Title: Realizing supersymmetry in condensed matter systems

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Abstract: <p>Supersymmetry (SUSY) has not been verified so far as a fundamental symmetry in particle physics. Emergent phenomena in condensed matter physics bring the possibility of realizing SUSY as an IR symmetry. We show that 2+1D N=2 Nf=2 supersymmetric quantum electrodynamics (SQED3) with dynamical gauge bosons and fermionic gauginos emerges naturally at the tricritical point of nematic pair-density-wave (PDW) quantum phase transition on the surface of a correlated topological insulator hosting three Dirac cones, such as the topological Kondo insulator SmB6. It provides a first example of emergent supersymmetric gauge theory in condensed matter systems. We also investigate the possibility of emergent 3+1D SUSY theory in lattice models. By constructing an explicit fermionic lattice model featuring two 3D Weyl nodes, we find a continuous PDW quantum phase transition as a function of attractive Hubbard interaction. We further show that N=1 3+1D SUSY emerges at the PDW transition, which we believe is the first realization of emergent 3+1D space-time SUSY in microscopic lattice models. Supersymmetry allows us to determine certain critical exponents and the optical conductivity at the strongly coupled fixed point exactly, which may be measured in future experiments.</p>

Realizing supersymmetry in condensed matter systems

Shao-Kai Jian
Institute for Advanced Study, Tsinghua University

References:
SKJ, Y.-F. Jiang, & H.Yao, PRL 114, 237001 (2015)
SKJ, C.-H. Lin, J. Maciejko, & H. Yao, PRL 118, 166802 (2017)
SKJ, S. Yin, & E.-G. Moon, in preparation

Collaborators:
Prof. Hong Yao (Tsinghua)
Prof. Joseph Maciejko (Alberta)
Prof. Eun-Gook Moon (KAIST)
Dr. Yi-Fan Jiang (Stanford)
Dr. Chien-Hung Lin (Minnesota)
Dr. Shuai Yin (Tsinghua)

Perimeter Institute
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Outline

- Emergent supersymmetric quantum electrodynamics (SQED) on the surface of topological insulators
- Emergent 3+1D supersymmetric (SUSY) theory in a lattice model
- Multi-universality in pair-density-wave transition of 3+1D Weyl semimetal

Symmetries in quantum phases of matter

- Broken symmetries:
  - spontaneous symmetry breaking phases,
    e.g. antiferromagnets, superfluids, superconductors
- Preserving symmetries:
  - long-range entangled topological phases,
    e.g. FQH states, quantum spin liquids
  - symmetry protected topological phases,
    e.g. topological insulators, topological superconductors
- Emergent symmetries:
  - symmetries in low energy are enlarged, usually occurring at critical points or in critical phases

Examples of emergent symmetries at critical points or in critical phases

- Emergent Lorentz symmetry
  - Dirac fermions in graphene
  - Weyl fermions in Weyl-semimetal
- Emergent gauge symmetry
  - Quantum spin liquid, e.g. emergent QED_3 even QCD_3 in frustrated magnetic systems
- These symmetries have their counterparts in particle physics.
- Can we realize in condensed matter systems the symmetries that haven't been discovered as fundamental symmetries in particle physics, like supersymmetry?

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Brief introduction to Supersymmetry

- Space-time SUSY is a fermionic symmetry:

$$\text{Fermions} \overset{\text{superpartners}}{\Longleftrightarrow} \text{Bosons}$$

- Space-time SUSY generators Q_α:

$$\{Q_\alpha, Q_\beta\} = 2P_\mu \Gamma^\mu_{\alpha\beta}, \qquad [Q_\alpha, P_\mu] = 0$$

  - α, β = spinor indices, $\mu = t, x, y, z$ (space-time indices)
  - different from quantum-mechanical SUSY: $\{Q, Q^\dagger\} = H$

- Necessary elements in a space-time SUSY theory:
  - the number of fermions = the number of bosons
  - bosons and fermions having identical masses:

$$[Q_\alpha, P^\mu P_\mu] = 0$$

  - Lorentz invariance and translational invariance

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Simple examples in 2+1D

- Complex boson ⇔ Dirac fermion

(A) $S_{\text{Gaussian}} = \int d^3x \left[|\partial_t\phi|^2 + v_b^2|\nabla\phi|^2 + i\,\bar{\psi}\gamma^t\partial_t\psi + i\,v_f\bar{\psi}\gamma^i\partial_i\psi\right]$

ϕ = complex boson, ψ = two-component spinor

- Lorentz symmetry requires $v_b = v_f$, SUSY transformation:

$$\delta\phi = \sqrt{2}\xi\psi$$

$$\delta\psi = i\sqrt{2}\gamma^\mu\bar{\xi}\partial_\mu\phi$$

(B) $S = \int d^3x \left[\left|\partial_\mu\phi\right|^2 + i\,\bar{\psi}\gamma^\mu\partial_\mu\psi + g(\phi\psi\sigma^y\psi + H.c.) + u|\phi|^4\right]$

- Supersymmetry requires: $u = g^2$, (Wess-Zunimo model)

- Gauge boson ⇔ Gaugino

- Pure gauge theory: $S = \int d^3x \left[-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda\right]$

$F_{\mu\nu}$ = gauge boson, φ = real scalar

Gaugino: λ = two-dimensional spinor

Why is SUSY interesting?

- For particle physics, SUSY is useful to solve many fundamental and mysterious issues, e.g.

  - solving the hierarchy problem

    small mass of Higgs bosons

  - grand unification

  - candidate for dark matters

    lightest supersymmetric particles

- But, evidence of SUSY in particle physics is lacking!

  - Recent LHC experiment seems negative in finding SUSY

- For condensed matter physics, SUSY puts more constraints and exact results can be obtained even in strongly interacting fixed point!

Emergent SUSY in Condensed Matter Systems

- Can SUSY emerge in condensed matter systems in low energies and long distances even though microscopic models are not supersymmetric?

  - Many known examples of emergent spacetime SUSY in 1+1D systems, e.g. the tricritical points in the 1+1D diluted Ising model

    Friedan, Z. Qiu, and Shenker, PRL 1984
    Foda, Nucl. Phys. B 1998
    Fendley, Schoutens, and de Boer, PRL 2003
    Bauer, Huijse, Berg, Troyer, & Schoutens, PRB 2013
    Huijse, Bauer, and Berg, PRL 2015

  - Boson-fermion mixtures in 2D

    S.-S. Lee, PRB 2007
    Y. Yu & K. Yang, PRL 2010

  - Surface of topological insulators/superconductors

    T. Grover & A. Vishwanath, 1206.1332
    P. Ponte & S.-S. Lee, New J. Phys. 2014
    T. Grover, D. N. Sheng, & A. Vishwanath, Science 2014
    Z.-X. Li, A. Vaezi, C. B. Mendl, & H. Yao, Science Advances 2018

Emergent SUSY in Condensed Matter Systems

Question#1: Can SUSY gauge theory, such as supersymmetric QED, emerge in condensed matter systems?

- Yes, $SQED_3$ emerges at a tricritical point on the surface of topological insulators.

Question#2: Can SUSY emerge in bulk of three dimensional lattice systems with only fermionic constituents?

- Yes, SUSY emerges at PDW transitions in 3D Weyl semimetals.

Emergent SUSY on the Surface of TI

- Dirac fermion on the surface of a TI

$$H = \sum_p \psi_p^\dagger \left(\sum_{j=1}^{2} \gamma^j p_j\right)\psi_p$$

  - γ^j = 2×2 Pauli matrices
  - ψ = two-component spinor
  - protected by time reversal symmetry and U(1) symmetry
- We need a complex boson to be a superpartner of Dirac fermion:
  - superconducting transition on the surface of topological insulators: U(1) order parameter

$$\phi \sim \langle \psi \sigma^y \psi \rangle$$

T. Grover & A. Vishwanath, 1206.1332
P. Ponte & S.-S. Lee, New J. Phys. 2014

  - effective field theory at fixed point: Wess-Zunimo model

$$S = \int d^3x \left[\left|\partial_\mu \phi\right|^2 + i\,\bar{\psi}\gamma^\mu \partial_\mu \psi + g(\phi\psi\sigma^y\psi + H.c.) + g^2|\phi|^4 \right]$$

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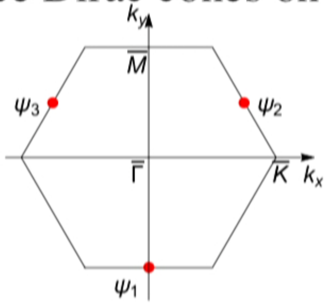
Can SUSY emerge on TI surface with 3 Dirac cones?

- Odd number of Dirac cones on the surface of a TI

  - three Dirac cones on 111 surface of SmB_6



F. Lu, J. Zhao, H. Weng, Z. Fang, & Xi Dai, PRL 2013
M. Ye, J. W. Allen, & K. Sun, 1307.7191

- Three gapless Dirac fermions ⇒ three complex bosons at criticality

  - Symmetry: $U(1)\times P\times C_3$

  - We need to break all symmetries: three complex bosons as order parameter (ϕ_1, ϕ_2, ϕ_3)

$$(\phi_1, \phi_2, \phi_3)$$

  - pair-density-wave state!

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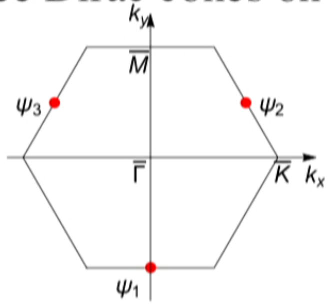

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$$\phi_1 + \phi_2 + \phi_3 \qquad\qquad (\phi_2 - \phi_3, 2\phi_1 - \phi_2 - \phi_3)$$

- pair-density-wave state!

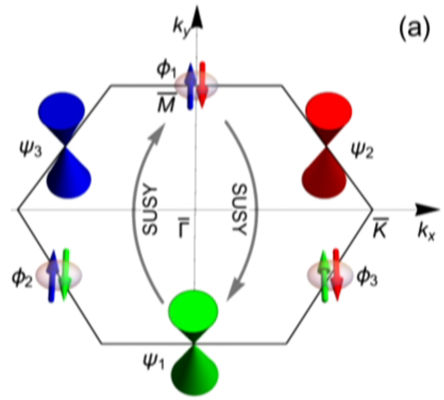
Pair-Density-Wave States

- Superconductivity with spin-singlet pairing

$$\Delta(\vec{r},\vec{r}') = \langle \psi_{\vec{r}} \sigma^y \psi_{\vec{r}'} \rangle = f(\vec{r}-\vec{r}')\Delta(\vec{R})$$

- Finite momentum pairing: $\Delta(\vec{R}) \neq \text{const}$
  - PDW: $\Delta(\vec{R}) = \Delta_{\vec{Q}}\, e^{i\,\vec{Q}\cdot\vec{R}} + \Delta_{-\vec{Q}}\, e^{-i\,\vec{Q}\cdot\vec{R}}$
  - breaking both U(1) and translational symmetries
- Paring states from three Dirac cones:
  - intravalley: $\phi \sim \sum_i \langle \psi_{K_i} \sigma^y \psi_{K_i} \rangle$
  - intervalley: $\phi_3^p \sim \langle \psi_1 \sigma^y \psi_2 \rangle, \ldots$
- Mode counting is good in PDW:
  - there Dirac fermions: ψ_1, ψ_2, ψ_3
  - three complex bosons:

$$\phi_1 \sim \langle \psi_2 \sigma^y \psi_3 \rangle, \phi_2 \sim \langle \psi_1 \sigma^y \psi_3 \rangle, \phi_3 \sim \langle \psi_1 \sigma^y \psi_2 \rangle$$



Possible PDW phases

- The boson potential up to fourth-order:

$$V_b = r\sum_{i=1}^{3}|\phi_i|^2 + u(|\phi_1|^2|\phi_2|^2 + c.p.) + u'(\phi_1^{*2}\phi_2^2 + c.p.) + u''\sum_{i=1}^{3}|\phi_i|^4$$

- not allow charge-2e Josephson coupling $\phi_1^*\phi_2$

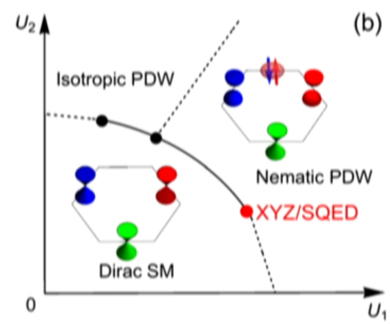
- u' is charge-4e Josephson coupling and can be absorbed into u

$$V_b = r\sum_{i=1}^{3}|\phi_i|^2 + u''\left(\sum_{i=1}^{3}|\phi_i|^2\right)^2 + (u - 2u'')(|\phi_1|^2|\phi_2|^2 + c.p.)$$

(1) $u < 2u''$, three magnitudes $|\phi_i|$ are the same to minimize potential: isotropic PDW

(2) $u > 2u''$, one direction is preferred: nematic PDW

(3) $u = 2u''$, bicritical point



- Tricritical point at Dirac SM-Nematic PDW transition

$u > 2u'', u'' = 0$, tricritical point at DSM-NPDW phase boundary

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RG analysis of the effective theory ($D = 4 - \epsilon$)

- Velocity flows: $v_x,\ \delta v = v_y - v_x,\ c_x,\ \delta c = c_y - c_x$

$$\frac{dv_x}{dl} = \frac{g^2}{(2\pi)^d}\left(\frac{16\pi^2(c_x - v_x)}{3c_x(c_x+v_x)^2} + \frac{2\pi^2(2c_x^2 - 3c_x v_x + 7v_x^2)}{3c_x v_x (c_x+v_x)^3}\delta v - \frac{8\pi^2(c_x^2 - 3c_x v_x - v_x^2)}{3c_x^2(c_x+v_x)^3}\delta c\right), \quad \text{(S14)}$$

$$\frac{dc_x}{dl} = \frac{g^2}{(2\pi)^d}\left(\frac{\pi^2(v_x^2 - c_x^2)}{c_x v_x^3} + \frac{\pi^2(2c_x^2 + v_x^2)}{2c_x v_x^4}\delta v\right), \quad \text{(S15)}$$

$$\frac{d\delta v}{dl} = \frac{g^2}{(2\pi)^d}\left(-\frac{4\pi^2(6c_x^2 + 33c_x v_x + 31v_x^2)}{15c_x v_x(c_x+v_x)^3}\delta v - \frac{16\pi^2(c_x^2 + 3c_x v_x + v_x^2)}{15c_x^2(c_x+v_x)^3}\delta c + \frac{16\pi^2}{3c_x(c_x+v_x)^3}\delta v^2 + \frac{8\pi^2(3c_x+v_x)}{3c_x^2(c_x+v_x)^3}\delta v\delta c\right), \quad \text{(S16)}$$

$$\frac{d\delta c}{dl} = \frac{g^2}{(2\pi)^d}\left(-\frac{\pi^2(c_x^2+v_x^2)}{c_x^2 v_x^3}\delta c - \frac{\pi^2}{c_x v_x^2}\delta v + \frac{\pi^2(2c_x^2 - v_x^2)}{2c_x^2 v_x^4}\delta v\delta c\right). \quad \text{(S17)}$$

  - stable fixed point $\delta v = \delta c = 0, v_x = c_x$

- Coupling constant flows

$$\frac{dg^2}{dl} = \epsilon g^2 - \frac{3\pi}{2}g^4,$$

$$\frac{du}{dl} = \epsilon u - \pi g^2 u + \pi g^4 - \frac{\pi}{2}(3u^2 + 16u'^2 + 8uu''),$$

$$\frac{du'}{dl} = \epsilon u' - \pi g^2 u' - \pi(u'^2 + 2uu' + 2u'u''),$$

$$\frac{du''}{dl} = \epsilon u'' - \pi g^2 u'' + \frac{\pi}{2}g^4 - \frac{\pi}{2}(u^2 + 4u'^2 + 10u''^2)$$

- Fixed points:

(1) stable fixed point: $(g^{*2}, u^*, u'^*, u''^*) = \left(\frac{2}{3\pi}\epsilon, 1 + \frac{\sqrt{57}}{21\pi}\epsilon, 0, ½ + \frac{\sqrt{57}}{42\pi}\epsilon\right)$

$u = 2u'', u' = 0$ $\Rightarrow$ bicritical point

(2) tricritical point: $(g^{*2}, u^*, u'^*, u''^*) = \left(\frac{2}{3\pi}\epsilon, \frac{2}{3\pi}\epsilon, 0, 0\right)$

$g^2 = u, u' = u'' = 0$ $\Rightarrow$ tricritical point

Emergent supersymmetric XYZ theory

- The "fixed point" action at tricritical point is

$$L_{XYZ} = \Sigma_{i=1}^{3}\left(i\,\psi_i^{\dagger}\gamma^{\mu}\partial_{\mu}\psi_i + \left|\partial_{\mu}\phi_i\right|^2\right) + g(\phi_1\psi_2\sigma^{y}\psi_3 + c.p.) + g^2(|\phi_1|^2|\phi_2|^2 + c.p.)$$

  - invariant under SUSY transformation

$$\delta\phi_i = \sqrt{2}\xi\psi_i$$

$$\delta\psi_1 = i\sqrt{2}\gamma^{\mu}\bar{\xi}\partial_{\mu}\phi_1 + g\sqrt{2}\xi\phi_2\phi_3, \dots$$

  ⇒ supersymmetric XYZ theory

- Why is it called SUSY XYZ theory?

  - chiral superfield

$$\Phi = \phi + \sqrt{2}\theta\psi + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\phi + \theta^2 f + \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\psi - \frac{1}{4}\theta^2\bar{\theta}^2\partial^{\mu}\partial_{\mu}\phi$$

  - The superpotential of XYZ theory is constructed by three chiral superfield X, Y, Z

$$L_{XYZ} = \int d^2\bar{\theta}d^2\theta(\bar{X}X + \bar{Y}Y + \bar{Z}Z) + g\left(\int d^2\theta XYZ + H.c.\right)$$

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SKJ, C.-H. Lin, J. Maciejko, & H. Yao, PRL 118, 166802 (2017)

Duality between XYZ theory and SQED

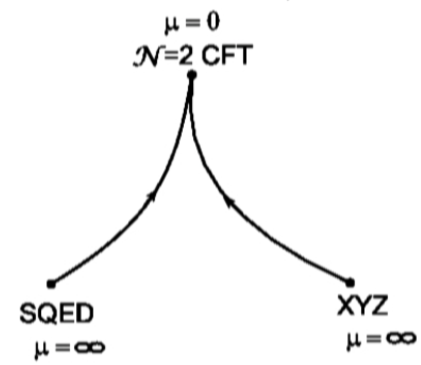
- $\mathcal{N} = 2, N_f = 1$, $SQED_3$ flows to IR fixed point with three chiral superfield $V_{\pm}$ and M

$$L_{SQED} = \int d^2\bar{\theta} d^2\theta (\bar{V}_+ V_+ + \bar{V}_- V_- + \bar{M} M) + g\left(\int d^2\theta\, M V_+ V_- + H.c.\right)$$

O. Aharony, A. Hanany, K. Intriligator, N. Seiberg, & M. J. Strassler, Nucl. Phys. B (1997).
M. J. Strassler, hep-th/0309149

- it is dual to SUSY XYZ model

$$L_{XYZ} = \int d^2\bar{\theta} d^2\theta (\bar{X}X + \bar{Y}Y + \bar{Z}Z) + g\left(\int d^2\theta XYZ + H.c.\right)$$



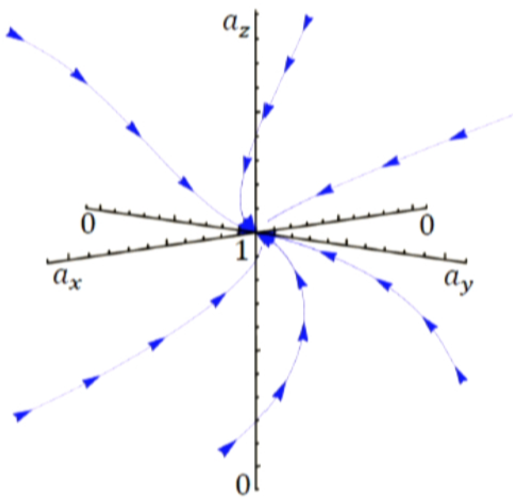
To the best of our knowledge, this is probably the first example of emergent supersymmetric gauge theory in condensed matter systems!

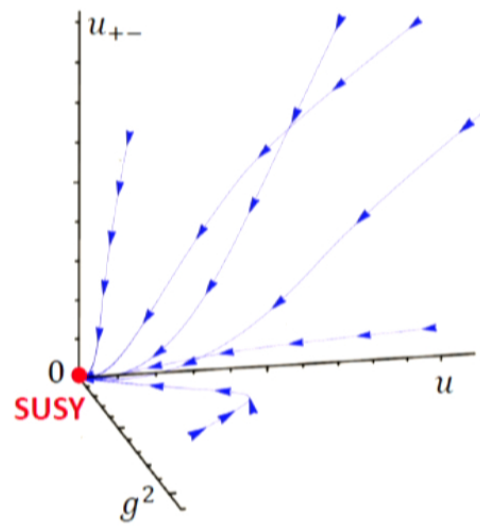
SKJ, C.-H. Lin, J. Maciejko, & H. Yao, PRL 118, 166802 (2017)

M. J. Strassler, hep-th/0309149 19

RG analysis at PDW transitions for 3+1D Weyl semimetals

- velocity flows: $a_j \equiv \frac{v_{bj}}{v_{fj}}, (j = x, y, z)$, and $w_j \equiv \frac{v_{fj}}{v_{fz}}, (j = x, y)$
  - plane of stable fixed points: $(a_j^* = 1,\ w_x^*,\ w_y^*)$
- coupling constant flows ($\epsilon = 0$) for 3+1D
  - one stable fixed points: $(g^{*2}, u^*, u_{+-}^*) = (0, 0, 0)$





SKJ, Y.-F. Jiang, & H.Yao, PRL 114, 237001 (2015)

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Possible ways to realize microscopic models

- The microscopic models may be potentially realized by ultracold atoms in an optical lattice

  - For cubic lattice models of Weyl fermions, strong attractive Hubbard interaction U may be achieved by Feshbach resonance.

- Solid state materials

  - ideal Weyl semimetal with two Weyl points
  - What is the effect of long-range Coulomb interaction?

The role of Coulomb interaction in Weyl semimetal: Marginal fermi liquid

- Thomas-Fermi screening:

$$V_{eff}(q) = \frac{4\pi e^2}{q^2 + \lambda^{-2}}$$

  - $\lambda = \left(4\pi e^2 \nu(0)\right)^{-1/2}$, $\nu(0)$ = density of state at Fermi surface

- in Weyl semimetal, the density of state

$$\nu(E) \propto E^2$$

  - at charge neutral point $\lambda \to \infty \Rightarrow$ fail to screen Coulomb interaction

- Marginal fermi liquid:

$$\alpha(\Lambda) = 4\left(\ln \frac{\Lambda_0}{\Lambda}\right)^{-1}$$

  - $\alpha = e^2/v_f$ fine structure constant

- Is supersymmetry stable against long-range Coulomb interaction?

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Effective action with Coulomb interaction

- The effective action at PDW transition point with the presence of Coulomb interaction is

$$S = S_f + S_b + S_I + S_e$$

$$S_f = \int d^4x \sum_{n=\pm}[\psi_n^\dagger \partial_\tau \psi_n + \sum_{j=1}^3 i v_f \psi_n^\dagger \gamma_n^j \partial_j \psi_n]$$

$$S_b = \int d^4x \left\{\sum_{n=\pm}\left[|\partial_\tau \phi_n|^2 + \sum_{j=1}^3 v_b^2 \left|\partial_j \phi_n\right|^2 + u|\phi_n|^4\right] + u_{+-}\, |\phi_+|^2 |\phi_-|^2\right\}$$

$$S_I = \int d^4x\, g \sum_{n=\pm}[\phi_n \psi_n \sigma^y \phi_n + H.c.]$$

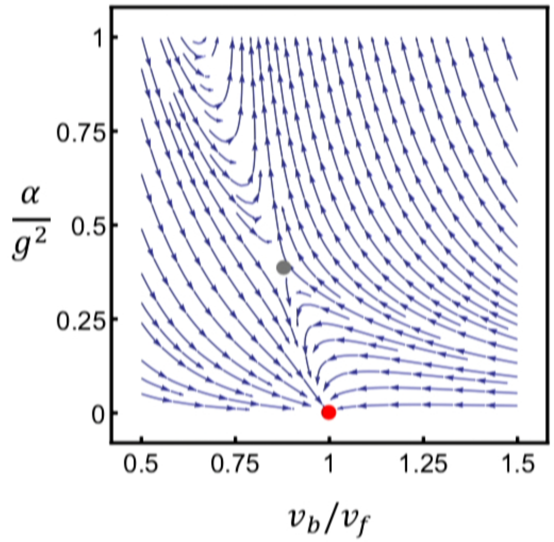
$$S_e = \int d\tau d^3x d^3y \frac{e^2}{4\pi}\frac{\rho(\tau,\vec{x})\rho(\tau,\vec{y})}{|\vec{x}-\vec{y}|}$$

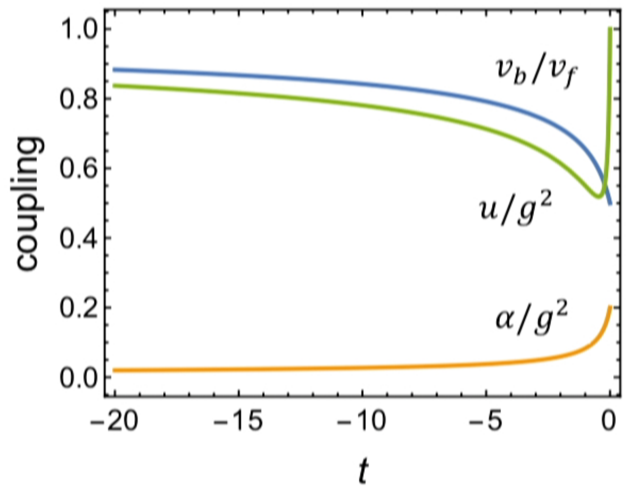
- $\rho(x) = \sum_{n=\pm} \psi_n^\dagger(x)\psi_n(x)$ is charge density
- velocity ratio v_f/v_b
- g, u, u_{+-}
- $\alpha \equiv e^2/v_f$

- RG flows of velocities and coupling constants at $D = 4 - \epsilon$

Fate of SUSY against Coulomb interaction

- SUSY is stable against small long-range Coulomb interaction:





- $\alpha/g^2 \to 0$
- $v_b/v_f \to 1$
- $u/g^2 \to 1$

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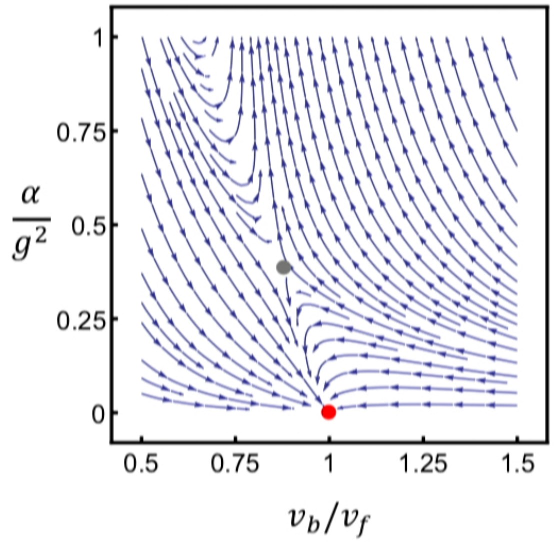

- Fixed point action

$$S_{3+1D-SUSY} = \sum_{n=\pm} \int d^4x \left(\left|\partial_\mu \phi_n\right|^2 + i\psi_n^\dagger \gamma^\mu \partial_\mu \psi_n \right)$$

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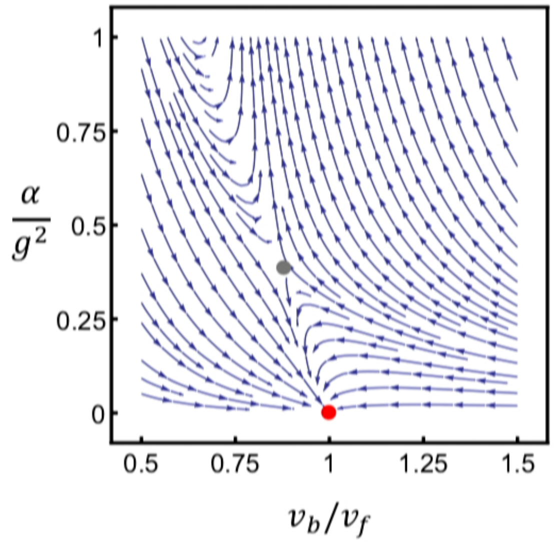

- Fixed point action

$$S_{3+1D-SUSY} = \sum_{n=\pm} \int d^4x \left(\left|\partial_\mu \phi_n\right|^2 + i\psi_n^\dagger \gamma^\mu \partial_\mu \psi_n \right)$$

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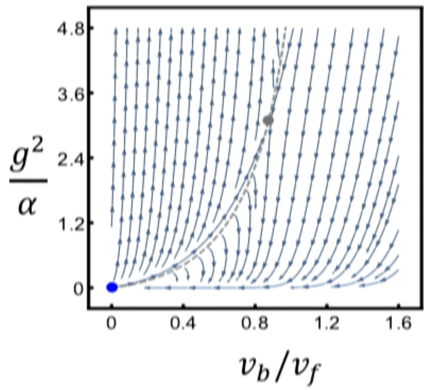
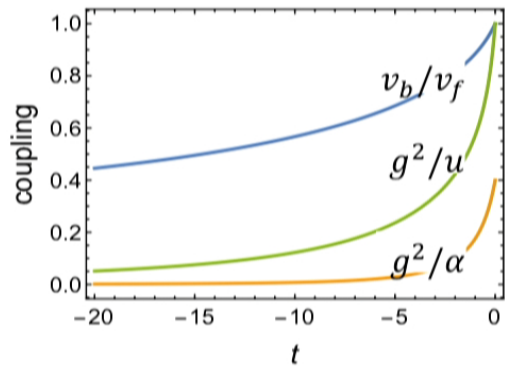
Critical MFL fixed point

- Another stable fixed point at critical surface!





- No emergent SUSY

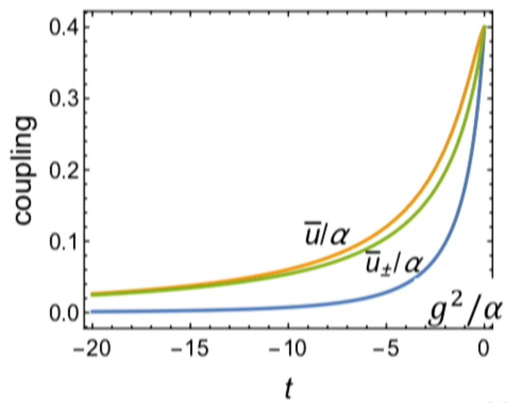
  - $v_b/v_f \to 0$

- Dominant by Coulomb interaction:

  - MFL behavior

$$\tau^{-1} = c\omega \left(\ln \frac{\Lambda_0}{\omega}\right)^{-2}$$

  - critical MFL fixed point



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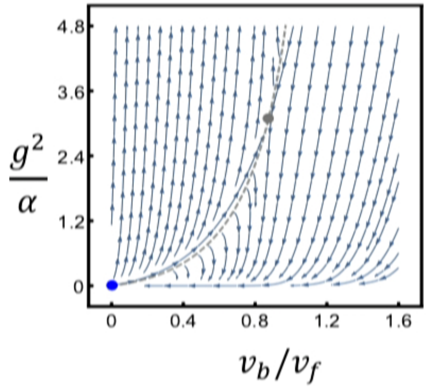
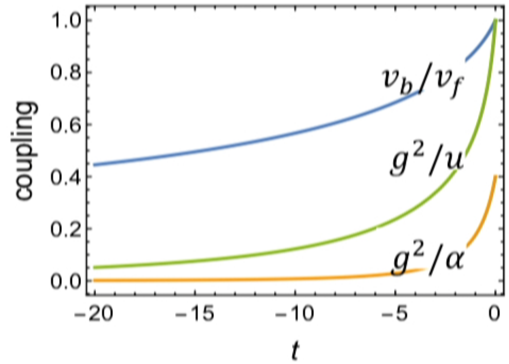
Critical MFL fixed point

- Another stable fixed point at critical surface!





- No emergent SUSY

  - $v_b/v_f \to 0$

- Dominant by Coulomb interaction:

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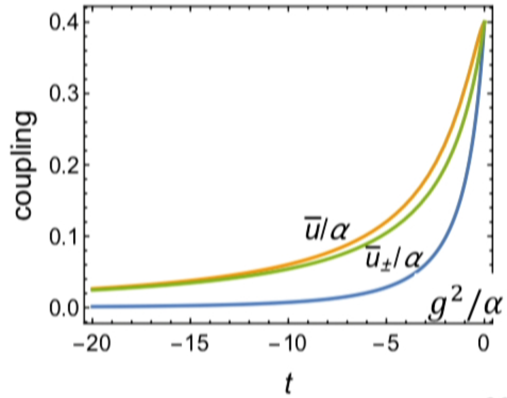
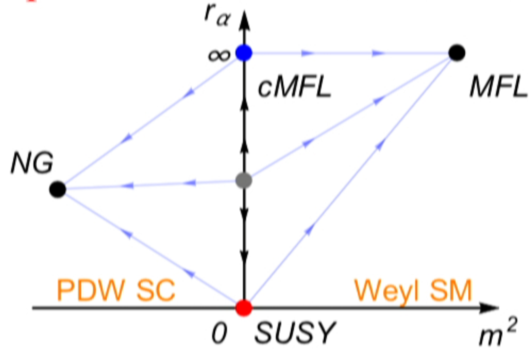


Multiuniversality

- Two stable fixed points at the critical surface!



- Fixed point action:

$$S = \int d^3x \left[|\partial_t \phi|^2 + v_b^2 |\nabla \phi|^2 + i\, \bar{\psi}\gamma^t \partial_t \psi + i\, v_f \bar{\psi}\gamma^i \partial_i \psi \right]$$

  - SUSY fixed point $v_b = v_f$
  - cMFL fixed point $v_b \ll v_f$

- Multiuniversality: a single quantum phase transition exhibits distinct critical phenomena!

Z. Bi & T. Senthil, arXiv:1808.07465

SKJ, S. Yin, & E.-G. Moon, in preparation

Conclusions

- $SQED_3$ emerges at the tricritical point of nematic PDW transition on the surface of topological insulator hosting three Dirac cones.
- Spacetime SUSY emerges at PDW transitions in 3D Weyl semimetals.
- Microscopic lattice models of Weyl semimetals featuring PDW transitions and their experimental realization/consequence are proposed.
- Interplay between PDW fluctuation and long-range Coulomb interaction leads to multi-universality.