Title: Quantum Error-Correcting Codes in the Low Energy Subspaces

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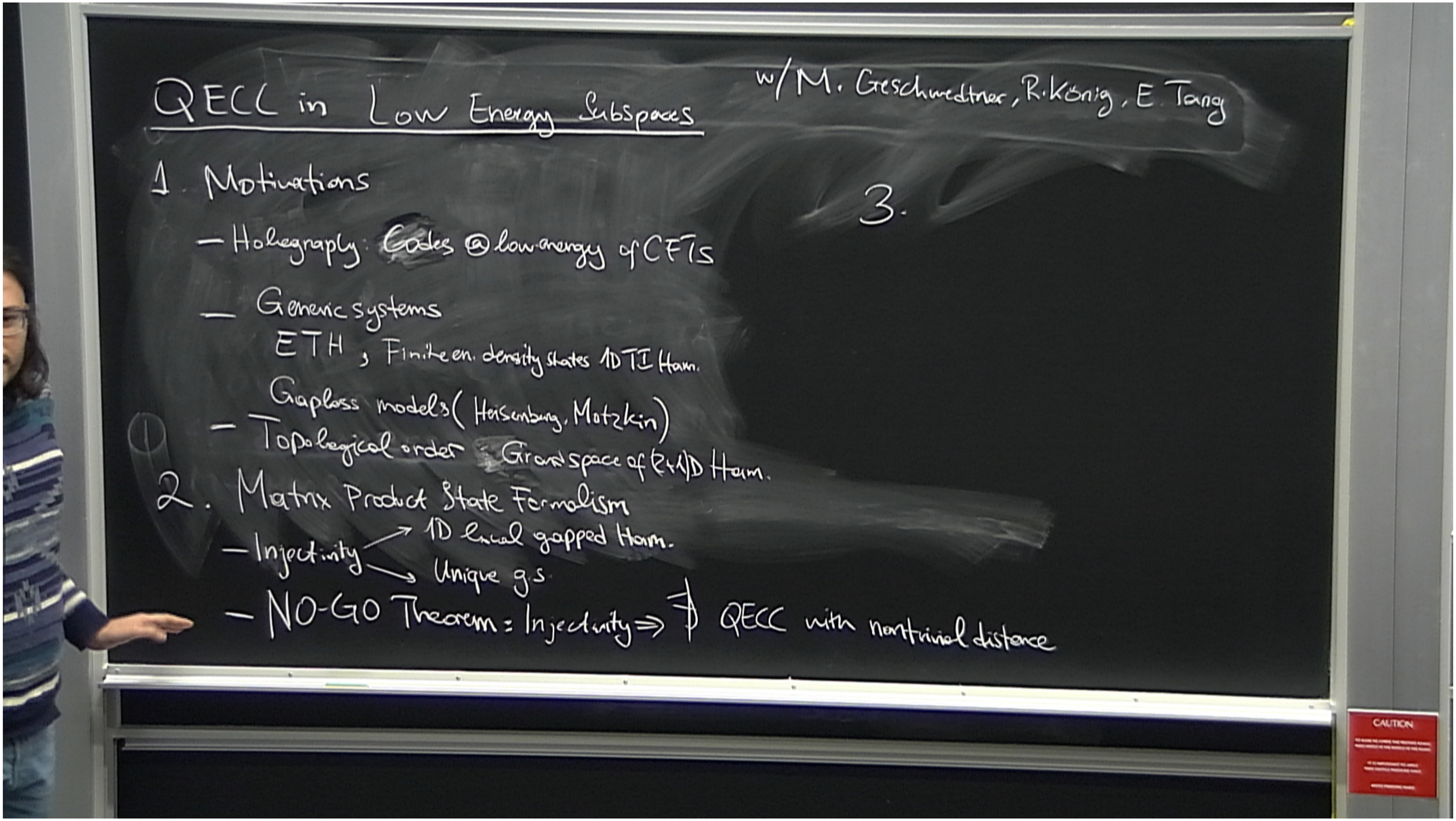
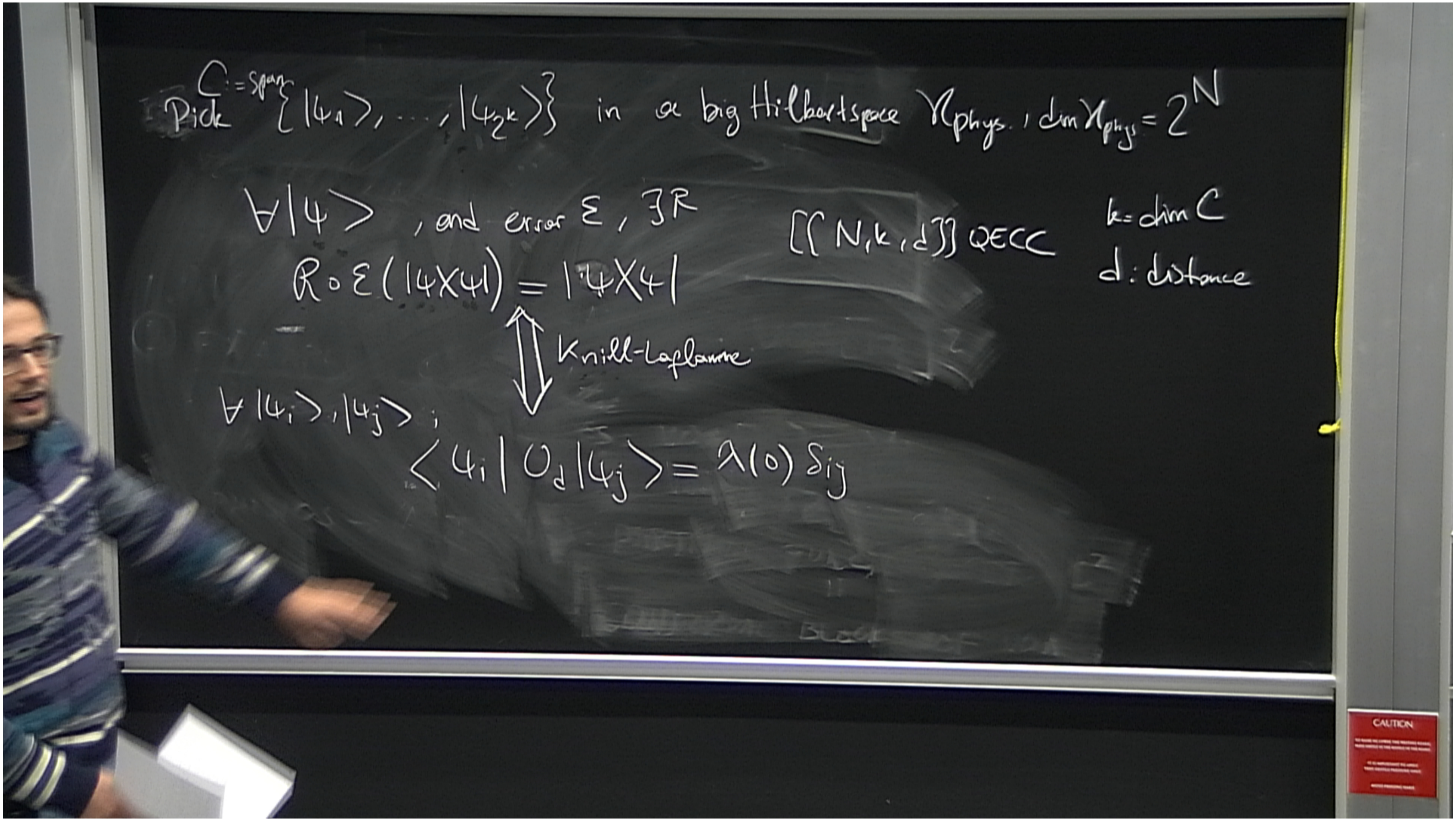
Abstract: <p>Recent years have shown that error correction is one of the most fundamental ingredients of various physical phenomena, from topological order to holography. However, only toy models and fixed point ground states could have been studied, even though the error-correcting properties are expected to hold generally in the low energy subspace.</p>

<p>In this talk, we will employ Matrix Product State formalism and see how low energy eigenstates of 1D translationally invariant Hamiltonians can form quantum error-correcting codes. Before diving into the results, we will review the necessary basics of quantum error correction and matrix product states.</p>

<p>Joint work with Martina Gschwendtner, Robert Koenig and Eugene Tang.</p>


QECC in Low Energy Subspaces
w/ M. Geschwedtner, R. König, E. Tang
1. Motivations
— Holography: Codes @ low-energy of CFTs
— Generic systems
ETH, Finite en. density states 1D TI Ham.
Gapless models (Heisenberg, Motzkin)
— Topological order: Ground space of (2+1)D Ham.
2. Matrix Product State Formalism
— Injectivity → 1D local gapped Ham.
→ Unique g.s.
— NO-GO Theorem: Injectivity ⇒ ∄ QECC with nontrivial distance
3.



C := span
Pick {|ψ₁⟩, …, |ψ_{2^k}⟩} in a big Hilbert space H_phys., dim H_phys = 2^N
∀|ψ⟩, and error E, ∃R
R ∘ E(|ψ⟩⟨ψ|) = |ψ⟩⟨ψ|
Knill-Laflamme
∀|ψ_i⟩, |ψ_j⟩ ;
⟨ψ_i| O_d |ψ_j⟩ = λ(O) δ_ij
[[N, k, d]] QECC
k = dim C
d : distance
CAUTION

Pick $\mathcal{C} := \mathrm{span}\{|\psi_1\rangle, \dots, |\psi_{2^k}\rangle\}$ in a big Hilbertspace $\mathcal{H}_{phys.}$, $\dim \mathcal{H}_{phys.} = 2^N$

$\forall |\psi\rangle$, and error $\mathcal{E}$, $\exists R$

$F(|\psi\rangle\langle\psi|, R \circ \mathcal{E}(|\psi\rangle\langle\psi|)) \geq 1 - \varepsilon$

$[[N, k, d, \varepsilon]]$ AQECC

$k = \dim \mathcal{C}$

d: distance

$\Updownarrow$ Knill-Laflamme

$\forall |\psi_i\rangle, |\psi_j\rangle$:

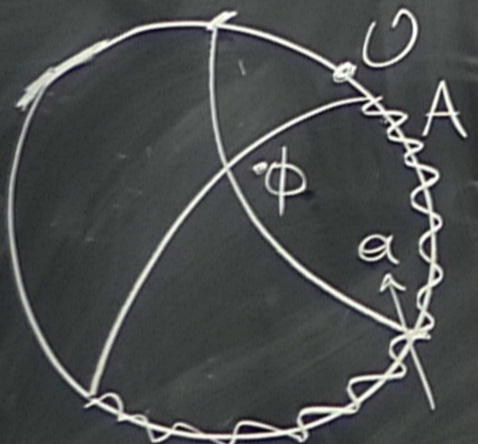
$$\langle \psi_i | O_d | \psi_j \rangle = \lambda(O)\, \delta_{ij} + \varepsilon_{ij}$$

$$\varepsilon \leq 2^{2k+d} \max_{i,j} \sqrt{\varepsilon_{ij}}$$

$i = j$: local indistinguishability

$i \neq j$: No d-local op. $\overset{\in \mathcal{C}}{|\psi_i\rangle} \to \overset{\in \mathcal{C}}{|\psi_j\rangle}$

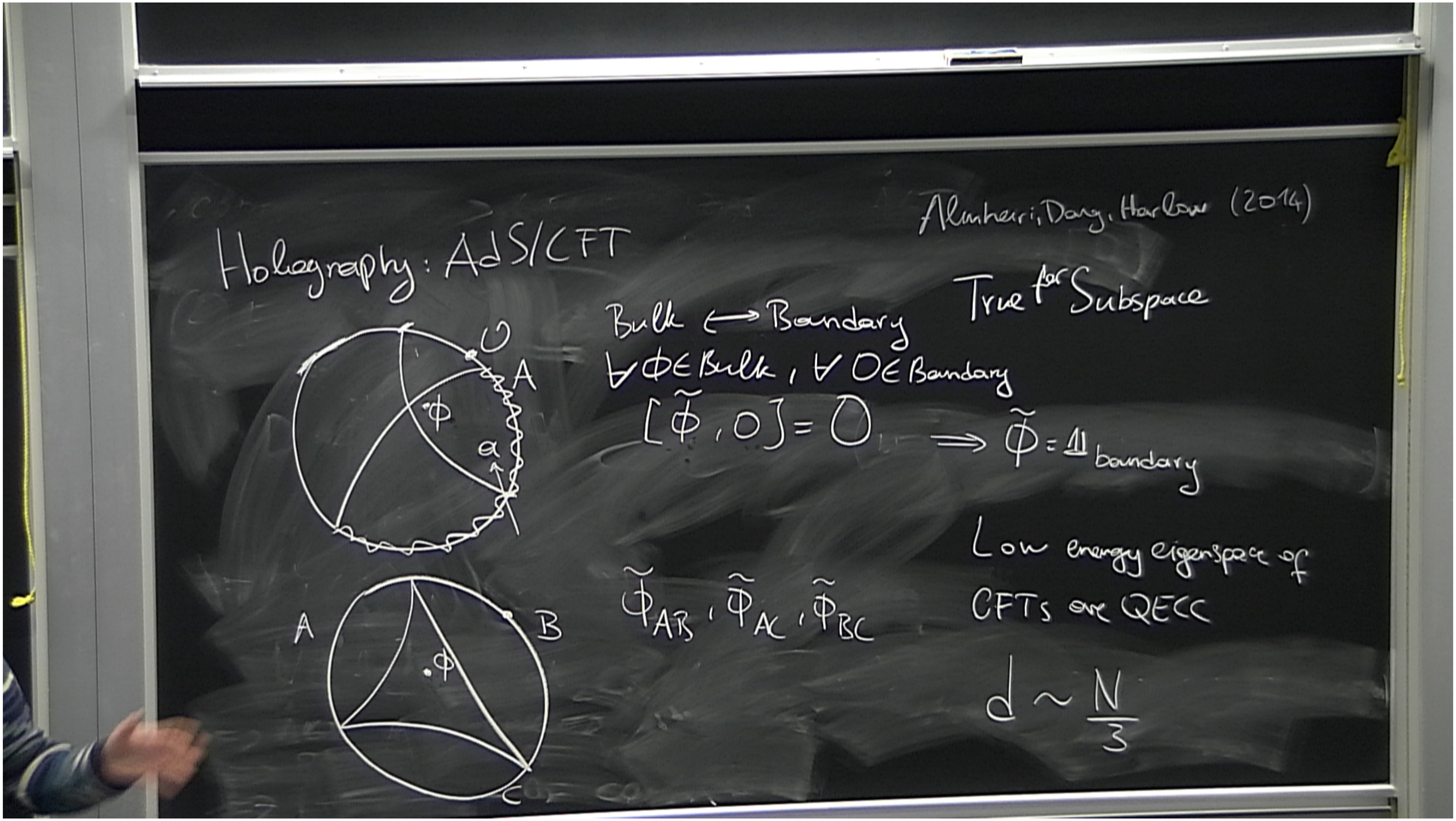
Holography: AdS/CFT

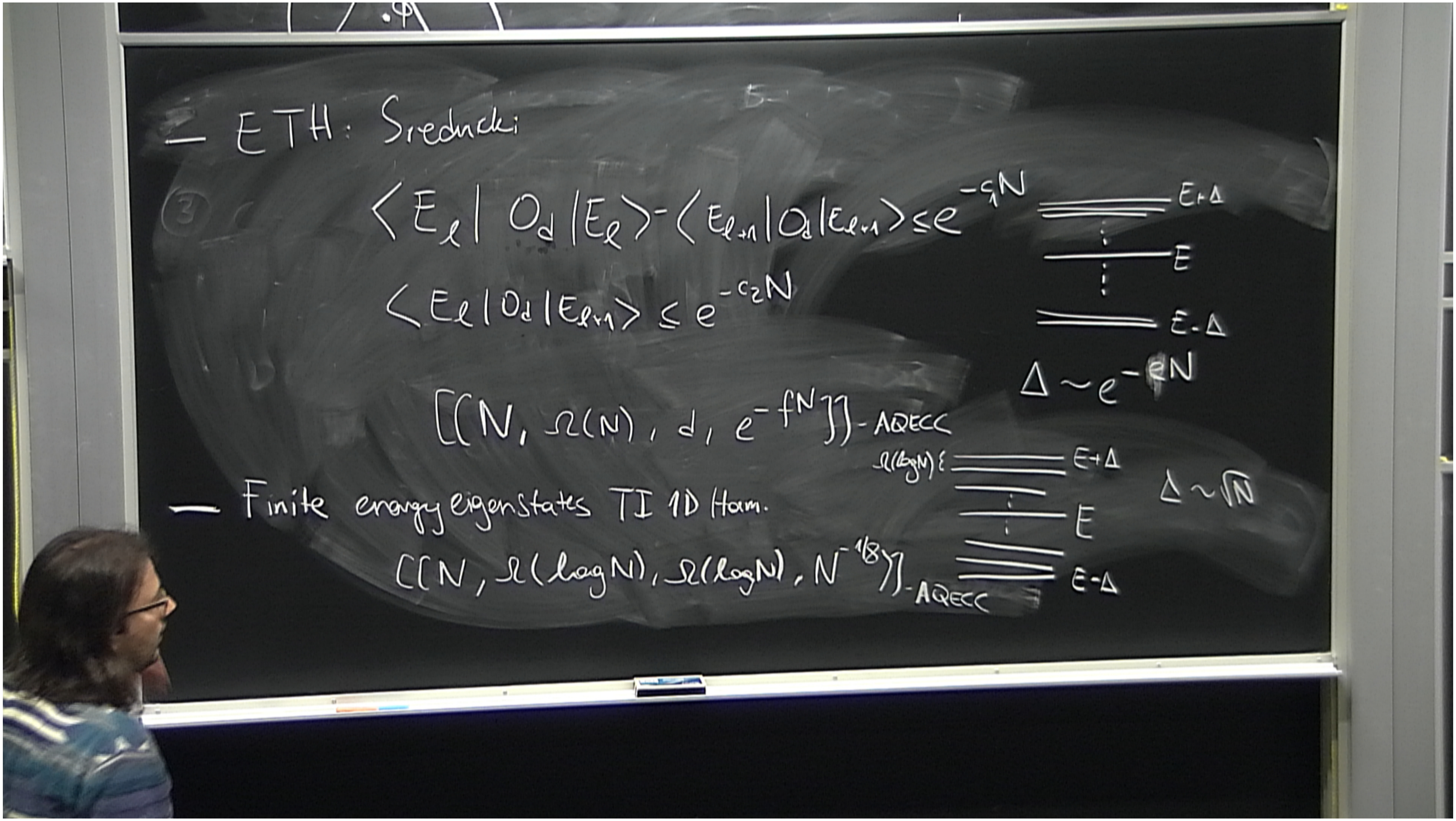


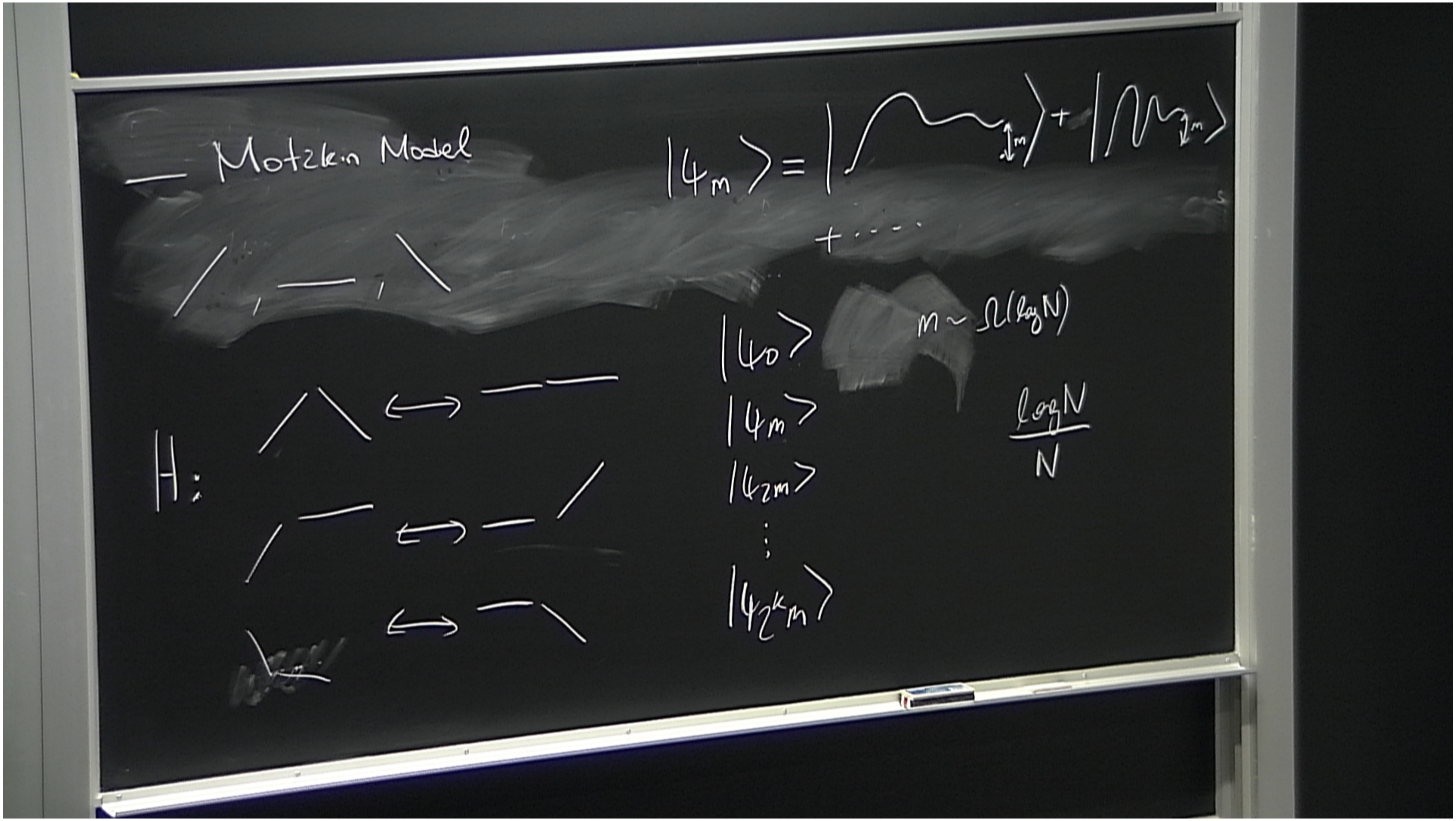
Bulk $\longleftrightarrow$ Boundary

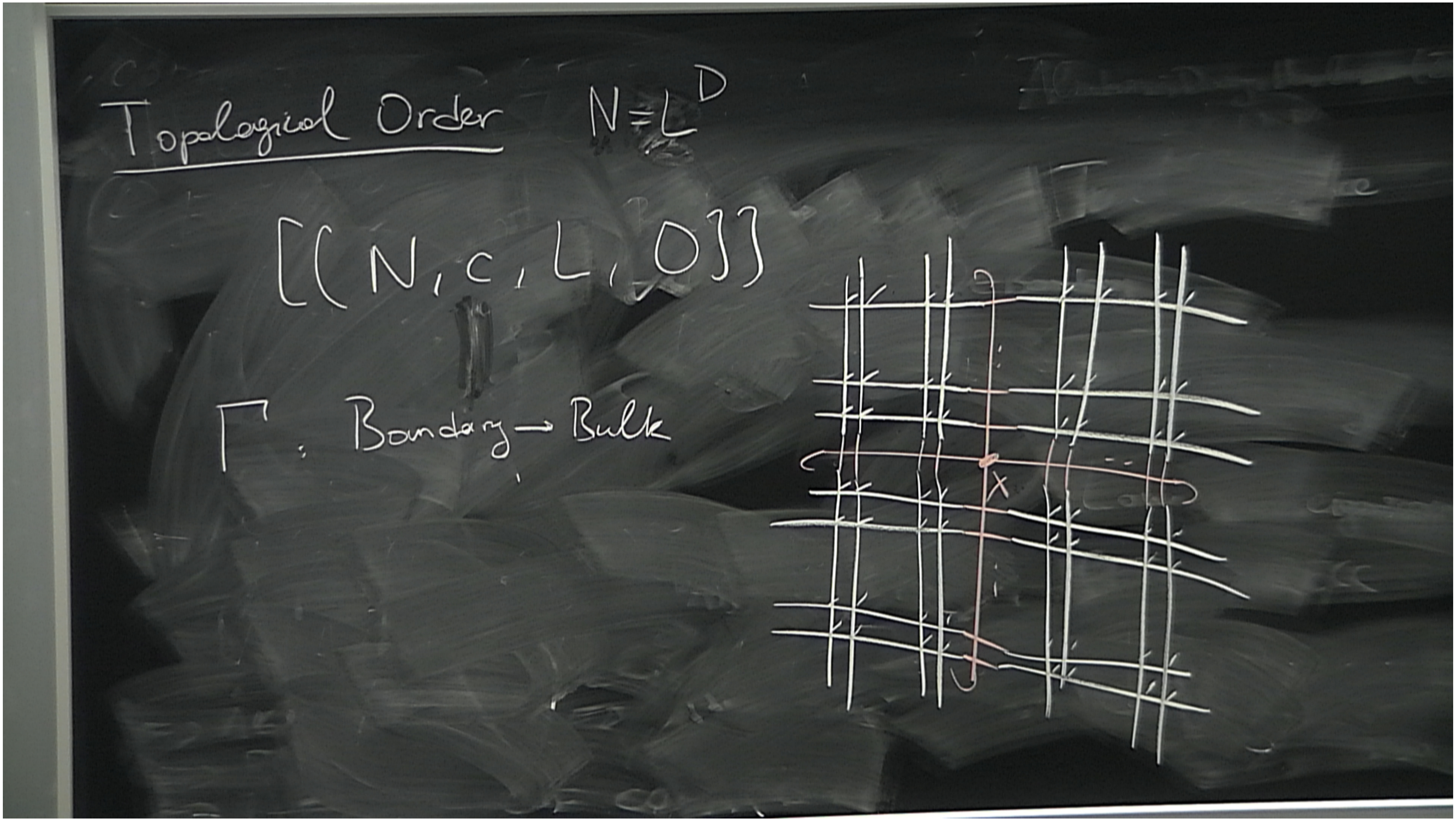
$\forall\, \Phi \in$ Bulk, $\forall\, O \in$ Boundary

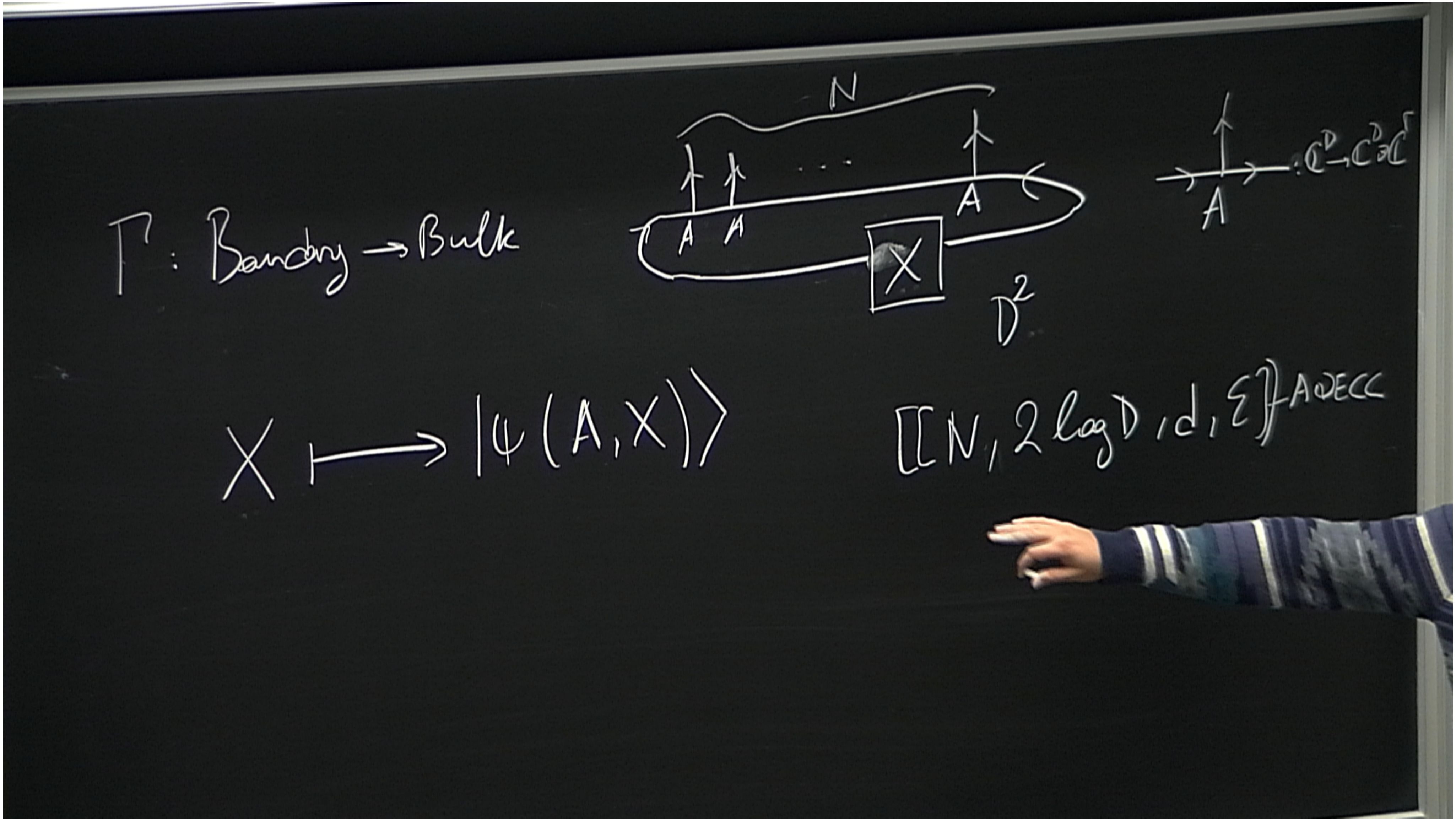
$$[\tilde{\Phi}, O] = 0 \Longrightarrow \tilde{\Phi} = \mathbb{1}_{\text{boundary}}$$

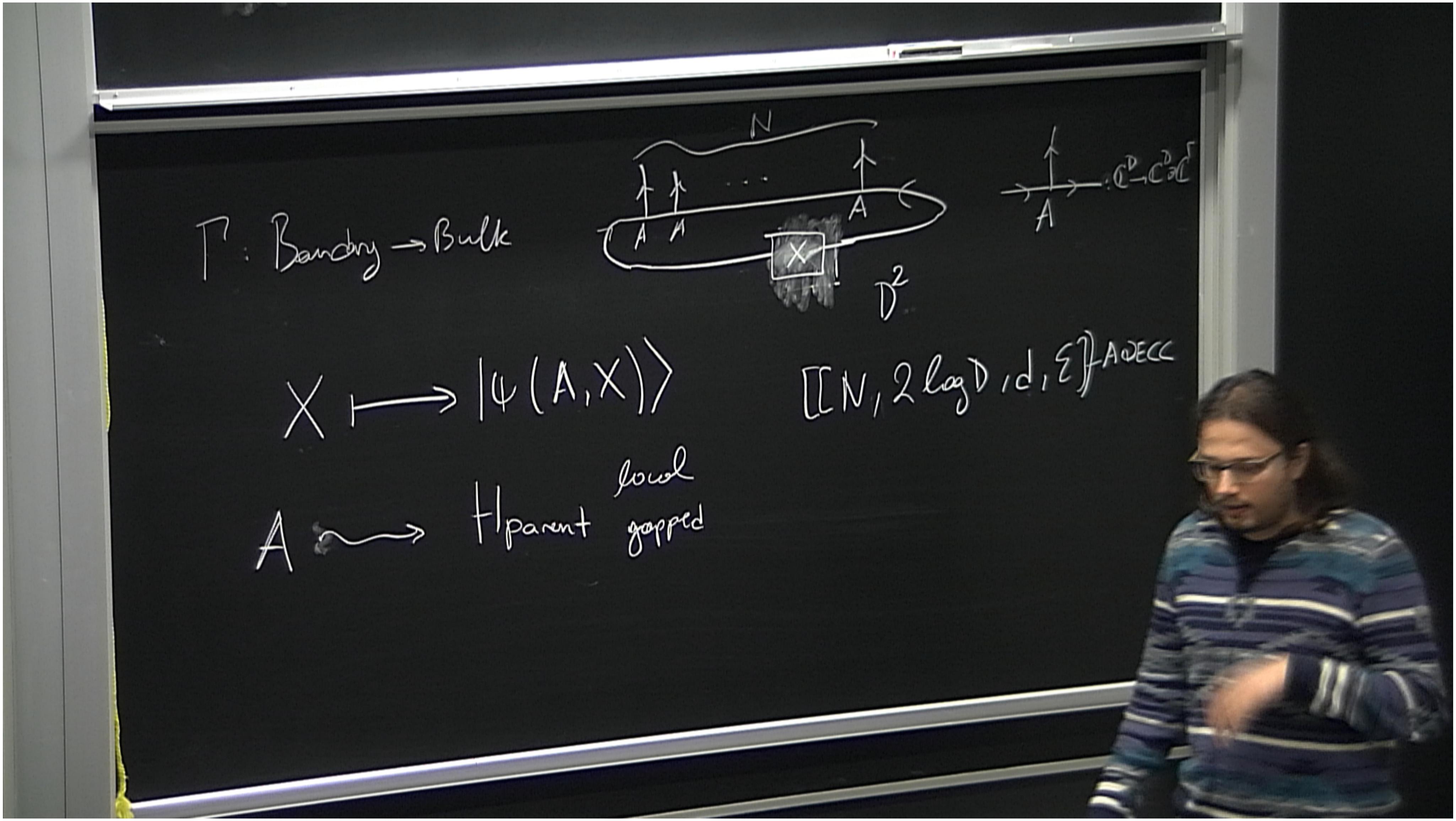

Almheiri, Dong, Harlow (2014)
Holography: AdS/CFT
True for Subspace
Bulk ⟷ Boundary
∀ ϕ ∈ Bulk, ∀ O ∈ Boundary
[ϕ̃, O] = 0 ⟹ ϕ̃ = 𝟙 boundary
O
A
ϕ
a
A
B
C
ϕ
ϕ̃AB, ϕ̃AC, ϕ̃BC
Low energy eigenspace of
CFTs are QECC
d ~ N/3

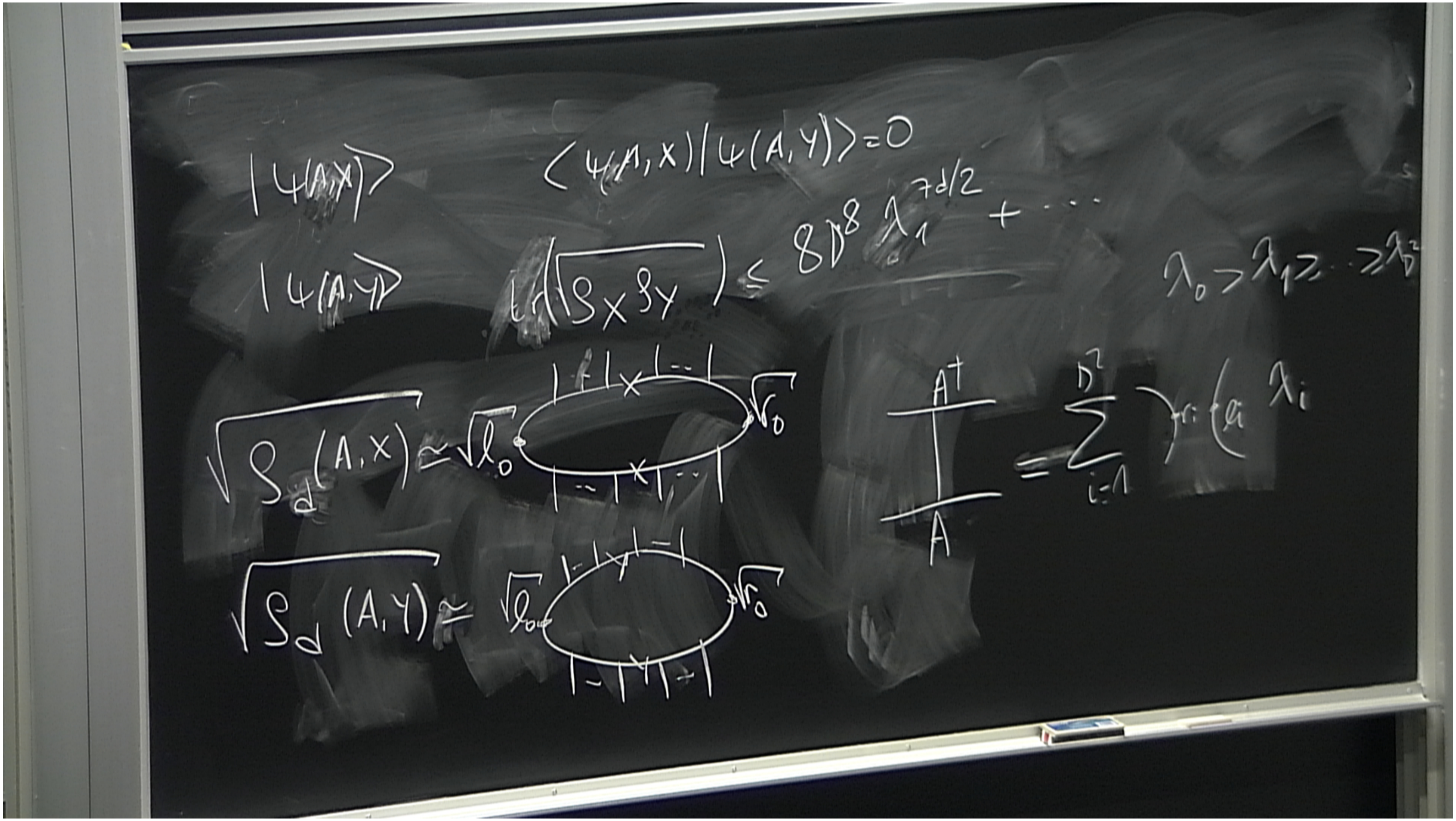

– ETH: Srednicki
$\langle E_\ell | O_d | E_\ell \rangle - \langle E_{\ell+1} | O_d | E_{\ell+1} \rangle \leq e^{-c_1 N}$
$\langle E_\ell | O_d | E_{\ell+1} \rangle \leq e^{-c_2 N}$
$E+\Delta$
E
$E-\Delta$
$\Delta \sim e^{-eN}$
$[[N, \Omega(N), d, e^{-fN}]]$ - AQECC
$\Omega(\log N)$ {
$E+\Delta$
$\Delta \sim \sqrt{N}$
– Finite energy eigenstates TI 1D Ham.
E
$[[N, \Omega(\log N), \Omega(\log N), N^{-1/8}]]$ - AQECC
$E-\Delta$

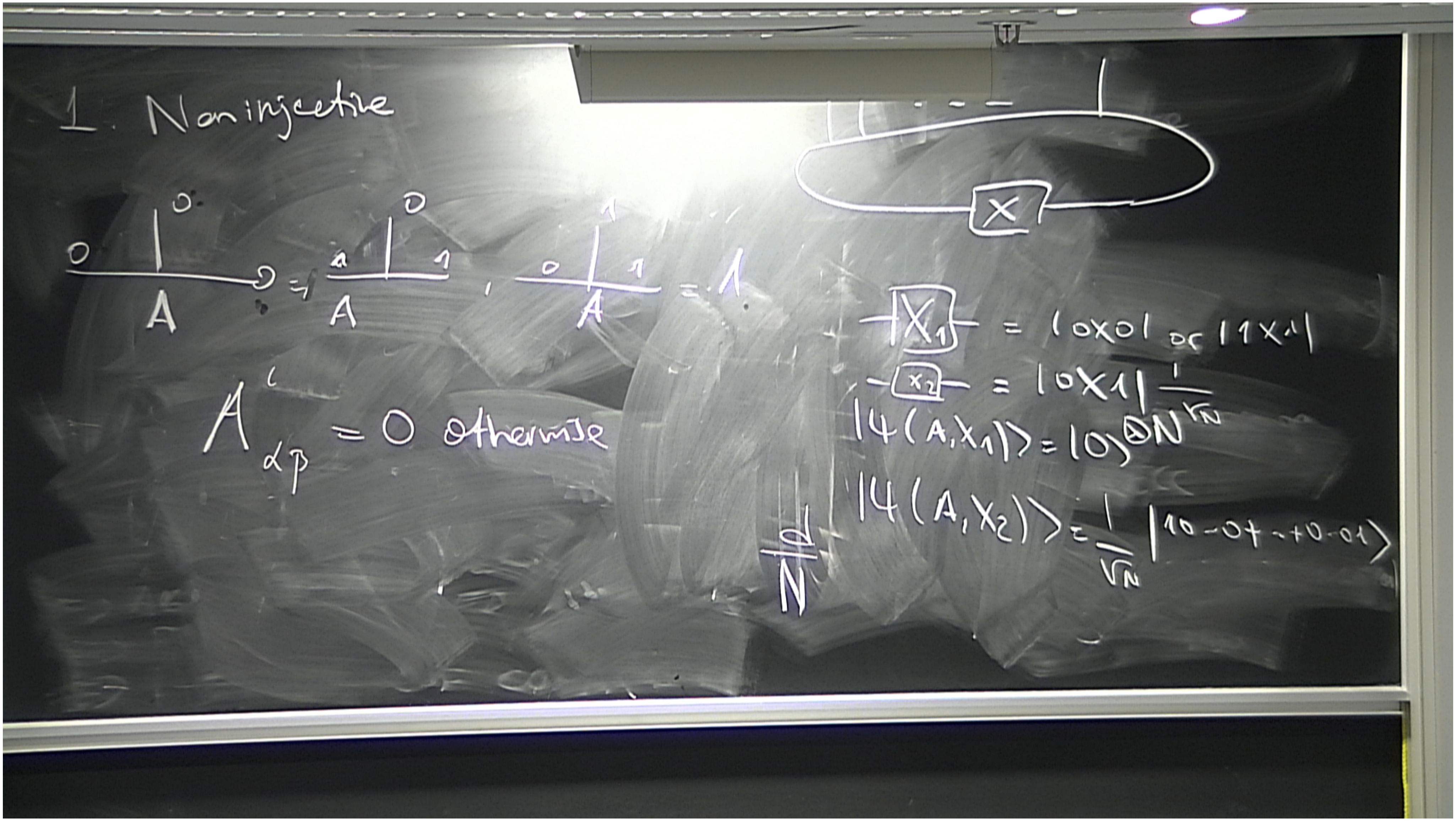

Motzkin Model
$|\psi_m\rangle = | \ \rangle + | \ \rangle + \cdots$
H:
$|\psi_0\rangle$
$|\psi_m\rangle$
$|\psi_{2m}\rangle$
$\vdots$
$|\psi_{2^k m}\rangle$
$m \sim \Omega(\log N)$
$\frac{\log N}{N}$


Topological Order
N = L^D
[(N, c, L, O)]
Γ: Boundary → Bulk


Γ : Boundary → Bulk
N
A A . . . A
X
D²
A
X ⟼ |ψ(A,X)⟩
[[N, 2 log D, d, ε]] AQECC


Γ : Boundary → Bulk
N
A A A
D²
X ⟼ |ψ(A,X)⟩
[[N, 2 log D, d, ε]] AQECC
A ⇝ H parent local gapped


|ψ(A,X)⟩
|ψ(A,Y)⟩
⟨ψ(A,X)|ψ(A,Y)⟩=0
tr(√(ρX ρY)) ≤ 8D^8 λ1^(7d/2) + ...
λ0 > λ1 ≥ ... ≥ λD²
√ρd(A,X) ≃ √l0 √r0
√ρd(A,Y) ≃ √l0 √r0
A†
A
= Σ_{i=1}^{D²} ri li λi


1. Noninjective
A = 1
$A^{i}_{\alpha\beta} = 0$ otherwise
$X_1 = |0\rangle\langle 0|$ or $|1\rangle\langle 1|$
$X_2 = |0\rangle\langle 1| \frac{1}{\sqrt{N}}$
$|\psi(A, X_1)\rangle = |0\rangle^{\otimes N}$
$|\psi(A, X_2)\rangle = \frac{1}{\sqrt{N}} |10\ldots0 + \ldots + 0\ldots01\rangle$

$$|\psi(A)\rangle = \underset{A}{|}\ \cdots\ \underset{A}{|}$$

$$|\psi_j(A,B)\rangle = \underset{A}{|}\ -\ \underset{A}{|}\ \overset{j}{\underset{B}{|}}\ \underset{A}{|}\ \cdots\ \underset{A}{|}$$

$$|\Phi_p(A,B)\rangle = \sum_j e^{ipj}\,|\psi_j(A,B)\rangle$$

$$\langle \Phi_p(A,B)|O_d|\Phi_{p'}(A,B)\rangle \leq 2\,\frac{d}{N}$$

$$\langle \Phi_p(A,B)|O_d|\Phi_{p'}(A,B)\rangle \leq 2\,\frac{d}{N}$$

$$[[N, \Omega(\log N), \Omega(\log N), N^{\alpha}]] \quad \alpha < \frac{1}{2}$$