

Title: Anomalies, 2-groups, 6d susy

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Abstract:

Part 1 is about anomalies and how they can deform generalized global symmetry into 2-group global symmetry. This is illustrated with simple QFT examples in 4d. Part 2 is about 6d.

6d theories with 2-group symmetry exist, but cannot be conformal. 6d superconformal theories (SCFTs) cannot have 2-group or higher-form, generalized global symmetries. This requires cancellation of mixed gauge and global terms in the anomaly polynomial. SCFT relations between conformal and $\mathcal{N}=(2,0)$ Hooft anomalies will also be discussed. Based on papers with Cordova and Dumitrescu.

Anomalies, 2-groups, 6d susy

Ken Intriligator (UCSD)

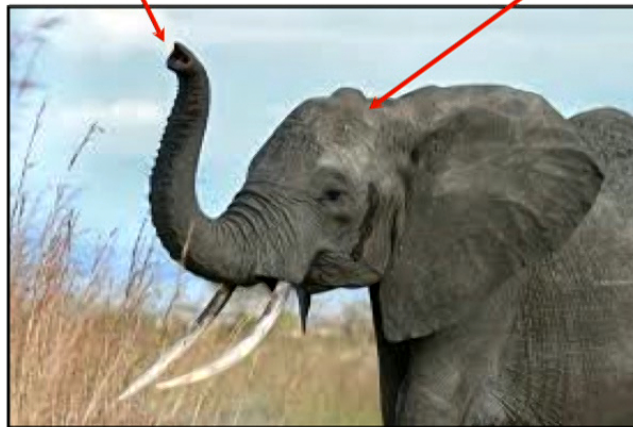
Based on work with **Clay Cordova** (IAS / U. Chicago)
and **Thomas Dumitrescu** (UCLA).

“What is QFT?”

(See N. Seiberg's website for a talk with this title.)

Perturbation theory
around free field
Lagrangian theories

**5d & 6d* SCFTs, +
deformations,
compactifications**



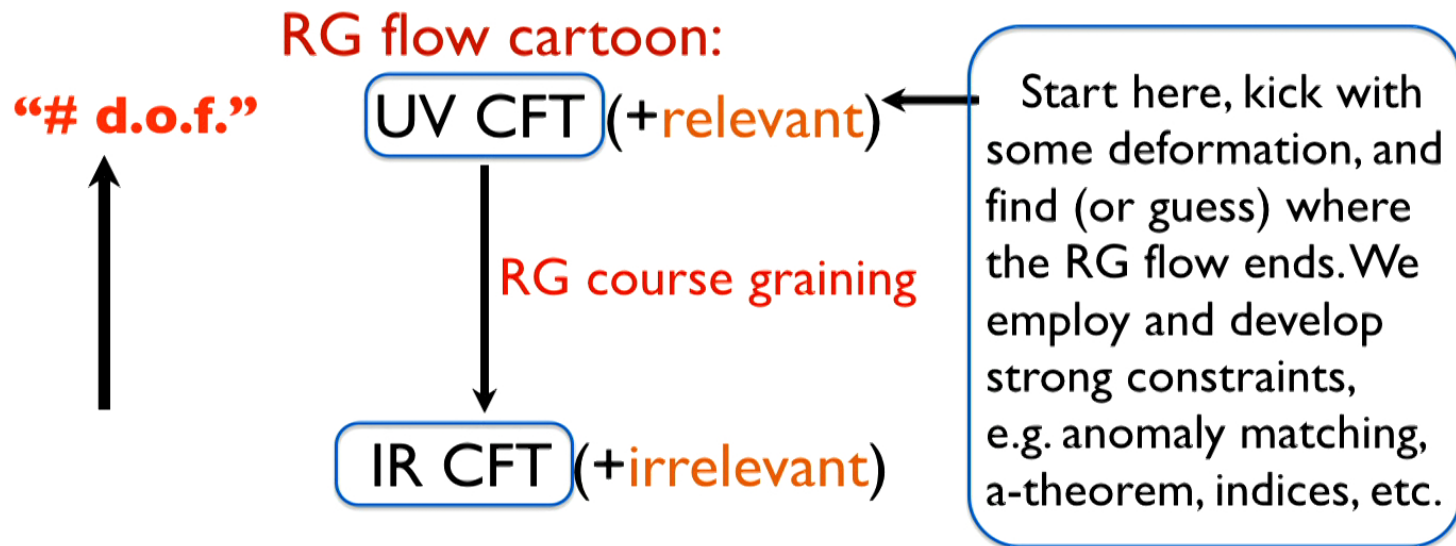
**Explore parts of the
space of QFTs via
CFTs + perturbations**

**(?unexplored...something
crucial for the future?)**

* **d=6** is largest d of SCFTs.

RG flows, universality

In extreme UV or IR, masses become unimportant or decoupled. Enhanced, conformal symmetry in these limits. E.g. QCD: UV-free quarks and gluons in UV, and IR-free pions or mass gap in IR. Now many examples of non-trivial, interacting CFTs and especially with SUSY. Can deform them to find new QFTs.




RG flow constraints

- 't Hooft **anomaly** matching for global symmetries + gravity. They must be constant on RG flows; match at endpoints.
- Reducing # of d.o.f. intuition. For $d=2,4$ (& $d=6$ susy) : a-theorem

$$a_{UV} \geq a_{IR} \quad a \geq 0 \quad \text{For unitary thys}$$

conformal
anomaly: $\langle T_{\mu}^{\mu} \rangle \sim aE_d + \sum_i c_i I_i$



a-theorem proof of
Komargodski + Schwimmer via
conf'l anomaly matching.

(d =odd: via sphere partition function / entanglement entropy.)

- Additional power from supersymmetry. Supermultiplets and supermultiplets of **anomalies**.

q-form global currents

- Conserved flavor current: $\partial^\mu J_\mu^a = 0$. Source: A_μ^a bkgd.
 = “q=0-form” global symmetry. (a = g Lie alg. index) $\delta A_\mu^a = (D_\mu \lambda)^a$
- Conserved **higher q-form global symmms:**
 Gaiotto, Kapustin, Seiberg, Willett and refs therein.

$$j_{[\mu_1 \dots \mu_{q+1}]^{(q+1)}} \quad \text{with} \quad \partial^{\mu_1} j_{[\mu_1 \dots \mu_{q+1}]^{(q+1)}} = 0. \quad \text{i.e.} \quad d * j^{(q+1)} = 0$$

$$Q(\Sigma_{d-q-1}) = \int_{\Sigma_{d-q-1}} *j^{(q+1)} \quad \Delta_{\text{exact}}(j^{q+1}) = d - q - 1$$

q>0: only abelian, $U(1)^{(q)}$ (or discrete subgroup).

E.g. 4d $u(1)$ gauge theory with charged matter has $U(1)^{(1)}$
global symmetry with $*j^{(2)} = F_{u(1)}$

Couple all currents to background fields

- Poincare': Source = bkgd metric $g_{\mu\nu} = \delta_{ab} e_{\mu}^a e_{\nu}^b$
 $\delta e^{(1)a} = -\theta_b^{(0)a} e^{(1)b}$
- Conserved flavor current: $\partial^{\mu} J_{\mu}^a = 0$. Source: A_{μ}^a bkgd.
- Conserved $q > 0$ current: Invariance: $\delta A_{\mu}^a = (D_{\mu} \lambda)^a$

$$S \supset \int B^{\mu_1 \dots \mu_{q+1}} j_{[\mu_1 \dots \mu_{q+1}]} dV = \int B^{(q+1)} \wedge \star j^{(q+1)}$$

$$\delta B^{(q+1)} = d\Lambda^q \quad \text{invariance since} \quad d \star j^{(q+1)} = 0$$

$$\text{E.g. for 4d } u(1) \text{ gauge theory: } \int B^{(2)} \wedge F_{u(1)}^{(2)}$$

Background gauge invariance encodes conservation laws.

Recall anomalies

Effective action as fn of background fields:

$$W[\mathcal{B}] = -\log\left(\int [d\psi][dA] e^{-S[\mathcal{B}, \psi, A]/\hbar}\right)$$

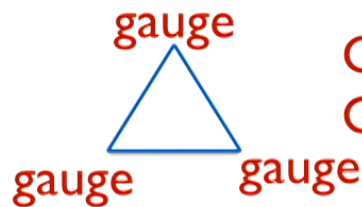
$$W[\mathcal{B} + \delta\mathcal{B}] - W[\mathcal{B}] = \mathcal{A}[\mathcal{B}] = 2\pi i \int \mathcal{I}^{(d)}[\mathcal{B}, \delta\mathcal{B}] \quad \text{(descent procedure)}$$

$$d\mathcal{I}^{(d)}[\mathcal{B}, \delta\mathcal{B}] = \delta\mathcal{I}^{(d+1)}[\mathcal{B}], \quad d\mathcal{I}^{(d+1)}[\mathcal{B}] = \mathcal{I}^{(d+2)}[\mathcal{B}]$$

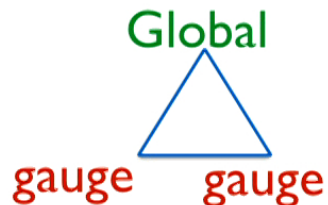
For $d=2n$, the matter content must be gauge anomaly free. Anomalies encoded in a topological $d+2$ form in gauge **and global** background field strength Chern classes, and Pontryagin classes for the background metric curvature. Compute via $(n+1)$ -gon diagram, or inflow, etc. Calculable via various methods.

We discuss mixed gauge+ global anomalies. They quantum-deform the global symmetry group into a “2-group.”

Anomalies (4d case)



Gauge anomalies must vanish for a healthy theory.
Constrains chiral matter content.



ABJ anomaly, only for global U(1)s. If non-zero, global U(1) is just not a symm (explicitly broken by instantons, perhaps to a discrete subgroup).



't Hooft anomalies. **Useful** if non-zero: must be constant along RG flow, match at ends.



Does not violate either symmetry! **Deforms** global symmetry to a **2-group** symmetry.

$$d * j_{\text{global}}^{(1)} = \frac{\kappa}{(2\pi)^2} F_{\text{global}} \wedge F_{\text{gauge}} = \frac{\kappa}{2\pi} F_{\text{global}} \wedge *J_B^{(2)}$$

“2-group” global symmetry

Non-trivial structure function interplay between conserved $q=2$ -form current and ordinary currents. Like the Green-Schwarz mechanism for the background fields. We find simple examples, use to explore and clarify many aspects.

(See e.g. Kapustin and Thorngren papers, and refs therein.)

Global symmetry:
bkgd gauge transfs

$$G^{(0)} \times_{\hat{\kappa}} U(1)^{(1)}$$

$$\delta A_{\mu}^a = (D_{\mu} \lambda)^a$$

$$\delta B^{(2)} = d\Lambda^{(1)} + \frac{\hat{\kappa}}{2\pi} \lambda^{(0)} dA^{(1)}$$

+ analog for Poincare' $SO(4)$ frame rotation of spin connection: $+\frac{\hat{\kappa}_{\mathcal{P}}}{16\pi} \text{tr}(\theta^{(0)} d\omega^{(1)})$

$$H^{(3)} = dB^{(2)} - \frac{\hat{\kappa}_A}{2\pi} CS(A) - \frac{\hat{\kappa}_{\mathcal{P}}}{16\pi} CS(\omega), \quad \text{dH sourced by background gauge \& gravity instanton.}$$

4d QED example

Consider a 4d (non-susy) QED, i.e. $u(1)$ gauge theory, with N flavors of massless Dirac Fermion (IR free, needs a UV cutoff).

Global symmetry: $SU(N)_L^{(0)} \times SU(N)_R^{(0)} \times U(1)_B^{(1)}$

$U(1)_A^{(0)}$ broken by ABJ anomaly.

$U(1)_V^{(0)} \rightarrow u(1)_{\text{gauge}}$

$j_B^{\mu\nu} \propto \epsilon^{\mu\nu\rho\sigma} f_{\rho\sigma}$
 global current dyn. $u(1)$ gauge field.



Non-zero mixed anomaly. No broken symmetry.
 Deforms the global symmetry to a 2-group symm.:

$$\left(SU(N)_{L-R}^{(0)} \times_{\kappa=1} U(1)_B^{(1)} \right) \times SU(N)_{L+R}^{(0)}$$

Chiral toy model examples

Consider a 4d (non-susy) theory with two 0-form flavor symms $U(1)_A$ and $U(1)_C$ and matter chiral Fermions with charges (q_A, q_C) .

	q_A	q_C
ψ_1	1	3
ψ_2	1	4
ψ_3	-1	5
ψ_4	0	-6

$$\kappa_{A^3} = \text{Tr}U(1)_A^3 = 1 \quad \text{'t Hooft}$$

$$\kappa_{A^2C} = \text{Tr}U(1)_A^2 U(1)_C = 12 \quad \text{mixed}$$

$$\kappa_{AC^2} = \text{Tr}U(1)_A U(1)_C^2 = 0 \quad \text{ABJ}=0$$

$$\kappa_{C^3} = \text{Tr}U(1)_C^3 = 0 \quad \text{gauge}=0$$



Take A =global and C =gauge symmetry. Non-zero 't Hooft and mixed anomaly. $\mathcal{I}_6^{\text{mixed}} = (\kappa_{A^2C} c_2(F_A) + q_{C,tot} p_1(T)) \wedge c_1(f_c)$



2-group structure constants

Global 0-form and 1-form symmetries: $G^{(0)}$ $G^{(1)}$

$\beta \in H^3(G^{(0)}, G^{(1)})$ we call them $\hat{\kappa}_{G^{(0)}}$, $\hat{\kappa}_{\mathcal{P}}$.

Kapustin & Thorngren: Postnikov class. We also call them 2-group structure constants.

Coefficients of CS terms in invariant field strength $H^{(3)}$. For quantized charges, compact global groups, these coefficients must be integers: $\hat{\kappa}_{G^{(0)}}$, $\hat{\kappa}_{\mathcal{P}} \in \mathbf{Z}$. They are scheme independent physical properties of the QFT. Can only arise at tree-level level or one-loop. Mixed anomaly terms give this symmetry.

E.g.: $U(1)_A^{(0)} u(1)_C^{(0)} \longrightarrow U(1)_A^{(0)} \times_{\hat{\kappa}_A, \hat{\kappa}_{\mathcal{P}}} U(1)_B^{(1)}$

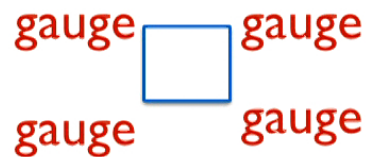
GLOBAL \nearrow gauge

$$\hat{\kappa}_A = -\frac{1}{2}\kappa_{A^2 C} \in \mathbf{Z}$$

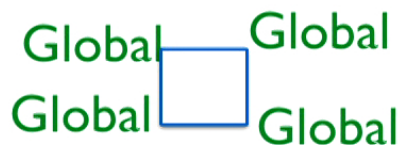
$$\hat{\kappa}_{\mathcal{P}} = -\frac{1}{6}\kappa_{\mathcal{P}^2 C} \in \mathbf{Z}$$

“Mixed anomaly”
coeffs. give 2-group
with no anomaly.

Likewise 6d anomalies



Gauge anomalies must vanish. Can use a dyn GSWs mechanism to cancel reducible parts.



't Hooft anomalies. Useful if non-zero. Must be constant along RG flow, match at ends.



Does not violate any symmetry. Deforms global symmetry to a 2-group symmetry. Here the gauge group can be non-Abelian. (In 4d, there is only one gauge vertex, so it must be $u(1)$.)

Example: small $SO(32)$ instanton theory (Witten '95)

$$\mathcal{I}_8^{\text{mixed}} = c_2(F_{sp(N)}) (c_2(F_{SO(32)}) + (16 + N)p_1(T))$$

Mixed gauge/global anomalies and 2-groups

$$U(1)_B^{(1)} \quad 4d: \quad \star j^{(2)} = c_1(f_{\text{gauge}}) = \frac{f_{\text{gauge}}}{2\pi}, \quad q_J = \int_{\Sigma_2} \star j^{(2)} \in \mathbf{Z}$$

$$U(1)_B^{(1)} \quad 6d: \quad \star j^{(2)} = c_2(f_{\text{gauge}}) = \frac{1}{8\pi^2} \text{Tr} f_{\text{gauge}} \wedge f_{\text{gauge}}, \quad q_J = \int_{\Sigma_2} \star j^{(2)} \in \mathbf{Z}$$

Conserved since $d \star j^{(2)} = 0$, charged objects = e.g. ANO vortex strings (4d), instanton strings (6d).

Couple the 1-form global symmetries to 2-form background gauge fields B. $S_{4d,6d} \supset \int B \wedge \star j$

The mixed “anomaly” means that B shifts under a bkgd flavor or metric gauge transformation

$$\begin{aligned} A' &= A + d\lambda_A, \\ B' &= B + d\Lambda + \frac{\kappa}{2\pi} \lambda_A F_A \end{aligned}$$

2-group affects reducible 't Hooft anomaly matching

E.g. $U(1)_A^{(0)} \times_{\hat{\kappa}_A} U(1)_B^{(1)}$ only has $\text{Tr}U(1)_A^3$ 't Hooft anomaly matching mod $6\hat{\kappa}_A$, because of a possible counterterm: $S_{SG} = \frac{in}{2\pi} \int B^{(2)} \wedge F_A^{(2)}$, $U(1)_B^{(1)}$: $n \in \mathbf{Z}$
 $\kappa_A^3 \rightarrow \kappa_A^3 + 6n\hat{\kappa}_A$ E.g. can gap if $\text{Tr}U(1)_A^3 = 0 \pmod{6\hat{\kappa}_A}$

TQFTs can give similar, but physical (non-counterterm) terms with fractional n. They can be used to match 't Hooft anomalies via a gapped TQFT. E.g. $u(1)_C$ gauge theory broken to \mathbf{Z}_{q_C} TQFT by Higgs mechanism of field with charge $q_C > 1$. Allows $\text{Tr}U(1)_A^3 \neq 0$ to be matched by gapped TQFT if $\text{Tr}U(1)_A^3 = 0 \pmod{6n\hat{\kappa}_A}$, $q_C n \in \mathbf{Z}$

2-group symmetry can be IR emergent, accidental.

E.g. embed 4d QED model, with 2-group symmetry, into an $su(2)$ gauge group UV completion, where $su(2)$ is broken to $u(1)$ by the vev of an adjoint Higgs scalar Georgi-Glashow model. The global $U(1)^{(1)}$ of QED theory is an accidental symmetry, explicitly broken in the UV completion by monopole operators. Note subgroups:

$$G^{(0)} \times_{\hat{\kappa}} U(1)^{(1)} \supset U(1)^{(1)} \quad \text{but} \quad G^{(0)} \times_{\hat{\kappa}} U(1)^{(1)} \not\supset G^{(0)}$$

$$\delta A_{\mu}^a = (D_{\mu} \lambda)^a \quad \delta B^{(2)} = d\Lambda^{(1)} + \frac{\hat{\kappa}}{2\pi} \lambda^{(0)} dA^{(1)} + \frac{\hat{\kappa} \mathcal{P}}{16\pi} \text{tr}(\theta^{(0)} d\omega^{(1)})$$

Affects breaking / emerging patterns: the 1-form symmetry must emerge before the 0-form symmetry if $\hat{\kappa} \neq 0$

2-group vs CFT

With background gauge field $d * j_G^{(1)A} = \frac{\hat{\kappa}_A}{2\pi} F_G^{(2)A} \wedge * J_B^{(2)}$

W/o backgrounds, Ward identity contact terms e.g.:

$$\frac{\partial}{\partial x_\mu} \langle j_\mu^A(x) j_\nu^A(y) J_{\rho\sigma}^B(x) \rangle = \frac{\hat{\kappa}_A}{2\pi} \partial^\lambda \delta^{(4)}(x-y) \langle J_{\nu\lambda}^B(y) J_{\rho\sigma}^B(z) \rangle$$

Derivative of delta function: does not alter the charge algebra. Implies non-zero 3-point function also at separated points. Incompatible with additional constraints of CFT (modulo caveats for special cases that we discuss in detail). Tension between 2-group vs CFT. Indeed the examples with 2-group symmetry are IR free. CFT in IR or UV only if 2-group symmetry is IR spontaneously broken or UV emergent.

2-group vs CFT in $d > 4$

1-form symmetry has conserved 2-form current, $\Delta[j_{\mu\nu}] = d - 2$

Exists as a short rep of the conf'l gp, and for $d > 4$ it is not necessarily free (it is co-closed, but not also closed as in 4d).

Using conservation laws we show that, as in 4d

$$\langle T^{\mu\nu}(x) T^{\kappa\lambda}(y) j^{\rho\sigma}(0) \rangle = 0 \quad \text{So no 2-group with metric diffs.}$$

But there were not enough constraints to rule out

$$\langle J^\mu(x) J^\nu(y) j^{\rho\sigma}(z) \rangle \neq 0 \quad \text{Possibly 2-group with global symms.}$$

But only w/o susy and we don't now know about non-susy 6d QFTs.

No 6d SCFT 2-group

Not even global $U(1)^{(1)}$ (CDI): **no** unitary 6d SCFT reps contain global, conserved 2-form currents. So no 2-group symmetry nor mixed gauge, global anomalies can occur for 6d SCFTs. If it is a SCFT, any apparent such mixed anomalies must be cancelled by the GSWS mechanism, along with the reducible gauge anomalies. Justifies prescription of **Ohmori, Shimizu, Tachikawa, and Yonekura**.

$$\mathcal{I}_8^{\text{total}} = \mathcal{I}_8^{\text{"1-loop"}} + \mathcal{I}_8^{\text{GSWS}} = \mathcal{I}_8^{\text{gauge}} + \mathcal{I}_8^{\text{mixed}} + \mathcal{I}_8^{\text{global}}$$

Only if **dynamical**
2-form B gauge field(s).

$\neq 0$

$= 0$, if
SCFT

This affects 't Hooft anomaly coefficients for e.g. $SU(2)_R$ in SCFT examples with gauge multiplets. Turns out to be crucial for ensuring positivity of the conf'l anomaly \mathbf{a}_{SCFT} computed via 't Hooft anomalies.

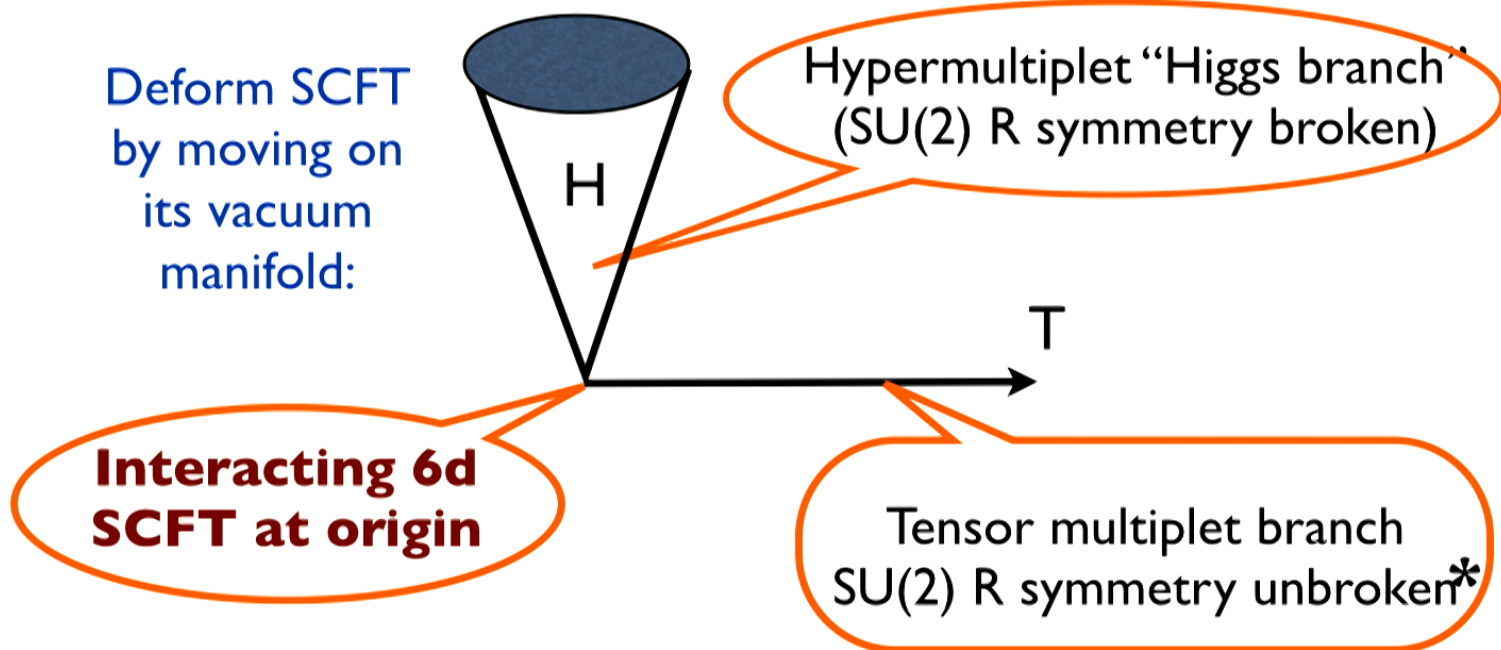
6d susy gauge theories

Example: N small $SO(32)$ instantons theory (Witten '95)
 $sp(N)$ gauge group, with matter s.t. both irreducible and reducible sp gauge anomalies = zero. No tensor multiplet. Exists with little string UV completion. Has a conserved 2-form current from $\star c_2(F_{sp(N)})$ so cannot be conformal.

$$\mathcal{I}_8^{\text{mixed}} = c_2(F_{sp(N)}) (c_2(F_{SO(32)}) + (16 + N)p_1(T)) \quad \text{so 2-group symmetry.}$$

Example: gauge group and matter s.t. 0 irred gauge anomaly, with **non-0** reducible gauge anomaly, cancelled by GSWS mechanism via a dynamical 2-form gauge field. Eliminates 2-form current & mixed anomalies. SCFT.

Part 2: 6d (1,0) SCFTs



* Easier case. Just dilaton, no NG bosons. Dilaton = tensor multiplet. Study anomaly matching in EFT.

6d anomalies & susy

$$\langle T_{\mu}^{\mu} \rangle = a \text{Euler} + \sum_{i=1}^3 c_i I_i(C) + \kappa (\nabla F)^2 + \hat{\kappa} CFF + f f_{abc} \text{Tr}(F^a F^b F^c)$$

Conformal anomaly in presence of background metric and gauge fields coupling to all global conserved currents (flavor and $SU(2)_R$). Equivalent to contact terms in energy momentum tensor and current correlation functions. Determined by operator correlation functions at separated points, e.g. the 3 c anomalies encode 3 independent structures of energy momentum tensor 2-point and 3-point functions. We study susy relations between these anomaly coefficients and 't Hooft anomaly coefficients (= exactly calculable). $\mathcal{I}_8^{\text{gravity+global}} \supset \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T)$

E.g. for the (1,0) SCFT of N small E8 instantons, via M-theory inflow (N M5s @ M9 Horava-Witten wall.)

**Ohmori, Shimizu,
Tachikawa**

$$\mathcal{E}_N : (\alpha, \beta, \gamma, \delta) = (N(N^2 + 6N + 3), -\frac{N}{2}(6N + 5), \frac{7}{8}N, -\frac{N}{2})$$

6d (1,0) tensor LEEFT

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6} \quad \text{Elvang, Myers, et. al.}$$

Our deformation classification implies that b=D-term and we argue

$$\Delta a = \frac{98304\pi^3}{7}b^2 > 0 \quad \text{Proves 6d a-theorem for susy tensor branch flows.}$$

b-term susy-completes to terms in $X_4 = \sqrt{\Delta I_8}$
 $b=(y-x)/2$ $X_4 \equiv 16\pi^2(xc_2(R) + yp_1(T))$ **By recycling a 6d SUGRA analysis from Bergshoeff, Salam, Sezgin '86**

Upshot:
 CDI'15


$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

So exact 't Hooft anomaly coefficients give the **exact conformal anomaly**. E.g. using this and **OST** for the anomalies get:

$$a(\mathcal{E}_N) = \frac{64}{7}N^3 + \frac{144}{7}N^2 + \frac{99}{7}N$$

a, for 6d SCFTs with gauge flds:

E.g. SU(N) gauge group, 2N flavors, 1 tensor + anomaly cancellation for reducible gauge + mixed gauge + R-symmetry anomalies. Use $a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$


gauge  gauge = 0* (via GSWS)
 R-symm R-symm

$$a_{SCFT} = (N^2 - 1)\left(-\frac{251}{210}\right) + 2N^2\left(\frac{11}{210}\right) + \frac{199}{210} + \frac{96}{7}N^2 > 0.$$

V H T *AC :GSWS

Again, cannot be a conserved 2-form current in SCFT at the origin, despite apparent $c_2(f_{\text{gauge}})$: it sources dH and is believed to become part of a (poorly understood) non-Abelian version so not gauge invariant current at the origin.

Other conformal anomalies via 't Hooft anomalies

$$\langle T_{\mu}^{\mu} \rangle = a \text{Euler} + \sum_{i=1}^3 c_i I_i(C) + \kappa (\nabla F)^2 + \hat{\kappa} CFF + f f_{abc} \text{Tr}(F^a F^b F^c)$$


On tensor branch: $\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b \frac{(\partial\varphi)^4}{\varphi^3} + \Delta a \frac{(\partial\varphi)^6}{\varphi^6}$

Δc_i likewise = 6 derivative terms involving Weyl curvature (we assume a linear relation to 't Hooft anomalies). Unitarity implies that $b \geq 0$ with $b=0$ iff the dilaton theory is free. Then all anomaly matching terms must be prop'l to $b=(y-x)/2$, which susy-completes to terms in $\mathcal{L}_{GSWS} \sim B \wedge X_4$

$$X_4 \equiv 16\pi^2(xc_2(R) + yp_1(T)) \qquad X_4 = \sqrt{\Delta I_8}$$

Find: $\Delta c_1 \propto (y-x)\left(\frac{4}{3}y-x\right), \quad \Delta c_2 \propto (y-x)\left(\frac{2}{3}y-x\right), \quad \Delta c_3 \propto (y-x)(2y-x)$

Used free hyper, free tensor, and (2,0) results to get coeffs, not yet directly derived from higher R (1,0) SUGRA at 6-derivative order (gives predictions).

Susy relations for SCFT

$$\langle T_{\mu}^{\mu} \rangle = a\text{Euler} + \sum_{i=1}^3 c_i I_i(C) + \kappa(\nabla F)^2 + \hat{\kappa} CFF + f f_{abc} \text{Tr}(F^a F^b F^c)$$

$$c_1 = 4\alpha - \frac{14}{3}\beta + \frac{16}{3}\gamma + \frac{8}{3}\delta$$

$$c_2 = 4\alpha - \frac{10}{3}\beta + \frac{8}{3}\gamma + \frac{10}{3}\delta$$

$$c_3 = 4\alpha - 6\beta + 8\gamma + 2\delta$$

Agrees with results already found in literature **Beccaria, Tseytlin; Yankielowicz, Zhou** (via fitting to known examples + non-unitary SCFT of a free gauge field with higher derivative couplings. We instead used $\Delta c_i \propto b \sim (y-x)$

$$c_1 = \frac{1}{2}(c_2 + c_3)$$

As claimed by **de Boer, Kulaxizi, Parnachev**. We prove it via SCFT constraints on $\langle TTT \rangle$: 2 indep. structures.


We also use SCFT constraints on 3-point functions + tensor branch EFT to get $\kappa_{\text{flavor}} = \hat{\kappa}_{\text{flavor}} = \tau_{FF} = 2\alpha_{F^2 T^2} - 2\alpha_{F^2 R^2}, \quad f_{\text{flavor}} = 0.$


Current 2-point coefficient

$$\mathcal{I}_8 \supset \alpha_{F^2 T^2} c_2(F) p_1(T) + \alpha_{F^2 R^2} c_2(F) c_2(R)$$

SCFT 2 & 3-point functions

$$\langle \mathcal{O}_{1,L_1}^{R_1}(z_1) \mathcal{O}_{2,L_2}^{R_2}(z_2) \mathcal{O}_{3,L_3}^{R_3}(z_3) \rangle = F_{\text{fixed}}(\Delta, L, R, \chi_{13}, \chi_{23}, u_{13}, u_{23}) \cdot H_{\Delta, L, R}(X, \Theta)$$


 Supermultiplets of ops
 in superspace $z = (x_{[\alpha\beta]}, \theta_\alpha^r)$


 Coordinate built from 3 points
 in superspace

E.g. 3-point function of energy momentum tensor supermultiplets

$$\langle \mathcal{T}(x_1, \theta_1) \mathcal{T}(x_2, \theta_2) \mathcal{T}(x_3, \theta_3) \rangle \sim H^{TTT}(Z) = C_1^{TTT} + C_2^{TTT} \frac{(X\Theta)^2}{X^6} + C_3^{TTT} \frac{\Theta^8}{X^4}$$

Shortening conditions on ops on LHS constrain C_i constants on RHS.
 For energy momentum tensor 3-point function this gives 1 constraint,
 so 2 indep. C_i on the RHS, corresponding to 2 independent c_i anomalies.
 Likewise, we consider 3-point functions involving conserved currents.
 Find all 3-point functions have one additional constraint from SUSY, one
 fewer independent structure. Constrains the conformal anomalies.

E.g. small E_8 instanton SCFT

Global flavor symmetry $SU(2)_L \times SU(2)_R \times E_8$

Exact \mathcal{I}_8 and the susy relations give the 2-point and 3-point correlation functions of the energy momentum tensor and conserved currents. Find e.g.

$$\tau_{SU(2)_L}^2 = 16N^3 + 6N^2 - 21N$$

$$\tau_{SU(2)_R}^2 = c_3 = 16N^3 + 42N^2 + 22N$$

$$\tau_{E_8}^2 = 12(2N^2 + 3N)$$

On the tensor branch, pull 1M5 away from other $(N-1)M5s+M9$. Then

$b \sim (y - x) \sim N + \frac{1}{4}$ The change in all anomaly coefficients, including the above, is proportional to this factor, since the dilaton EFT must trivialize if $x=y$.

Summary, Conclude

- Explore space of QFTs via enhanced symmetry lamp posts and RG flows.
- 2-group symmetry in simple QFTs in $d=4, 6$ via mixed anomalies. 2-group vs CFT.
- Exact results for SQFTs and SCFTs in 6d.
- Thank you!