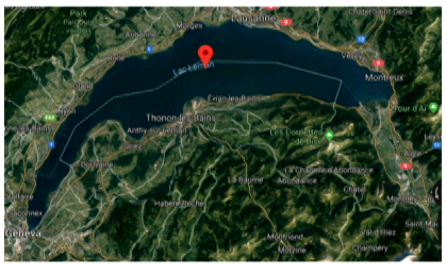
Title: The Pion Lake and the Peninsula

Date: Nov 01, 2018 02:30 PM

URL: http://pirsa.org/18110060

Abstract: <p>In this talk I will consider the S-matrix bootstrap of four dimensional scattering amplitudes with O(3) symmetry and no bound states and apply the formalism to pion physics. I will discuss the remarkable location where QCD seems to lie in the multi-dimensional space of zeros, scattering length and resonance mass values, based on various experimental and theoretical expectations.</p>

Bootstrapping QCD:
the Lake, the Peninsula and the Kink



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1st November, 2018

arXiv: 1810.12849

with J. Penedones and P. Vieira

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Plan of the talk

Q: Do Pion Scattering amplitudes take a special place in the space of consistent S-matrices?

Introduction

Pions
Setup and Conformal mapping
Experimental data/Resonances
Chiral effective theory

QCD Geography

What next?

Pions (anti-historical picture)

UV

$SU(3)$ **Gauge Theory**

$SU(2)_L \times SU(2)_R$ **Fermionic matter**

$\downarrow$

Spontaneous symmetry breaking

$$\langle \bar{\psi}\psi \rangle \neq 0 \qquad \text{Pions} \in \frac{SU(2)_L \times SU(2)_R}{SU(2)_V}$$

Explicit soft symmetry breaking

$m\bar{\psi}\psi$ **Ward-identities preserved**

$\downarrow$

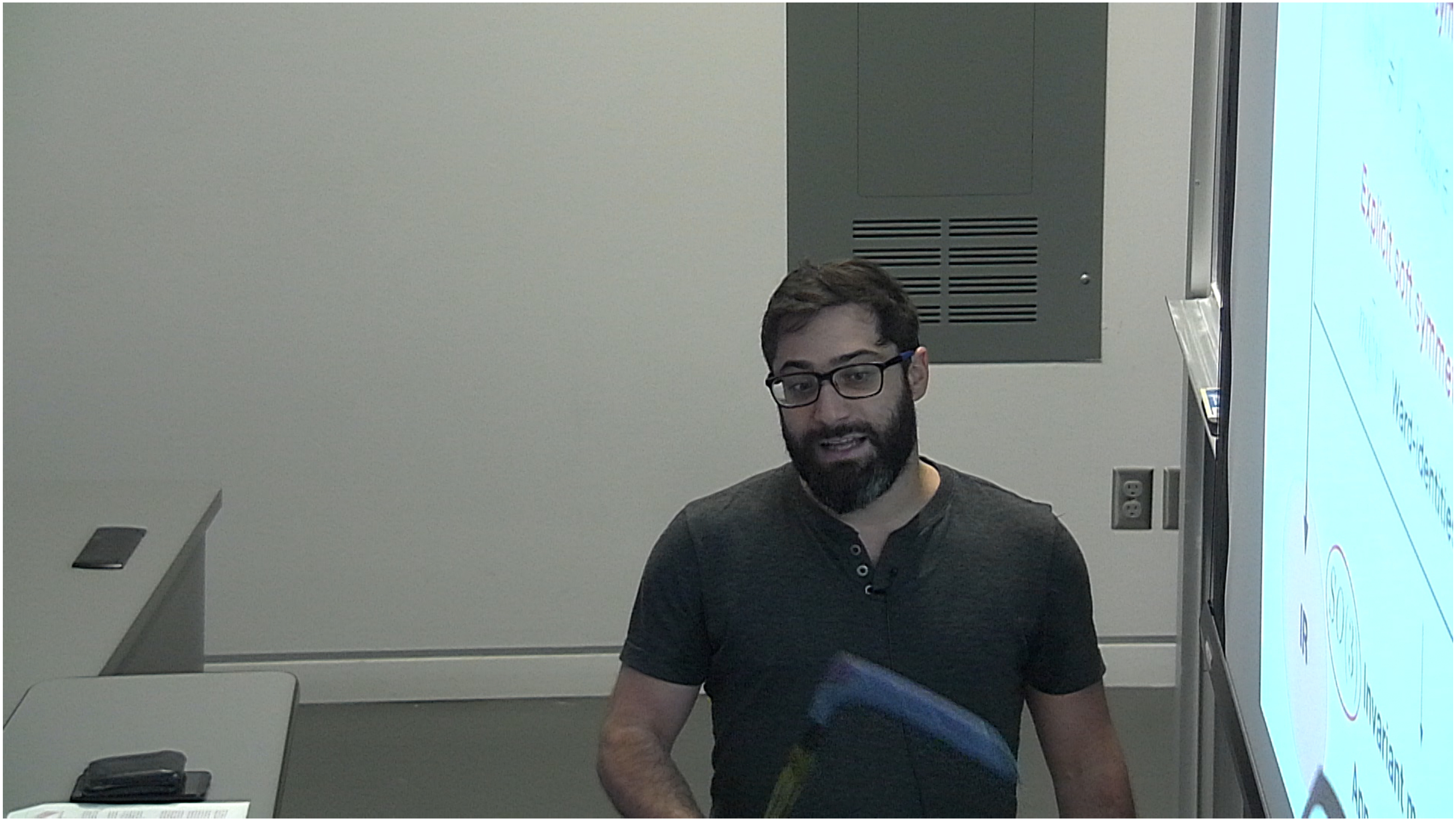
Loosely constraining

$SO(3)$ **Invariant massive scalar particles**

IR

Approximate Goldstone Bosons

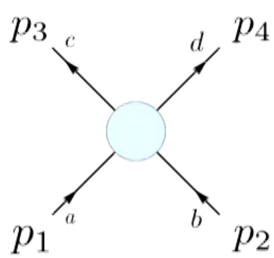
Super Powerful!!!



Bootstrap Setup: O(N) + Crossing

Basis with manifest Crossing Symmetry

$$\mathcal{T}(s,t,u)_{ab}^{cd} = A(s|t,u)\delta_{ab}\delta^{cd} + A(t|s,u)\delta_a^c\delta_b^d + A(u|s,t)\delta_a^d\delta_b^c$$

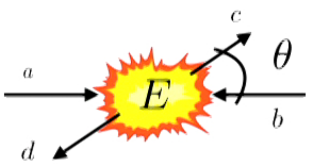


Theoretical Physicist view

With

$$A(s|t,u) = A(s|u,t)$$

$$u = 4 - s - t$$



Experimental Physicist view

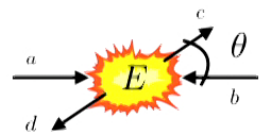
Basis with manifest Unitarity

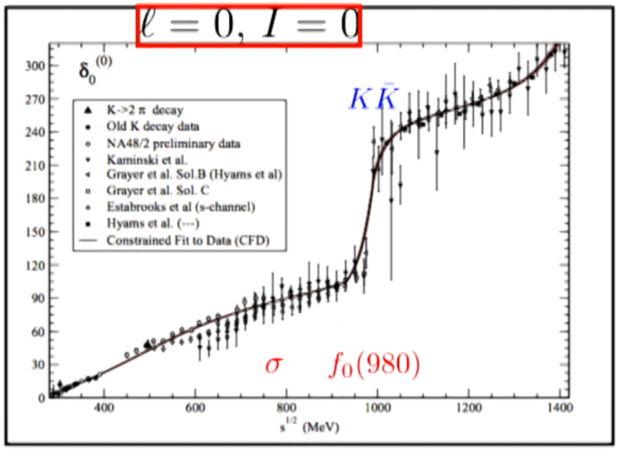
$$\mathcal{T}(s,t,u) = (NA(s|t,u) + A(t|s,u) + A(u|s,t))\mathbb{P}_0 + (A(t|s,u) + A(u|s,t))\mathbb{P}_2 + (A(t|s,u) - A(u|s,t))\mathbb{P}_1$$

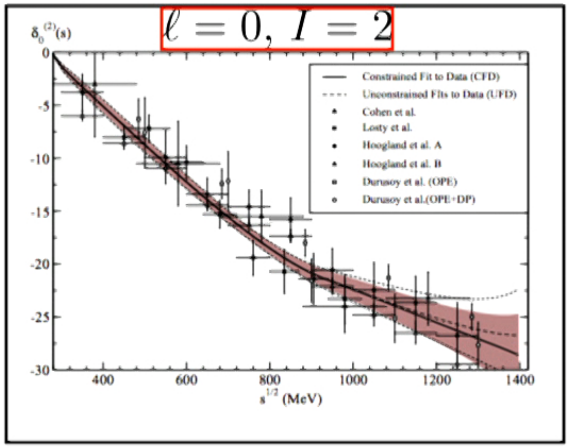
$$= \frac{16\pi i\sqrt{s}}{\sqrt{s-4}} \sum_{I=0,1,2} \mathbb{P}_I \sum_{\ell} (2\ell+1)\left(1 - S_\ell^{(I)}(s)\right) P_\ell\left(\frac{u-t}{u+t}\right)$$

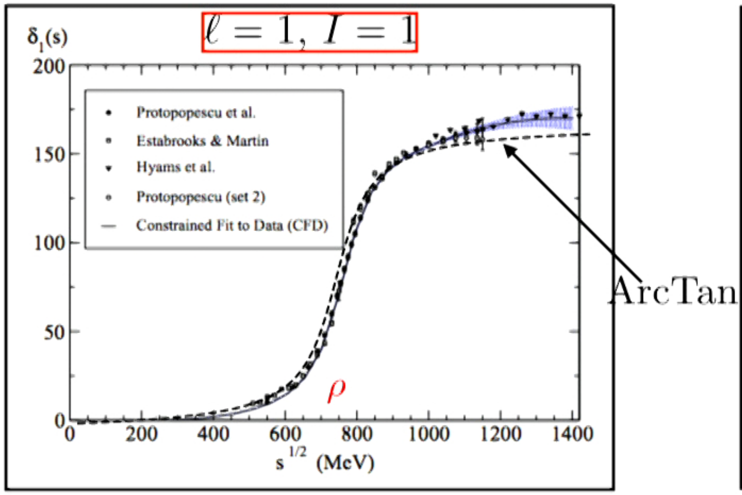
$$S_\ell^{(I)}(s) = e^{2i\delta_\ell^{(I)}(s)}$$

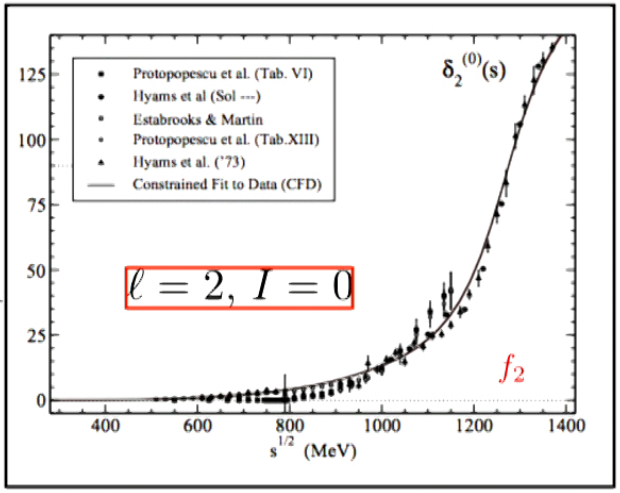
Experimental Data



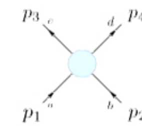








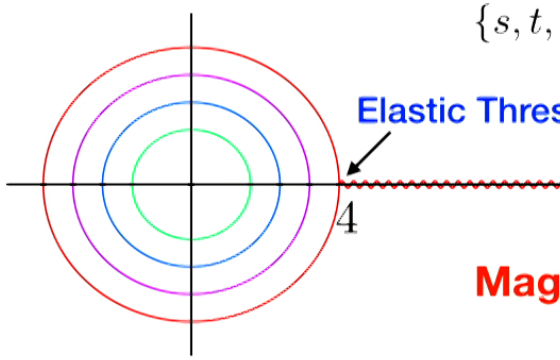
Bootstrap Setup: analytic ansatz

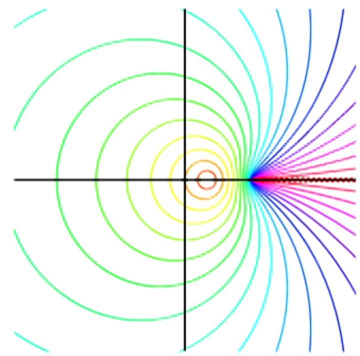


$$\mathcal{T}(s,t,u)_{ab}^{cd} = A(s|t,u)\delta_{ab}\delta^{cd} + A(t|s,u)\delta_a^c\delta_b^d + A(u|s,t)\delta_a^d\delta_b^c$$

Analytic Extension

$$\{s,t,u\} \in \mathbb{C}^3/\mathrm{cuts}$$





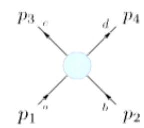
Magic variables

$$\rho(z) = \frac{\sqrt{4-z_0} - \sqrt{4-z}}{\sqrt{4-z_0} + \sqrt{4-z}}$$

Real Analytic also!

$$A(s|t,u) = \sum_{n\le m}^{\infty} a_{nm}(\rho_t^n\rho_u^m + \rho_t^m\rho_u^n) + \sum_{n,m}^{\infty} b_{nm}(\rho_t^n + \rho_u^n)\rho_s^m$$

Bootstrap Setup: the numerics



$$S_\ell^{(I)}(s) = 1 + \frac{i}{32\pi}\sqrt{\frac{s-4}{s}}\int_{-1}^{1} dx\, P_\ell(x)\mathcal{T}^{(I)}(s,x)$$

Linear in the Ansatz coefficients

Crossing and Analyticity Encoded

Cutoff in the Ansatz N_{max}

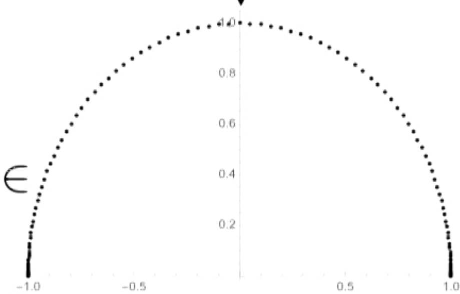
Unitarity imposed numerically

s cut === disk boundary

$$\boxed{|S_\ell^{(I)}(s)|^2 \leq 1}$$

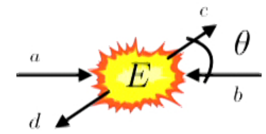
$$\ell \in \{0,\ldots,L_{max}\}$$

$$s(e^{i\phi}),\ \phi \in$$



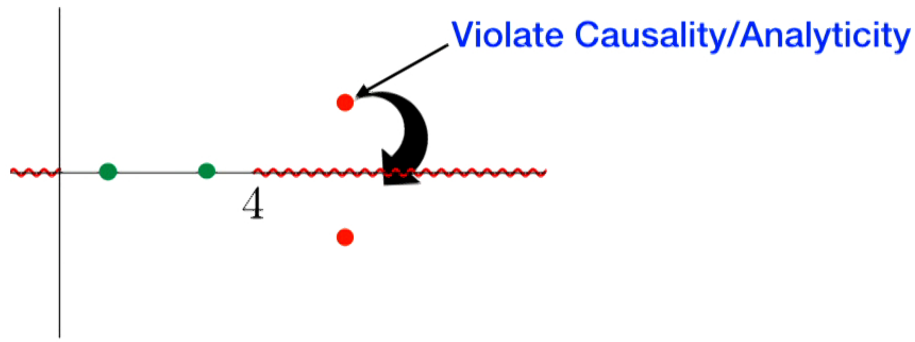
Bootstrap Problem: Max/Min a linear target given unitarity

Bootstrap Setup: Resonances



Bound States: real poles = stable particles

Resonance States: complex poles = unstable particles



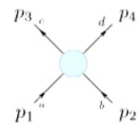
Unitarity + Real Analyticity for 2 to 2 scattering

$$S_\ell^{(I)}\,\hat{S}_\ell^{(I)} = 1$$

Zeros of the S-matrix in the Physical Sheet!

$$\boxed{S_\ell^{(I)}(m_R^2) = 0}$$

Chiral EFT



Implement the non-linearly realized chiral symmetry

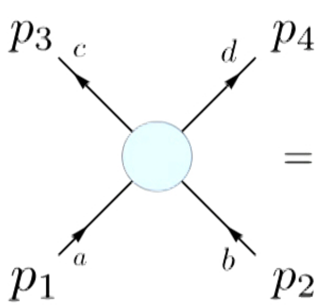
Hints from Perturbation Theory:

$$\mathcal{L}_2 = \frac{f^2}{4}\,\mathrm{tr}(\partial U^\dagger \partial U) + \frac{f^2}{4}(\chi U^\dagger + U\chi^\dagger) \qquad U = e^{i\phi\cdot t/f} \in SU(2)$$

$$\big\downarrow$$

$$\mathcal{L}_2 = \mathcal{L}_{free} + \frac{1}{24 f^2}\left(\mathrm{tr}([\phi, \partial\phi]\phi\partial\phi) + m^2\mathrm{tr}(\phi^4)\right) + \mathcal{O}(f^{-4})$$

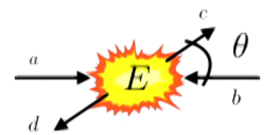
$A(s|t,u)$



$$= \frac{s-m^2}{f^2}\delta_{ab}\delta^{cd} + \frac{t-m^2}{f^2}\delta_a^c\delta_b^d + \frac{u-m^2}{f^2}\delta_b^c\delta_a^d$$
$$+\mathcal{O}(p^4, m^4, m^2p^2)$$

NL realization —> Soft theorems

Chiral Zeros and Threshold Parameters”



$$\boxed{\mathcal{S}_\ell^{(I)} = 1 + 2i\sqrt{1-\frac{4}{s}}\mathcal{T}_\ell^{(I)}}$$

Tree-level amplitude:

Chiral Zeros

$\ell = 0,\ I = 0$	$\mathcal{T}_0^{(0)} = \frac{2s - m^2}{32\pi f^2}$	$s_0 = \frac{1}{2}m^2$
$\ell = 0,\ I = 2$	$\mathcal{T}_0^{(2)} = \frac{2m^2 - s}{16\pi f^2}$	$s_2 = 2m^2$
$\ell = 1,\ I = 1$	$\mathcal{T}_1^{(1)} = \frac{s - 4m^2}{96\pi f^2}$	$s_1 = 4m^2$

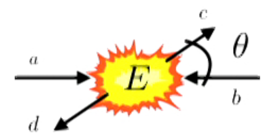
Resonances: zeros in $S_\ell^{(I)}(s_R) = 0$

Chiral: zeros in $\mathcal{T}_\ell^{(I)}(s_I) = 0 \ \leftrightarrow \ S_\ell^{(I)}(s_I) = 1$

Weak Coupling Condition

Strong deformation

Chiral Zeros and Threshold Parameters”



$$\boxed{\mathcal{S}_\ell^{(I)} = 1 + 2i\sqrt{1-\frac{4}{s}}\,\mathcal{T}_\ell^{(I)}}$$

Resonances: zeros in $S_\ell^{(I)}(s_R) = 0$

Chiral: zeros in $\mathcal{T}_\ell^{(I)}(s_I) = 0 \leftrightarrow S_\ell^{(I)}(s_I) = 1$

Expansion at threshold: s=4

$$\mathrm{Re}[\mathcal{T}_\ell^{(I)}] = k^{2\ell}(a_\ell^{(I)} + k^2 b_\ell^{(I)} + \mathcal{O}(k^4))$$

Scattering Lengths

Effective ranges (scattering slopes)

Example: QCD values

I	$\mathcal{O}(k^0)$	$\mathcal{O}(k^2)$
0	$a_0^{(0)} = 0.2196 \pm 0.0034$	$b_0^{(0)} = 0.276 \pm 0.006$
2	$a_0^{(2)} = -0.0444 \pm 0.0012$	$b_0^{(2)} = -0.0803 \pm 0.0012$
1		$a_1^{(1)} = 0.038 \pm 0.002$

Mid-term Summary

Crossing Symmetric Ansatz

$$\mathcal{T}(s,t,u)_{ab}^{cd} = A(s|t,u)\delta_{ab}\delta^{cd} + A(t|s,u)\delta_a^c\delta_b^d + A(u|s,t)\delta_a^d\delta_b^c$$

Mandelstam Analyticity + Real Analyticity

$$A(s|t,u) = \sum_{n\le m}^{\infty} a_{nm}(\rho_t^n\rho_u^m + \rho_t^m\rho_u^n) + \sum_{n,m}^{\infty} b_{nm}(\rho_t^n + \rho_u^n)\rho_s^m$$

$$\rho(z) = \frac{\sqrt{4-z_0} - \sqrt{4-z}}{\sqrt{4-z_0} + \sqrt{4-z}}$$

Check Unitarity Numerically

$$S_\ell^{(I)}(s) = 1 + \frac{i}{32\pi}\sqrt{\frac{s-4}{s}}\int_{-1}^{1} dx\, P_\ell(x)\mathcal{T}^{(I)}(s,x)$$

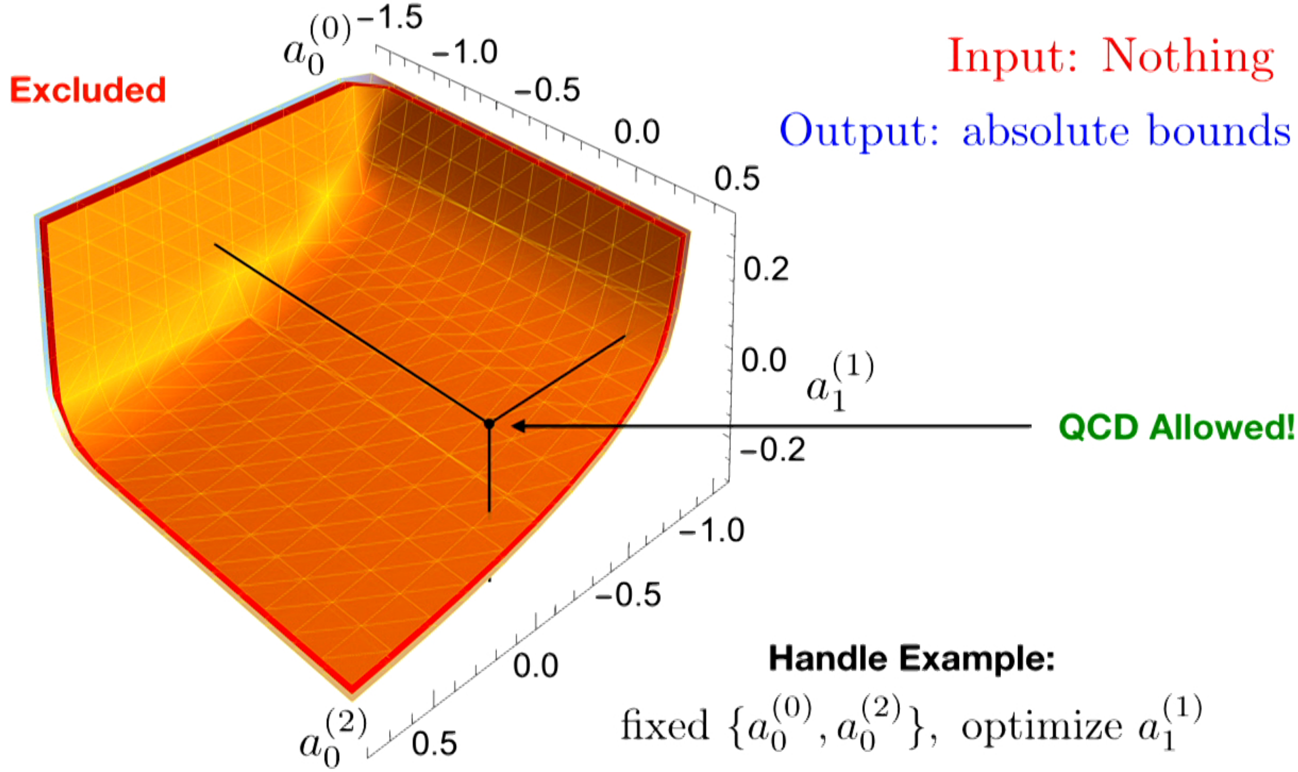
$$\boxed{|S_\ell^{(I)}(s)|^2 \le 1}$$

Parameters of the Game

$a_\ell^{(I)}$	$b_\ell^{(I)}$	$(s_0, s_2) : \mathcal{T}_0^{(I)}(s_I) = 0$	$S_\ell^{(I)}(s_R) = 0$

Step1: Absolute bounds on scattering lengths

Optimization Problem: $\min_{a,b\in\text{Ansatz}} \; a_\ell^{(I)}$



Input: Nothing

Output: absolute bounds on $a_\ell^{(I)}$

Handle Example:

fixed $\{a_0^{(0)}, a_0^{(2)}\}$, optimize $a_1^{(1)}$

QCD Geography: the Lake problem

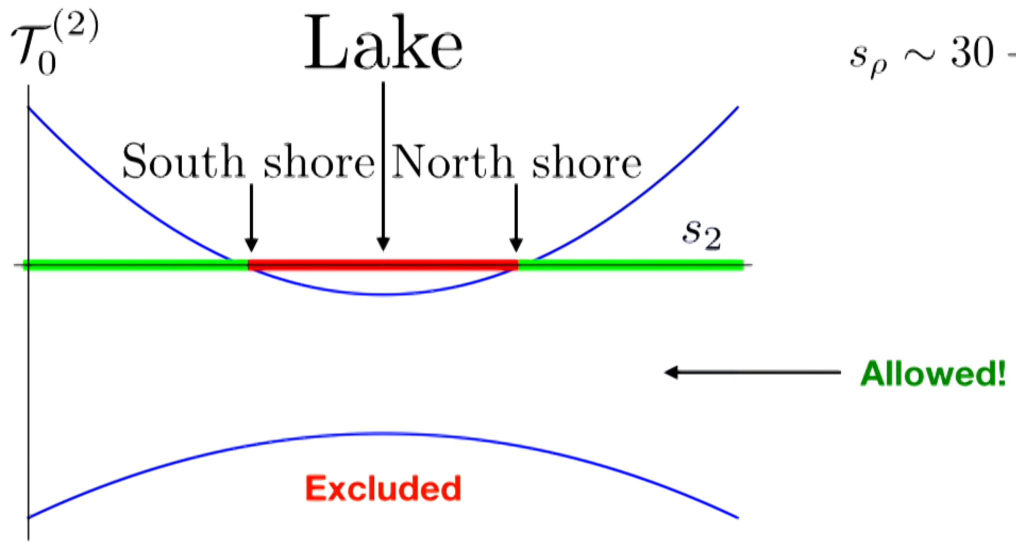
Optimization Problem: $\underset{a,b\in \text{Ansatz}}{max/min}\ \mathcal{T}_0^{(2)}(s) \quad s \in [0,4]$

Handle:
$\mathcal{T}_0^{(0)}(s_0) = 0$ fixed

$\text{Input}: \rho \text{ resonance} \leftrightarrow S_1^{(1)}(s_\rho) = 0$

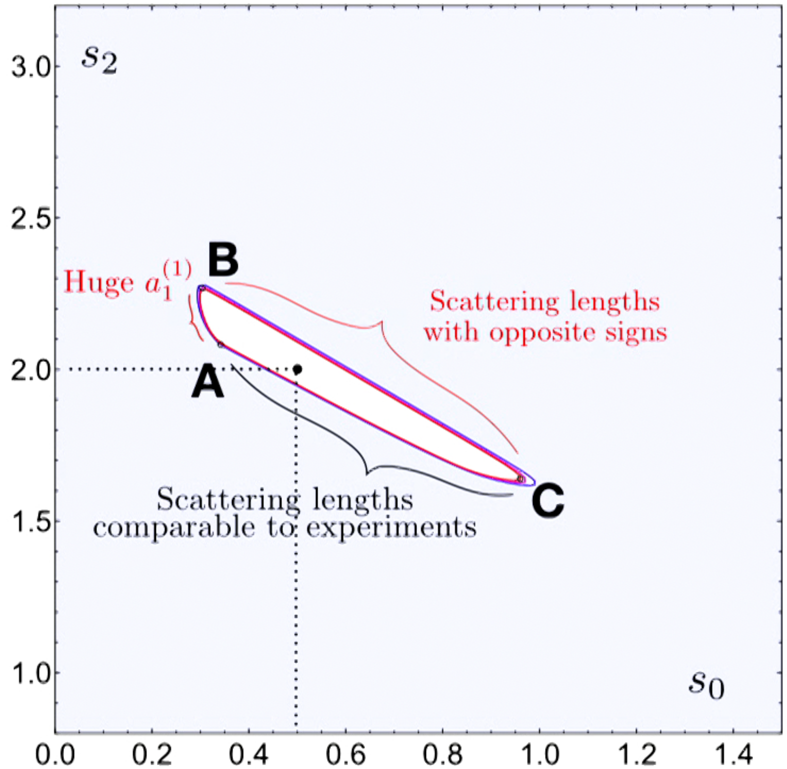
$\text{Output: exclusion region for } (s_0, s_2)$

$s_\rho \sim 30 + 6\,i$



QCD Geography: the Lake shape

Optimization Problem: $\underset{a,b\in \text{Ansatz}}{max/min}\ \mathcal{T}_0^{(2)}(s) \quad s\in[0,4]$



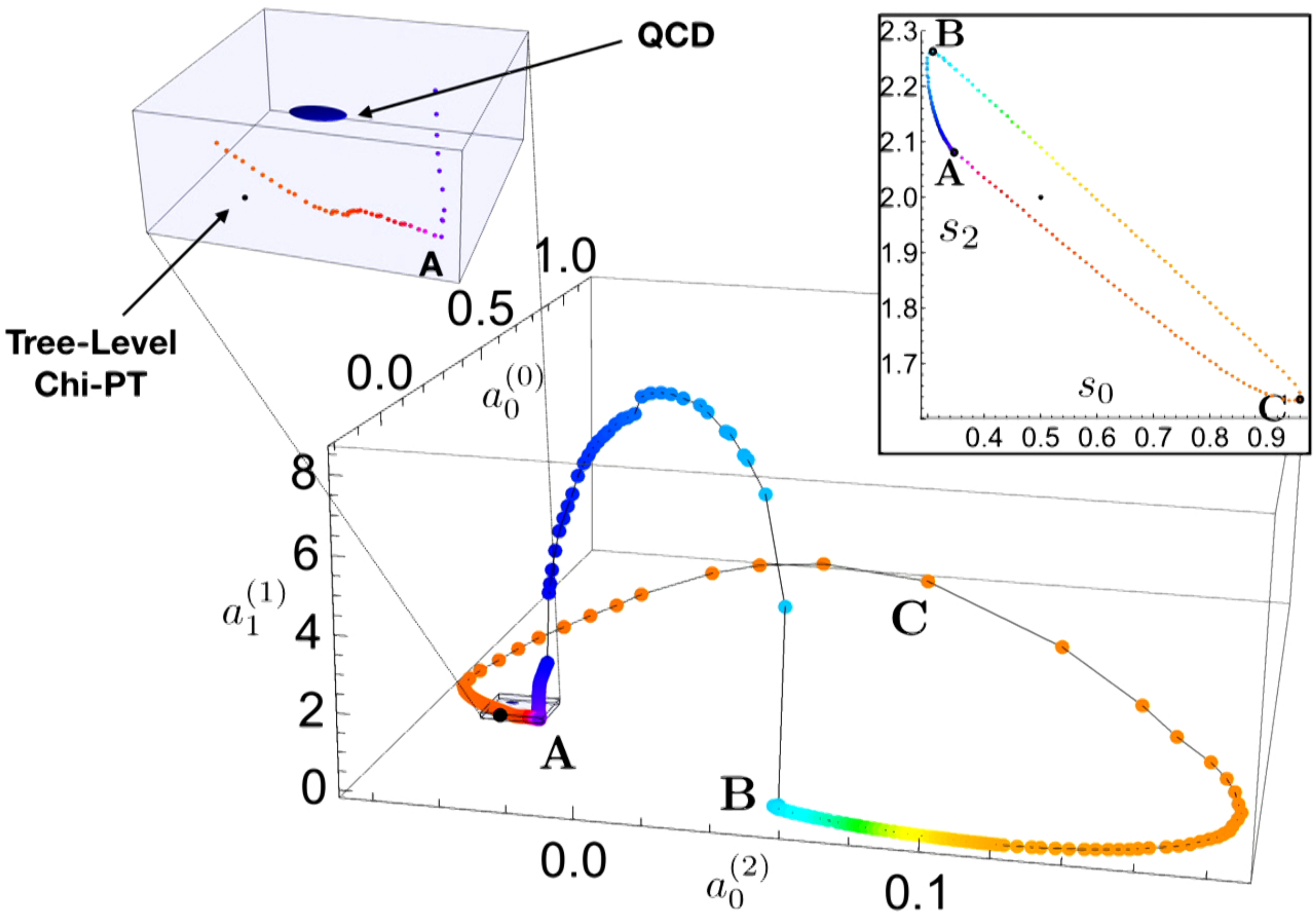
Handle:

$$\mathcal{T}_0^{(0)}(s_0) = 0 \ s_0 \in [0,4]$$

Input : ρ resonance $\leftrightarrow S_1^{(1)}(s_\rho) = 0$

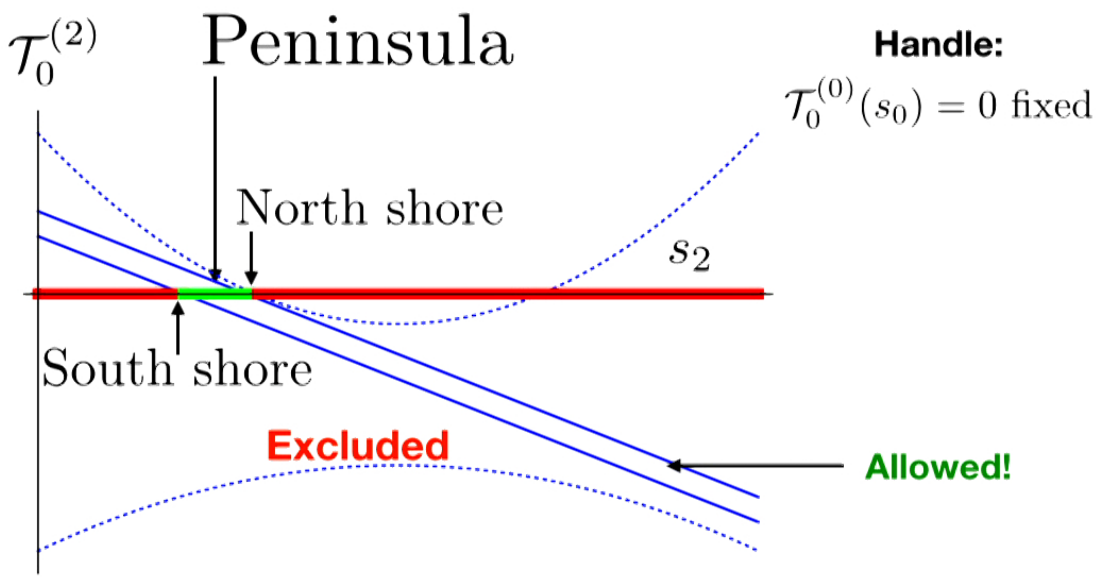
Output: exclusion region for (s_0, s_2)

QCD Geography: walking around the Lake



QCD Geography: the Peninsula problem

Optimization Problem: $\underset{a,b\in \text{Ansatz}}{max/min}\ \mathcal{T}_0^{(2)}(s) \quad s \in [0,4]$

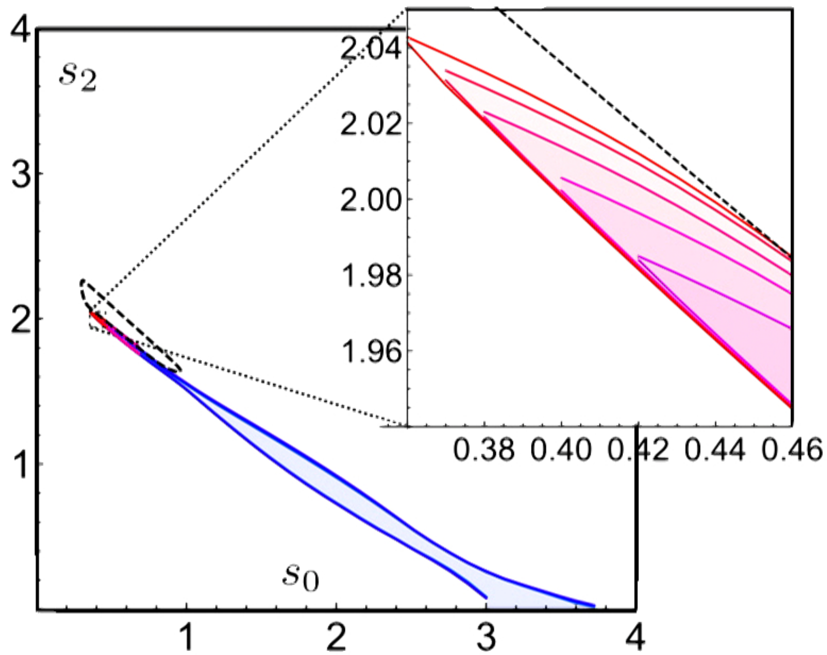


$$\text{Input}: \rho \text{ resonance} \leftrightarrow S_1^{(1)}(s_\rho) = 0$$

$$|a_\ell^{(I)} - \text{exp. value}| \leq \Delta_{\text{exp. value}}$$

QCD Geography: the Peninsula shape

Optimization Problem: $\max_{a,b\in\text{Ansatz}}/\min \mathcal{T}_0^{(2)}(s) \quad s\in[0,4]$

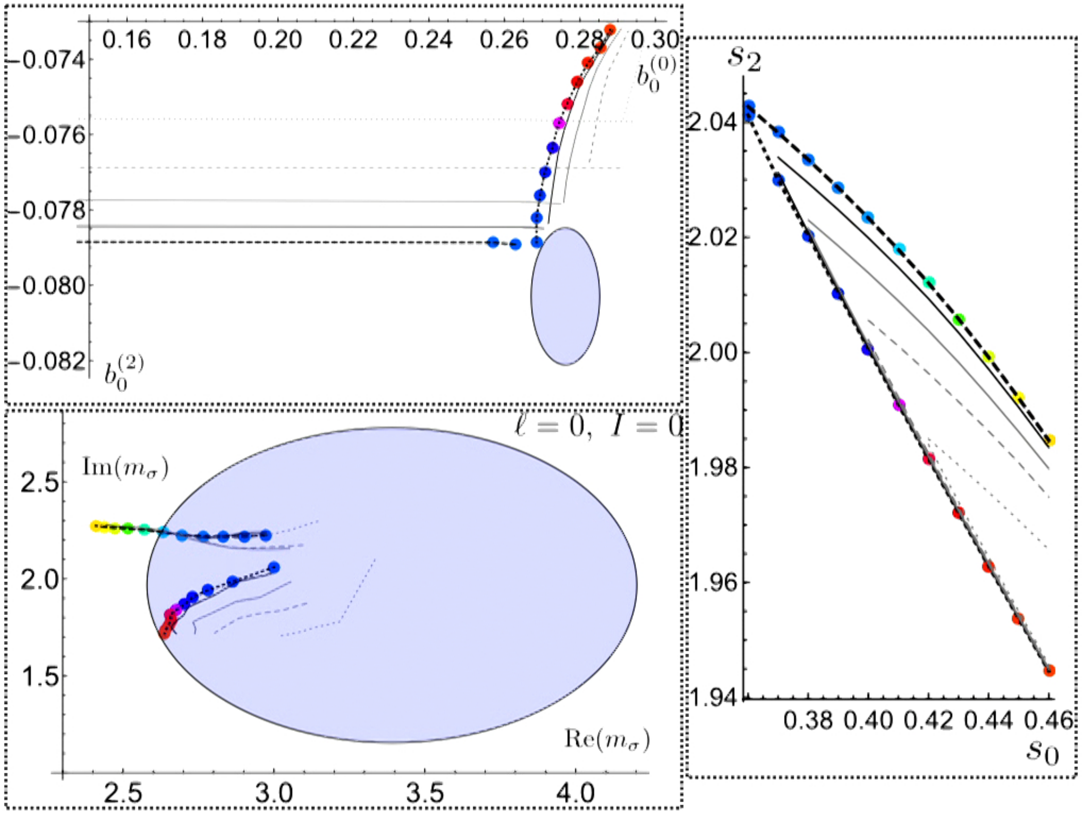


Handle:

$\mathcal{T}_0^{(0)}(s_0) = 0 \;\; s_0 \in [0,4]$

Output 1: allowed region in (s_0, s_2)

QCD Geography: the shore of the Peninsula

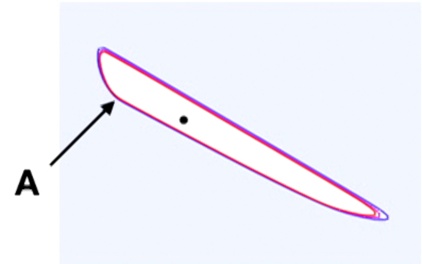


Output 2: $(b_0^{(0)}, b_0^{(2)})$ and m_σ

QCD Geography: last summary

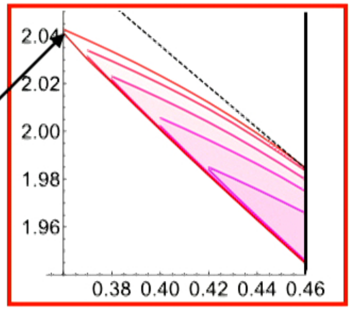
Bonus: constraint on the zeros position

1) Rho Resonace : kink A



2) Rho Resonace + scattering lengths: tip of the Peninsula

$s_0 = 0.36 \; s_2 = 2.04$

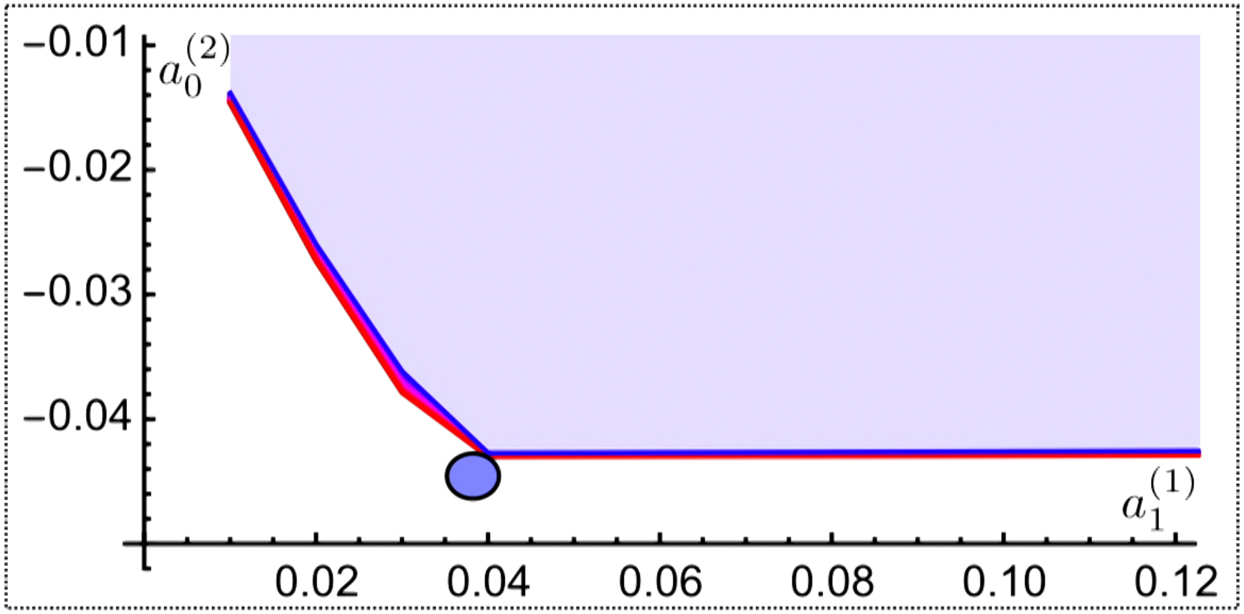


QCD Geography: the Kink

Optimization Problem: $\underset{a,b\in \text{Ansatz}}{min} \quad a_\ell^{(I)}$

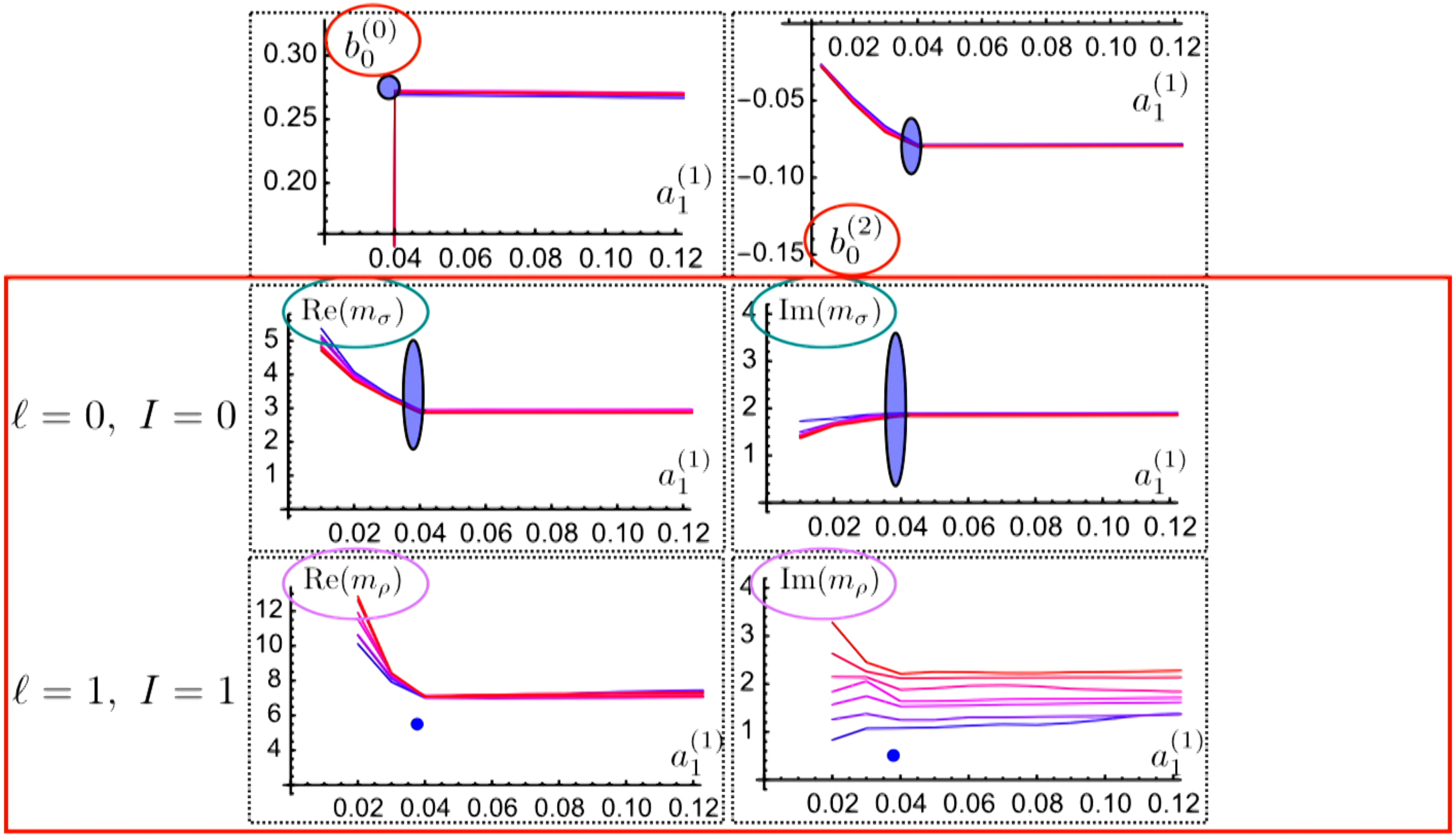
$\text{Input} : s_0 = 0.36 \ , s_2 = 2.04$

Handle:



QCD Geography: correlated Kinks

Spectrum not imposed: emerging?



What next?

1. Different symmetry breaking patterns: massless scattering in 1+1

2. Massless scattering in 3+1 and Reggeization

3. (hep-ph project) adapt our ansatz to experimental data

4. Precision Physics Test: impose chirality and bound low energy params

5. Use QCD as a window to understand the predictability power of the S-matrix Bootstrap.