Title: Inflation from spontaneously broken Weyl invariance in theories with torsion

Date: Nov 20, 2018 11:00 AM

URL: http://pirsa.org/18110066

Abstract: <p>Introduction of the torsion tensor on the space-time manifold leads to a Weyl invariant geometry, in which t e torsion trace acts as a "U(1)" compensating field for the conformal transformations. Such symmetry can be extended to the whole matter sector included in the standard model by coupling the Higgs scalar (and all other possible fundamental scalars) to the torsion. A relevant question is then whether such framework can be used to describe cosmic inflation, as a spontaneous symmetry breaking phenomenon. It turns out that this is indeed possible because the Weyl invariance imposes a distinct field space geometry, which introduces a slow roll plateau in the otherwise too steep quartic potential dictated by Weyl symmetry. The model can have observable consequences that distinguish it from other inflationary scenarios, at least under some choices of initial conditions.</p>

Inflation from spontaneously broken Weyl invariance in theories with torsion

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Part I: Weyl invariant gravity with torsion

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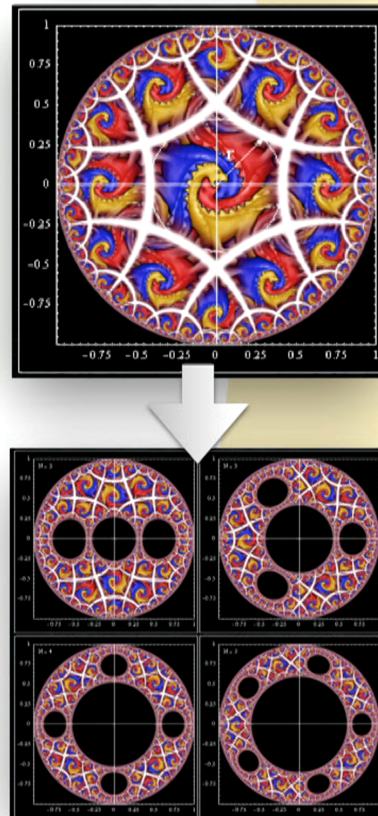
Weyl symmetry and motivation

- Renormalisable theories posses an UV fixed point of RG flow, where the theory becomes conformally invariant.

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$$

- As the Newton constant has classical dimension 2, we need large quantum corrections to it in the UV.
- Conformally invariant models, might be reachable perturbatively.
- Are there any low energy signatures of this symmetry breaking? New dofs?



Gravity with torsion

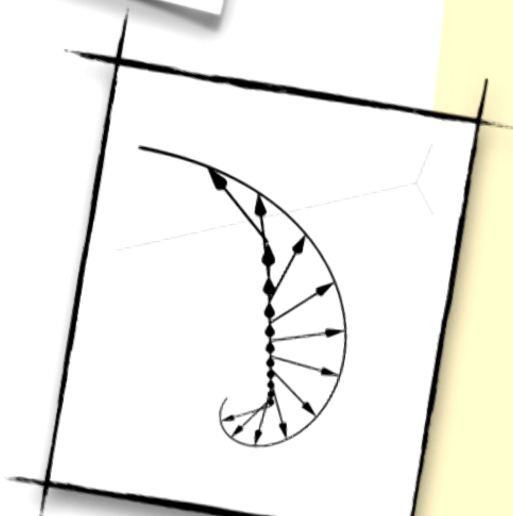
$$\nabla_X Y - \nabla_Y X - [X,Y]$$

$$e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$$

$$\Gamma^\lambda_{[\mu\nu]}$$

$$T^a = \mathrm{d}e^a + \omega^a_b \wedge e^b$$

- An additional geometrical structure, separated from curvature, linked to “twisting” of space-time.
- Misconception: torsion is not just an external field. Parallel transport works differently if torsion is present.



Geometry, Topology and Physics, Nakahara, 1989

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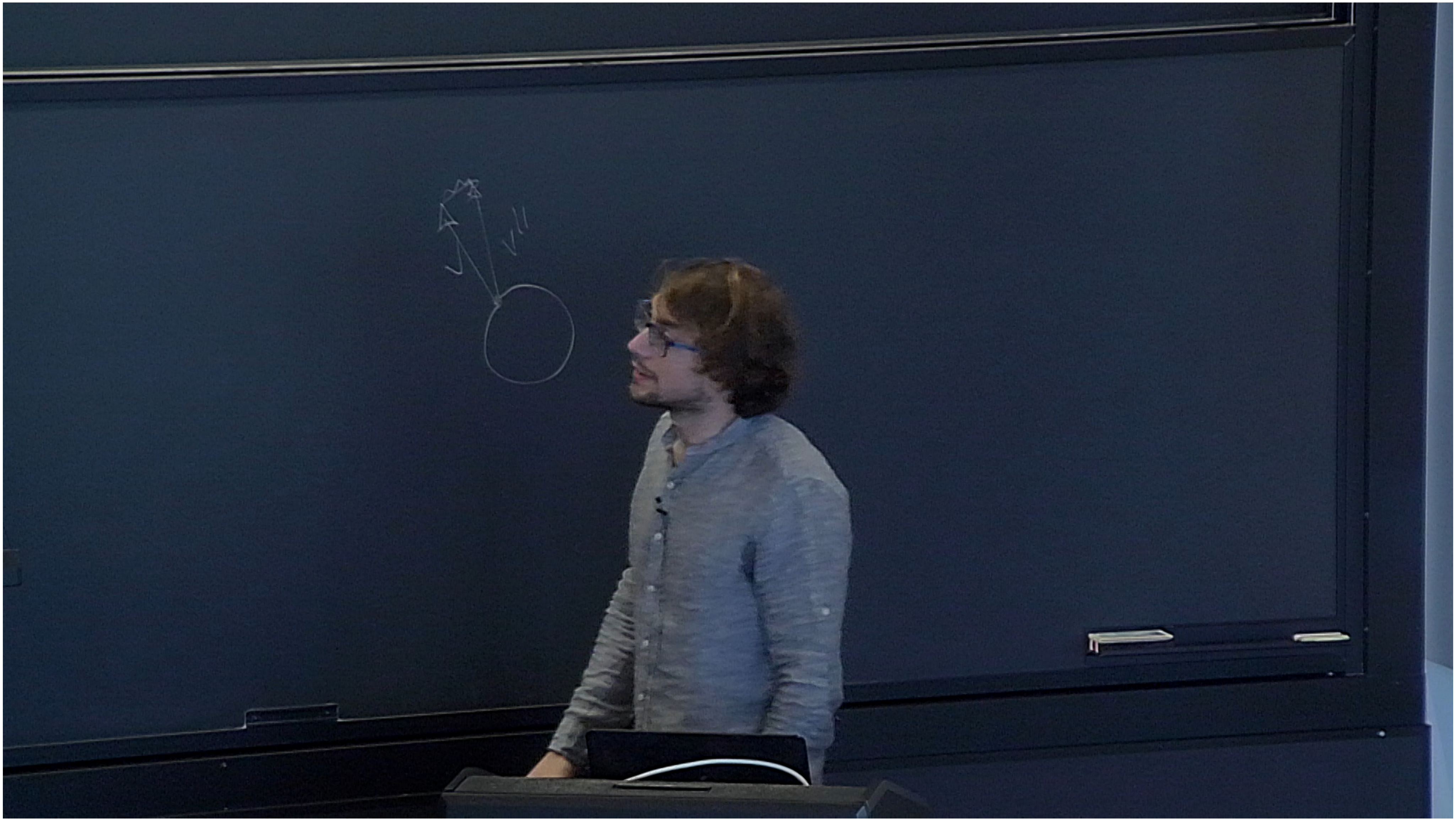
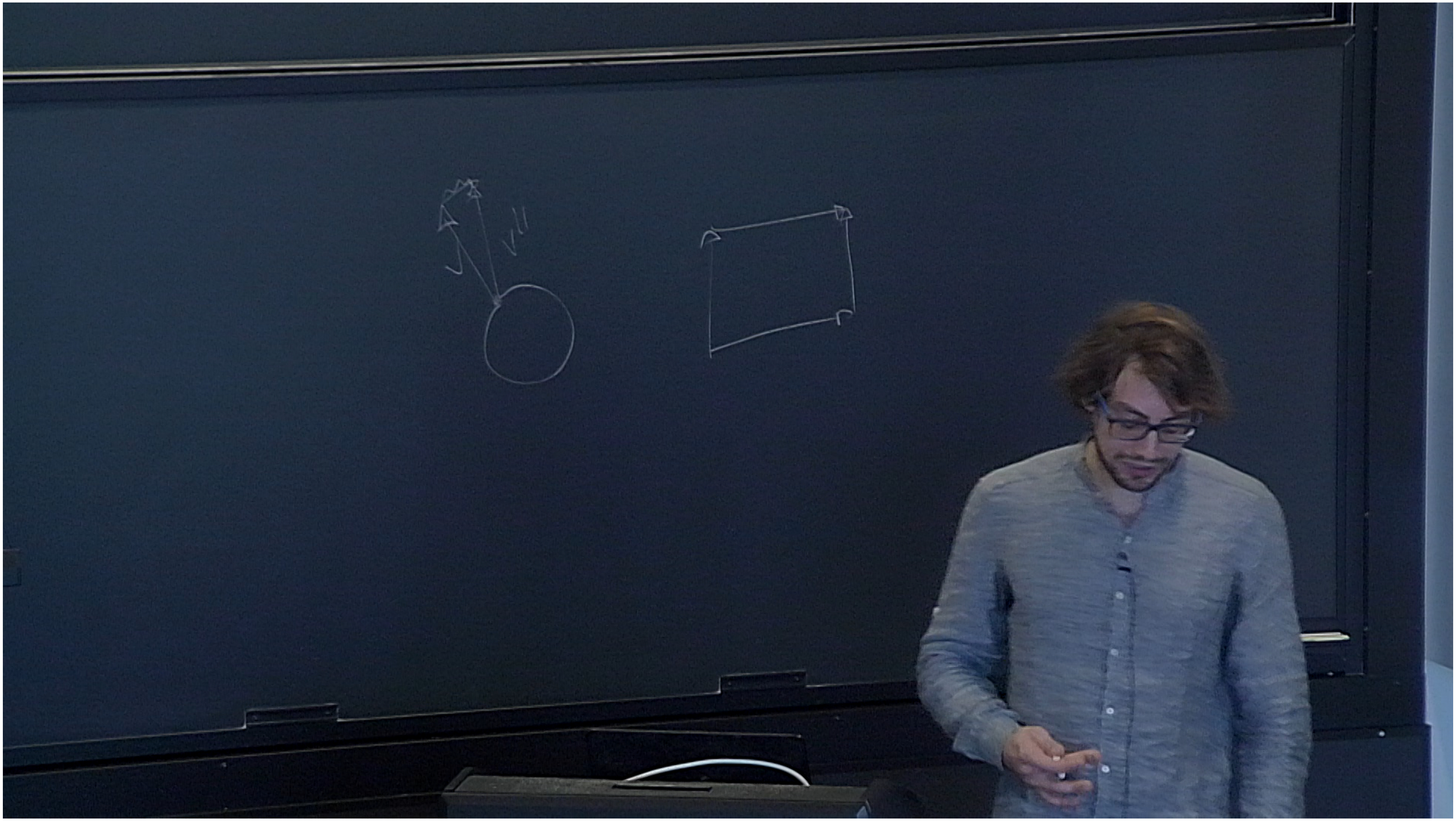


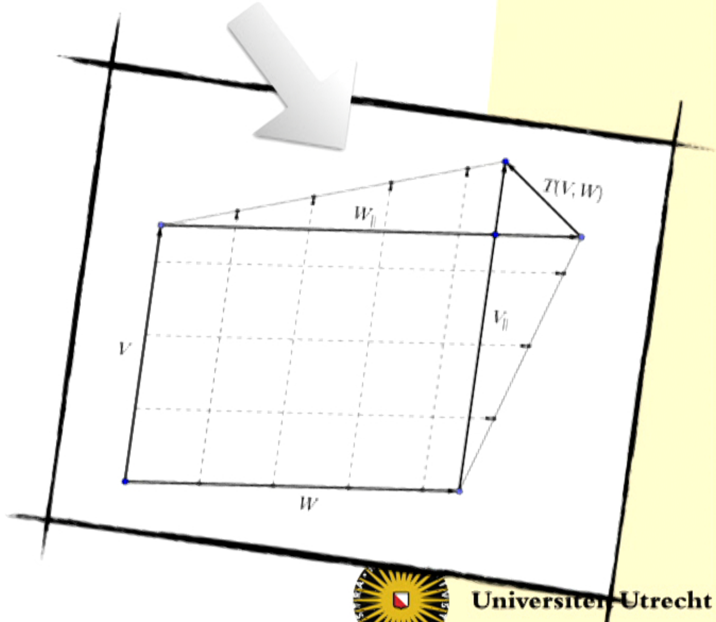


The link between torsion and Weyl symmetry

- Why should torsion be linked to Weyl symmetry?

$$\omega^a_b \to \omega^a_b \qquad e^a_\mu \to e^{\theta(x)} e^a_\mu$$

$$T^a \to T^a + e^a \wedge \mathrm{d}\theta$$



- The torsion trace is naturally linked to scale transformations.
- Transforming torsion and vierbein leaves the Cartan connection invariant.

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Geometrical properties

- Riemann curvature and geodesics trajectories are frame invariant.

$$R^{\lambda}{}_{\sigma\mu\nu} \to R^{\lambda}{}_{\sigma\mu\nu}$$

$$\nabla_{\dot\gamma}\dot\gamma^{\mu} \to e^{-\theta(x)}\nabla_{\dot\gamma}\dot\gamma^{\mu}$$

Proper time reparametrization

$$\mathrm{d}\tau_{g.i.} = \Phi \times \mathrm{d}\tau$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} \to e^{-(D-2)\theta(x)} T_{\mu\nu}$$

$$\kappa \equiv \frac{\alpha}{\Phi^2(x)} \to e^{(D-2)\theta(x)} \frac{\alpha}{\Phi^2(x)}$$

- Trajectories of free falling bodies invariant up to a reparametrization of time.
- Absence of dimension-full parameters requires dynamical Planck Mass.

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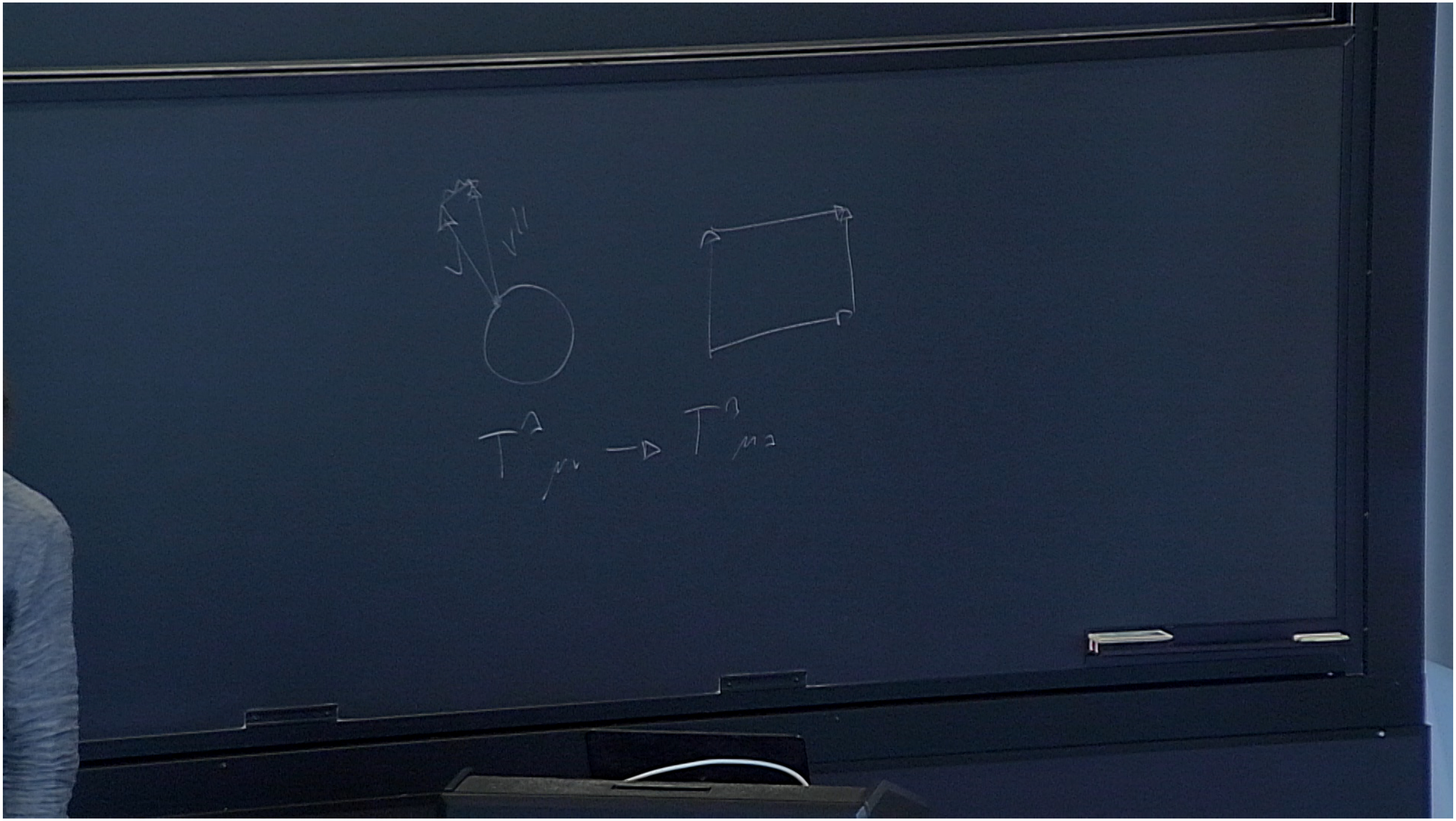

Scale symmetry and dilatation current

- Scale invariant theory possess a Noether charge, the dilatation current

$$\Pi^\mu = -\frac{D-2}{2}\phi\partial^\mu\phi$$

- If the theory is scale invariant, the equation of motion imply

$$T^\mu_\mu = -\partial_\mu \Pi^\mu$$

$$\Longrightarrow \exists\, \Theta_{\mu\nu} = T_{\mu\nu} + \frac{g_{\mu\nu}}{2}\nabla_\alpha \Pi^\alpha - \nabla_{(\mu}\Pi_{\nu)}$$

$$\Theta^\mu_\mu = 0$$

Scale and conformal invariance in QFT, J.Polchinski, 1988

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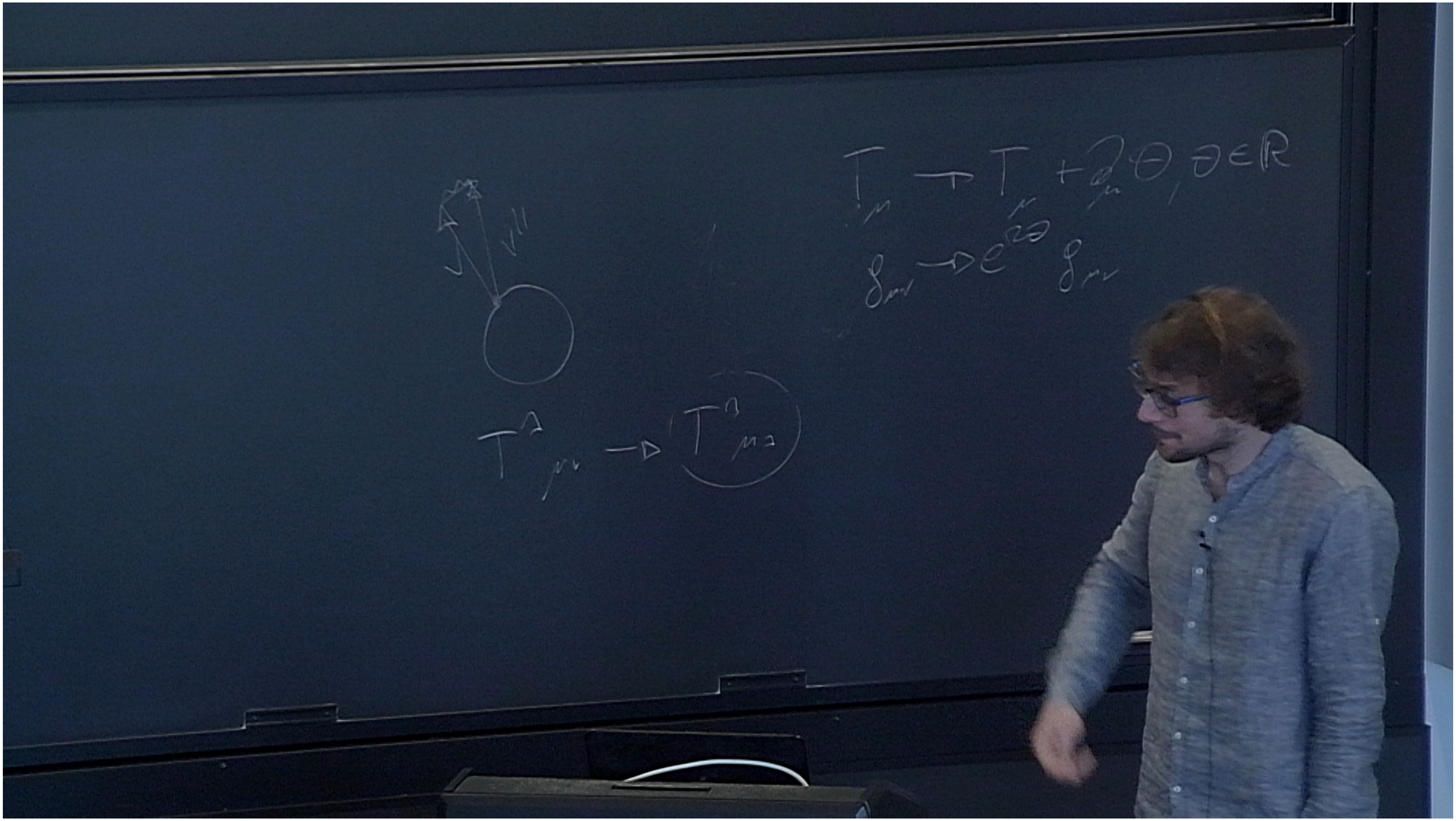
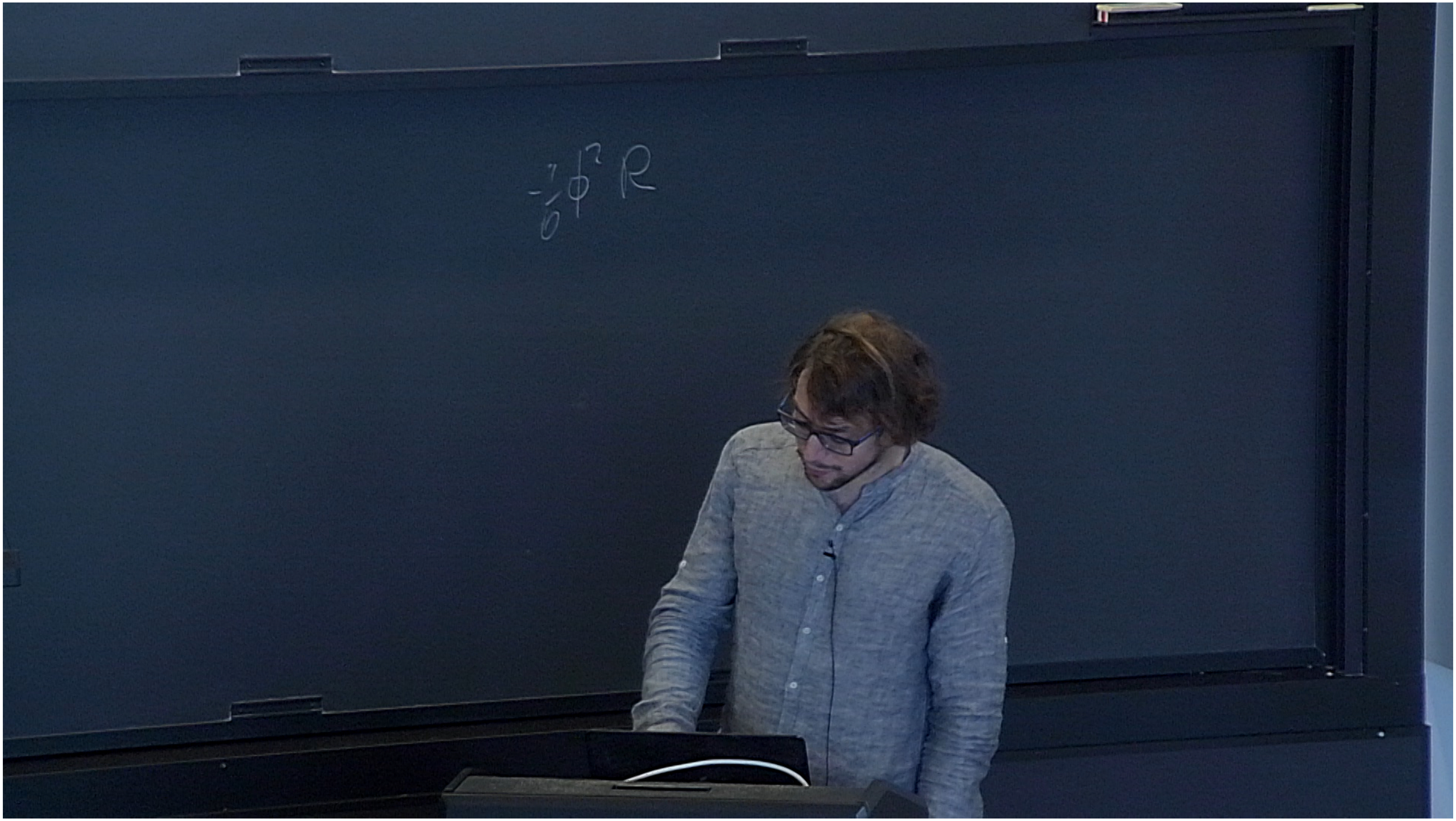


T_μ → T_μ + ∂_μ Θ, Θ ∈ ℝ
g_μν → e^{2Θ} g_μν
T^Λ_μν → T^Λ_μν


$-\frac{1}{6}\phi^2 R$

Interactions in scalar theory

- For O(N) scalars the flat space dilatation current is:

$$\Pi^\mu = \frac{D-2}{2}\phi_I \partial^\mu \phi^I$$

- Idea: couple dilatation current to torsion trace (and complete theory by requiring symmetry).

$$\mathcal{L}_{kin} = -\frac{1}{2}(\partial_\mu\phi + T_\mu\phi)^2 = -\frac{1}{2}(\nabla_\mu\phi)^2$$

- Extension of gravitational field sources. Equation of motion imply the fundamental equation:

$$\Pi^\mu = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu} \qquad T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\nabla_\mu \Pi^\mu + T^\mu_\mu = 0$$

The Polchinski condition becomes a gauge constrain

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Scale and conformal invariance in QFT, J.Polchinski, 1988

$$\Theta^\mu_\mu = 0$$

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What did we gain?

- "Local" anomaly:

$$\langle \hat{T}^{\mu}_{\mu} \rangle = C\left(\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta}\right) \neq 0$$

- Including torsion trace this is compensated, and does not violate the fundamental Ward identity.

$$T^{\mu}_{\mu} + \nabla_{\mu}\Pi^{\mu} = 0$$

- This is because the Gauss Bonnet integral is a boundary term, and gets absorbed in the divergence of the dilatation current.

$$\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta} = \nabla_{\mu}\mathcal{V}^{\mu}$$

$$\Longrightarrow \Pi^{\mu} \to \Pi^{\mu} + \mathcal{V}^{\mu}$$

Is conformal symmetry really anomalous?, S.Lucat, T.Proko[illegible]16

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Interactions in scalar theory

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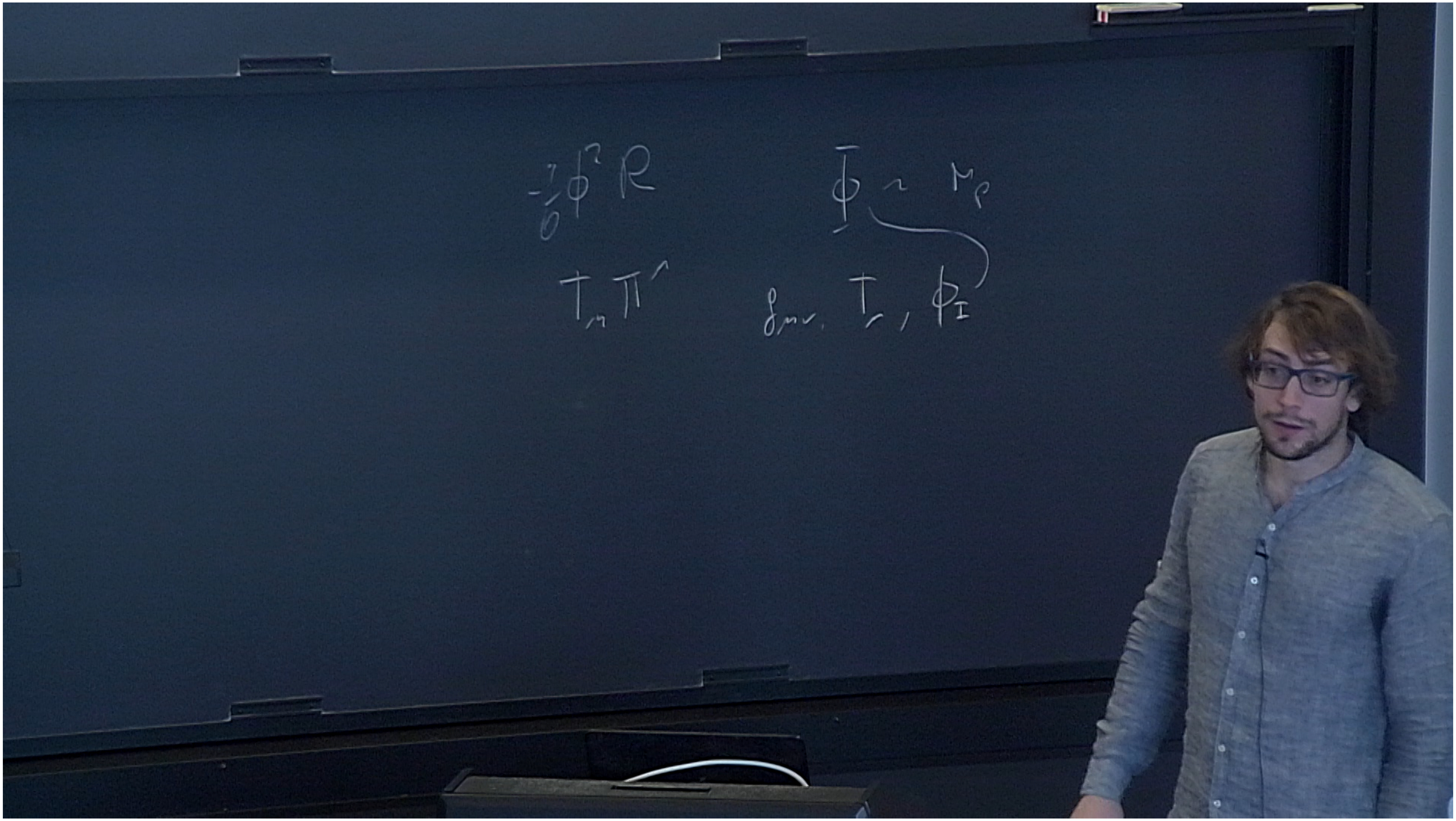
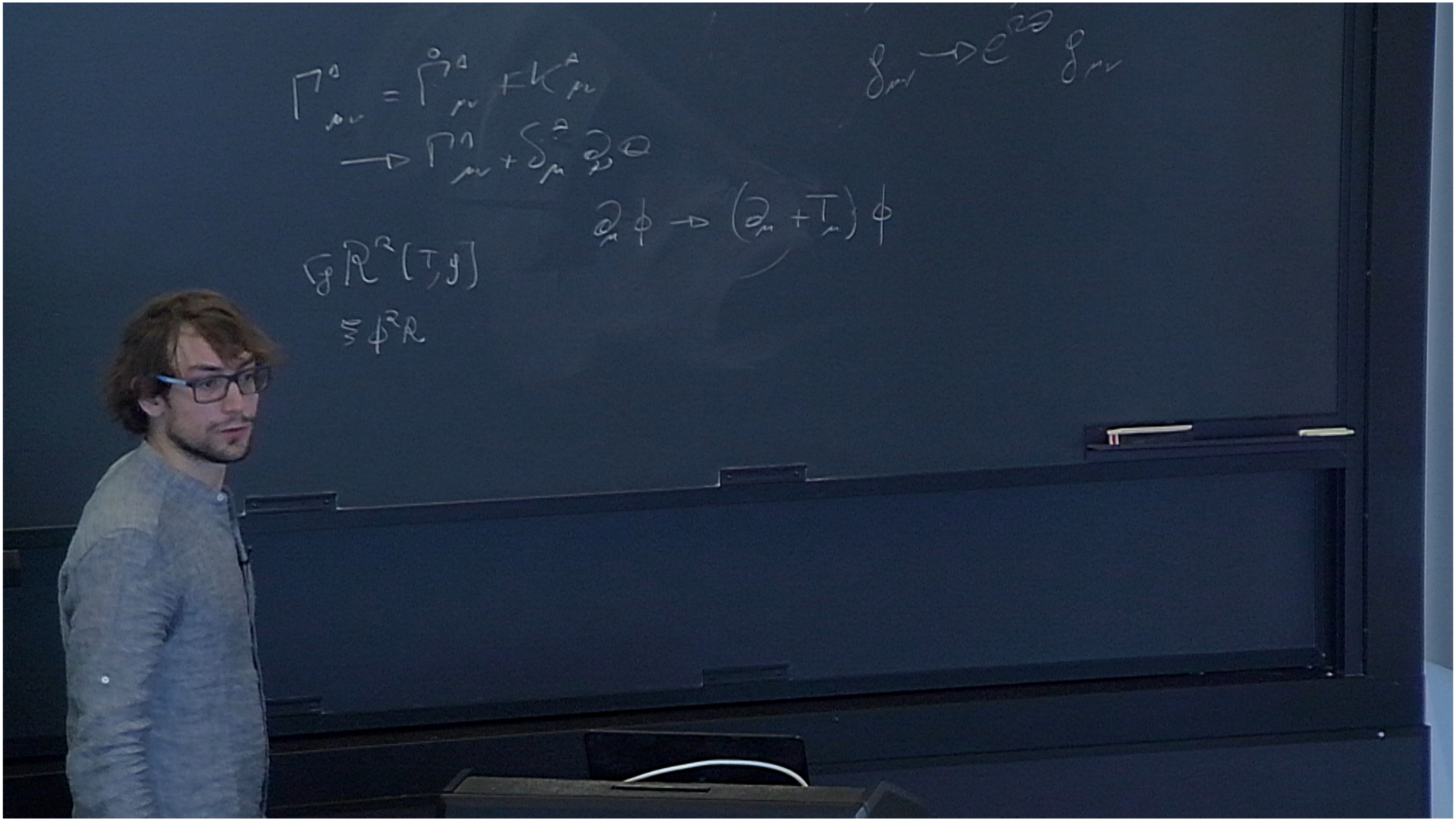
Part II: the inflationary model




$\Gamma^{\lambda}_{\mu\nu} = \mathring{\Gamma}^{\lambda}_{\mu\nu} + K^{\lambda}_{\mu\nu}$
$\longrightarrow \Gamma^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu} \partial_{\nu} \theta$
$g_{\mu\nu} \longrightarrow e^{2\theta} g_{\mu\nu}$
$\partial_\mu \phi \longrightarrow (\partial_\mu + T_\mu) \phi$
$\sqrt{g} R^2 [T, g]$
$\xi \phi^2 R$

Conformal symmetry breaking

- At the conformal point we conjecture the existence of a self consistent Weyl invariant theory.

- What is the UV theory? Maybe SO(2,4) local. Symmetry breaking:

$$SO(2,4) \to SO(1,4) \to SO(1,3)$$

$$\mathcal{M} = \Omega^{-1}\mathrm{d}\Omega\,,\ \Omega \subset SO(2,4)/SO(1,3)$$

$$[K^a, P^b] = 2i\eta^{ab}D - 2i\Sigma^{ab}\,,$$
$$P^b\,, \Sigma^{ab}\,, K^a\,, D \in SO(2,4)$$

$$e^a_\mu = \exp(\phi^0)\delta^a_\mu\,, T^a = e^a \wedge \mathrm{d}\tilde{\phi}^0$$

Low energy degrees of freedom

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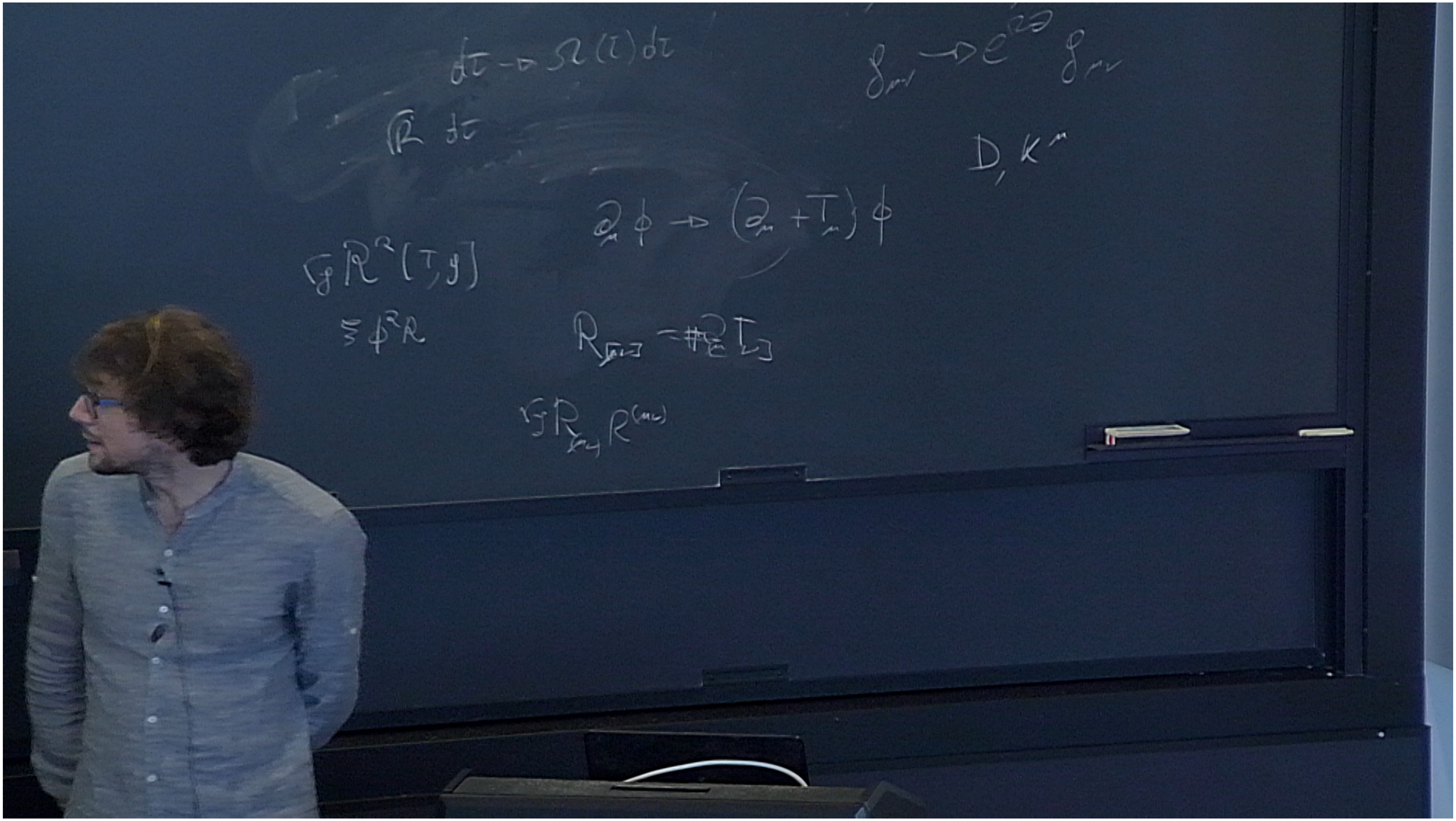

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Low energy degrees of freedom

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Modified gravity theory

- By symmetry all operators are marginal (dimensionless couplings)

$$\mathcal{L}_\phi = -\frac{1}{2}(\bar{\nabla}\phi)^2 + \frac{\xi}{2}\phi^2\bar{R} + \lambda\phi^4$$

- Any curvature coupling is allowed, as long as the coupling constant is dimensionless

$$\mathcal{L}_{grav} = \alpha\bar{R}^2 + \beta\bar{R}_{\mu\nu}\bar{R}^{\mu\nu} + \gamma F_{\mu\nu}F^{\mu\nu}$$

- Second order in curvature possible and automatically satisfy the constrain

$$T^{grav}_{\mu\nu}g^{\mu\nu} + \nabla_\mu \Pi^\mu_{grav} = 0$$

- The rest of the SM is ok in D=4!

$$\frac{i}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\nabla}_\mu\psi + \phi\bar{\psi}\psi + \mathrm{Tr}\left(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)$$

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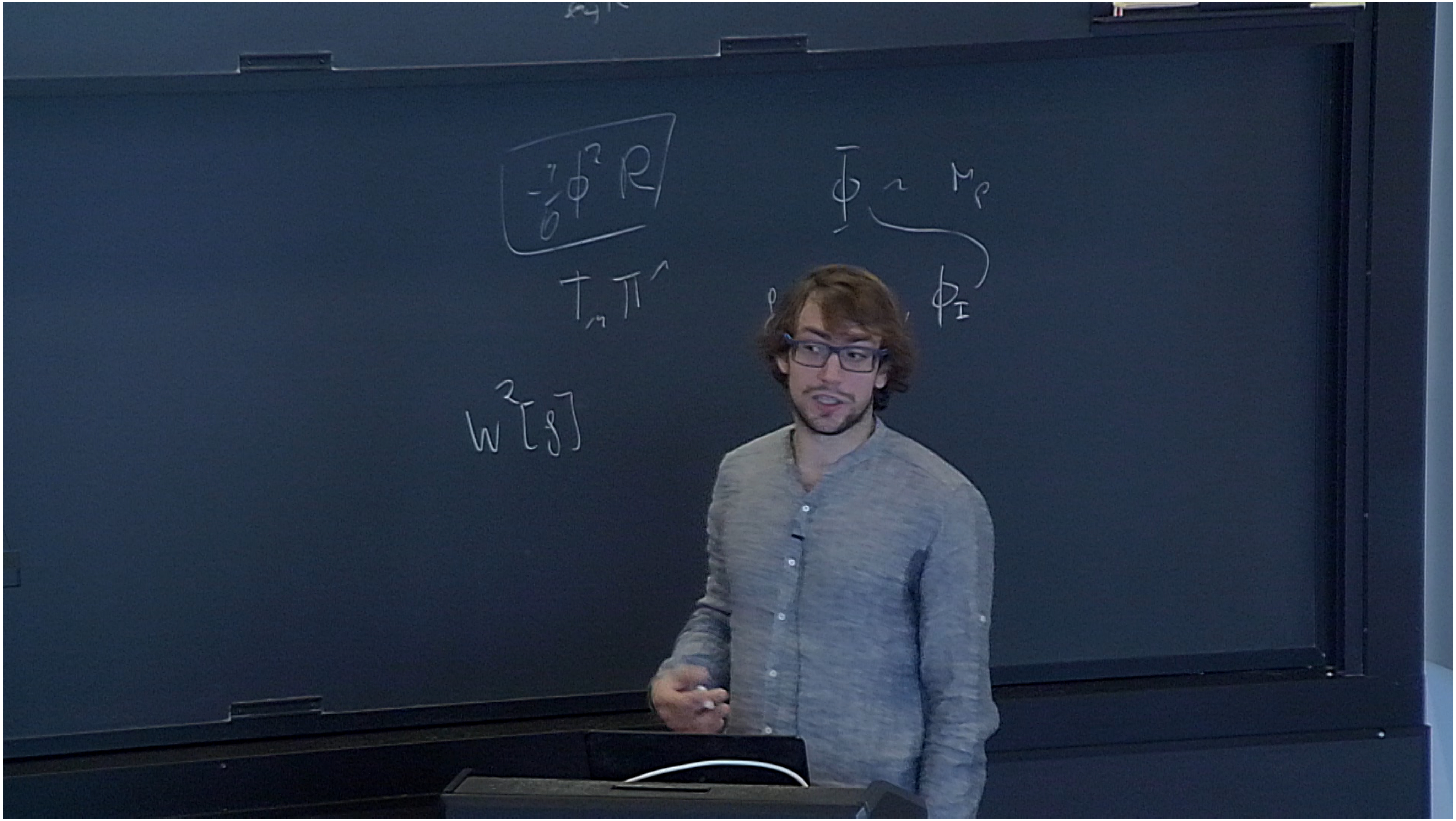
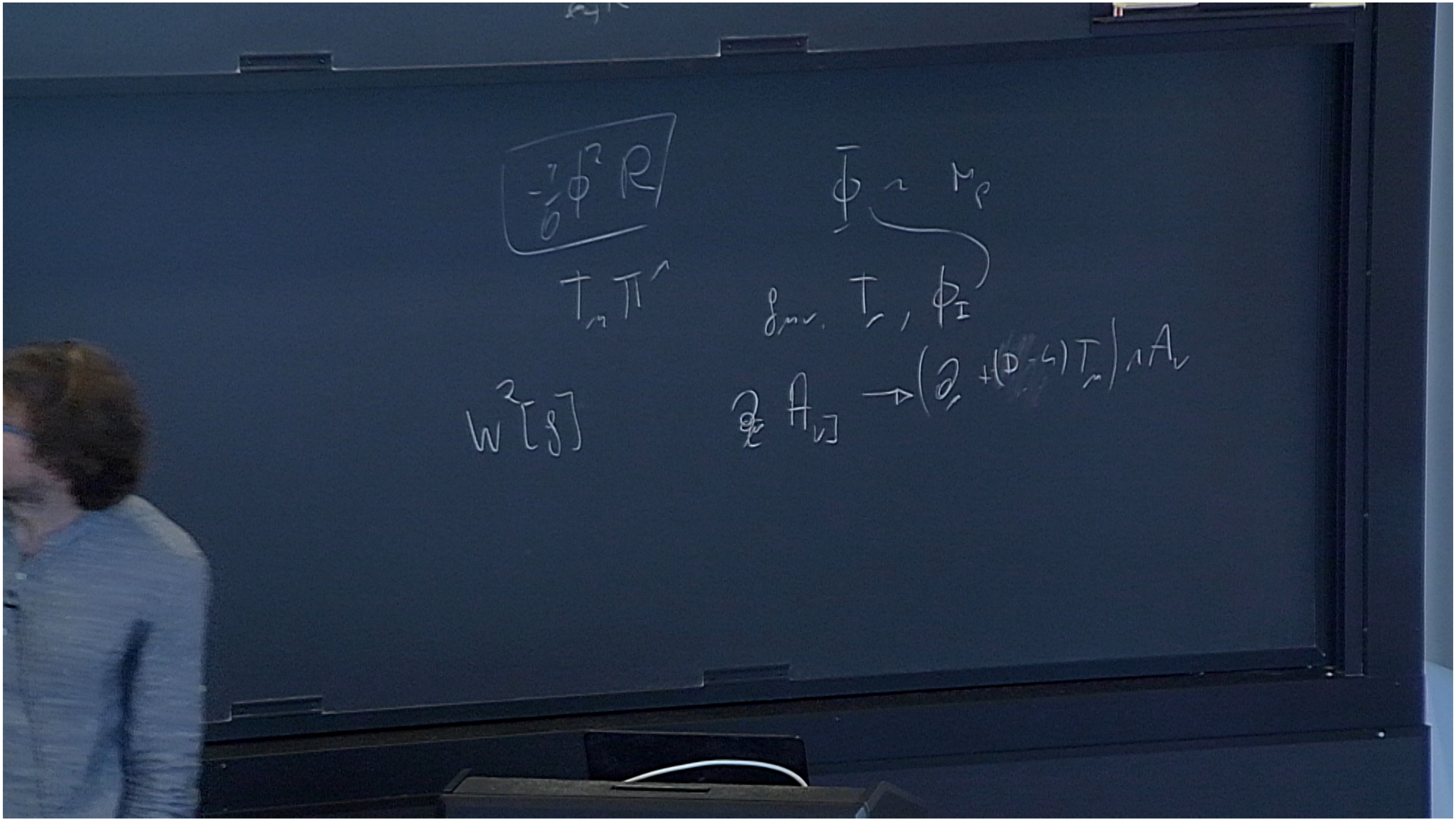


W²[g]



Einstein frame and degrees of freedom

- First we define an Einstein's frame, by introducing lagrange multiplier and gauge fixing it:

$$S_E = \int \mathrm{d}^4x\sqrt{-g}\Big[-\Big(\frac{\xi^2}{16\alpha}+\lambda\Big)\left(\delta_{IJ}\phi^I\phi^J\right)^2 + \frac{\xi}{8\alpha}\omega^2\delta_{IJ}\phi^I\phi^J + \frac{\omega^2}{2}\Big(\mathring{R}+6\mathring{\nabla}^2\phi^0-6\partial_\mu\phi^0\partial^\mu\phi^0\Big) - \frac{\omega^4}{16\alpha} - \frac{1}{2}\delta_{IJ}g^{\mu\nu}\left(\partial_\mu+\partial_\mu\phi^0\right)\phi^I\left(\partial_\nu+\partial_\nu\phi^0\right)\phi^J\Big]$$

$$\omega^2 \to M_P^2$$

Not a dynamical field

- Hamiltonian analysis reveals N+1 dynamical scalars, and one constrain,

$$\phi^I P_I + \omega P_\omega - 2P^{ij}\gamma_{ij} - P_0 = 0\,,\; P_\Psi = \frac{\delta\mathcal{L}}{\delta\dot{\Psi}}$$

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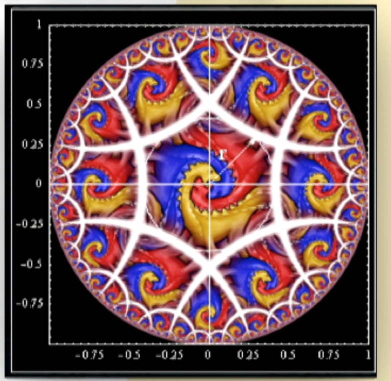
The geometry of the scalar sector

- In the Einstein frame, by definition, the dynamics is as described by general relativity, with Hamiltonian,

$$N\mathcal{H} + N_i\mathcal{H}^i$$

$$P_A P_B \frac{\mathcal{G}^{AB}}{2\sqrt{\gamma}} \subset \mathcal{H}\,, A\,, B = 0\,, 1\,, \cdots\,, N$$



$$\mathcal{G}^{AB} = \begin{pmatrix} \frac{1}{6M_P^2} & -\frac{\phi^I}{6M_P^2} \\ -\frac{\phi^I}{6M_P^2} & \delta^{IJ} + \frac{\phi^I\phi^J}{6M_P^2} \end{pmatrix}$$

Has curvature

$$\mathcal{R} = -\frac{\mathcal{N}(\mathcal{N}+1)}{6M_P^2}$$

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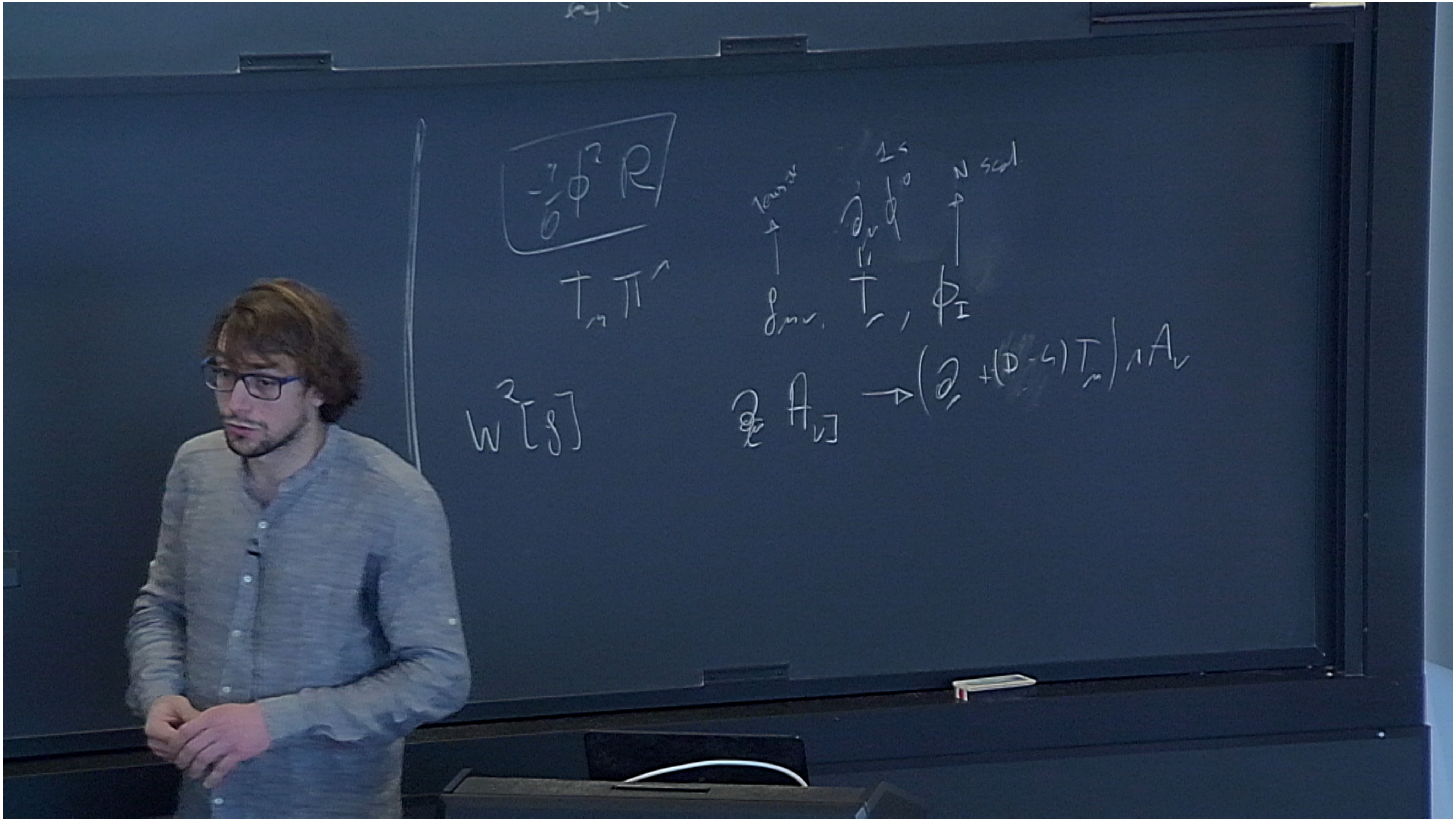

The geometry of the scalar sector II

- The dilaton kinetically mixes with the other scalars, and corresponds to a flat direction, like other goldstones, but importantly it is non compact,

$$\phi^0 = \chi - \frac{1}{2}\log\left(1 + \frac{\rho^2}{6M_P^2}\right)\,, \rho = \sqrt{\phi^I\phi^J\delta_{IJ}} = \sqrt{6}M_P\sinh\psi$$

$$\mathcal{G}_{AB}\mathrm{d}\phi^A\mathrm{d}\phi^B = 6M_P^2\left[\mathrm{d}\psi^2 + \left(\cosh^2(\psi)\mathrm{d}\chi^2 + \sinh^2(\psi)\mathrm{d}\Omega_{\mathcal{N}-1}\right)\right]$$

- This corresponds to the Poincaré disk of N+1 dimension.

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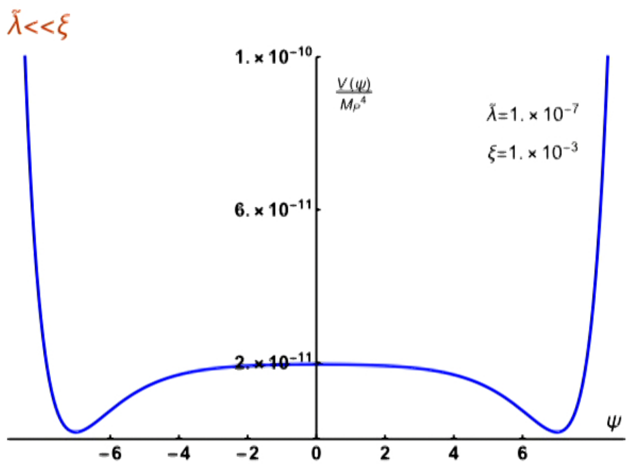
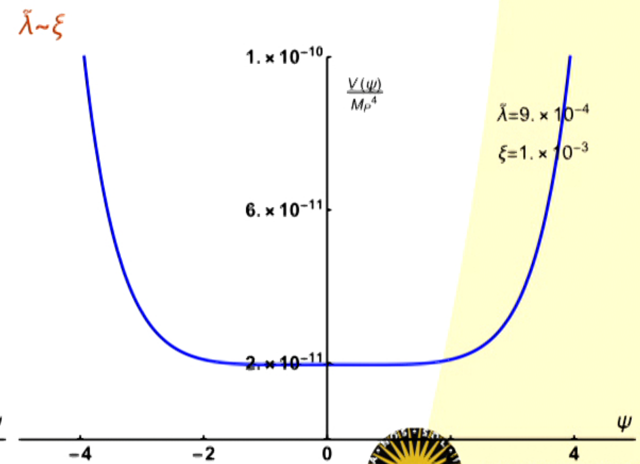
Einstein's frame potential

$$\alpha \leftrightarrow R^2\,, \lambda \leftrightarrow \phi^4\,, \xi \leftrightarrow \phi^2 R$$

Jordan frame parameters

- The potential in the Einstein's frame has a maximum and two minimum, but requires a hierarchy to have a chance of exiting inflation:





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Inflationary dynamics

- The quartic coupling in the Jordan frame controls the (positive) vacuum energy left at the end of inflation. If the Higgs mass parameter is also spontaneously generated,

$$\frac{V(\psi_m)}{M_P^4} = \frac{\lambda}{\xi^2 + 16\alpha\lambda} \simeq \frac{V_{EW}(H_m)}{M_P^4}$$

$$\Longrightarrow \Lambda_{eff} = V(\psi_m) - V_{EW}(H_m) \simeq 0$$

- This can be realised by requiring:

$$\xi^2 \gg 16\alpha\lambda$$

$$\lambda \ll \xi^2$$

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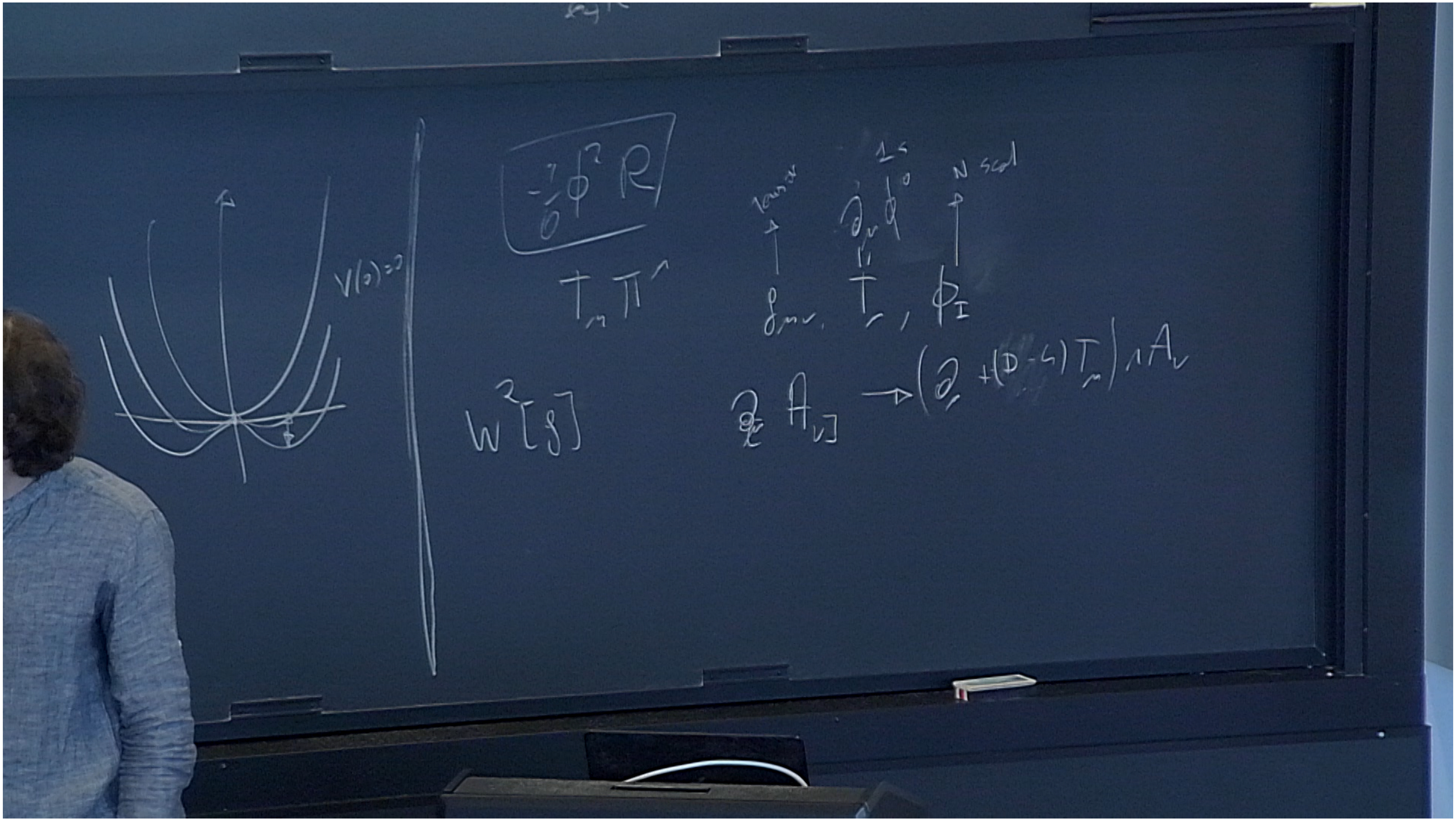

V(φ)=0
-1/6 φ² R
T_μ Π^μ
g_μν, T_μ, φ_I
N scal
W²[g]
A_ν] → (∂_μ + (D-4) T_μ) A_ν

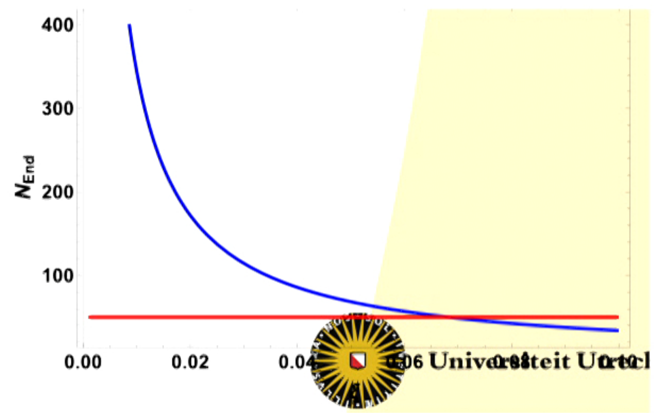
Inflationary dynamics II

- The curvature squared coupling controls the size of scalar perturbations,

$$\Delta_s^2(k=k^*) = 3.089 \times 10^{-10} \implies \alpha \simeq 10^9$$

- Non minimal coupling and initial value of the radial field controls the length of inflation, to get N>50 e-folds,

$$\psi_0 \simeq 10^{-6} \implies \xi < 0.07$$



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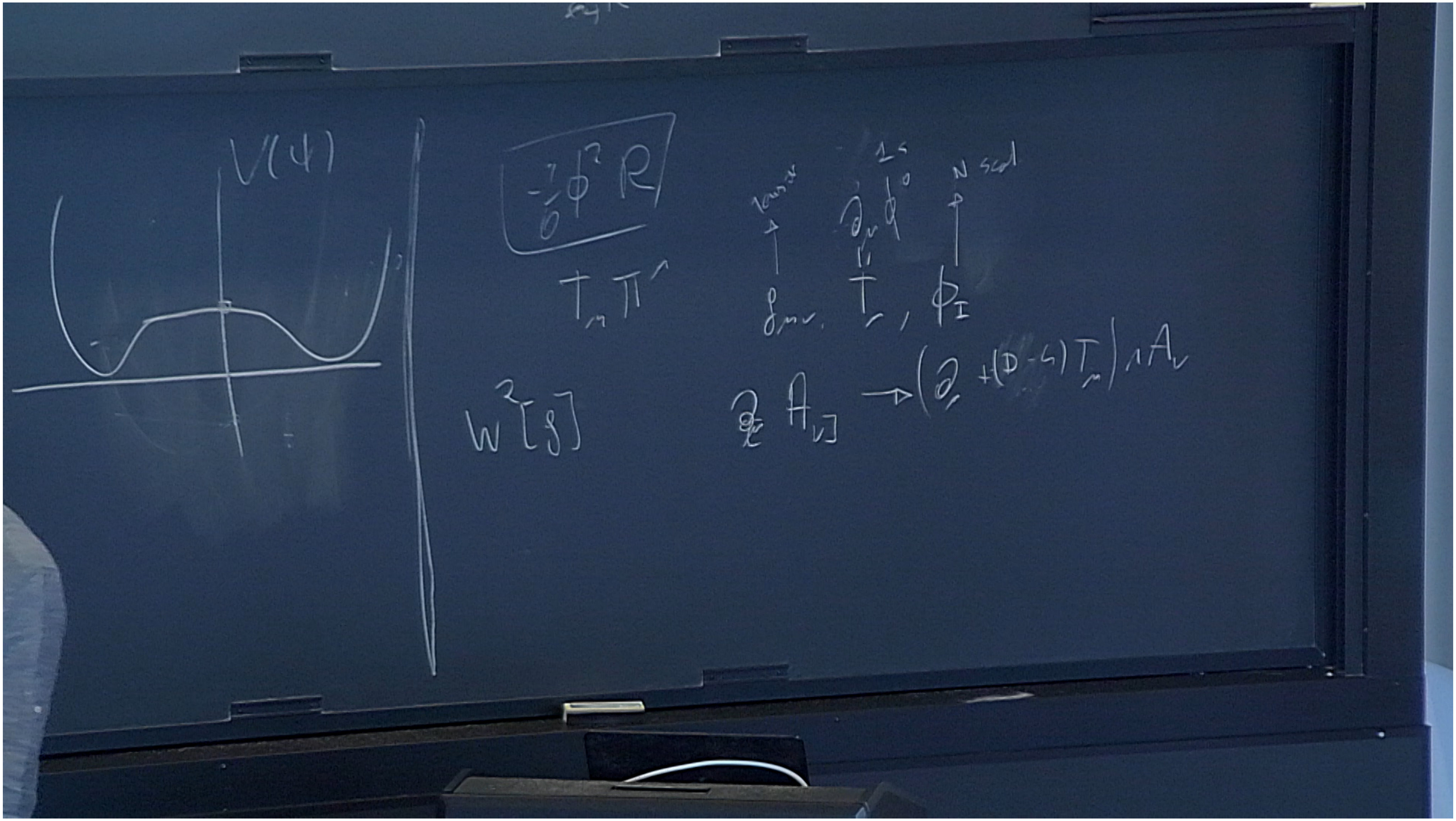
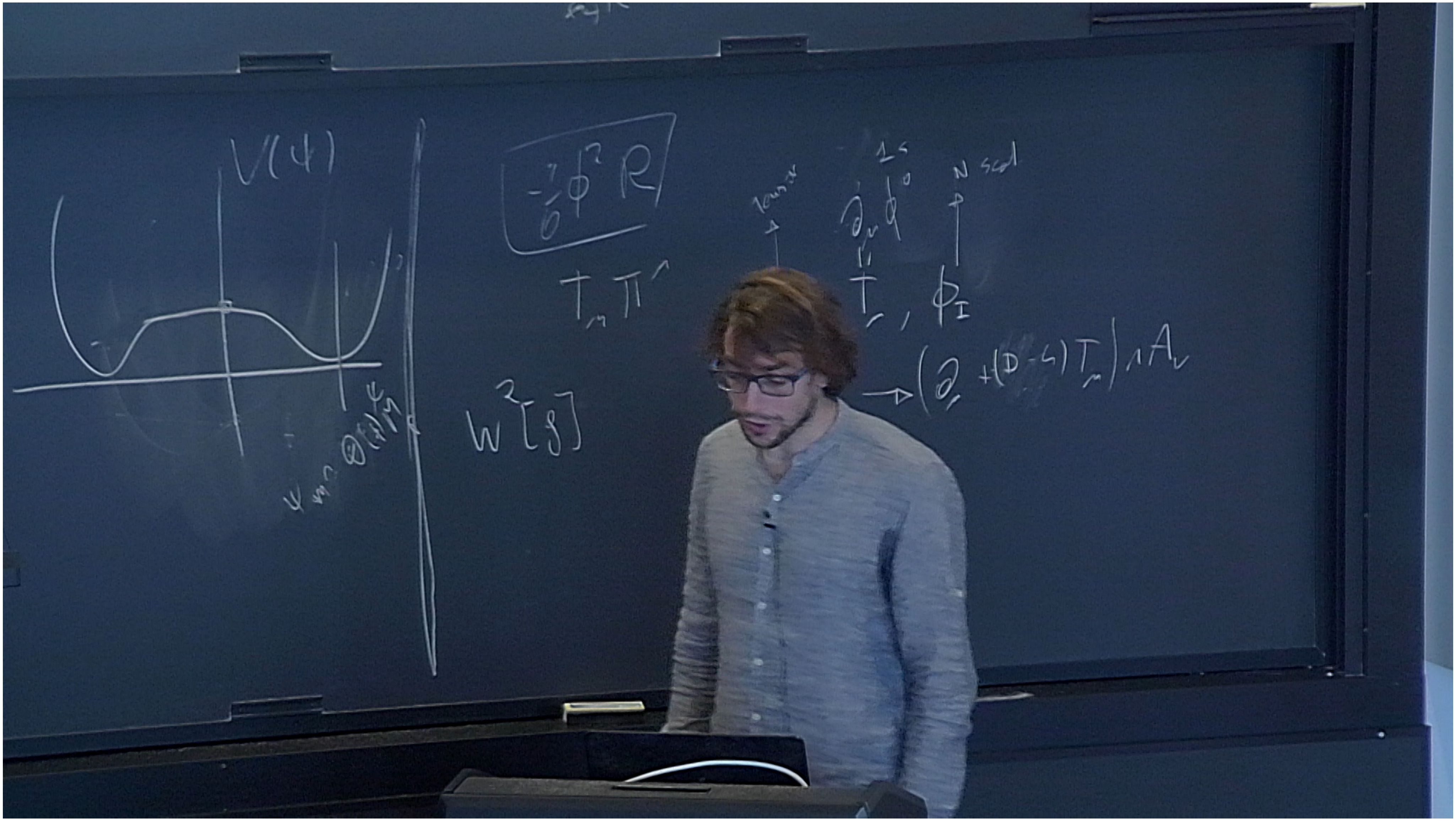
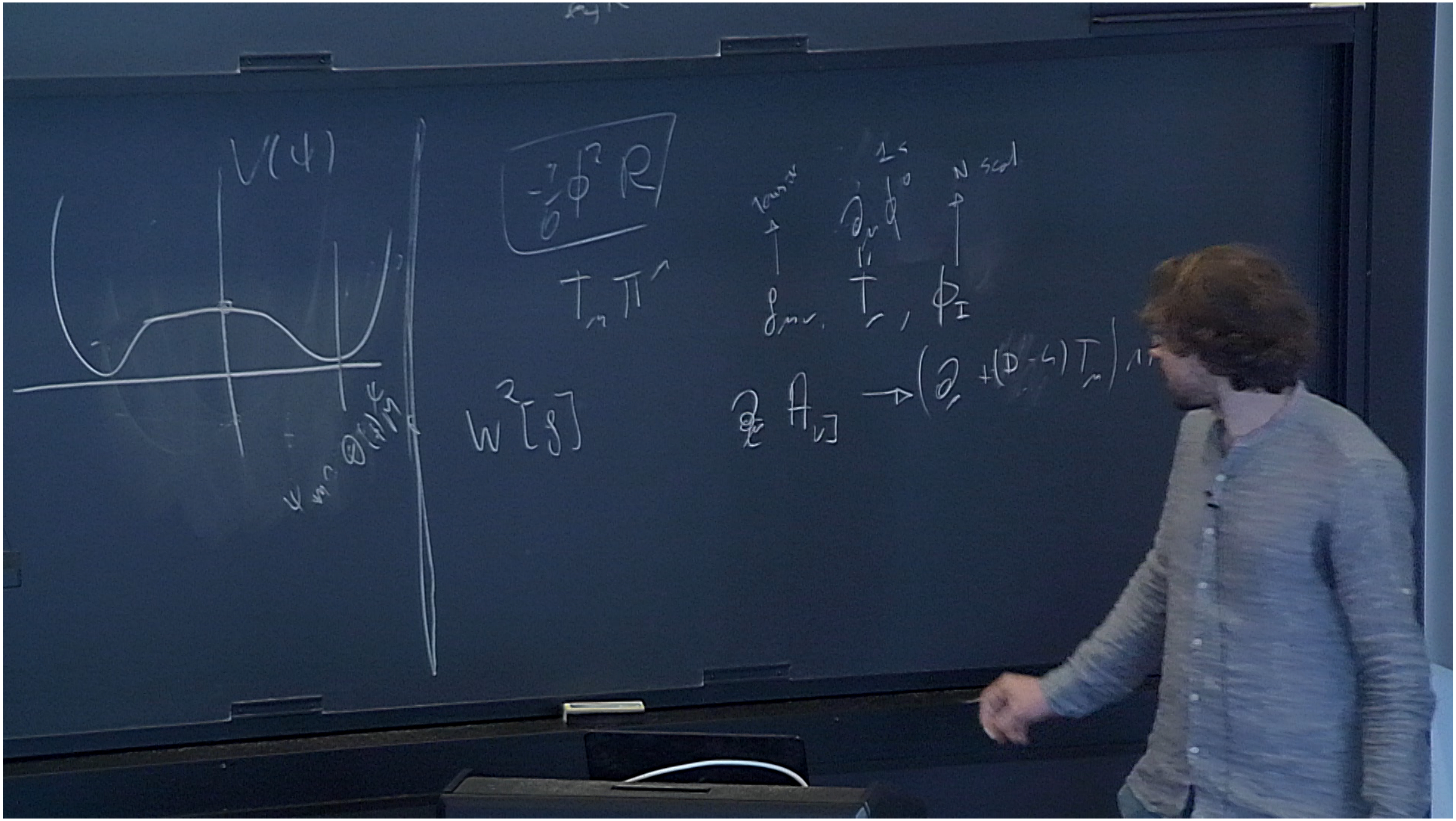

V(ϕ)
N scal
W²[g]





Part III: results and possible experimental signatures

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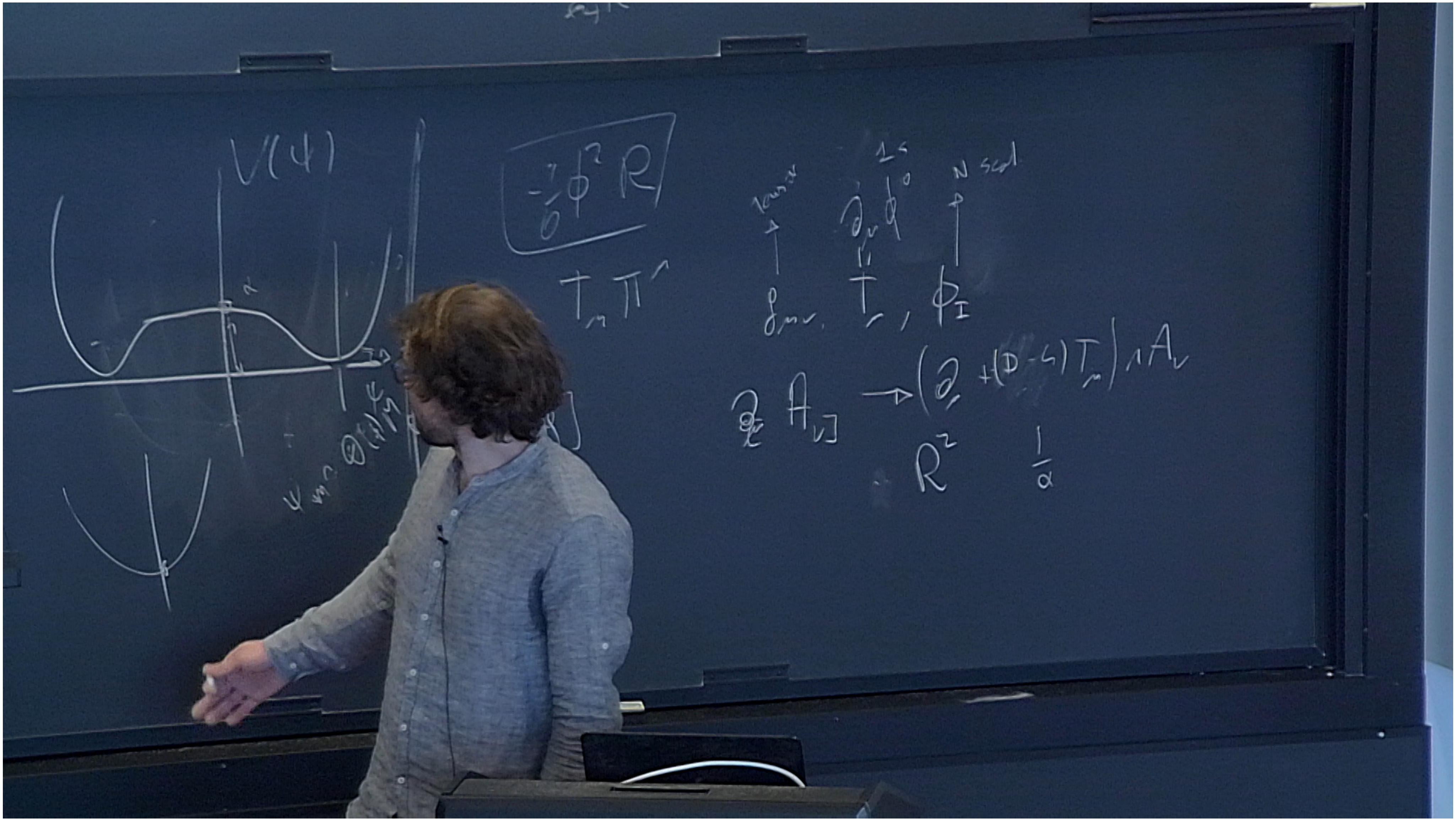
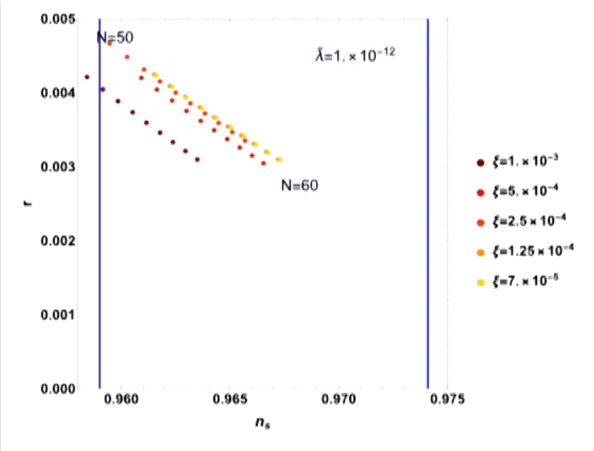
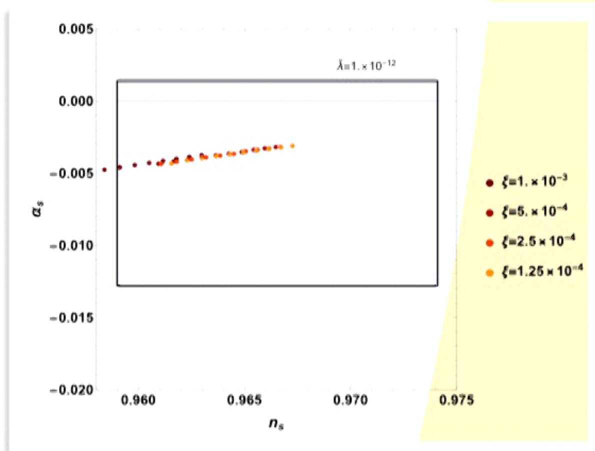


Inflationary predictions

- The predictions of this model are similar to Starobinsky's, with a small tensor-to-scalar ratio, in the n_s - r plane, and n_s - α_s plane





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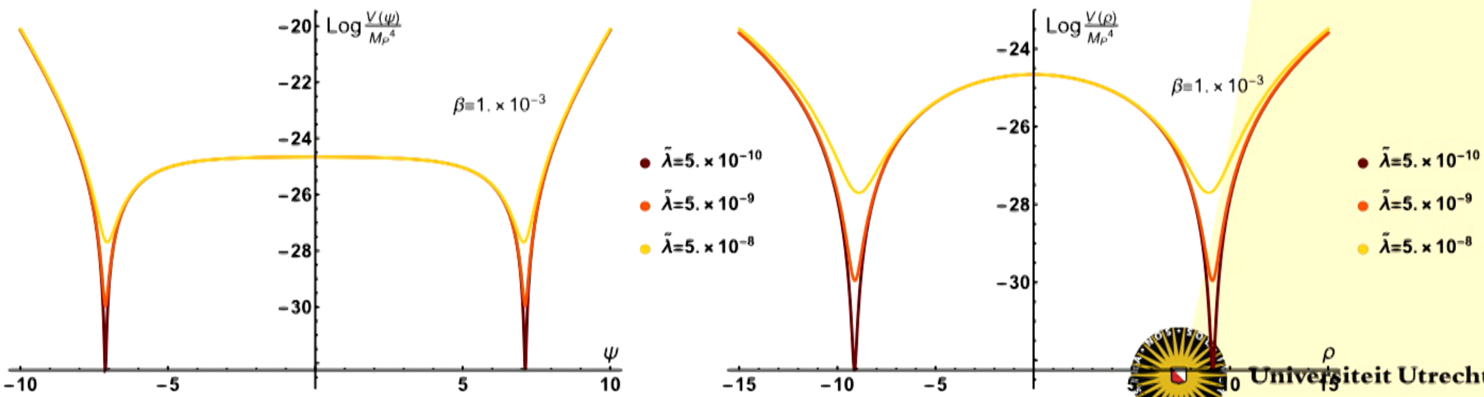
The geometrical effect on the potential

$$V(\rho) = \left(\frac{\xi^2}{16\alpha} + \lambda\right)\rho^4 - \frac{\xi}{8\alpha} M_P^2 \rho^2$$

$$V(\sqrt{6} M_P \sinh(\psi))$$



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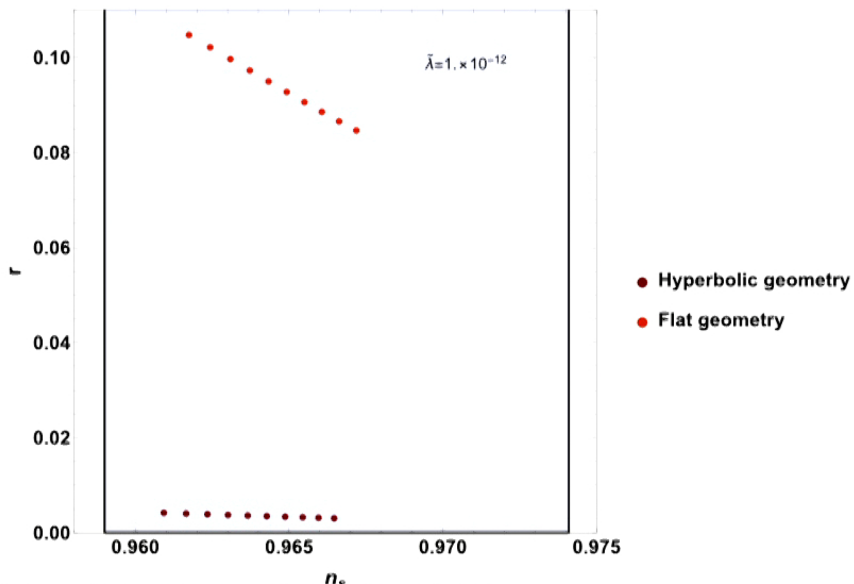
The geometrical effect on the tensor to scalar ratio

- Due to the negatively curved geometry, the potential is more "flat", predicting a lower tensor to scalar ratio,



- The slow roll parameter are generically bigger in the quartic potential, requiring fine tuning. The hyperbolic potential does not suffer from this

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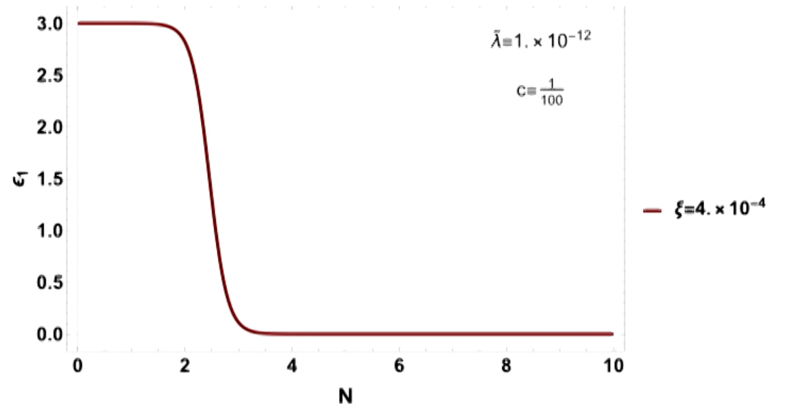
The role of the flat direction

- The main consequence of the Weyl symmetry is that the potential does not depend on the dilaton,

$$\frac{\partial V}{\partial \chi} = 0 \implies \chi' = \frac{c}{H e^{3N} \cosh^2(\psi)}$$



- If c is large, this leads to an initial kinaton dominated era, in which kinetic energy dominates over the potential

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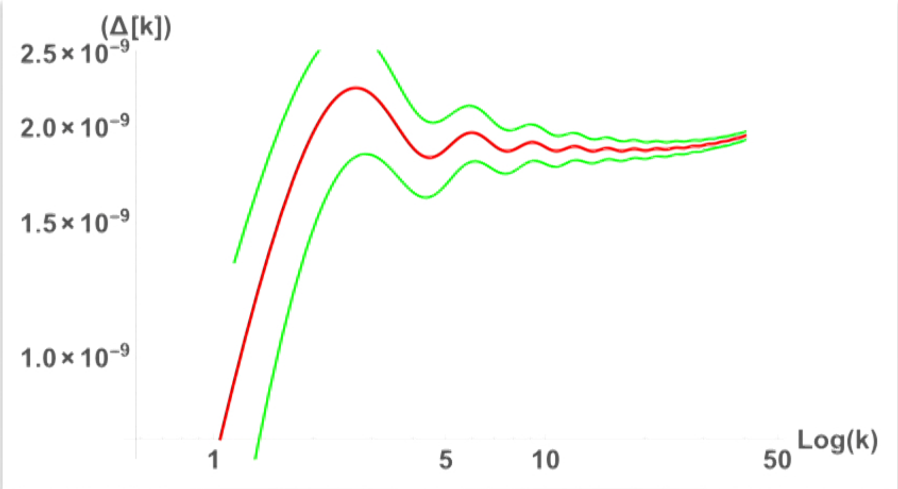
Observational consequences

- In the CMB power spectrum:



- The initial kinaton domination can appear in the CMB spectrum, as it lowers the power on large angular scales.

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Quantum corrections and fine tuning

- As we saw, we must require a hierarchy between coupling constants to obtain a viable model, namely,

$$\lambda \ll \xi \ll \alpha$$

- The one loop result suggests that this is possible to achieve,

$$\delta\alpha = \frac{(\xi+\frac{1}{6})^2}{16\pi^2}\log\left[\frac{3\lambda\phi^2+\left(\xi+\frac{1}{6}\right)R}{\mu^2}\right]$$

$$\delta\xi = \frac{(\xi+\frac{1}{6})\lambda}{8\pi^2}\log\left[\frac{3\lambda\phi^2+\left(\xi+\frac{1}{6}\right)R}{\mu^2}\right]$$

$$\delta\lambda = -\frac{\lambda^2}{8\pi^2}\log\left[\frac{3\lambda\phi^2+\left(\xi+\frac{1}{6}\right)R}{\mu^2}\right]$$

$$\Longrightarrow \delta\lambda \ll \delta\xi \ll \delta\alpha$$

- But the role of quantum gravity corrections still unclear

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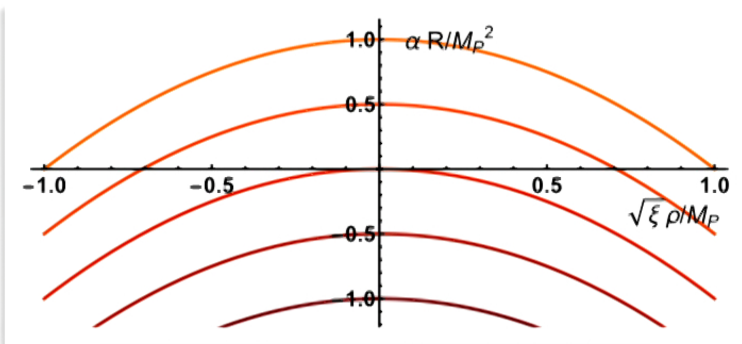
Spontaneous symmetry breaking?

- Because of the constrain due to Weyl symmetry the motion is always confined to a line in the curvature scalar field plane,



$\Longrightarrow \; R > 0\,, \rho = 0\,, \mathrm{dS}$ space-time,
$\Longrightarrow \; R < 0\,, \rho = 0\,, \mathrm{AdS}$ space-time,
$\Longrightarrow \; R = 0\,, \rho = 0\,,$ flat space-time.

- Our model describes a "dS relaxation" phenomenon, where initial conditions are such that a quasi dS space-time emerges as the matter fields slowly rolls towards the minimum.

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Conclusions

- We constructed a theory of gravity that is Weyl invariant, but metric compatible
- The theory contains in general a massless dilaton mode, whose coupling to O(N) scalars can generate inflationary dynamics.
- This is a model of spontaneous breaking of conformal (Weyl) symmetry.
- The model is testable, in principle, and can explain the lack of power on the largest CMB scales
- Our approach is general and suggest a more profound relation between Weyl symmetry and inflationary predictions.

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Thanks for attention

Questions?

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