Title: Renormalization and Effective Field Theory - Lecture 10

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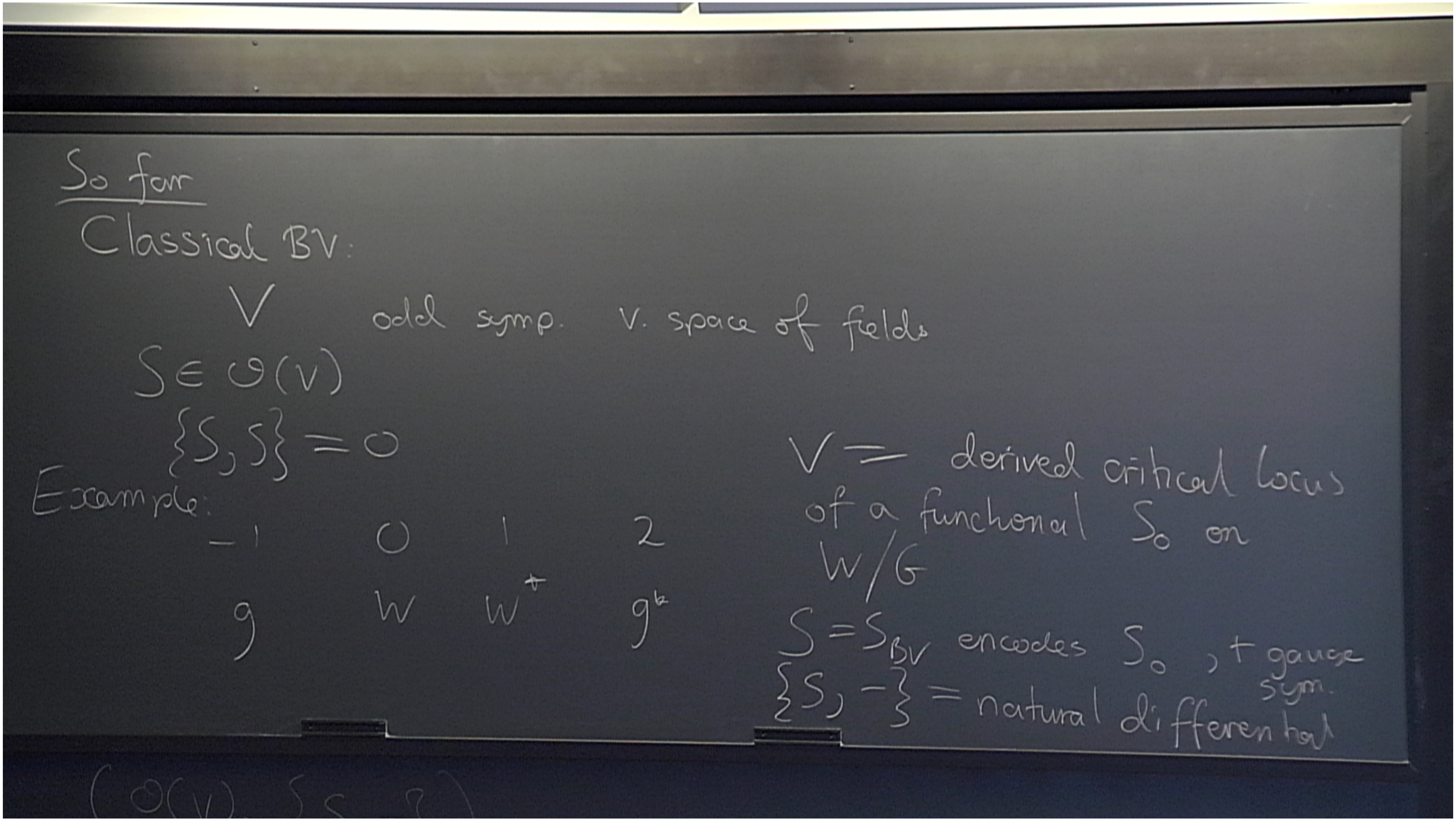
Abstract:


So far
Classical BV:
V odd symp. v. space of fields
$S \in \mathcal{O}(V)$
$\{S, S\} = 0$
Example:
-1 0 1 2
$\mathfrak{g}$ W W^* $\mathfrak{g}^*$
$V =$ derived critical locus of a functional S_0 on W/G
$S = S_{BV}$ encodes S_0, + gauge sym.
$\{S, -\} =$ natural differential
$(\mathcal{O}(V), \{S, ?\})$

$(\mathcal{O}(V), \{S, -\})$ computes functions on crit. lo
S_0 on W/G

$\{S,S\}=0$ automatically in these examples.

Quantum master equation

Choose Darboux coords x_i, η^i on V

x_i even, η^i odd, $\{x_i, \eta^j\} = \delta_i^j$

Then we can write

$$\Delta = \sum \frac{\partial}{\partial x_i} \frac{\partial}{\partial \eta^i}$$

This is independent of basis

Then we say S satisfies the quantum master eq^n if

$$\frac{1}{2}\{S, S\} + \hbar \Delta S = 0$$

Meaning Suppose we're in a gauge theory situation.

$$V^{\leq 0} = \left\{ \begin{matrix} -1 & 0 \\ \mathfrak{g} & W \end{matrix} \right\}$$

$(\mathcal{O}(V), \hbar\Delta)$ = divergence complex for Lebesgue on $V^{\leq 0}$ (viewed as a linear space)

$(\mathcal{O}(V), \hbar\Delta + \{S, -\})$ = should be divergence com
for the measure $e^{S_0/\hbar}\, d\mu$ on $\mathfrak{g}[-1] \oplus V$

QME $\iff (\hbar\Delta + \{S,-\})^2 = 0$

$\hbar\Delta) =$ divergence complex for Lebesgue measure

≤ 0 (viewed as a linear space)

$\hbar\Delta + \{S, -\}) =$ should be divergence complex for the measure

$e^{S_0/\hbar}\, d\mu$ on $\mathfrak{g}[-1] \oplus W = W/\mathfrak{g}$

$, -\})^2 = 0$

$\mathfrak{g}$ Lie algebra

$\mathfrak{g}$ acts on W linearly

$\mathfrak{g}$-invariant function S_0 on W to be

Does QME hold?

$$S_{BV} = \tfrac{1}{2} b c^2 + c x \eta =$$

-1	0	1	2
$\mathfrak{g}$	W	W^*	$\mathfrak{g}^*$
c	x	η	b

BV Laplacian

$$\Delta = \sum \frac{\partial}{\partial b_a} \frac{\partial}{\partial c^a} + \frac{\partial}{\partial x_i} \frac{\partial}{\partial \eta}$$

unction S_0 on W to be $S_0 = 0$

d?

2

$$S_{BV} = \tfrac{1}{2} b c^2 + c x \eta = \tfrac{1}{2} b_a c^b c^c f_a^{bc} + c^a x_i \eta^j \rho_a{}^i{}_j$$

BV Laplacian

$$\Delta = \sum \frac{\partial}{\partial b_a} \frac{\partial}{\partial c^a} + \frac{\partial}{\partial x_i} \frac{\partial}{\partial \eta^i}$$

$$\{S, S\} = 0 \checkmark$$

$$\Delta S = \sum c^b f^a_{ba} - c^b \rho^i_{bi}$$

$t^b \in g$

$$\mathrm{Tr}_g\, t^b - \mathrm{Tr}_W\, t^b$$

t^b changes dVol on g

t^b changes dVol$_W$

Canonical Transformations

V odd symplectic

$S \in \mathcal{O}(V)$ satisfies classical master eq^n

$T \in \mathcal{O}(V)$ degree -1

$S \longrightarrow S + \varepsilon\{S, T\}$ This satisfies CME

This is a change of coords by Hamiltonian v. field

$\{T, -\}$

A first order def. $S \longrightarrow S + \varepsilon S'$

satisfies CME if $\{S, S'\} = 0$

First order def. / Change of coordinates

$$= H^0(\mathcal{O}(V), \{S, -\})$$

If S satisfies QME, changes of coordinates act by

$$S \longrightarrow S + \varepsilon\{S,T\} + \hbar\Delta T$$

Action of $\{T,-\}$ on the measure $e^{S/\hbar}d\mu$

Deformations

$$= H^0\left(\mathcal{O}(V)[[\hbar]], \{S,-\}+\hbar\Delta\right)$$

Gauge Fixing

V = as usual

$$\overset{-1}{g} \quad \overset{0}{W} \quad \overset{1}{W} \quad \overset{2}{g}$$

And $S = BV$ action assoc. to g-invariant fn S_0 on W

Original Integral

$$\int_{V\le 0} e^{S_0/\hbar} = \int_{V\le 0} e^{S/\hbar}$$

Gauge Fixing

V = as usual

$$\mathfrak{g}^{-1} \quad W^{0} \quad W^{1} \quad \mathfrak{g}^{2}$$

And S = BV action assoc. to $\mathfrak{g}$-invariant fn S_0 on W

Original Integral

$$\int_{V \leq 0} e^{S/\hbar} = \int_{V \leq 0} e^{S/\hbar}$$

Problematic: Not a Gaussian

Lemma

If L_t is a family of Linear Lagrangians in V

then, $\int_{L_t} e^{S/\hbar}$ is independent of t

if S satisfies QME

$$V = L_0 + L_0^*[-1]$$

L_t is obtained as the graph of a quadratic function f_t on L_0 f_t degree -1

$$f_t = tf + O(t^2)$$

Work mod t^2

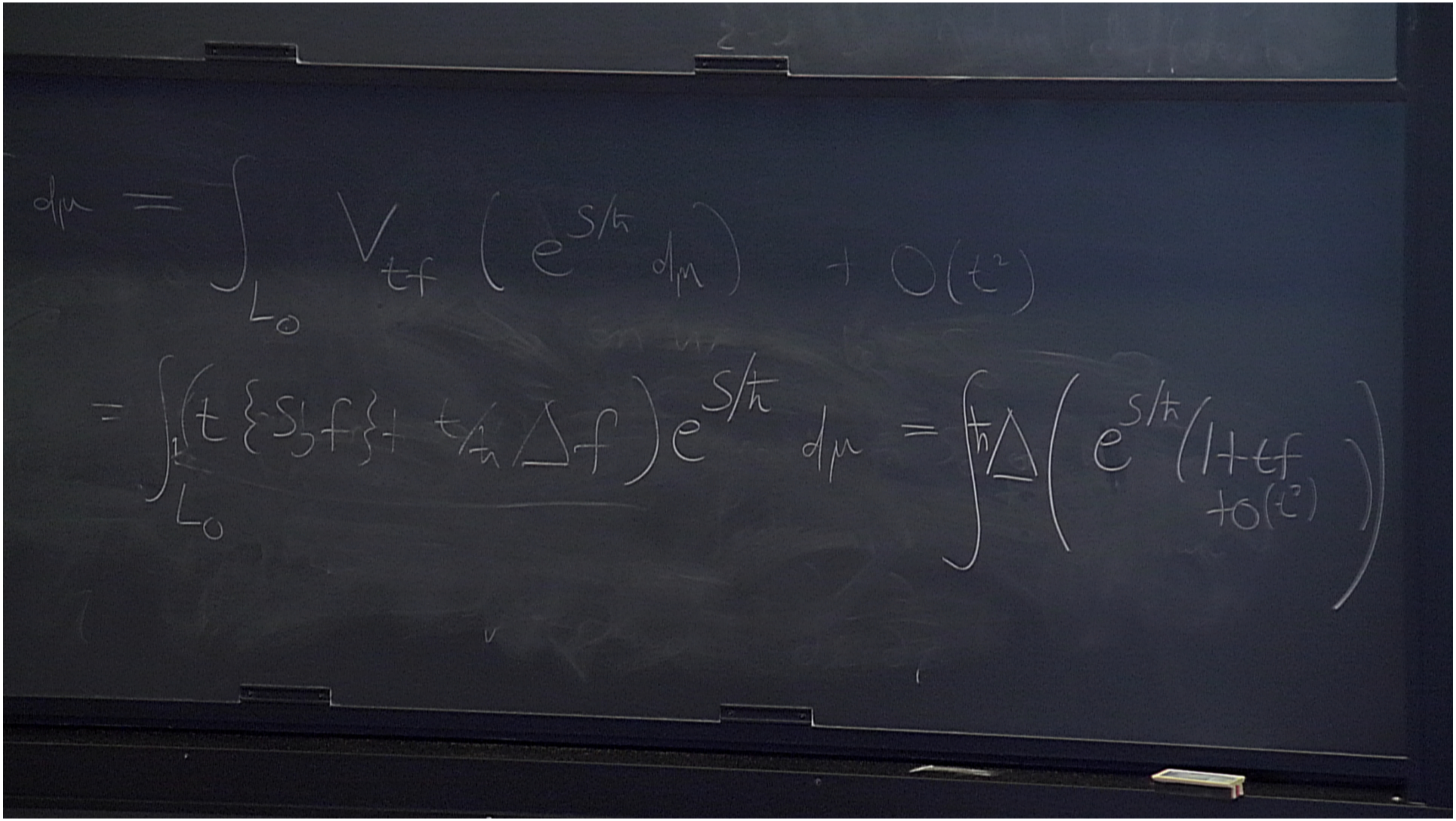
L_t is obtained by applying Hamiltonian transf. generated by f_t to L_0

$$\int_{L_t} e^{S/\hbar}\, d\mu = \int_{L_0} V_{tf}\left(e^{S/\hbar}\, d\mu\right) + O(t^2)$$

$$= \int_{L_0} \left(t\{S, f\} + t/\hbar\, \Delta f\right) e^{S/\hbar}\, d\mu$$

$$\int_{L_t} e^{S/\hbar}\, d\mu = \int_{L_0} V_{tf}\left(e^{S/\hbar}\, d\mu\right) + O(t^2)$$

$$= \int_{L_0} \underbrace{\left(t\{S,f\} + \tfrac{t}{\hbar}\Delta f\right)}_{\text{This is a divergence}} e^{S/\hbar}\, d\mu$$


$d\mu = \int_{L_0} V_{tf}\left(e^{S/\hbar}\, d\mu\right) + O(t^2)$
$= \int_{L_0} \left(t\{S, f\} + \tfrac{t}{\hbar}\Delta f\right) e^{S/\hbar}\, d\mu = \int \hbar\Delta\left(e^{S/\hbar}\left(1+tf + O(t^2)\right)\right)$

$$\ldots tf \left(e^{S/\hbar} \, d\mu \right) + O(t^2)$$

$$\ldots + t\hbar\Delta f \big) e^{S/\hbar} \, d\mu = \int \hbar\Delta\left(e^{S/\hbar}\left(1+tf + O(t^2)\right)\right) - \int (tf)\left(\tfrac{1}{2}\{S,S\} + \hbar\Delta S\right)e^{S/\hbar}$$

Chern-Simons Theory

M 3-manifold

CS action

$$A \in \Omega^1(M, \mathfrak{g})$$

$$S_{CS}(A) = \frac{1}{2}\int \langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$$

Fields in the BV formalism are

$$\begin{array}{cccc} -1 & 0 & 1 & 2 \\ \Omega^0(M,\mathfrak{g}) & \Omega^1(M,\mathfrak{g}) & \Omega^2(M,\mathfrak{g}) & \Omega^3(M,\mathfrak{g}) \end{array}$$

Fields are $\mathcal{E} = \Omega^*(M,\mathfrak{g})[1]$

If $\alpha \in \mathcal{E}$

$$S_{BV}(\alpha) = \frac{1}{2}\int \langle \alpha, d\alpha \rangle + \frac{1}{6}\int \langle \alpha, [\alpha, \alpha] \rangle$$

$$\alpha = \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3$$

$$\Omega^0 \qquad \Omega^1 \qquad \Omega^2 \qquad \Omega^3$$

"$\frac{1}{2} c^2 b$"

$$S_{BV}(\alpha) = S_{CS}(\alpha^1) + \underbrace{\int \alpha^0 d\alpha^2}_{A \to A + d\chi} + \underbrace{\int \alpha^0 \alpha^1 \alpha^2}_{A \to A + [A, \chi]} + \int \frac{\alpha^0 \alpha^0}{2} \alpha^3 + \cdots$$

$$S_{CS}(\alpha) \ldots \int_{A \to A + d\chi} \alpha\, d\alpha + \int_{A \to A + [A,\chi]} \frac{\alpha}{2}[\alpha,\alpha] + \ldots$$

BV Kernel

$$\delta_\Delta\left(\sum t_a \otimes t_a\right) \in \Omega^3(M \times M) \otimes \mathfrak{g} \otimes \mathfrak{g}$$

$$= \mathcal{E} \otimes \mathcal{E}$$

If F, G are functions of fields

$\{F, G\}$ is given by contracting, one leg of F, one of G, with BV Kernel

BV Kernel

$$\delta_\Delta \otimes \Big(\underbrace{\sum t_a \otimes t_a}_{\in C}\Big) \in \Omega^3(M \times M) \otimes \mathfrak{g}$$

$$= \mathcal{E} \otimes \mathcal{E}$$

If F, G are functions of fields

$\{F, G\}$ is given by contracting, one leg of F, one of G, with

$$S = S_q + I$$

$\{S_q, -\}$ is the odd symmetry of $\mathcal{E}$ given by d

$$\int \alpha \, d\alpha$$

$$\{S_q, S_q\} = 0 \iff d^2 = 0$$

$$\{S_a, \underline{I}\} \stackrel{?}{=} 0 \qquad \alpha =$$

$$\{\underline{I}, \underline{I}\} \stackrel{?}{=} 0$$

$$\underline{I}(\alpha) = \int \langle \alpha, [\alpha, \alpha] \rangle = \int \alpha_a$$

$$\underline{I}(\alpha) + \varepsilon \{S_a, \underline{I}(\alpha)\} \stackrel{?}{=} \underline{I}(\alpha + \varepsilon d\alpha) = \underline{I}(\alpha)$$

$$\underline{I}\} \stackrel{?}{=} 0 \qquad \alpha = \sum \alpha_a \otimes t_a$$

$$\underline{I}\} \stackrel{?}{=} 0$$

$$\alpha) = \int \langle \alpha, [\alpha, \alpha] \rangle = \int \alpha_a \wedge \alpha_b \wedge \alpha_c \, f^{abc}$$

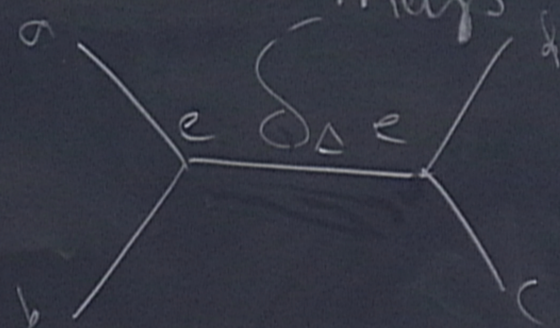
$$(\alpha) \stackrel{?}{=} \underline{I}(\alpha + \varepsilon d\alpha) = \underline{I}(\alpha)$$ Stokes theorem

$I(\alpha)^T$:

$$\int \alpha\wedge\beta\wedge\gamma : \Omega^\cdot(M)^{\otimes 3} \longrightarrow \mathbb{R}$$

is a cochain map

$\{I, I\}$



$\{I, I$

$=$

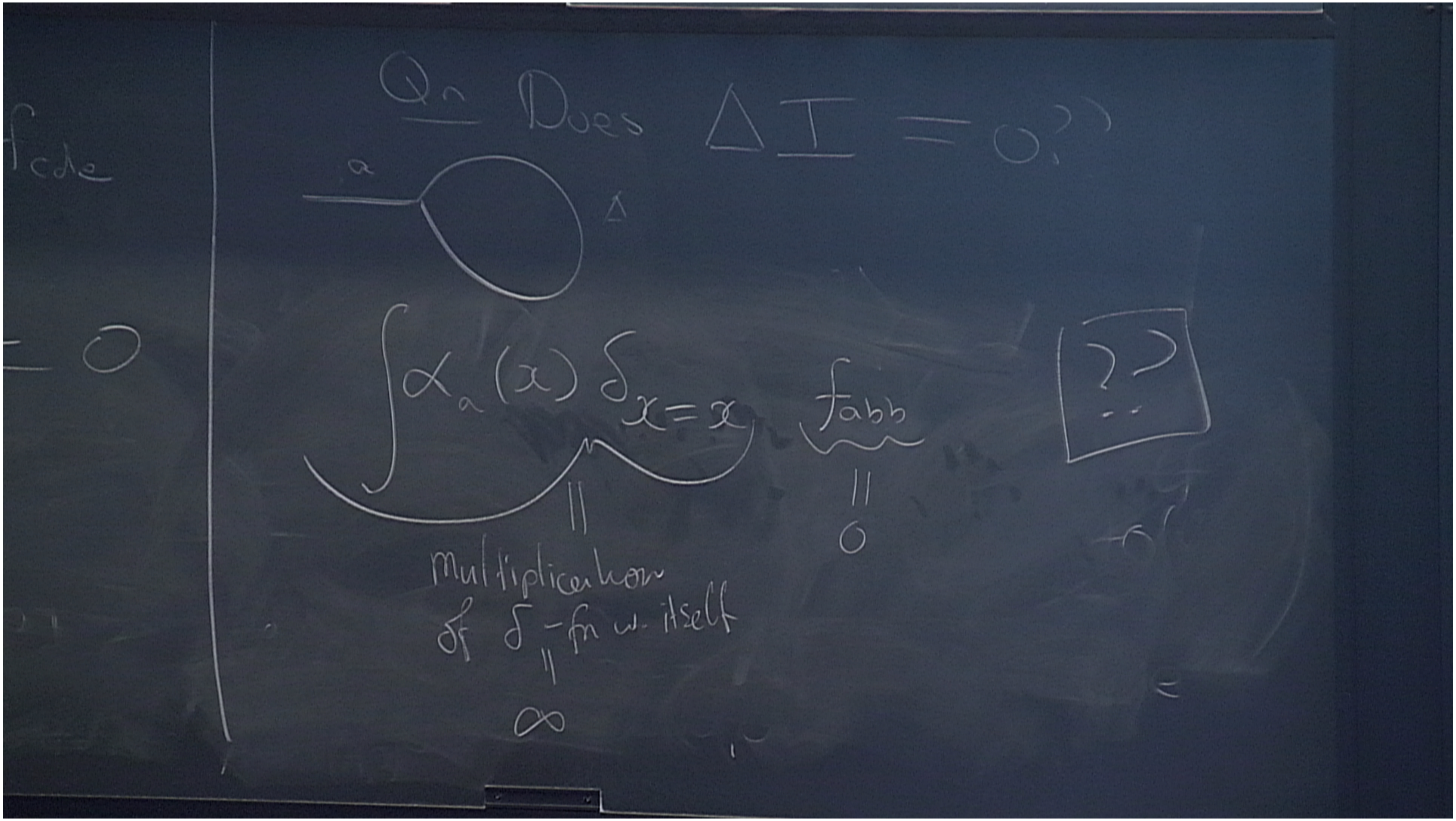
$\to \mathbb{R}$

$$\{I, I\}(\alpha) = \int_{M \times M} (\alpha_a \wedge \alpha_b)(x)\, \alpha_c \wedge \alpha_d (x')\, \delta_{x=x'}\, f_{abe} f_{cde}$$

$$\int_M \underbrace{\alpha_a \wedge \alpha_b \wedge \alpha_c \wedge \alpha_d}_{\text{Graded antisymmetric}} f_{abe} f_{cde}$$

By Jacobi, $= 0$

Qn: Does


Qn Does ΔI = 0??
∫α_a(x) δ_{x=x}
f_abb
multiplication
of δ-fn w. itself
∞
0
??