Title: Towards a T-dual cosmology

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Abstract: <p>As a proxy for a Quantum Gravity theory, String Theory introduces new degrees of freedom and symmetries that should play a major role in the physics of the early universe. We motivate the introduction of these ingredients as we review some of the issues of the inflationary paradigm. Among these new ingredients, T-duality may help us to solve the Big Bang singularity and yield a different cosmological scenario. Thus, we review how this has been thought in the context of Double Field Theory, which aims to build a covariant field theory under T-duality's group, and consider its cosmological applications. </p>

Towards a T-dual cosmology

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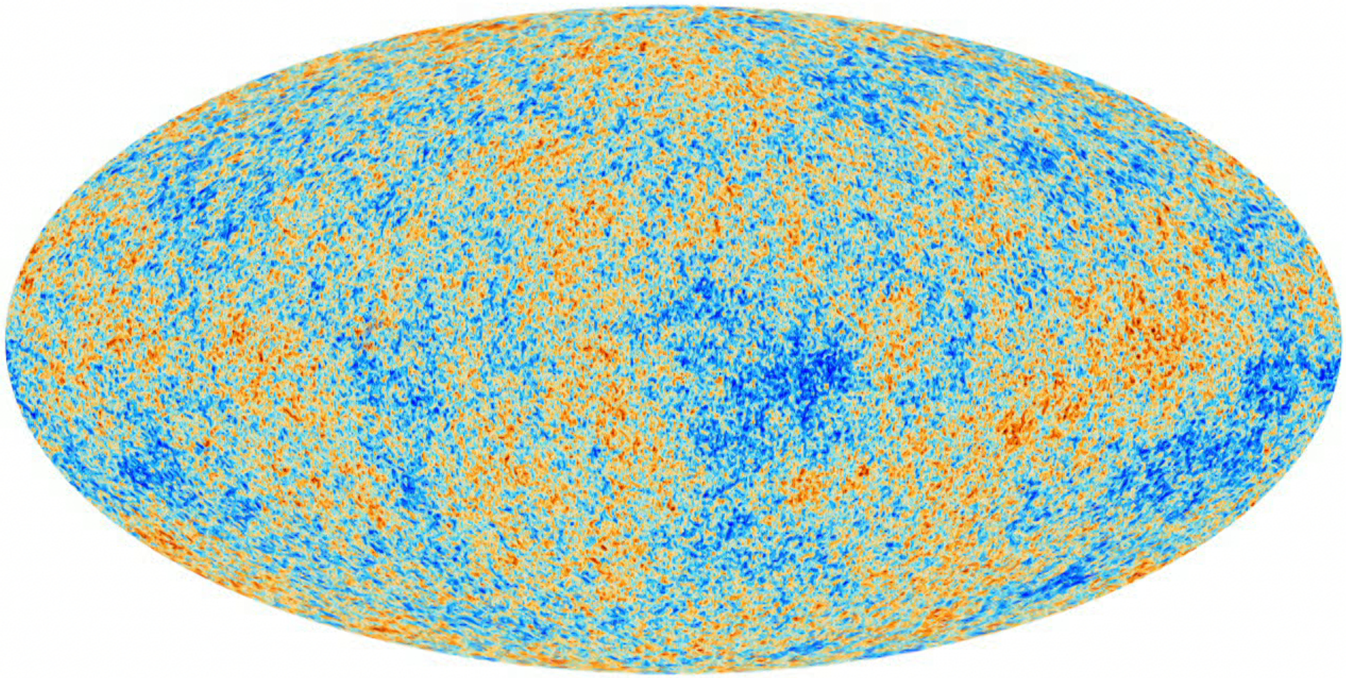
hep-th/1809.03482

Outline

- The Concordance Model and Inflation
- Problems with Inflation
- Supergravity and T-duality
- String Gas Cosmology
- Introduction to Double Field Theory (DFT)
- $O(D,D)$ Cosmological Completion
- Summary and Future Directions



Black Body Radiation with average $T \sim 2.7$ K and fluctuations of order $\Delta T/T \sim 10^{-5}$ (Planck Collaboration 2013)

ΛCDM Model

- With only 6 parameters is able to explain all the current cosmological data
- Among these parameters, two are related to the initial fluctuations that gave rise to the CMB

$$P(k) = \Delta_R^2 k^{n_s - 1},$$

where $\Delta_R^2 \sim 2.5 \times 10^{-9}$ and $n_s \sim 0.9667$. The small value of $n_s - 1$ encodes the almost scale-invariance of the power spectrum

Inflation

- First causal scenario that yields such physics by invoking **a phase of quasi-de Sitter expansion**
- It predicts an almost scale-invariant power spectrum for the adiabatic fluctuations with a red tilt (Chibisov, Mukhanov 1981)
- It also generally predicts an almost scale-invariant power spectrum for the gravitational waves with a red tilt

Problems with Inflation

- Several issues:
  - non-fundamental scalar field
  - trans-planckian problem
  - eternal inflation and multiverse
  - **singularity** (A. Borde and A. Vilenkin '94)
- Standard Cosmological Model remains incomplete
- Singularities are classical, thus finding a good QG theory should do it: **string theory**[2]
- Strings allow for new degrees of freedom and introduce new symmetries/dualities

[2] No convincing embedding of inflation in string theory so far, e.g. swampland conjectures (Vafa et al 2018)

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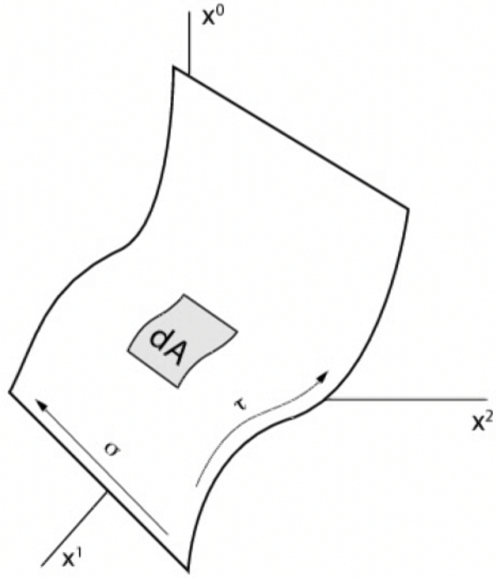
Look! A string!

$$S_{string} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\gamma^{ab}g_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu$$



$\{x^\mu\}$: spacetime coordinates (target space), $\{\tau, \sigma\}$: worldsheet coordinates

How SUGRA was born?

- We could have added a topological term and a 2-form field:

$$S_{\phi,b} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{\gamma}\left[i\epsilon^{ab}b_{\mu\nu}(X)\,\partial_a X^\mu\partial_b X^\nu + \alpha'\phi(X)\,R^{(2)}\right]$$

- Weyl-inv. classically ($T^a_a = 0$), but contain QM anomalies:

$$T^a_a = -\frac{1}{2\alpha'}\beta^g_{\mu\nu}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu - \frac{i}{2\alpha'}\beta^b_{\mu\nu}\epsilon^{ab}\partial_a X^\mu\partial_b X^\nu - \frac{1}{2}\beta^\phi R^{(2)}$$

- Using RG flow:

$$\beta^g_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \frac{1}{4}H_{\mu\lambda\omega}H_\nu{}^{\lambda\omega} + \mathcal{O}\left(\alpha'^2\right)$$

$$\beta^b_{\mu\nu} = -\frac{1}{2}\nabla^\omega H_{\omega\mu\nu} + \nabla^\omega\phi H_{\omega\mu\nu} + \mathcal{O}\left(\alpha'^2\right)$$

$$\beta^\phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\phi + \alpha'\nabla_\omega\phi\nabla^\omega\phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \mathcal{O}\left(\alpha'^2\right)$$

where $H_{\mu\nu\rho} \equiv 3\partial_{[\mu}b_{\nu\rho]}$

- Weyl invariance: $\beta^g = \beta^b = \beta^\phi = 0$

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$$S_{SUGRA} = \int d^D x \sqrt{g} e^{-2\phi} \left[R + 4 (\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk} \right]$$

- Symmetries:
  - Diffeomorphisms: $L_\lambda g_{ij} = \lambda^k \partial_k g_{ij} + g_{kj} \partial_i \lambda^k + g_{ik} \partial_j \lambda^k$
  - Gauge: $b_{ij} \to b_{ij} + \partial_i \tilde{\lambda}_j - \partial_j \tilde{\lambda}_i$
- Equations of motion:

$$R_{ij} - \frac{1}{4} H_i{}^{pq} H_{jpq} + 2\nabla_i \nabla_j \phi = 0$$

$$\frac{1}{2} \nabla^p H_{pij} - H_{pij} \nabla^p \phi = 0$$

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T-duality 101

T-duality is a symmetry of string theory relating winding modes in a given compact space with momentum modes in another (dual) compact space.

- Mass spectrum of a closed string on a circle of radius R:

$$M^2 = \left(N + \tilde{N} - 2\right) + p^2 \frac{l_s^2}{R^2} + w^2 \frac{R^2}{l_s^2}$$

- The mass spectrum is invariant under:

$$\begin{cases} \frac{R}{l_s} & \leftrightarrow \frac{l_s}{R} \\ p & \leftrightarrow w \end{cases}$$

- For $M = 0$, then $\{p, w\} = 0$, $\left\{N, \tilde{N}\right\} = 1 \Rightarrow \{\phi, g_{\mu\nu}, b_{\mu\nu}\}$[3]

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String Gas Cosmology[4]

- We consider a thermodynamical gas of closed strings. Since we know the string's spectrum, we can write

$$\rho = \frac{1}{a^{D-1}} \sum_s N_s E_s$$

$$E_s^2 = \left(N + \tilde{N} - 2\right) + p^2 \frac{l_s^2}{a^2} + w^2 \frac{a^2}{l_s^2}$$

where $s = \left\{p, w, N, \tilde{N}\right\}$

- The pressure is given by,

$$p = -\frac{\partial\left(\rho V\right)}{\partial V} = -\frac{1}{D-1} a^{1-D} \sum_s \frac{N_s}{l_s^2} \left(-\frac{l_s^2}{a^2} n^2 + \frac{a^2}{l_s^2} w^2\right)$$

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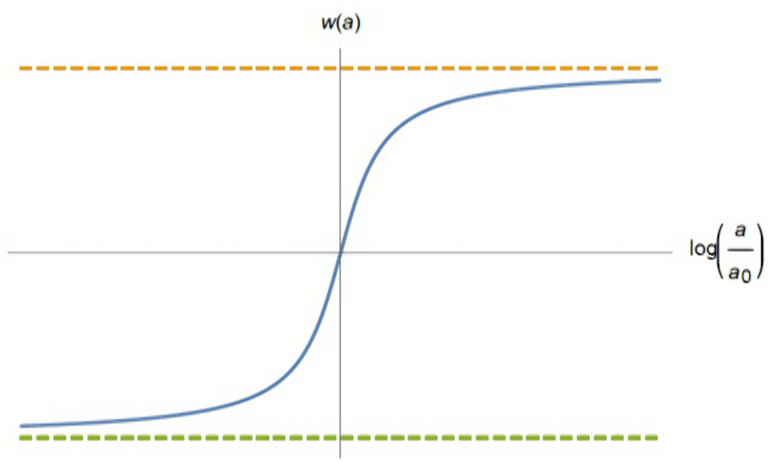
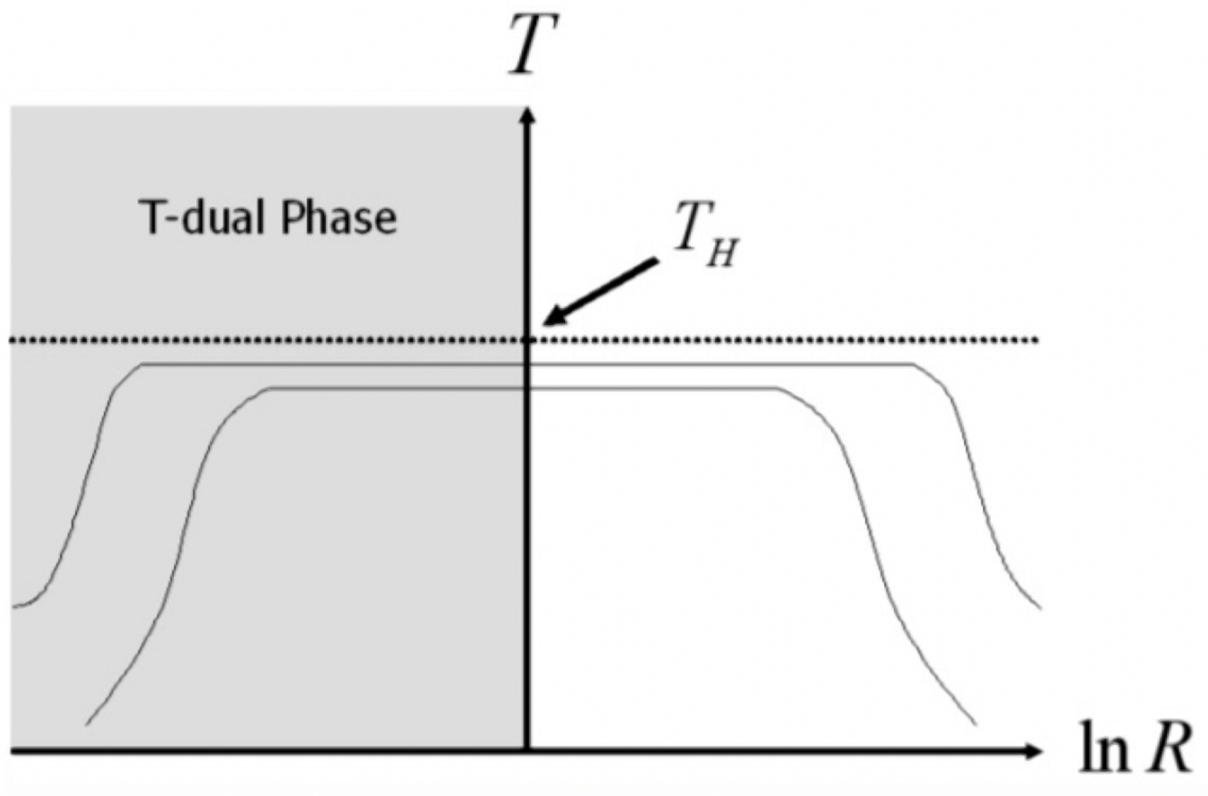
small box ($a \ll l_s$)	self-dual ($a \sim l_s$)	large box ($a \gg l_s$)
$\omega = -1/(D-1)$	$\omega = 0$	$\omega = 1/(D-1)$

$$\omega(a) = \frac{2}{\pi(D-1)} \arctan\left(\beta \ln\left(\frac{a}{a_0}\right)\right)$$



Note that $\omega(a^{-1}) = -\omega(a)$.


T
T-dual Phase
T_H
ln R

It remains a kinematical proposal, with no dynamics accounting for such picture of the early universe. Supergravity is not enough and still singular (Veneziano, Gasperini 2002).

T-duality and double space

- QM in a box:

$$|x\rangle = \sum_p e^{ipx} |p\rangle, \qquad p \in \mathbb{Z}$$

- Since the winding modes are dual to momentum modes through T-duality, one could argue for the existence of the following operator:

$$|\tilde{x}\rangle = \sum_w e^{iw\tilde{x}} |w\rangle, \qquad w \in \mathbb{Z}$$

- Thus, string states in general could be seen as point particles propagating in a **doubled space**

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- **Objective:** To T-dual covariantize SUGRA
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Qualitative Understanding

- Double coordinates: for compact dimensions, momentum and winding modes. Momenta are dual to y^m, windings dual to $\tilde{y}_m$ (new coordinates)
- **In the second quantization (String Field Theory) this is not even an option**
- Formally, DFT has the following coordinate dependence:

$$X^M = (\tilde{x}_\mu, \tilde{y}_m, x^\mu, y^m)\,, \qquad \tilde{x}_\mu : \text{aesthetic}$$

$m = 1, ..., n$ for compact and $\mu = 1, ..., d$ for non-compact dimensions

- As $R \to \infty$, the dual dependence should disappear, since winding modes are suppressed. Conversely, in the T-dual description, momentum modes are suppressed and only dependence on dual coordinates

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- The T-duality group associated to string toroidal compactification on T^n is $O(n,n)$. We enhance this symmetry to the full duality group $O(D,D)$
- Degrees of freedom: bosonic massless[6] sector of the string:

$$\phi,\ g_{\mu\nu},\ b_{\mu\nu}$$

must become $O(D,D)$ objects. In the decompactification limit, their action is the bosonic sector of SUGRA
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“Gold Standard” Model

$$S = -\frac{1}{4\pi}\int d\sigma d\tau \left(\eta^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j G_{ij} + \epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij}\right)$$

- Define

$$G_{ij} = \begin{pmatrix} \hat{G}_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} \hat{B}_{ab} & 0 \\ 0 & 0 \end{pmatrix}$$

Define also,

$$\hat{E}_{ab} = \hat{G}_{ab} + \hat{B}_{ab}$$

- Note that

$$\hat{E}' = h(\hat{E}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \hat{E} \equiv (a\hat{E} + b)(c\hat{E} + d)^{-1}$$

$a, b, c, d \in M_{d\times d}$. This is a **linear fractional transformation**

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  $a, b, c, d \in M_{d\times d}$. This is a **linear fractional transformation**

"Gold Standard" Model

$$S = -\frac{1}{4\pi}\int d\sigma d\tau \left(\eta^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j G_{ij} + \epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij}\right)$$

- Define

$$G_{ij} = \begin{pmatrix} \hat{G}_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} \hat{B}_{ab} & 0 \\ 0 & 0 \end{pmatrix}$$

  Define also,

$$\hat{E}_{ab} = \hat{G}_{ab} + \hat{B}_{ab}$$

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- The Hamiltonian:

$$H_{string} = \frac{1}{2} Z^t \mathcal{H}(\hat{E}) Z + N + \bar{N}$$

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with w^i, $p_i \in \mathbb{Z}$ with $p_i = n/R$ and $w^i = mR/l_s^2$

- Imposing LMC,

$$L_0 - \bar{L}_0 = 0 = N - \bar{N} - p_i w^i$$

then

$$N - \bar{N} = p_i w^i = \frac{1}{2} Z^t \eta Z, \quad \eta = \begin{pmatrix} 0 & 1_{d\times d} \\ 1_{d\times d} & 0 \end{pmatrix}$$

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$$Z' = hZ$$

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Double space and generalized fields

- How to represent d.o.f. as T-dual object?
- Consider the generalized metric defined as

$$\mathcal{H} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix}, \quad g(x), b(x)$$

$$\mathcal{H} \in O(D,D), \quad \mathcal{H}^{MN} = \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}, \quad \mathcal{H}_{MP}\mathcal{H}^{PN} = \delta_M^N$$

- The dilaton appears together with g,

$$e^{-2d} = \sqrt{g}e^{-2\phi}$$

defining a $O(D,D)$-scalar

- **Note: det $\mathcal{H} = 1$!**

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Where do fields live?

- Fundamental representation of $O(D,D)$ has dimension $2D$, but only D coordinates
- Fix this by introducing new coordinates $\tilde{x}_i$, so that the generalized coordinates are

$$X^M = (\tilde{x}_i, x^i)$$

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- Intuition: these coord. correspond to the Fourier duals to the generalized momenta $\mathcal{P}^M \equiv Z^M$
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Is that even possible?

- No, we cannot just double our space and leave it like that
- **Section Condition:**

$$\boxed{\eta^{MN}\partial_M\partial_N(...) = 0}$$

trivial-unique solution: $\tilde{\partial}(...) = 0$ (or any $O(D,D)$ rotation of it). This section is called supergravity frame

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Generalized Lie Derivative

- Gauge + diff.

$$\xi^M = \left(\tilde{\lambda}_i, \lambda^i\right)$$

- Generalized Lie Derivative:

$$\mathcal{L}_\xi e^{-2d} = \partial_M \left(\xi^M e^{-2d}\right)$$

$$\mathcal{L}_\xi \mathcal{H}_{MN} = L_\xi \mathcal{H}_{MN} + \boxed{\partial_M \xi^R \mathcal{H}_{RN} + \partial_N \xi^R \mathcal{H}_{MR}}$$

$$\mathcal{L}_\xi \eta_{MN} = 0$$

- These transf. + strong constraint = diff.'s and gauge transformations

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The Action

$$S = \int dX e^{-2d} \mathcal{R}$$

$$\begin{aligned}\mathcal{R} &= \frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_K \mathcal{H}_{NL} \\ &+4\mathcal{H}^{MN}\partial_M \partial_N d - 4\mathcal{H}^{MN}\partial_M d \partial_N d + 4\partial_M \mathcal{H}^{MN}\partial_N d \\ &-\partial_M \partial_N \mathcal{H}^{MN}\end{aligned}$$

i) terms up to 2nd order derivatives; ii) recover SUGRA in the supergravity frame; iii) respect the gauge symmetries

DFT with single time

- For cosmological background w/ a single time parameter, the DFT's EOM = SUGRA's. Any solution from SUGRA can be embedded into DFT space, given

$$dS^2 = -dt^2 + \mathcal{H}_{MN} dX^M dX^N = -dt^2 + a^2(t)\, d\vec{x}^2 + a^{-2}(t)\, d\tilde{x}^2$$

- This has been considered[7] and singularities cannot be avoided
- Is it possible to do something else?

[7] R. Brandenberger, R. Costa, GF, A. Weltman: Phys.Rev. D87 (2013) no.6, 063530

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Interlude: on clocks and rods

- We live in space and time, and all the measurements can be made in terms of clocks and rods[8]
- If there is a constant speed for all the possible observers (SR), thus we only need either rods or clocks, since light rays follow null geodesics:

$$\Delta s^2 = 0 \Rightarrow \Delta t^2 = \frac{\Delta x^2}{c^2}$$

- However, if the world is made of closed strings, we could have used winding modes for building our rods, such that

$$\Delta \tilde{x} = \frac{l_s^2}{\Delta x}$$

where α' is the string tension

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- Thus, using these rods,

$$|\Delta\tilde{x}| = \left|\frac{l_s^2}{c\Delta t}\right|$$

- Now, for a truly T-dual universe,

$$|\Delta\tilde{x}| = |\tilde{c}\Delta\tilde{t}|$$

Thus, it is also natural to propose a "winding-clock" that is dual to the momentum-one by,

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DFT's EOM with 2-time parameters

- From previous argument, the DFT EOMs with double-time in vacuum are[9]:

$$2\bar{\phi}'' - \bar{\phi}'^2 - (D-1)\tilde{H}^2 + 2\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 - (D-1)\,H^2 = 0$$
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- In the presence of matter[10]

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where $p\left(t,\tilde{t}\right)$ and $\rho\left(t,\tilde{t}\right)$, in principle

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SUGRA frame: large box

- For a constant dilaton, EOMs are

$$2\left(\tilde{H}' + \dot{H}\right) + D\left(\tilde{H}^2 + H^2\right) = 0$$

$$\left(\tilde{H}^2 - H^2\right) + \left(\tilde{H}' - \dot{H}\right) = \frac{1}{2(D-1)} G\rho$$

$$\left(\tilde{H}' + \dot{H}\right) + (D-1)\left(\tilde{H}^2 + H^2\right) = \frac{G}{2} p$$

- For SUGRA frame, only t-dependence should be relevant, given the $\tilde{t}$−was introduced exactly to tackle the winding modes

It implies,

$$w \equiv \frac{p}{\rho} = \frac{1}{D-1}$$

Thus,

$$\rho(a) \propto a^{-D}$$
$$a(t) = a_0 t^{2/D}$$

This is a radiation-like solution, as we had before in SUGRA with constant dilaton.

Winding-frame (small box)

- Winding modes dominate: $\tilde{t}$-dependence is kept,

$$2\tilde{H}' + D\tilde{H}^2 = 0$$

$$\tilde{H}^2 + \tilde{H}' = \frac{1}{2(D-1)} G\rho$$

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- Implying EOS:

$$w = -\frac{1}{D-1}$$

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The Friedmann-like equation has a minus sing,

$$\tilde{H}^2 = -\frac{G}{(D-2)(D-1)}\rho$$

Considering $\tilde{H}$ complex ,

$$a(\tilde{t}) = \tilde{A}(\tilde{t})\, e^{i\theta(\tilde{t})}$$

s.t.,

$$\tilde{H}_{\tilde{A}}^2 - \theta'^2 = -g\rho_0 \tilde{A}^{-D}\cos(D\theta)$$
$$2\tilde{H}_{\tilde{A}}\theta' = g\rho_0 \tilde{A}^{-D}\sin(D\theta)$$

where $g \equiv G/(D-2)(D-1)$. For $\theta = \pi/D$, the 2nd equation vanishes and the 1st equation gives:

$$\boxed{a(\tilde{t}) = \tilde{a}_0 \tilde{t}^{2/D} e^{i\pi/D}}$$

Interpretation

- For momenta:

$$a_m(t) = a_0 t^{2/D} e^{i\theta_m}$$

where $\theta_m = 0$.

- Thus,

$$\begin{cases} a_m(t) & = a_0 t^{2/D} \\ a_w(\tilde{t}) & = \tilde{a}_0 \tilde{t}^{2/D} e^{i\pi/D} \end{cases}$$

- Momentum's and winding's scale factor are dual:

$$a_m \to a_w^{-1}$$

- The solutions are dual given,

$$\boxed{t \to \tilde{t}^{-1} e^{-i\pi/2}}$$

- This is a Wick rotation of the reciprocal of the time coordinate. Since θ is dynamical, this rotation happens dynamically.

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where $\theta_m = 0$.

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$$\begin{cases} a_m(t) & = a_0 t^{2/D} \\ a_w(\tilde{t}) & = \tilde{a}_0 \tilde{t}^{2/D} e^{i\pi/D} \end{cases}$$

- Momentum's and winding's scale factor are dual:

$$a_m \to a_w^{-1}$$

- The solutions are dual given,

$$\boxed{t \to \tilde{t}^{-1} e^{-i\pi/2}}$$

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General case,

$$\tilde{H}_{\tilde{A}}^2 - \theta'^2 = -g\rho_0 \tilde{A}^{-D} \cos(D\theta)$$
$$2\tilde{H}_{\tilde{A}}\theta' = g\rho_0 \tilde{A}^{-D} \sin(D\theta)$$

The solutions are,

$$\theta(\tilde{t}) = \pm\frac{2}{D}\arccos\left[\left(\frac{\tilde{A}}{\tilde{A}_0}\right)^{-D/2}\right]$$
$$\tilde{A}(\tilde{t}) = \left[\tilde{A}_0^D + \frac{D^2}{4} g\rho_0 \tilde{t}^2\right]^{1/D}$$

Note that for large $\tilde{t}$ limit,

$$\tilde{A}(\tilde{t}) \to \tilde{t}^{2/D}$$

and

$$\theta(\tilde{t}) \to \pm\frac{2}{D}\arccos\left(\frac{1}{\tilde{t}}\right) \xrightarrow[\tilde{t}\to\infty]{} \pm\frac{\pi}{D}.$$

Hence, deep in the winding regime the oscillations cease to exist. This shows that the temporal duality appears due to the dynamics of our solutions.

DFT's EOM with 2-time parameter

Dual-clock:

$$|\Delta\tilde{t}| = \left|\frac{\alpha'^2}{c\tilde{c}\Delta t}\right|$$

Physically there is a single clock. When only winding or momentum modes are cheap, the existence of a unique time coordinate is clear. Around the self-dual radius, we need a prescription

(Physical Clock Constraint)

$$\boxed{|\tilde{t}| \to \frac{1}{|t|}}$$

Physical Clock Constraint

$$\tilde{t} \to \frac{1}{t} \qquad\qquad \frac{d}{d\tilde{t}} \to -t^2 \frac{d}{dt}$$

Physical time is defined as,

$$dt^2 + d\tilde{t}^2 \to \boxed{dt^2 \left(1 + \frac{1}{t^4}\right) \equiv dt_p^2}$$

As $t \to 0$,

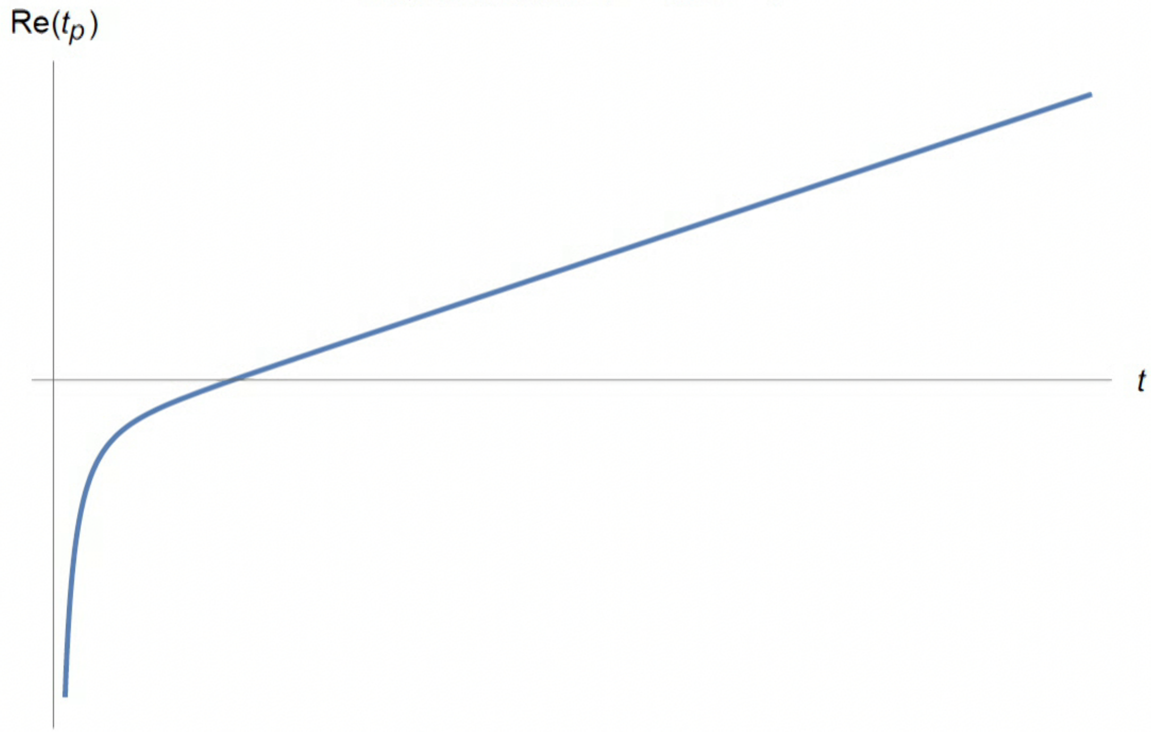
$$dt_p \sim \frac{1}{t^2} dt \quad \to \quad t_p \sim -\frac{1}{t}$$

so that $t_p \to -\infty$. When $t \to \infty$, then

$$dt_p \sim dt \quad \to \quad t_p \sim t$$

so that $t_p \to \infty$. Thus, $t_p \in (-\infty, \infty)$.

Physical Time $\tau = 1,\ s = 1$



Physical clock definition: $dt_p \equiv \sqrt{1 + \frac{1}{t^4}}\, dt.$

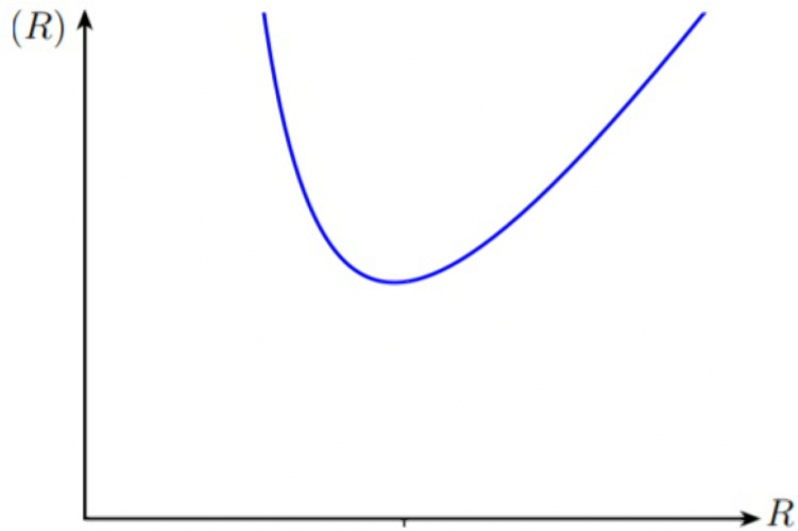


FIG. 9: Physical length (vertical axis) as a function of the coordinate length (horizontal axis).

Dynamics

- Equations of motion in terms of the physical time,

$$2\ddot{\bar{\phi}}_p - \dot{\bar{\phi}}_p^2 - (D-1)\,H_p^2 = \boxed{\frac{4}{t_p}\dot{\bar{\phi}}_p\sigma\left(1-2\sigma^2\right)}$$

$$-(D-1)H_p^2 + \ddot{\bar{\phi}}_p = \frac{1}{2}e^{\bar{\phi}}\bar{\rho}\boxed{\sigma^2\left(\frac{1-\sigma^2}{1-2\sigma^2}\right)} + \boxed{\frac{2}{t_p}\frac{\sigma}{1-2\sigma^2}\left(1-2\sigma^2+2\sigma^4\right)\dot{\bar{\phi}}_p}$$

$$\dot{H}_p - H_p\dot{\bar{\phi}}_p = \frac{1}{2}e^{\bar{\phi}}\bar{p}_p\boxed{\sigma^2\left(1-\sigma^2\right)} + \boxed{\frac{2}{t_p}\sigma\left(1-2\sigma^2\right)H_p}$$

- Asymptotically ($|t_p| \to \pm\infty$),

$$2\ddot{\bar{\phi}}_p - \dot{\bar{\phi}}_p^2 - (D-1)\,H_p^2 \to 0$$

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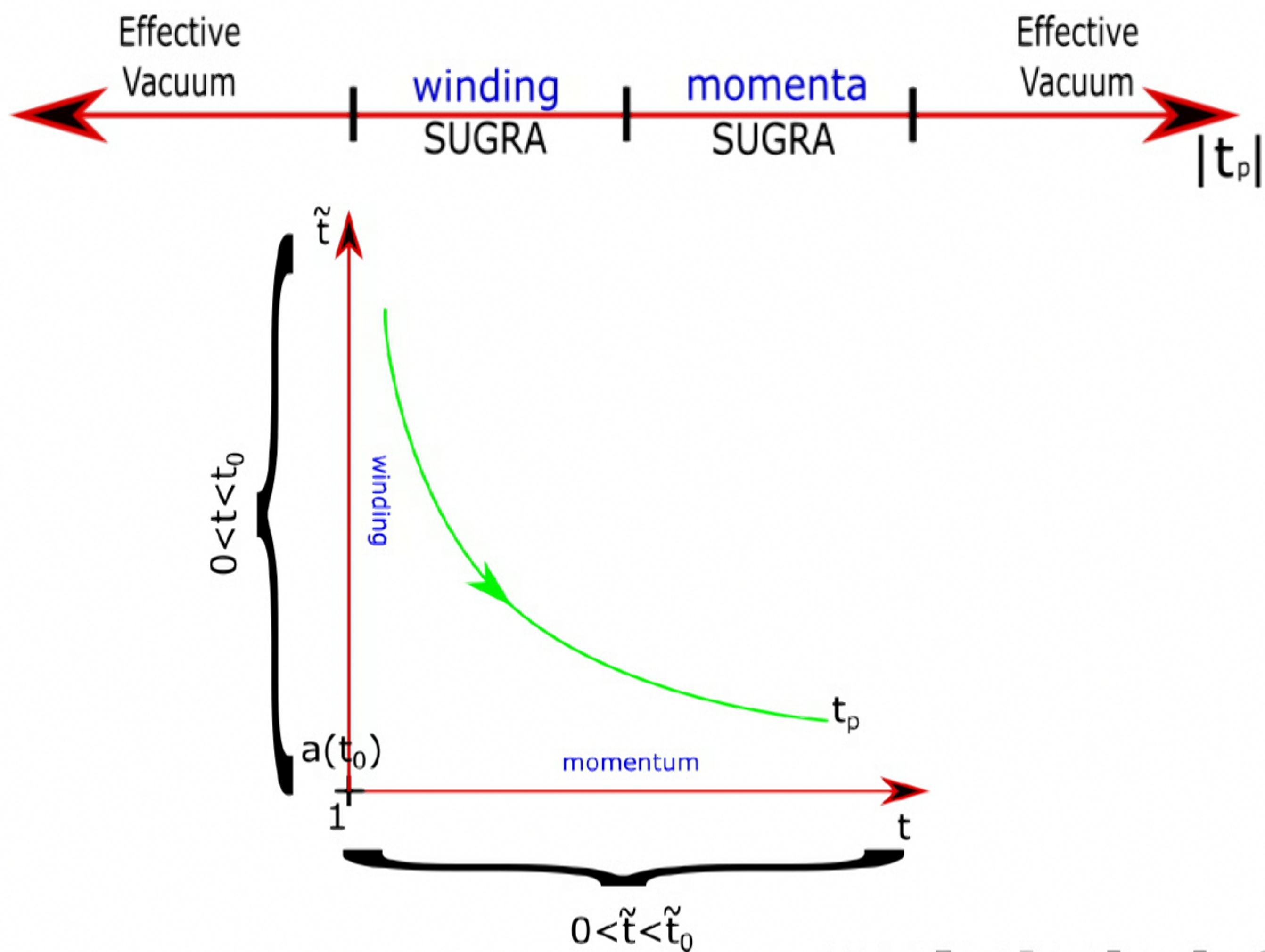
Effective
Vacuum
winding
SUGRA
momenta
SUGRA
Effective
Vacuum
$|t_p|$
$\tilde{t}$
$0<t<t_0$
winding
t_p
$a(t_0)$
momentum
1
t
$0<\tilde{t}<\tilde{t}_0$

$O(D,D)$ Cosmological Completion[11]

- SUGRA and matter:

$$S = \int d^D x \sqrt{g} e^{-2\phi} \left[R + 4 (\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk} \right] + \int d^D x \sqrt{g} \mathcal{L}_m$$

- $O(D,D)$ completion in the supergravity frame:

$$S = \int d^D x \sqrt{g} e^{-2\phi} \mathcal{L}_{SUGRA} + \int d^D x \sqrt{g} \boxed{e^{-2\phi}} \mathcal{L}_m$$

[11] Stephen Angus, Kyoungho Cho, GF, Jeong-Hyuck Park, Shinji Mukohyama (in preparation)

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[11] Stephen Angus, Kyoungho Cho, GF, Jeong-Hyuck Park, Shinji Mukohyama (in preparation)

$$3H^2 - 6H\dot{\phi} + 2\dot{\phi}^2 = e^{2\phi}\rho$$

$$\dot{H} + 4H\dot{\phi} - 2\dot{\phi}^2 = -e^{2\phi}(\rho - p) + \boxed{\frac{T_{(0)}}{2}}$$

$$\ddot{\phi} + 3H\dot{\phi} - 2\dot{\phi}^2 = -\frac{e^{2\phi}}{2}(\rho - 3p) + \boxed{\frac{T_{(0)}}{2}}$$

Imposing the dilaton to be constant, one recovers Friedmann equations for any matter content!

Summary and Future Directions

- Cosmological Standard Model still lacks a clear picture of the early universe
- Inflation albeit successful presentes many conceptual problems
- If string theory is the correct quantum gravity theory, T-duality is key for understanding early stages of the Universe
- Double Field Theory may provide a better description of the background for string cosmology as well perturbations
- Recent results using DFT and extensions could result in a non-singular picture of the early universe
- Non-commutativity of closed string theory
- Lots of work to do!

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