Title: Emergent Dirac fermions in Composite Fermi Liquids

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Abstract: <p>Interacting electrons in high magnetic fields exhibit rich physical phenomena including the gapped fractional quantum Hall effects and the gapless states. The composite Fermi liquids (CFLs) are gapless states that can occur at even denominator Landau level fillings. Due to the celebrated work of Halperin, Lee and Read (94), the CFLs were understood as Fermi liquids of composite fermions, which are bound states of electron and electromagnetic flux quanta. However, at 1/2 filling, it is not obvious why the HLR description is consistent with the particle hole symmetry. Motivated by this, recently Son (15) proposed an alternative description for CFLs at 1/2, according to which the composite fermions are instead emergent Dirac fermions. Importantly, Sonâ€™s theory predicts a Pi Berry curvature singularity at the composite Fermi sea center. In the first part of this talk [2,3], I will present our numerical work about detecting this Z2 Berry phase at 1/2 filling. In the second part [1], I will present how and why Dirac fermions can emerge at all the other filling fractions (1/2m and 1-1/2m when m is integer) even without the particle hole symmetry.</p>

<p> </p>

<p>[1] arXiv 1808.07529. JW.</p>

<p>[2] arXiv 1711.07864. Geraedts, JW, Rezayi, Haldane.</p>

<p>[3] arXiv 1710.09729. JW, Geraedts, Rezayi, Haldane.</p>

Emergent Dirac Fermions in Composite Fermi Liquids

Jie Wang

Physics Department, Princeton University

Based on,

- A Dirac Fermion Hierarchy of Composite Fermi Liquids, **J. Wang**. *arXiv:1808.07529 (submitted to PRL).*
- Berry Phase and Model Wavefunction in the Half-filled Landau level, S. Geraedts, **J. Wang**, E.H. Rezayi and F.D.M. Haldane. *arXiv:1711.07864 (PRL 121,147202).*
- Lattice Monte Carlo for Quantum Hall States on a Torus, **J. Wang**, S. Geraedts, E.H. Rezayi and F.D.M. Haldane. *arXiv:1710.09729 (submitted to PRB).*

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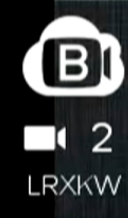
PRINCETON UNIVERSITY



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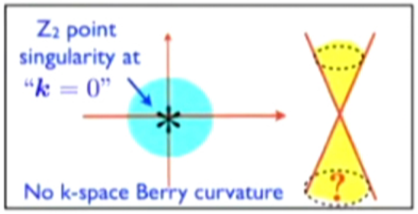
1. Intro. to Composite-Fermi-Liquids.
   - Halperin-Lee-Read Theory. — HLR (94).
   - Son's Dirac Fermion theory ($\nu = \frac{1}{2}$). — Son (15).
2. Many-Body Berry Phase in a half filled LL.



JW w/ Geraedts, Rezayi, Haldane. 1711.07864 & 1710.09729.

3. Generalized Dirac fermion theory.

PH symmetric, Dirac fermion theory.

$$\nu = \cdots \frac{1}{2m}, \cdots \frac{1}{6}, \cdots \frac{1}{4}, \cdots \frac{1}{2} \cdots \frac{3}{4}, \cdots \frac{5}{6}, \cdots 1 - \frac{1}{2m}, \cdots$$

Flux-attached Dirac fermion theory.

JW. 1808.07529.
see also, H. Goldman & E. Fradkin. 1808.09314.

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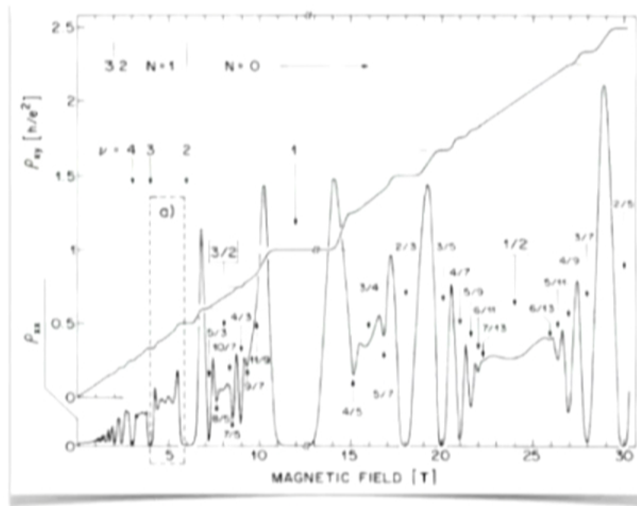
to Quantum Hall effect

Quantum Hall physics: 2D (interacting) electrons in magnetic field.
Interactions + Landau level fillings ($\nu \equiv N_e/N_\phi$) $\Rightarrow$ different phases.

- Gapped state: e.g. Laughlin, Moore-Read.
- Gapless state: e.g. composite-Fermi liquid (CFL).



Gapped quantum Hall phases have rich physics,
but I will focus on the gapless CFL phase.

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...posite Fermi Liquid State (CFL)

Halperin-Lee-Read (HLR): CFL = Fermi sea of composite fermions,

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Halperin-Lee-Read (HLR): CFL = Fermi sea of composite fermions,

The effective action at $\nu = \frac{1}{2m}$ is,

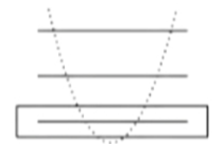
$$
\begin{aligned}
\mathcal{L}_{HLR} &= \psi^\dagger_{CF}(iD_t - \mu + \frac{\vec{D}\cdot\vec{D}}{2m})\psi_{CF} - \frac{ada}{8m\pi} + \mathcal{L}_{int}. \\
D_\mu &= \partial_\mu + i(a_\mu + A_\mu) \\
\nabla \times \vec{a} &= -2m\pi\bar{\psi}_{CF}\psi_{CF} = -2m\pi n_{el}(\vec{r}).
\end{aligned}
$$

Mean field ground state is <u>a filled Fermi sea of composite fermions</u>.

icle Hole Symmetry

Typically, the Coulomb interaction energy scale is much less then the separation of Landau levels. The problem thus is a degenerate perturbation problem in a single Landau level, for non-commutative guiding centers (a,b=x,y).

$$H = \sum_{i,j} V_2(\vec{R}_i - \vec{R}_j), \quad [R_i^a, R_j^b] = -il_B^2 \epsilon^{ab} \delta_{ij}.$$



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PH transformation is an anti-unitary transformation ($\alpha \to \alpha^*$) that exchanges particles and holes ($c_i^\dagger \leftrightarrow c_i$). Thus maps Landau level filling $\nu \to 1 - \nu$.

PH is an **Exact** symmetry at 1/2 filling as long as the projected Hamiltonians is purely two-body type.

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It is NOT obvious how HLR theory is PH symmetric at 1/2. Superficially, CFL$\neq$CHL.

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c Fermion Action

Son's Dirac theory for half filled Landau level problem, Son(15).

$$S_\psi = \int i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{2}\frac{1}{2\pi}adA + \frac{1}{2}\frac{1}{4\pi}AdA.$$

- **Fermionic particle-vortex duality**. Son; C. Wang; Metlitski; Mross; Karch...

$$\rho_\psi = \frac{1}{2}\frac{dA}{2\pi}, \quad \rho_A = \frac{1}{2}\frac{dA}{2\pi} - \frac{1}{2}\frac{da}{2\pi}.$$

Dirac fermion **density** ρ_ψ $\leftrightarrow$ physical fermion magnetic field $\frac{dA}{2\pi}$.

Dirac fermion magnetic field $\frac{da}{2\pi}$ $\leftrightarrow$ physical fermion **density** ρ_A.

- **Explicitly PH symmetric**.

PH: $\psi \to T\psi T^{-1}$, T = Time-Reversal.

- π **Berry phase**.

Fermi sea of Dirac fermions.

ni sea Berry Phase

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- Son's theory predicted a π phase (due to spin-1/2).
- The relation between **Fermi sea Berry phase** Φ_{FS} and **Hall conductivity** σ_H in fact can be more general,

$$\boxed{\sigma_H = \frac{e^2}{\hbar}\frac{\Phi_{FS}}{2\pi}, \quad \Phi_{FS} = 2\pi\nu.}$$

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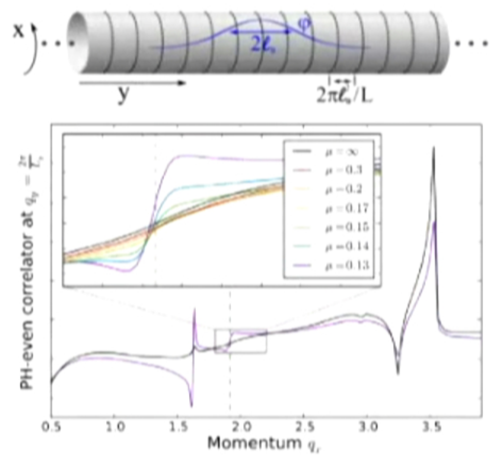
- First argued by Haldane (04) in the content of anomalous Hall effect. Due to the *anomalous velocity term* related to Berry curvature (Qian Niu et.al.),

$$\hbar\frac{dk_a}{dt} = eE_a + eF_{ab}\frac{dr^b}{dt}, \quad \hbar\frac{dr^a}{dt} = \nabla_k^a \epsilon_n(k) + \underline{\hbar\tilde{F}_{ab}(k)}\frac{dk_b}{dt}.$$

erical observation of π Berry phase

- The π Fermi sea Berry phase was first **indirectly** observed in a density-matrix-renormalization-group study (Geraedts et.al. 15), through PH-even correlation functions: **backscattering is forbidden** by particle-hole symmetric scatters.



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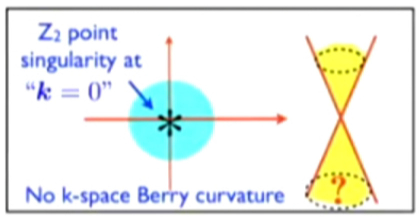

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y-Body Berry Phase

Recall textbook Berry phase for Bloch waves $|\psi_k\rangle$ ($|u_k\rangle$ is its periodic part),

$$\prod_{closed\ path} \langle u_{k_1}|u_{k_2}\rangle = \prod_{closed\ path} \langle \psi_{k_1}| e^{i(k_1-k_2)\cdot r} |\psi_2\rangle,$$

$$\psi_k(r) = e^{ik\cdot r} u_k(r),$$

$$\rho(\vec{k}) \equiv e^{i\vec{k}\cdot\vec{r}}.$$

Conventional single-body Berry phase can be thought of as density operators matrix elements.

y-Body Berry Phase

ur Ansatz for many-body Berry phase,

$$\prod_{closed\ path} \langle \Psi_{K_1} | \sum_i^N e^{i(K_1-K_2)\cdot R_i} | \Psi_{K_2} \rangle, \quad \tilde{\rho}(q) = \sum_i^N e^{iq\cdot R_i}.$$

- $|\Psi_K\rangle$ is many-body state. K is many-body momentum. $\tilde{\rho}(q)$ is density operator projected into a single Landau level.

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- A crucial difference is: $\tilde{\rho}(K)$ are not commutative, but satisfies the Girvin-Macdonald-Platzman algebra,

$$[\tilde{\rho}(K_1), \tilde{\rho}(K_2)] = 2i\sin(\frac{K_1 \times K_2}{2})\tilde{\rho}(K_1 + K_2).$$

- In this work, $|\Psi\rangle$ are found either from exact-diagonalization or from model wavefunctions.

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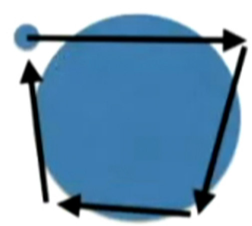
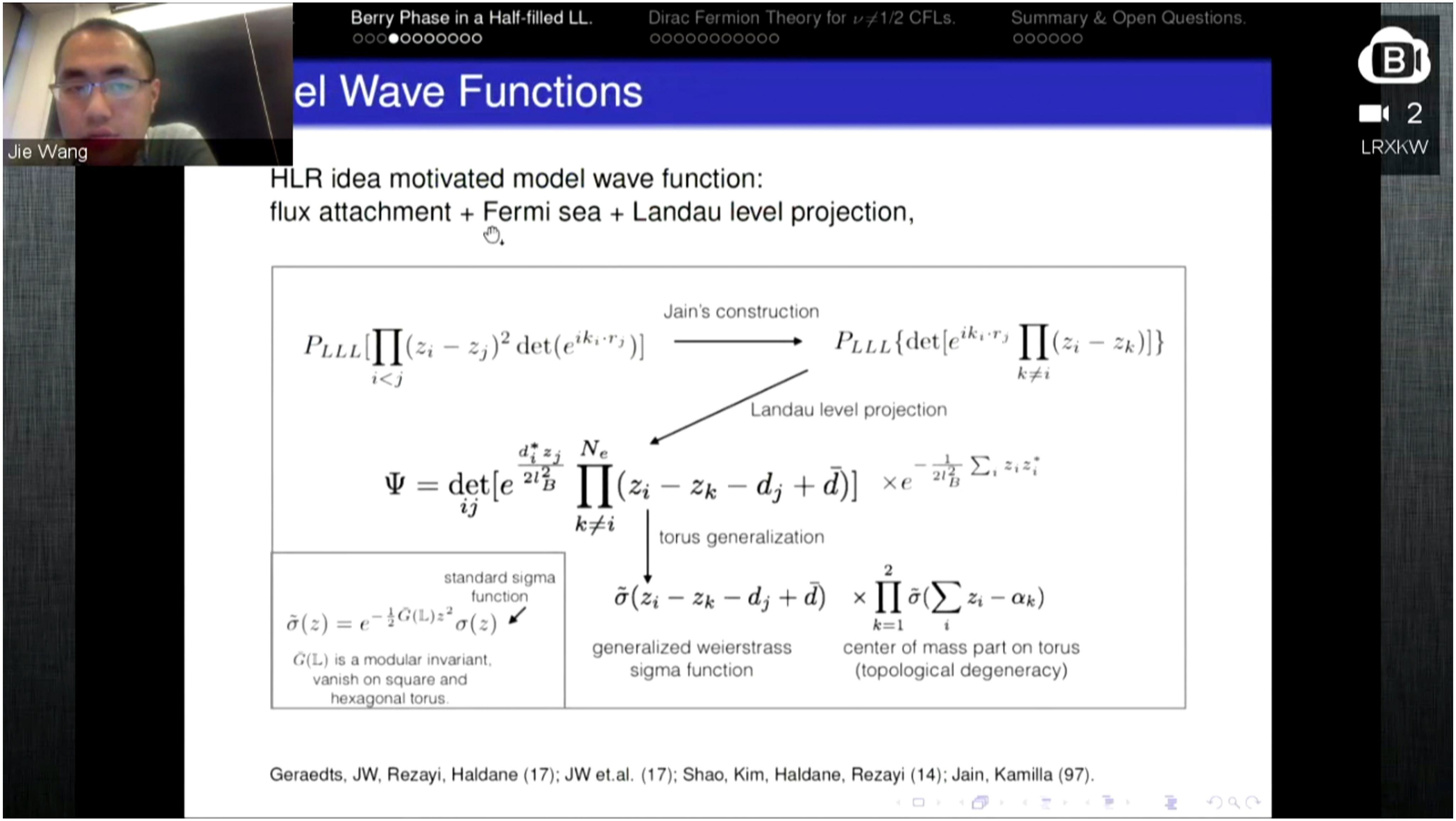


Geraedts, JW, Rezayi, Haldane (17).


Jie Wang
Berry Phase in a Half-filled LL.
Dirac Fermion Theory for $\nu \neq 1/2$ CFLs.
Summary & Open Questions.
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el Wave Functions
HLR idea motivated model wave function:
flux attachment + Fermi sea + Landau level projection,
$P_{LLL}[\prod_{i<j}(z_i - z_j)^2 \det(e^{ik_i \cdot r_j})]$
Jain's construction
$P_{LLL}\{\det[e^{ik_i \cdot r_j} \prod_{k \neq i}(z_i - z_k)]\}$
Landau level projection
$\Psi = \det_{ij}[e^{\frac{d_i^* z_j}{2l_B^2}} \prod_{k \neq i}^{N_e}(z_i - z_k - d_j + \bar{d})] \times e^{-\frac{1}{2l_B^2}\sum_i z_i z_i^*}$
torus generalization
standard sigma function
$\tilde{\sigma}(z) = e^{-\frac{1}{2}\bar{G}(\mathbb{L})z^2}\sigma(z)$
$\bar{G}(\mathbb{L})$ is a modular invariant, vanish on square and hexagonal torus.
$\tilde{\sigma}(z_i - z_k - d_j + \bar{d})$
$\times \prod_{k=1}^{2} \tilde{\sigma}(\sum_i z_i - \alpha_k)$
generalized weierstrass sigma function
center of mass part on torus (topological degeneracy)
Geraedts, JW, Rezayi, Haldane (17); JW et.al. (17); Shao, Kim, Haldane, Rezayi (14); Jain, Kamilla (97).

el Wave Functions

Model wave function at $\nu = 1/2m$ is a transcription of **HLR idea**.

$$\Psi(\{d\},\{\alpha\},\{z\}) = \det_{ij}[e^{\frac{d_i^* z_j}{2l_B^2}} \prod_{k\neq i}^{N} \tilde{\sigma}(z_i - z_k - d_j + \bar{d})] \prod_{i<j}^{N} \tilde{\sigma}^{2m-2}(z_i - z_j) \prod_{k=1}^{2m} \tilde{\sigma}(\sum_i z_i - \alpha_k).$$

- $\{z\}$ are electron coordinates. $\{\alpha\}$ are center-of-mass zeros, specify the topological sector ($2m$ fold degeneracy on torus).

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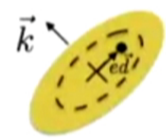
- $\{z\}$ are electron coordinates. $\{\alpha\}$ are center-of-mass zeros, specify the topological sector ($2m$ fold degeneracy on torus).
- The wavefunction dependents on a set of discrete variational parameter $\{d\}$, composite fermion dipoles.
- Pretty fundamentally, any dipolar electrons move perpendicular to the dipole direction in magnetic fields: dipole-momentum locking,

$$\frac{\boldsymbol{B} \times e\boldsymbol{d}_i}{\hbar l_B^2} = \boldsymbol{k}_i.$$



- Like momentum quantization on torus, dipoles are quantized. Like momentums form Fermi sea in crystals, dipole form Fermi sea in CFLs.

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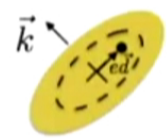

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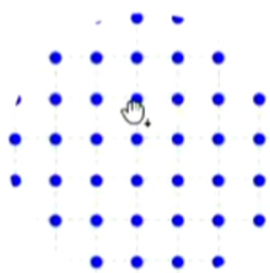
el Wave Functions

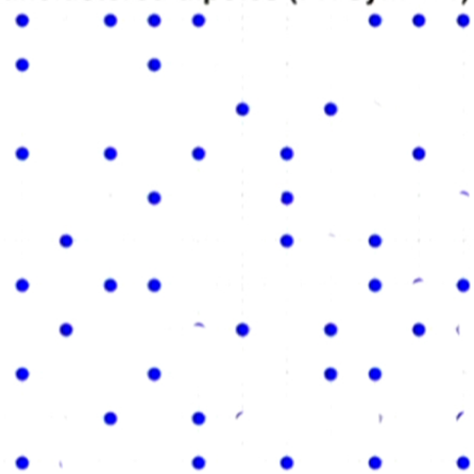
Unexpected result:

Model wave function has a very large overlap with ED ground state (98.%), and is almost particle-hole symmetric ($>$ 99.%), but only when dipoles are clustered to form a Fermi sea.

compact Fermi sea (PH sym ≈ 1)



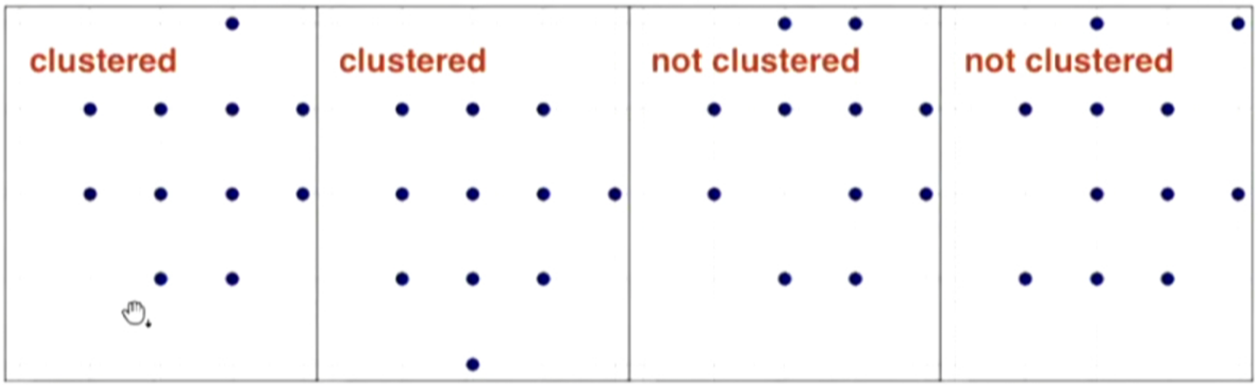
unclustered dipoles (PH sym ≪ 1)



...parison with ED ground state (LLL Coulomb)

Overlap with ED ground state:

0.981955	0.992142	0.226429	0.276141

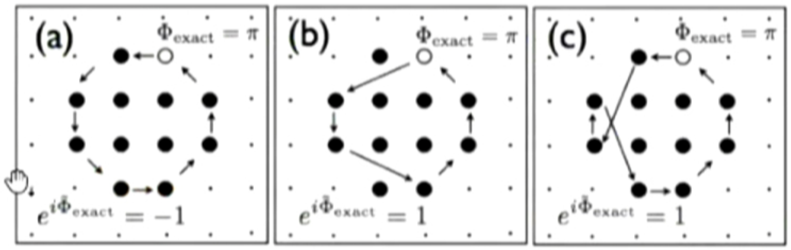


PH symmetry:

0.993543	0.995312	0.856664	0.964223

Geraedts, JW, Rezayi, Haldane (17).

erry Phase in Half Filled Landau Level



$\tilde{\Phi}_{\text{model}} = -0.930\pi$ $\quad$ $\tilde{\Phi}_{\text{model}} = 0.034\pi$ $\quad$ $\tilde{\Phi}_{\text{model}} = 0.034\pi$

Key result: $e^{i\tilde{\Phi}} = (i)^{N_+ - N_-} e^{i\Phi}$

path dependence $\quad$ Z_2 phase, +1 or -1

- The path-dependence is argued to be related to the non-commutativity of $\tilde{\rho}(K)$; the Z_2 feature supports the Son-Dirac theory.

Geraedts, JW, Rezayi, Haldane (17). JW, Geraedts, Rezayi, Haldane (17).

ce Representation

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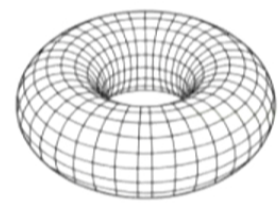
- Lattice representation: any integrations on torus can be replaced by discrete lattice summation on $N_\phi \times N_\phi$ grid. Suppose $\hat{O}$ is some observable, then,

$$\langle \Psi_1 | \hat{O} | \Psi_2 \rangle = C \sum_{z_i \in \text{lattice}} \Psi_1^* \Psi_2 \hat{O}^{LAT}(\{z\}).$$



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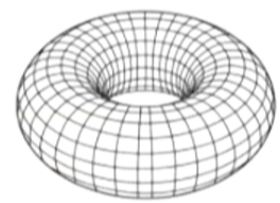


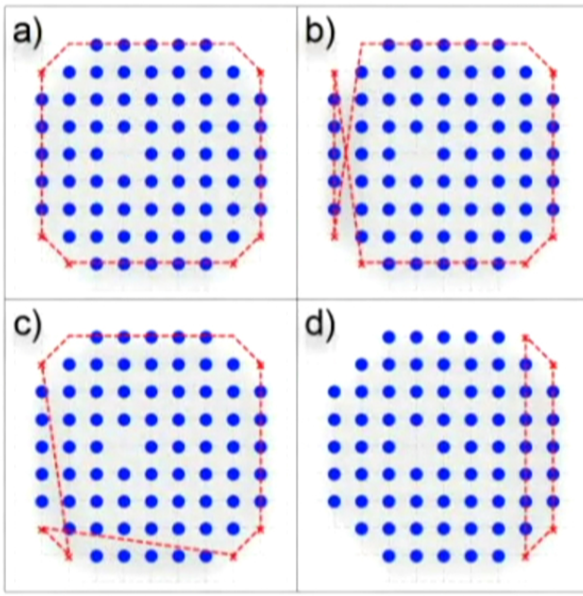
lattice= $N_\phi \times N_\phi$ evenly spaced grid

- Prove based on magnetic translation groups, thus true for all clean quantum Hall states (gapped and gapless). Has an application in Monte Carlo.

Haldane (18); JW, Geraedts, Rezayi, Haldane (17).

ce Monte Carlo results

$N = 69$ Fermi sea



JW, Geraedts, Rezayi, Haldane (17).

Expected from Z_2,

$$\Phi/\pi = \begin{cases} a), & 1 \\ b), & 1 \\ c), & 1 \\ d), & 0 \end{cases}$$

From Monte Carlo,

$$\text{“}\Phi/\pi\text{''} = \begin{cases} a), & 1.097 \pm 0.012 \\ b), & 1.057 \pm 0.015 \\ c), & 1.033 \pm 0.012 \\ d), & 0.005 \pm 0.009 \end{cases}$$

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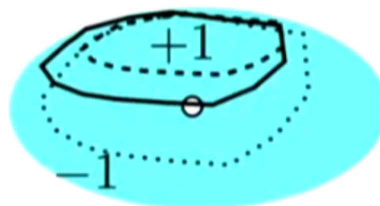
in of Z_2: parity doublet



Path passing through center is not well defined.
If one hole in the center, we have an inversion symmetric Fermi sea with **K=0, and Ne=even**. In this case, PH (plus center-of-mass translation) swaps parity partner,

$$PH\,|\pm\rangle = |\mp\rangle$$

By **particle-hole symmetry**, states will be double degenerate, and become **orthogonal** to its particle-hole conjugate.

Geraedts, JW, Rezayi, Haldane (17).

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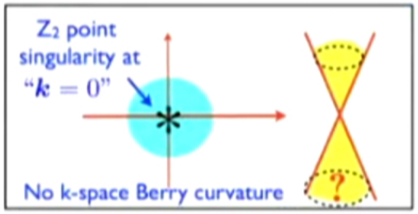

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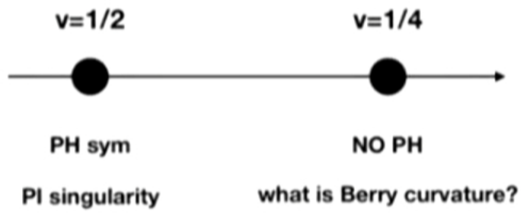
...e Comments on the Berry Curvature



- Just from the def. $e^{i\Phi} \equiv \prod_{\text{path}} \langle \Psi_{K_1} | \rho(K_1 - K_2) | \Psi_{K_2} \rangle$, Berry curvature must be quantized (cannot be uniform) at one half by PH sym.
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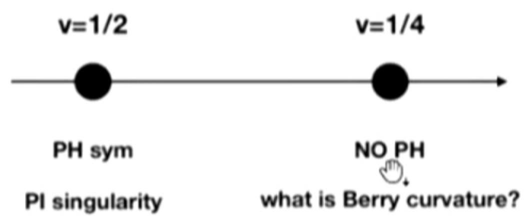

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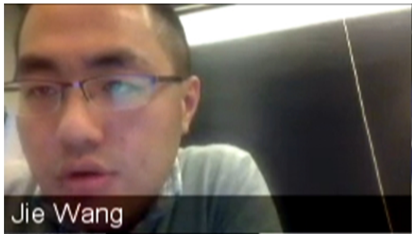
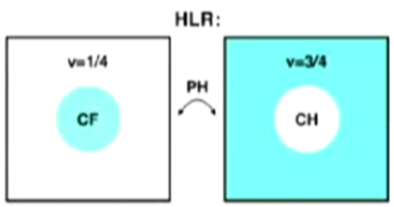
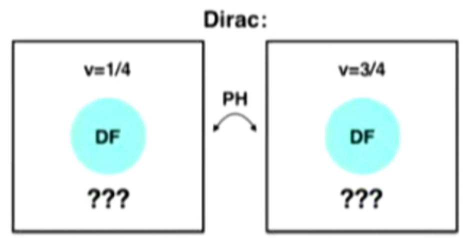
- Berry curvature is a hidden property in HLR formulation. It also characterize the CFL phase.
- The question is: what is this feature at e.g. $1/4$ CFLs? It should tell us something about the CFL phase at 1/4.



…ure of Fermi sea at $\nu\neq1/2$?

- Let's think of ν=1/4 & 3/4 CFLs first.
- The 1/4 state is a Fermi sea of **composite-fermion** while the 3/4 state is a Fermi sea of **composite-hole**.





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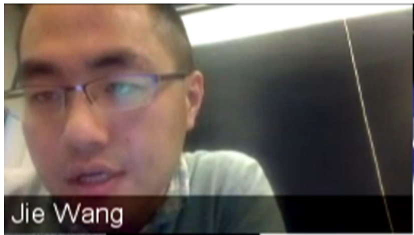

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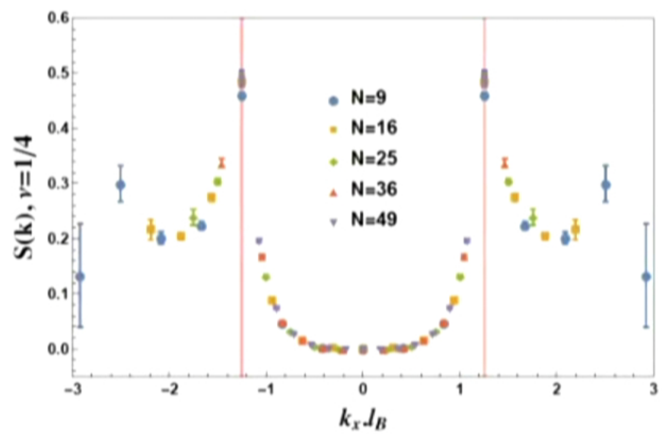
- Independent on the "nature of composite fermions", the 1/4 and 3/4 CFLs should have the same size of Fermi sea, equals to the physical electron density, according to the Luttinger Hypothesis.
- In Landau Fermi liquids, Luttinger theorem says the Fermi sea size is invariant under weak interactions. Since CFL is not Landau Fermi liquid, it is called Luttinger hypothesis. This can be tested in numerics (next slides).

erical Test of Luttinger Hypothesis

- Red lines: $2k_F$ predicted from Luttinger hypothesis.
- Dots: $S(k_x)$ with $k_y = 0$ computed from 1/4 model wavefunctions on different system sizes.



c Fermion action without PH Sym

What happens in the Son-Dirac picture, if start from 1/2 and then decrease or increase the electron density to 1/4 or 3/4?

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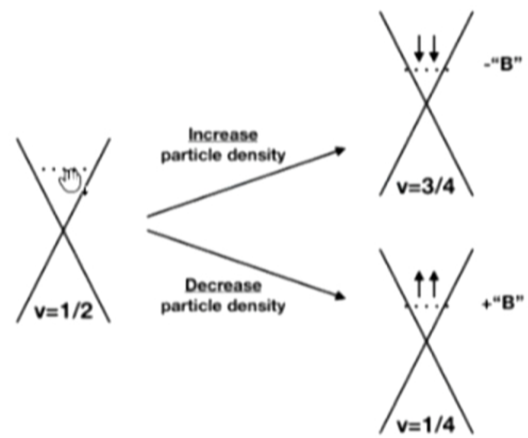

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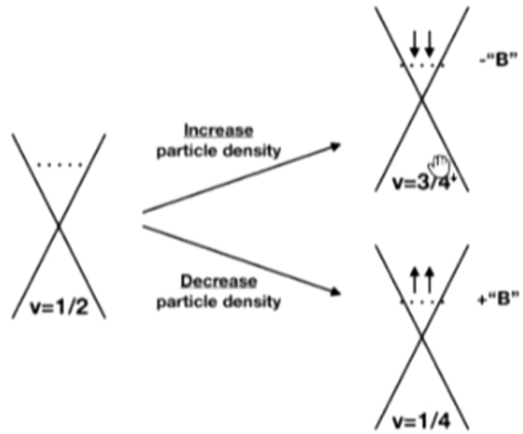
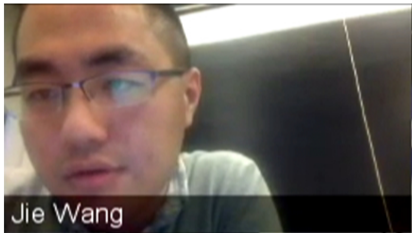


We expect,

- In both cases, Fermi sea shrinks (Luttinger hypothesis).
- Dirac fermion feels a "magnetic field" (because electron density is no longer 1/2). Sign of magnetic field dependents on density below or above one half.



c Fermion action without PH Sym

In Son-Dirac action, to the lowest order, mass term and CS term are allowed when PH symmetry is gone at ν=1/2m or 1−1/2m,

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi - \frac{1}{2}\frac{1}{2\pi}adA - \mathcal{K}\frac{1}{4\pi}ada + \mathcal{M}\bar{\psi}\psi .$$

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Just by Luttinger hypothesis, the CS level $\mathcal{K}$ is found to have a unique solution,

$$\rho_{\psi} = \frac{1}{2m}\frac{1}{2\pi l_B^2} \quad \Rightarrow \quad \mathcal{K} = \begin{cases} +\frac{1}{2} & \nu < 1/2, \\ 0 & \nu = 1/2, \\ -\frac{1}{2} & \nu > 1/2. \end{cases}$$

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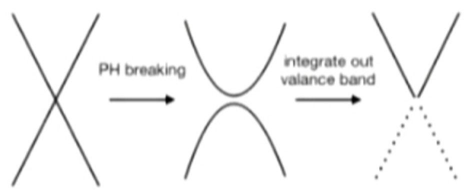
Interpretation: PH breaking opens a gap, the valance band needs to be integrated out.



In the end, I will argue the band curvature is vanishingly small, $|\mathcal{M}| \approx 0$!

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c Fermion action without PH Sym

At ν=1/2m and 1 − 1/2m,

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi - \frac{1}{2}\frac{1}{2\pi}adA - \mathcal{K}\frac{1}{4\pi}ada + \mathcal{M}\bar{\psi}\psi .$$

Because it is not 1/2 filling, DFs perceive a "magnetic field". The filling fraction for DFs is again an even denominator fraction,

$$\nu_{DF} = \begin{cases} \frac{1}{2m-2}, & \text{if} \quad \nu < 1/2, \\ \frac{-1}{2m-2}, & \text{if} \quad \nu > 1/2. \end{cases}$$

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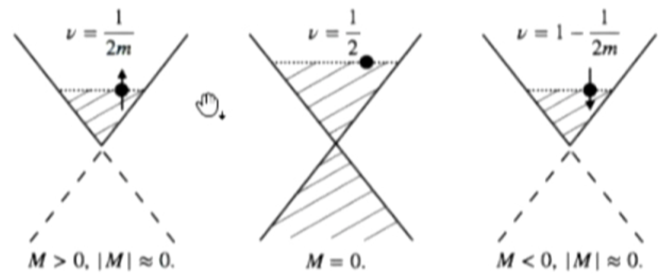
-attached Dirac Fermion Picture

Flux-attached DF picture for CFLs:
(extension of Son-Dirac)

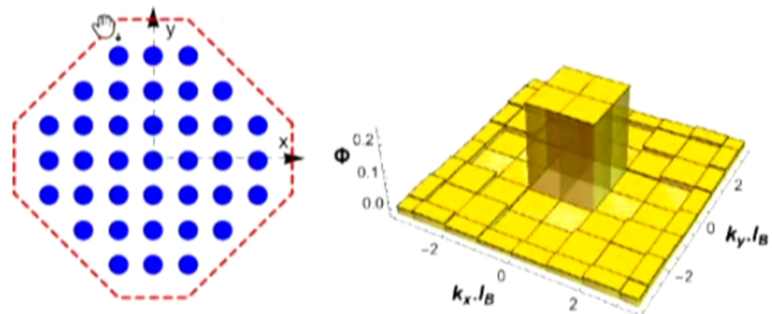
JW, 1808.07529.



- Composite Fermions are internal gauge flux attached Dirac fermions.

y Curvature from a ν=1/4 Model wavefunction

Berry Curvature Plot on N=37 Fermi Sea
(obtained by a linear regression of many closed paths near Fermi sea).



Key messages:

JW, 1808.07529.

- Moving a single dipole (composite fermion) along the red line gives $2\pi\nu = \pi/2$ Berry phase. Agree with $\sigma_H = \frac{e^2}{h}\frac{\Phi_{FS}}{2\pi}$.

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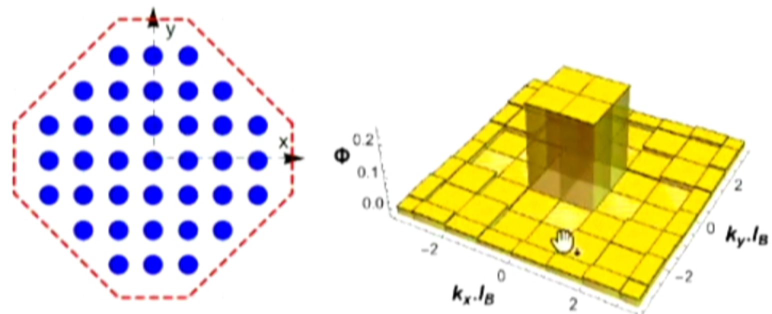

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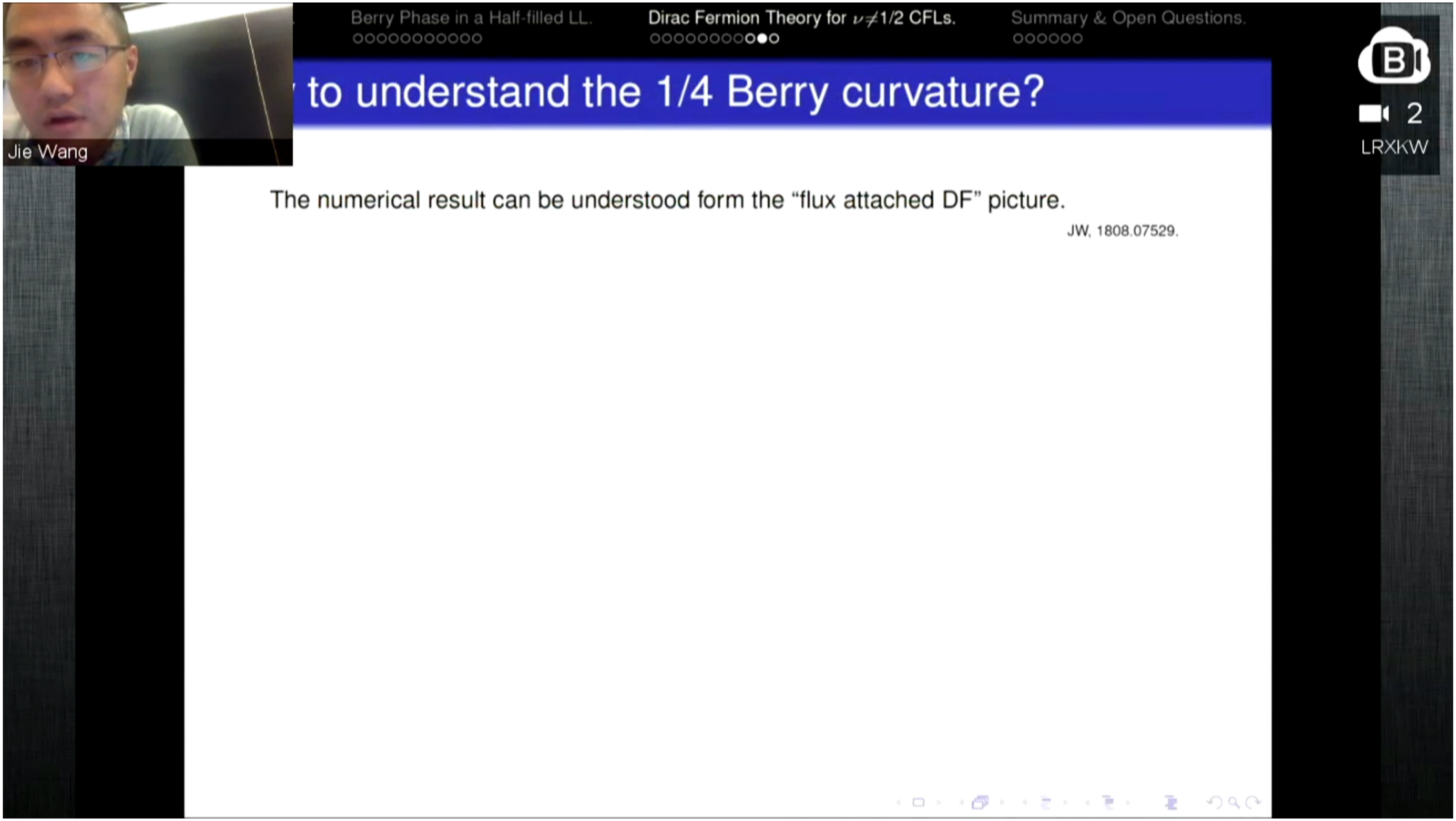
- Moving a single dipole (composite fermion) along the red line gives $2\pi\nu = \pi/2$ Berry phase. Agree with $\sigma_H = \frac{e^2}{h}\frac{\Phi_{FS}}{2\pi}$.
- The Berry curvature distribution is interesting: A π peak in the center, the rest $-\pi/2$ phase is uniformly distributed.


Jie Wang
Berry Phase in a Half-filled LL.
Dirac Fermion Theory for $\nu \neq 1/2$ CFLs.
Summary & Open Questions.
to understand the 1/4 Berry curvature?
The numerical result can be understood form the “flux attached DF” picture.
JW, 1808.07529.
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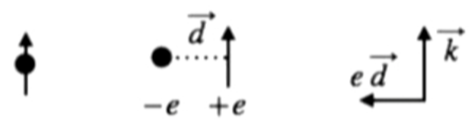
Flux-attached Dirac fermion (after LLL projection) has spin-orbit locking and dipole-momentum locking.



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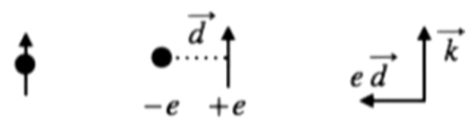
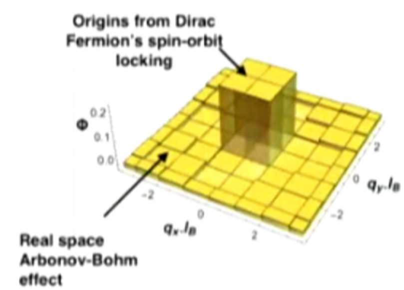


- The spin-orbit locking $\Rightarrow \pi$ singularity at center.
- The dipole-momentum locking $\Rightarrow$ uniform Berry curvature due to Arbonov-Bohm effect.



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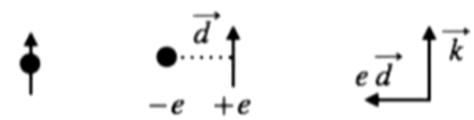

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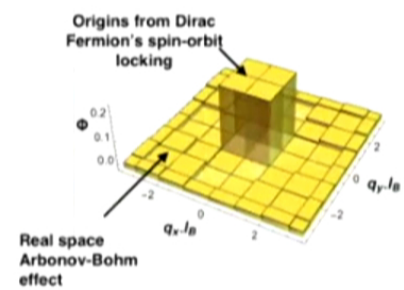
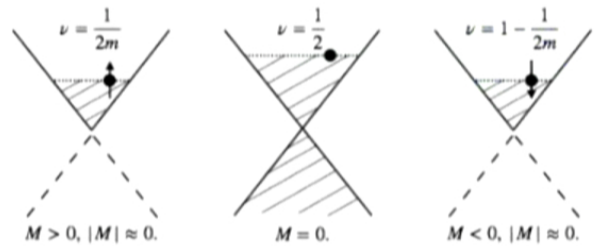


<u>Comment on the mass term</u>: constrained by the Fermi sea Berry phase, the $|\mathcal{M}| \to 0$ (dipole-momentum locking gives $2\pi\nu - \pi$; $\Phi_{FS} = 2\pi\nu$).

-attached Dirac Fermion Picture

A unified Dirac fermion picture for ν=1/2m & 1 − 1/2m CFLs,



The flux-attached DF effective action, with $\langle da/2\pi \rangle = 0$,

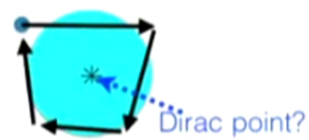
$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{2m}\frac{1}{2\pi}adA + \frac{sgn(\mathcal{M})}{2m}\frac{1}{4\pi}ada + [\frac{1}{2} - \frac{sgn(\mathcal{M})}{2}\frac{m-1}{m}]\frac{1}{4\pi}AdA + \mathcal{M}\bar{\psi}\psi.$$

JW, 1808.07529.

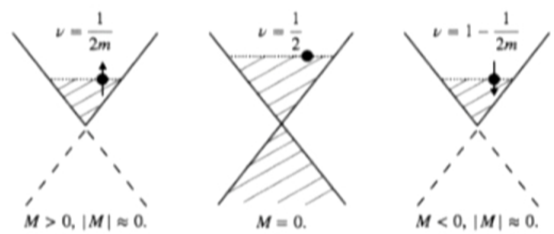
- Identical action obtained by H. Goldman & E. Fradkin motivated by the "emergent reflection symmetry". H. Goldman and E. Fradkin, 1808.09314.

mary

- At ν=1/2 (PH sym), Geraedts, **JW**, Rezayi, Haldane, 1711.09864. **JW**, Geraedts, Rezayi, Haldane, 1710.09729.
  - Defined "many-body" Berry phase.
  - Studied in detail the model wavefunction motivated by HLR's idea.
  - Developed Lattice representation.
  - Detected a Z_2 Berry phase. Suggesting "HLR=Dirac".



- At $\nu\neq$1/2, **JW**, 1808.07529.
  - Studied Berry curvature of a ν=1/4 model wavefunction. Found π phase at center, together with uniform Berry curvature.
  - Proposed flux-attached Dirac fermion theory, unifying all CFLs (underlying particles are fermions) in one picture.



re Directions

Some questions for future:

- Compute observables from HLR and flux-attached DF theory. It is a conjecture that they are equivalent.

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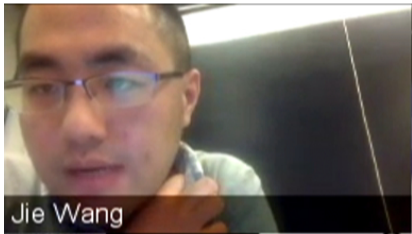

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- Understanding the PH symmetric T-Pfaffian state. Bulk gap, boundary modes.
- Is it possible that all PH symmetric quantum Hall states are gapless?

Thank you for your attention!

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