Title: Reproducibility despite exponential divergence in the Newtonian few-body problem

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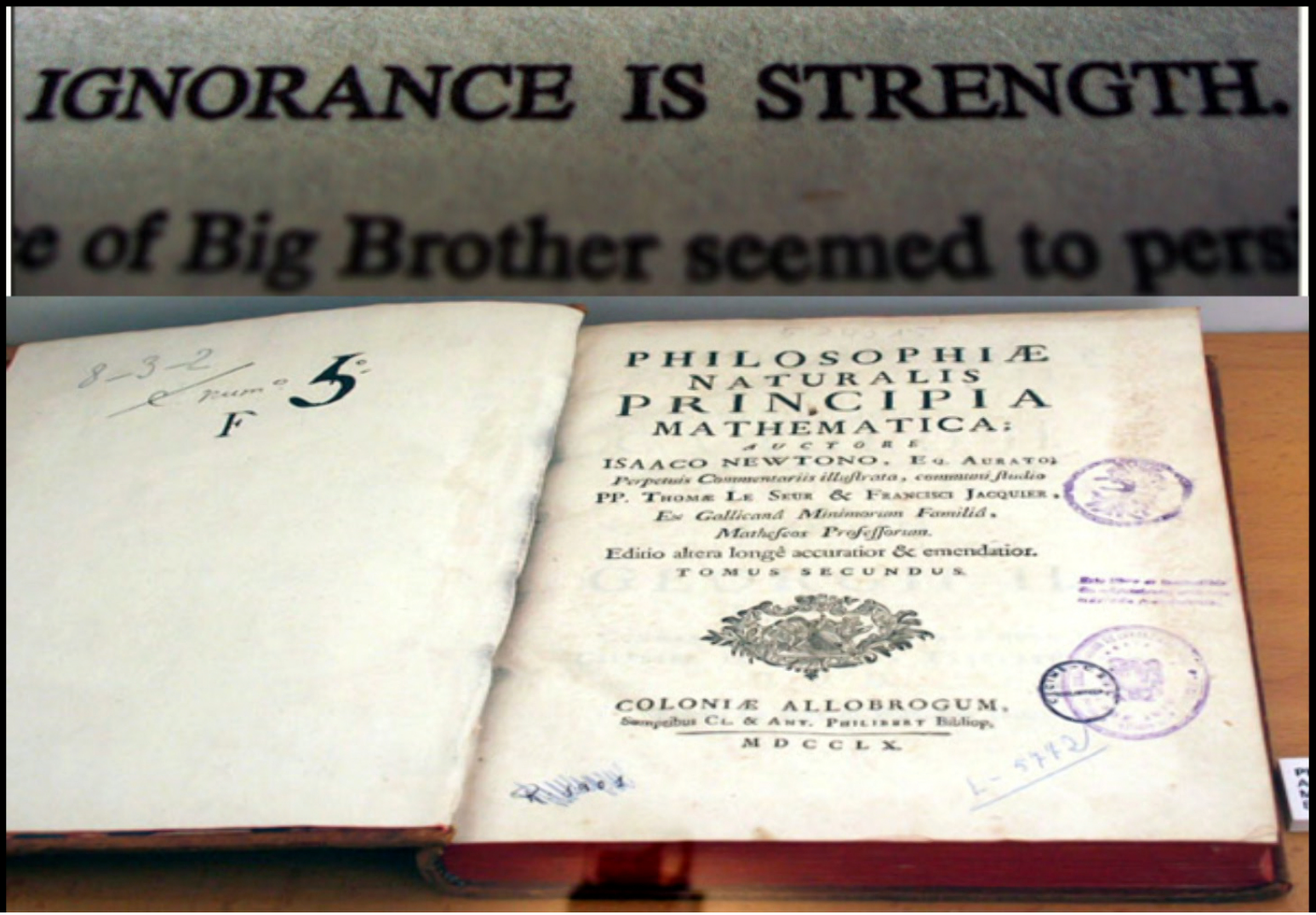
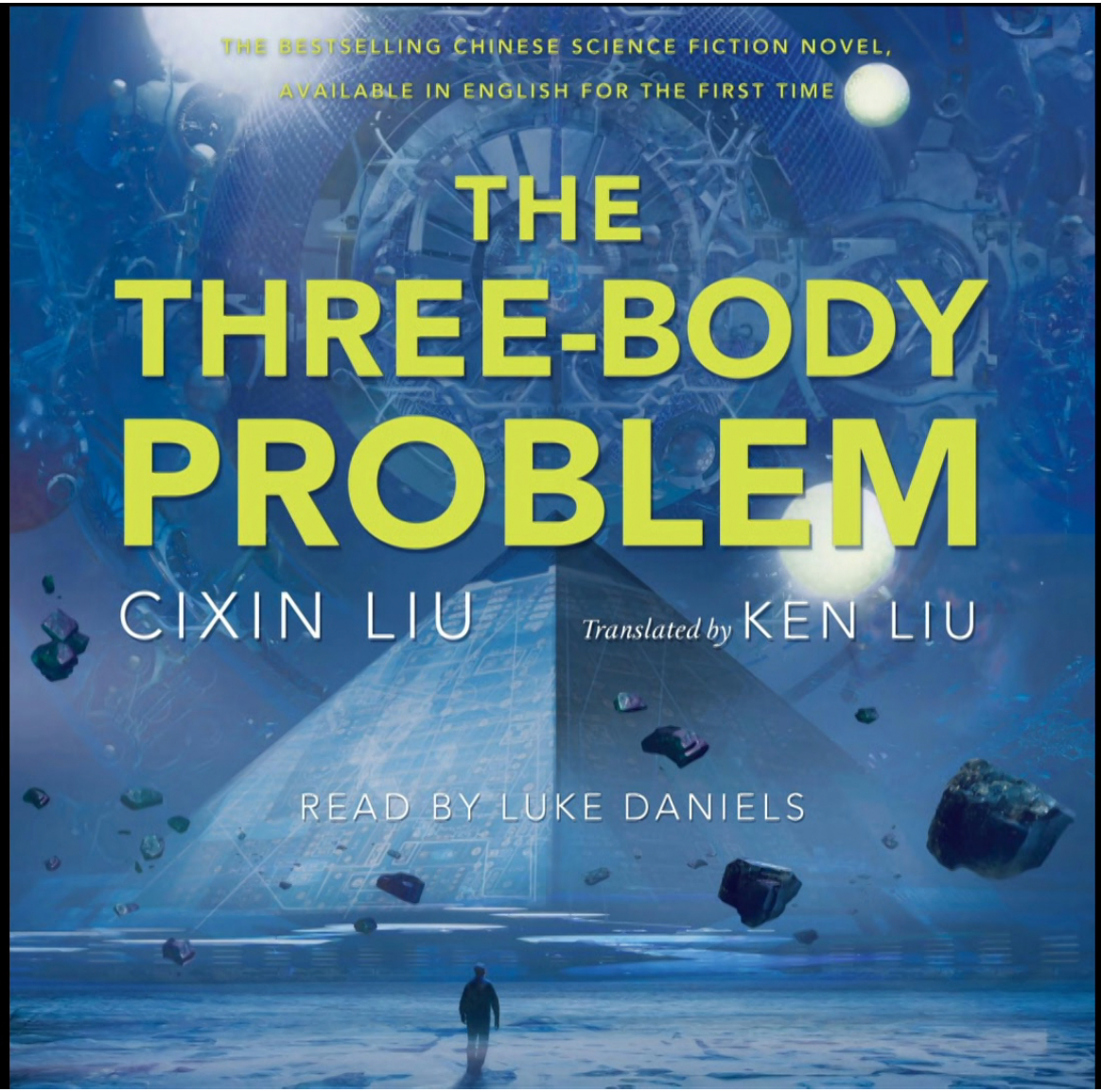
Abstract: Energy and momentum are conserved in Newton's laws of gravitation.
Numerical integration of the equations of motion should comply to
these requirements in order to guarantee the correctness of a
solution, but this turns out to be insufficient. The steady growth of
numerical errors and the exponential divergence, renders numerical
solutions over more than a dynamical time-scale meaningless. Even
time reversibility is not a guarantee for finding the definitive
solution to the numerical few-body problem. As a consequence,
numerical N-body simulations produce questionable results. Using
brute force integrations to arbitrary numerical precision I will
demonstrate empirically that the statistics of an ensemble of resonant
3-body interactions is independent of the precision of the numerical
integration, and conclude that, although individual solutions using
common integration methods are unreliable, an ensemble of approximate
3-body solutions accurately represent the ensemble of true solutions.


Reproducibility despite exponential divergence in the Newtonian few-body problem
Simon Portegies Zwart
Sterrewacht Leiden

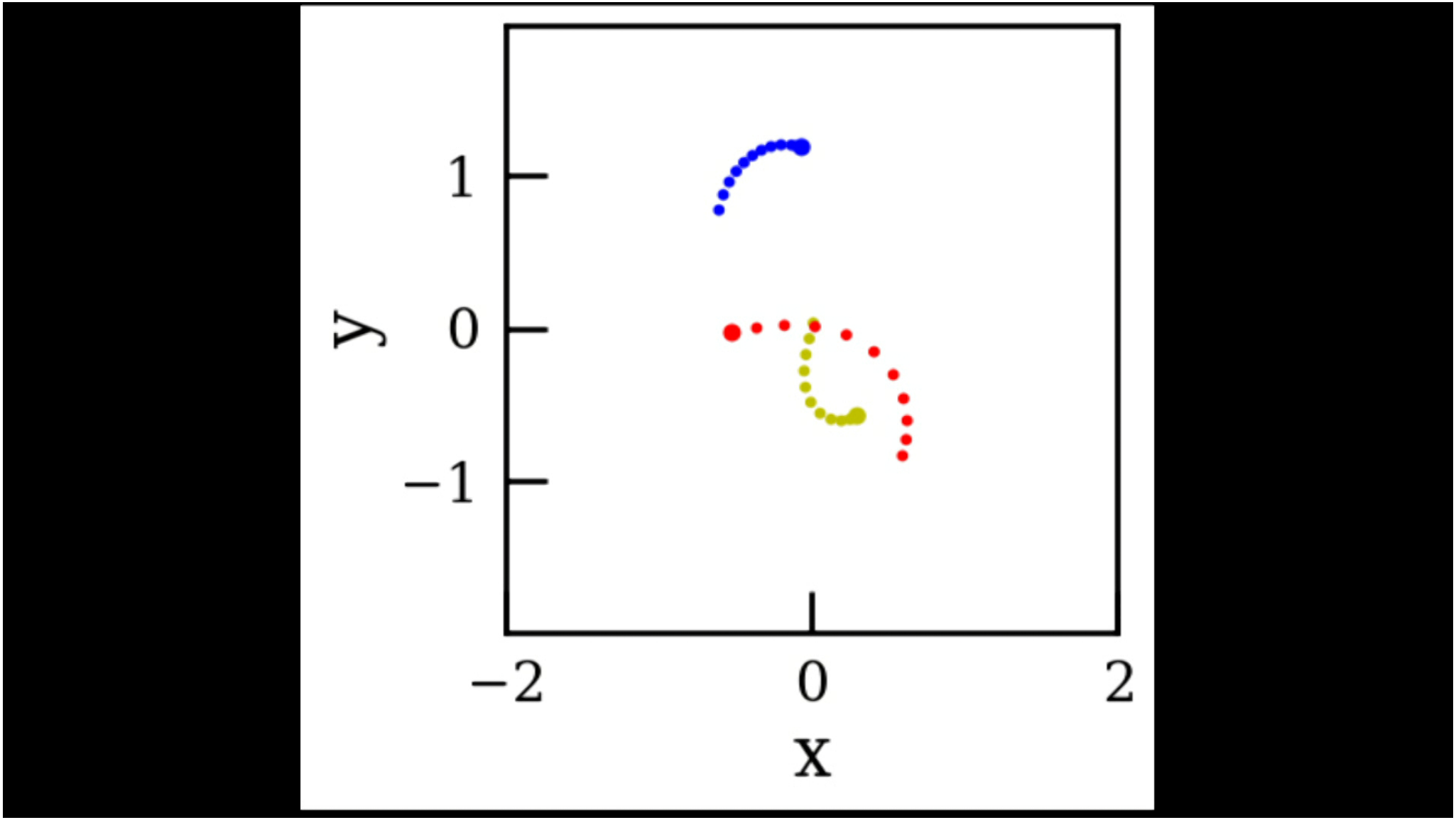

IGNORANCE IS STRENGTH.
e of Big Brother seemed to pers
8-3-2
pum.o 5.
f.
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA:
AUCTORE
ISAACO NEWTONO, Eq. Aurato;
Perpetuis Commentariis illustrata, communi studio
PP. Thomæ Le Seur & Francisci Jacquier,
Ex Gallicana Minimorum Familiâ,
Matheseos Professorum.
Editio altera longè accuratior & emendatior.
TOMUS SECUNDUS.
COLONIÆ ALLOBROGUM,
Sumptibus Cl. & Ant. Philibert Bibliop.
MDCCLX.


Jungle-verse
Size scale covers anything from:
13.8 billion light years to km-size
that covers 24 orders of magnitude
13.8 billion years to seconds
that covers 17 orders of magnitude


THE BESTSELLING CHINESE SCIENCE FICTION NOVEL,
AVAILABLE IN ENGLISH FOR THE FIRST TIME
THE
THREE-BODY
PROBLEM
CIXIN LIU
Translated by KEN LIU
READ BY LUKE DANIELS

Little Green Machine
four pizzaboxes with super powers

Volkskrant 5 April 2017


1
0
−1
y
−2
0
2
x

"*Errors in calculations of n-body systems grow exponentially … and may therefore invalidate the results …*" (Miller 1964)

BRUTUS

a brute force arbitrary-precision N-body code

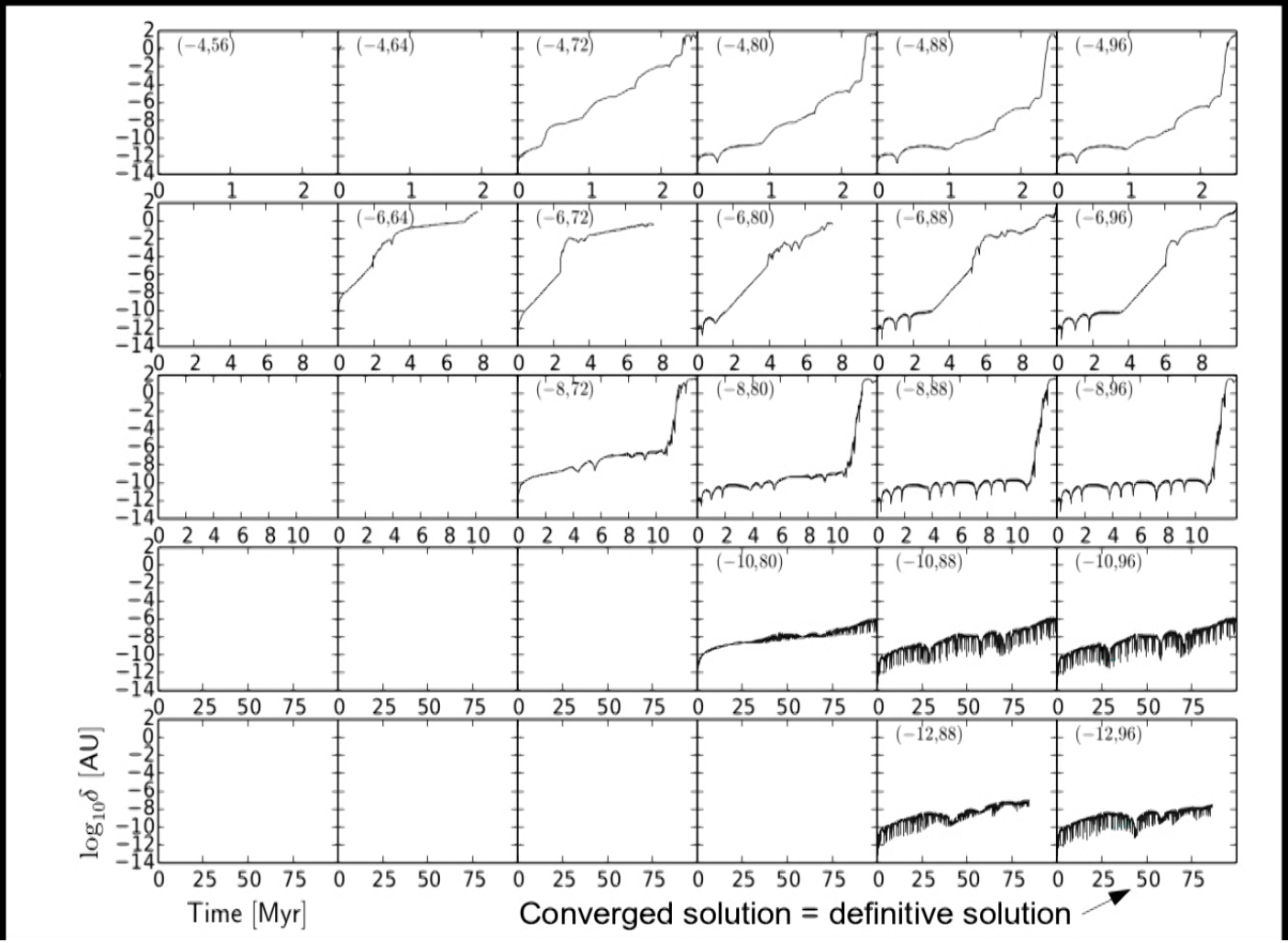
- Two ingredients:
  - Gragg-Bulirsch-Stoer method
    - Modified midpoint method
    - Richardson extrapolation
    - Tolerance parameter
  - Arbitrary-Precision arithmetic
    - Number of significant digits

Tjarda Boekholt 2015

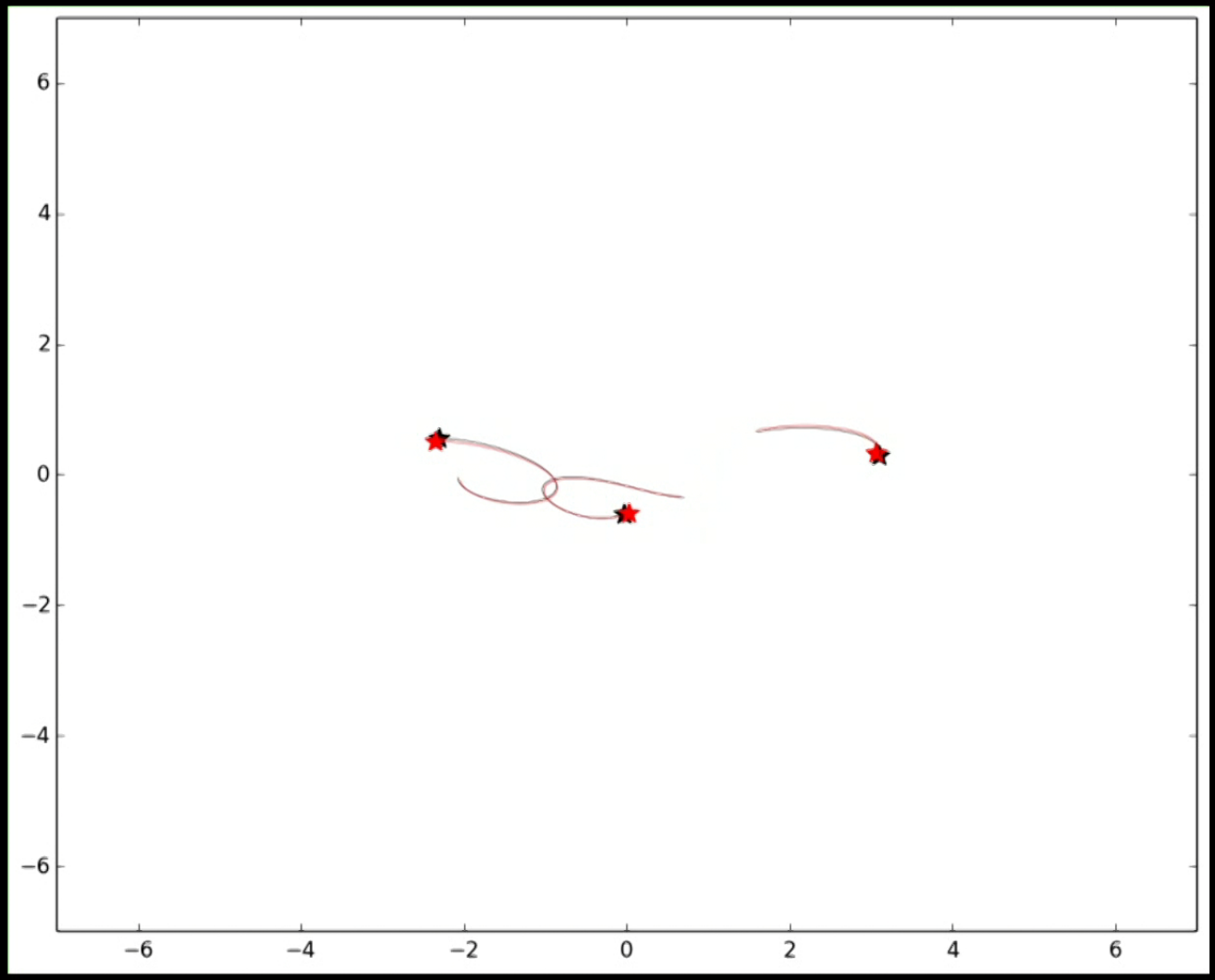

Lucie Filion

Precision

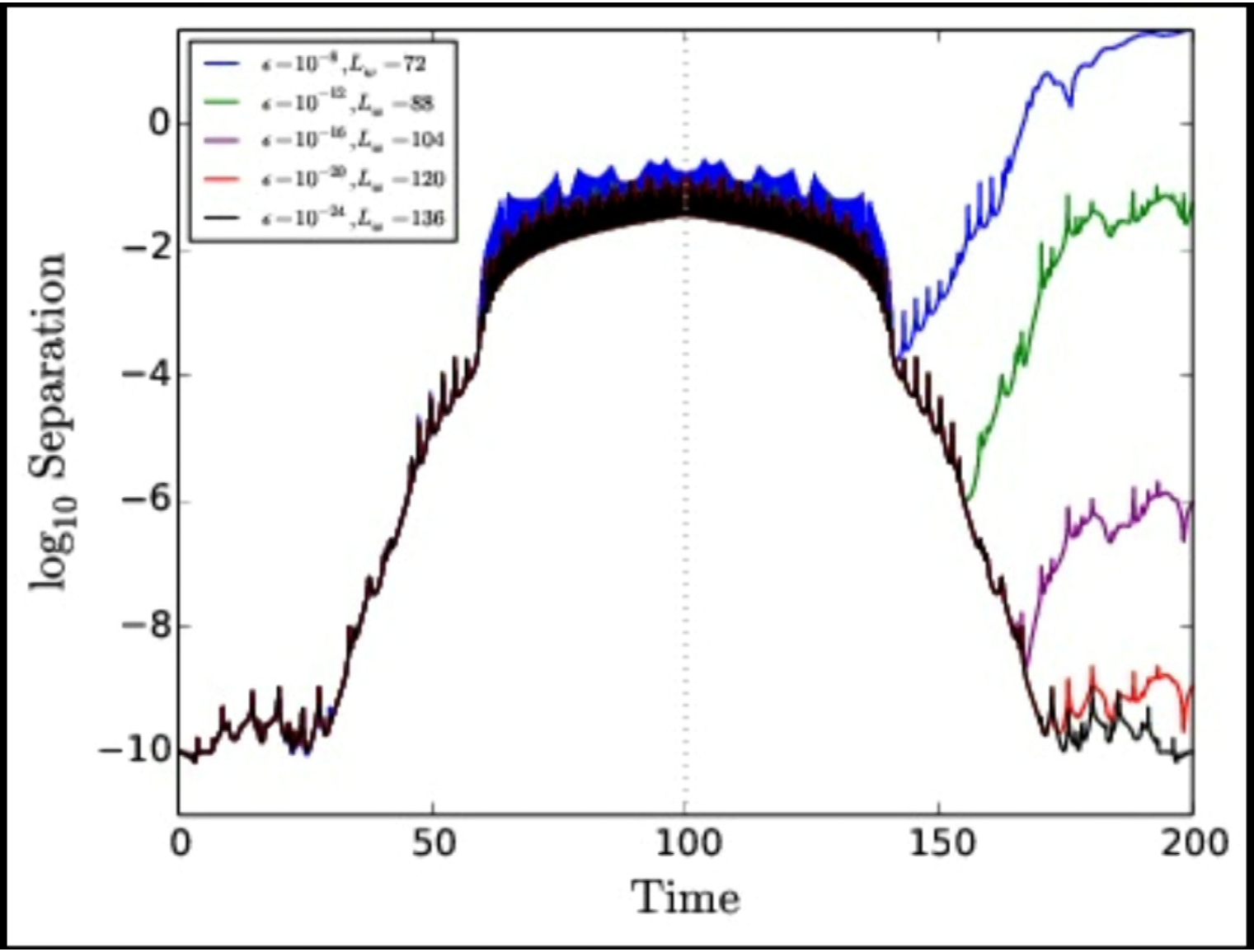
Accuracy



black: dE/E $<10^{-74}$ Red: dE/E $<10^{-11}$

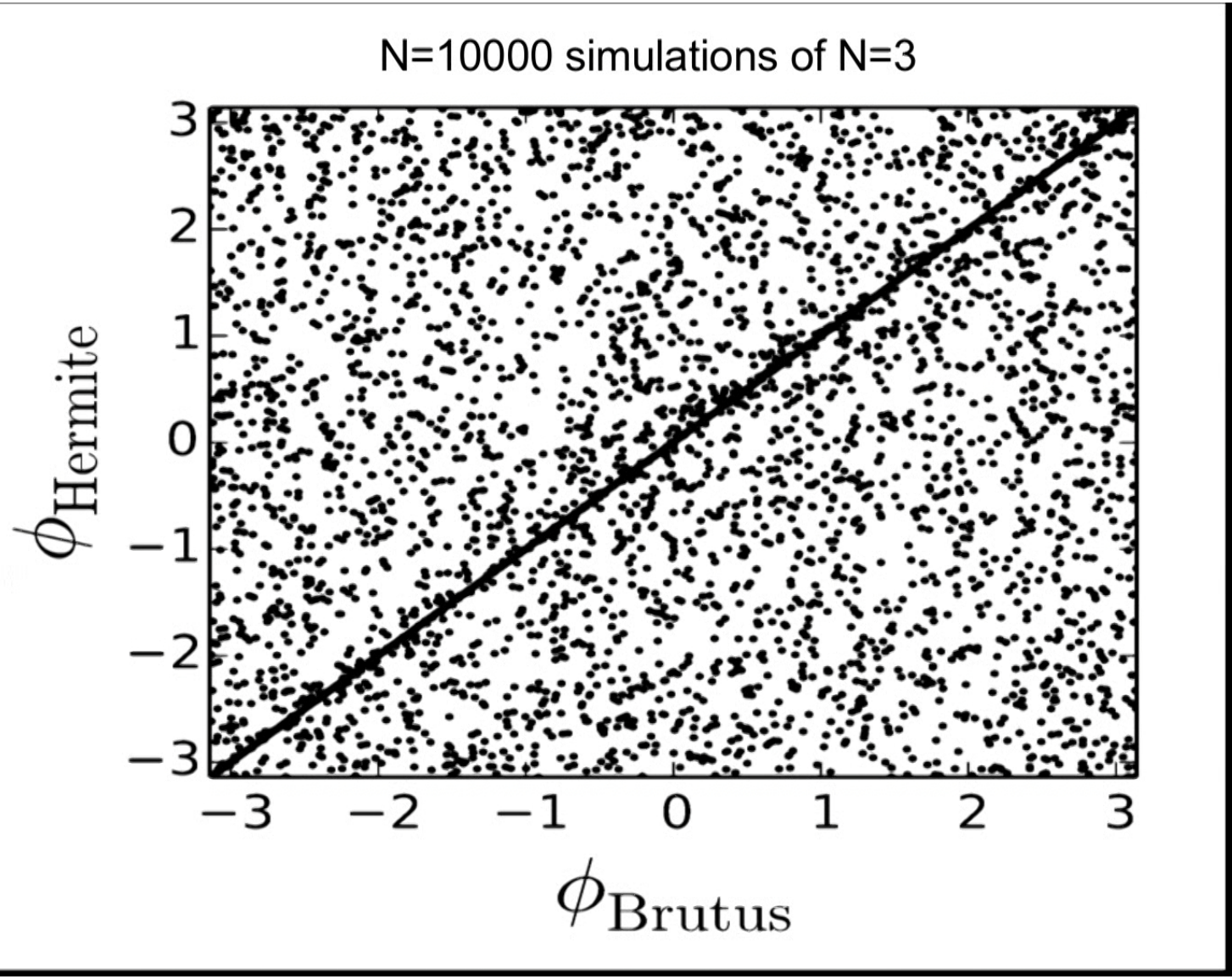


• **Time reversibility:** The ability of a numerical integrator to recover the initial realization from reversing the final realization. Time reversibility does not guarantee that the solution is converged as it can already be obtained by insisting time symmetry in numerical errors without resolving close encounters.

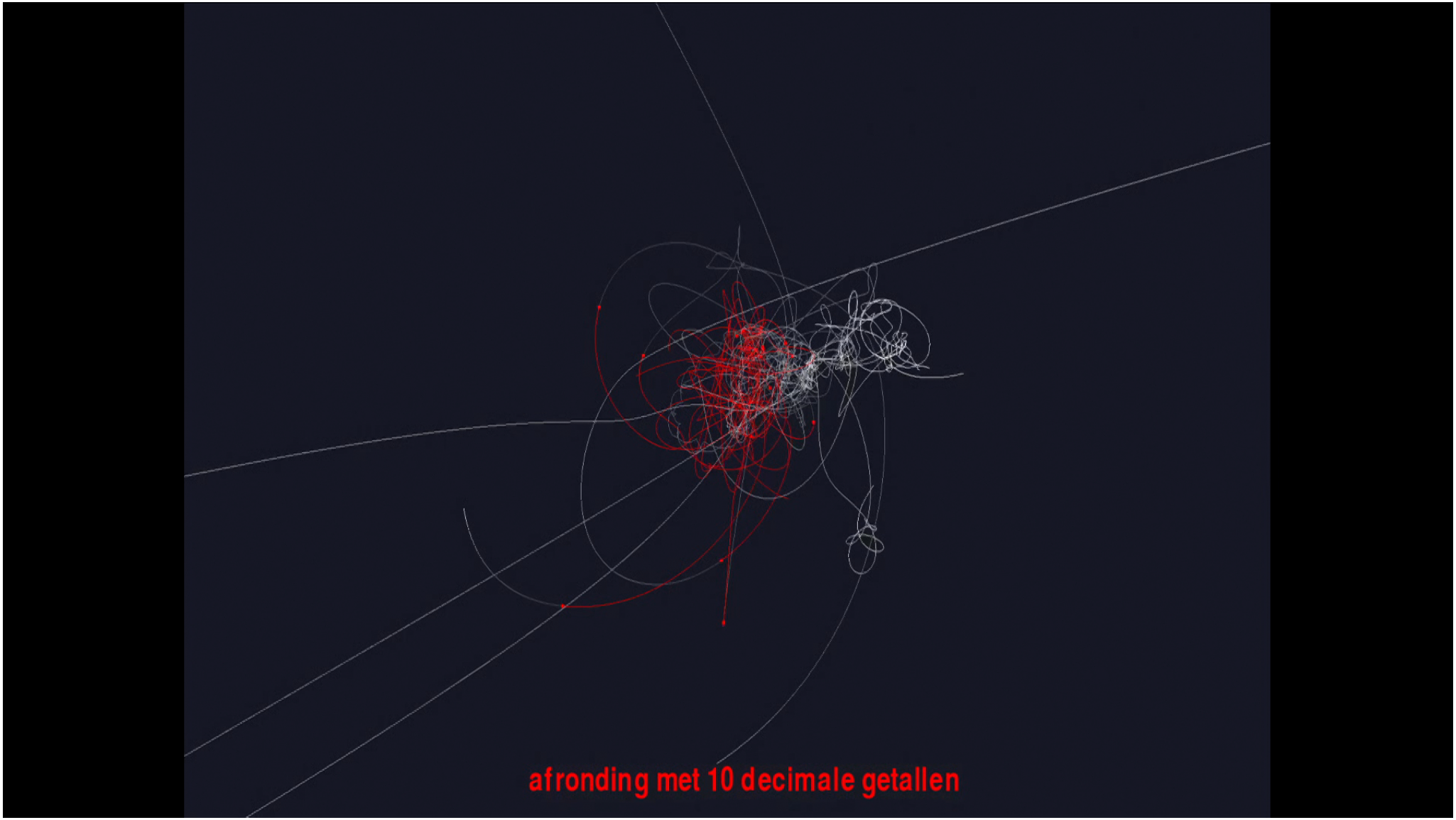

$\epsilon = 10^{-8}, L_w = 72$
$\epsilon = 10^{-12}, L_w = 88$
$\epsilon = 10^{-16}, L_w = 104$
$\epsilon = 10^{-20}, L_w = 120$
$\epsilon = 10^{-24}, L_w = 136$
$\log_{10}$ Separation
0
−2
−4
−6
−8
−10
0
50
100
150
200
Time

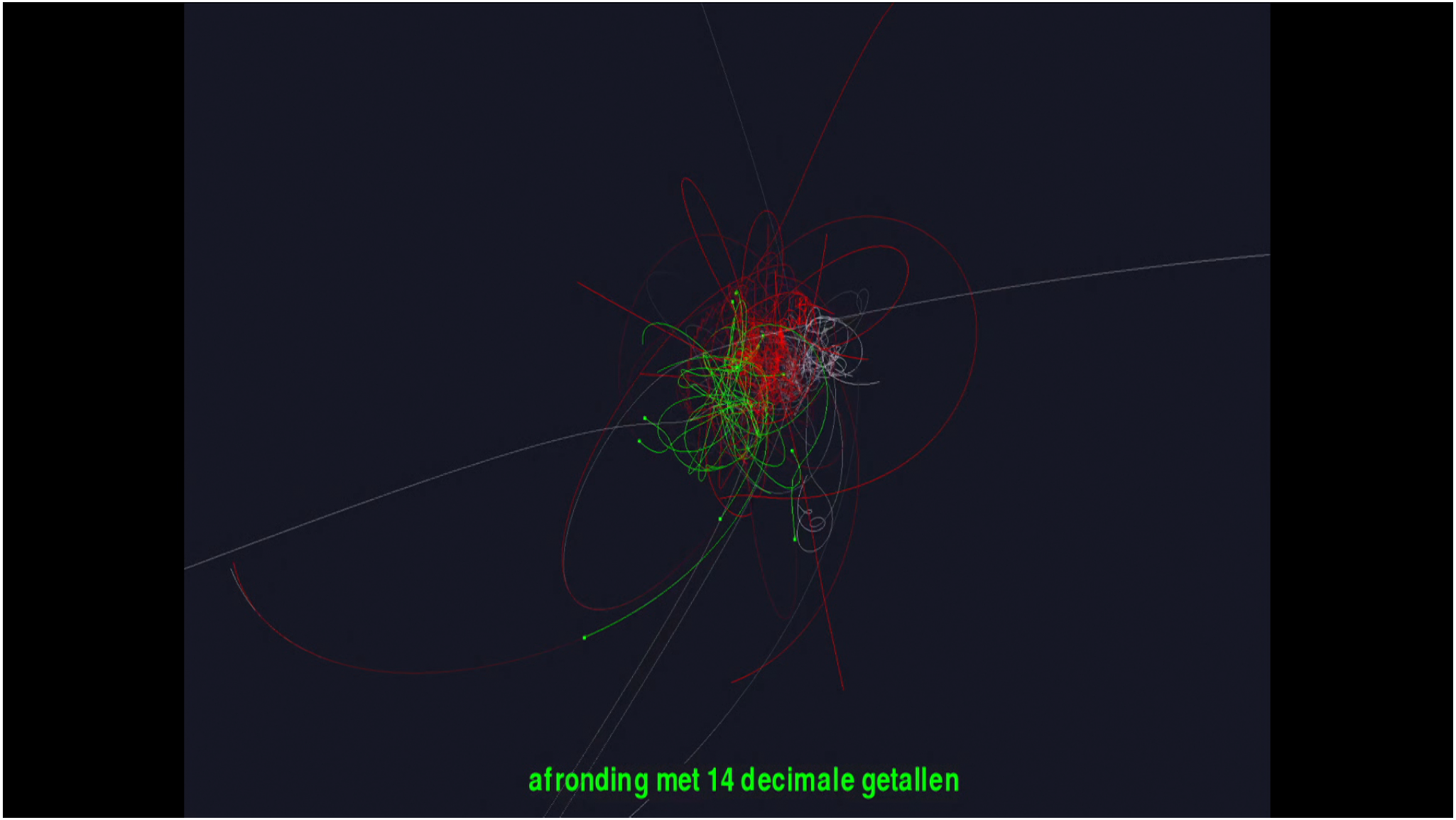
•**Reprehensible**: Solution to Newton's equations of motion for which the accumulation of numerical errors and the system's response exceeds the exponential growth of the initial offset.

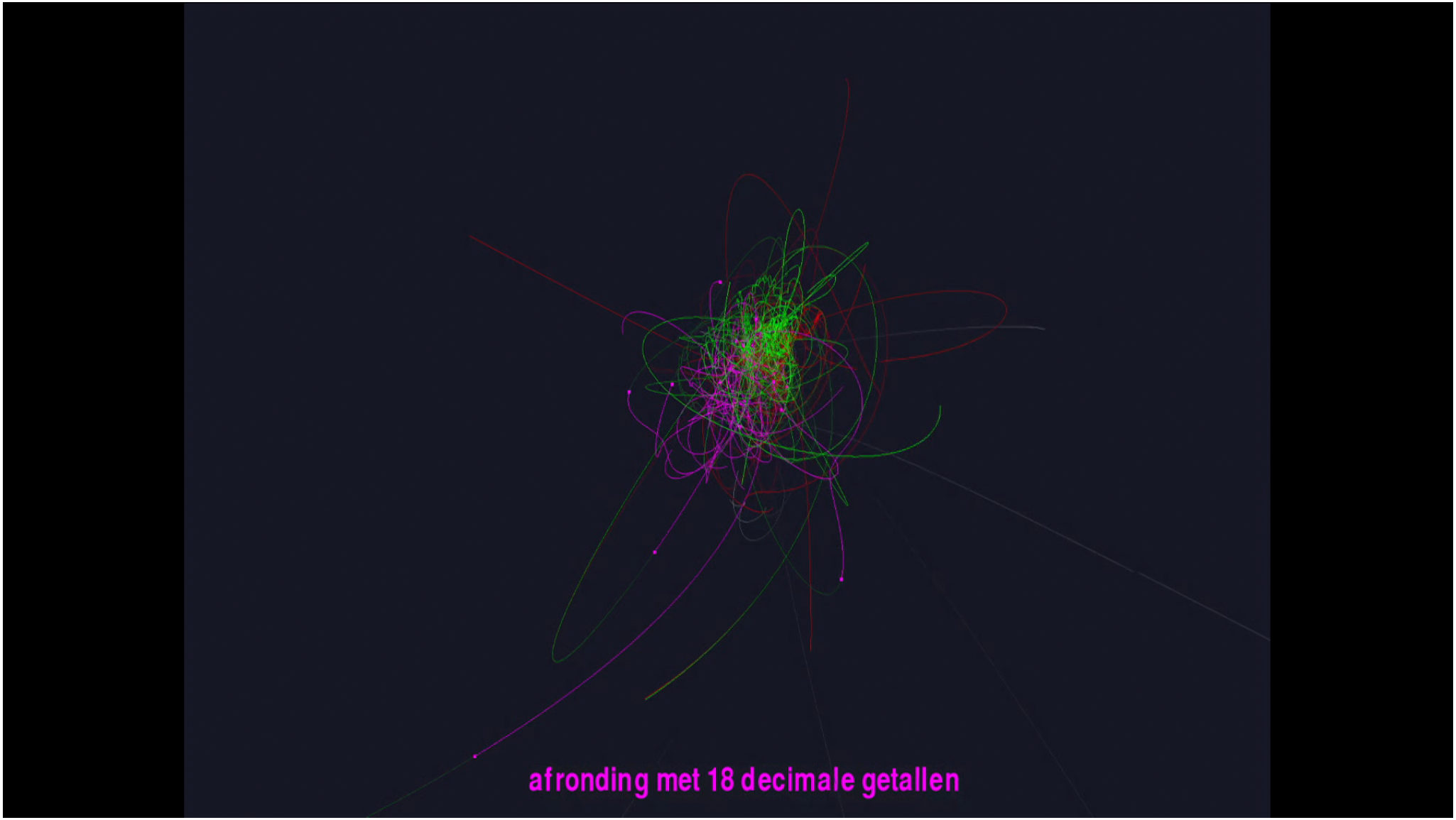
- **Convergence**
- **Definitive solution**
- **nag Hoch**: The concept that an ensemble of random initial realizations in a wide range of parameters gives statistically the same result as the converged solutions of the same ensemble realizations. This concept is a quality of the numerical method, but we speculate that this quality also applies to real systems.

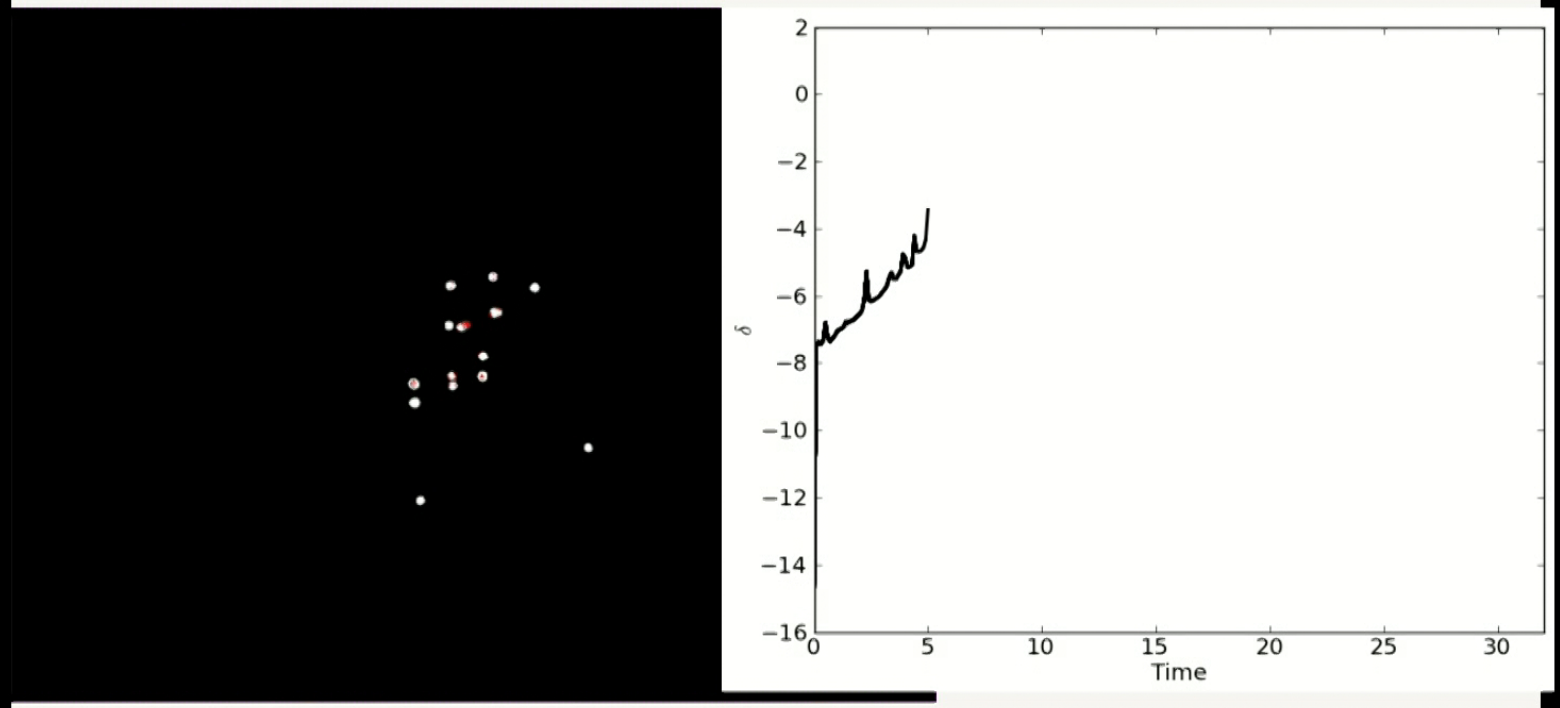

N=10000 simulations of N=3
ϕ_{Hermite}
ϕ_{Brutus}
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

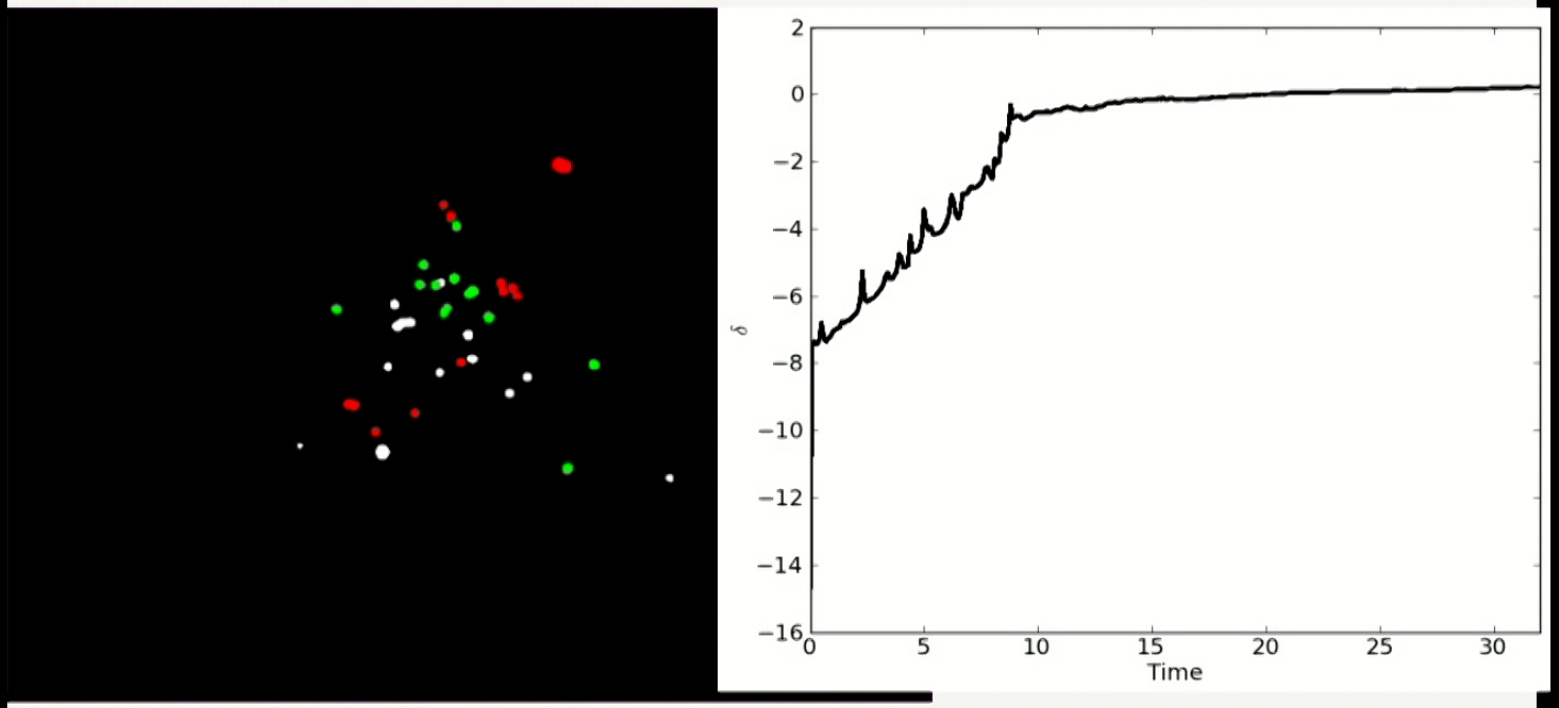

afronding met 6 decimale getallen


afronding met 10 decimale getallen


afronding met 14 decimale getallen

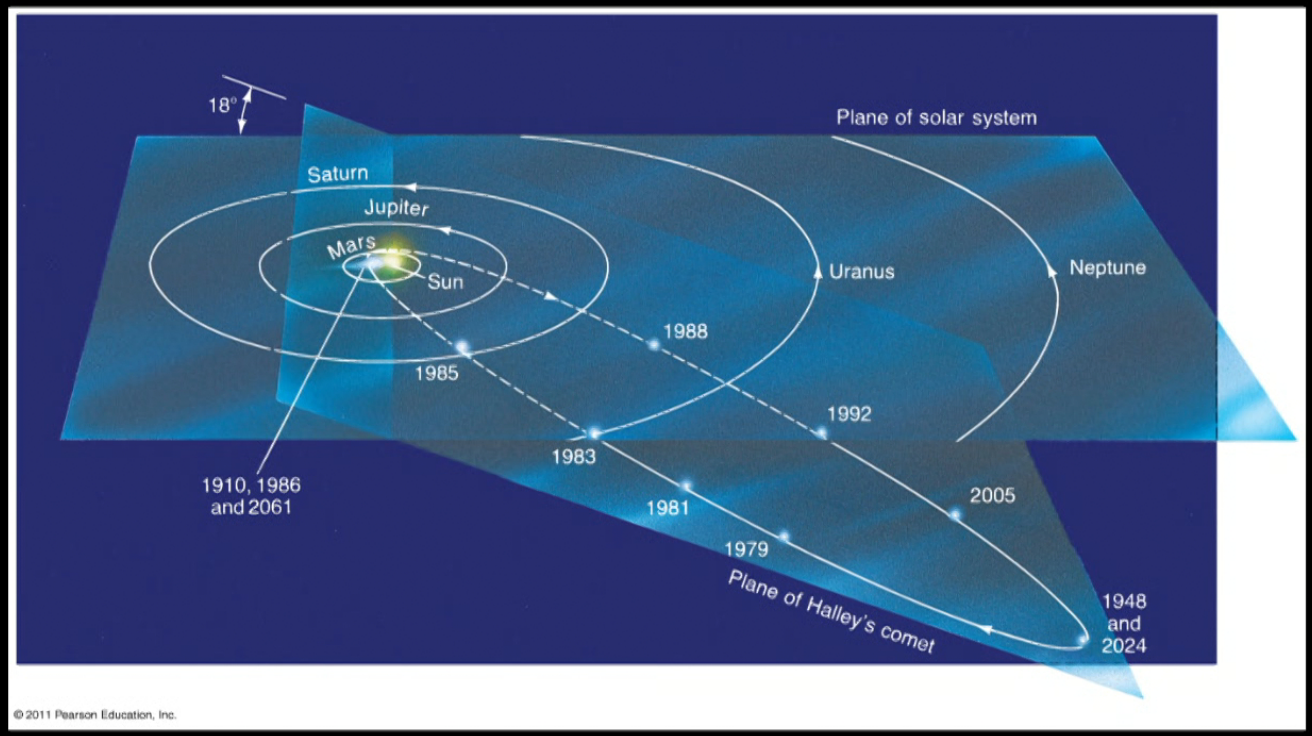

afronding met 18 decimale getallen


2
0
−2
−4
−6
−8
−10
−12
−14
−16
δ
0
5
10
15
20
25
30
Time


2
0
−2
−4
−6
−8
−10
−12
−14
−16
δ
0
5
10
15
20
25
30
Time

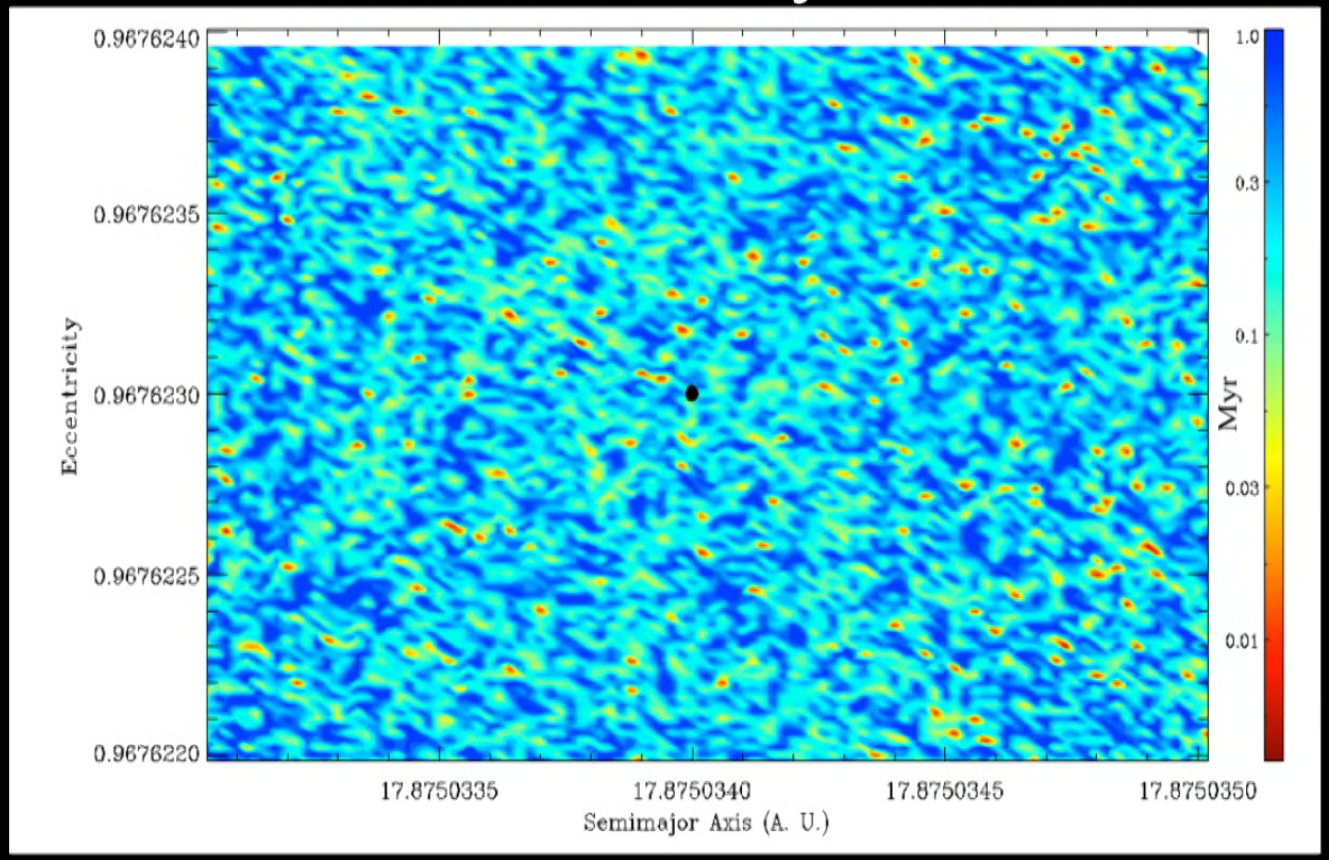
Halley's comet

Orbit of comet Halley



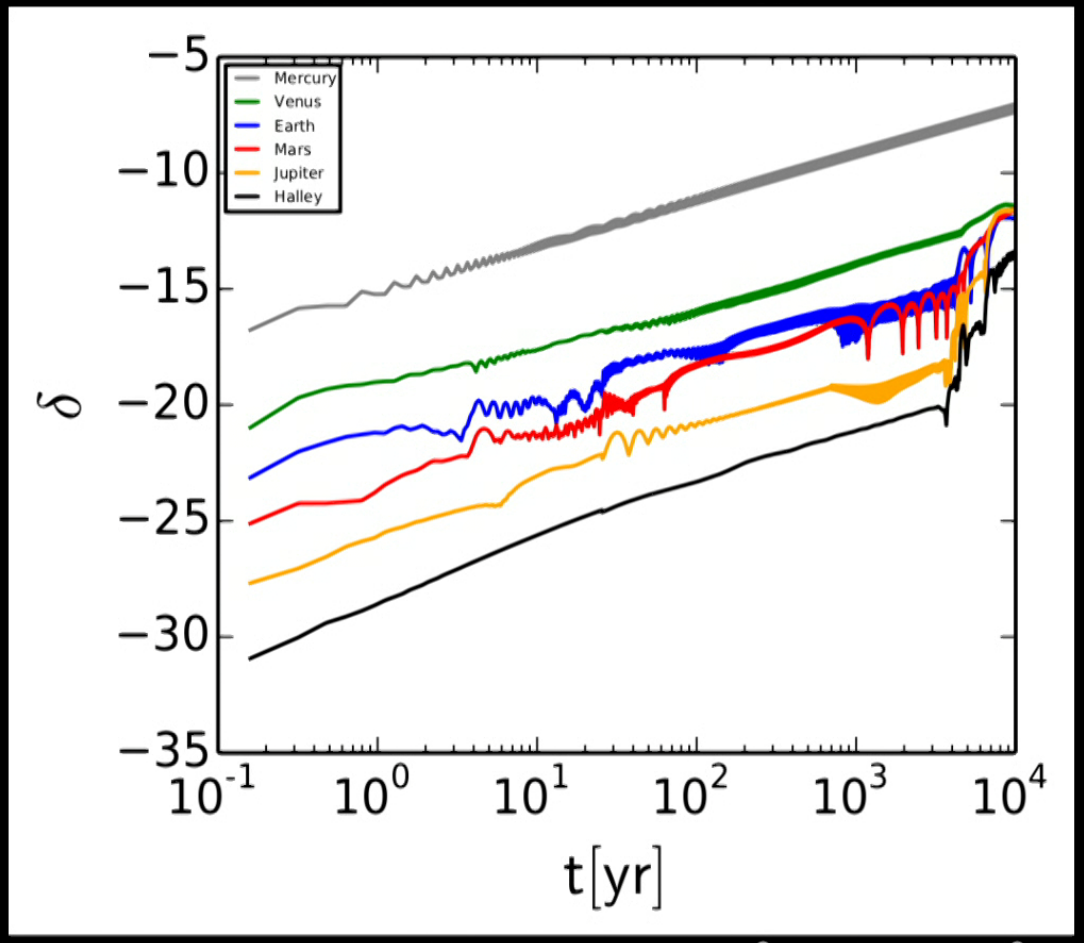
•**veracious**: the concept that a limited and bounded variation of the initial conditions near a pre-selected initial realization gives statistically an indistinguishable ensemble average as a single *converged* solution. A veracious solution can be perceived as the experimental observation of the true solution. The presumption that Newtonian simulations are *veracious* is used for example in stability studies of the Solar System.

Chaos in Halley's comet

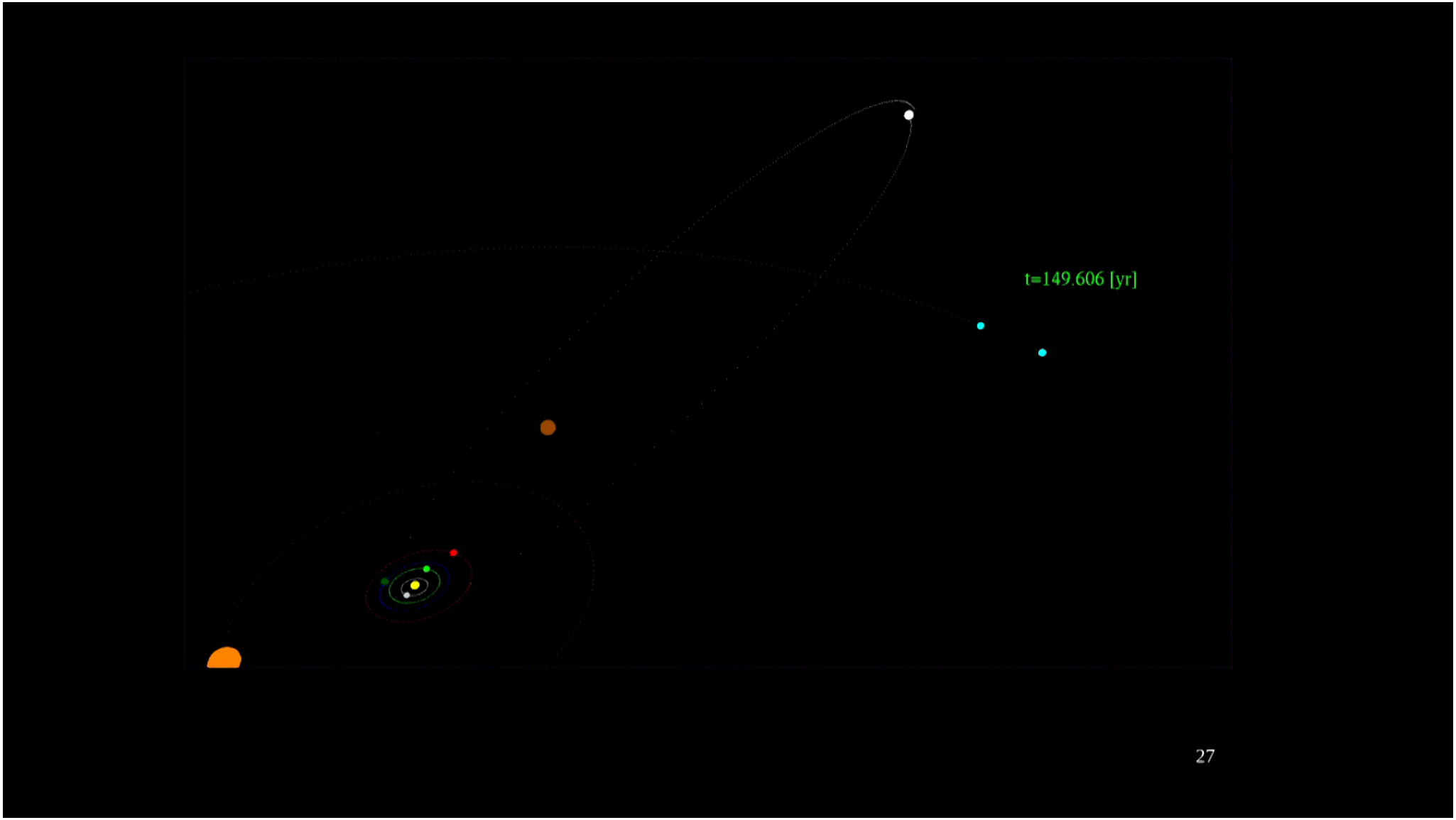


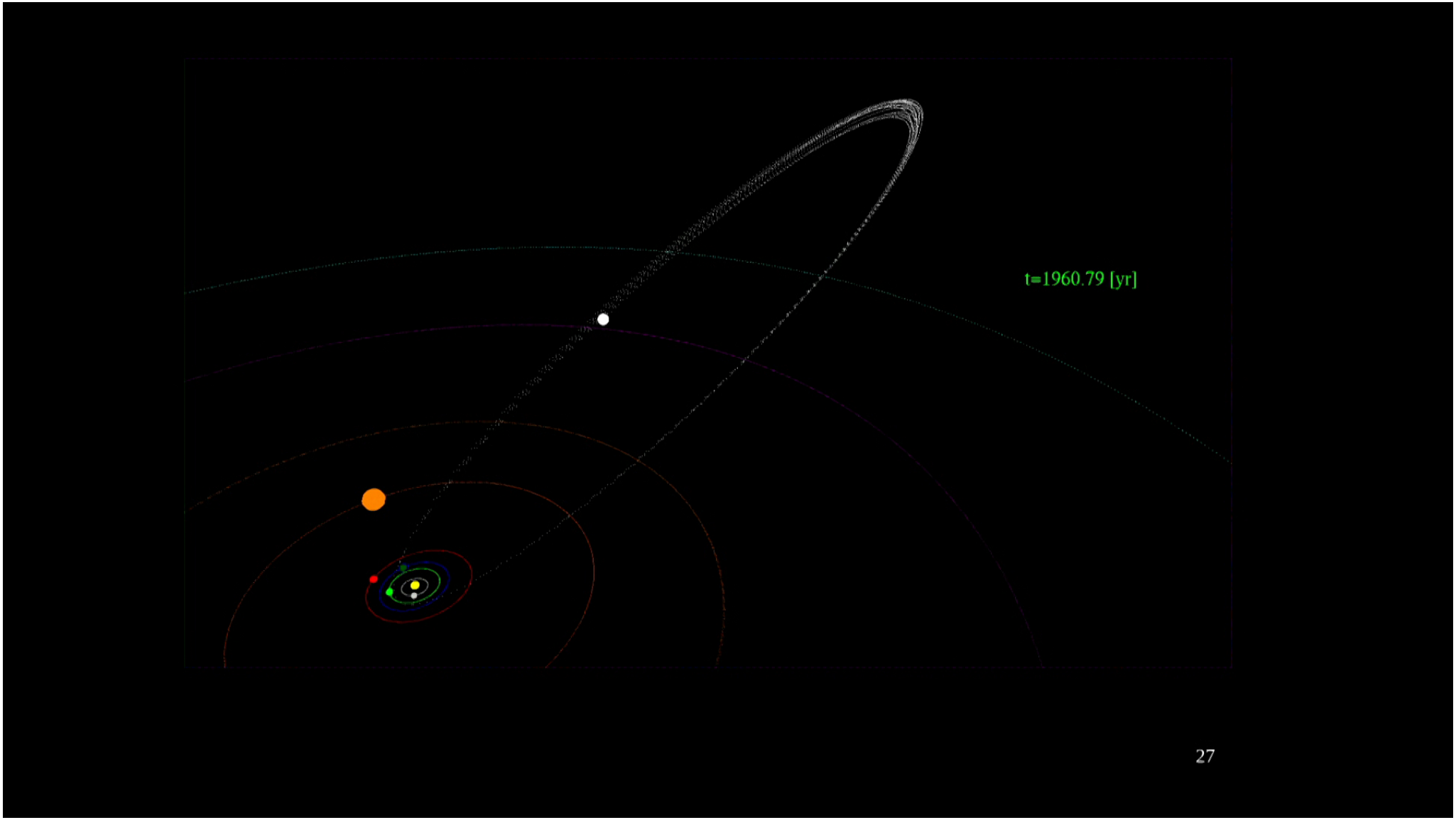
Munoz-Gutierrez etal 2014

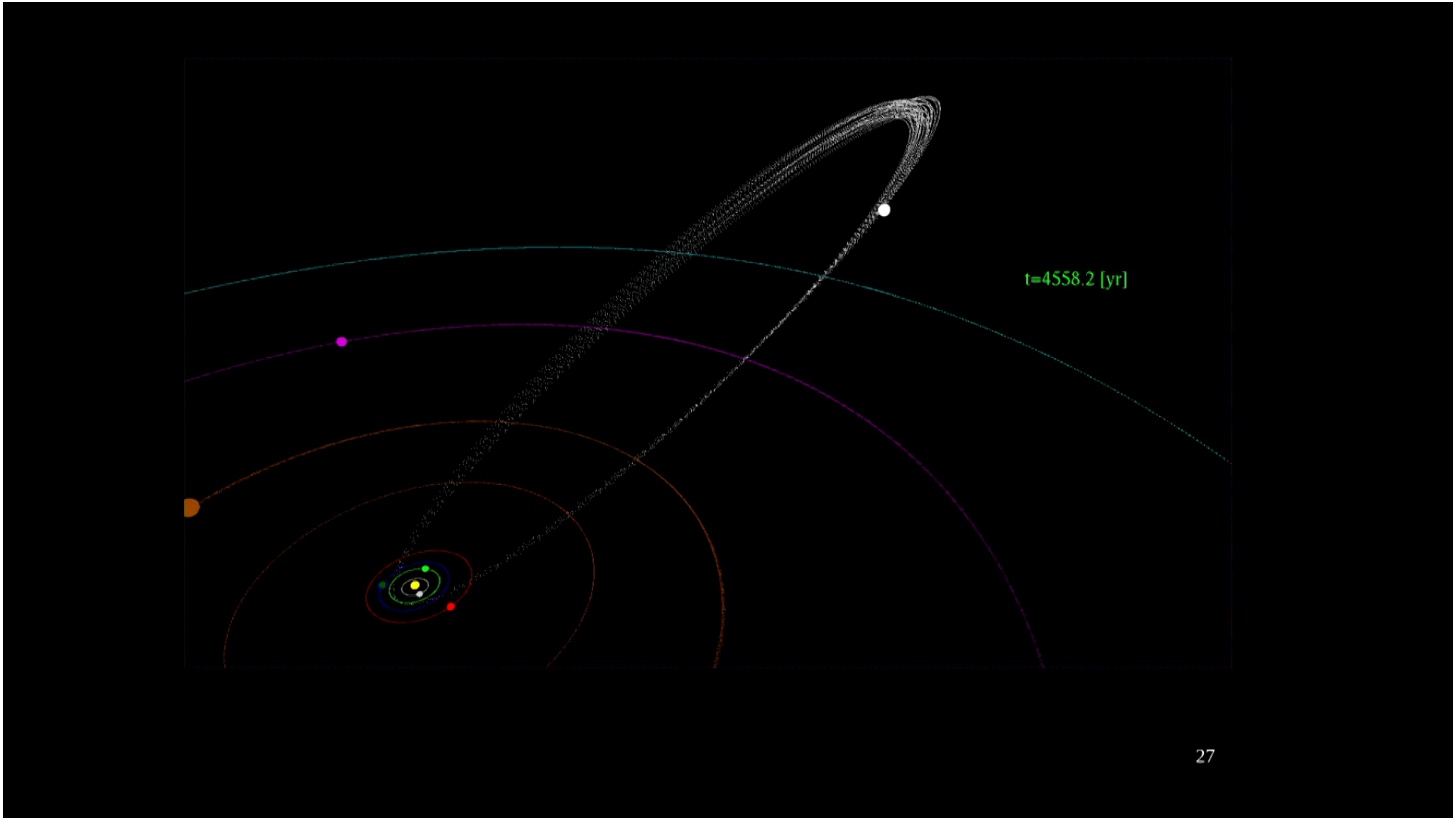
Growth of the error



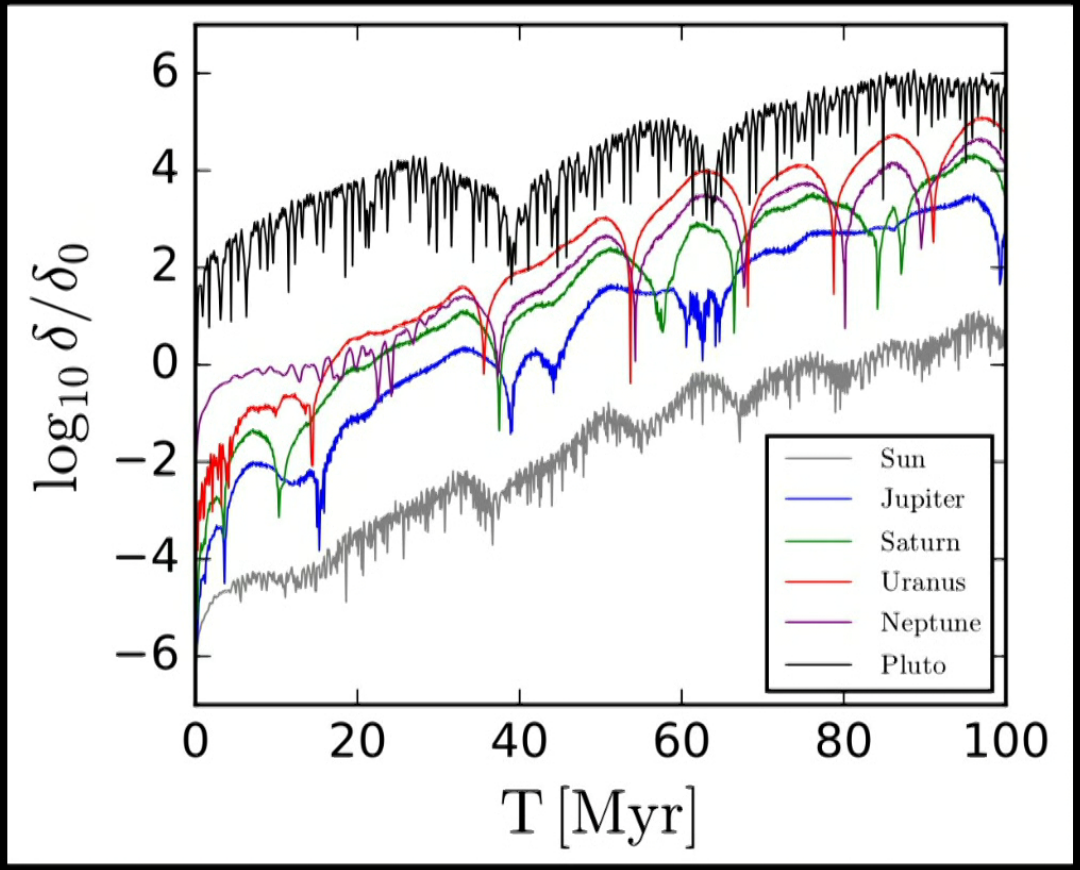
$\delta = 0.5 \log_{10} 1/(6N) \sum (x_2-x_1)^2 + (v_2-v_1)^2$

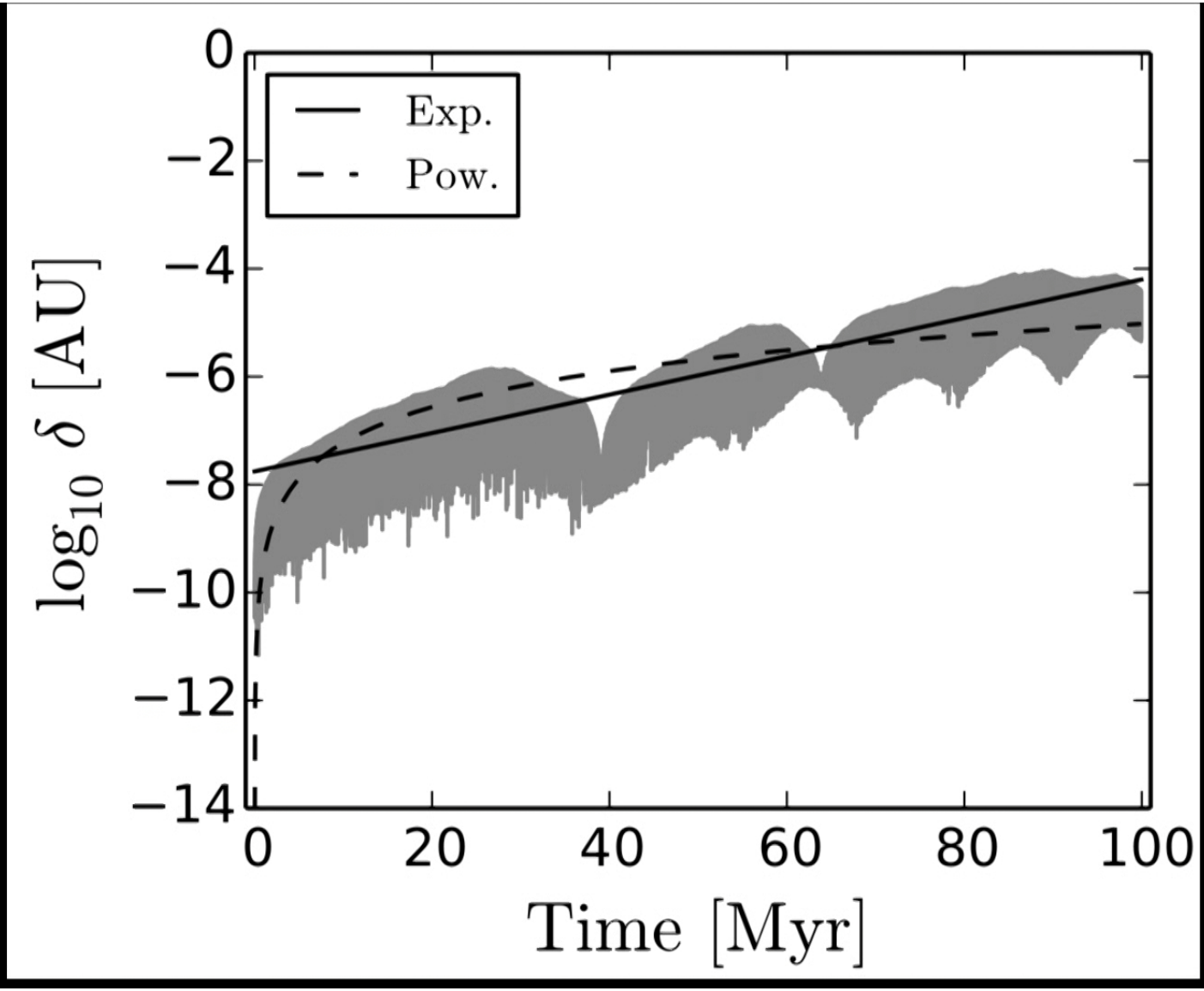

t=149.606 [yr]
27

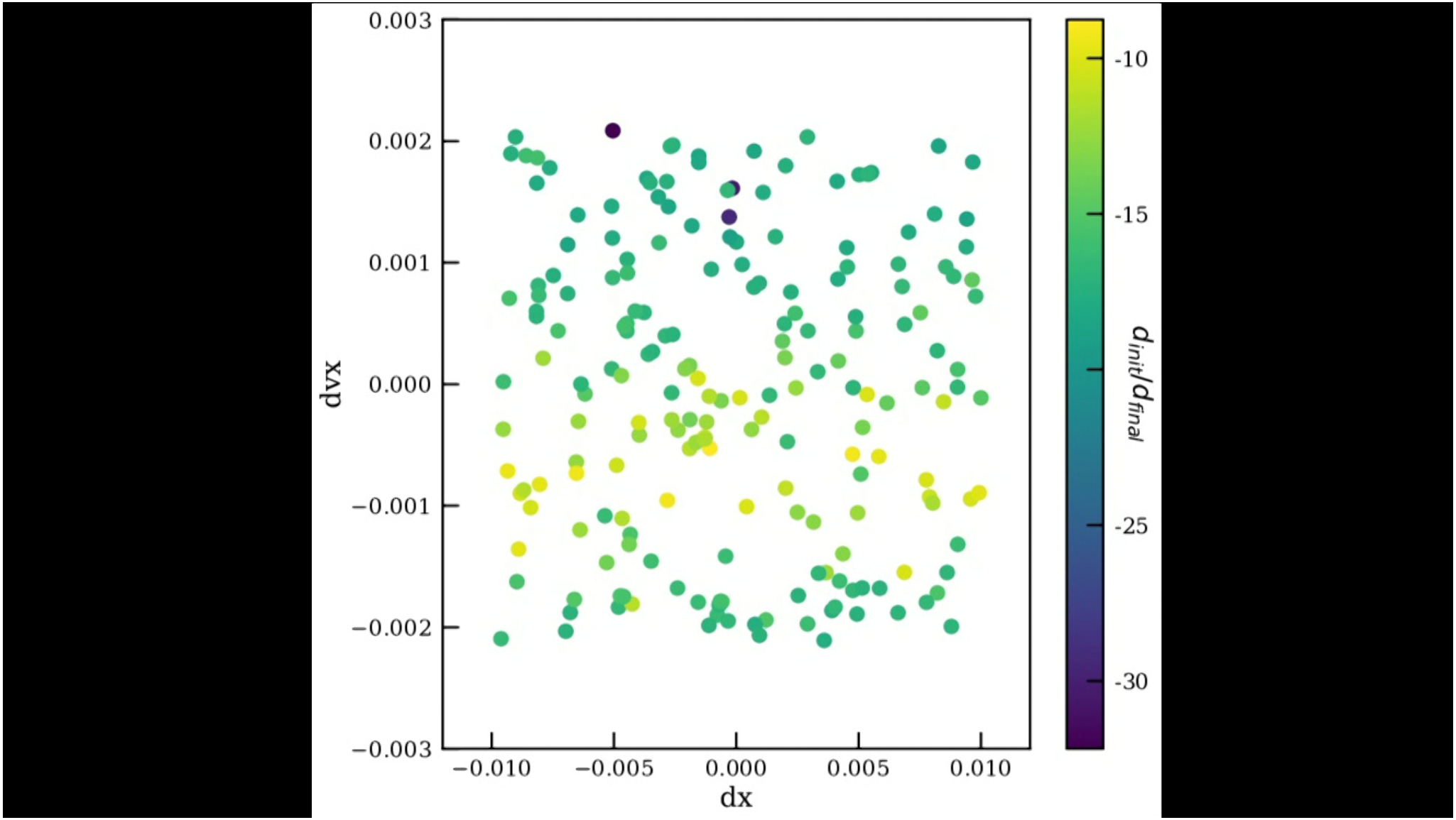

t=1960.79 [yr]
27


t=4558.2 [yr]
27

The exponential divergence for the Solar System (WI=256, $\varepsilon=10^{-24}$)




Exp.
Pow.
$\log_{10}\delta$ [AU]
0
−2
−4
−6
−8
−10
−12
−14
0
20
40
60
80
100
Time [Myr]


0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
dvx
−0.010
−0.005
0.000
0.005
0.010
dx
-10
-15
-25
-30
d_{init}/d_{final}

Messages

• Chaos (*hypersensitivity to initial conditions*) does not prevents accurate & precise calculations. But they require major resources and patience.

•A statistical ensamble of simulations seems to give the correct phase-space characteristics of the physical system.

• Variations in the orbit of the comet Halley are currently driven by Venus, but about 3000 years from now Jupiter will become the dominant perturbing body.