

Title: Trapped ions for multi-valued quantum information

Date: Nov 07, 2018 02:00 PM

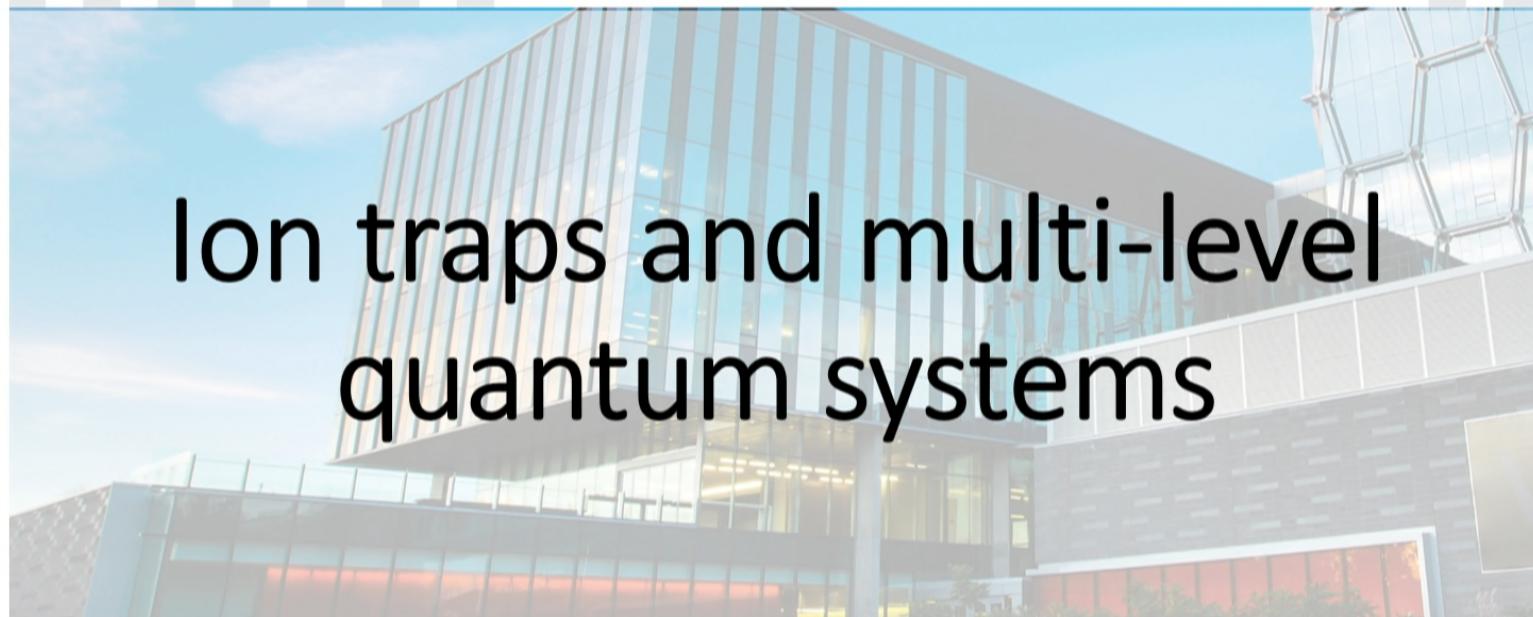
URL: <http://pirsa.org/18110059>

Abstract: <p>I give an overview of trapped ion quantum information experiments and discuss prospects for implementing multi-valued quantum logic using trapped ions. Qudits (the multi-state generalization of qubits) are attractive for quantum computing because they enable a much larger Hilbert space for the same number of trapped ions, which may allow us to improve the information capacity of a quantum processor. I describe possible advantages and disadvantages of using qudits in place of qubits, and lay out some of the protocols that my lab will test for implementing measurements, single-qudit operations, and two-qudit operations in a trapped ion system.</p>



Institute for Quantum Computing

Ion traps and multi-level quantum systems

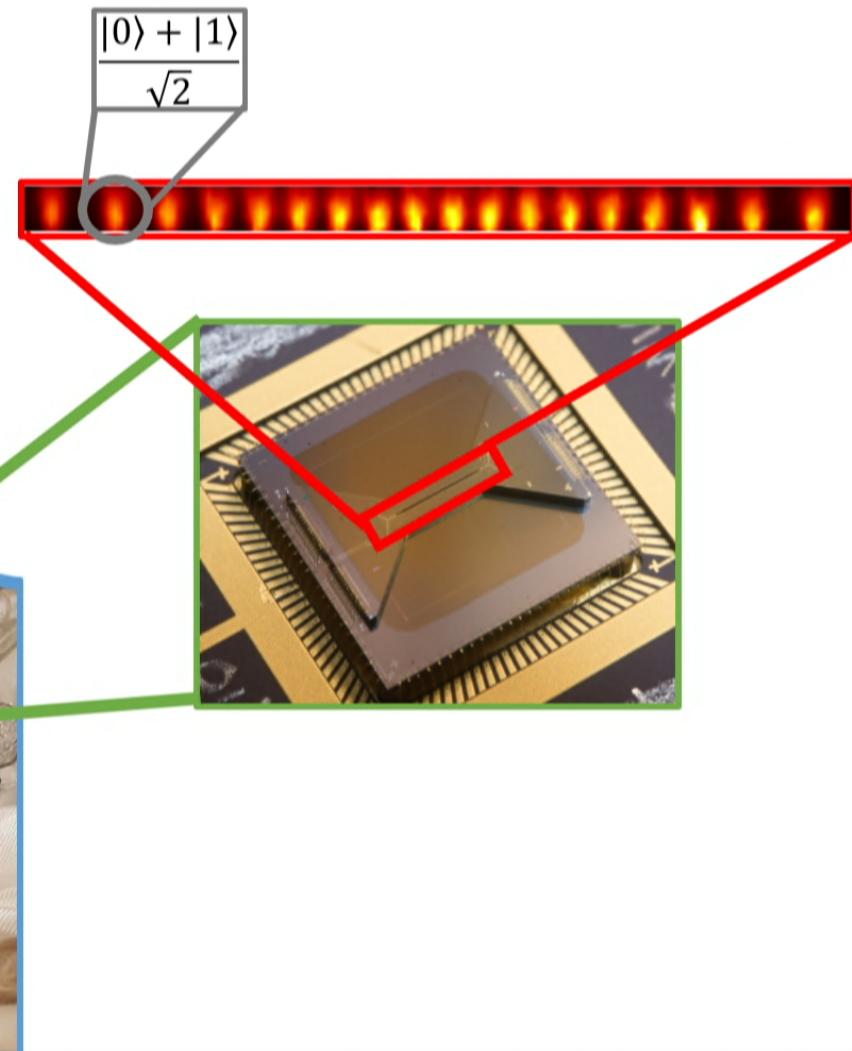
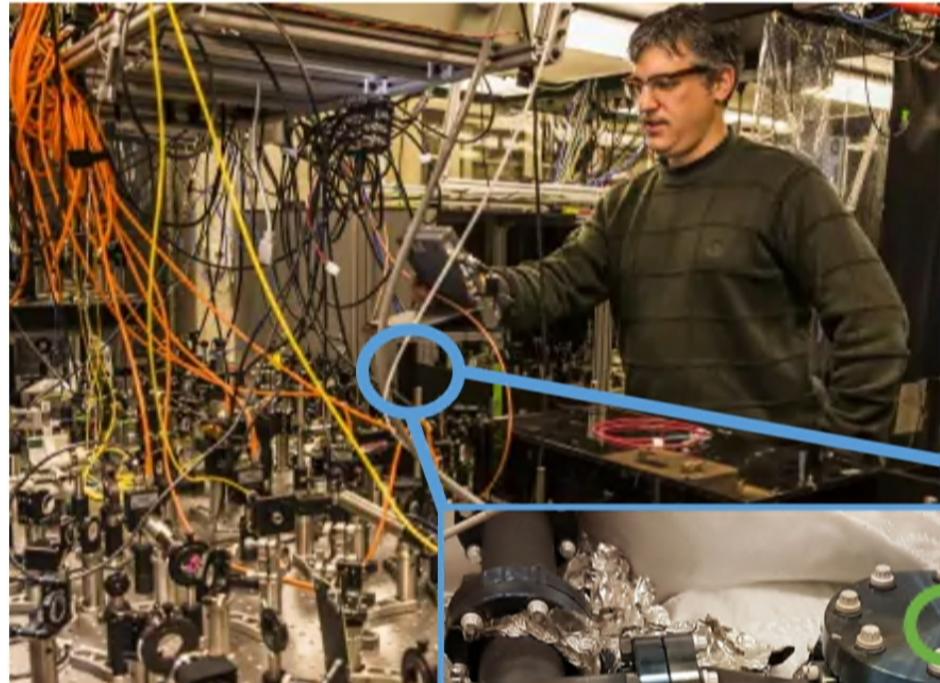


Crystal Senko

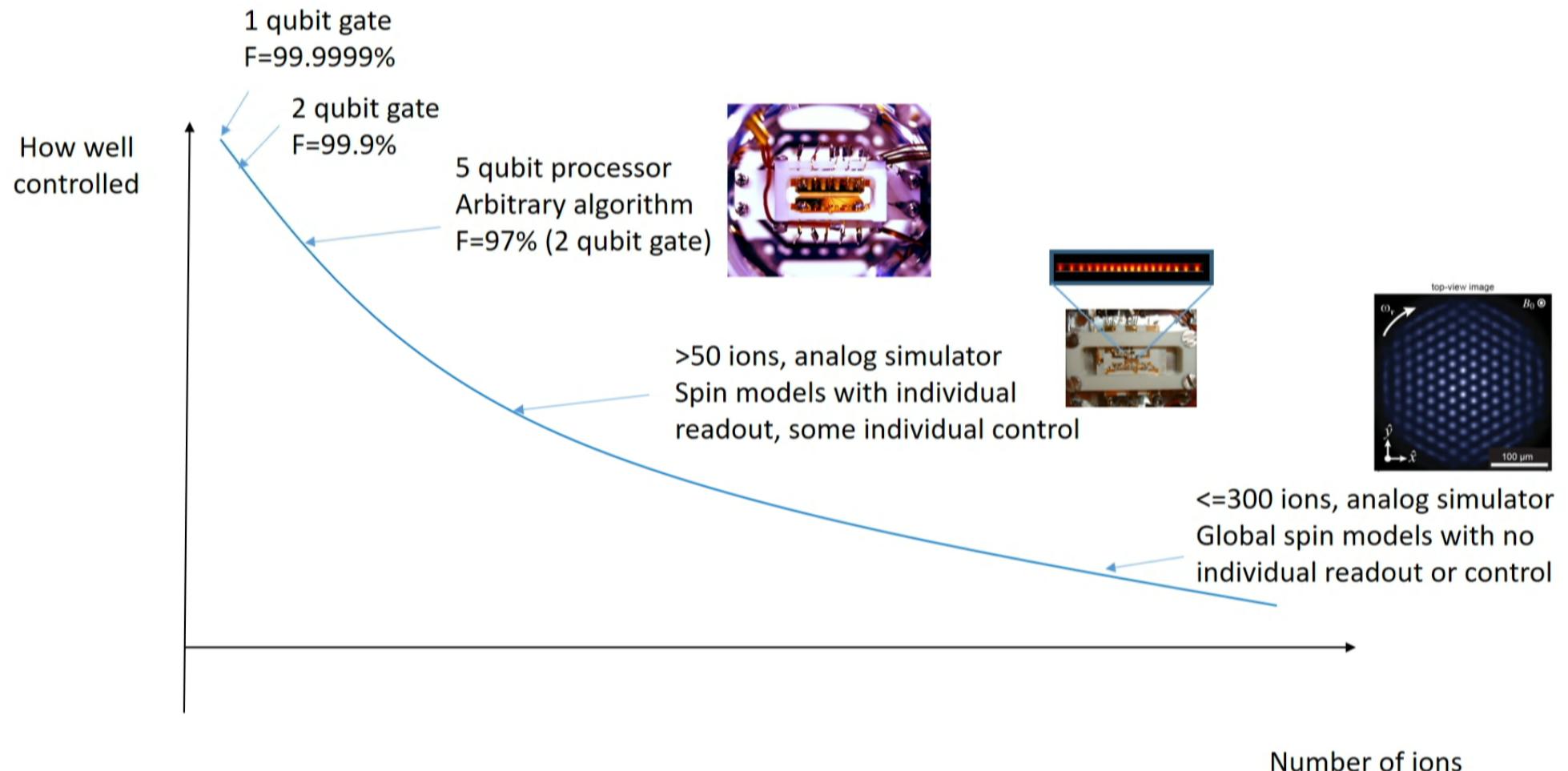
Perimeter Institute colloquium, Nov. 7, 2018



What does a trapped ion quantum computer look like?

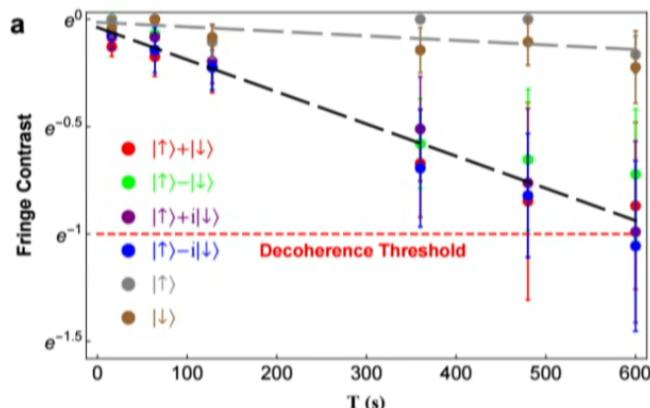


What can we do with ions?



For computation: care about precision, information capacity

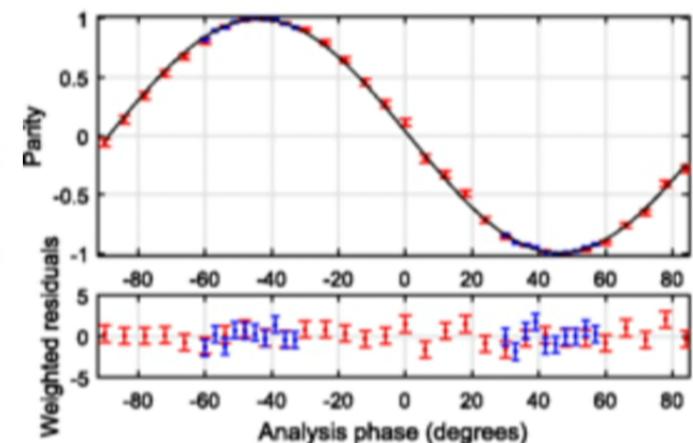
10 minute coherence time



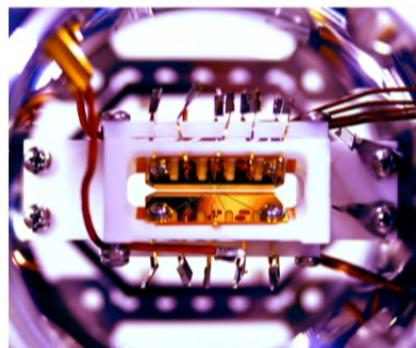
Nature Photonics 11, 646-650 (2017) - Kim group

High fidelity quantum gates

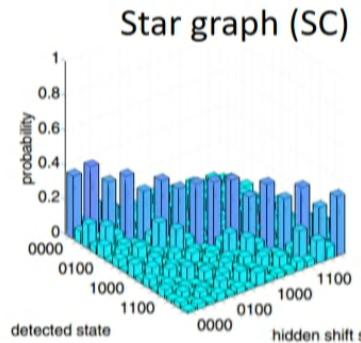
Two-qubit: **99.9%**
PRL 117, 060504 (2016)
Single qubit: **99.9999%**
PRL 113, 220501 (2014)
Lucas group



Fully connected programmable quantum processor

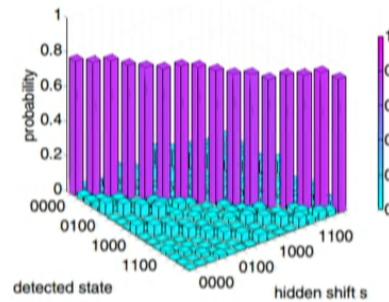


Nature 563, 63 (2016) - Monroe group



PNAS 114, 13 (2017) - Monroe group

Fully connected (ions)

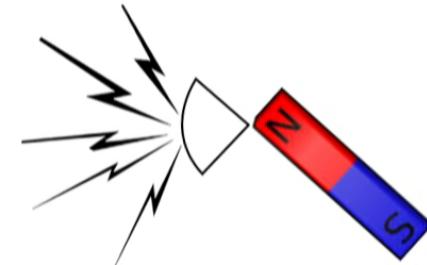


We want to use qudits: like qubits, with more states



Why the experiments haven't been done?

- Coherence more limited by magnetic field noise



My 7-level qudit is NOT in the state $|3\rangle$! ☺

Gee, how informative...

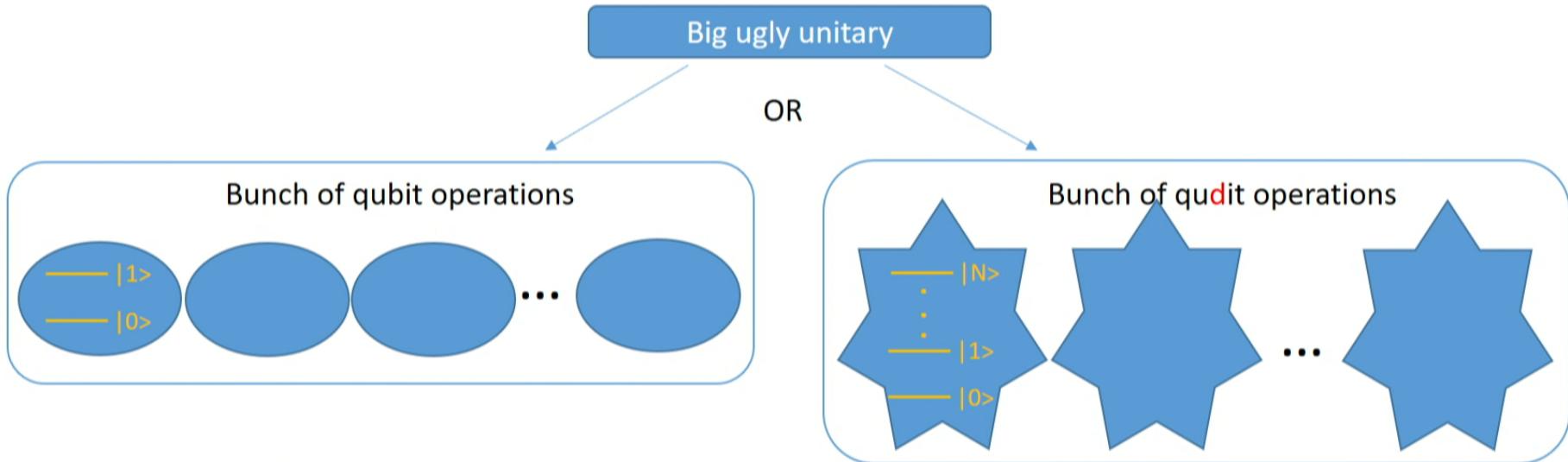


- Experiments hard to interpret without full readout

- More control knobs required

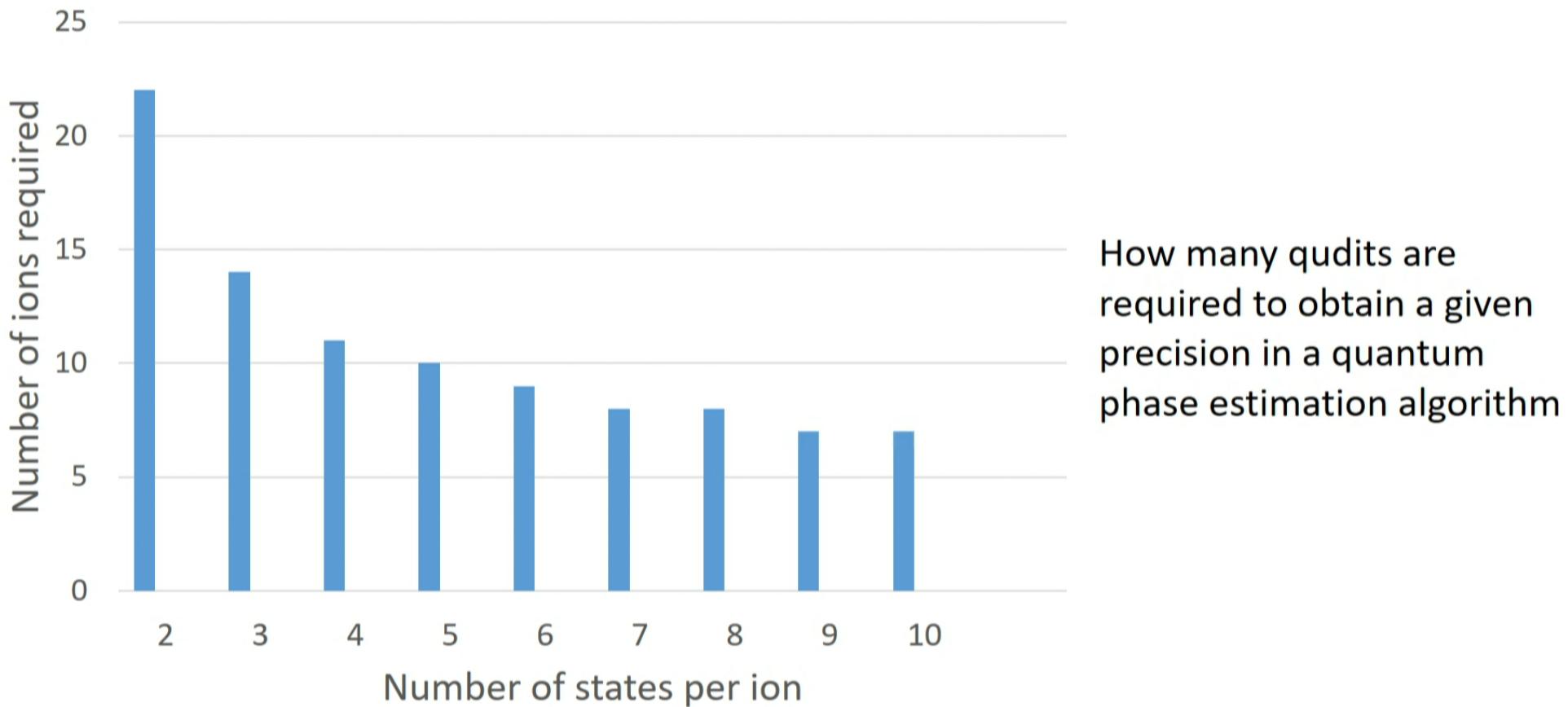


Why qudits?



- Simple components
- Complexity is entirely in the number of qubits and how they're wired
- Components are more difficult
- Could simplify other aspects... maybe

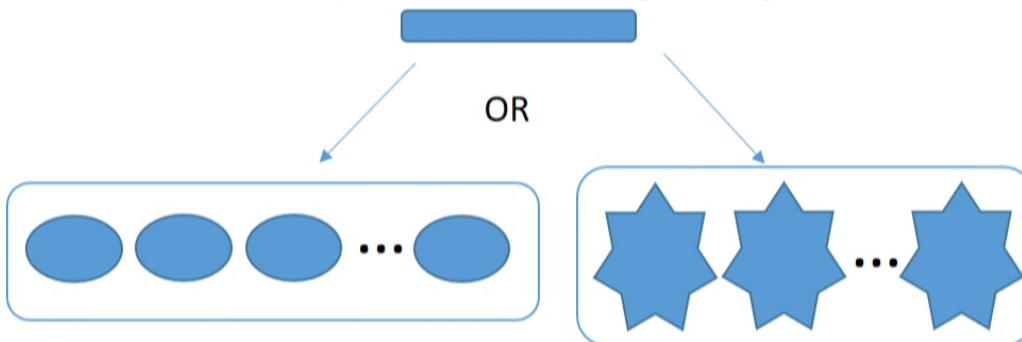
Computational gains from a larger Hilbert space?



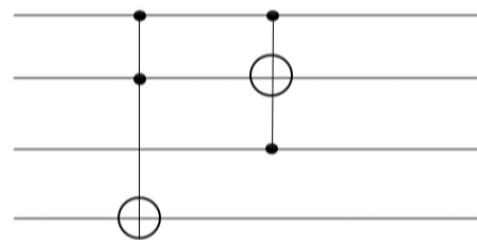
Parasa and Perkowski, 2011 IEEE International Symposium on Multivalued Logic

Why qudits?

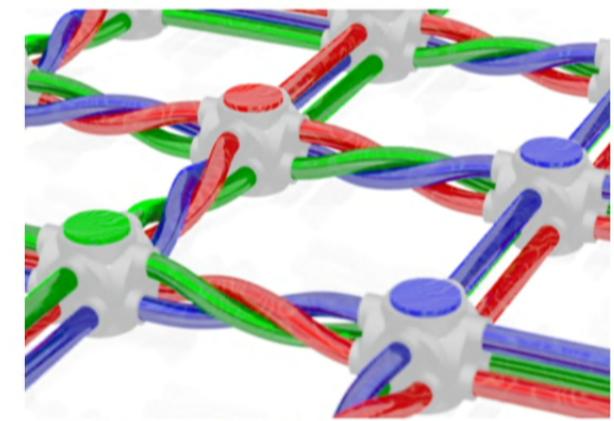
Higher dimensionality another
tradeoff for complexity



Specific improvements
e.g. simplifying algorithms

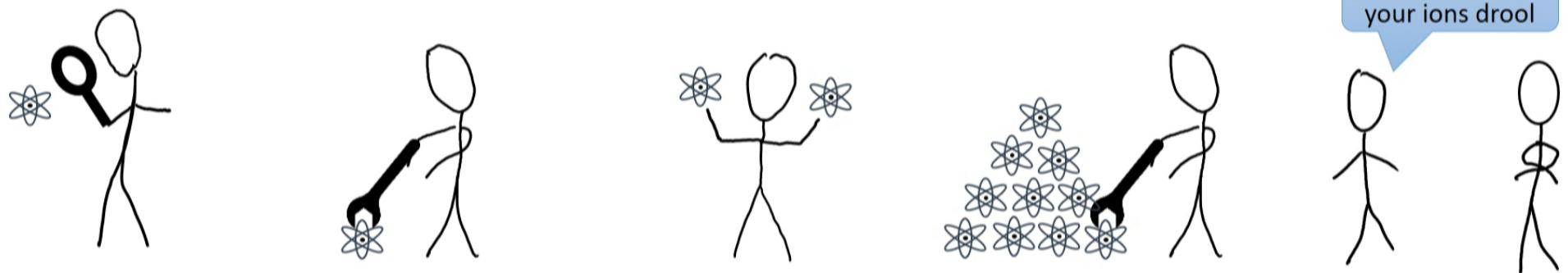


Specific improvements
e.g. error correcting code
thresholds



PRA 91, 042331

Central question: Can qudits offer computational advantages?



P E R I O D I C T A B L E

Atomic Properties of the Elements

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs
speed of light in vacuum c $299\,792\,458 \text{ m s}^{-1}$ (exact)
Planck constant h $6.6261 \times 10^{-34} \text{ J s}$ ($\hbar = h/2\pi$)
elementary charge e $1.6022 \times 10^{-19} \text{ C}$
electron mass m_e $9.1094 \times 10^{-31} \text{ kg}$
$m_e c^2$ 0.5110 MeV
proton mass m_p $1.6726 \times 10^{-27} \text{ kg}$
fine-structure constant α $1/137.036$
Rydberg constant R_∞ $10\,973\,732 \text{ m}^{-1}$
$R_\infty c$ $3.289\,84 \times 10^{15} \text{ Hz}$
$R_\infty hc$ 13.6057 eV
Boltzmann constant k $1.3807 \times 10^{-23} \text{ J K}^{-1}$

Atomic Number Ground-state Level

Symbol	58	1G_4
Name	Ce	Cerium
Atomic Weight [†]	140.116	[Xe]4f ₁₅ d ₆ s ₂
		5.5387

Ground-state Configuration Ionization Energy (eV)

57	² D _{3/2}	58	¹ G _{5/2}	59	¹ I _{9/2}	60	⁵ I ₄	61	⁹ H _{1/2}	62	⁷ F ₀	63	⁸ S _{7/2}	64	⁹ D _{5/2}	65	⁹ H _{5/2}	66	⁵ I ₆	67	¹ I _{15/2}	68	³ H ₆	69	² F _{7/2}	70	¹ S ₀	71	² D _{5/2}	
La	Lanthanum 136.9055	Ce	Cerium 140.116	Pr	Praseodymium 140.90765	Nd	Neodymium 144.2	Pm	Promethium (145)	Sm	Samarium 150.36	Eu	Europium 151.964	Gd	Gadolinium 157.25	Tb	Terbium 159.02534	Dy	Dysprosium 162.55	Ho	Holmium 164.93032	Er	Erbium 167.26	Tm	Thulium 168.93421	Yb	Ytterbium 173.04	Lu	Lutetium 174.967	
	[Xe]5s ²		[Xe]5f ⁶ 6s ²			[Xe]4f ⁷ 6s ²			[Xe]4f ⁷ 6s ²		[Xe]4f ⁷ 6s ²		[Xe]4f ⁷ 6s ²		[Xe]4f ⁷ 6s ²		[Xe]4f ⁷ 6s ²		[Xe]4f ¹¹ 6s ²		[Xe]4f ¹¹ 6s ²		[Xe]4f ¹⁴ 6s ²		[Xe]4f ¹⁴ 5d ² 6s ²					
	5.5769		5.5837			5.4735			5.5250		5.5820			5.5820		5.4704		5.1801		5.9389		6.0215		6.1077		6.1943		6.2542		5.4259
89	³ D _{3/2}	90	⁵ F ₃	91	¹ K _{11/2}	92	⁹ I ₆	93	⁶ I _{11/2}	94	⁷ F ₀	95	⁸ S _{7/2}	96	⁹ D _{7/2}	97	⁹ H _{5/2}	98	⁵ I ₆	99	¹ I _{15/2}	100	³ H ₆	101	² F _{7/2}	102	¹ S ₀	103	³ P _{1/2}	
Ac	Actinium (227)	Th	Thorium 232.0381	Pa	Protactinium 231.03588	U	Uranium 238.0289	Np	Neptunium (237)	Pu	Plutonium (244)	Am	Americium (243)	Cm	Curium (247)	Bk	Berkelium (247)	Cf	Californium (251)	Es	Einsteinium (252)	Fm	Fermium (257)	Md	Mendelevium (259)	No	Nobelium (262)	Lr	Lawrencium (262)	
	[Rn]5d ⁷ s ²		[Rn]5f ⁷ s ²			[Rn]5f ⁷ s ²			[Rn]5f ⁷ s ²		[Rn]5f ⁷ s ²			[Rn]5f ⁷ s ²		[Rn]5f ⁷ s ²		[Rn]5f ⁷ s ²		[Rn]5f ⁷ s ²		[Rn]5f ⁷ s ²		[Rn]5f ¹⁴ s ²		[Rn]5f ¹⁴ s ²				
	5.17		6.3067			5.89			6.1941		6.2652			6.0262		5.9738		5.9915		6.1879		6.2817		6.42		6.58		6.9		4.9

PERIODIC TABLE
Atomic Properties of the Elements

Group

IA																		
1	H																	
Hydrogen 1.00794																		
^1s																		
13.5984																		
IIA																		
2	Be																	
Beryllium 9.01218																		
$^1\text{s}^2 \text{ } ^2\text{s}^2$																		
6.941																		
5.3017																		
3	Li																	
Lithium 7.01600																		
$^1\text{s}^2 \text{ } ^2\text{s}^1$																		
11.92																		
13.5917																		
4	Mg																	
Magnesium 24.3050																		
$[Ne]3s^2$																		
5.1391																		
11.92																		
12.50																		
13.5917																		
III A																		
5	K																	
Potassium 39.0983																		
$[Ar]4s^1$																		
4.3407																		
19.92																		
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Period																		

Frequently used fundamental physical constants

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$R_{c,c}$ 3.289 84 $\times 10^{15}$ Hz

R_{hc} 13.6057 eV

Boltzmann constant k 1.3807 $\times 10^{-23}$ J K⁻¹

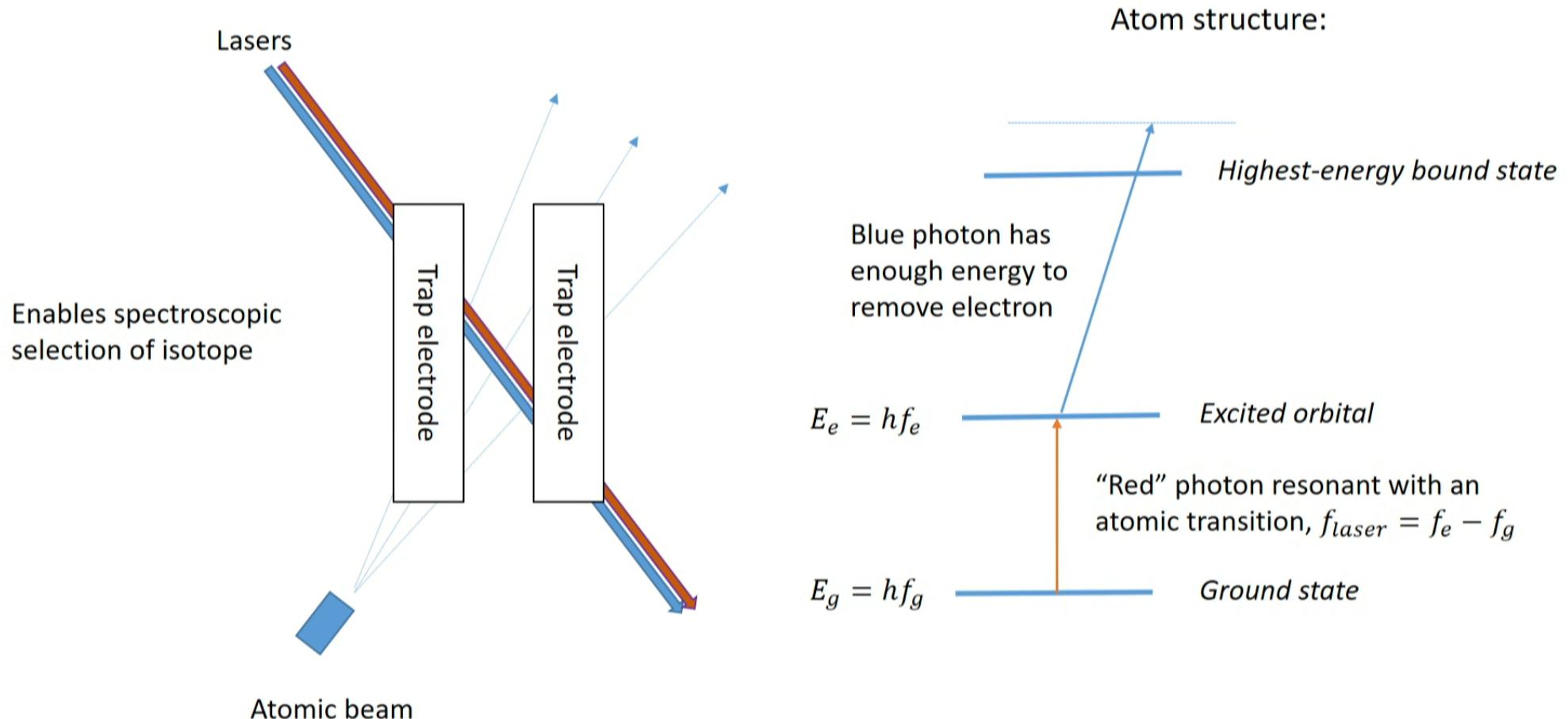
IIIB **IVB** **V B** **VIB** **VIIIB**

5	B																	
Boron 10.811																		
$^1\text{s}^2 \text{ } ^2\text{p}_1$																		
12.0107																		
$^1\text{s}^2 \text{ } ^2\text{p}_2$																		
12.1797																		
13.5181																		
13.5974																		
14.2228																		
14.5431																		
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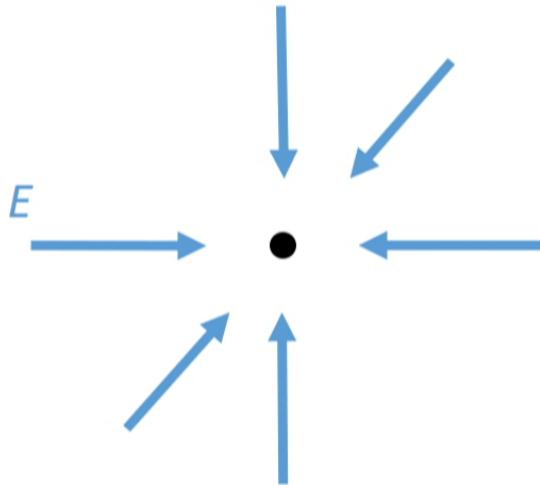
■ Solids
■ Liquids
■ Gases
■ Artificially Prepared

For a description of the atomic data, visit physics.nist.gov/atomic

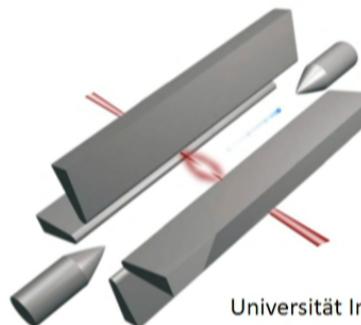
Making ions to trap



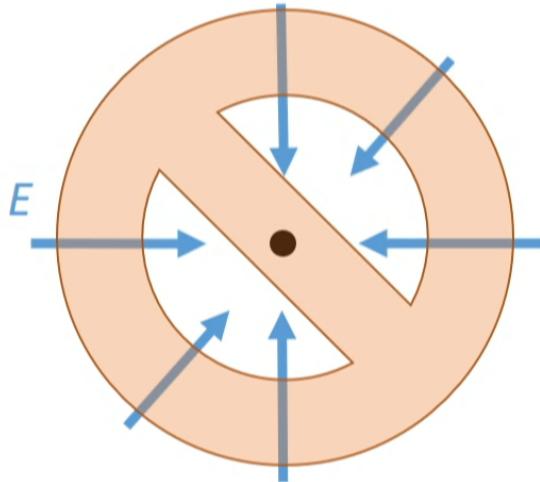
Trapping ions with a Paul trap



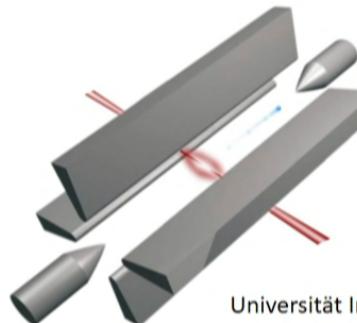
- Electric fields exert a force on a charged particle



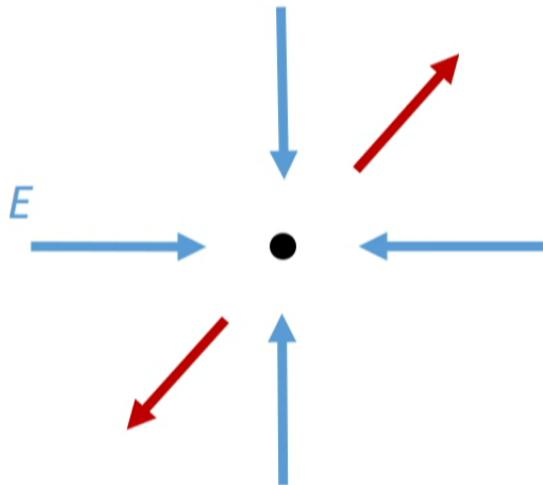
Trapping ions with a Paul trap



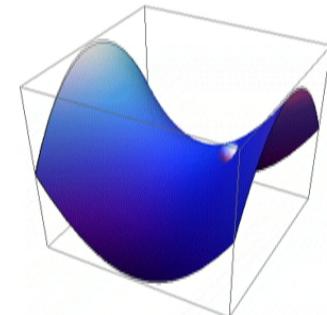
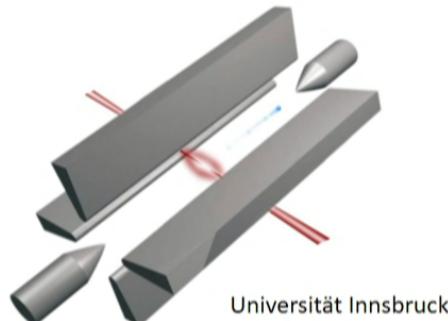
- Electric fields exert a force on a charged particle
- Maxwell's equations, Earnshaw theorem: $\vec{\nabla} \cdot \vec{E} = 0$
- Static electric fields cannot trap a charged particle



Trapping ions with a Paul trap



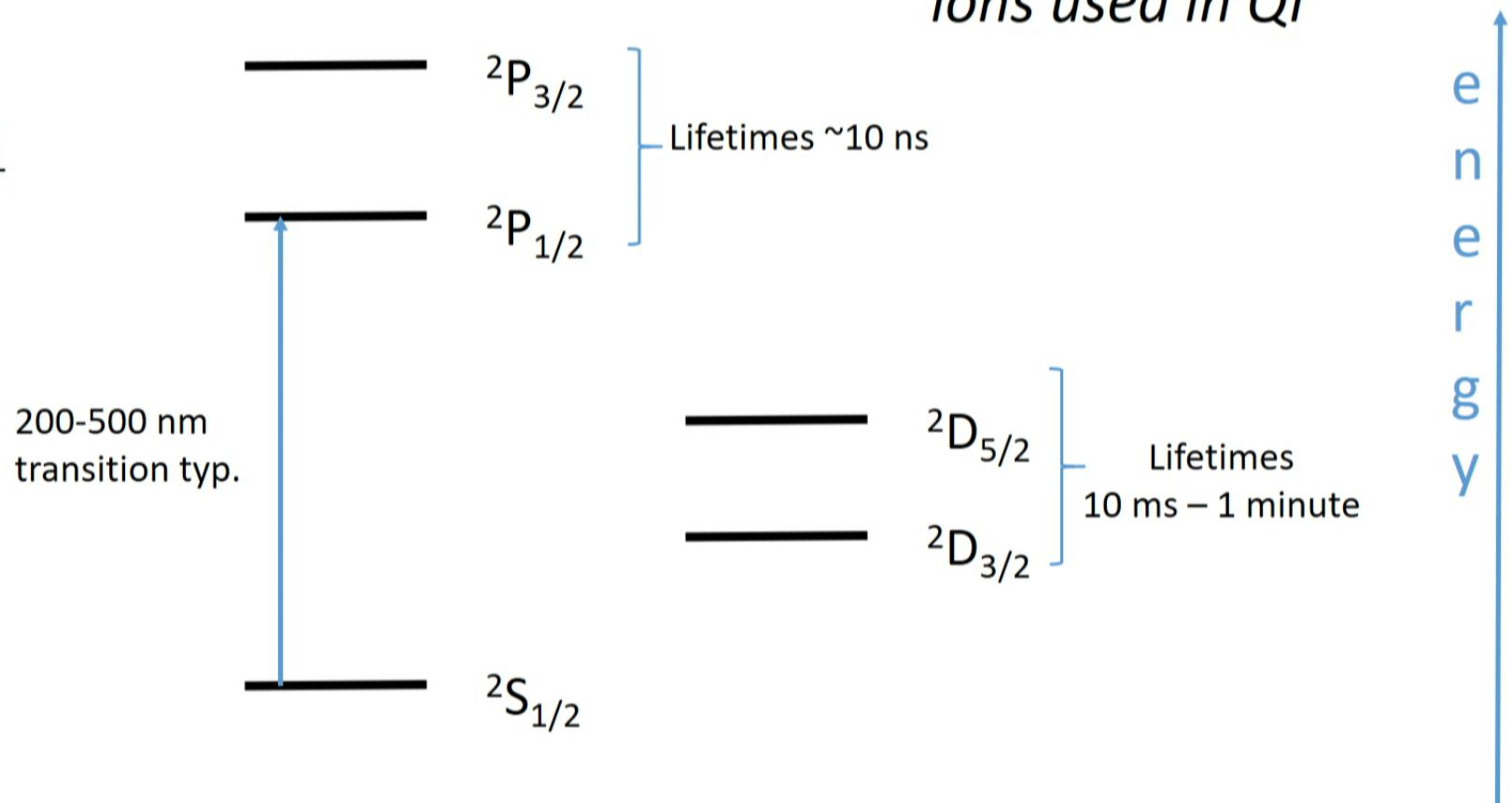
- Electric fields exert a force on a charged particle
- Maxwell's equations, Earnshaw theorem: $\vec{\nabla} \cdot \vec{E} = 0$
- Static electric fields cannot trap a charged particle
- Time-varying electric fields can!



Typical atomic structure for ions used in QI

Which ion?

Be^+ , Mg^+ ,
 Ca^+ , Sr^+ , Ba^+ ,
 Hg^+ , Cd^+ , Yb^+



From 2 to more: qudit development

Tools we need, a la DiVincenzo

Qudit encoding (multi-level system)
with acceptable coherence

State preparation (includes
trapping and cooling ions)

Measurement

Single-qudit gates

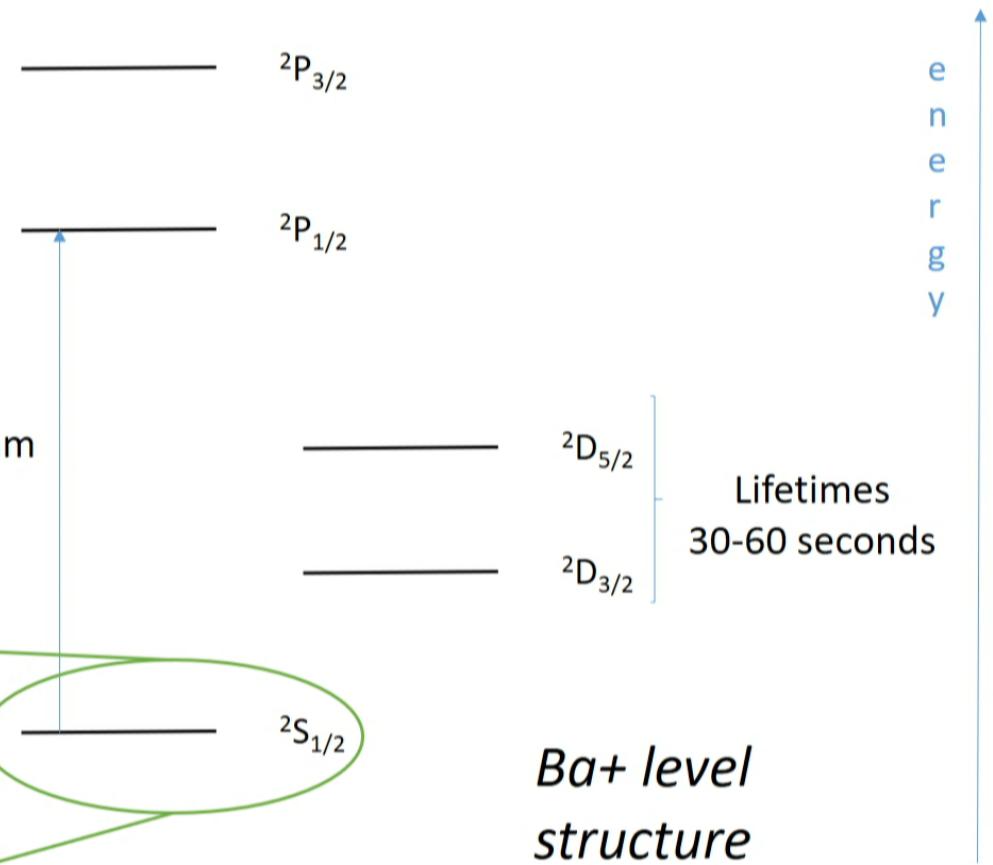
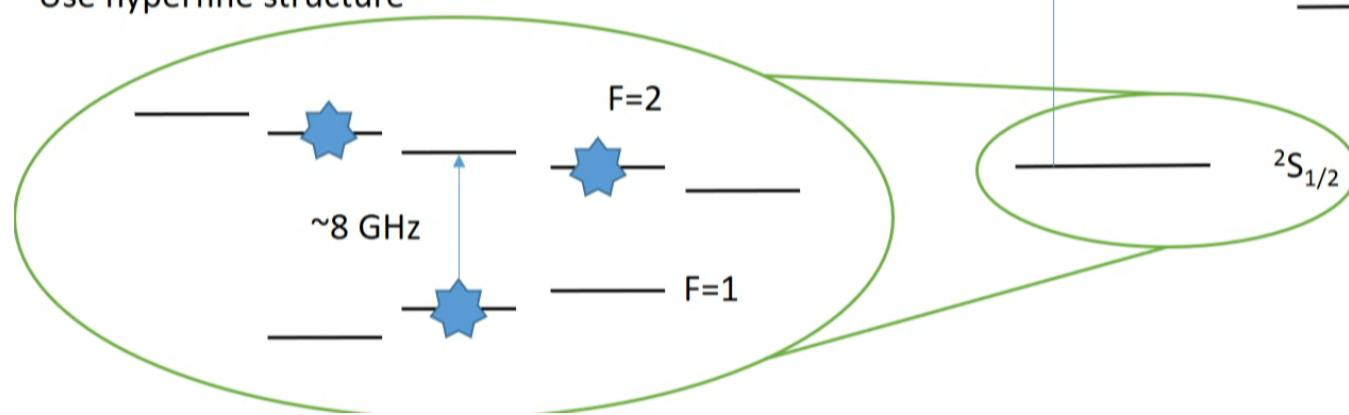
Two-qudit gates

Disclaimer:

Fidelity limitations are preliminary estimates
Initial experiments will probably not reach these numbers
Always assumes state-of-the-art technology

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation (includes trapping and cooling ions)
Measurement
Single-qudit gates
Two-qudit gates

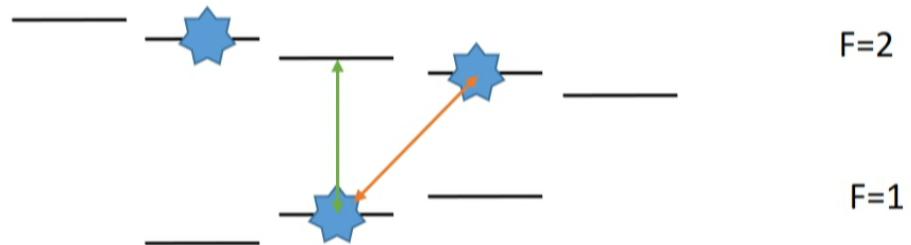
Use hyperfine structure



Ba+ level structure

Tools we need

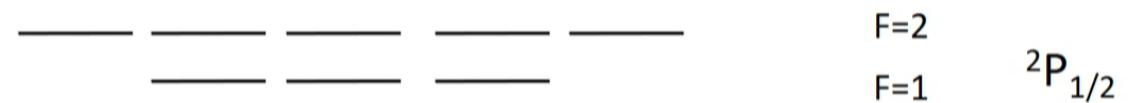
- Qudit encoding (multi-level system) with acceptable coherence
- State preparation (includes trapping and cooling ions)
- Measurement
- Single-qudit gates
- Two-qudit gates



$$\Delta E = g_F m_F \mu_B \Delta B \approx h * 0.7 \text{ MHz}/G$$

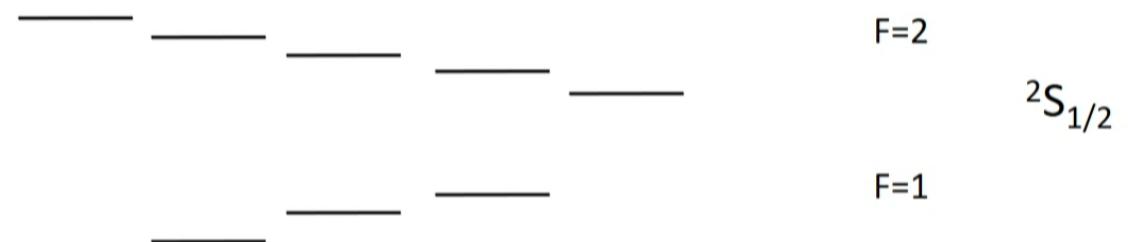
Coherence times:
10 minutes, m=0 to m=0
5 seconds (projected), m=0 to m=1

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation
Measurement
Single-qudit gates
Two-qudit gates



$^2P_{1/2}$

Optical pumping
similar to qubit case



$F=1$

Tools we need

Qudit encoding (multi-level system)
with acceptable coherence

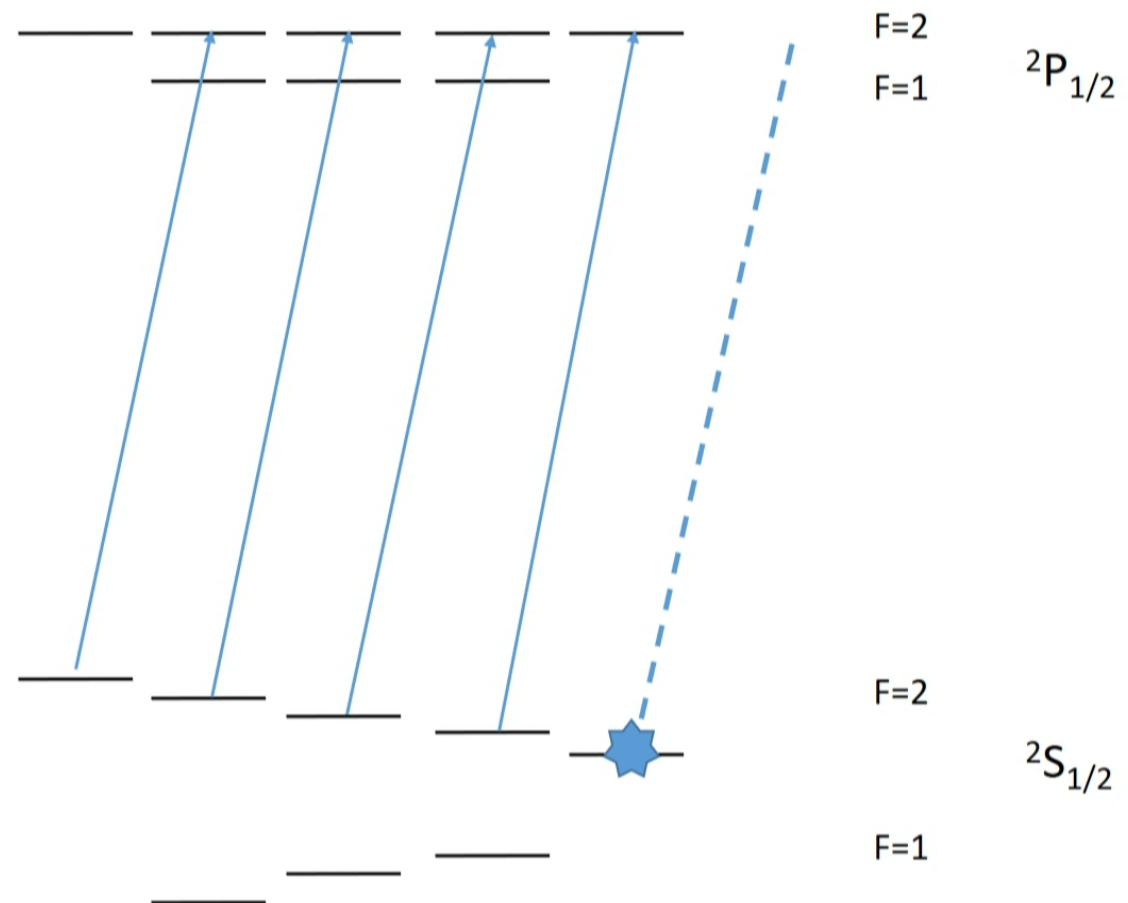
State preparation

Measurement

Single-qudit gates

Two-qudit gates

Optical pumping
similar to qubit case



Tools we need

Qudit encoding (multi-level system)
with acceptable coherence

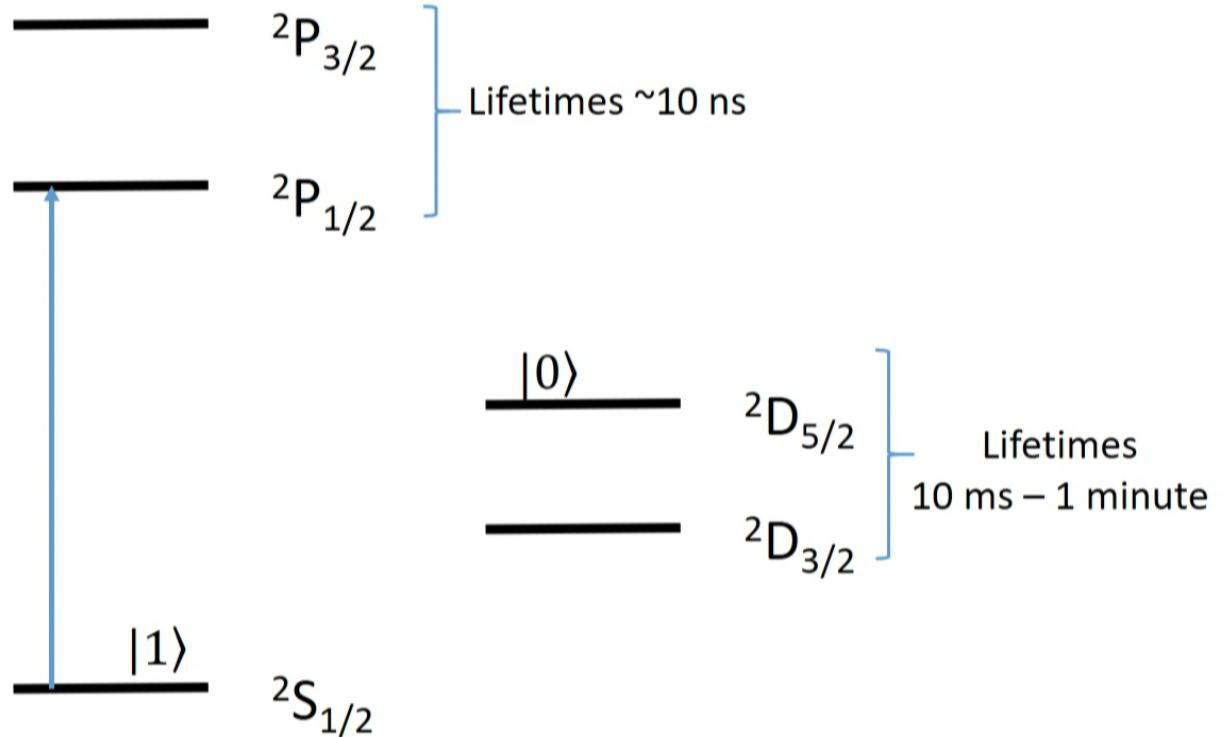
State preparation

Measurement

Single-qudit gates

Two-qudit gates

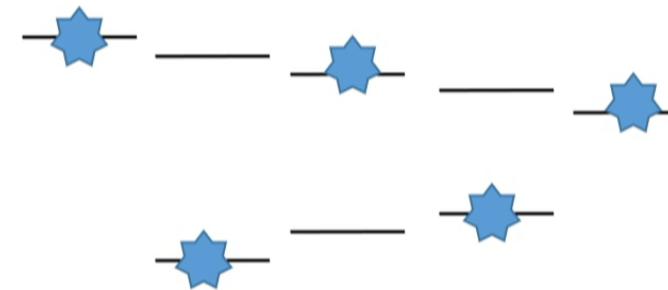
Apply blue laser:
 $|1\rangle$ fluoresces
 $|0\rangle$ does not



Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation
Measurement
Single-qudit gates
Two-qudit gates



$^2P_{1/2}$

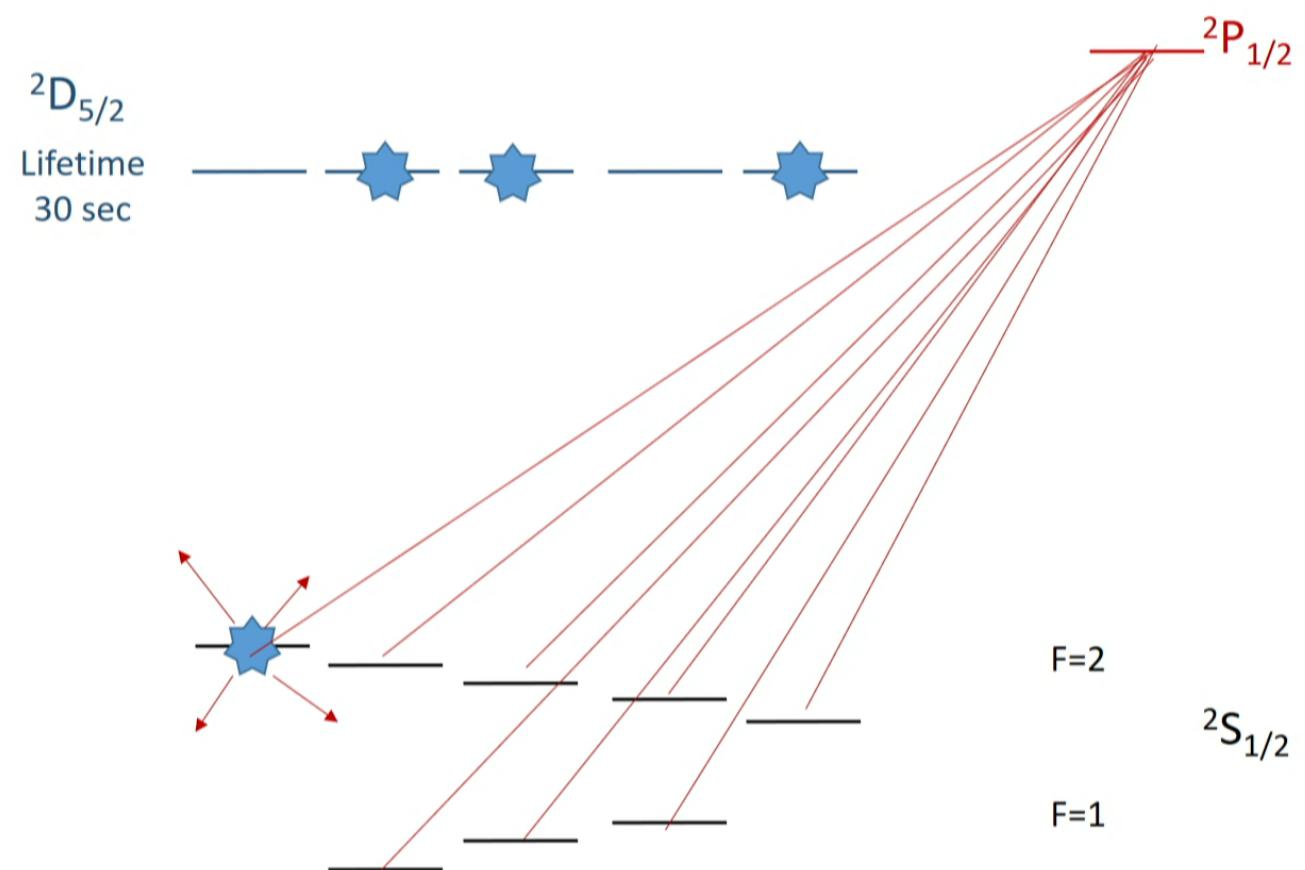


$F=2$

$^2S_{1/2}$

$F=1$

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation
Measurement
Single-qudit gates
Two-qudit gates

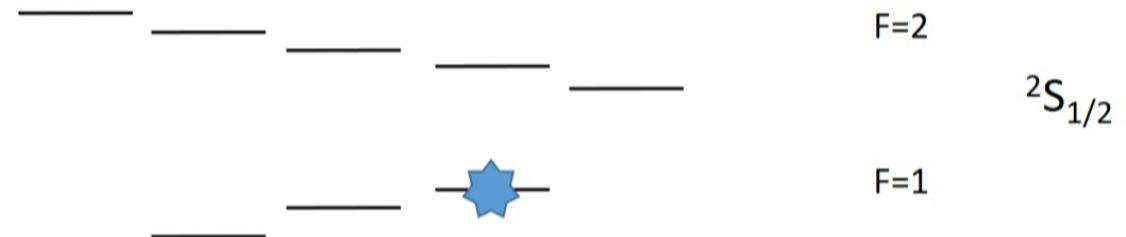


Tools we need	
Qudit encoding (multi-level system)	
with acceptable coherence	
State preparation	
Measurement	
Single-qudit gates	
Two-qudit gates	



Limiting factor = fidelity of transfer between S and D states
 Simple RAP protocol: limit ~0.998 for single transfer

Qudit dimension, d	Fidelity of measurement
3	0.996
5	0.984
7	0.976



Tools we need

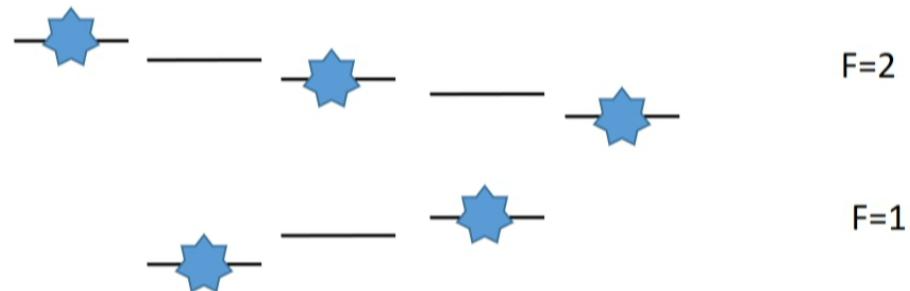
Qutrit encoding (three-level system) with acceptable coherence

State preparation

Measurement

Single-qudit gates

Two-qudit gates



Synthesize unitaries from Givens rotations:

$$H_{jk} = e^{i\phi}|j\rangle\langle k| + e^{-i\phi}|k\rangle\langle j|$$

We would like: Connected graph; frequency addressing

Tools we need

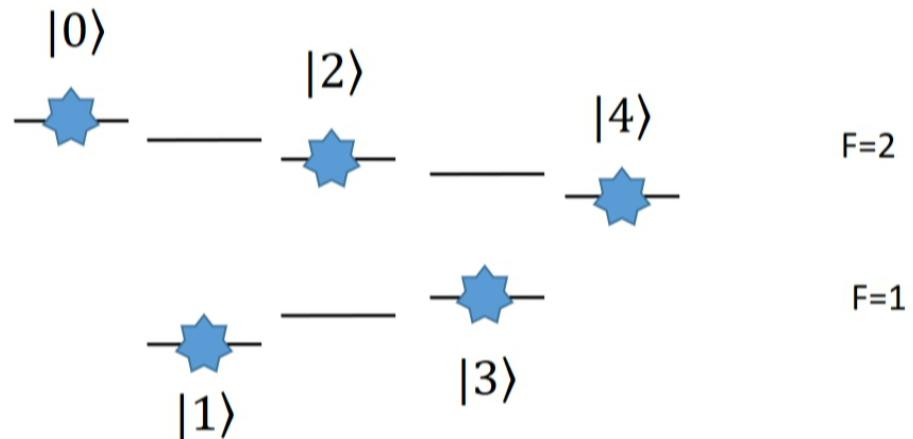
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Two-qudit gates



Error rate for single Givens rotation (from decoherence):
10⁻⁶

Error rate for single-qudit gate:
10⁻⁴ achievable

Example: X gate

$$X|i\rangle = |i + 1 \text{ mod } d\rangle$$

Tools we need

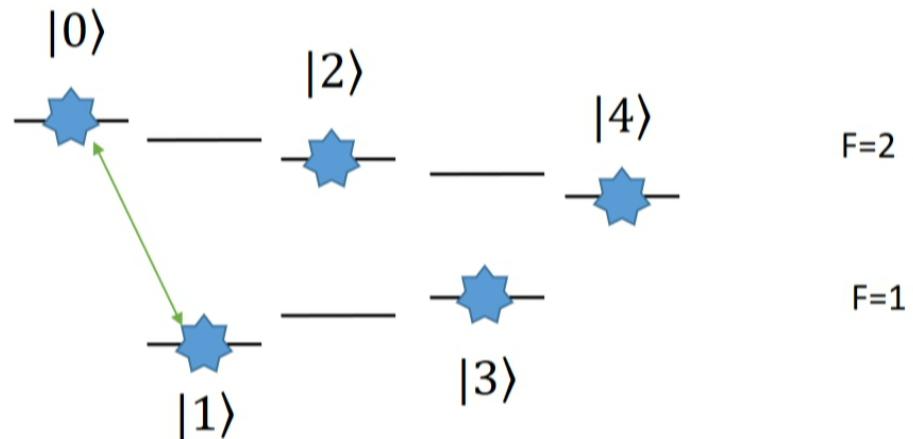
Qutrit encoding (three-level system) with acceptable coherence

State preparation

Measurement

Single-qudit gates

Two-qudit gates



Error rate for single Givens rotation (from decoherence):
 10^{-6}

Error rate for single-qudit gate:
 10^{-4} achievable

Example: X gate

$$X|i\rangle = |i + 1 \text{ mod } d\rangle$$

Tools we need

Qudit encoding (multi-level system)
with acceptable coherence

State preparation

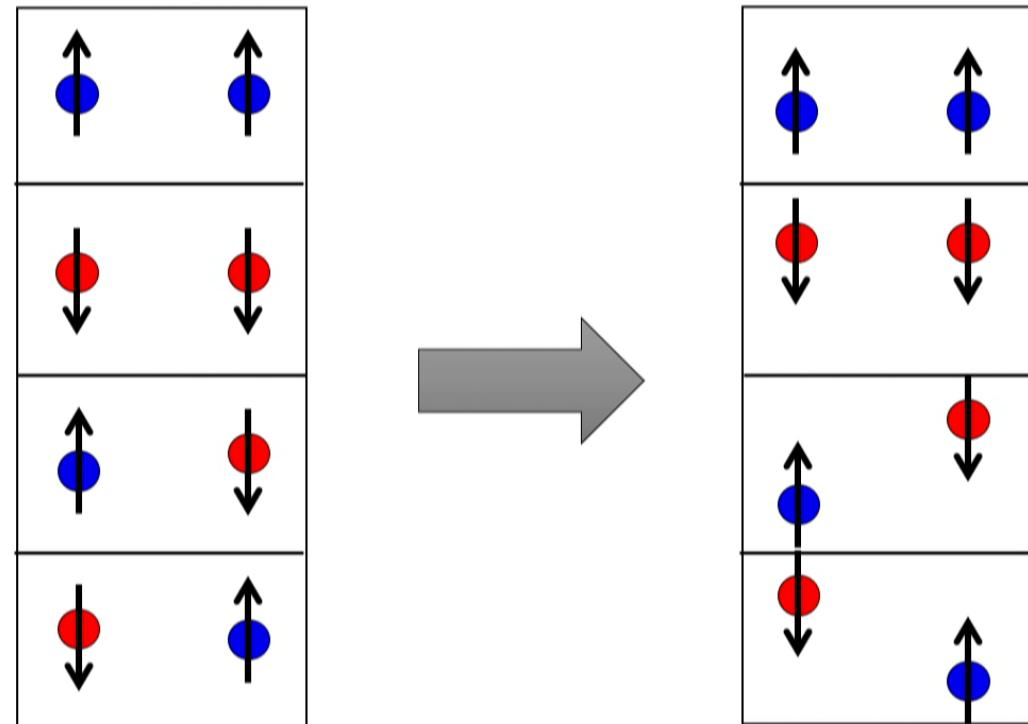
Measurement

Single-qudit gates

Two-qudit gates

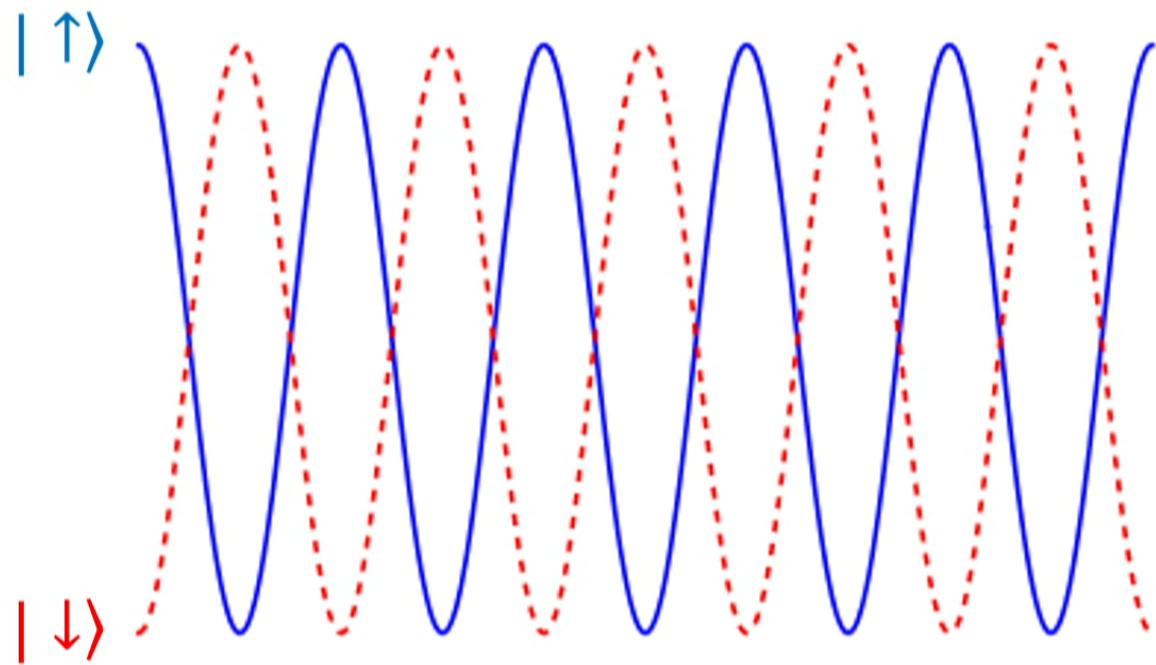
**Spin dependent force +
Coulomb interaction =
spin-spin interaction**

$$H = F \cdot x \propto \sigma_x [x_0(a^\dagger + a)]$$



Spin-dependent potential from laser standing wave

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation
Measurement
Single-qudit gates
Two-qudit gates



Two lasers interfere to create potential
Detuning one laser → potential oscillates in time

Drive a harmonic oscillator off resonance → excite and de-excite the motion

Tools we need

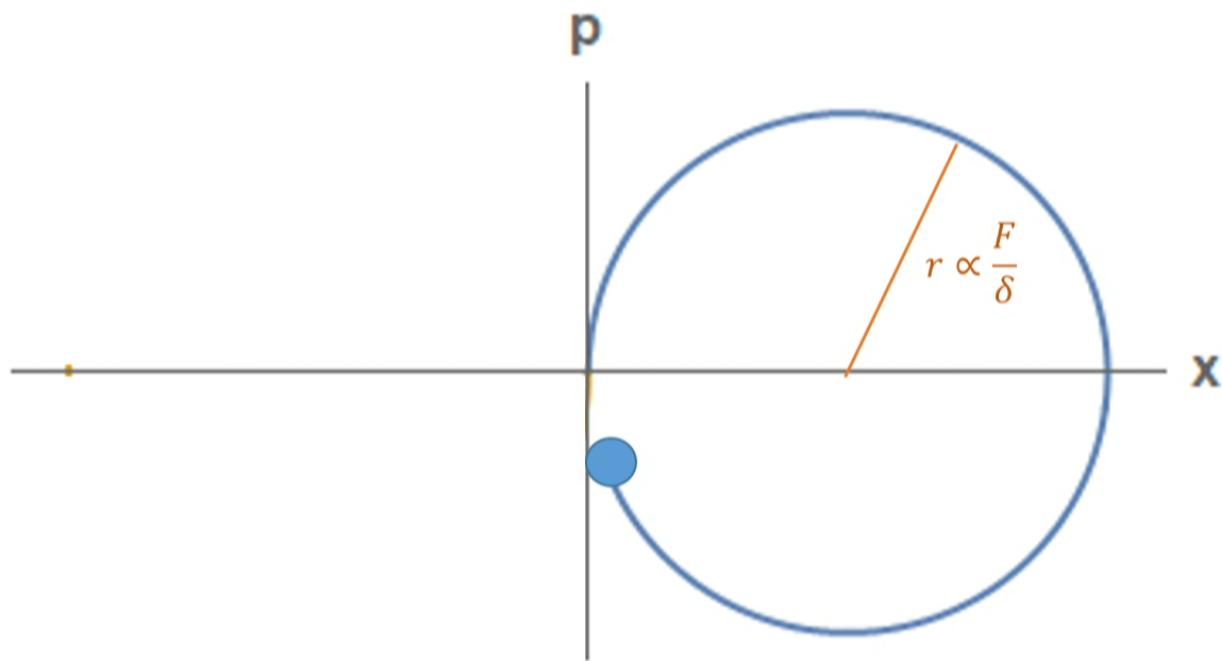
Qudit encoding (multi-level system)
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State preparation

Measurement

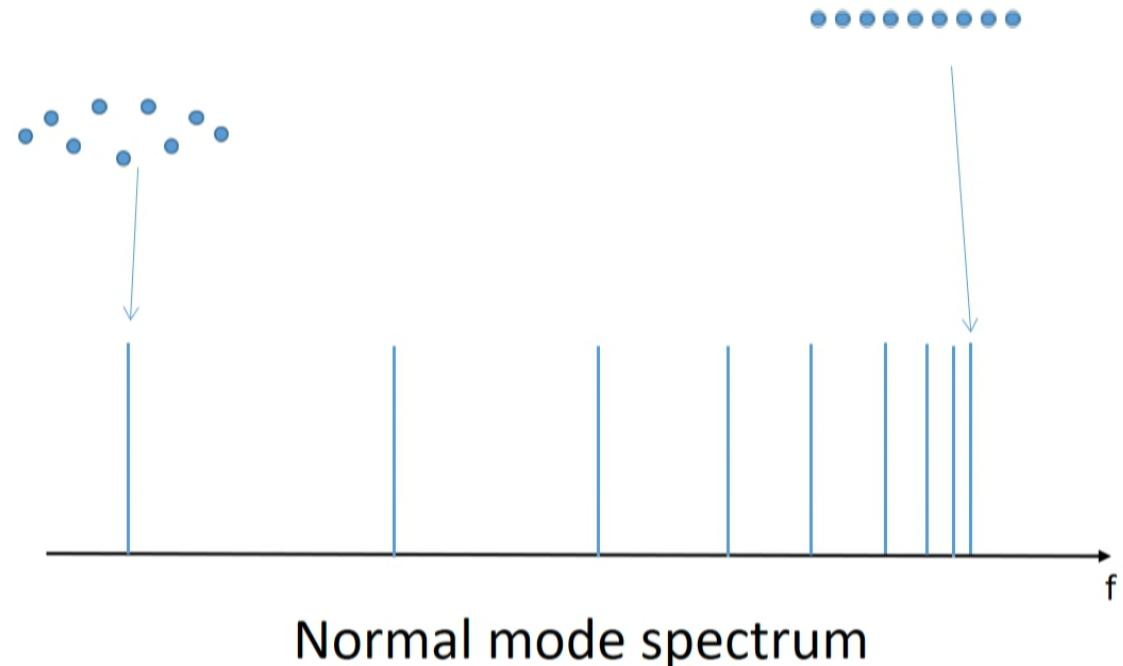
Single-qudit gates

Two-qudit gates



Ions in a harmonic trap = harmonic oscillators!

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
State preparation
Measurement
Single-qudit gates
Two-qudit gates



Two ions: Excite center of mass mode

Tools we need

Qudit encoding (multi-level system)
with acceptable coherence

State preparation

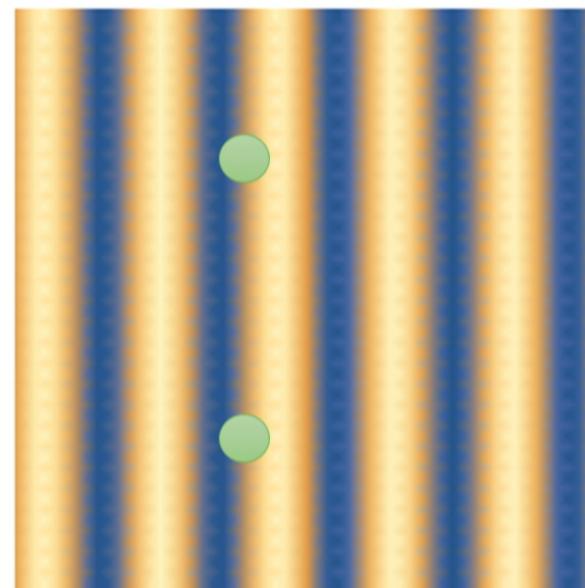
Measurement

Single-qudit gates

Two-qudit gates

Ions in same spin state: mode is excited

Ions in different states: mode is not excited



Geometric phase gates: obtained by driving harmonic oscillator off resonance

Tools we need

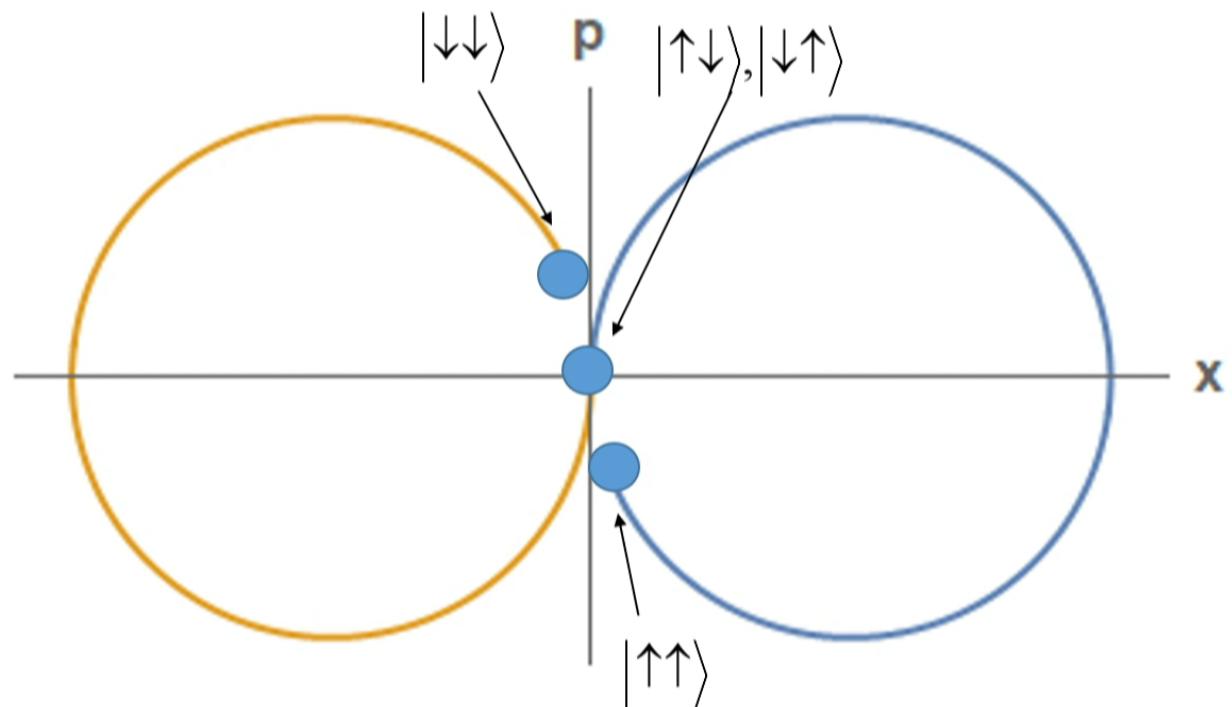
Qudit encoding (multi-level system)
with acceptable coherence

State preparation

Measurement

Single-qudit gates

Two-qudit gates



Geometric phase gates: work for larger spins too!

Tools we need

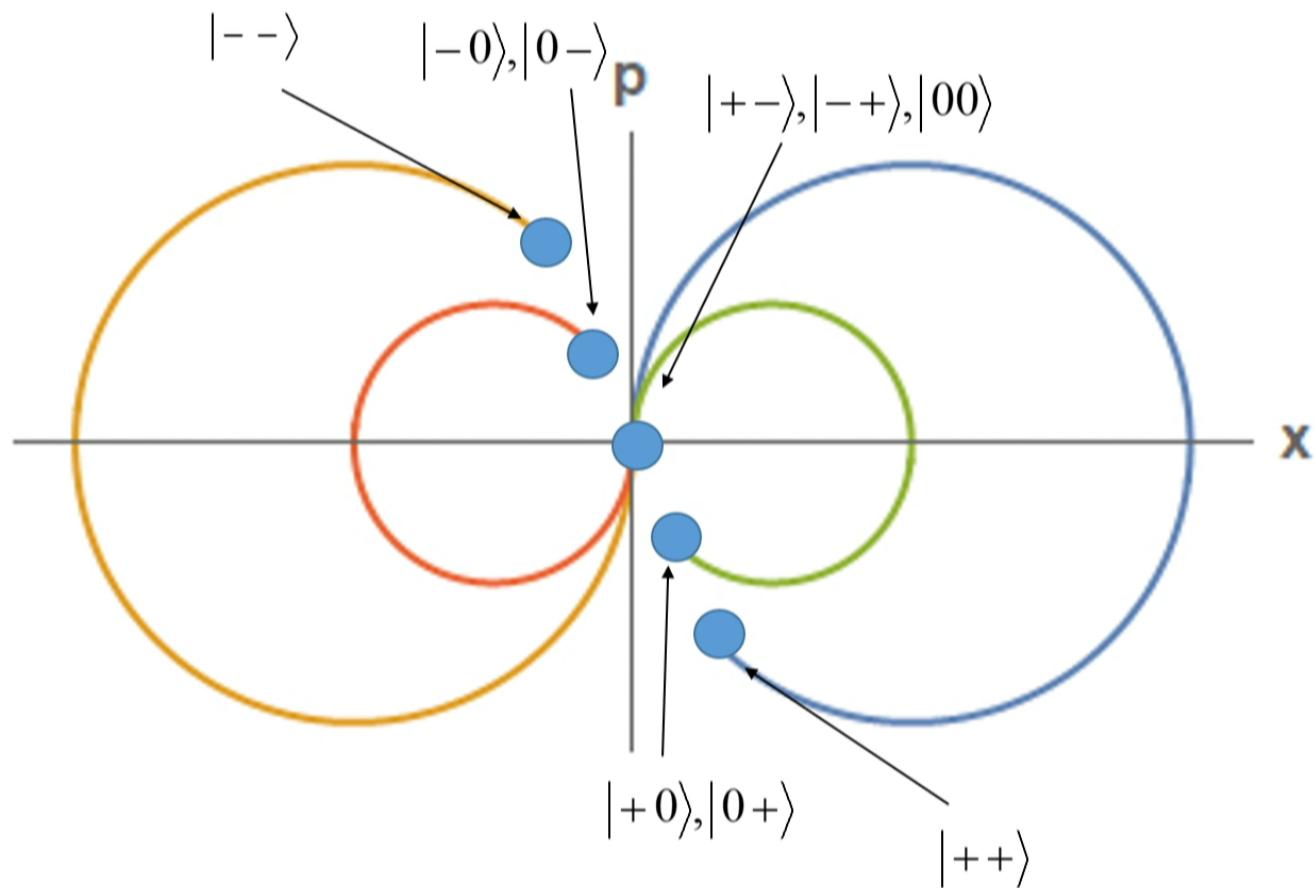
Qudit encoding (multi-level system)
with acceptable coherence

State preparation

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Single-qudit gates

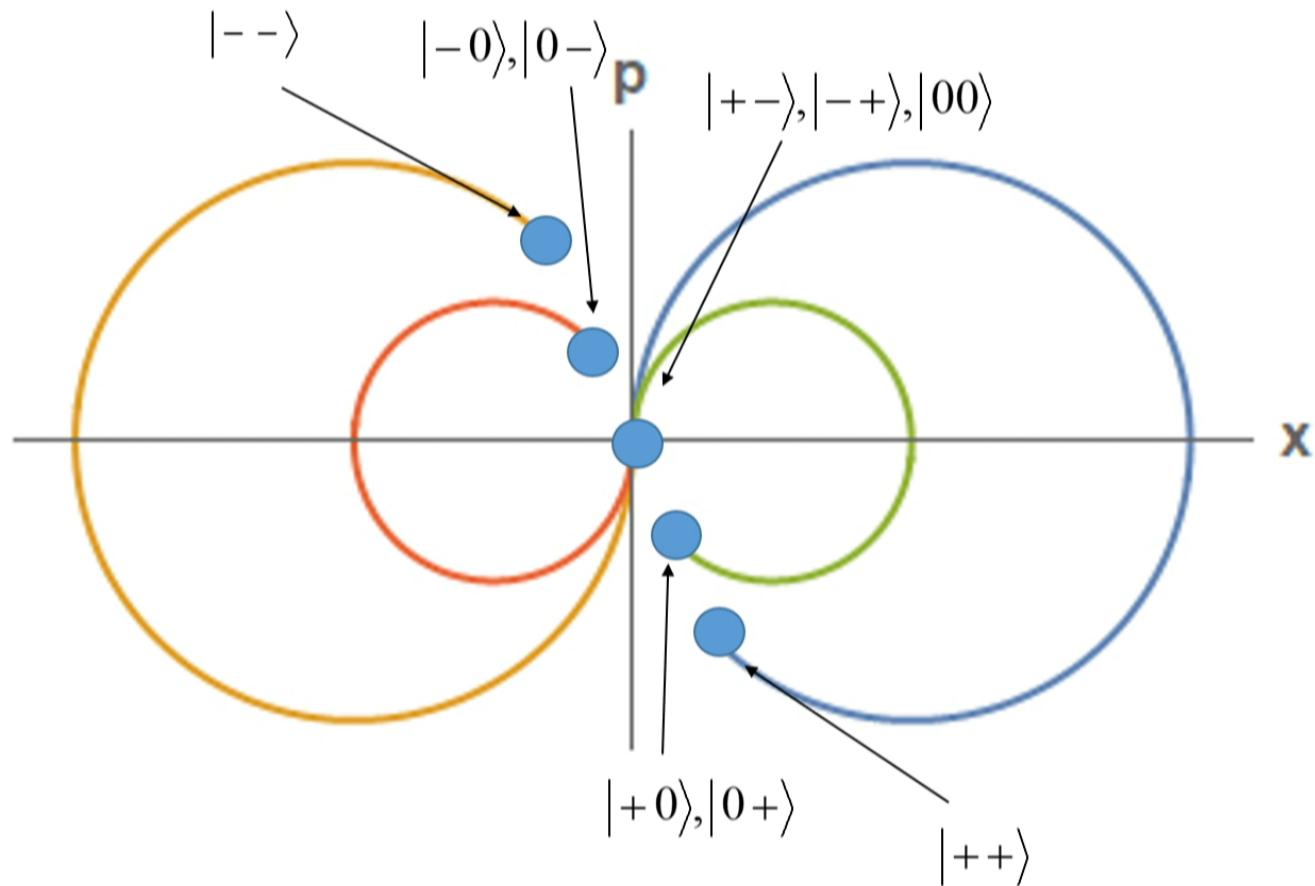
Two-qudit gates



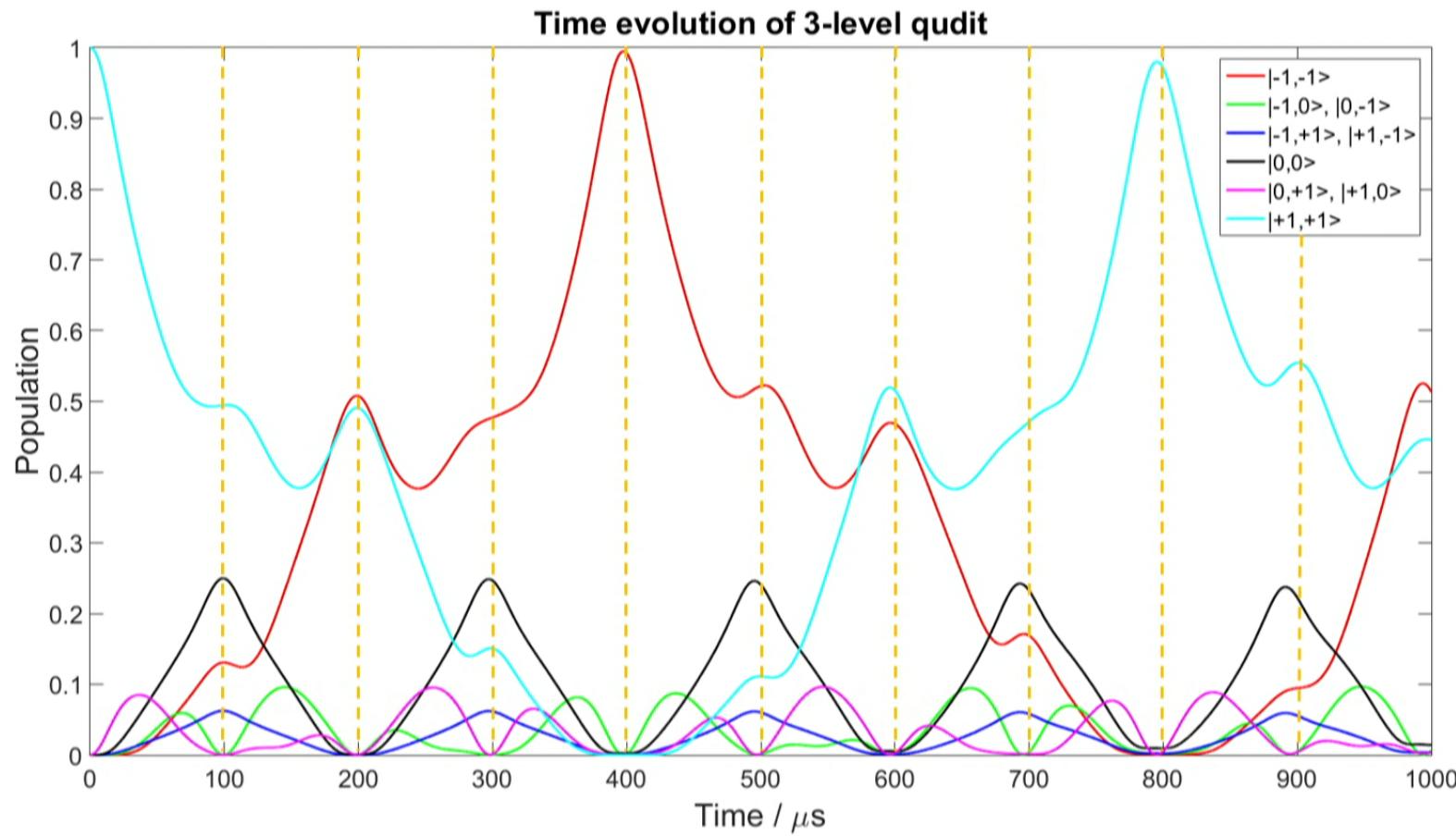
Geometric phase gates: work for larger spins too!

Tools we need
Qudit encoding (multi-level system) with acceptable coherence
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Measurement
Single-qudit gates
Two-qudit gates

$|m_1, m_2\rangle$ acquires phase proportional to $(m_1 + m_2)^2$

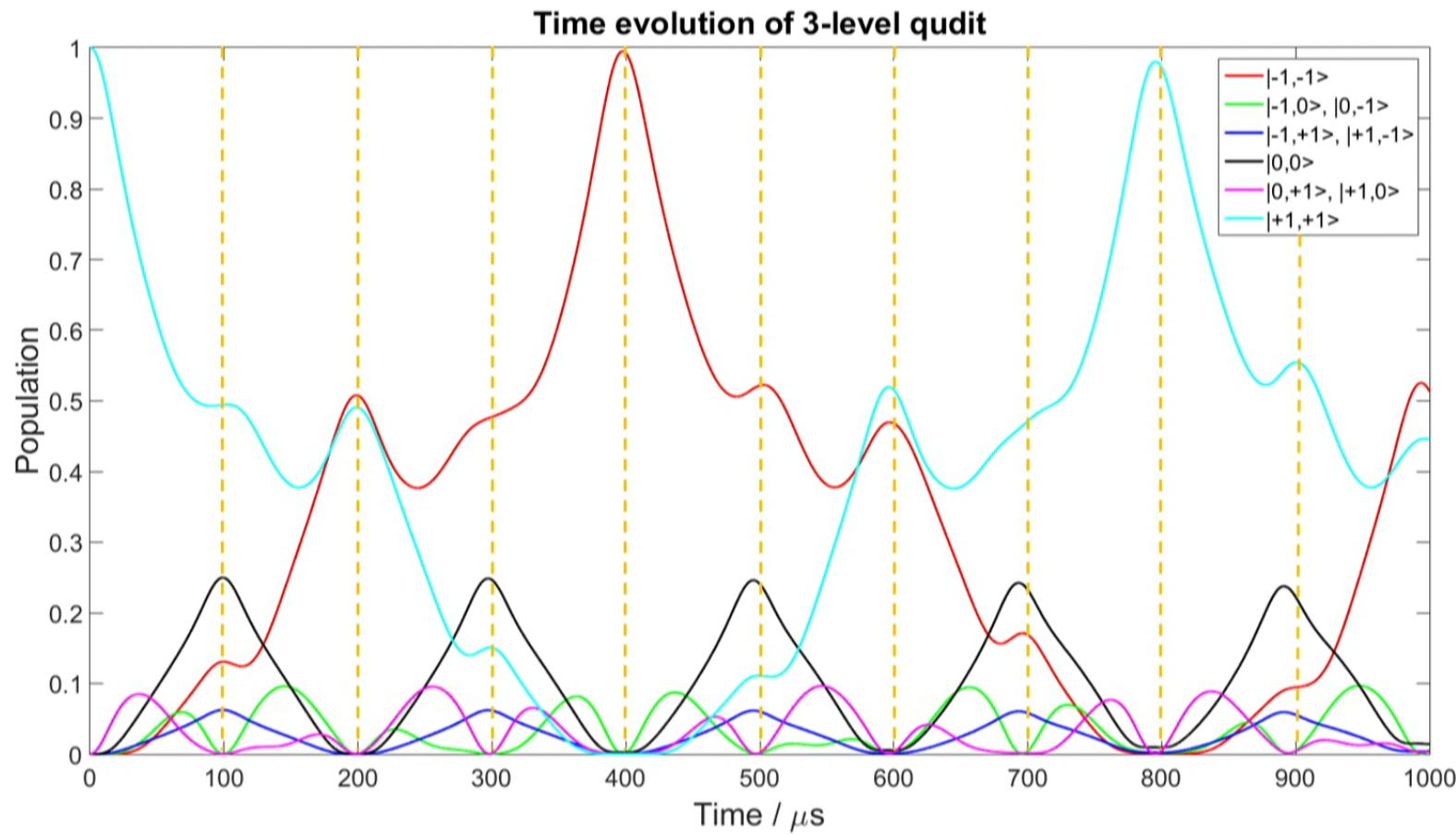


Visualizing errors in entangling gates?

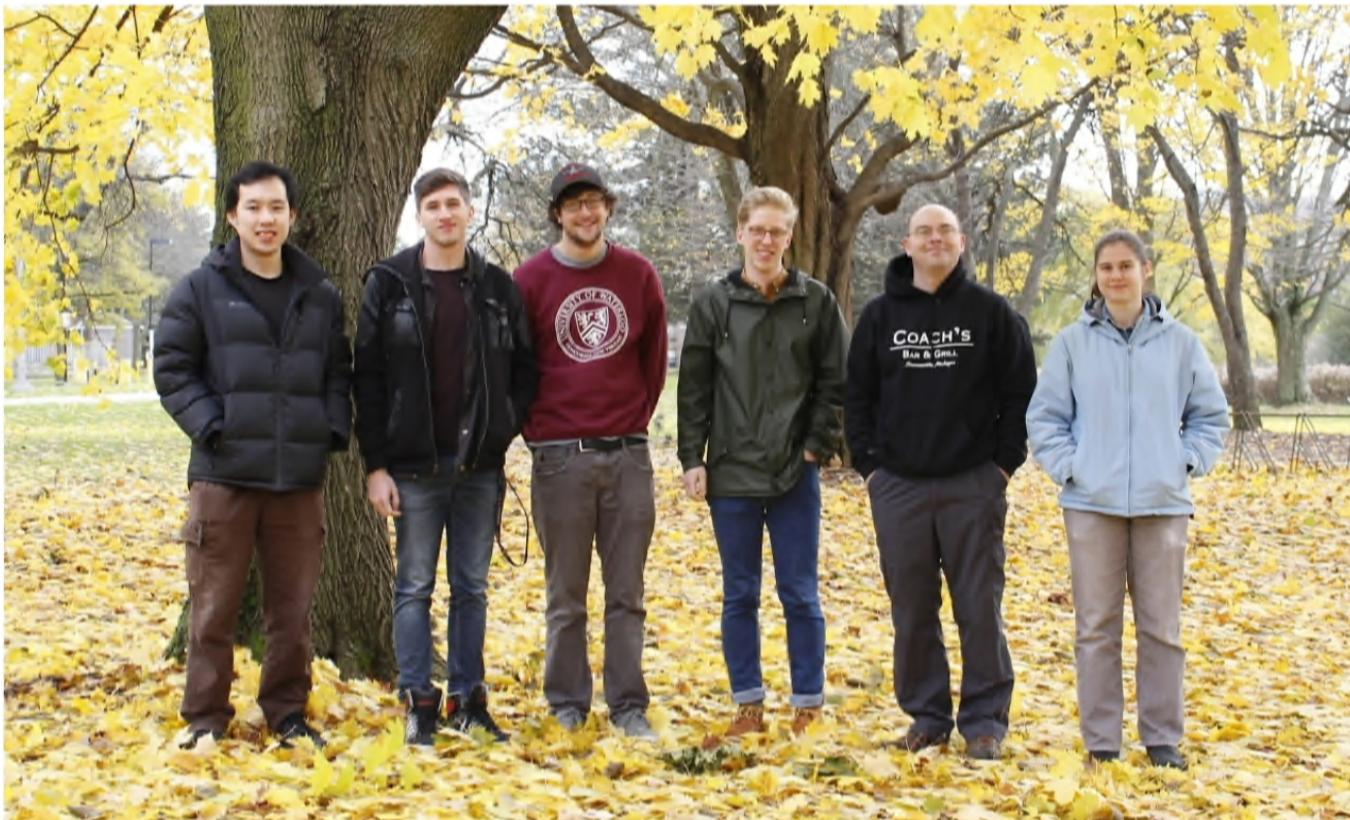


Yellow lines =
entangling gates =
integer number of
complete circles

Visualizing errors in entangling gates?



Yellow lines =
entangling gates =
integer number of
complete circles



From left: Pei Jiang Low, Brendan Bramman, Noah Greenberg,
Dr. Matt Day, Richard Rademacher

- Protocols for all needed qudit operations
- Fidelity limitations seem promising
- Experiments coming soon!





Thank you

Contact Crystal
Senko or Rajibul
Islam to learn more
about the IQC Ions
crew.



Transformative
Quantum
Technologies



UNIVERSITY OF
WATERLOO

IQC Institute for
Quantum
Computing

Detection of spin states: Only binary readout

