Title: Measuring Color Memory in a Color Glass Condensate at Electron-Ion Colliders

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Abstract: <p>We review the color memory effect which is the non-abelian gauge theory analog of the gravitational memory effect, in which the passage of color radiation induces a net relative SU(3) color rotation of a pair of nearby quarks. Then, we show how the color memory effect arises in Regge limit scattering processes and propose that this effect can be measured in the Regge limit of deeply inelastic scattering at electron-ion colliders.</p>

Measuring Color Memory in a Color Glass Condensate at Electron-Ion Colliders

Monica Pate

Harvard University

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Based on:

1707.08016/hep-th with A. Raclariu and A. Strominger

1805.12224/hep-ph with A. Ball, A. Raclariu, A. Strominger and R. Venugopalan

Measuring Color Memory in a Color Glass Condensate at Electron-Ion Colliders

"Color Memory"

- Memory effects are universal feature of gauge theory in the infrared.
- Underlying reason: They are the observable consequence of symmetry.
- Focus on memory effect in classical Yang-Mills theory

"Color Glass Condensate"

- "Color" $\to$ degrees of freedom are deconfined (*i.e.* carry color)
- "Glass" $\to$ separation of timescales in the Regge limit
- "Condensate" $\to$ heavy ion wavefunction heavily populated with gluons

The color memory effect is a key process that controls observables constructed in the Color Glass Condensate.

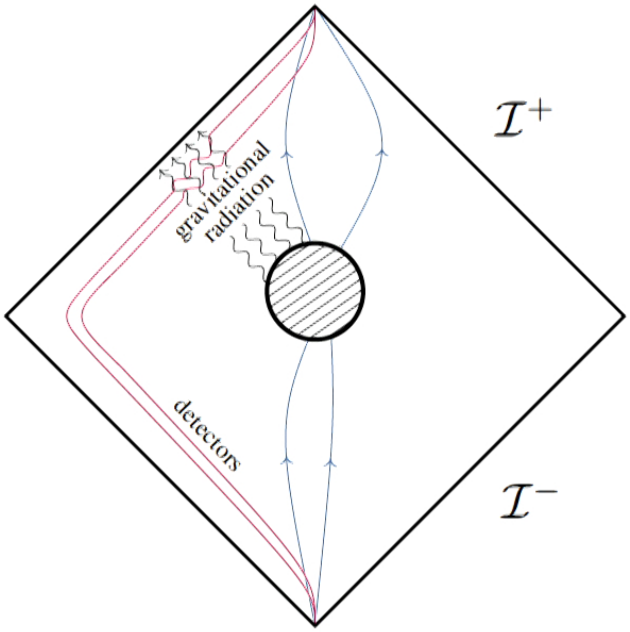
The infrared sector of gauge and gravity theories is governed by asymptotic symmetries, whose observable consequence is the memory effect.

Color memory is the net relative color rotation of a pair of nearby quarks induced by the passage of color radiation.

The classical color memory effect arises in Regge-limit scattering of heavy ions and can be measured at particle colliders.

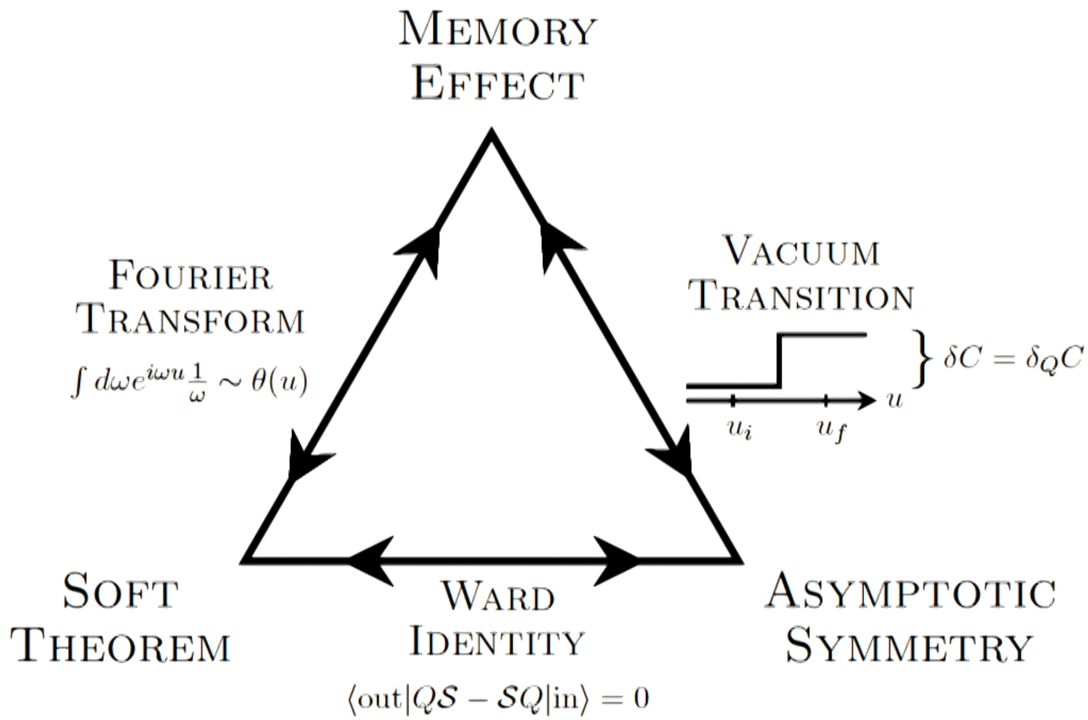
A Triad of Equivalence in Asymptotically Flat Spacetimes

- **Supertranslations**
  - Abelian subgroup of the BMS symmetry group
- **Weinberg's soft graviton theorem**
  - relates scattering amplitudes with and without a soft graviton
- **Gravitational memory effect**
  - permanent shift relative displacement of inertial observers induced by transit of gravitational radiation



[Strominger, hep-th/1312.2229
He, Lysov, Mitra, & Strominger, hep-th/1401.7026
Strominger & Zhiboedov, hep-th/1411.5745]

The Infrared Triangle



- Asymptotic symmetry $\Rightarrow$ conservation laws (in fact, an infinite number)
- Soft theorem $\Rightarrow$ constraints on soft radiation
- Memory effect $\Rightarrow$ observable consequence

The infrared sector of gauge and gravity theories is governed by asymptotic symmetries, whose observable consequence is the memory effect.

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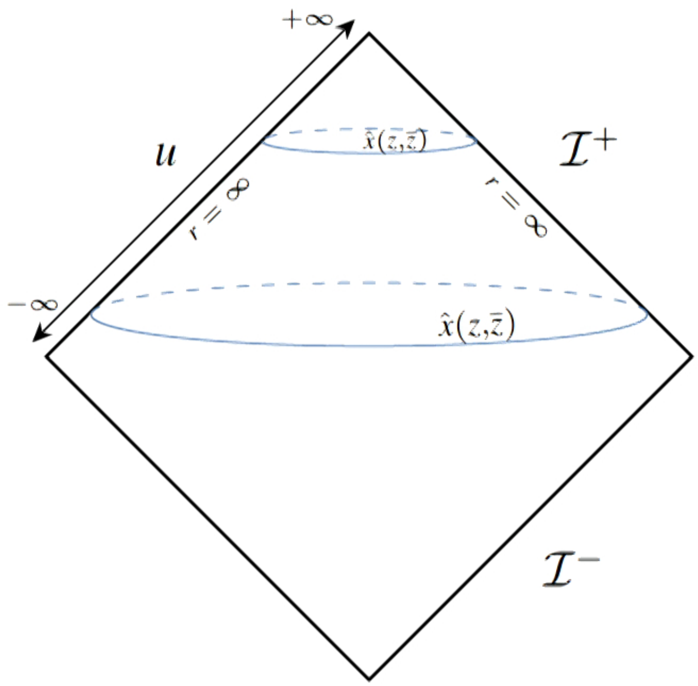
Asymptotic Symmetries of Yang-Mills theory

- Asymptotic symmetry analysis:

$$\begin{array}{c}\text{asymptotic}\\ \text{symmetry group}\end{array} = \frac{\text{allow gauge symmetries}}{\text{trivial gauge symmetries}}$$

- Asymptotic structure of Minkowski spacetime:



Coordinate system for studying $\mathcal{I}^+$:

$$u = t - r, \quad r^2 = \vec{x}^2, \quad \vec{x} = r\hat{x}(z,\bar{z})$$

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

- $\mathcal{I}^+$: $r \to \infty$ at fixed $(u,z,\bar{z})$
- u is observer time on $\mathcal{I}^+$.
- $(z,\bar{z})$ are stereographic coordinates on the *celestial sphere*.

Asymptotic Symmetries of Yang-Mills theory (continued)

- Physically motivated boundary conditions:

$$\text{Coulombic electric field} \sim \mathcal{F}_{ru}, \qquad \text{energy density} \sim \frac{\gamma^{z\bar{z}}}{r^2}\mathcal{F}_{uz}\mathcal{F}_{u\bar{z}} + \cdots$$

Finite, non-vanishing charge and energy configurations require

$$\mathcal{F}_{uz} \sim \mathcal{O}(1), \qquad \mathcal{F}_{ru}, \mathcal{F}_{rz} \sim \mathcal{O}(r^{-2})$$

- Suggests boundary conditions for gauge fields:

$$\mathcal{A}_z \sim \mathcal{O}(1), \quad \mathcal{A}_r \sim \mathcal{O}(r^{-2}), \quad \mathcal{A}_u \sim \mathcal{O}(r^{-1})$$

- Identify allowed symmetries:

$$g_{\text{bulk}}(u,r,z,\bar{z}) = g(z,\bar{z}) + \mathcal{O}(r^{-1}), \qquad \mathcal{A}_\mu \to g\mathcal{A}_\mu g^{-1} + ig\partial_\mu g^{-1}$$

- To obtain physical symmetries, must quotient by trivial symmetries.

Charges

Canonical analysis

- Physical symmetries have non-trivial action on radiative phase space.
- Introduce asymptotic expansion:

$$\mathcal{F}_{ru} = \frac{1}{r^2}F_{ru} + \mathcal{O}(r^{-3}), \qquad \mathcal{F}_{uz} = F_{uz} + \mathcal{O}(r^{-1}), \qquad \mathcal{A}_z = A_z + \mathcal{O}(r^{-1})$$

- Identify variables on radiative phase space at $\mathcal{I}^+$:

$$\Gamma^+ \supset \{A_z \ \ (\text{including zero mode})\}$$

- Construct charges that generate the symmetry

$$Q_\varepsilon^+ = \frac{1}{g_{\mathrm{YM}}^2}\int_{\mathcal{I}_-^+} d^2z\gamma_{z\bar{z}}\ \mathrm{Tr}[\varepsilon F_{ru}]$$

where $g(z,\bar{z}) = 1 + i\varepsilon(z,\bar{z}) + \mathcal{O}(\varepsilon^2)$.

- Quotient by trivial transformations $\Rightarrow$ identifies symmetry transformations $g_{\mathrm{bulk}}(u,r,z,\bar{z})$ that differ at subleading order in $1/r$.

Radiative Vacua

- Under the symmetry

$$\delta_\varepsilon A_z = \partial_z \varepsilon - i[A_z, \varepsilon]$$

- Symmetry action on classical vacuum gauge configurations (pure gauge):

$$A_z = iU\partial_z U^{-1} \qquad \rightarrow \qquad \delta_\varepsilon A_z = U\partial_z\left(U^{-1}\varepsilon U\right)U^{-1}$$

- Charges act non-trivially on vacuum configurations
  $\iff$ Vacuum is charged under the symmetry
  $\iff$ Symmetry is spontaneously broken

- *Space of inequivalent vacua is the space of flat connections on S^2.*

  Claim:

  - Color flux through $\mathcal{I}^+$ induces vacuum transitions.
  - Vacuum transitions produce an observable shift in the relative colors of a pair "test quarks" located at different points on the celestial sphere.
    $\Rightarrow$ *Color memory effect*

Conservation Laws

- Analogous construction of charges on $\mathcal{I}^-$

$$Q_\varepsilon^- = \frac{1}{g_{\rm YM}^2}\int_{\mathcal{I}_+^-} d^2z\gamma_{z\bar{z}}\,\mathrm{Tr}[\varepsilon F_{rv}]$$

- Matching condition implies conservation law

$$Q_\varepsilon^+ = Q_\varepsilon^- .$$

(True for arbitrary $\varepsilon \Rightarrow$ local conservation law on celestial sphere)

- Recall Gauss constraint

$$\partial_u F_{ru} + \gamma^{z\bar{z}}(\partial_z F_{u\bar{z}} + \partial_{\bar{z}} F_{uz}) = -J_u,$$
$$J_u = i\gamma^{z\bar{z}}([A_z, F_{\bar{z}u}] + [A_{\bar{z}}, F_{zu}]) + g_{\rm YM}^2 \lim_{r\to\infty}\left[r^2 j_u^M\right].$$

- Obtain expression for conservation in terms of fluxes through $\mathcal{I}$

$$Q_\varepsilon^+ = \underbrace{\frac{1}{g_{\rm YM}^2}\int_{\mathcal{I}^+} dud^2z\,\mathrm{Tr}[\varepsilon(\partial_z\partial_u A_{\bar{z}} + \partial_{\bar{z}}\partial_u A_z)]}_{\text{encodes vacuum transition}} + \underbrace{\frac{1}{g_{\rm YM}^2}\int_{\mathcal{I}^+} dud^2z\gamma_{z\bar{z}}\,\mathrm{Tr}[\varepsilon J_u]}_{\text{color flux}}$$

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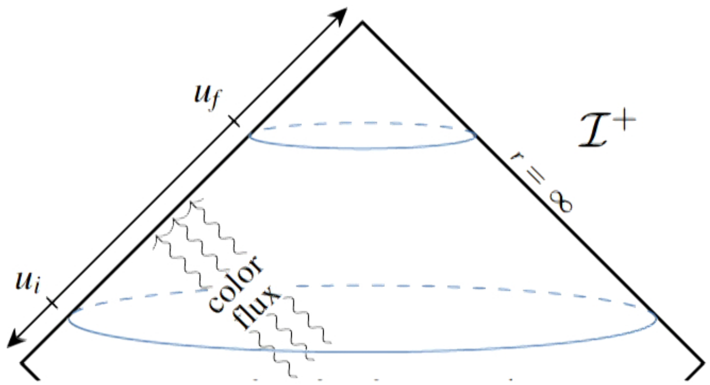
The Color Memory Effect

- Scenario with vacuum transition
  - Radiative vacuum before u_i and after u_f

$$F_{uz} = F_{z\bar{z}} = 0.$$
$$\Rightarrow A_z = iU\partial_z U^{-1}$$

  - Color flux at intermediate times



- Work in temporal gauge

$$A_u = 0.$$

- Rearrange conservation law

$$(D^z \Delta A_z + D^{\bar{z}} \Delta A_{\bar{z}}) = \Delta F_{ru} - \int_{u_i}^{u_f} du\, J_u.$$

- Find vacuum transition as a function of color flux. ("memory")
- **Claim:** Vacuum transition induces permanent rotation in relative colors of 'test' quarks.

The Color Memory Effect (continued)

Vacuum transition induces permanent relative color rotation of quark pair.

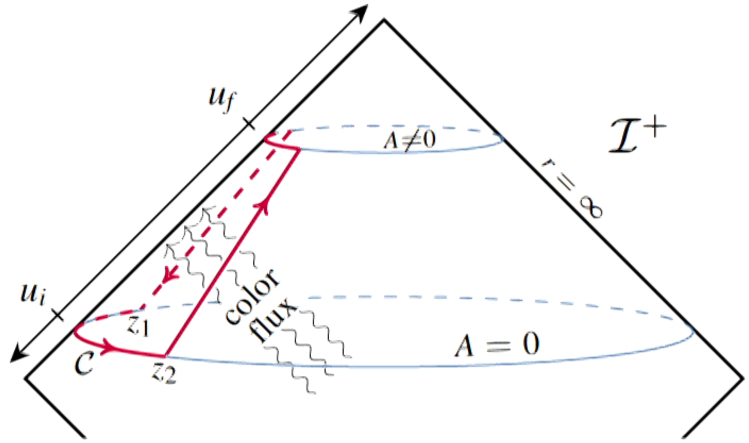
- Begin in color singlet at u_i (*i.e.* colors align upon parallel transport to common point on S^2).
- Fix quarks at distinct angles on S^2, allow to evolve from u_i to u_f,
- At u_f parallel transport $2 \to 1$ to compare colors .
- Quark color evolves according to

$$u^\mu(\partial_\mu q - iA_\mu q) = 0.$$

- If $A_z(u_i) = 0$, then generically $A_z(u_f) = iU\partial_z U^{-1} \neq 0$
- Quarks acquire relative color rotation

$$U(z_1)U^{-1}(z_2) = \mathcal{P}\exp\left(i\int_{z_2}^{z_1} A\Big|_{u_f}\right)$$

$\Rightarrow$ measures vacuum transition



- Relative color rotation is gauge covariant & transforms by conjugation.
- Trace to obtain gauge invariant object

$$\mathcal{W}_\mathcal{C} \equiv \frac{1}{N_c}\mathrm{Tr}\,\mathcal{P}\exp\left(i\oint_\mathcal{C} A\right)$$

where $\mathcal{C}$ is closed contour on $\mathcal{I}^+$.

Kinematics in the Regge Limit

- The Regge limit is a limit of fixed momentum transfer, with center-of-mass energy taken to infinity

$$t \text{ fixed}, \quad s \to \infty.$$

- In deeply inelastic scattering, we find

$$x_{\mathrm{Bj}} \equiv -\frac{q^2}{2P \cdot q} \sim \frac{t}{s} \to 0$$

where q is momentum of exchanged photon and P is the hadron momentum.

- Since x_{Bj} is fixed by kinematics to be the longitudinal momentum fraction carried by the struck parton,

$$p_{parton} = x_{\mathrm{Bj}} \, P_{hadron},$$

the Regge limit probes partons (gluons) carrying a small fraction x of the hadron momentum.

Kinematics in the Regge Limit (continued)

- Next from longitudinal spread
$$\Delta x^- \sim \frac{1}{k^+} = \frac{1}{xP^+}$$
find large-x d.o.f. are highly localized in x^-.
- For purposes of small-x dynamics, fixing $A_+ = 0$ gauge, we can approximate large-x d.o.f. by a color shockwave traveling in the x^+ direction
$$g^2 J_M^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{x}).$$
  - Resembles localized color flux through $\mathcal{I}^+$ that induces vacuum transition.
- Taking static configurations, $A_- = 0$ & no long. mag. fields ($F_{ij} = 0$), integral of only non-trivial component of the YM equations becomes
$$-\partial_i \Delta A_i = \int_{x_i^-}^{x_f^-} dx^- J_-, \qquad J_- = -\delta(x^-)\rho(\vec{x}) - i[A_i, \partial_- A_i].$$
- Resembles vacuum transition memory formula, but to make precise, must place at $\mathcal{I}^+$.

Color Memory in the Regge Limit

First must identify analogue of IMF for $\mathcal{I}^+$ observer

- LC coordinates are nice coordinates for the IMF because they transform simply under boosts in x^3 direction

$$(x^+, x^-) \to (\lambda x^+, \lambda^{-1} x^-).$$

- The IMF is reached by taking $\lambda \to \infty$. In LC coordinates, can readily obtain IMF configurations from configurations in other inertial frames.
- *Example*: Current sourced by particle of constant 4-velocity U^μ

$$j^\mu(x) = Q \frac{U^\mu}{U^+} \delta(x^- - U^- \tau) \delta^2(\vec{x} - \vec{U}\tau)|_{\tau = x^+/U^+}$$

Under boost

$$j^\mu(x) \to j'^\mu(x) = j^\mu(x)|_{U=U'}, \qquad (U'^+, U'^-, \vec{U}') = (\lambda U^+, \lambda^{-1} U^-, \vec{U}).$$

In limit $\lambda \to \infty$, only j^+ remains and is proportional to $\delta(x^-)$.

The Regge Limit and Null Infinity

- $(u,r,z,\bar{z})$ do not transform nicely (*i.e.* by scaling) under boosts in x^3 direction.
- *Trick:* obtain nice coordinates by singular coordinate transformation

$$(r,u,z,\bar{z}) \to (\lambda r, \lambda^{-1}u, \lambda^{-1}z, \lambda^{-1}\bar{z}), \qquad \lambda \to \infty.$$

- In new coordinates, further rescalings are boosts in x^3.
- S^2 is flattened to transverse plane.
- Related to LC coordinates by

$$x^+ = \sqrt{2}r, \qquad x^- = \frac{1}{\sqrt{2}}(u + rz\bar{z}), \qquad x^1 + ix^2 = 2rz.$$

- Can identify $r \to \infty$ limit with $x^+ \to \infty$ limit!

Color Memory in the Regge Limit

- All that remains is to take $x^+ \to \infty$ limit of

$$-\partial_i \Delta A_i = \int_{x_i^-}^{x_f^-} dx^- J_-, \qquad J_- = -\delta(x^-)\rho(\vec{x}) - i[A_i, \partial_- A_i].$$

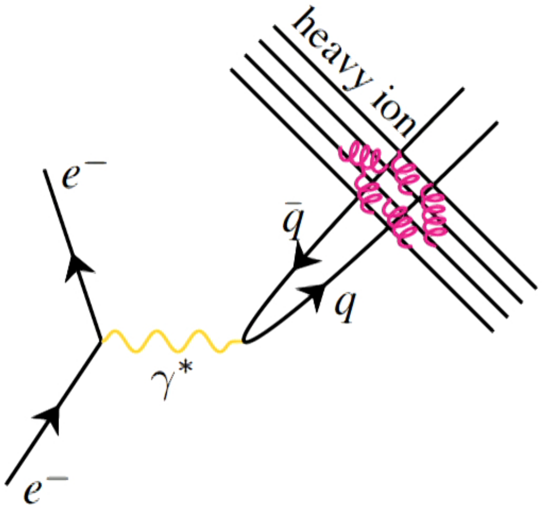
- However, vacuum transition formula was x^+-independent
  $\Rightarrow$ can place at $\mathcal{I}^+$ for free!

Small-x gluon field configurations are the vacuum-to-vacuum field configurations governed by the conservation law for large gauge symmetry.

- Can we measure these?
- In other words, what are the analogues of the "test" quarks?

Color Memory in Collider Observables

- *Goal:* seek observables sensitive to vacuum-to-vacuum configurations.
- Consider electron-ion deep-inelastic scattering.



- In Regge limit, electron emits virtual photon which splits into singlet quark-antiquark pair.
- In eikonal approximation, shockwave induces color rotation on each quark.
- Forward scattering amplitude:

$$\mathcal{S}(\vec{x}_1, \vec{x}_2) = \frac{1}{N_c} \mathrm{Tr}\left[U(\vec{x}_1) U^\dagger(\vec{x}_2)\right] = \mathcal{W}_C.$$

Identify amplitude with quark dipole color rotation!

Color Memory in Collider Observables (continued)

- To obtain observable, must average over color sources (CGC).
  - Large random rotations average to zero, small rotations approx. identity
  - Emergent scale: size of dipole where transition occurs.
- *Dipole cross-section*: (via optical theorem)

$$\sigma_{\text{dipole}}(x,\vec{r}) = 2\int d^2\vec{b}\,[1 - \langle \text{Re}\,\mathcal{S}(\vec{x}_1,\vec{x}_2)\rangle],$$

  where $\vec{r} = \vec{x}_1 - \vec{x}_2$ and $\vec{b} = (\vec{x}_1 + \vec{x}_2)/2$.
- *Inclusive DIS virtual photon-heavy ion cross-section*:

$$\sigma_{\gamma^* \text{ion}}(x,Q^2) = \int_0^1 dz \int d^2\vec{r}\,|\Psi(z,\vec{r},Q^2)|^2_{\gamma^*\to q\bar{q}}\sigma_{\text{dipole}}(x,\vec{r}),$$

  where $|\Psi(z,\vec{r},Q^2)|^2_{\gamma^*\to q\bar{q}}$ is probability for $\gamma^* \to q\bar{q}$ with dipole of size $\vec{r}$ and quark carrying momentum fraction z.

Conclusions and Outlook

Summary

- The infrared sector of gauge and gravity theories is governed by asymptotic symmetries, whose observable consequence is the memory effect.
- Color memory is the net relative color rotation of a pair of nearby quarks induced by the passage of color radiation.
- The classical color memory effect arises in Regge-limit scattering of heavy ions and can be measured at particle colliders.

Future Directions

- Interplay of constraints on QCD scattering amplitudes from asymptotic symmetries arising in the infrared and Regge limits.
- General implications of asymptotic symmetries for Regge-limit physics.