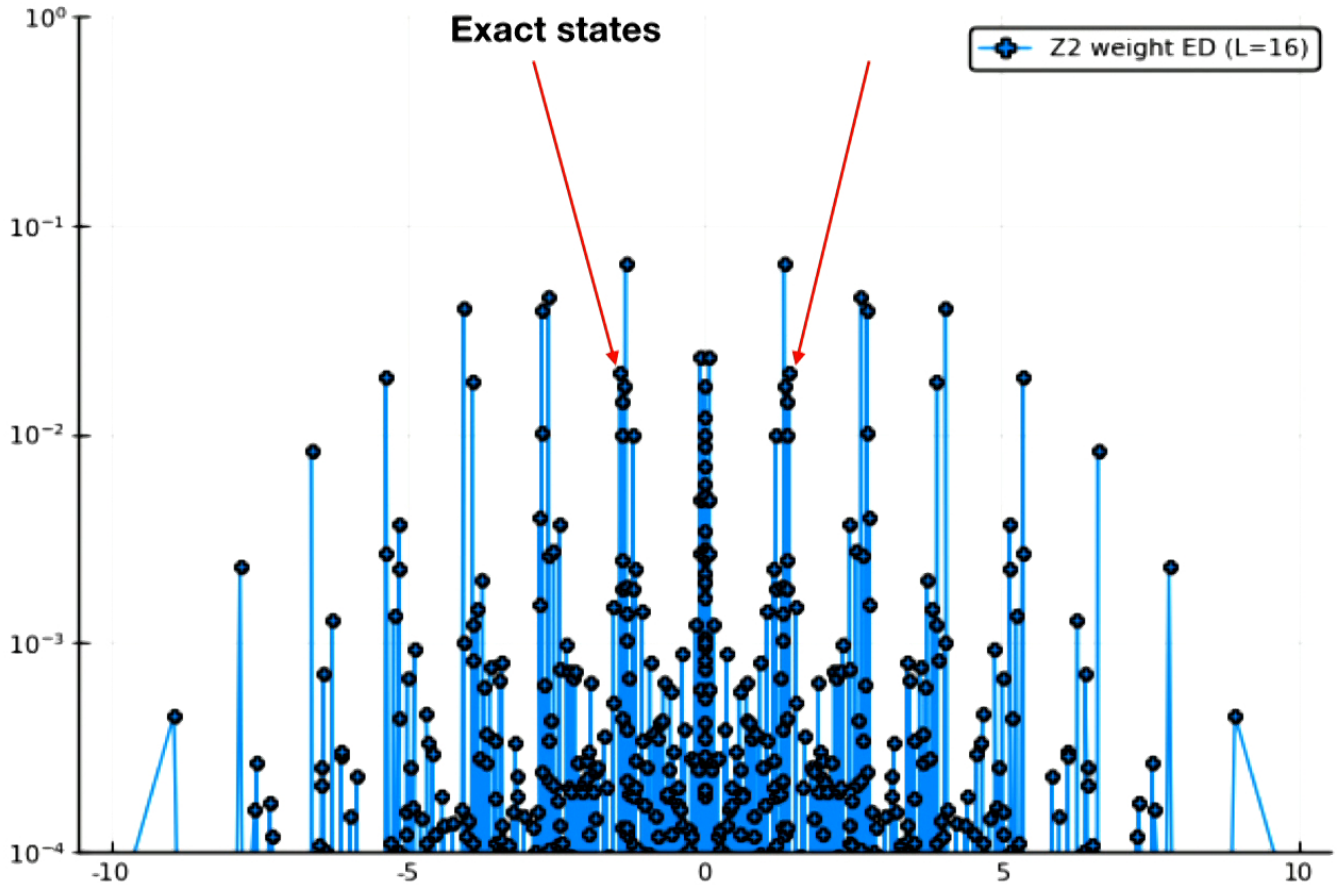
FIG. 1. Energy density profiles $\langle X_j \rangle_{\alpha,\beta}$ in the four exact eigenstates $|\Gamma_{\alpha,\beta}\rangle$ in the OBC system of size $L = 50$.

Inversion Symmetry and Particle-hole

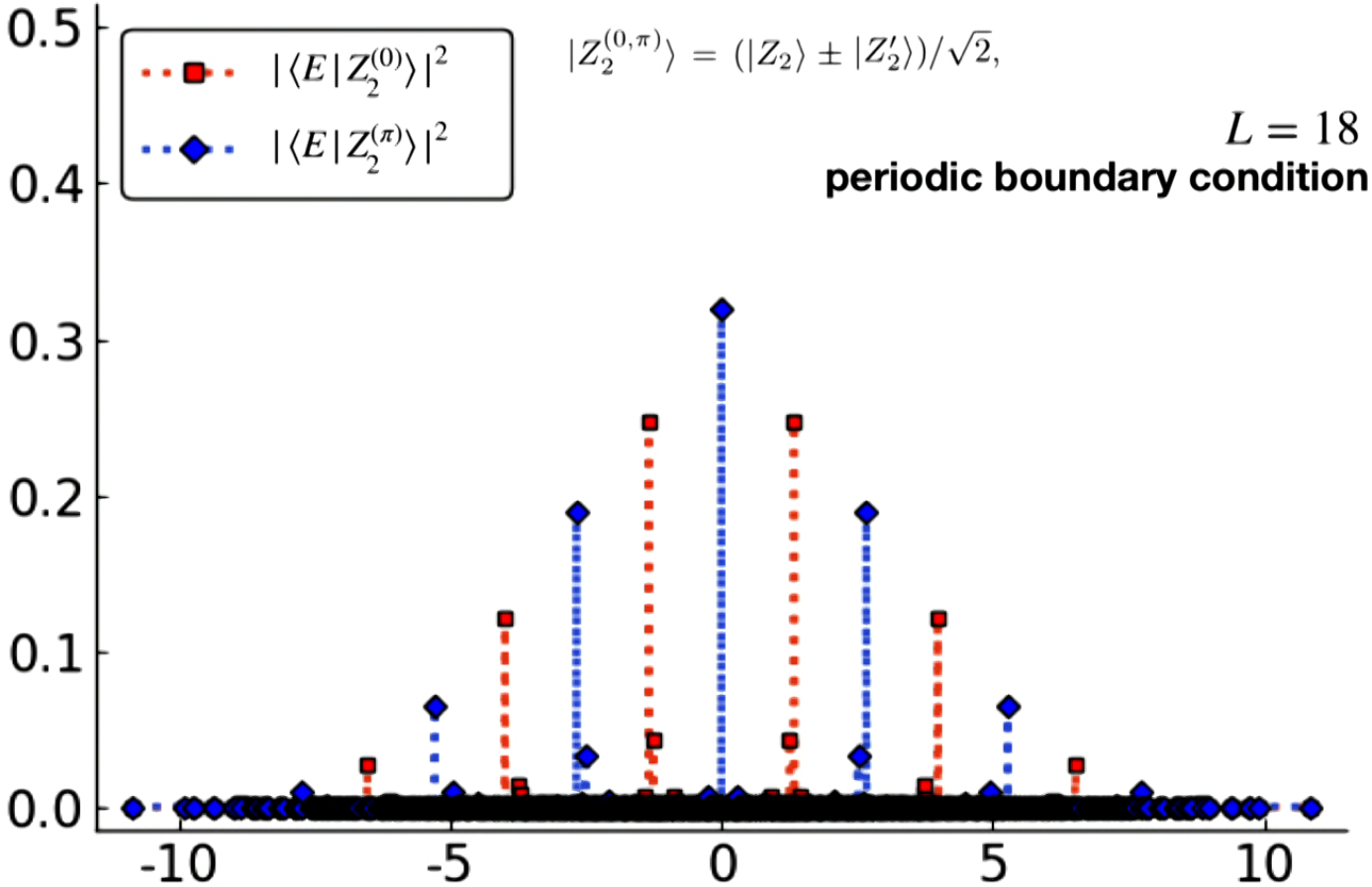
	E	I	$\mathscr{C}$
$\lvert\Gamma_{1,2}\rangle$	$\sqrt{2}$	$(-1)^{L_b-1}$	$(-1)^{L_b}\lvert\Gamma_{2,1}\rangle$
$\lvert\Gamma_{2,1}\rangle$	$-\sqrt{2}$	$(-1)^{L_b-1}$	$(-1)^{L_b}\lvert\Gamma_{1,2}\rangle$

	E	I	$\mathscr{C}$
$\lvert\Gamma_{1,1}\rangle$	0	$(-1)^{L_b}\lvert\Gamma_{2,2}\rangle$	$(-1)^{L_b}\lvert\Gamma_{2,2}\rangle$
$\lvert\Gamma_{2,2}\rangle$	0	$(-1)^{L_b}\lvert\Gamma_{1,1}\rangle$	$(-1)^{L_b}\lvert\Gamma_{1,1}\rangle$

Recap the Z2 scar states (here in OBC L=16)



Let us put the excitation in the bulk!
Single Mode Approximation (SMA)



$L = 18$, we have $|\langle Z_2^{(\pi)}|\Phi_{K=\pi}\rangle|^2/\langle\Phi_{K=\pi}|\Phi_{K=\pi}\rangle \approx 0.081.$ $|\Phi_{K=0/\pi}\rangle = |\Phi_1\rangle \pm |\Phi_2\rangle$

Has the same K and I quantum number dependence as Z2 E=0 scar states.

SMA Wavefunction I

$$|M\rangle = \sum_{b=1}^{L_b} \mathrm{Tr}[B^{\sigma_1} C^{\sigma_2} \ldots M^{\sigma_{2b-1}\sigma_{2b}} \ldots B^{\sigma_{L-1}} C^{\sigma_L}] |\sigma_1 \ldots \sigma_L\rangle$$

"Quasiparticle" on top of the exact MPS

$$|\Xi_1\rangle = (|M\rangle - (-1)^{L_b} T_x |M\rangle)/\xi_1$$

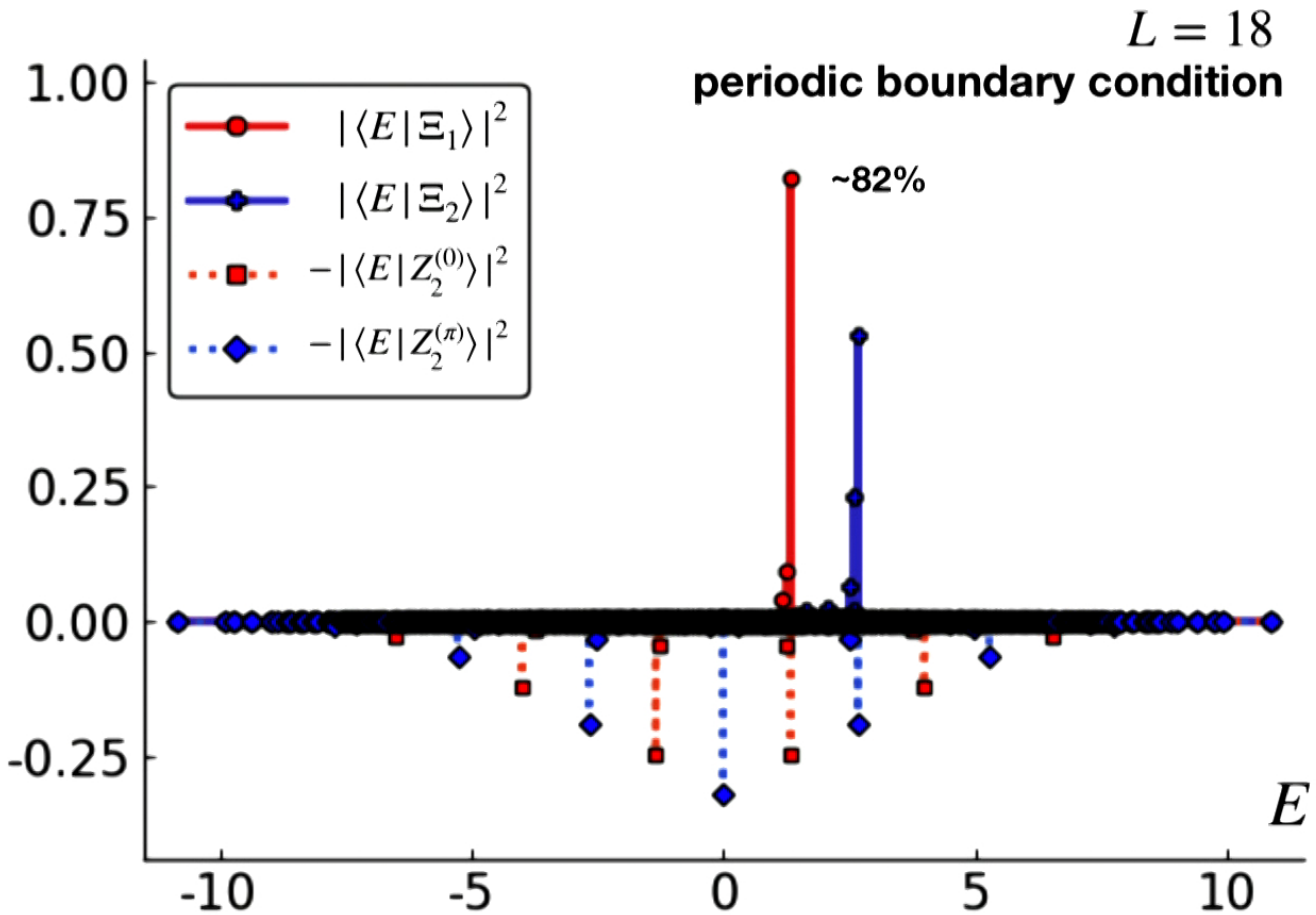
$$M^{00} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ , \ M^{01} = \begin{pmatrix} \mu_1 & 0 \\ \mu_2 & 0 \end{pmatrix} \ , \ M^{10} = \begin{pmatrix} 0 & 0 \\ -\mu_2 & \mu_1 \end{pmatrix}$$

Chosen such that $I|M\rangle = (-1)^{L_b-1}|M\rangle, \ \text{hence} \ I|\Xi_1\rangle = (-1)^{L_b-1}|\Xi_1\rangle$

Finding the optimal parameter: minimizing energy variance

$$f_1(\mu_1, \mu_2) = \langle \Xi_1 | H^2 | \Xi_1 \rangle - \langle \Xi_1 | H | \Xi_1 \rangle^2 \ \text{at fixed} \ L$$

$$L = 18 \qquad \begin{aligned} \mu_1 &= 1.08457 \\ \mu_2 &= -0.6318 \\ f_1 &= 0.025 \\ \langle \Xi_1 | H | \Xi_1 \rangle &= 1.32 \end{aligned}$$


$L = 18$
periodic boundary condition
~82%
$|\langle E|\Xi_1\rangle|^2$
$|\langle E|\Xi_2\rangle|^2$
$-|\langle E|Z_2^{(0)}\rangle|^2$
$-|\langle E|Z_2^{(\pi)}\rangle|^2$
1.00
0.75
0.50
0.25
0.00
-0.25
-10
-5
0
5
10
E

SMA Wavefunction II

$$|M\rangle = \sum_{b=1}^{L_b} \mathrm{Tr}[B^{\sigma_1} C^{\sigma_2} \ldots M^{\sigma_{2b-1}\sigma_{2b}} \ldots B^{\sigma_{L-1}} C^{\sigma_L}] |\sigma_1 \ldots \sigma_L\rangle$$

"Quasiparticle" on top of the exact MPS

$$|\Xi_2\rangle = (|N\rangle + (-1)^{L_b} T_x |N\rangle)/\xi_2$$

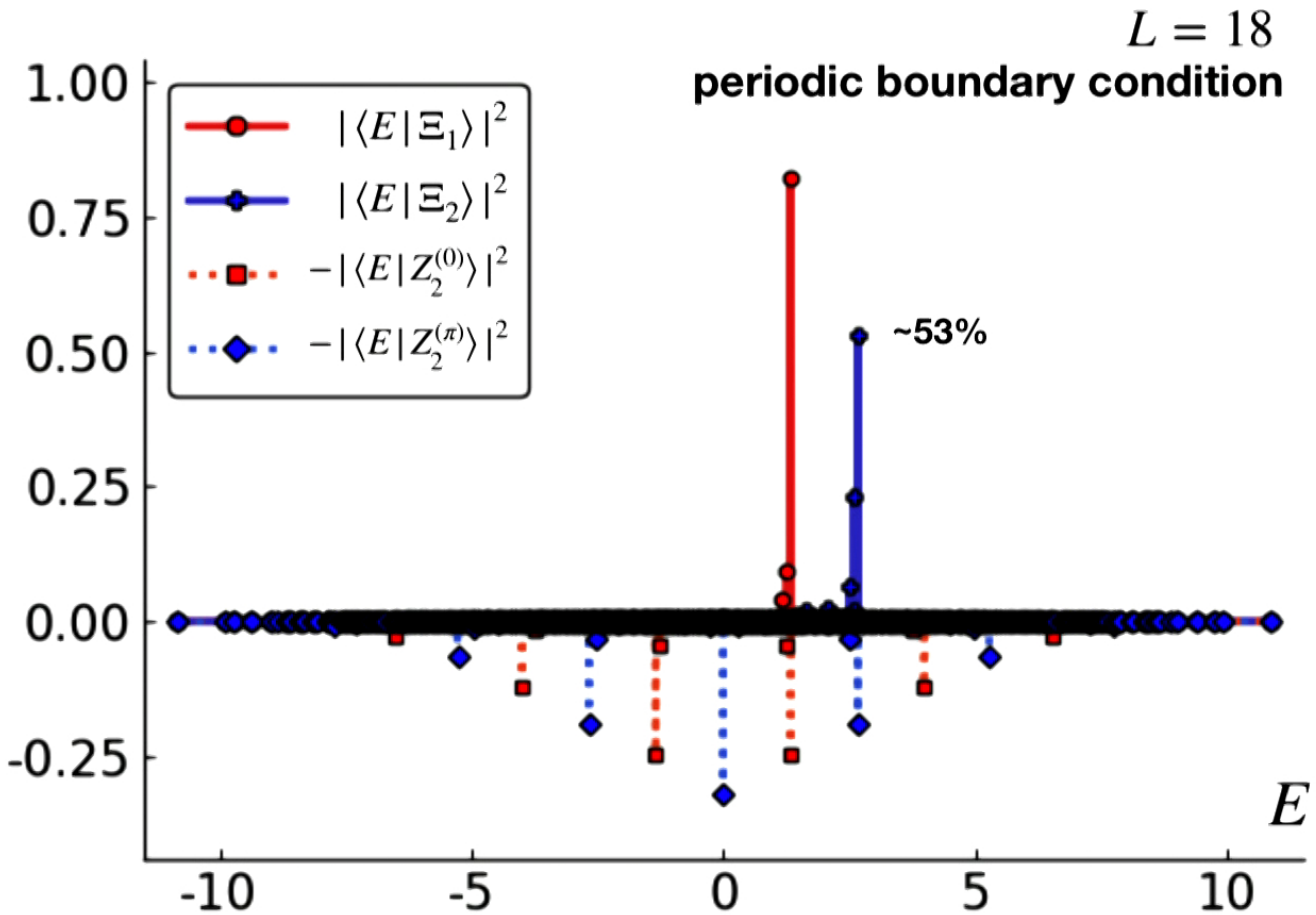
$$N^{00} = \begin{pmatrix} \nu & \sqrt{2} \\ -\sqrt{2} & -\nu \end{pmatrix}, \; N^{01} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \; N^{10} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

Chosen such that $I|N\rangle = (-1)^{L_b}|N\rangle$

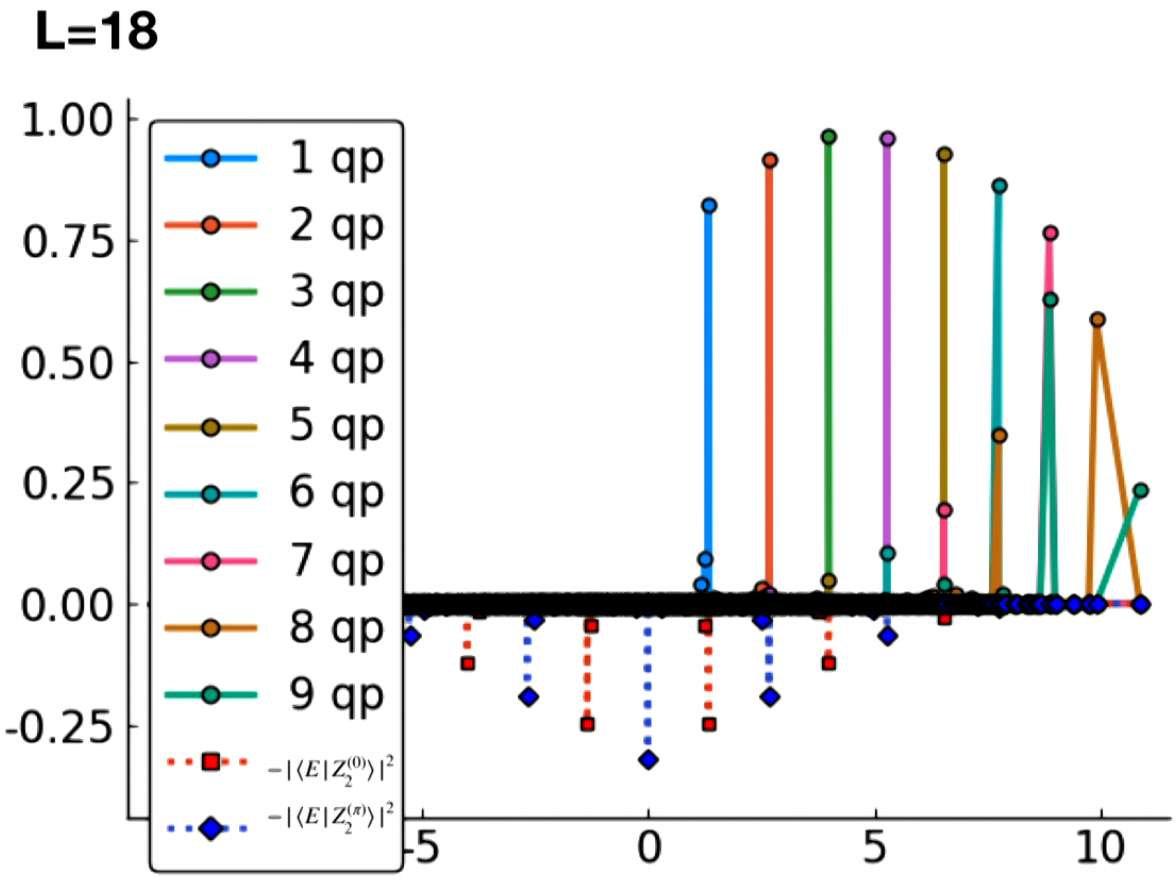
Finding the optimal parameter: minimizing energy variance

$$f_2(\nu) = \langle \Xi_2 | H^2 | \Xi_2 \rangle - \langle \Xi_2 | H | \Xi_2 \rangle^2$$

$$L = 18 \qquad \nu = 2.381 \qquad f_2 = 0.33 \text{ and } \langle \Xi_2 | H | \Xi_2 \rangle = 2.47$$


$L = 18$
periodic boundary condition
$|\langle E|\Xi_1\rangle|^2$
$|\langle E|\Xi_2\rangle|^2$
$-|\langle E|Z_2^{(0)}\rangle|^2$
$-|\langle E|Z_2^{(\pi)}\rangle|^2$
~53%
1.00
0.75
0.50
0.25
0.00
-0.25
-10
-5
0
5
10
E

Multiparticle



Multiparticle Ansatz

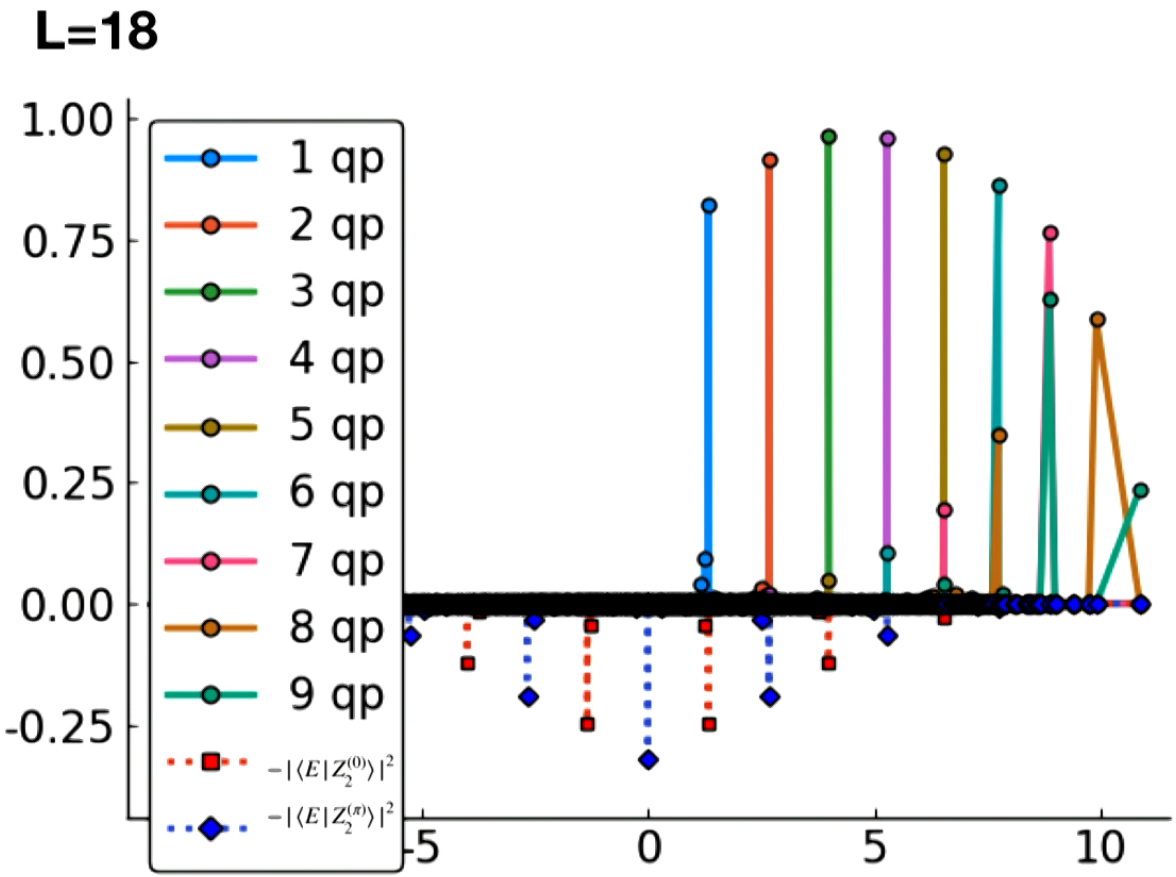
$$|\Phi_n(M)\rangle = \sum_{i_1 \ldots i_n} \mathbf{Tr}[A \ldots M_{i_1} A \ldots M_{i_n} \ldots A]$$

$$A = BC$$

Summing over no repeating i's
(Noninteracting scattering states)

$$\sim |\Phi_n(M)\rangle \pm T_x |\Phi_n(M)\rangle)$$

Multiparticle



Outlook

- Z2 scar states (at least nearby E=0) can be viewed as quasiparticle excitations = frequency observed in the experiment.
- Can we say something about thermodynamic limit without an exact SMA?
- Deformation of PXP such that SMA is exact?
- Criterions for the existence of many-body scars?
- Possible proximate integrable point?
- Other models with finite-energy-density exact eigenstates? Exact or approximate quasiparticle description?

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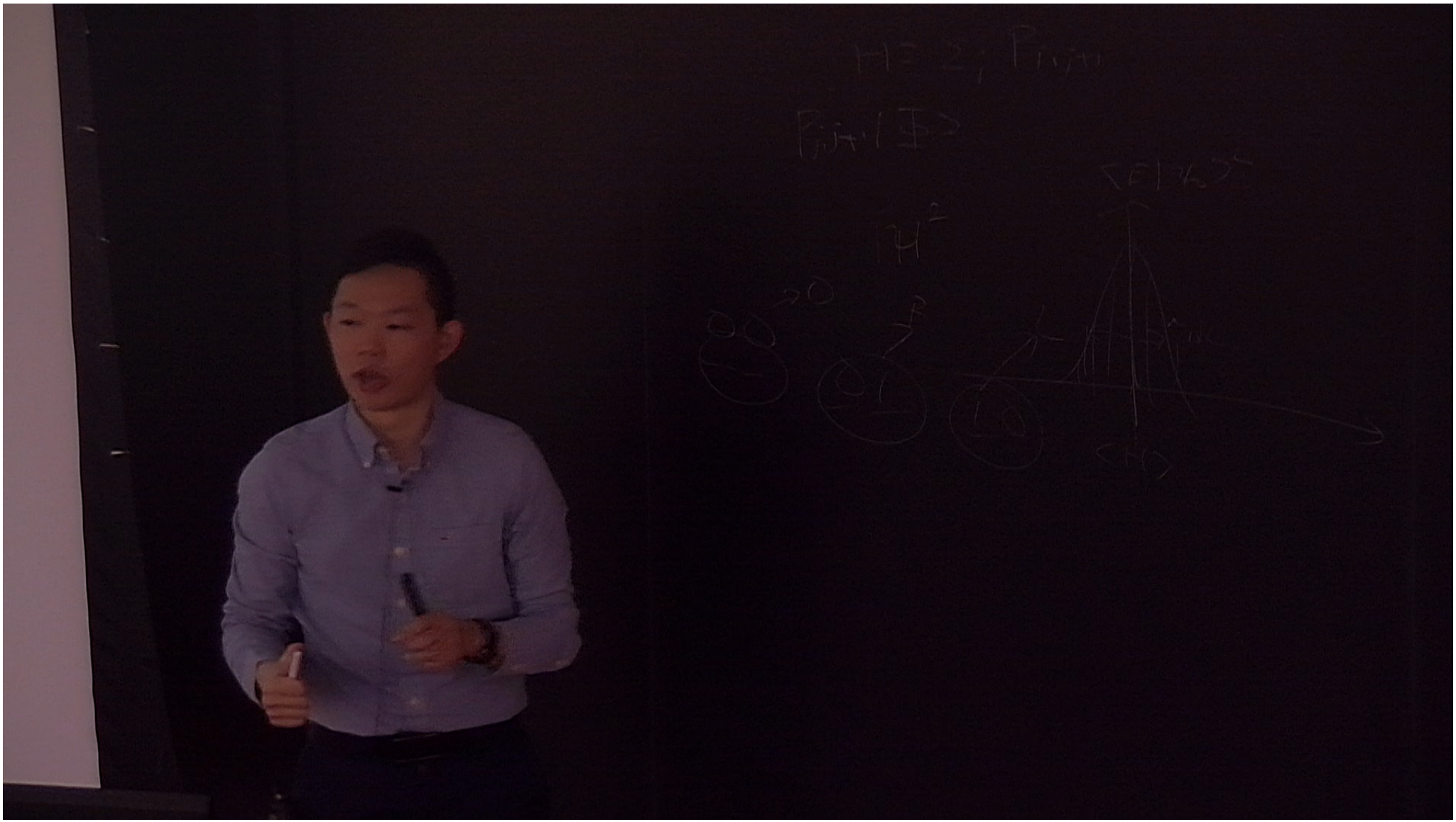
Thank you for your attention!