Title: Natural dynamics of the cosmological constant

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Abstract: <p>We consider a natural extension of general relativity, by the addition of a term which is a topological invariant when the cosmological constant is in fact constant. Allowing the cosmological constant to vary, we discover it is endowed with a natural dynamics, which also includes a mechanism to suppress its value.   A key role in hiding the consequences of a dynamical cosmological constant is played by the torsion of the spacetime connection.</p>

<p>As a dynamical variable, the varying cosmological constant turns out to be canonically conjugate to a measure of intrinsic, cosmological time, given by the Chern-Simons invariant of the Ashtekar connection (which we originally studied with Chopin Soo). As a result, a small quantum universe that "knows what time it is" does not know the value of the cosmological constant, and visa versa.   This leads to some novel scenarios for the early universe.</p>

<p>This is joint work in progress with Stephon Alexander, Joao Magueijo and Robert Sims.</p>

Natural dynamics for the cosmological constant

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November 16 2018

Joint work in progress with Stephon Alexander, Joao Magueijo, and Robert Sims, also 1994 work with Chopin Soo.

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The hidden chirality of general relativity

General relativity is parity even but there are several indications that there is a hidden chirality

- The simplest way of writing the action and field equations is the Plebanski action, which is cubic, so the EOM are quadratic, but it is chiral (L-R asymmetric.) These are closely related to the Ashtekar variables.
- Twistor theory reveals a beautiful chiral complex geometry of spacetime.
- These are related to the near-topological form of GR:
  - GR=TFT+ constraints

Might this hidden chirality manifest itself in quantum gravity?

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Previous attempts:

- Parity odd CMB observable (T-B mixing)
- Parity odd modifications of dispersion relations

Might this hidden chirality manifest itself in quantum gravity?

Here we propose a natural extension of GR that yields a time dependent Λ, which is constrained by its dynamics so that:

$$\frac{\Lambda}{3} = \left(\frac{16\pi^2}{3}\right)^{\frac{1}{2}} \sqrt{\frac{\nabla_\mu J_5^\mu}{\sqrt{-g}}}$$

Note: no free parameters or dimensional parameters. ie natural!!

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Note: no free parameters or dimensional parameters. ie natural!!

+ small damped oscillations around the fixed point.

The Chern-Simons Intrinsic-time

The Ashtekar variables are complex coordinates for real, Lorentzian spacetimes:

$$A_{ai} = \text{3d spin connection}_{ai} + \frac{\imath}{\sqrt{q}} K_{ab} E^b_i$$

$$q q^{ab} = E^{ai} E^b_i \qquad K_{ab} \approx \dot{q}_{ab}$$

$$\{A^i_a(x), E^b_j(y)\} = \imath G \delta^b_a \delta^i_j \delta^3(y,x)$$

$$I^{GR} = \int dt \int_\Sigma \imath E^{ai} \dot{A}_{ai} - N\mathcal{H} - N^a H_a - w_i \mathcal{G}^i$$

Constraints generate gauge transformations:

Gauss's law for SU(2): $\mathcal{G}^i = \mathcal{D}_a E^{ai}$

Diffeomorphism constraint $\mathcal{H}_a = E^b_i F^i_{ab}$

All constraints are cubic!

Hamiltonian constraint: $\mathcal{H} = \epsilon_{ijk} E^{ai} E^{bj} (F^k_{ab} + \frac{\Lambda}{3} \epsilon_{abc} E^{ck})$

Equations of motion:

$$\dot{A}_{ai} = \{A_{ai}, \int N\mathcal{H}\} = N\imath G \epsilon_{ijk} E^{bj} (2F^k_{ab} + \Lambda \epsilon_{abc} E^{ck})$$

$$\dot{E}^{ai} = \{E^{ai}, \int N\mathcal{H}\} = \imath G \epsilon^{ijk} \mathcal{D}_b (N E^a_j E^b_k)$$

Self-dual solutions: $\boxed{F^i_{ab} = -\frac{\Lambda}{3} \epsilon_{abc} E^{ci}}$

All eom are quadratic!

Explicit deSitter solution:

$$F^i_{ab} = -\frac{\Lambda}{3}\epsilon_{abc}E^{ci}$$

deSitter spacetime is (was) the unique lorentzian self-dual solution:

We make the spatially flat ansatz:

$$A_{ai} \approx \text{3d spin connection}_{ai} + \imath \dot{e}_{ai}$$

$$A_{ai} = \imath\sqrt{\Lambda/3}f(t)\delta_{ai} \quad \mapsto F_{abi} = -f^2(t)\frac{\Lambda}{3}\epsilon_{abi}$$

The self-dual condition implies:

$$E^{ai} = f^2\delta^{ai} \quad \mapsto \quad e_{ai} = f\delta_{ai}$$

To fix the solution fix the lapse N

$$N \approx det(e)^{-1} = f^{-3}$$

The equations of motion give:

$$\dot{f} = \sqrt{\Lambda/3}Nf^4 = \sqrt{\Lambda/3}f$$

This gives the dS metric:

$$ds^2 = -dt^2 + e^{2\sqrt{\Lambda/3}t}(dx^a)^2$$

Hamilton-Jacobi, deSitter and Chern-Simons theory

Let us solve the constraints with a Hamilton-Jacobi function S(A).

The momenta are given by $$E^{ai} = -\frac{\delta S(A)}{\delta A_{ai}}$$

To get deSitter we impose the self-dual condition:

$$F^i_{ab} = -\frac{\Lambda}{3}\epsilon_{abc}E^{ci} = \frac{\Lambda}{3}\epsilon_{abc}\frac{\delta S(A)}{\delta A_{ci}}$$

This has the unique solution: $$S_{CS} = \frac{2}{3\Lambda}\int Y_{CS}$$

Chern-Simons invariant:

$$Y_{CS} = Tr(A \wedge dA + \tfrac{2}{3}A^3)$$

$$\frac{\delta \int Y_{cs}}{\delta A_{ai}} = 2\epsilon^{abc}F^i_{bc}$$

The Kodama State

Hence the H-J function for dS is:

$$S_{CS} = \frac{2}{3\Lambda} \int Y_{CS}$$

This suggests as an ansatz the state:

$$\Psi_K(A) = \mathcal{N} e^{\frac{3}{2\lambda} \int Y_{CS}}$$

Here we are using the connection representation:

$$< A|\Psi >= \Psi(A) \quad E^{ai} = -\hbar G \frac{\delta}{\delta A_{ai}}$$

In fact, with a certain choice of operator ordering,
this is an exact solution to the quantum constraints:

The Kodama State

$$\Psi_K(A) = \mathcal{N} e^{\frac{3}{2\Lambda}\int Y_{CS}}$$

$$S_{CS} = \frac{2}{3\Lambda}\int Y_{CS}$$

Its transform to the spin network representation is exact:

$$\Psi[\Gamma] = \int dA \; T[\Gamma, A] e^{\frac{\kappa}{4\pi} S_{CS}(A)}$$

for A Euclidean, this is the Kauffman bracket or Jone's polynomial of the network.

—> Requires framed spin networks labeled with $SU_q(2)$ reps.

—> The level, k, is related to Λ:

$$k = \frac{6\pi}{\hbar G \Lambda}$$

Thermality of the exact quantum theory on Σ=S³

Recall:

- The KMS condition. Thermal states are periodic in imaginary time.
- The natural time coordinate is: $T_{CS} = \mathcal{I}m \int Y_{CS}(A)$
- The Euclidean continuation has A_a real

Hence the natural Euclidean time coordinate is $T_{ECS} = \int Y_{CS}(A)$

But this is a periodic coordinate on the configuration space.

Under large gauge transformations:

$$\int Y_{CS}(A) \rightarrow \int Y_{CS}(A) + 8\pi^2 n$$

Hence there is a dimensionless temperature. $\mathcal{T}_{dimless} = \frac{1}{8\pi^2}$

Hence, the whole quantum theory of gravity with Σ=S³ is thermal!

$$\mathcal{T}_{dimless} = \frac{1}{8\pi^2}$$

To connect this with the deSitter temperature we scale on a trajectory corresponding to an S^3 slicing of dS:

The relation between the two time coordinates is given by

$$\frac{\partial T_{CS}}{\partial t} = \int_{S^3} N\{T_{CS}(A), \mathcal{H}\} = 4\pi\sqrt{\frac{\Lambda}{3}}$$

This leads to the dimensional Gibbons-Hawking temperature:

$$\mathcal{T}_{dS} = \frac{1}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

Note: this does not just say that QFT on dS is thermal. It says quantum gravity with a positive CC is intrinsically thermal.

The Lorentzian Chern-Simons time in the homogeneous case:

$$A_{ai} = \imath\delta_{ai}\dot{a} = \imath\delta aiHa$$

$$T_{CS} = \int_{S^3} \mathcal{I}mTrA^3 = H^3a^3$$

This is the number of co-moving volumes in an Horizon volume.

T_{CS} <1 "comoving volume is within the horizon"

T_{CS} >1 " comoving volume is outside the horizon"

Topological dynamics of Λ

We consider the action:

$$S = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon^{abcd}\left\{e_a \wedge e_b \wedge R_{cd}(A) - 2\Lambda e_a \wedge e_b \wedge e^c \wedge e^d\right\} + \frac{3}{2\Lambda} R^{ab} \wedge R_{ab}$$

Plebanski form*:*

$$S^{Pl} = \frac{\imath}{8\pi G}\int_{\mathcal{M}}\left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6}\Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2}\Phi_{ABCD}\Sigma^{AB} \wedge \Sigma^{CD} - \frac{3}{2\Lambda} R^{AB} \wedge R_{AB}\right) + c.c$$

$$dY_{CS} = R^{AB} \wedge R_{AB}$$

Topological dynamics of Λ

The new term*:*

$$S^{CS} = -\frac{\imath}{16\pi G}\int_{\mathcal{M}} \frac{3}{\Lambda}(R^{AB}\wedge R_{AB} - R^{A'B'}\wedge R_{A'B'}) = -\frac{\imath}{16\pi G}\int_{\mathcal{M}} \frac{3}{\Lambda} d\mathcal{I}m(Y_{CS})$$

$$= -\frac{\imath}{8\pi G}\int_{\Sigma_{final}} \frac{3}{2\Lambda}\mathcal{I}mY_{CS} + \frac{\imath}{8\pi G}\int_{\Sigma_{initial}} \frac{3}{2\Lambda}\mathcal{I}mY_{CS}$$

Reproduces the Im part of the Kodama state on initial and final surfaces

$$+\frac{\imath}{16\pi G}\int_{\mathcal{M}} d(\frac{3}{\Lambda})\mathcal{I}mY_{CS}$$

Topological dynamics of Λ

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No free parameters!!! Coefficient of RR term fixed by the condition that e^S match the Kodama state.

A duality symmetry for Λ

Look at the terms in the action containing Λ.

$$S^{\Lambda} = \frac{-\imath}{16\pi\hbar G}\int_{\mathcal{M}} \left\{ \frac{\Lambda}{3}\Sigma_{AB}\wedge\Sigma^{AB} + \Sigma_{AB}\wedge R^{AB} + \frac{3}{\Lambda}R_{AB}\wedge R^{AB} \right\}$$

Consider the formal on shell symmetry.

$$R^{AB} \to \frac{\Lambda}{3}\Sigma^{AB} \qquad \Sigma^{AB} \to \frac{3}{\Lambda}R^{AB}.$$

The fixed points are the self-dual solutions.

$$R^{AB} = \frac{\Lambda}{3}\Sigma^{AB}, \quad \Phi_{ABCD} = 0,$$

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Surprise: with the new term Λ doesn't have to be constant.

Self-dual solutions $\Lambda = constant$ ***(CDJ)***

Pick an SU(2) connection, A^{AB}, such that F^{AB} satisfies $\mathcal{D}F^{AB} = 0$

$$F^{(AB} \wedge F^{CD)} = 0$$

Pick next a constant, Λ and *define:*

$$\Sigma^{AB} \equiv \frac{3}{\Lambda} F^{AB}$$

This satisfies: $\Sigma^{(AB} \wedge \Sigma^{CD)} = 0$ *and* $\mathcal{D}\Sigma^{A'B'} = 0$

ie Torsion vanishes.

so there exists a frame field $e^{AA'}$, such that***:***

$$\Sigma^{AB} = e^{A'A} \wedge e^{B}_{A'}$$

Example: de Sitter or AdS

Self-dual solutions Λ = *variable*

Pick an SU(2) connection, A^{AB}, such that F^{AB} satisfies $\mathcal{D}F^{AB} = 0$

$$F^{(AB} \wedge F^{CD)} = 0$$

Pick next a variable, Λ and *define:*

$$\Sigma^{AB} \equiv \frac{3}{\Lambda} F^{AB}$$

This satisfies: $\Sigma^{(AB} \wedge \Sigma^{CD)} = 0$

But now there is torsion:

$$\mathcal{D}\Sigma^{AB} \equiv S^{AB} = \mathcal{D}(\frac{3}{\Lambda(x)} F^{AB}) = -\frac{3}{\Lambda^2} d\Lambda \wedge F^{AB} = -\frac{1}{\Lambda} d\Lambda \wedge \Sigma^{AB}$$

there still exists a frame field e $^{AA'}$, such that***:*** $\Sigma^{AB} = e^{A'A} \wedge e^{B}_{A'}$

still, solves the Einstein eq with***:*** $\Phi_{ABCD} = 0$

$$F_{AB} = \frac{\Lambda}{3} \Sigma_{AB} + \Phi_{ABCD} \Sigma^{CD}$$

Check of Λ-variable solutions in Palatini

$$R_a = \frac{1}{6}\epsilon_{abcd} e^b \wedge R^{cd}$$

$$e_a = \frac{1}{6}\epsilon_{abcd} e^b \wedge e^c \wedge e^d$$

Einstein eq's in terms of 3-forms:

$$R_a - \frac{1}{2} e_a R = -\Lambda e_a \quad \text{(E)}$$

This is solved by, with variable Λ:

$$R^{ab} = \frac{\Lambda}{3} e^a \wedge e^b \quad \text{(SD)}$$

When the torsion is defined by:

$$\mathcal{D} e^a \equiv T^a = \frac{1}{2\Lambda} d\Lambda \wedge e^a \quad \text{(T)}$$

To show this, take covariant curl of both sides of (SD):

D LHS =0, D RHS =0 using the definition of torsion (T)

To show consistency, take curl again and use (T) again.

More on torsion

The definition of torsion is as a 2-form: $\mathcal{D}e^{BA'} = T^{BA'}$

Hence: $S^{AB} = 2e^{(A}{}_{A'} \wedge \mathcal{D}e^{B)A'} = 2e^{(A}{}_{A'} \wedge T^{B)A'}$

Note that as is required for consistency: $\mathcal{D} \wedge \mathcal{D}\Sigma^{AB} = 0$

Go back to Lorentz indices $T^a = \mathcal{D}e^a = de^a + A^a{}_b \wedge e^b$

The connection is a 1-form: $A^{ab} = \omega^{ab}(e) + K^{ab}$

K^{ab} is the contortion 1-form, related to the torsion 2-form: $T^a = K^a{}_b \wedge e^b$

We also introduced the 3-form: $\mathcal{D}\Sigma^{ab} = S^{ab} = 2T^{[a} \wedge e^{b]}$

Which we found was: $S^{ab} = \frac{3}{2\Lambda^2} d\Lambda \wedge F^{ab} = \frac{1}{2\Lambda} d\Lambda \wedge e^a \wedge e^b = 2T^{[a} \wedge e^{b]}$

Thus, for self-dual solutions: $T^a = \frac{1}{\Lambda} d\Lambda \wedge e^a$

Λ ***kinetic energy from torsion***

On self-dual solutions to the A^{ab} ***equations of motion,***

$$A^{ab} = \omega^{ab}(e) + K^{ab} \qquad K_\alpha^{bc} = -\frac{1}{2\Lambda} e_\alpha^{[b} e^{c]\beta} \partial_\beta \Lambda$$

$$R^{ab}(A) = \tilde{R}^{ab}[\omega(e)] + \mathcal{D}K^{ab} + K^a{}_c \wedge K^{bc}$$

The effective action has new term in $(d\Lambda)^2$

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} -e e_a^\alpha e_b^\beta R_{\alpha\beta}{}^{ab}(A)$$

$$S^{new} = \frac{1}{8\pi G} \left(\int_{\mathcal{M}} \frac{3}{\Lambda^2} e g^{\alpha\beta} \partial_\alpha \Lambda \partial_\beta \Lambda + \int_{\partial\mathcal{M}} e^a \wedge e^b \wedge K_{ab} \right)$$

Effective dynamics for Λ

Effective dynamics for Λ

$$\tilde{\Box}\Lambda = \Sigma^{AB} \wedge \Sigma_{AB} \left[1 - (\frac{3}{\Lambda})^2 \frac{F^{AB} \wedge F_{AB}}{\Sigma^{AB} \wedge \Sigma_{AB}} \right]$$

Where

$$\tilde{\Box} = \frac{1}{\Lambda} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \frac{1}{\Lambda} \partial_\nu \right)$$

There are fixed points at

$$\partial_\mu \Lambda = 0; \qquad \frac{\Lambda}{3} = \sqrt{\frac{F^{AB} \wedge F_{AB}}{\Sigma^{AB} \wedge \Sigma_{AB}}}$$

Effective dynamics for Λ

There are fixed point at $\partial_\mu \Lambda = 0; \quad \frac{\Lambda}{3} = \sqrt{\frac{F^{AB} \wedge F_{AB}}{\Sigma^{AB} \wedge \Sigma_{AB}}}$

The gravitational chiral anomaly

$$\nabla_\mu J_l^\mu = \frac{3}{16\pi^2} R_{ab} \wedge R^{ab}$$

So the fixed points are at

$$\frac{\Lambda}{3} = \sqrt{\frac{16\pi^2}{3}} \sqrt{\frac{\nabla_\mu J_l^\mu}{\Sigma^{AB} \wedge \Sigma_{AB}}}$$

Note: $G\Lambda \approx (\Delta m_\nu)^4$

$\Delta m_\nu \approx 3 \times 10^{-3}$

A new uncertainty relation.

Another approach is to define a preferred slicing, and define Λ and the Chern-Simons time as a function of the slices.

$$T_{CS} = \mathcal{I}m \int_\Sigma Y_{CS}(A)$$

Then the new term in the action is

$$S^{new} = \frac{3}{16\pi\hbar G} \int dt \frac{\dot{\Lambda}}{\Lambda^2} \mathcal{I}m \int_\Sigma Y_{CS}(A)$$

This implies a new Poisson bracket and uncertainty relation.

$$\{\Lambda, \int_\Sigma \mathcal{I}m Y_{CS}(A)\} = \frac{16\pi G \Lambda^2}{3}$$

$$\Delta\Lambda\Delta\tau_{CS} \geq \frac{8\pi\hbar G}{3} \langle \hat{\Lambda^2} \rangle.$$

Effective dynamics for Λ

Effective equation of motion for Λ

$$S_{\rm eff} = -\frac{1}{8\pi G}\int d^4x e\left(\Lambda + \frac{b}{\Lambda}R\tilde{R} + \frac{c}{\Lambda^2}g^{\mu\nu}\partial_\mu\Lambda\partial_\nu\Lambda\right),$$

FRW solutions

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij}+h_{ij})dx^i dx^j\right]$$

$$\ddot{\Lambda} + \left(2\mathcal{H} - \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} - \left(\delta^{ij}+h^{ij}\right)\left(\partial_i\partial_j\Lambda - \frac{1}{\Lambda}\partial_i\Lambda\partial_j\Lambda\right) = -\frac{a^2}{2c}\left(\Lambda^2 - bR\tilde{R}\right)$$

deSitter solutions

$$\ddot{\Lambda} - \left(\frac{2}{\eta} + \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} + \frac{1}{2cH^2\eta^2}\left(\Lambda^2 - bR\tilde{R}\right) = 0.$$

time dependent potential

$$V(\phi) = \frac{\phi}{3\tilde{c}}\left(\phi^2 - 3\tilde{b}(x)\right).$$

$$\phi = \Lambda/M_p^2 = \rho\exp[\varphi]$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

Effective dynamics for Λ

$$\ddot{\Lambda} - \left(\frac{2}{\eta} + \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} + \frac{1}{2cH^2\eta^2}\left(\Lambda^2 - bR\tilde{R}\right) = 0$$

Potential for Λ

$$V(\Lambda) = \frac{\Lambda}{6c}\left(\Lambda^2 - 3bR\tilde{R}\right)$$

change to ϕ*=log* Λ *to make kinetic energy canonical:*

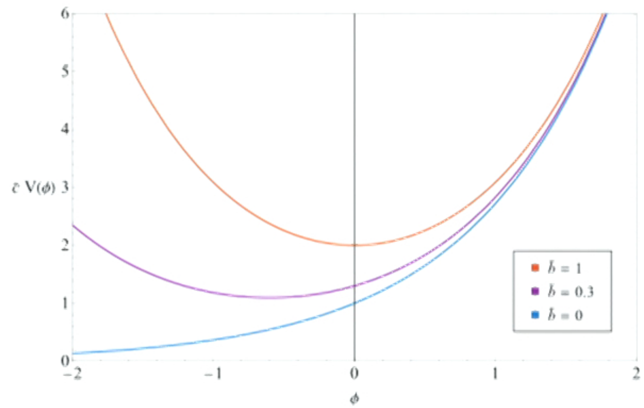
$$\phi = \Lambda/M_p^2 = \rho\exp[\varphi]$$

To find potential for ϕ*=log* Λ

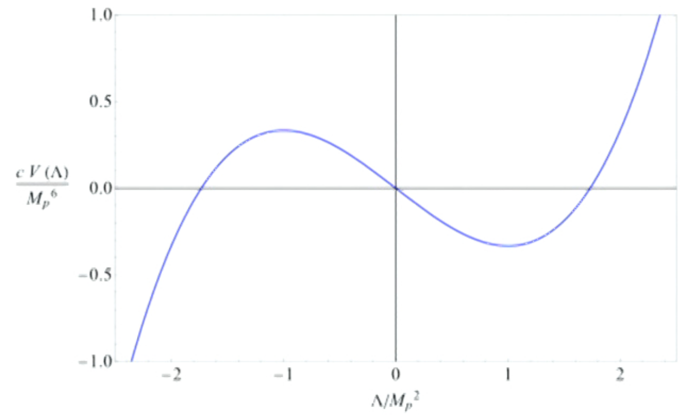
$$V(\phi) = \frac{1}{\tilde{c}}\left(e^{\phi} + \tilde{b}e^{-\phi}\right)$$

Effective Potential For Lambda

work in progress
Alexander, Sims, Smolin

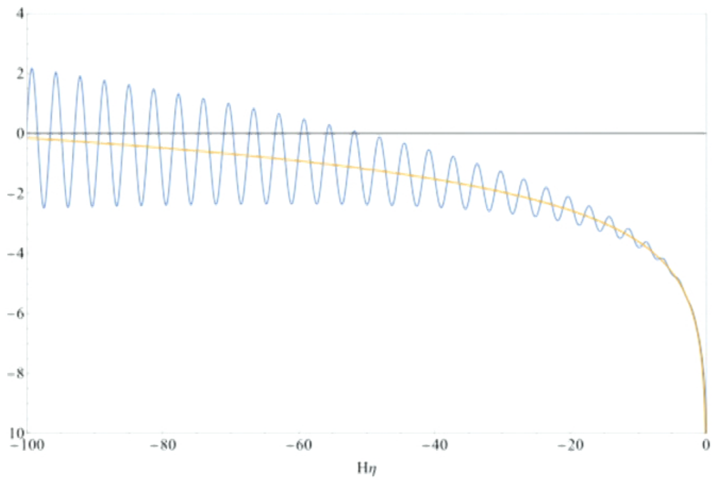


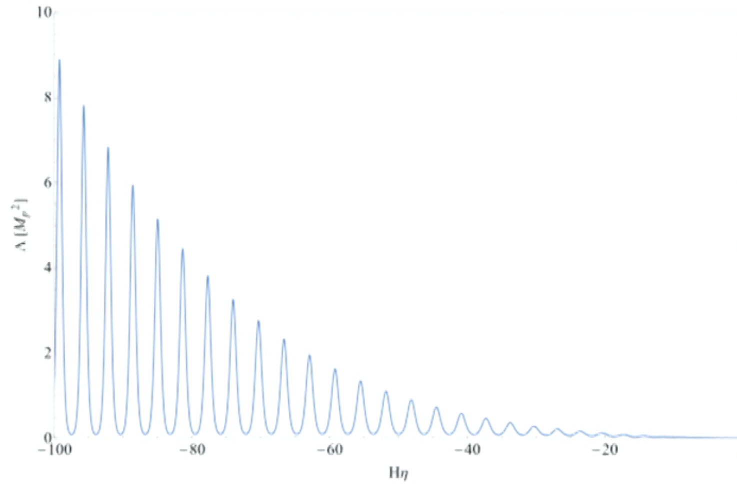
(b) Effective potential for field ϕ with various $\bar{b}$ values. For $\bar{b} = a(\eta)^{-n}$ for positive n (as we expect), propagating time forward results in smaller $\bar{b}$, thus the minima for $V(\phi)$ becomes more negative.



(a) Effective potential for the field Λ with $bR\tilde{R} = M_p^4$.

Assume falloff: $R\tilde{R} \propto a^{-n}$





Note: Λ does not change sign

$$\Lambda \lesssim H^2 \exp\left[-\mathcal{O}(10^{-1})H^2\left(\frac{B}{\sqrt{\delta}}\right)\right]$$

Coupled numerical evolution *Robert Sims, in progress.*

$$\ddot{\Lambda} + \left(3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} + \frac{1}{2c}\left(\Lambda^2 - bR\tilde{R}\right) = 0$$

$$bR\tilde{R} = t^{-3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left(\rho + \Lambda\right)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = -M_p^2\dot{\Lambda}.$$

Initial condition Λ_0 is at twice the fixed point.

Coupled numerical evolution *Robert Sims, in progress.*

$$\ddot{\Lambda} + \left(3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} + \frac{1}{2c}\left(\Lambda^2 - bR\tilde{R}\right) = 0 \qquad bR\tilde{R} = t^{-3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}(\rho + \Lambda)$$

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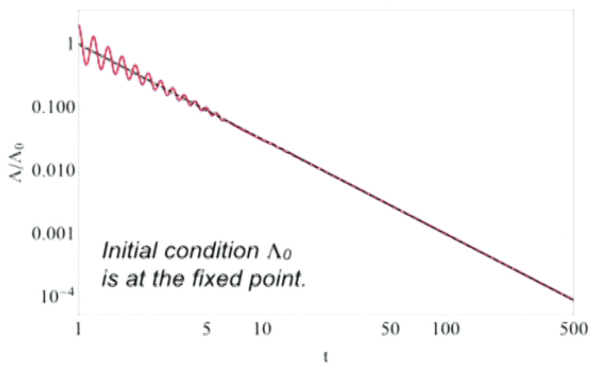
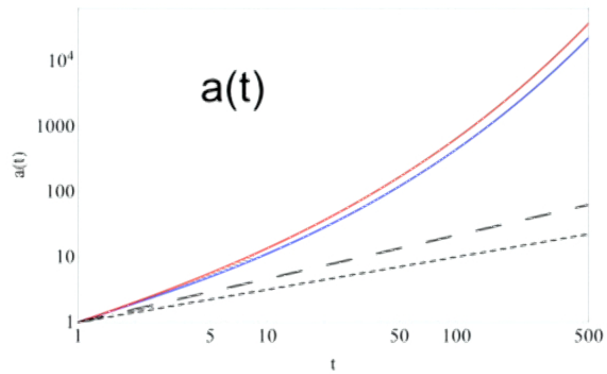
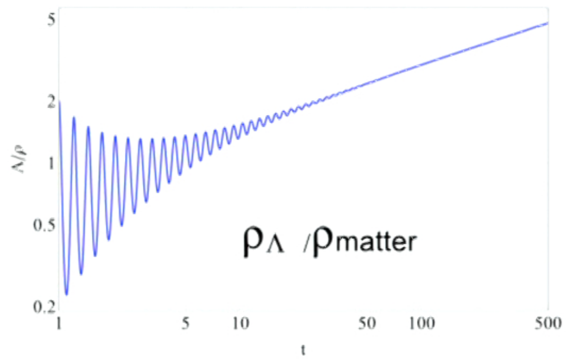
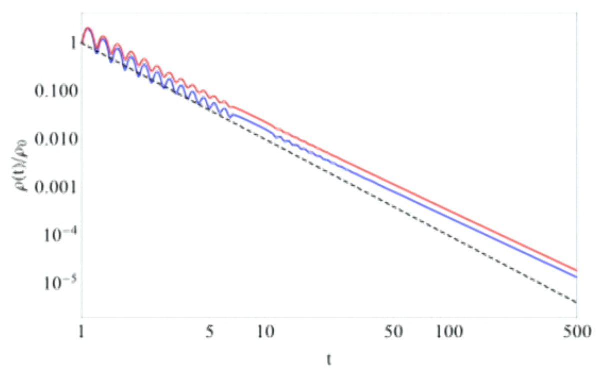
Λ oscillates around and tracks fixed point (black line)









Varying the initial H_0 *Robert Sims, in progress.*

$$\phi'' + \left(3\frac{a'}{a} - \frac{\phi'}{\phi}\right)\phi' + \frac{1}{\tilde{c}}\left(\frac{\phi^2 - \tilde{b}}{\tilde{\rho}_0^2}\right) = 0$$

$$\left(\frac{a'}{a}\right)^2 = \frac{\tilde{\rho} + \phi}{\tilde{\rho}_0}$$

$$\tilde{\rho}' + 3\frac{a'}{a}\tilde{\rho}(1+\omega) = -\phi'$$

Switch to Planck units:

$$\phi = \Lambda/M_p^2$$

$$\tilde{c} = 2c/9\,(M_p/H_0)^2$$

$$\tilde{\rho} = \rho/M_p^4$$

$$x = mt$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

Initial conditions:

Λ_0 *=twice fixed point*
H_0 *fixed by fixing c-tilde*
ρ_0 *is then found by solving the Friemann eq.*

Varying the initial H_0 *Robert Sims, in progress.*

$$\phi'' + \left(3\frac{a'}{a} - \frac{\phi'}{\phi}\right)\phi' + \frac{1}{\tilde{c}}\left(\frac{\phi^2 - \tilde{b}}{\tilde{\rho}_0^2}\right) = 0$$

$$\left(\frac{a'}{a}\right)^2 = \frac{1}{3}\left(\frac{M_p^2}{m^2}\right)(\tilde{\rho} + \phi)$$

$$\tilde{\rho}' + 3\frac{a'}{a}\tilde{\rho}(1+\omega) = -\phi'$$

Switch to Planck units:

$$\phi = \Lambda / M_p^2$$

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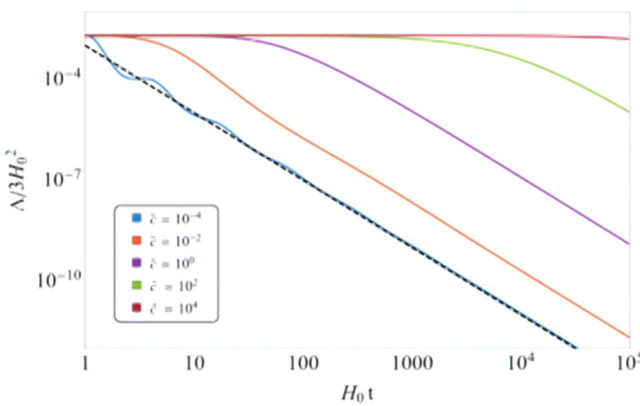
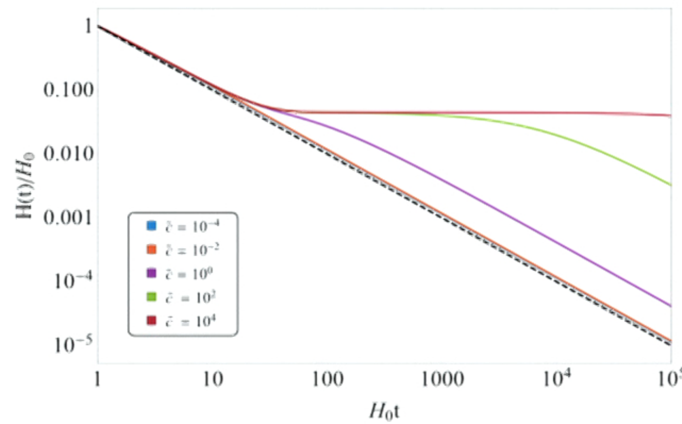
$$\tilde{\rho} = \rho / M_p^4$$

$$x = mt$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

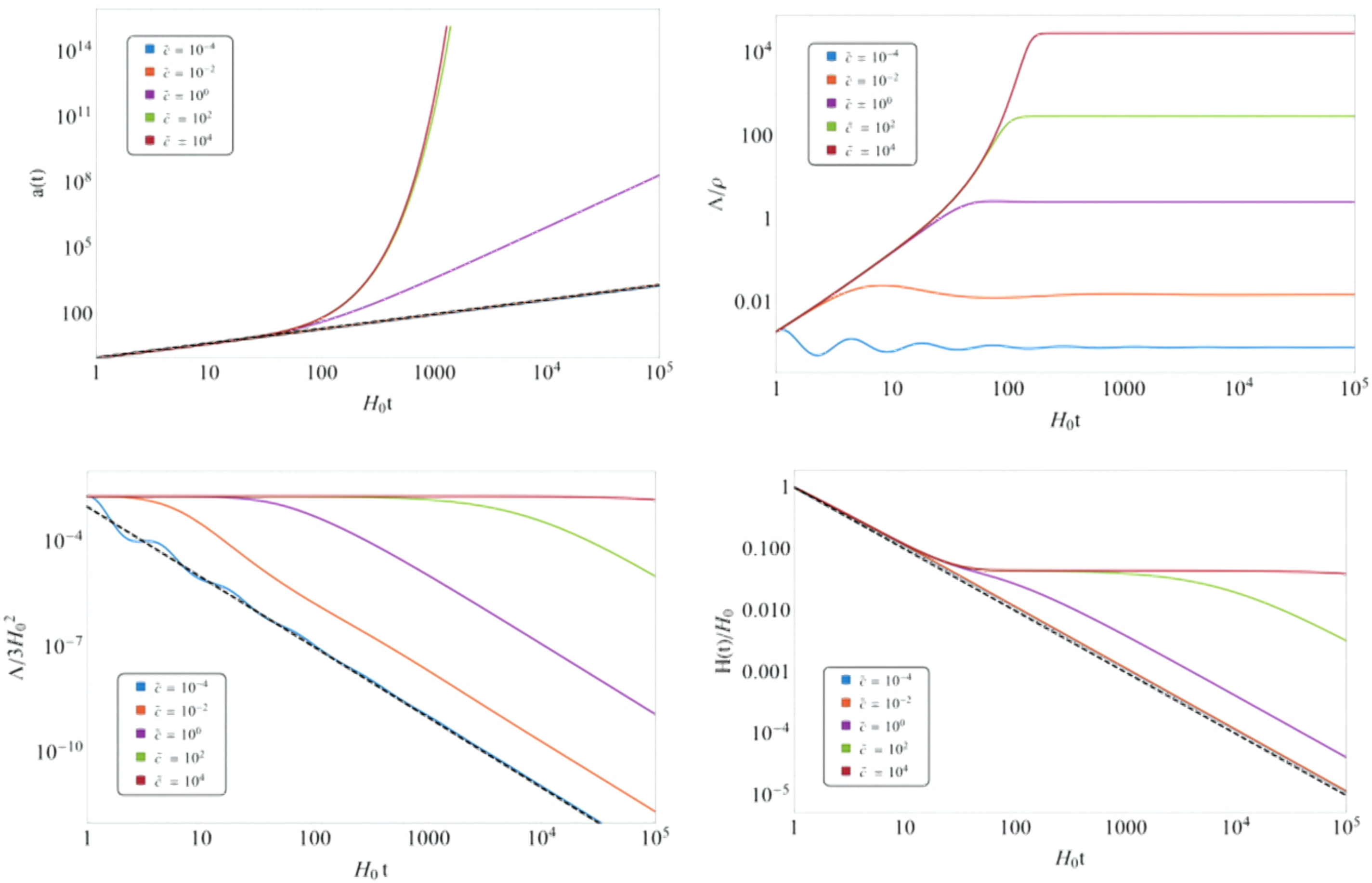




Varying the initial H₀ *Robert Sims, in progress.*

Switch to Planck units:

$$\tilde{c} = 2c/9\,(M_p/H_0)^2$$



Typical behaviours seen, depending on initial conditions:

- *Sign of* Λ *never changes.*
- Λ *goes to time dependent fixed point, which takes it into 0.*
- Λ *first shows damped oscillations around fixed point.*
- *Or* Λ *freezes out, leading to* Λ *domination.*