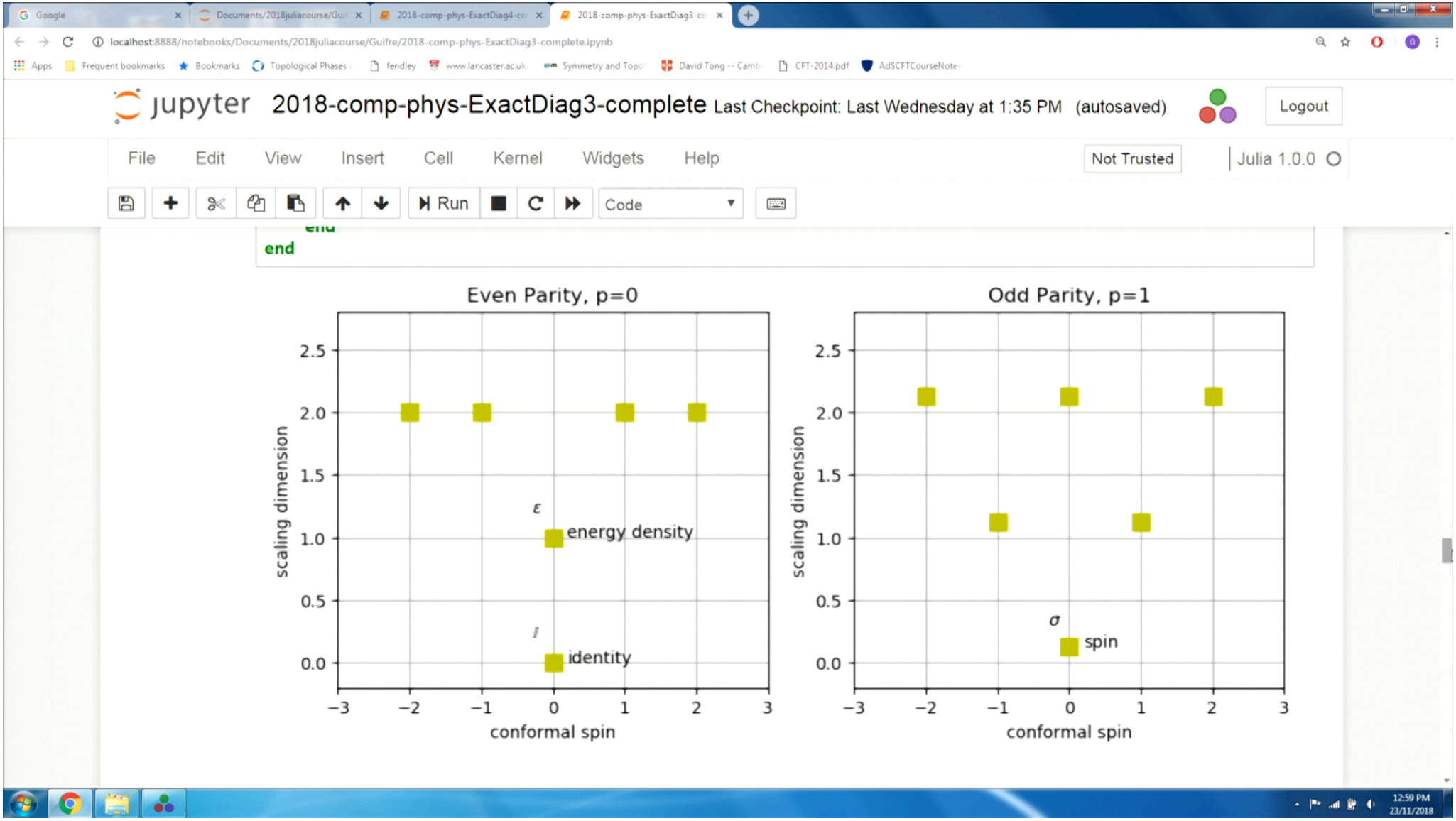
Title: Computational Physics - Lecture 22

Date: Nov 23, 2018 01:00 PM

URL: http://pirsa.org/18110048

Abstract: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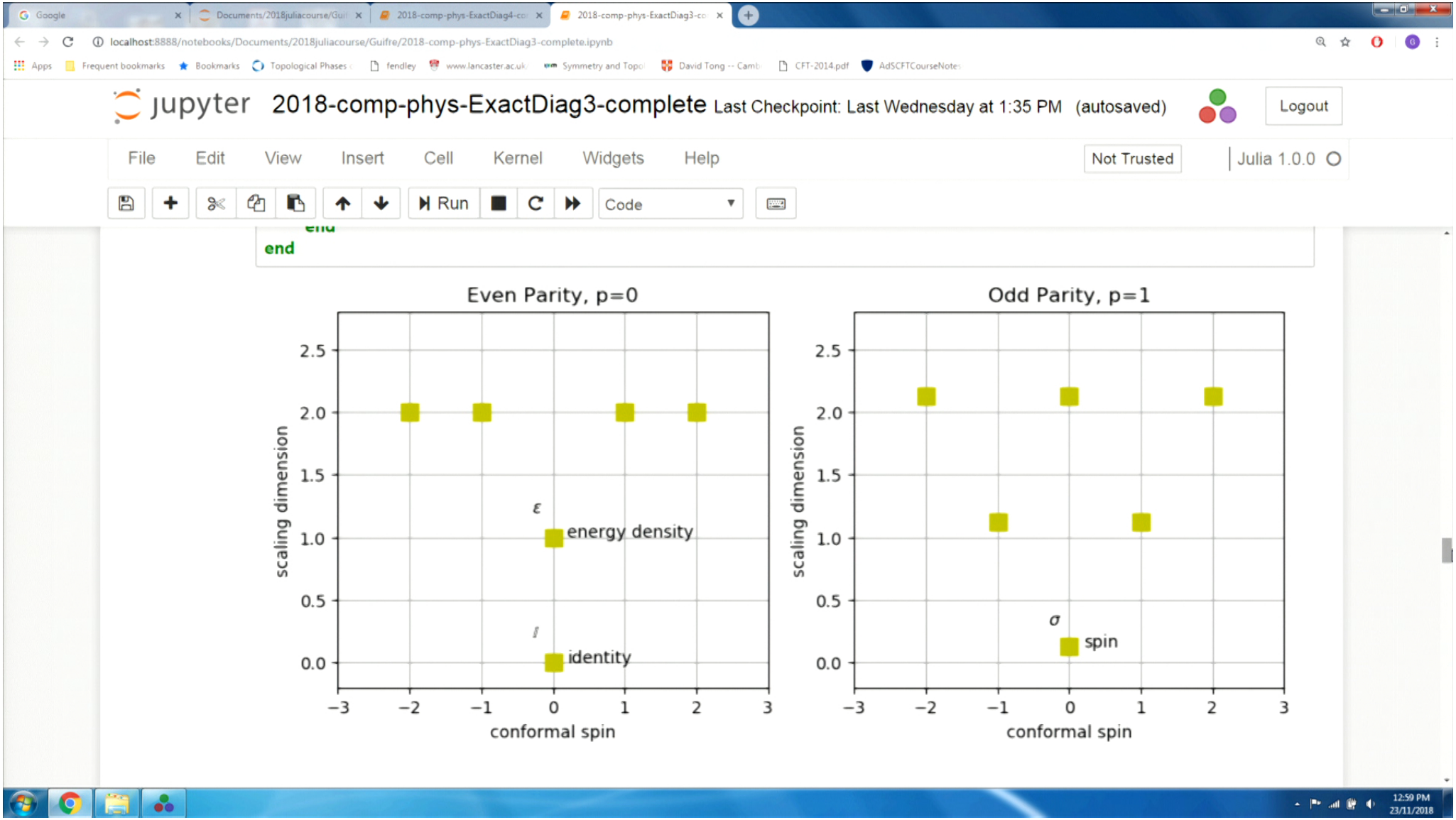
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
end
Even Parity, p=0
Odd Parity, p=1
scaling dimension
conformal spin
ε
energy density
I
identity
σ
spin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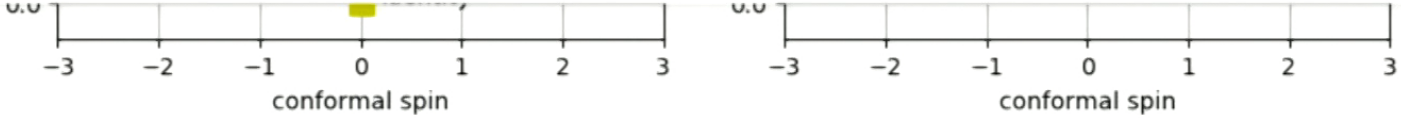
Not Trusted
Julia 1.0.0
Run
Code
end
Even Parity, p=0
Odd Parity, p=1
scaling dimension
conformal spin
ε
energy density
I
identity
σ
spin


Δ_D
$\Delta_\alpha \Rightarrow \Delta_\alpha = 2$
2



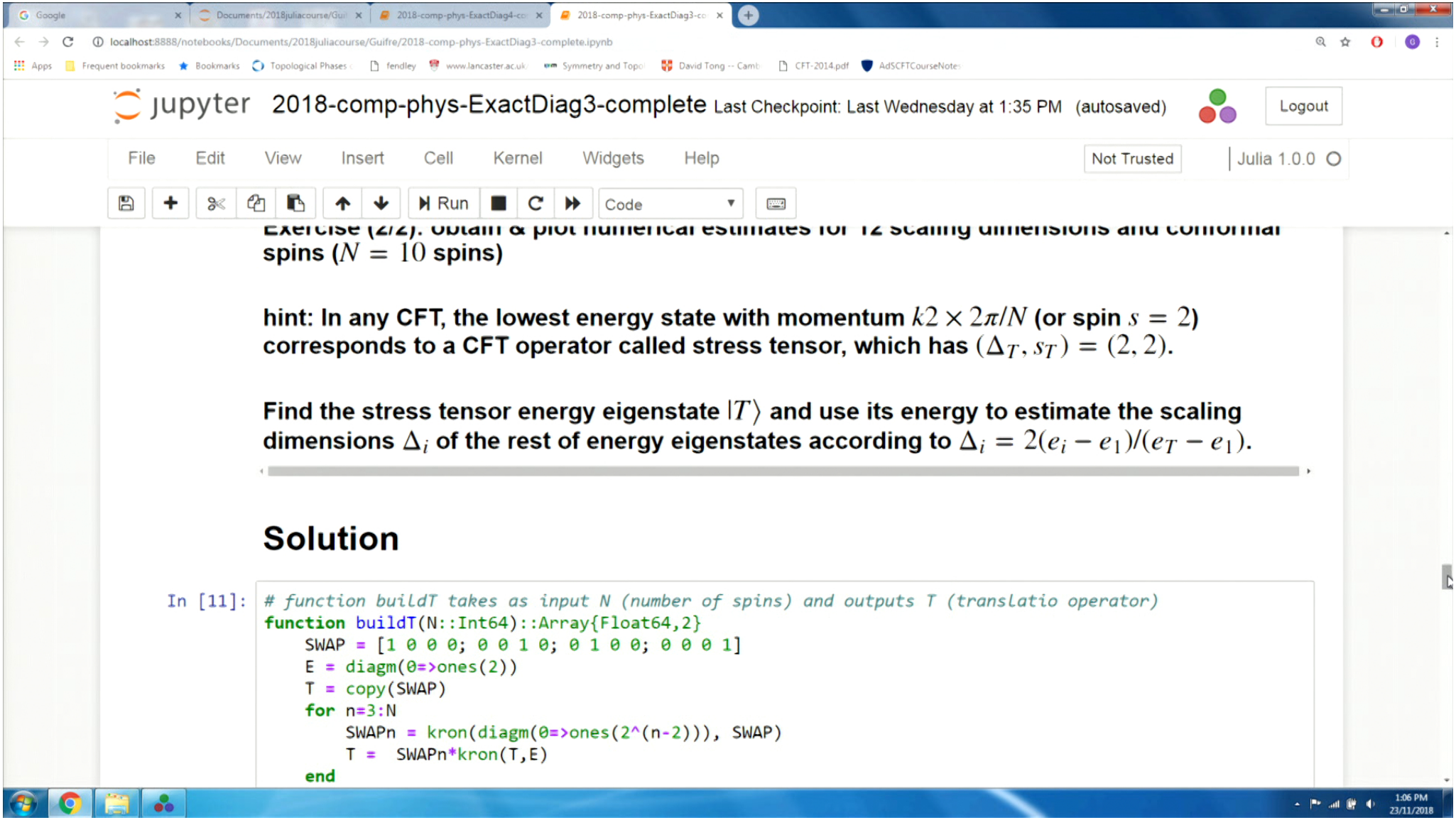
Exercise (2/2): obtain & plot numerical estimates for 12 scaling dimensions and conformal spins ($N = 10$ spins)

hint: In any CFT, the lowest energy state with momentum $k2 \times 2\pi/N$ (or spin $s = 2$) corresponds to a CFT operator called stress tensor, which has $(\Delta_T, s_T) = (2, 2)$.

Find the stress tensor energy eigenstate $|T\rangle$ and use its energy to estimate the scaling dimensions Δ_i of the rest of energy eigenstates according to $\Delta_i = 2(e_i - e_1)/(e_T - e_1)$.

Solution

In [11]:
```
# function buildT takes as input N (number of spins) and outputs T (translatio operator)
function buildT(N::Int64)::Array{Float64,2}
```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Julia 1.0.0
Run
Code
spins ($N = 10$ spins)
hint: In any CFT, the lowest energy state with momentum $k2 \times 2\pi/N$ (or spin $s = 2$) corresponds to a CFT operator called stress tensor, which has $(\Delta_T, s_T) = (2, 2)$.
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Solution
In [11]:
function buildT takes as input N (number of spins) and outputs T (translatio operator)
function buildT(N::Int64)::Array{Float64,2}
SWAP = [1 0 0 0; 0 0 1 0; 0 1 0 0; 0 0 0 1]
E = diagm(0=>ones(2))
T = copy(SWAP)
for n=3:N
SWAPn = kron(diagm(0=>ones(2^(n-2))), SWAP)
T = SWAPn*kron(T,E)
end
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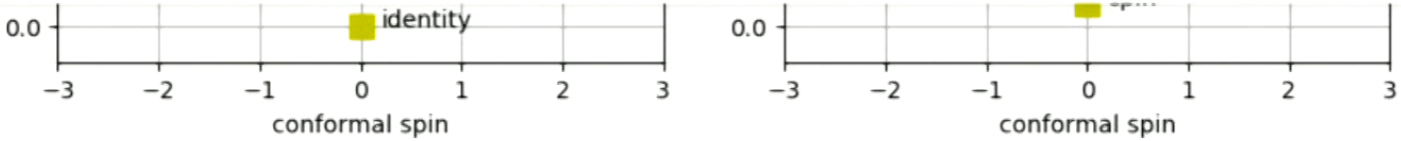
23/11/2018



Exercise (2/2): obtain & plot numerical estimates for 12 scaling dimensions and conformal spins ($N = 10$ spins)

hint: In any CFT, the lowest energy state with momentum $k2 \times 2\pi/N$ (or spin $s = 2$) corresponds to a CFT operator called stress tensor, which has $(\Delta_T, s_T) = (2, 2)$.

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Solution

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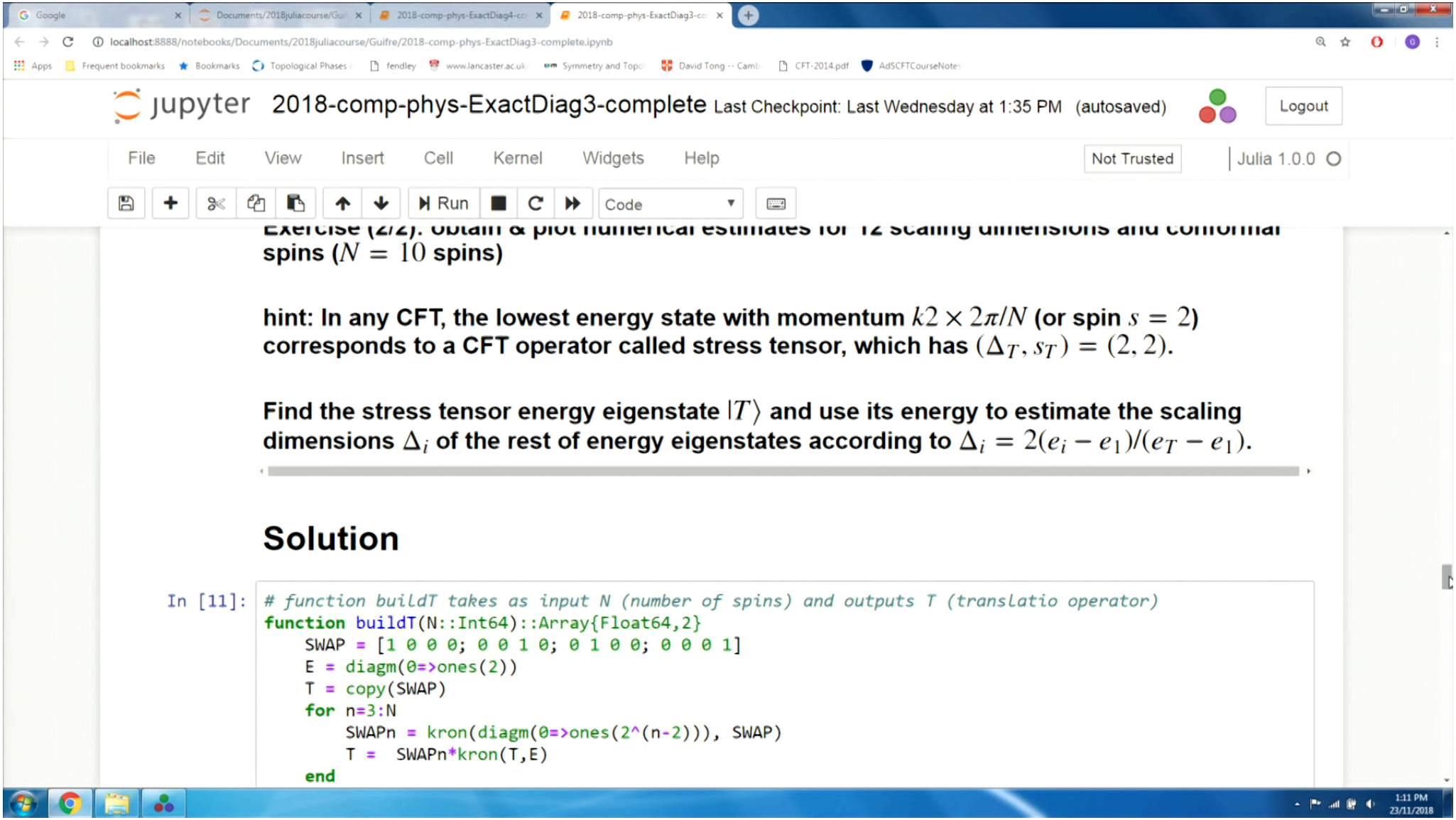
```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Julia 1.0.0
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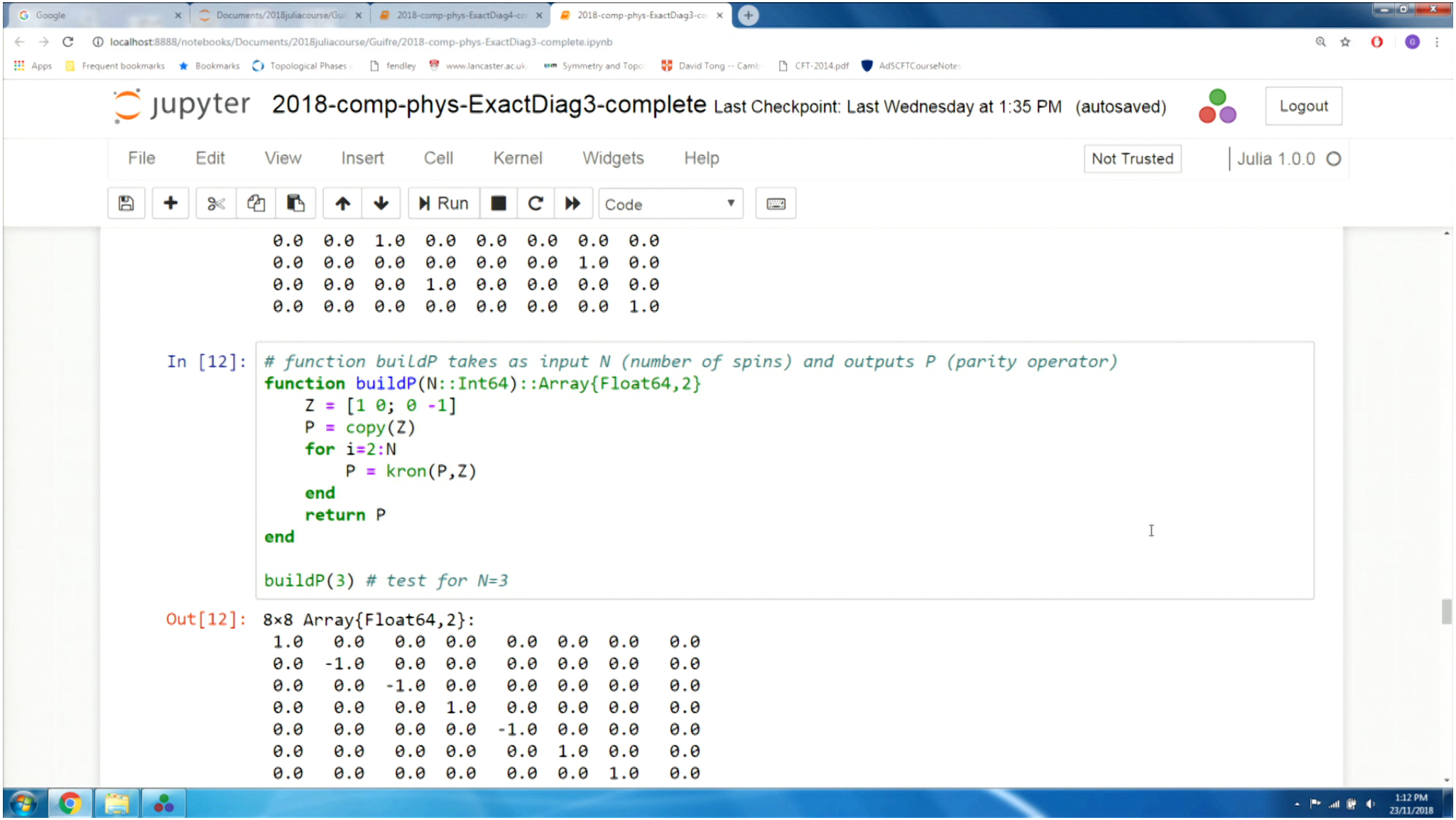
T = copy(SWAP)
for n=3:N
SWAPn = kron(diagm(0=>ones(2^(n-2))), SWAP)
T = SWAPn*kron(T,E)
end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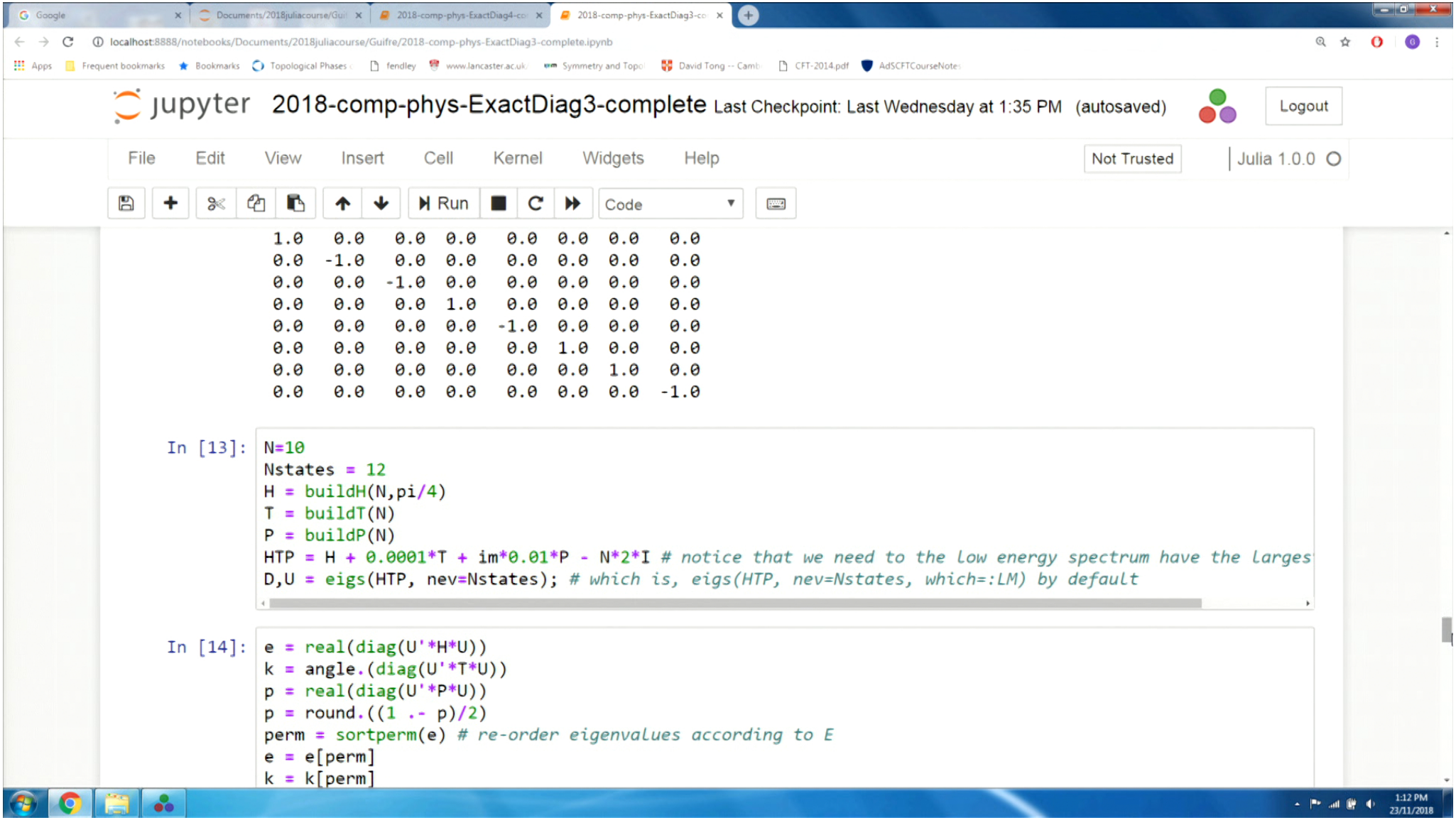
Julia 1.0.0
Run
Code
0.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0
In [12]:
function buildP takes as input N (number of spins) and outputs P (parity operator)
function buildP(N::Int64)::Array{Float64,2}
Z = [1 0; 0 -1]
P = copy(Z)
for i=2:N
P = kron(P,Z)
end
return P
end
buildP(3) # test for N=3
Out[12]:
8×8 Array{Float64,2}:
1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 -1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 -1.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0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Julia 1.0.0
Run
Code
1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 -1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 -1.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 -1.0
In [13]:
N=10
Nstates = 12
H = buildH(N,pi/4)
T = buildT(N)
P = buildP(N)
HTP = H + 0.0001*T + im*0.01*P - N*2*I # notice that we need to the low energy spectrum have the larges
D,U = eigs(HTP, nev=Nstates); # which is, eigs(HTP, nev=Nstates, which=:LM) by default
In [14]:
e = real(diag(U'*H*U))
k = angle.(diag(U'*T*U))
p = real(diag(U'*P*U))
p = round.((1 .- p)/2)
perm = sortperm(e) # re-order eigenvalues according to E
e = e[perm]
k = k[perm]
1:12 PM
23/11/2018

$|\psi_\alpha\rangle$ eigen state of H, T

$e_\alpha = A + B \underbrace{\frac{2\pi}{N}\left(\Delta_\alpha - \frac{c}{12}\right)}_{e_\alpha^{CFT}} + O\left(\frac{1}{N^{1+\gamma}}\right)$

e_α ↑ energy lattice

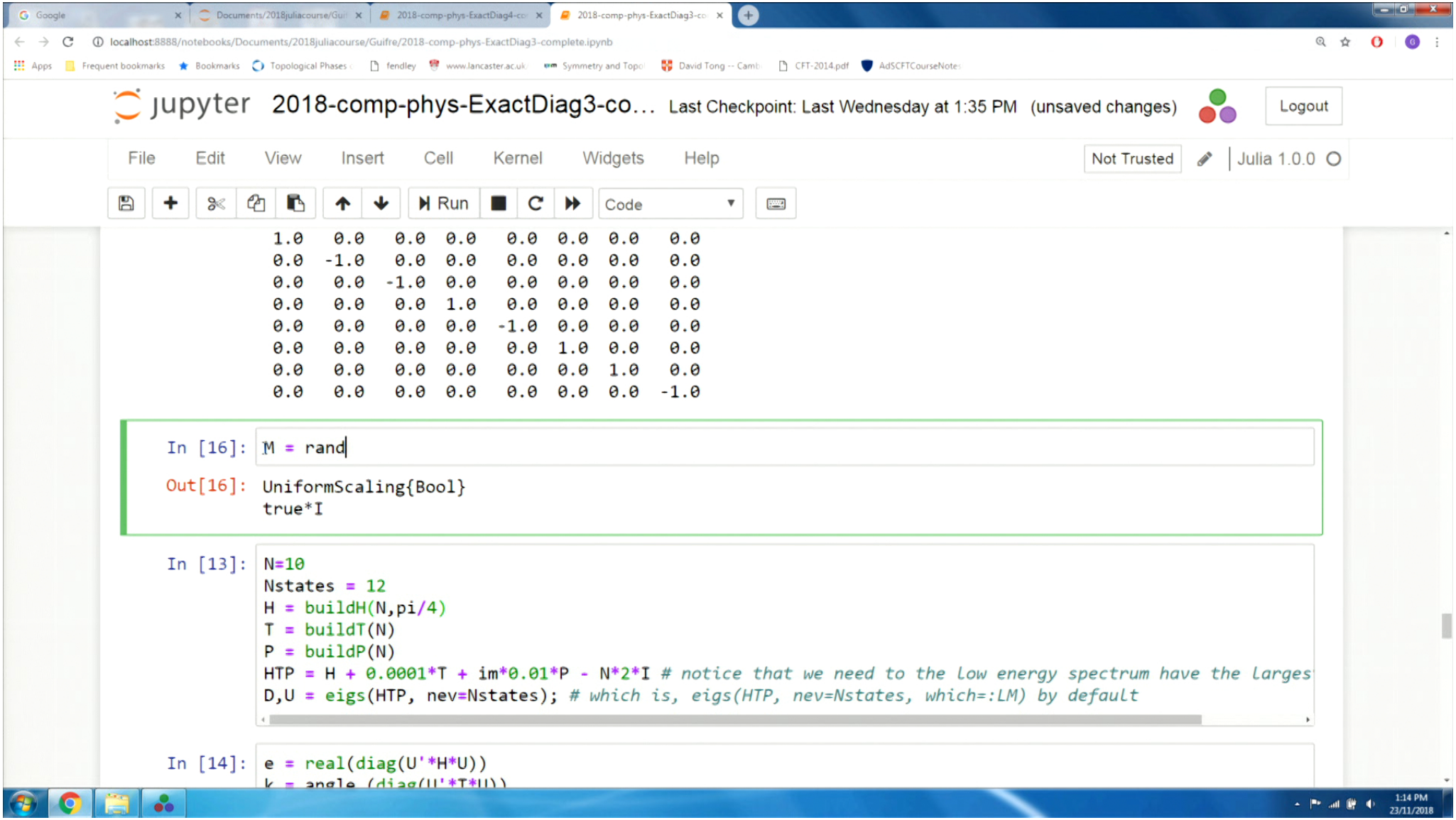
$K_\alpha = \frac{2\pi}{N} S_\alpha$

- identity state (= ground state) $|\mathbb{1}\rangle$ $(\Delta_{\mathbb{1}}, S_{\mathbb{1}}) = (0, 0)$

$$e_\alpha^{CFT} = \frac{2\pi}{N}\left(\Delta_\alpha - \frac{c}{12}\right)$$

- stress tensor state $|T\rangle$ $(\Delta_T, S_T) = (2, 2)$

$$\frac{e_\alpha - e_{\mathbb{1}}}{e_T - e_{\mathbb{1}}} = \frac{\Delta_\alpha - \Delta_{\mathbb{1}}}{\Delta_T - \Delta_{\mathbb{1}}} + \dots = \frac{\Delta_\alpha}{2} \Rightarrow \boxed{\Delta_\alpha = 2\,\frac{e_\alpha - e_{\mathbb{1}}}{e_T - e_{\mathbb{1}}}}$$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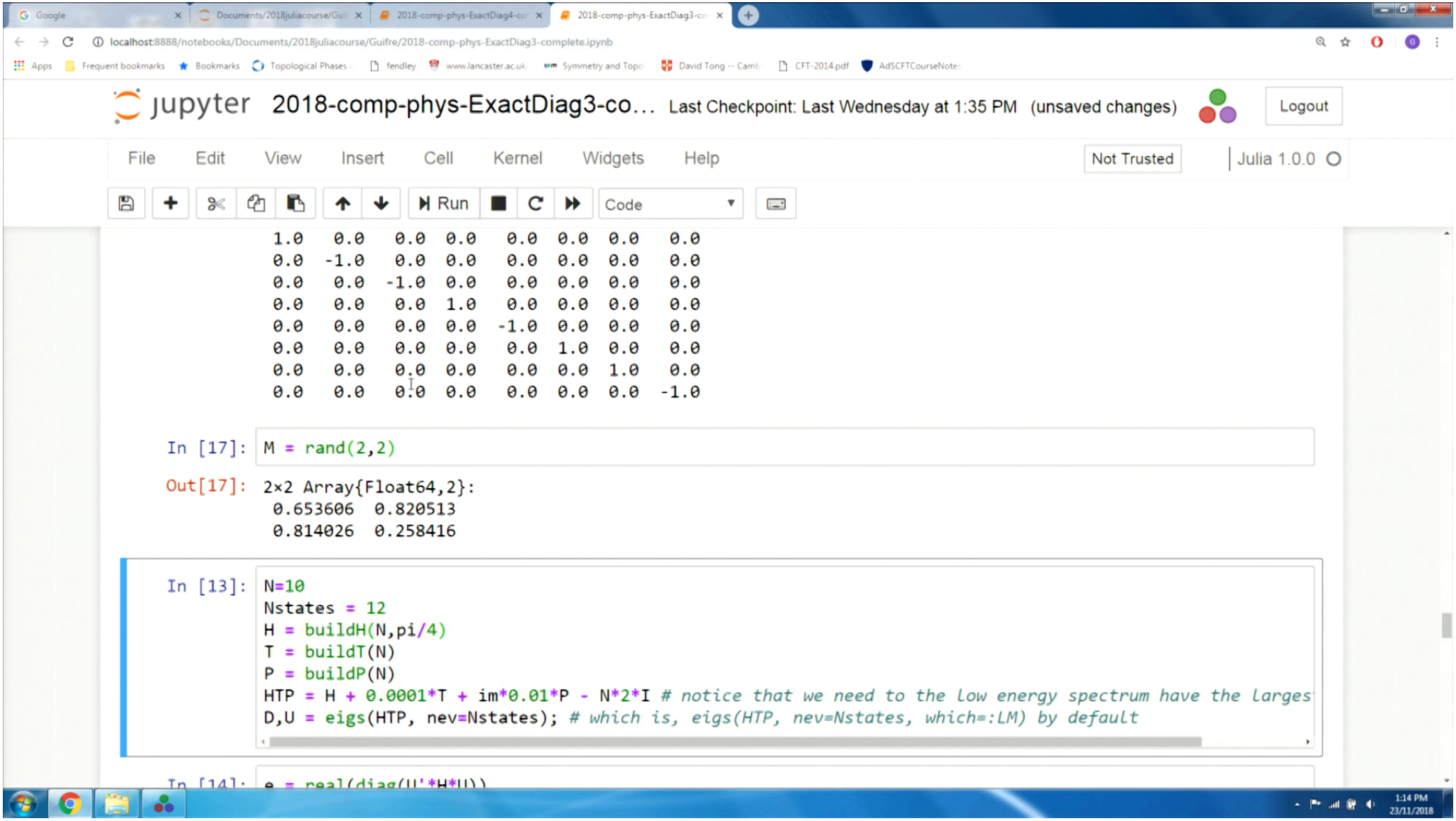
Julia 1.0.0
Run
Code
1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 -1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 -1.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 -1.0
In [16]: M = rand
Out[16]: UniformScaling{Bool}
true*I
In [13]: N=10
Nstates = 12
H = buildH(N,pi/4)
T = buildT(N)
P = buildP(N)
HTP = H + 0.0001*T + im*0.01*P - N*2*I # notice that we need to the low energy spectrum have the larges
D,U = eigs(HTP, nev=Nstates); # which is, eigs(HTP, nev=Nstates, which=:LM) by default
In [14]: e = real(diag(U'*H*U))
1:14 PM
23/11/2018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Julia 1.0.0
Run
Code
1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 -1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 -1.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 -1.0
In [17]: M = rand(2,2)
Out[17]: 2×2 Array{Float64,2}:
0.653606 0.820513
0.814026 0.258416
In [13]: N=10
Nstates = 12
H = buildH(N,pi/4)
T = buildT(N)
P = buildP(N)
HTP = H + 0.0001*T + im*0.01*P - N*2*I # notice that we need to the low energy spectrum have the larges
D,U = eigs(HTP, nev=Nstates); # which is, eigs(HTP, nev=Nstates, which=:LM) by default
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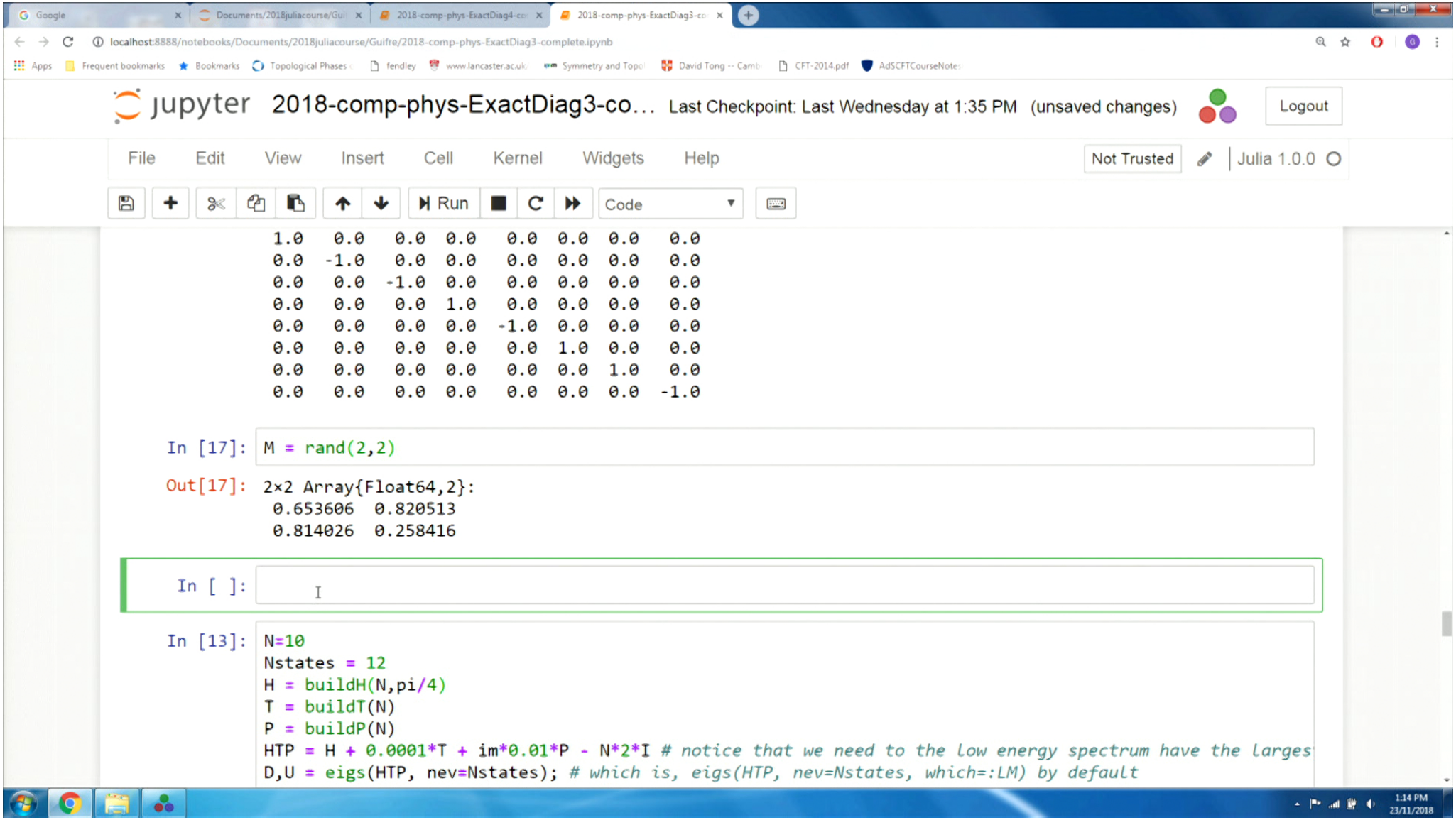
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Julia 1.0.0
Run
Code
1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 -1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 -1.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
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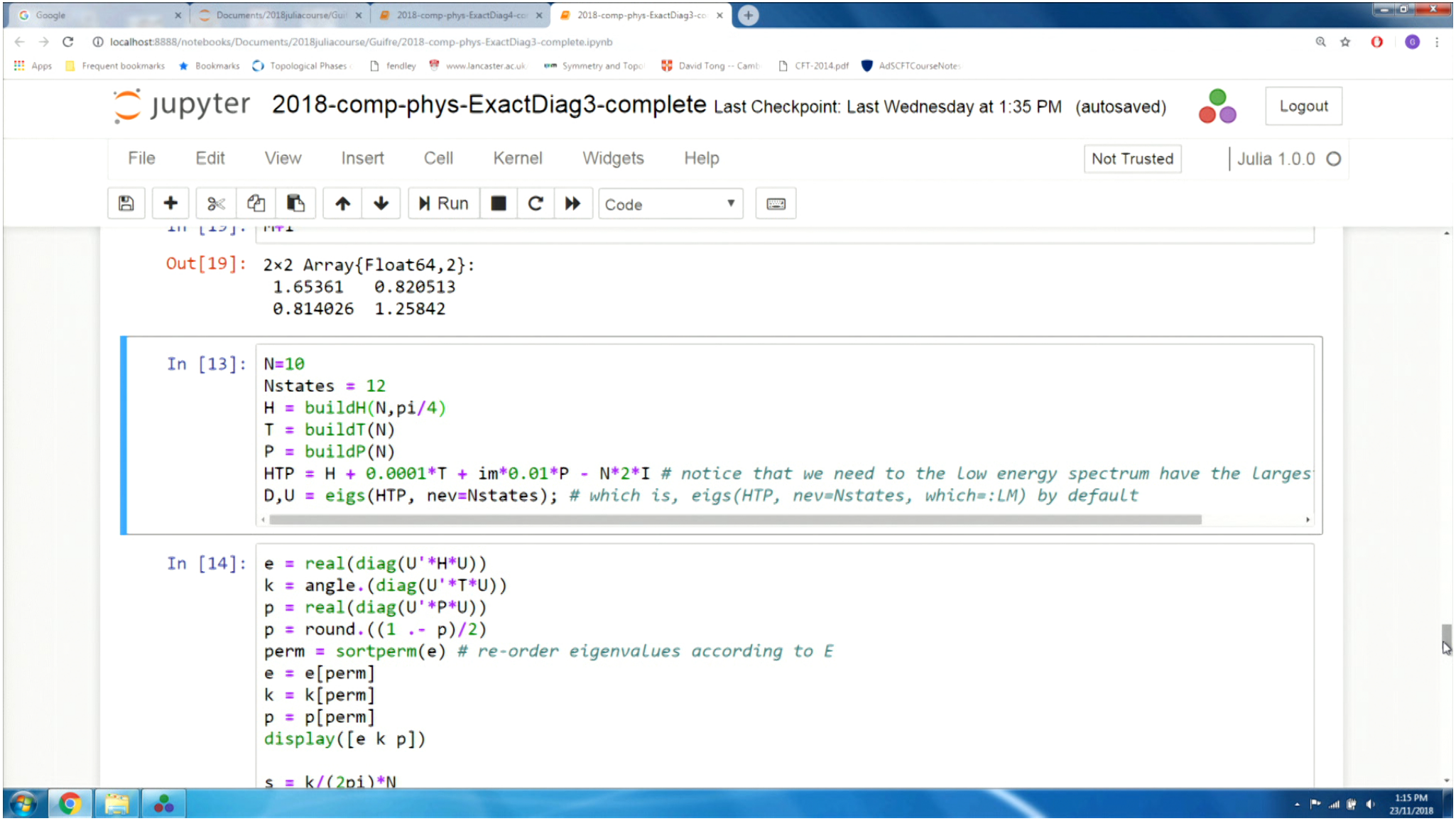
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Julia 1.0.0
Run
Code
Out[19]: 2×2 Array{Float64,2}:
1.65361 0.820513
0.814026 1.25842
In [13]:
N=10
Nstates = 12
H = buildH(N,pi/4)
T = buildT(N)
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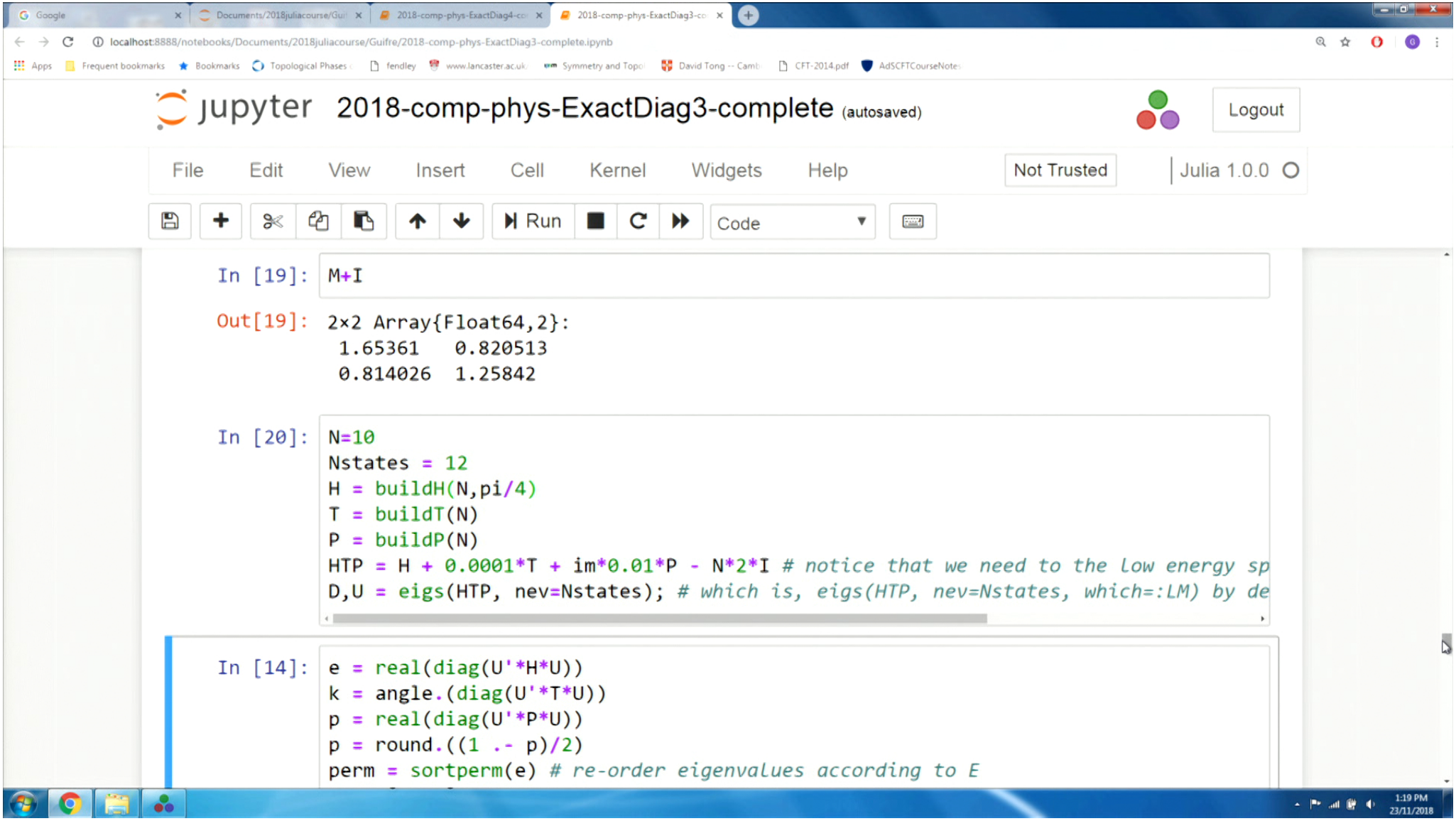
k = angle.(diag(U'*T*U))
p = real(diag(U'*P*U))
p = round.((1 .- p)/2)
perm = sortperm(e) # re-order eigenvalues according to E
e = e[perm]
k = k[perm]
p = p[perm]
display([e k p])
s = k/(2pi)*N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File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
In [19]:
M+I
Out[19]:
2×2 Array{Float64,2}:
1.65361 0.820513
0.814026 1.25842
In [20]:
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Nstates = 12
H = buildH(N,pi/4)
T = buildT(N)
P = buildP(N)
HTP = H + 0.0001*T + im*0.01*P - N*2*I # notice that we need to the low energy sp
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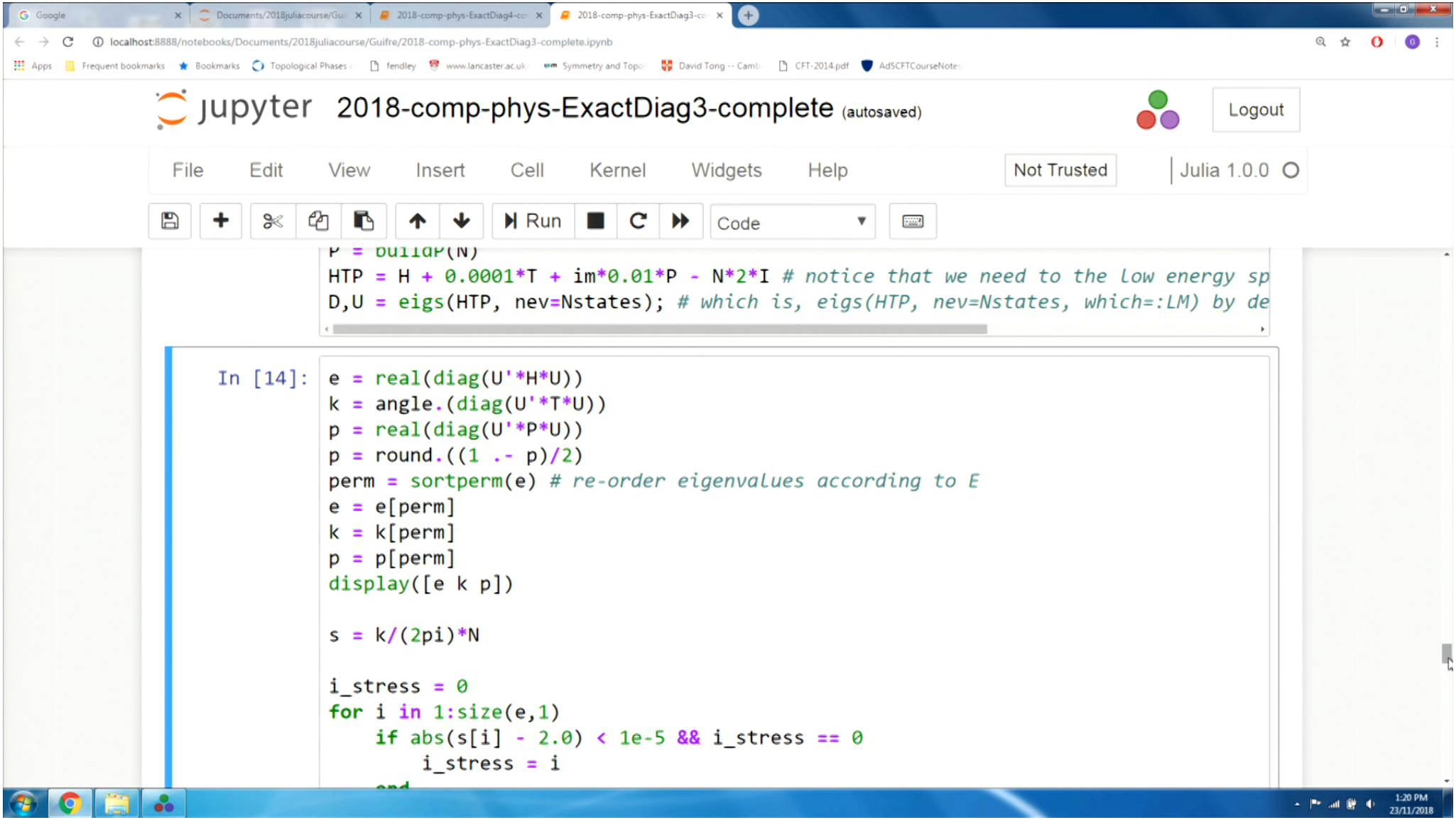
perm = sortperm(e) # re-order eigenvalues according to E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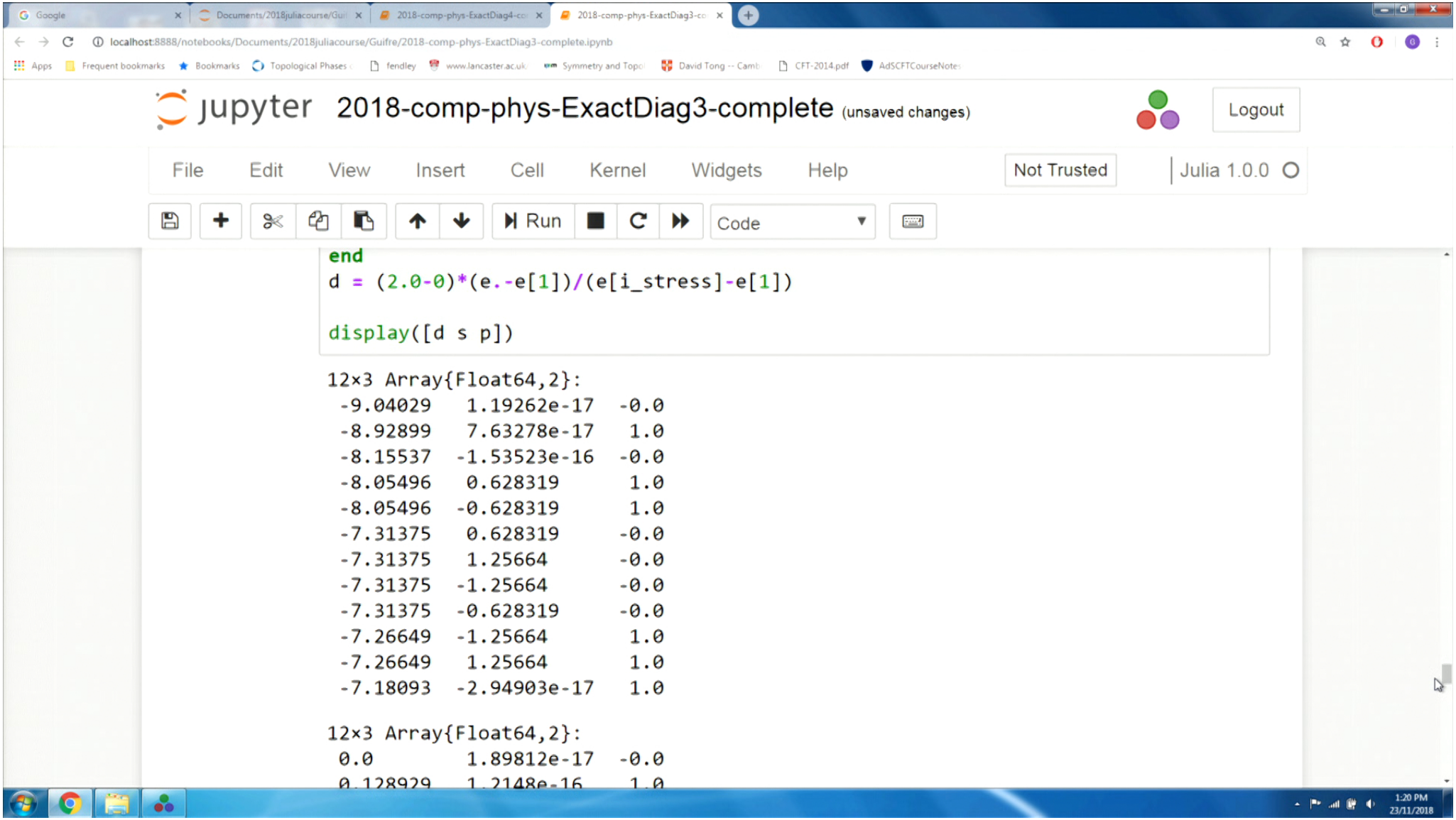
p = round.((1 .- p)/2)
perm = sortperm(e) # re-order eigenvalues according to E
e = e[perm]
k = k[perm]
p = p[perm]
display([e k p])
s = k/(2pi)*N
i_stress = 0
for i in 1:size(e,1)
if abs(s[i] - 2.0) < 1e-5 && i_stress == 0
i_stress = i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Julia 1.0.0
Run
Code
end
d = (2.0-0)*(e.-e[1])/(e[i_stress]-e[1])
display([d s p])
12×3 Array{Float64,2}:
-9.04029 1.19262e-17 -0.0
-8.92899 7.63278e-17 1.0
-8.15537 -1.53523e-16 -0.0
-8.05496 0.628319 1.0
-8.05496 -0.628319 1.0
-7.31375 0.628319 -0.0
-7.31375 1.25664 -0.0
-7.31375 -1.25664 -0.0
-7.31375 -0.628319 -0.0
-7.26649 -1.25664 1.0
-7.26649 1.25664 1.0
-7.18093 -2.94903e-17 1.0
12×3 Array{Float64,2}:
0.0 1.89812e-17 -0.0

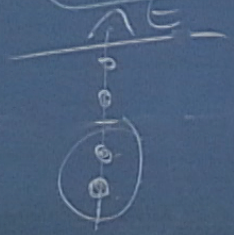
eigen state of H, T

energy

lattice

$K_\alpha = \frac{2\pi}{N} S_\alpha$

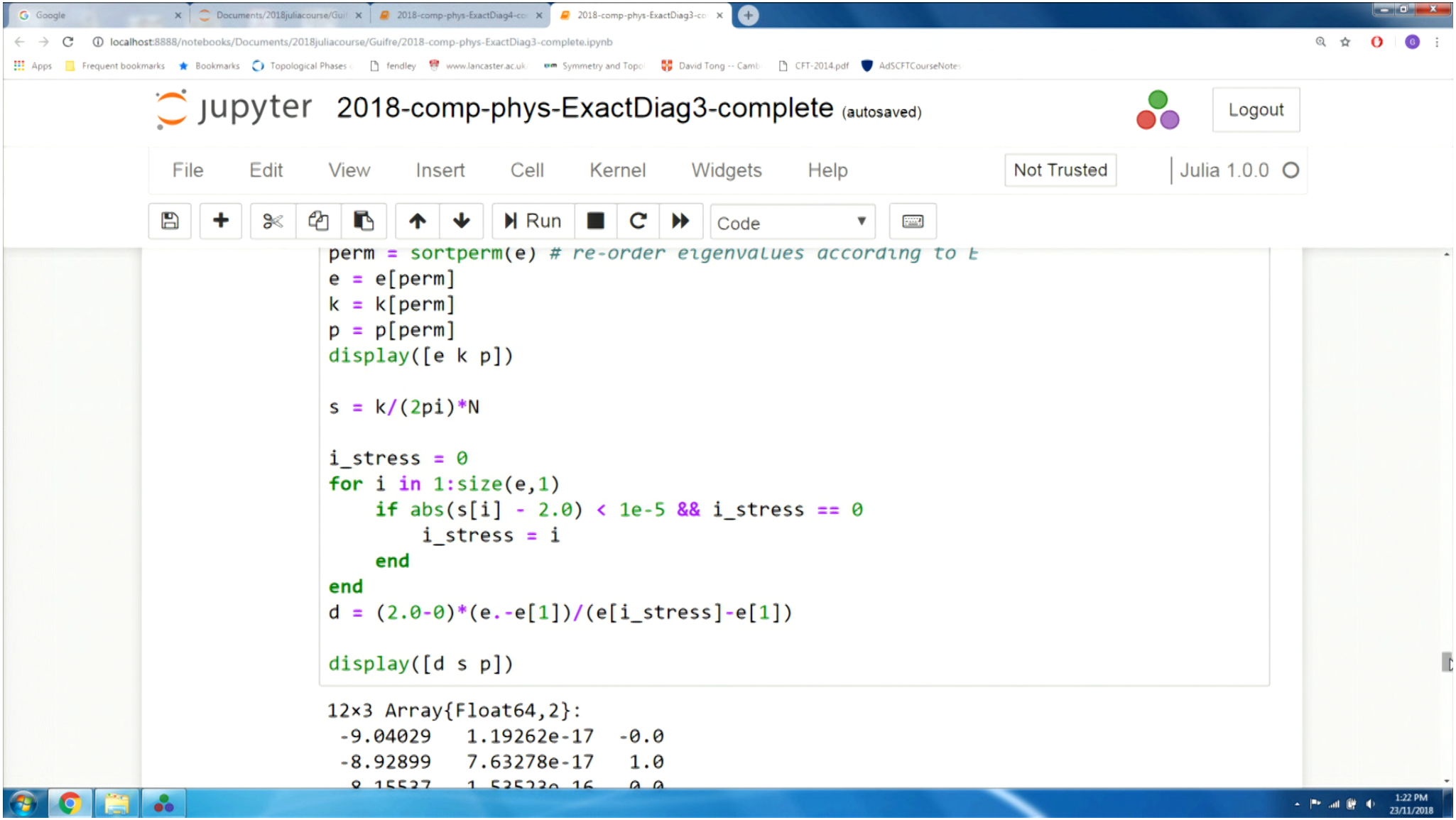
$e^{CFT}_\alpha = \frac{2\pi}{N}\left(\Delta_\alpha - \frac{c}{12}\right)$



- identity state (= ground state) $|\mathbb{1}\rangle$ $(\Delta_{\mathbb{1}}, S_{\mathbb{1}}) = (0, 0)$
- stress tensor state $|T\rangle$ $(\Delta_T, S_T) = (2, 2)$

$$\frac{e_\alpha - e_{\mathbb{1}}}{e_T - e_{\mathbb{1}}} = \frac{\Delta_\alpha - \Delta_{\mathbb{1}}}{\Delta_T - \Delta_{\mathbb{1}}} + \ldots = \frac{\Delta_\alpha}{2} \Rightarrow \boxed{\Delta_\alpha = 2\,\frac{e_\alpha - e_{\mathbb{1}}}{e_T - e_{\mathbb{1}}}}$$

eigs(M, nev = 2, which =: SR)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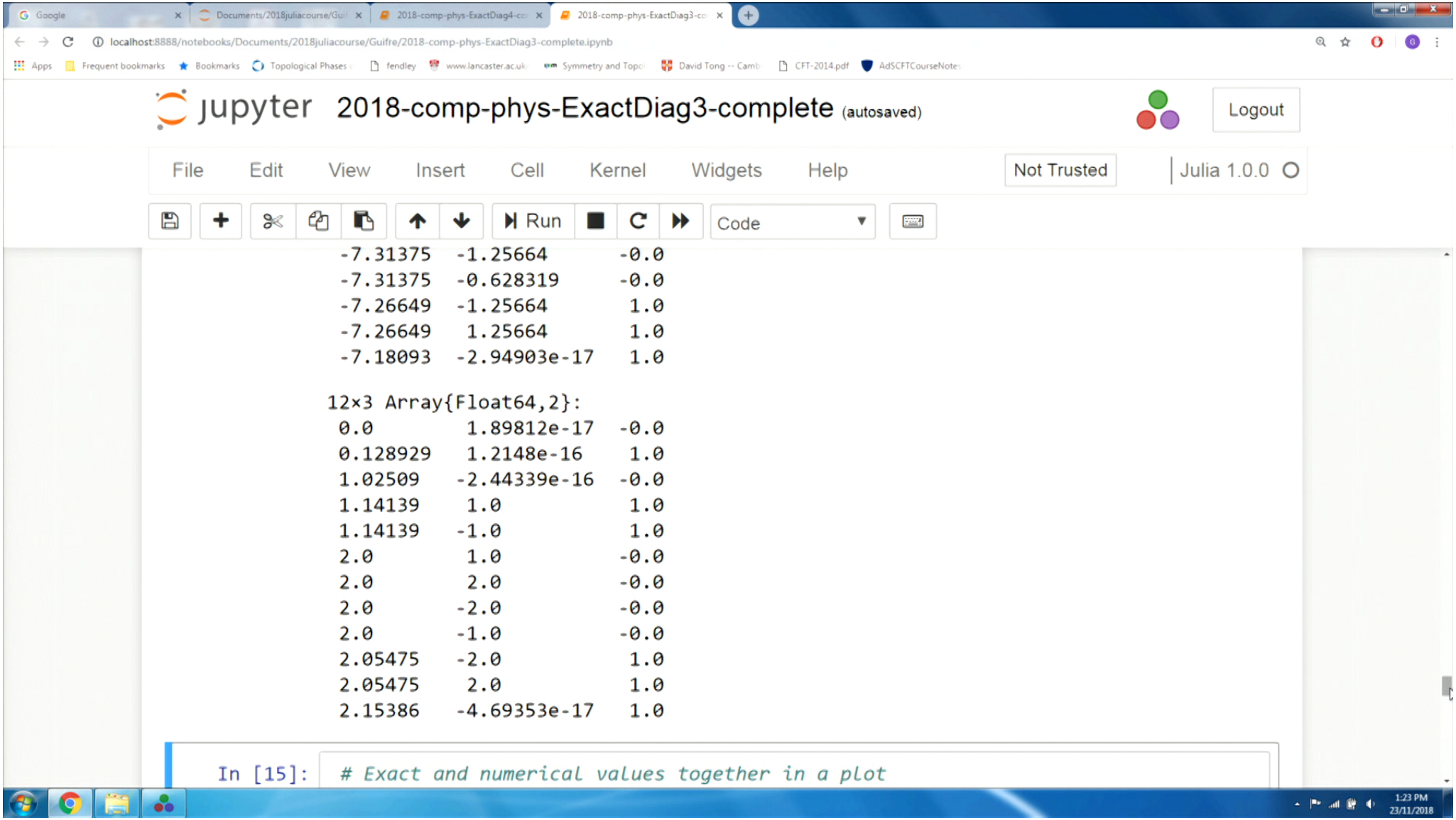
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
perm = sortperm(e) # re-order eigenvalues according to E
e = e[perm]
k = k[perm]
p = p[perm]
display([e k p])
s = k/(2pi)*N
i_stress = 0
for i in 1:size(e,1)
if abs(s[i] - 2.0) < 1e-5 && i_stress == 0
i_stress = i
end
end
d = (2.0-0)*(e.-e[1])/(e[i_stress]-e[1])
display([d s p])
12×3 Array{Float64,2}:
-9.04029 1.19262e-17 -0.0
-8.92899 7.63278e-17 1.0


jupyter 2018-comp-phys-ExactDiag3-complete (autosaved)
Logout
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
-7.31375 -1.25664 -0.0
-7.31375 -0.628319 -0.0
-7.26649 -1.25664 1.0
-7.26649 1.25664 1.0
-7.18093 -2.94903e-17 1.0
12×3 Array{Float64,2}:
0.0 1.89812e-17 -0.0
0.128929 1.2148e-16 1.0
1.02509 -2.44339e-16 -0.0
1.14139 1.0 1.0
1.14139 -1.0 1.0
2.0 1.0 -0.0
2.0 2.0 -0.0
2.0 -2.0 -0.0
2.0 -1.0 -0.0
2.05475 -2.0 1.0
2.05475 2.0 1.0
2.15386 -4.69353e-17 1.0
In [15]: # Exact and numerical values together in a plot

jupyter 2018-comp-phys-ExactDiag3-complete (unsaved changes)

Logout

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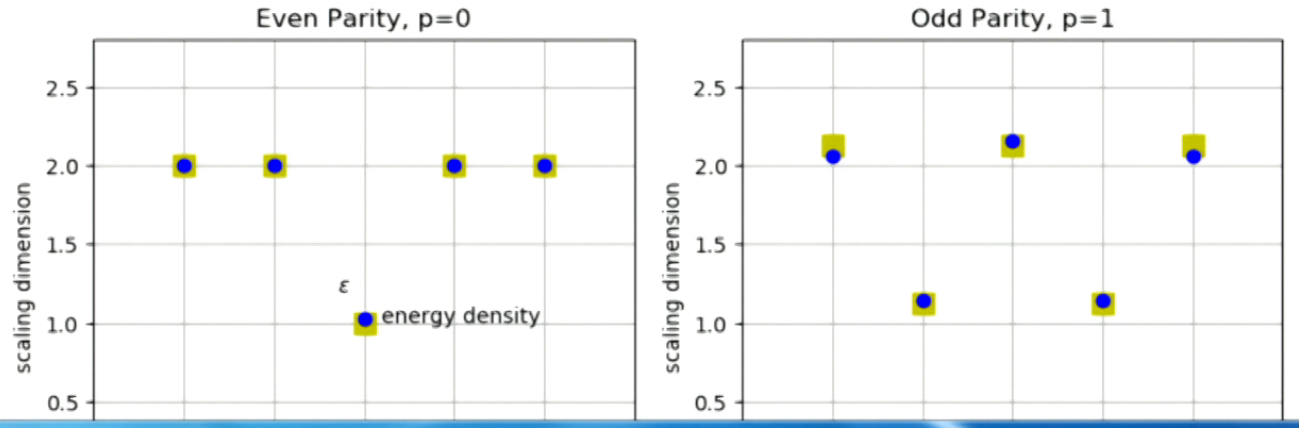
Run Code

```
        subplot(121)
        plot(s[i],d[i], marker = "o", color = "b")
    elseif p[i] == 1
        subplot(122)
        plot(s[i],d[i], marker = "o", color = "b")
    end
end
```




jupyter 2018-comp-phys-ExactDiag3-complete (unsaved changes)
Logout
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Julia 1.0.0
Run
Code
end
end
Even Parity, p=0
Odd Parity, p=1
scaling dimension
conformal spin
energy density
identity
spin
2.5
2.0
1.5
1.0
0.5
0.0
−3 −2 −1 0 1 2 3

lecture 4 Quantum entanglement

Q.M. bipartite system AB $\quad \mathbb{V}_{AB} \cong \mathbb{V}_A \otimes \mathbb{V}_B$

$\{|ij\rangle = |i_A\rangle \otimes |j_B\rangle\}$ $\quad \{|i_A\rangle\}$ $\quad \{|j_B\rangle\}$

A $\quad$ B

$=$ $\quad \otimes \quad =$

$|1\rangle$ $\quad$ $|1\rangle$

$|2\rangle$ $\quad$ $|2\rangle$

$|3\rangle$

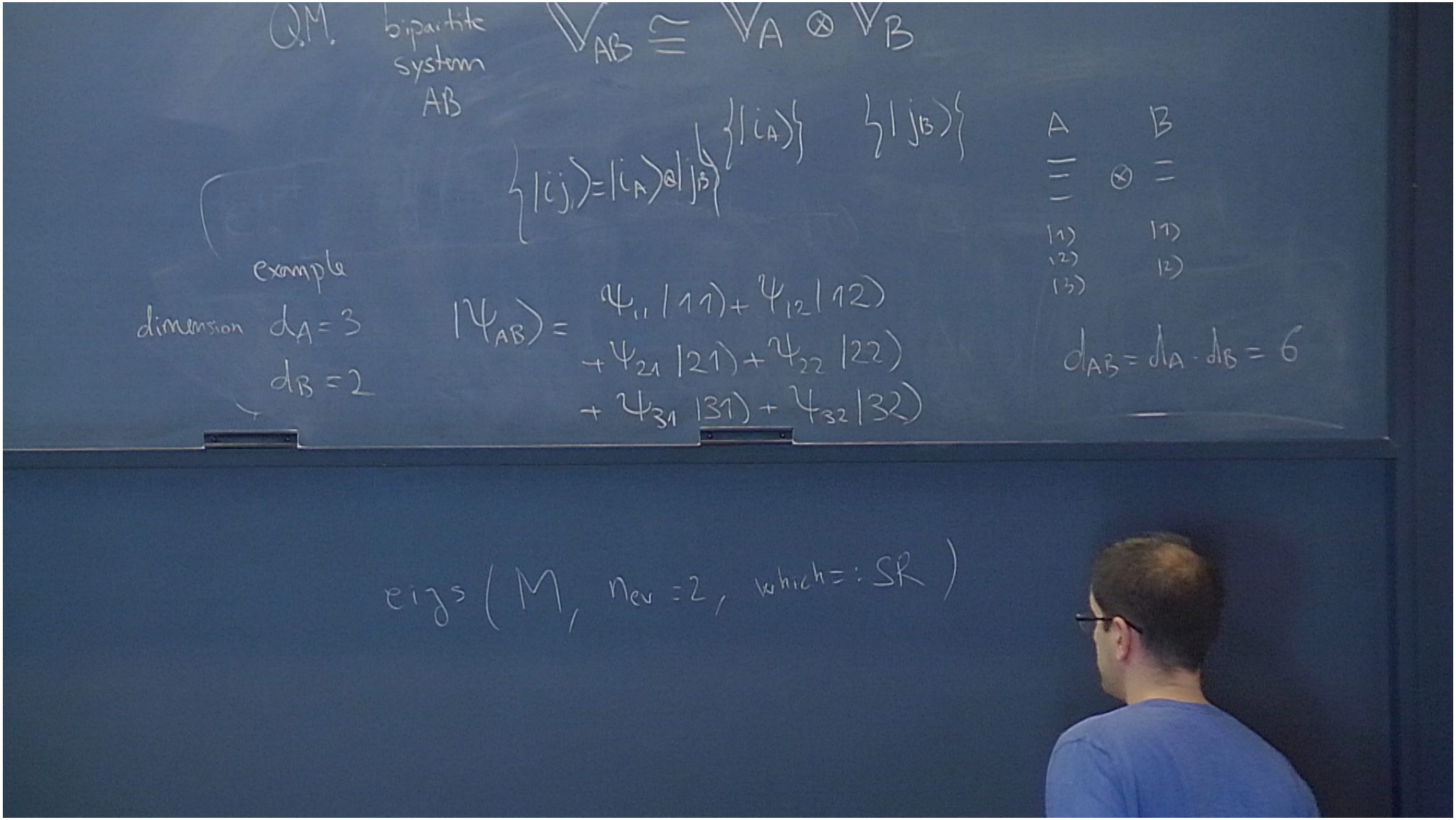
example

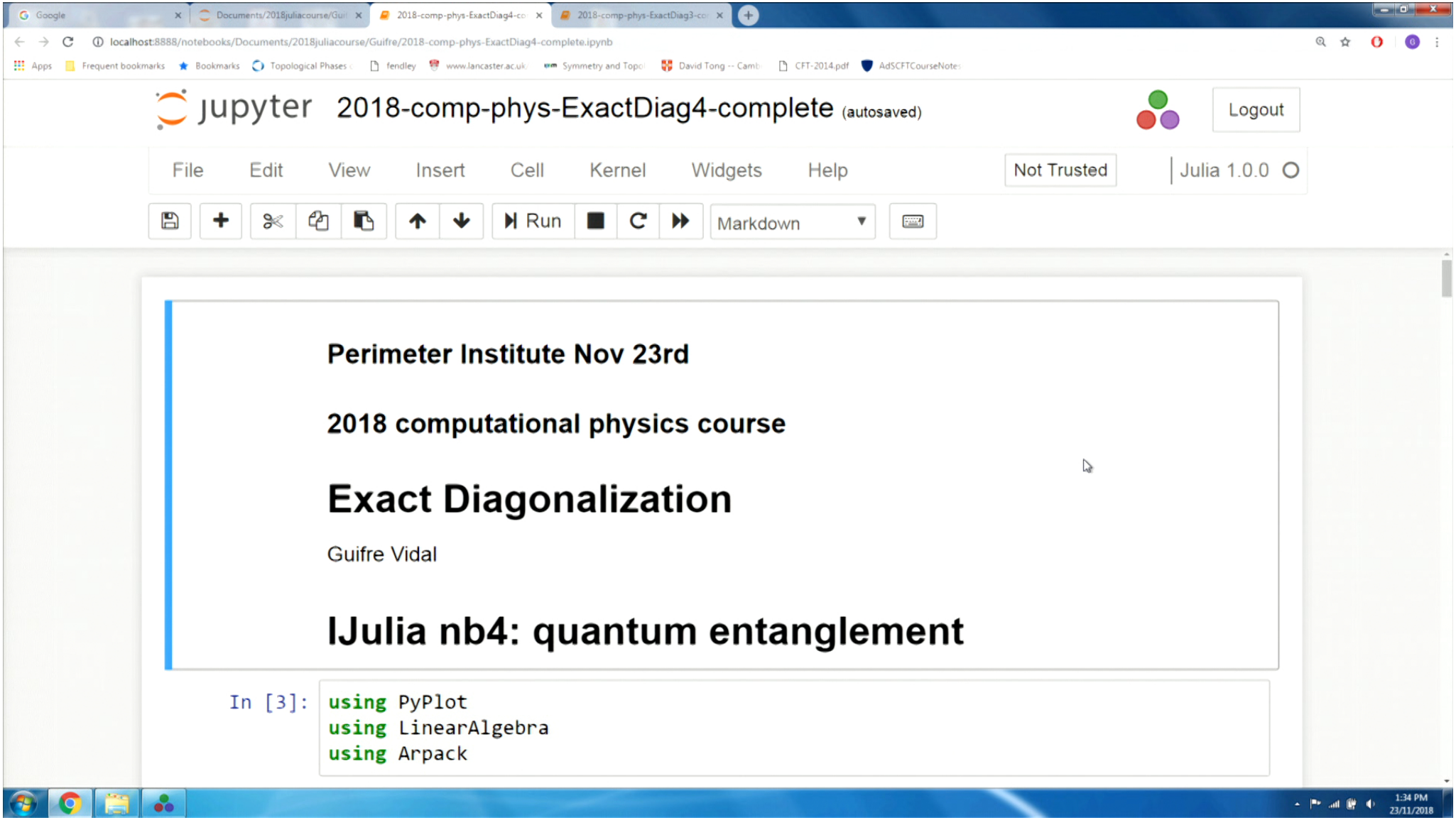
dimension $d_A = 3$

$d_B = 2$

$$|\psi_{AB}\rangle = \psi_{11}|11\rangle + \psi_{12}|12\rangle + \psi_{21}|21\rangle + \psi_{22}|22\rangle + \psi_{31}|31\rangle + \psi_{32}|32\rangle$$

$d_{AB} = d_A \cdot d_B = 6$


Q.M. bipartite system AB
$\mathcal{V}_{AB} \cong \mathcal{V}_A \otimes \mathcal{V}_B$
$\{|i_A\rangle\}$ $\{|j_B\rangle\}$
$\{|ij\rangle = |i_A\rangle \otimes |j_B\rangle\}$
A = ⊗ =
|1⟩ |1⟩
|2⟩ |2⟩
|3⟩
example
dimension $d_A = 3$
$d_B = 2$
$|\Psi_{AB}\rangle = \Psi_{11}|11\rangle + \Psi_{12}|12\rangle + \Psi_{21}|21\rangle + \Psi_{22}|22\rangle + \Psi_{31}|31\rangle + \Psi_{32}|32\rangle$
$d_{AB} = d_A \cdot d_B = 6$
eigs(M, nev=2, which=:SR)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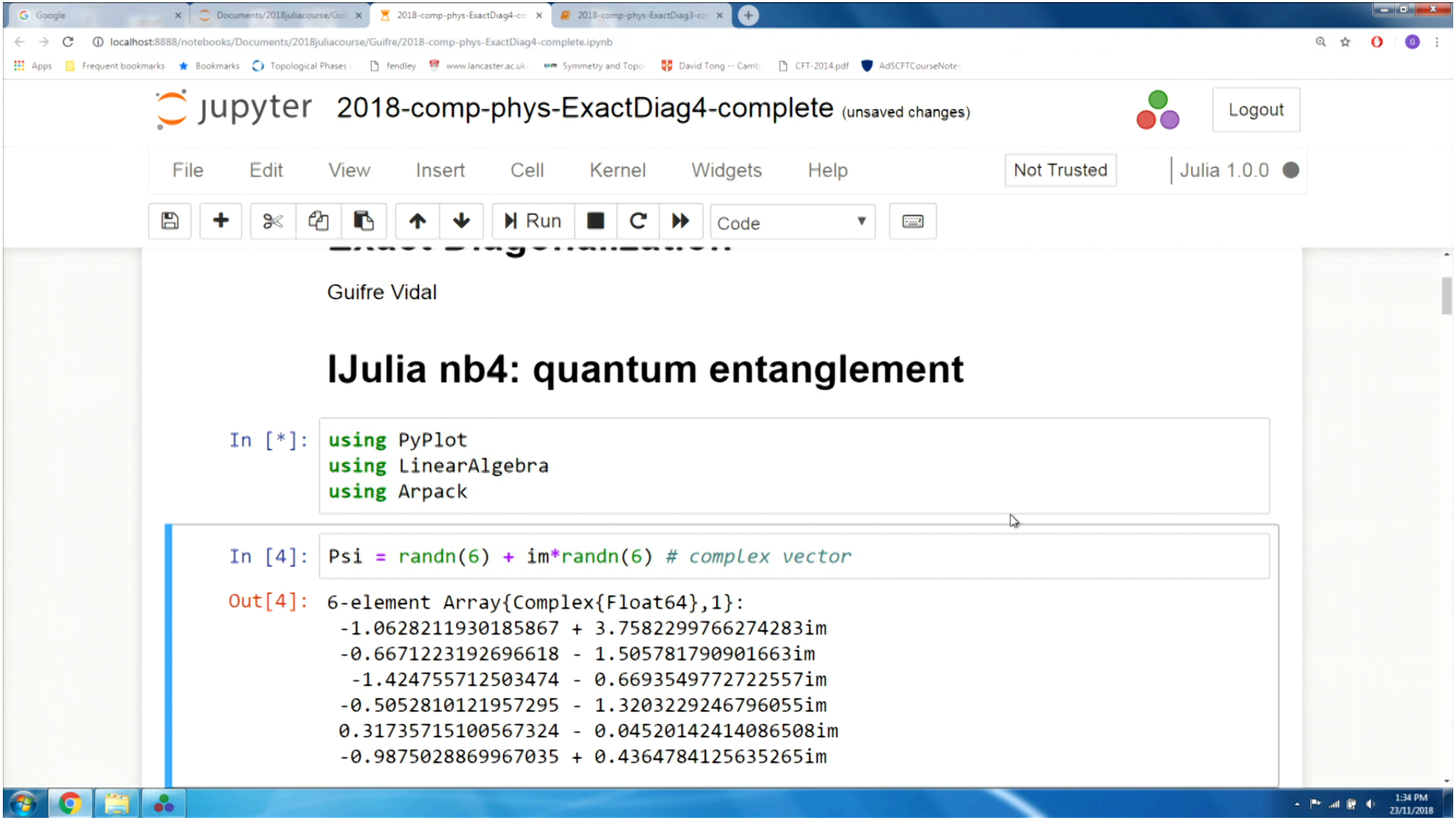
Julia 1.0.0
Run
Markdown
Perimeter Institute Nov 23rd
2018 computational physics course
Exact Diagonalization
Guifre Vidal
IJulia nb4: quantum entanglement
In [3]:
using PyPlot
using LinearAlgebra
using Arpack


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
Logout
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
Guifre Vidal
IJulia nb4: quantum entanglement
In [*]:
using PyPlot
using LinearAlgebra
using Arpack
In [4]:
Psi = randn(6) + im*randn(6) # complex vector
Out[4]:
6-element Array{Complex{Float64},1}:
-1.0628211930185867 + 3.7582299766274283im
-0.6671223192696618 - 1.505781790901663im
-1.424755712503474 - 0.6693549772722557im
-0.5052810121957295 - 1.3203229246796055im
0.31735715100567324 - 0.04520142414086508im
-0.9875028869967035 + 0.4364784125635265im

example

dimension $d_A = 3$

$d_B = 2$

$|\psi_{AB}\rangle = \psi_{11}|11\rangle + \psi_{12}|12\rangle + \psi_{21}|21\rangle + \psi_{22}|22\rangle + \psi_{31}|31\rangle + \psi_{32}|32\rangle$

$|1\rangle \quad |1\rangle$
$|2\rangle \quad |2\rangle$
$|3\rangle$

$d_{AB} = d_A \cdot d_B = 6$

---

def: product state $|\psi_{AB}\rangle = |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$

$|23\rangle = |2\rangle \otimes |3\rangle$

example $|11\rangle = |1\rangle \otimes |1\rangle$

$\frac{1}{2}(|11\rangle + |22\rangle + |21\rangle + |12\rangle)$

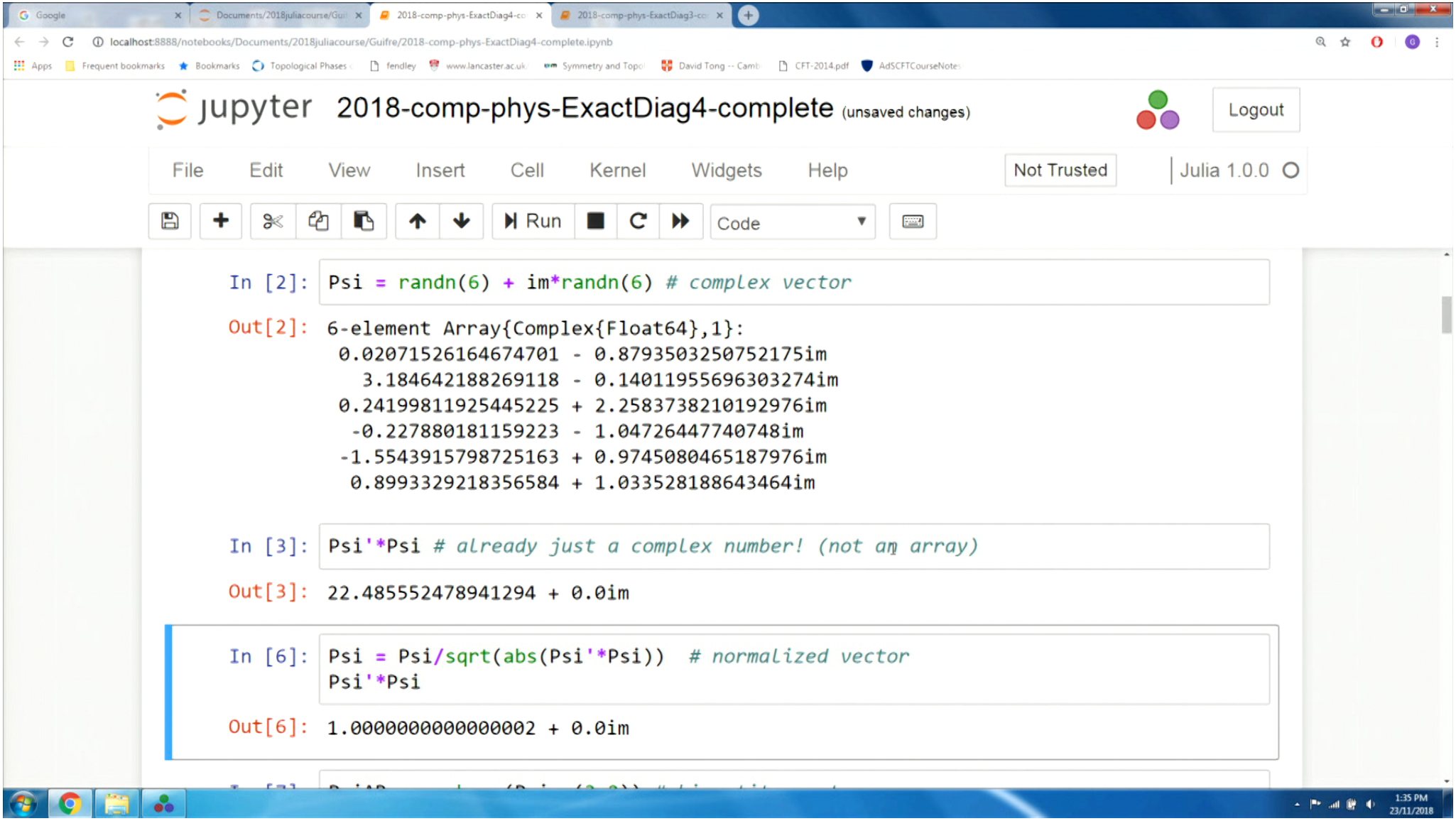
$= \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \otimes \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$

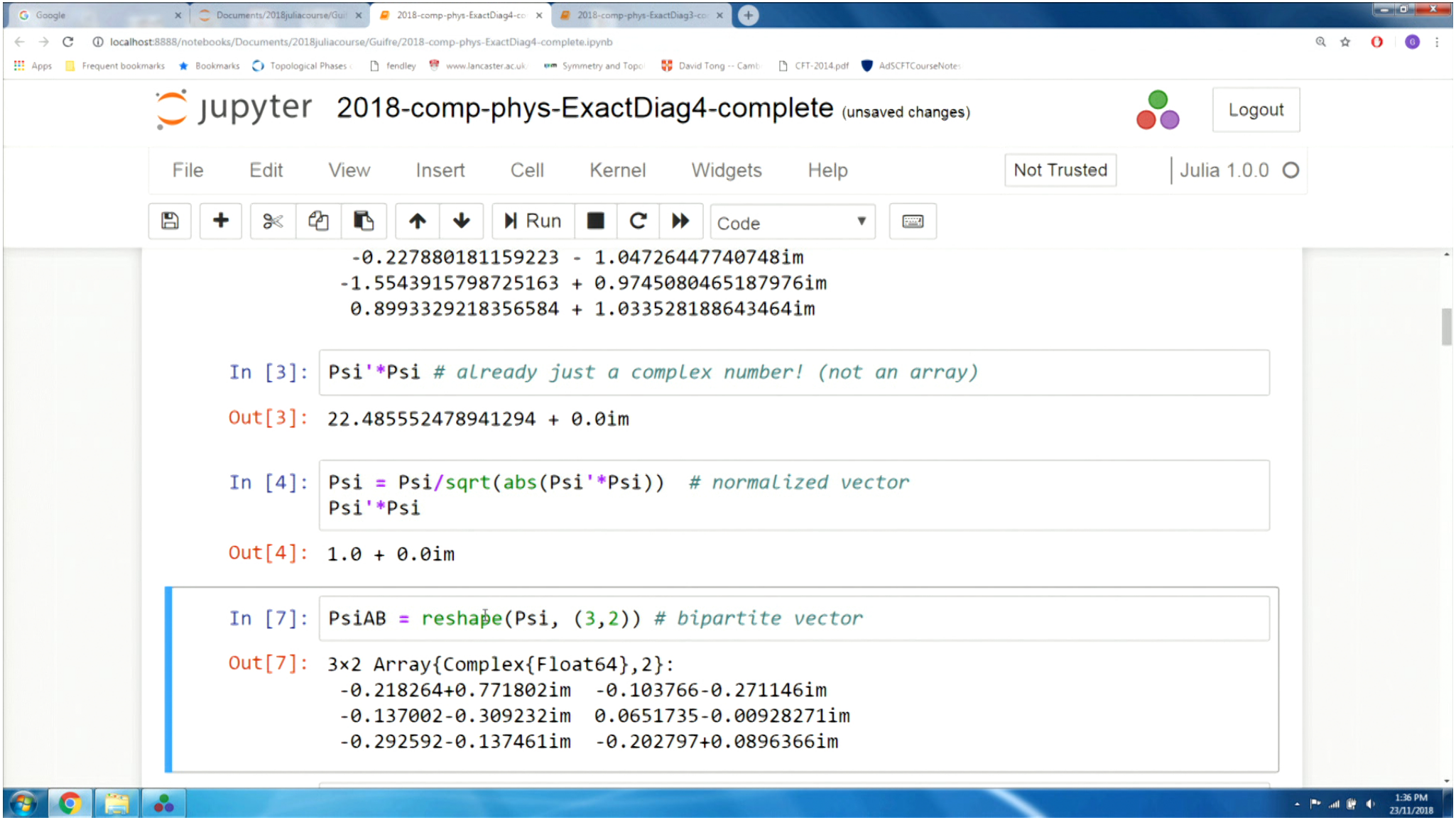
$\frac{1}{\sqrt{2}}(|11\rangle + |12\rangle) = |1\rangle \otimes \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$

def entangled state

$|\psi_{AB}\rangle \neq |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$

example $\frac{1}{\sqrt{2}}(|11\rangle + |22\rangle)$


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
Logout
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Code
In [2]: Psi = randn(6) + im*randn(6) # complex vector
Out[2]: 6-element Array{Complex{Float64},1}:
0.02071526164674701 - 0.8793503250752175im
3.184642188269118 - 0.14011955696303274im
0.24199811925445225 + 2.2583738210192976im
-0.227880181159223 - 1.047264477407 48im
-1.5543915798725163 + 0.9745080465187976im
0.8993329218356584 + 1.0335281886434 64im
In [3]: Psi'*Psi # already just a complex number! (not an array)
Out[3]: 22.485552478941294 + 0.0im
In [6]: Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[6]: 1.0000000000000002 + 0.0im


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
Logout
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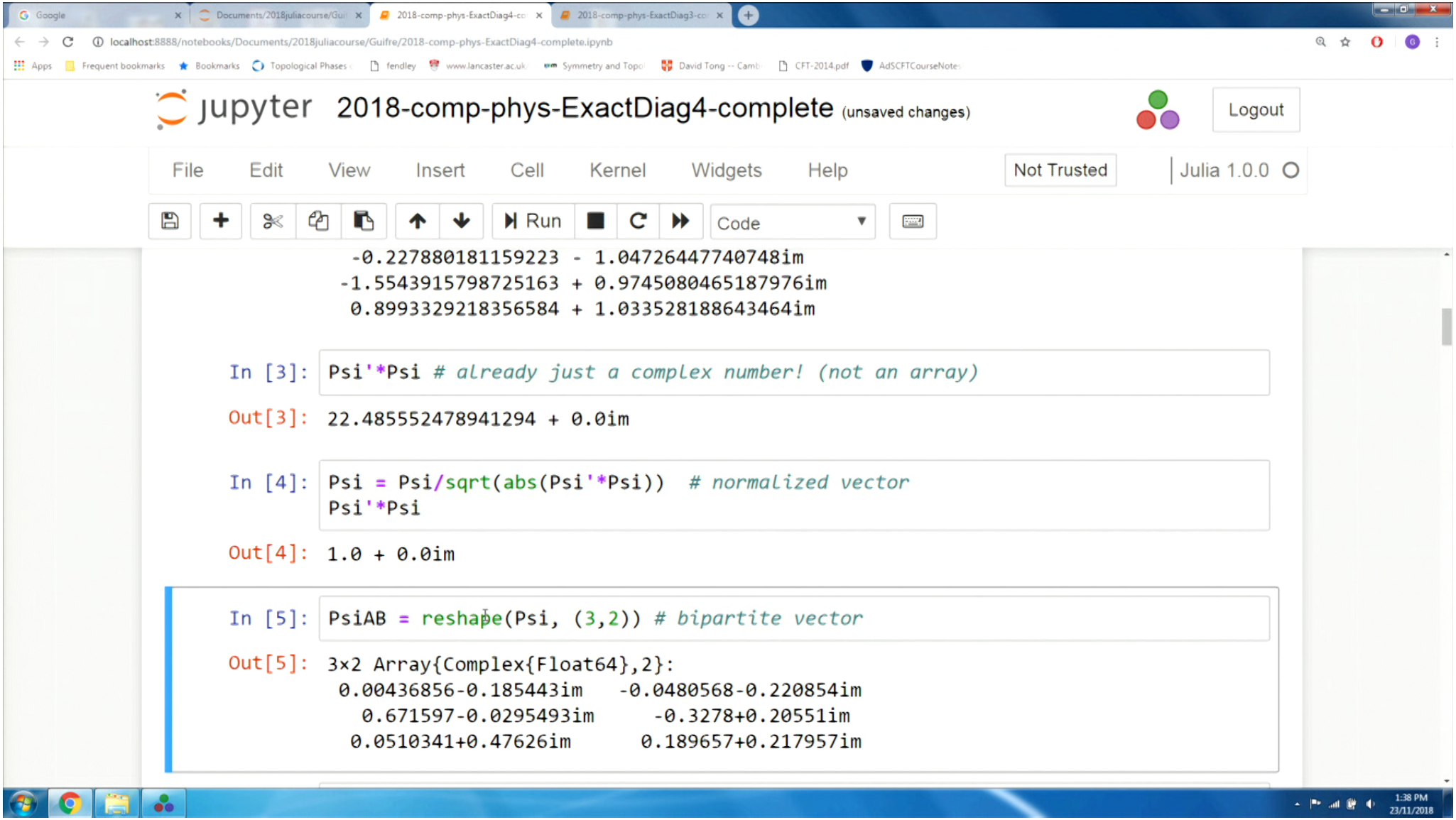
Julia 1.0.0
Run
Code
-0.227880181159223 - 1.047264477407 48im
-1.5543915798725163 + 0.9745080465187976im
0.8993329218356584 + 1.0335281886434 64im
In [3]:
Psi'*Psi # already just a complex number! (not an array)
Out[3]:
22.485552478941294 + 0.0im
In [4]:
Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[4]:
1.0 + 0.0im
In [7]:
PsiAB = reshape(Psi, (3,2)) # bipartite vector
Out[7]:
3×2 Array{Complex{Float64},2}:
-0.218264+0.771802im -0.103766-0.271146im
-0.137002-0.309232im 0.0651735-0.00928271im
-0.292592-0.137461im -0.202797+0.0896366im


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
Logout
File Edit View Insert Cell Kernel Widgets Help
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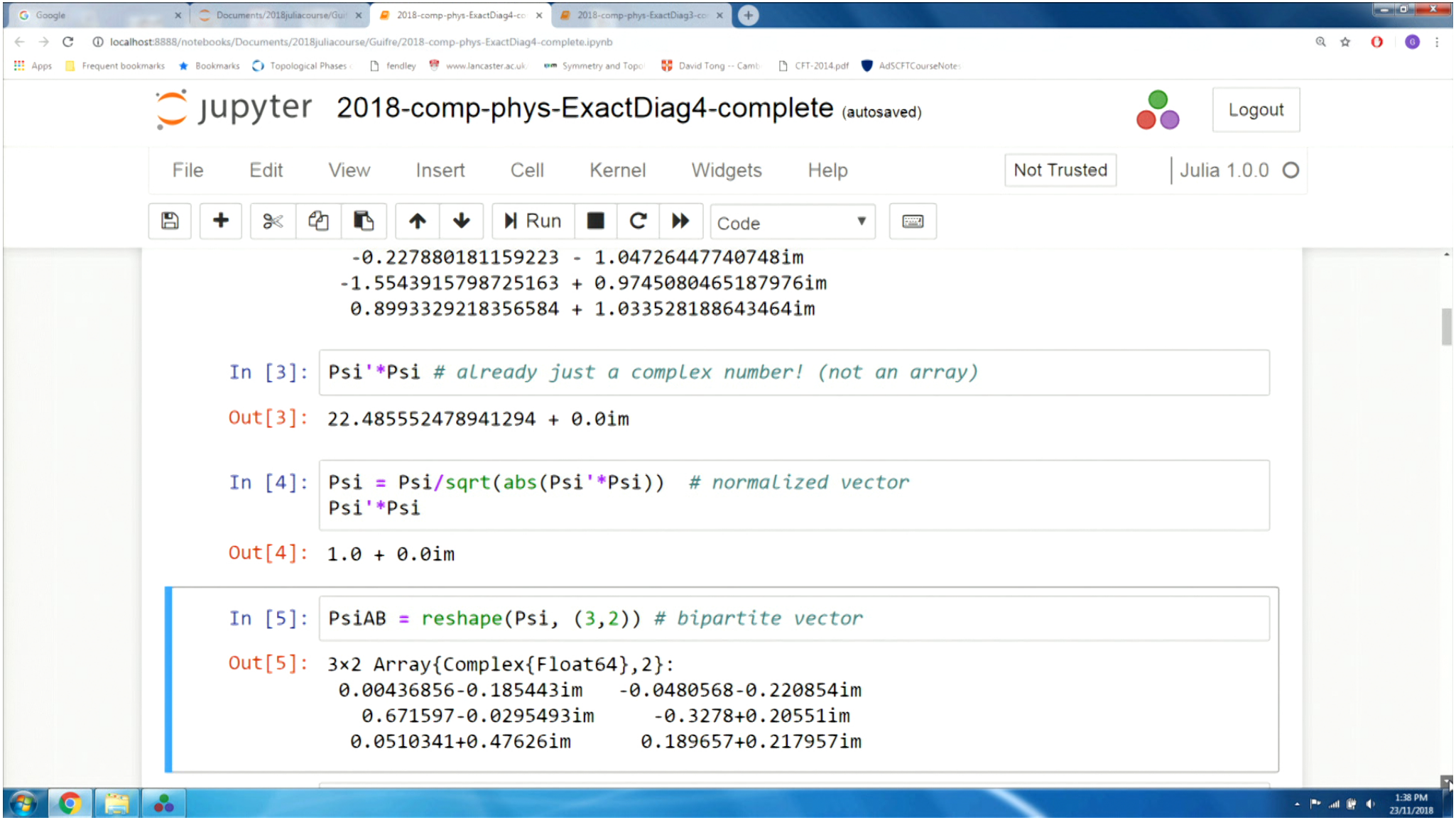
Julia 1.0.0
Run
Code
-0.227880181159223 - 1.04726447740748im
-1.5543915798725163 + 0.97450804651879 76im
0.8993329218356584 + 1.0335281886434 64im
In [3]:
Psi'*Psi # already just a complex number! (not an array)
Out[3]:
22.485552478941294 + 0.0im
In [4]:
Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[4]:
1.0 + 0.0im
In [5]:
PsiAB = reshape(Psi, (3,2)) # bipartite vector
Out[5]:
3×2 Array{Complex{Float64},2}:
0.00436856-0.185443im -0.0480568-0.220854im
0.671597-0.0295493im -0.3278+0.20551im
0.0510341+0.47626im 0.189657+0.217957im


jupyter 2018-comp-phys-ExactDiag4-complete (autosaved)
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File Edit View Insert Cell Kernel Widgets Help
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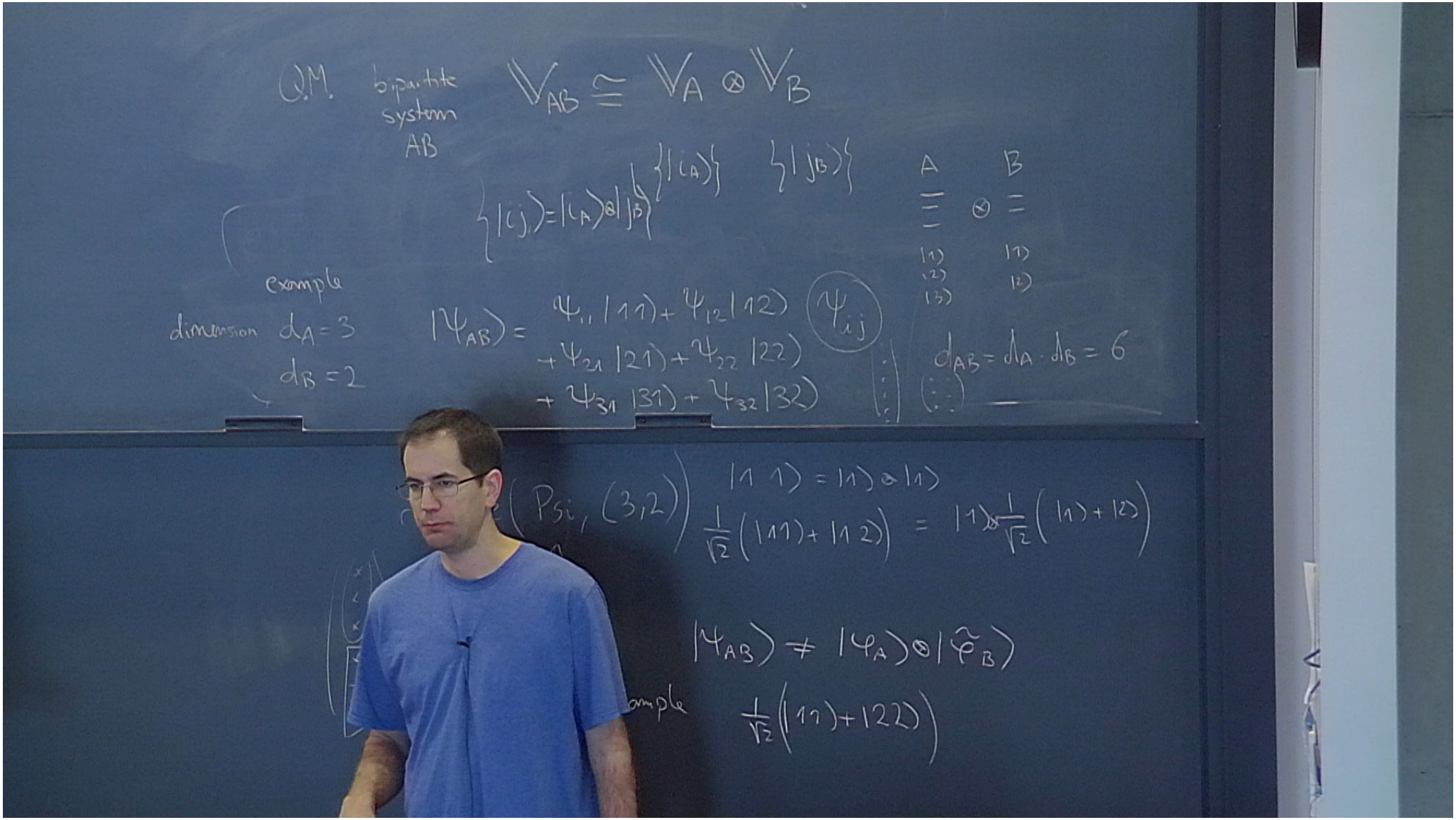
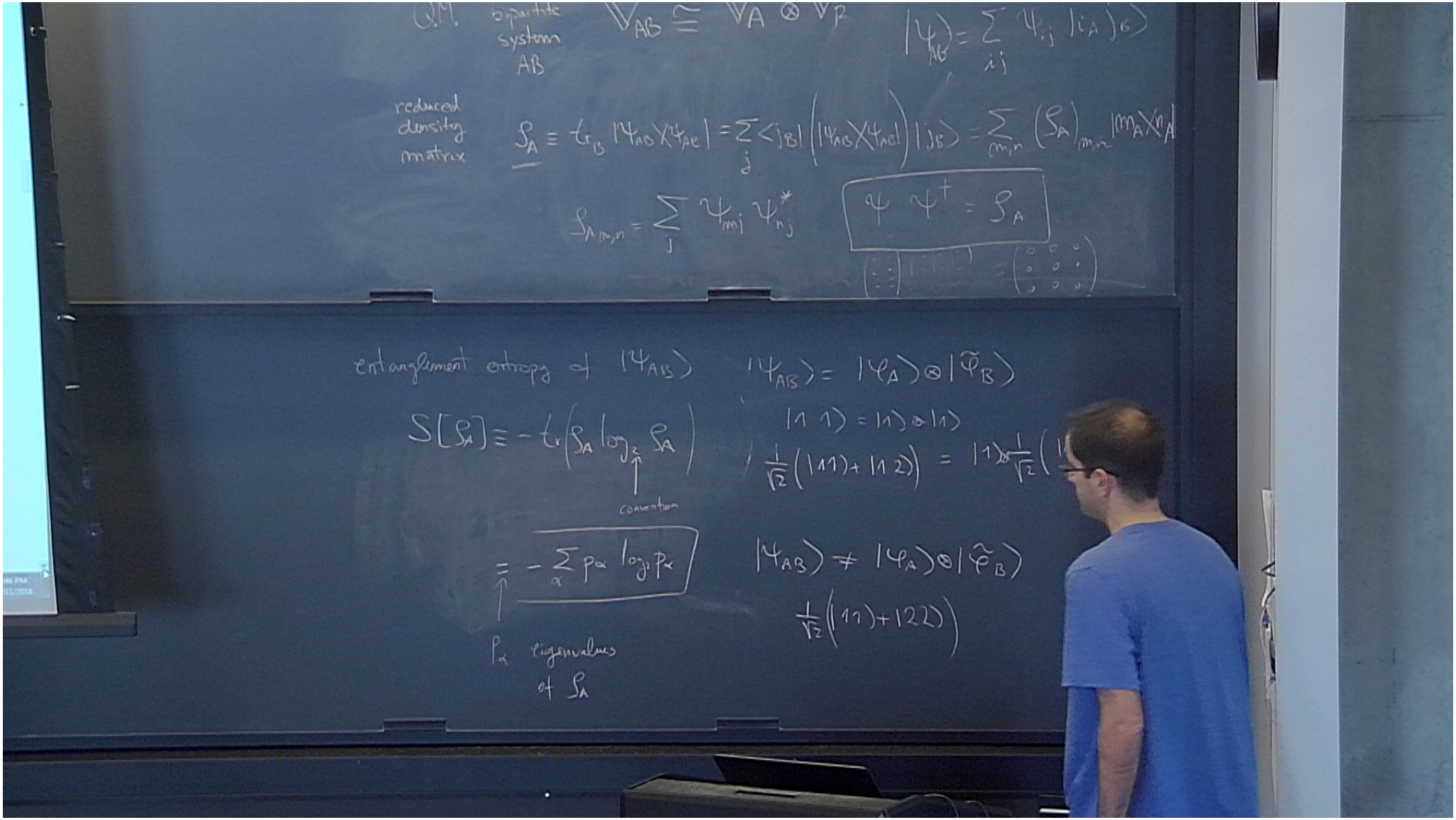
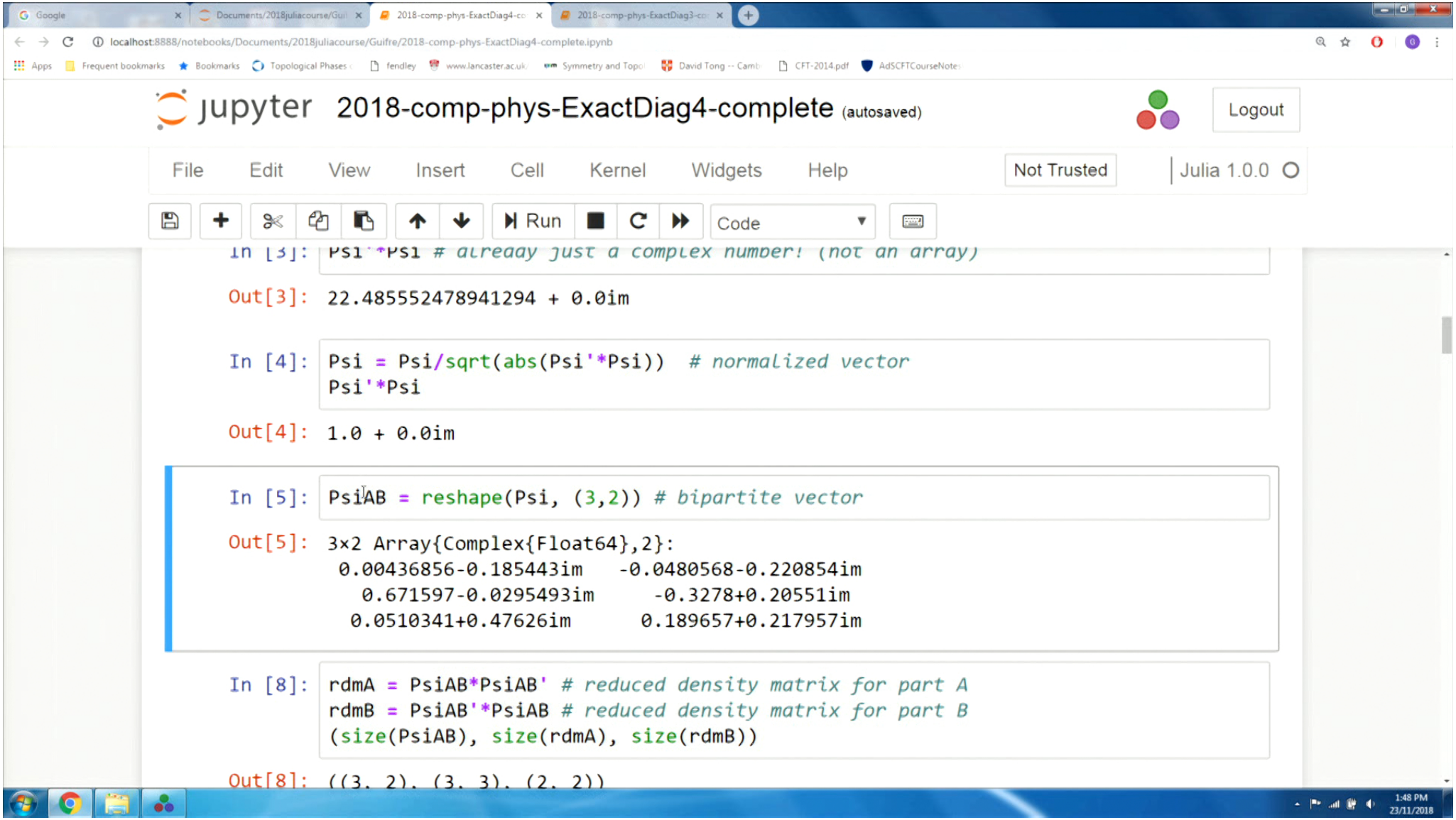
Julia 1.0.0
Run
Code
-0.227880181159223 - 1.04726447740748im
-1.5543915798725163 + 0.97450804651879 76im
0.8993329218356584 + 1.0335281886434 64im
In [3]:
Psi'*Psi # already just a complex number! (not an array)
Out[3]:
22.485552478941294 + 0.0im
In [4]:
Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[4]:
1.0 + 0.0im
In [5]:
PsiAB = reshape(Psi, (3,2)) # bipartite vector
Out[5]:
3×2 Array{Complex{Float64},2}:
0.00436856-0.185443im -0.0480568-0.220854im
0.671597-0.0295493im -0.3278+0.20551im
0.0510341+0.47626im 0.189657+0.217957im


Q.M. bipartite system AB
$\mathbb{V}_{AB} \cong \mathbb{V}_A \otimes \mathbb{V}_B$
$\{|i_A\rangle\}$ $\{|j_B\rangle\}$
$\{|i_j\rangle = |i_A\rangle \otimes |j_B\rangle\}$
A B
$|1\rangle$ $|1\rangle$
$|2\rangle$ $|2\rangle$
$|3\rangle$
example
dimension $d_A = 3$
$d_B = 2$
$|\Psi_{AB}\rangle = \Psi_{11}|11\rangle + \Psi_{12}|12\rangle + \Psi_{21}|21\rangle + \Psi_{22}|22\rangle + \Psi_{31}|31\rangle + \Psi_{32}|32\rangle$
Ψ_{ij}
$d_{AB} = d_A \cdot d_B = 6$
$|1\,1\rangle = |1\rangle \otimes |1\rangle$
$\frac{1}{\sqrt{2}}(|1\,1\rangle + |1\,2\rangle) = |1\rangle \otimes \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$
$|\Psi_{AB}\rangle \neq |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$
$\frac{1}{\sqrt{2}}(|11\rangle + |22\rangle)$


Q.M.
bipartite system AB
$\mathcal{V}_{AB} \cong \mathcal{V}_A \otimes \mathcal{V}_B$
$|\psi\rangle_{AB} = \sum_{ij} \psi_{ij} |i_A\rangle |j_B\rangle$
reduced density matrix
$\rho_A \equiv \mathrm{tr}_B |\psi_{AB}\rangle\langle\psi_{AB}| = \sum_j \langle j_B| \left(|\psi_{AB}\rangle\langle\psi_{AB}|\right) |j_B\rangle = \sum_{m,n} (\rho_A)_{m,n} |m_A\rangle\langle n_A|$
$\rho_{A\,m,n} = \sum_j \psi_{mj} \psi^*_{nj}$
$\psi \psi^\dagger = \rho_A$
entanglement entropy of $|\psi_{AB}\rangle$
$|\psi_{AB}\rangle = |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$
$|1\,1\rangle = |1\rangle \otimes |1\rangle$
$\frac{1}{\sqrt{2}}(|1\,1\rangle + |1\,2\rangle) = |1\rangle \otimes \frac{1}{\sqrt{2}}($
$S[\rho_A] \equiv -\mathrm{tr}\left(\rho_A \log_2 \rho_A\right)$
convention
$= -\sum_\alpha p_\alpha \log_2 p_\alpha$
$|\psi_{AB}\rangle \neq |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$
$\frac{1}{\sqrt{2}}(|1\,1\rangle + |2\,2\rangle)$
p_α eigenvalues of ρ_A


jupyter 2018-comp-phys-ExactDiag4-complete (autosaved)
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File Edit View Insert Cell Kernel Widgets Help
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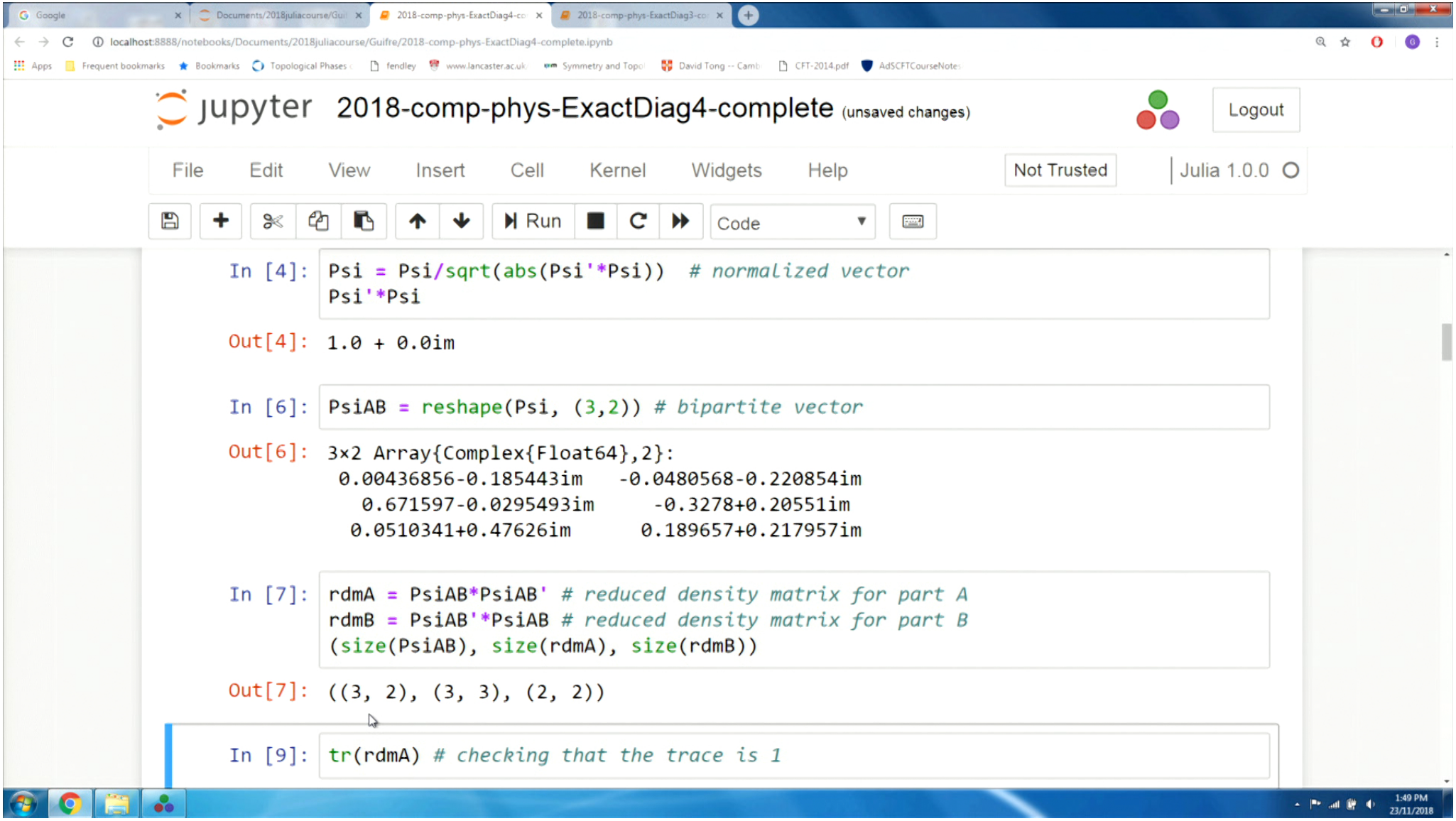
Julia 1.0.0
Run
Code
In [3]: Psi'*Psi # already just a complex number! (not an array)
Out[3]: 22.485552478941294 + 0.0im
In [4]: Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[4]: 1.0 + 0.0im
In [5]: PsiAB = reshape(Psi, (3,2)) # bipartite vector
Out[5]: 3×2 Array{Complex{Float64},2}:
0.00436856-0.185443im -0.0480568-0.220854im
0.671597-0.0295493im -0.3278+0.20551im
0.0510341+0.47626im 0.189657+0.217957im
In [8]: rdmA = PsiAB*PsiAB' # reduced density matrix for part A
rdmB = PsiAB'*PsiAB # reduced density matrix for part B
(size(PsiAB), size(rdmA), size(rdmB))
Out[8]: ((3, 2), (3, 3), (2, 2))


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
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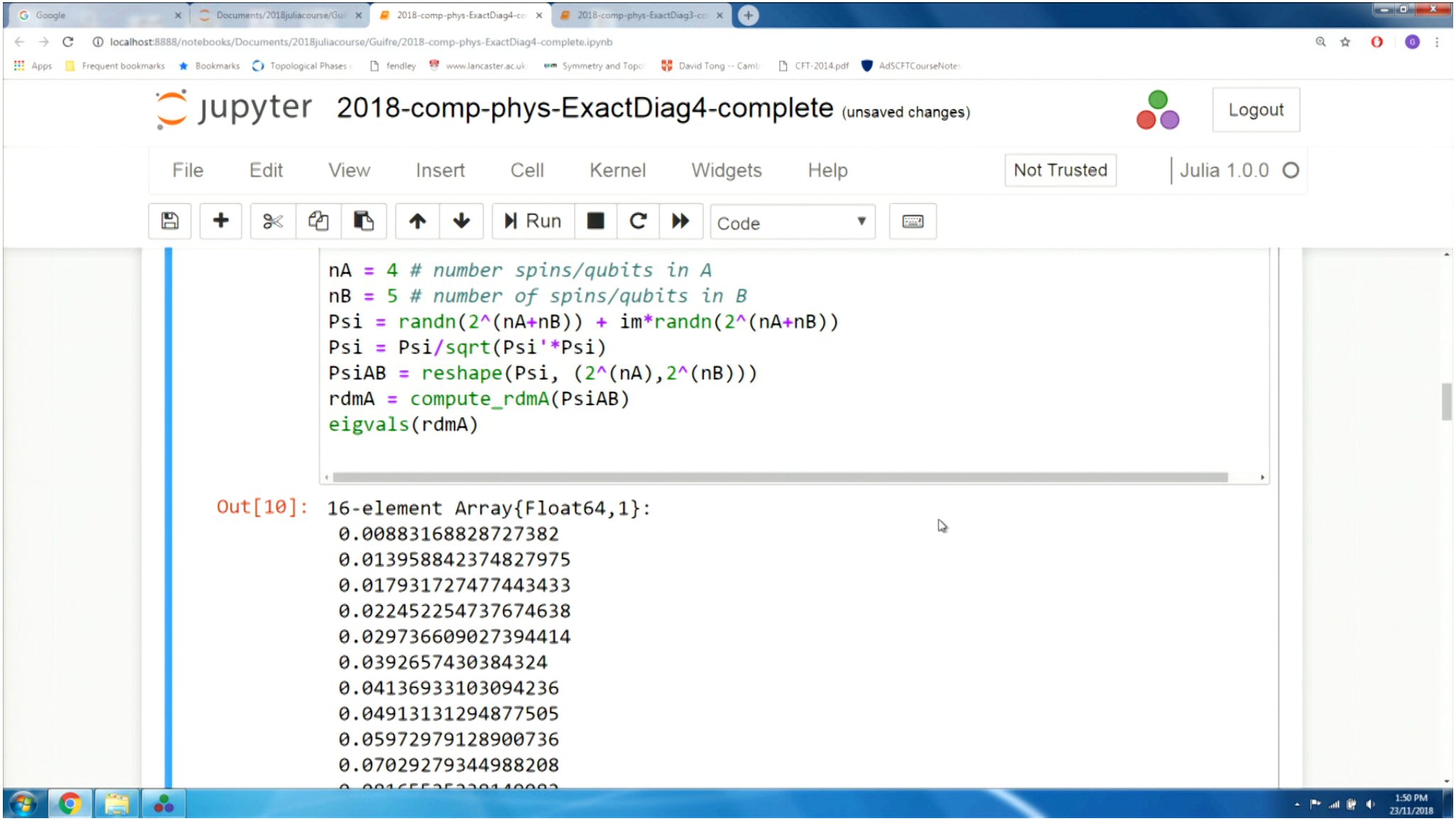
Julia 1.0.0
Run
Code
In [4]:
Psi = Psi/sqrt(abs(Psi'*Psi)) # normalized vector
Psi'*Psi
Out[4]:
1.0 + 0.0im
In [6]:
PsiAB = reshape(Psi, (3,2)) # bipartite vector
Out[6]:
3×2 Array{Complex{Float64},2}:
0.00436856-0.185443im -0.0480568-0.220854im
0.671597-0.0295493im -0.3278+0.20551im
0.0510341+0.47626im 0.189657+0.217957im
In [7]:
rdmA = PsiAB*PsiAB' # reduced density matrix for part A
rdmB = PsiAB'*PsiAB # reduced density matrix for part B
(size(PsiAB), size(rdmA), size(rdmB))
Out[7]:
((3, 2), (3, 3), (2, 2))
In [9]:
tr(rdmA) # checking that the trace is 1


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
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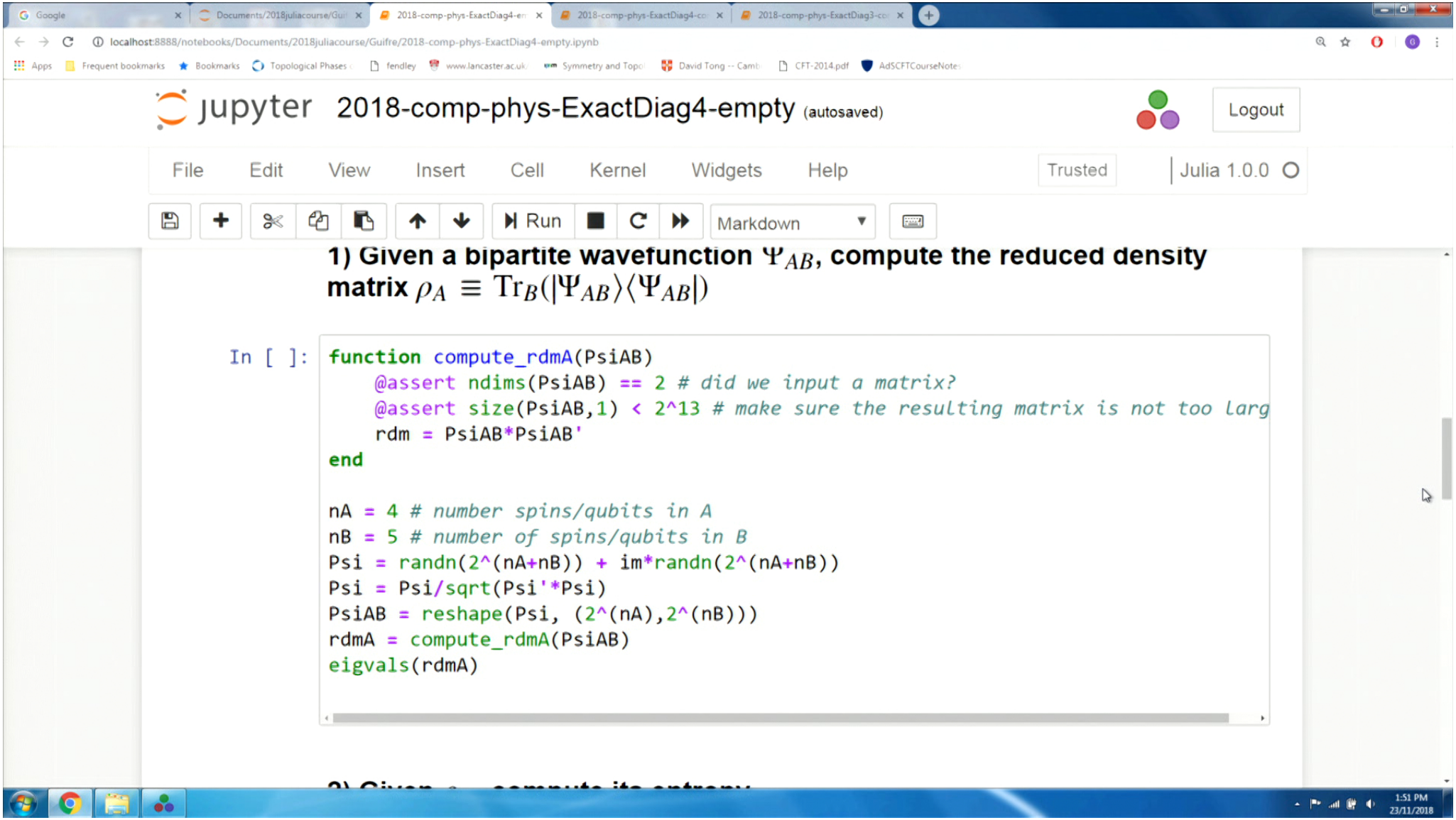
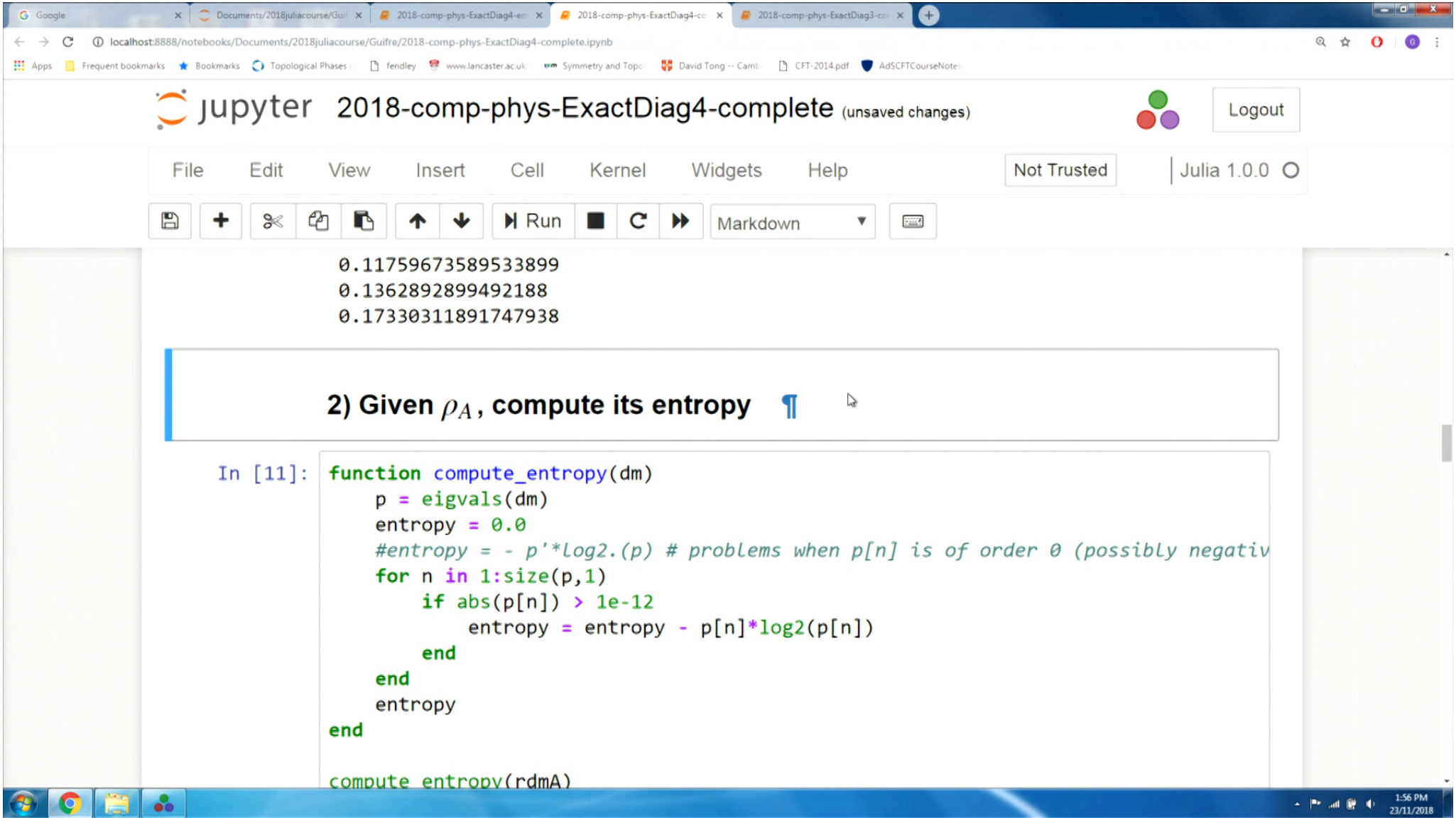
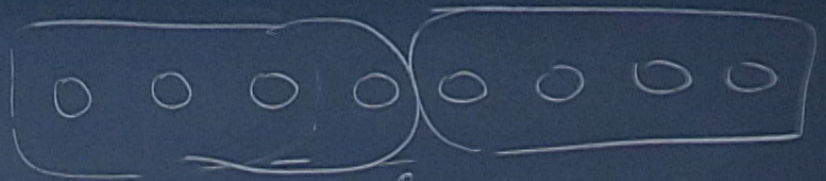
Julia 1.0.0
Run
Code
nA = 4 # number spins/qubits in A
nB = 5 # number of spins/qubits in B
Psi = randn(2^(nA+nB)) + im*randn(2^(nA+nB))
Psi = Psi/sqrt(Psi'*Psi)
PsiAB = reshape(Psi, (2^(nA),2^(nB)))
rdmA = compute_rdmA(PsiAB)
eigvals(rdmA)
Out[10]: 16-element Array{Float64,1}:
0.00883168828727382
0.013958842374827975
0.017931727477443433
0.022452254737674638
0.029736609027394414
0.0392657430384324
0.04136933103094236
0.04913131294877505
0.05972979128900736
0.07029279344988208


jupyter 2018-comp-phys-ExactDiag4-empty (autosaved)
Logout
File Edit View Insert Cell Kernel Widgets Help
Trusted
Julia 1.0.0
Run
Markdown
1) Given a bipartite wavefunction Ψ_{AB}, compute the reduced density matrix $\rho_A \equiv \mathrm{Tr}_B(|\Psi_{AB}\rangle\langle\Psi_{AB}|)$
In [ ]:
function compute_rdmA(PsiAB)
@assert ndims(PsiAB) == 2 # did we input a matrix?
@assert size(PsiAB,1) < 2^13 # make sure the resulting matrix is not too larg
rdm = PsiAB*PsiAB'
end
nA = 4 # number spins/qubits in A
nB = 5 # number of spins/qubits in B
Psi = randn(2^(nA+nB)) + im*randn(2^(nA+nB))
Psi = Psi/sqrt(Psi'*Psi)
PsiAB = reshape(Psi, (2^(nA),2^(nB)))
rdmA = compute_rdmA(PsiAB)
eigvals(rdmA)


jupyter 2018-comp-phys-ExactDiag4-complete (unsaved changes)
Logout
File Edit View Insert Cell Kernel Widgets Help
Not Trusted
Julia 1.0.0
Run
Markdown
0.11759673589533899
0.1362892899492188
0.17330311891747938
2) Given ρ_A, compute its entropy ¶
In [11]:
function compute_entropy(dm)
p = eigvals(dm)
entropy = 0.0
#entropy = - p'*log2.(p) # problems when p[n] is of order 0 (possibly negativ
for n in 1:size(p,1)
if abs(p[n]) > 1e-12
entropy = entropy - p[n]*log2(p[n])
end
end
entropy
end
compute_entropy(rdmA)
1:56 PM
23/11/2018

entanglement entropy of $|\Psi_{AB}\rangle$



N spins 1/2 / qubits

$$S[\rho_A] \equiv -\mathrm{tr}\left(\rho_A \log_2 \rho_A\right)$$

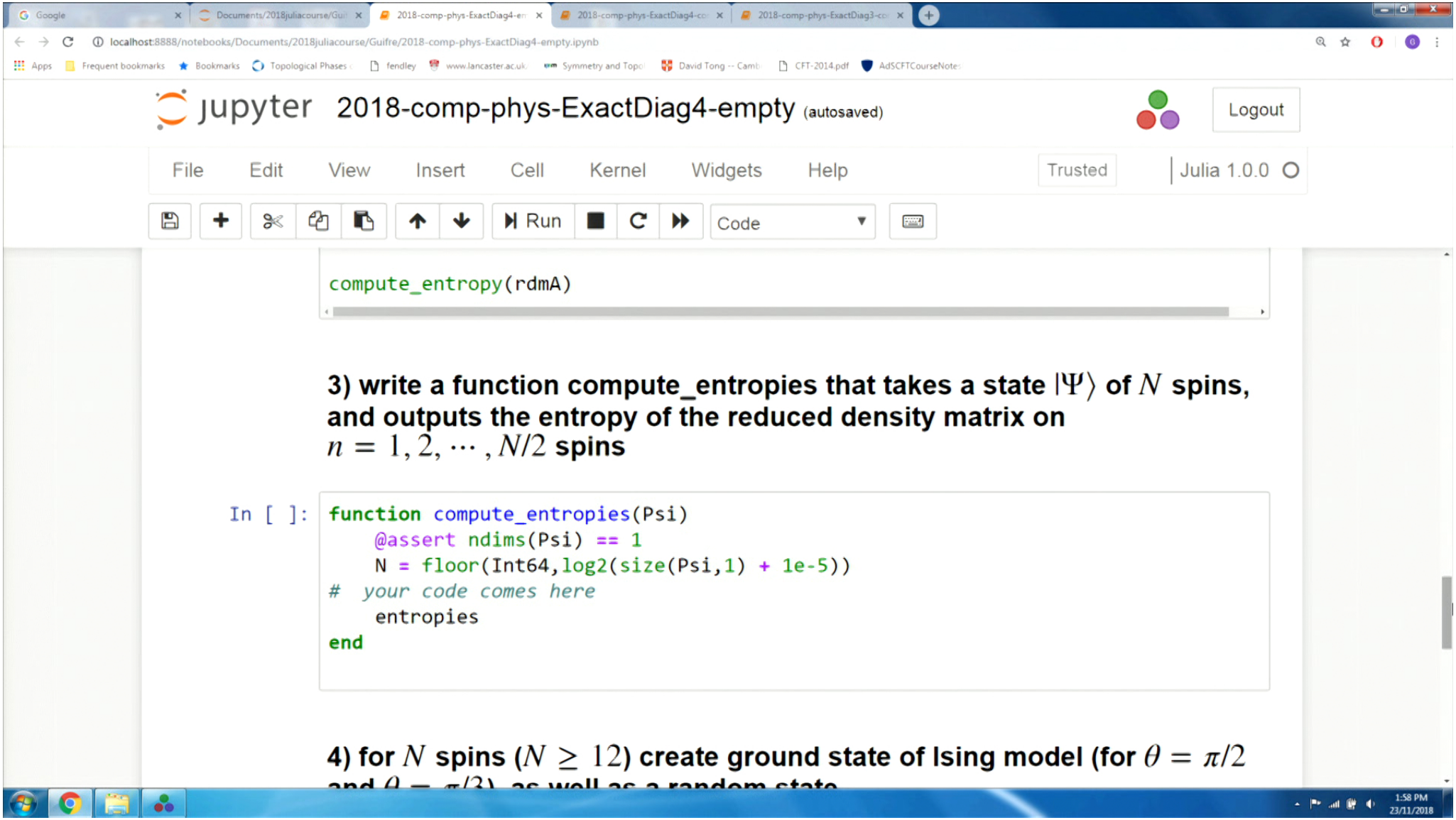
convention

$\lim_{\epsilon \to 0} \epsilon \log \epsilon = 0$,

$$= -\sum_\alpha p_\alpha \log_2 p_\alpha \geq 0 \qquad |\Psi_{AB}\rangle \neq |\varphi_A\rangle \otimes |\tilde{\varphi}_B\rangle$$

$$\frac{1}{\sqrt{2}}\left(|11\rangle + |22\rangle\right)$$

p_α eigenvalues of ρ_A


jupyter 2018-comp-phys-ExactDiag4-empty (autosaved)
Logout
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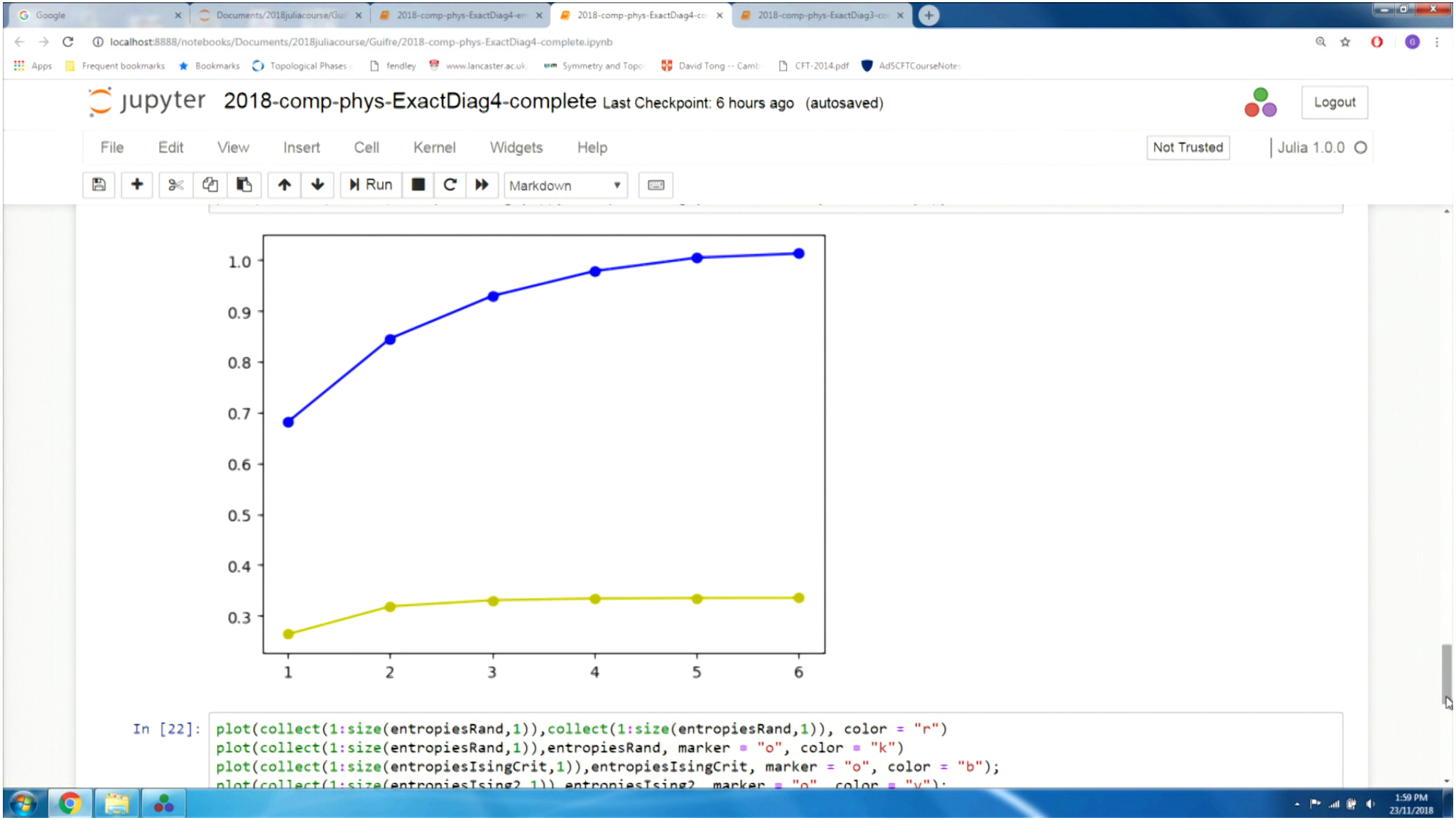
Trusted
Julia 1.0.0
Run
Code
compute_entropy(rdmA)
3) write a function compute_entropies that takes a state $|\Psi\rangle$ of N spins, and outputs the entropy of the reduced density matrix on $n = 1, 2, \cdots, N/2$ spins
In [ ]:
function compute_entropies(Psi)
@assert ndims(Psi) == 1
N = floor(Int64,log2(size(Psi,1) + 1e-5))
# your code comes here
entropies
end
4) for N spins ($N \geq 12$) create ground state of Ising model (for $\theta = \pi/2$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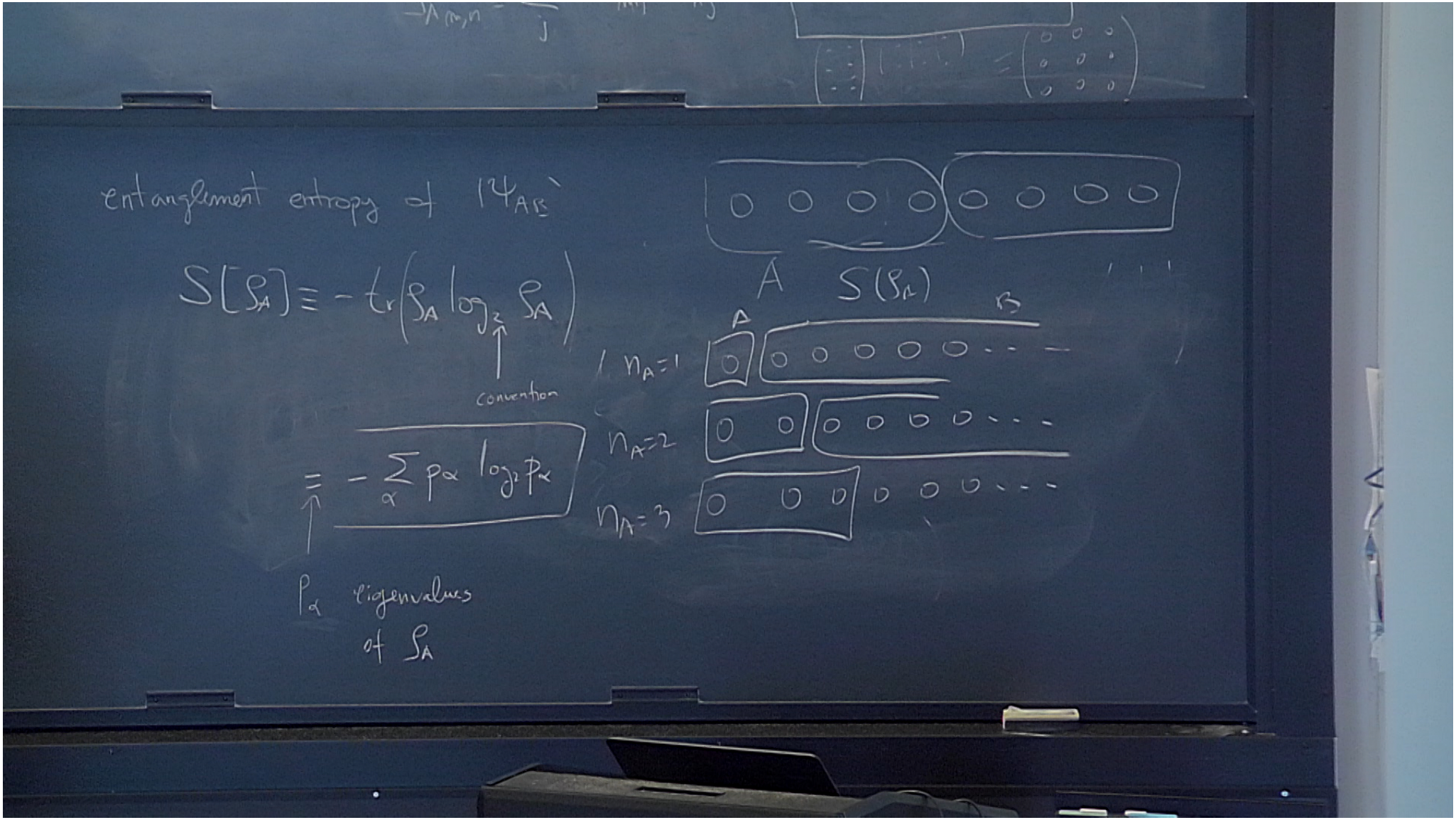
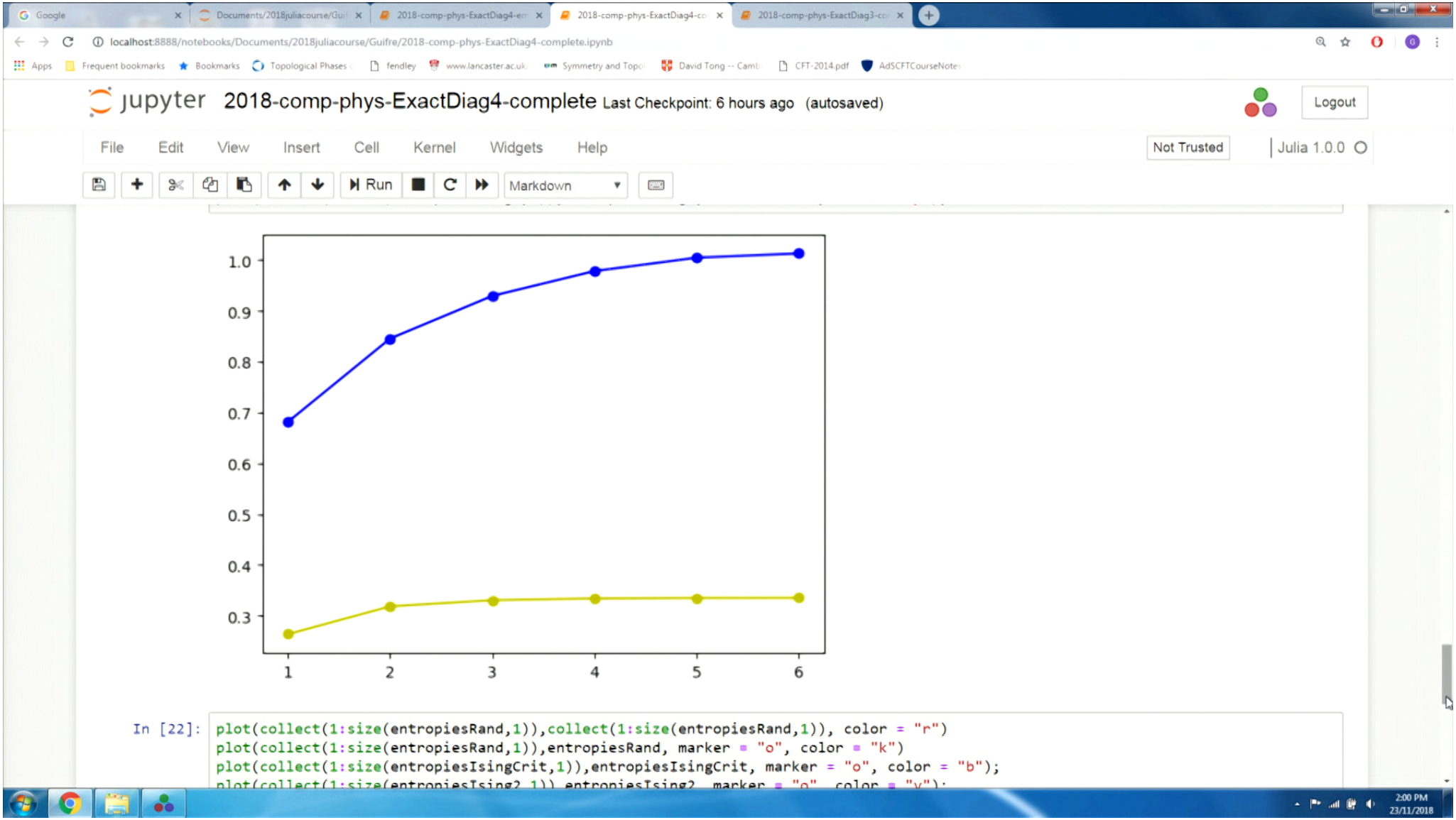
In [22]:
plot(collect(1:size(entropiesRand,1)),collect(1:size(entropiesRand,1)), color = "r")
plot(collect(1:size(entropiesRand,1)),entropiesRand, marker = "o", color = "k")
plot(collect(1:size(entropiesIsingCrit,1)),entropiesIsingCrit, marker = "o", color = "b");


entanglement entropy of $|\Psi_{AB}\rangle$
$S[\rho_A] \equiv -\mathrm{tr}(\rho_A \log_2 \rho_A)$
convention
$= -\sum_\alpha p_\alpha \log_2 p_\alpha$
p_α eigenvalues of ρ_A
A
$S(\rho_A)$
B
$n_A = 1$
$n_A = 2$
$n_A = 3$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Julia 1.0.0
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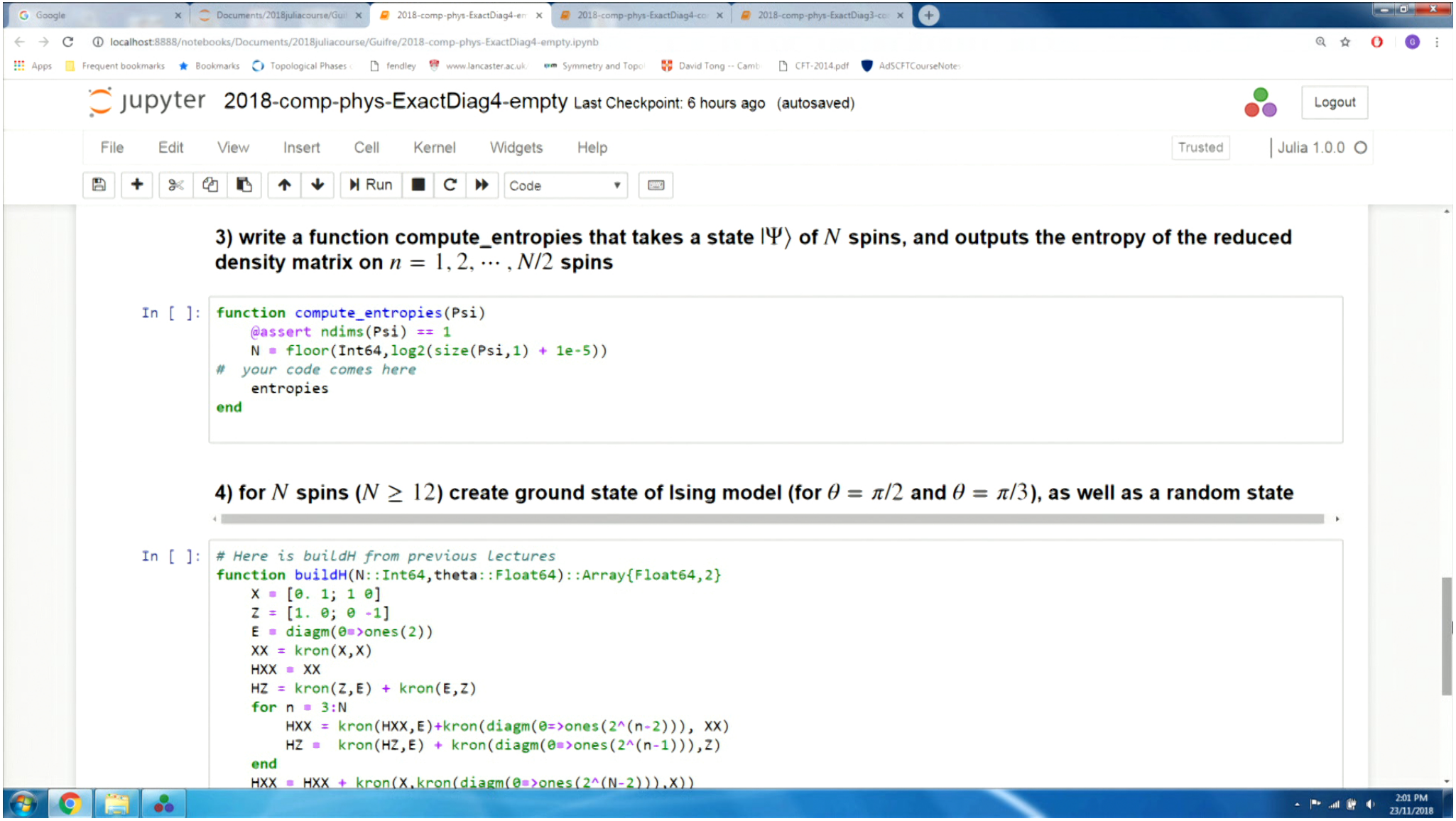
plot(collect(1:size(entropiesRand,1)),entropiesRand, marker = "o", color = "k")
plot(collect(1:size(entropiesIsingCrit,1)),entropiesIsingCrit, marker = "o", color = "b");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CFT-2014.pdf
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Run
Code
3) write a function compute_entropies that takes a state $|\Psi\rangle$ of N spins, and outputs the entropy of the reduced density matrix on $n = 1, 2, \cdots, N/2$ spins
In [ ]:
function compute_entropies(Psi)
@assert ndims(Psi) == 1
N = floor(Int64,log2(size(Psi,1) + 1e-5))
# your code comes here
entropies
end
4) for N spins ($N \geq 12$) create ground state of Ising model (for $\theta = \pi/2$ and $\theta = \pi/3$), as well as a random state
In [ ]:
Here is buildH from previous lectures
function buildH(N::Int64,theta::Float64)::Array{Float64,2}
X = [0. 1; 1 0]
Z = [1. 0; 0 -1]
E = diagm(0=>ones(2))
XX = kron(X,X)
HXX = XX
HZ = kron(Z,E) + kron(E,Z)
for n = 3:N
HXX = kron(HXX,E)+kron(diagm(0=>ones(2^(n-2))), XX)
HZ = kron(HZ,E) + kron(diagm(0=>ones(2^(n-1))),Z)
end
HXX = HXX + kron(X,kron(diagm(0=>ones(2^(N-2))),X))
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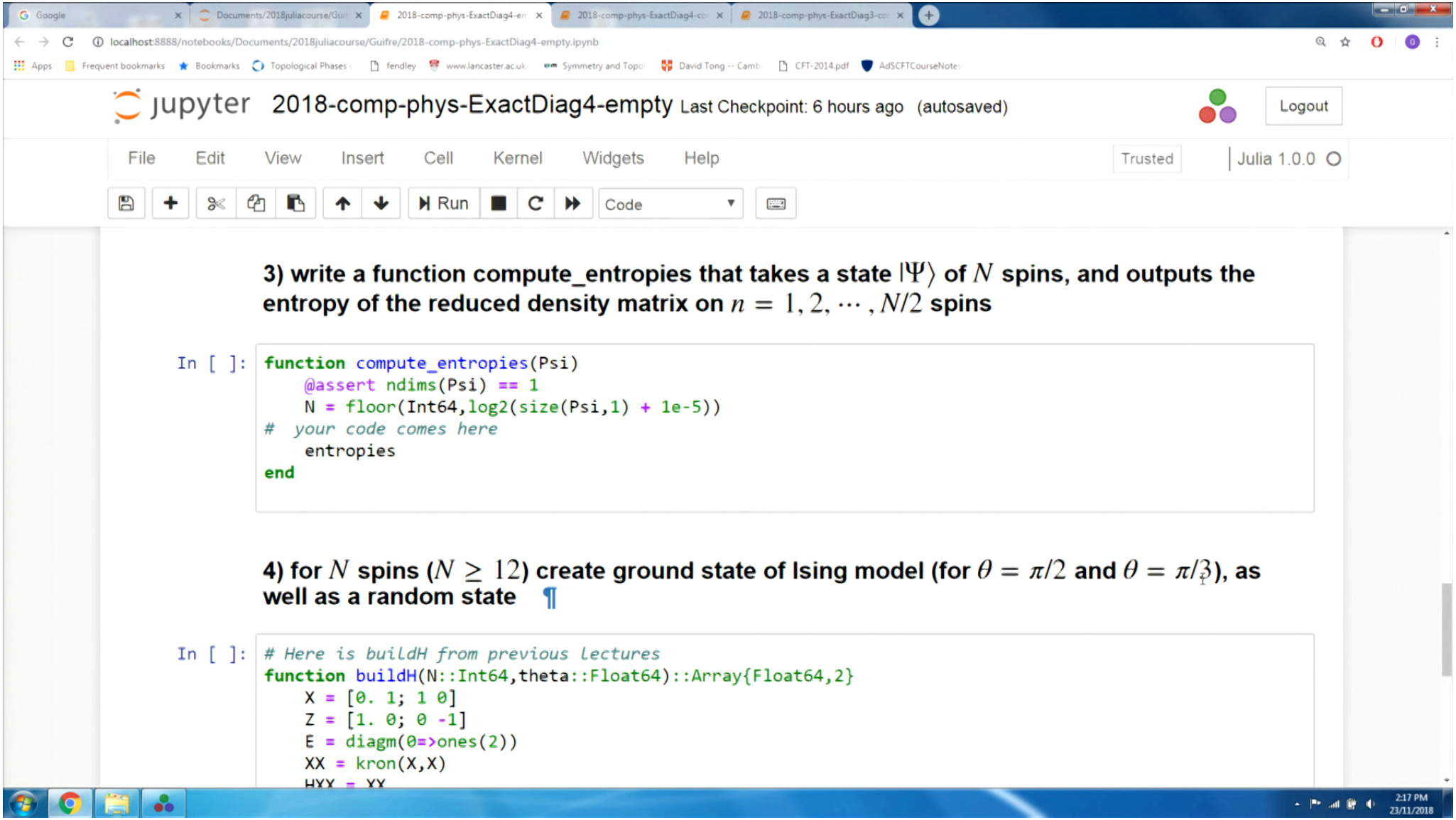
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David Tong -- Camb
CFT-2014.pdf
AdSCFTCourseNotes
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N = floor(Int64,log2(size(Psi,1) + 1e-5))
# your code comes here
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end
4) for N spins ($N \geq 12$) create ground state of Ising model (for $\theta = \pi/2$ and $\theta = \pi/3$), as well as a random state ¶
In [ ]:
Here is buildH from previous lectures
function buildH(N::Int64,theta::Float64)::Array{Float64,2}
X = [0. 1; 1 0]
Z = [1. 0; 0 -1]
E = diagm(0=>ones(2))
XX = kron(X,X)
HXX = XX
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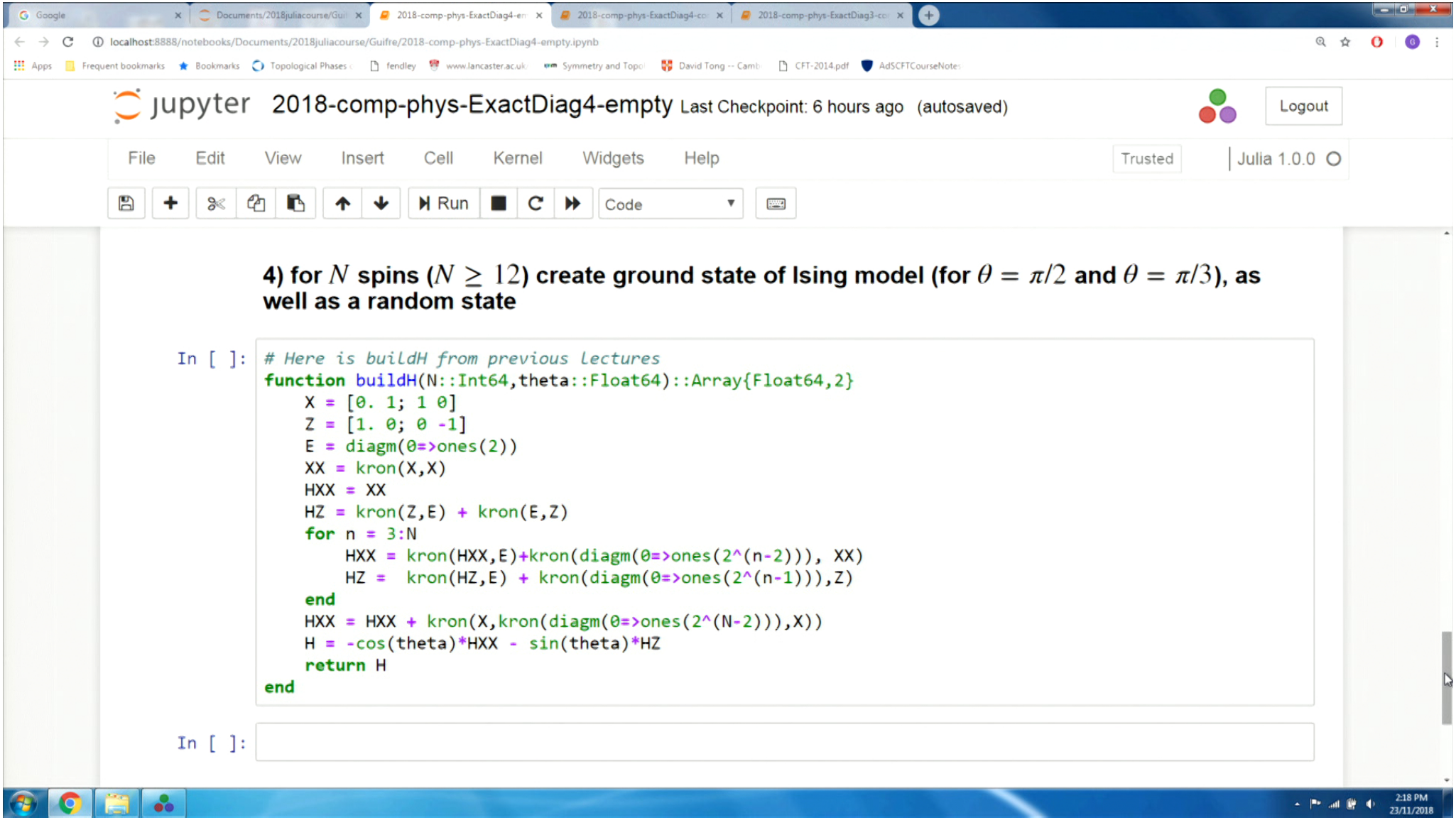
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Julia 1.0.0
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Code
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In [ ]:
Here is buildH from previous lectures
function buildH(N::Int64,theta::Float64)::Array{Float64,2}
X = [0. 1; 1 0]
Z = [1. 0; 0 -1]
E = diagm(0=>ones(2))
XX = kron(X,X)
HXX = XX
HZ = kron(Z,E) + kron(E,Z)
for n = 3:N
HXX = kron(HXX,E)+kron(diagm(0=>ones(2^(n-2))), XX)
HZ = kron(HZ,E) + kron(diagm(0=>ones(2^(n-1))),Z)
end
HXX = HXX + kron(X,kron(diagm(0=>ones(2^(N-2))),X))
H = -cos(theta)*HXX - sin(theta)*HZ
return H
end
In [ ]:
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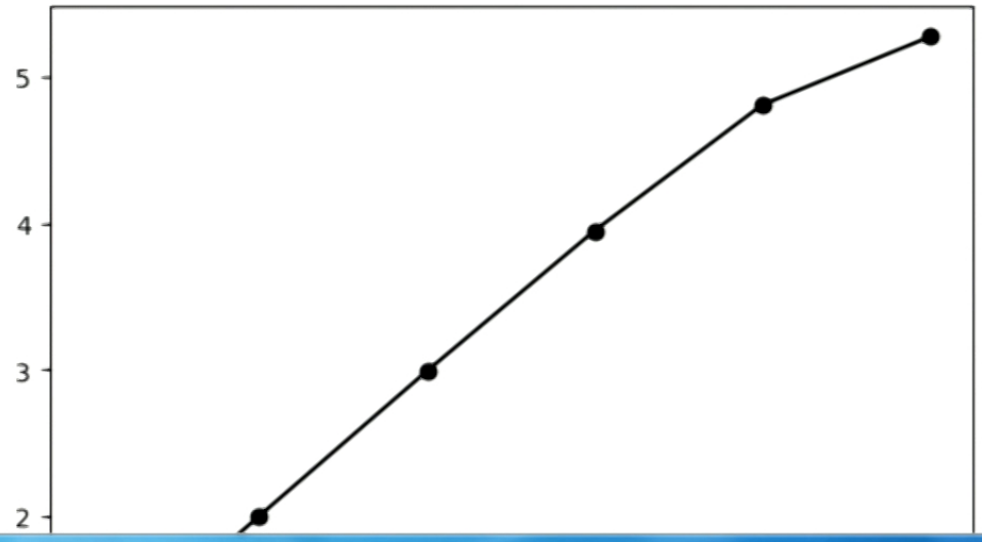
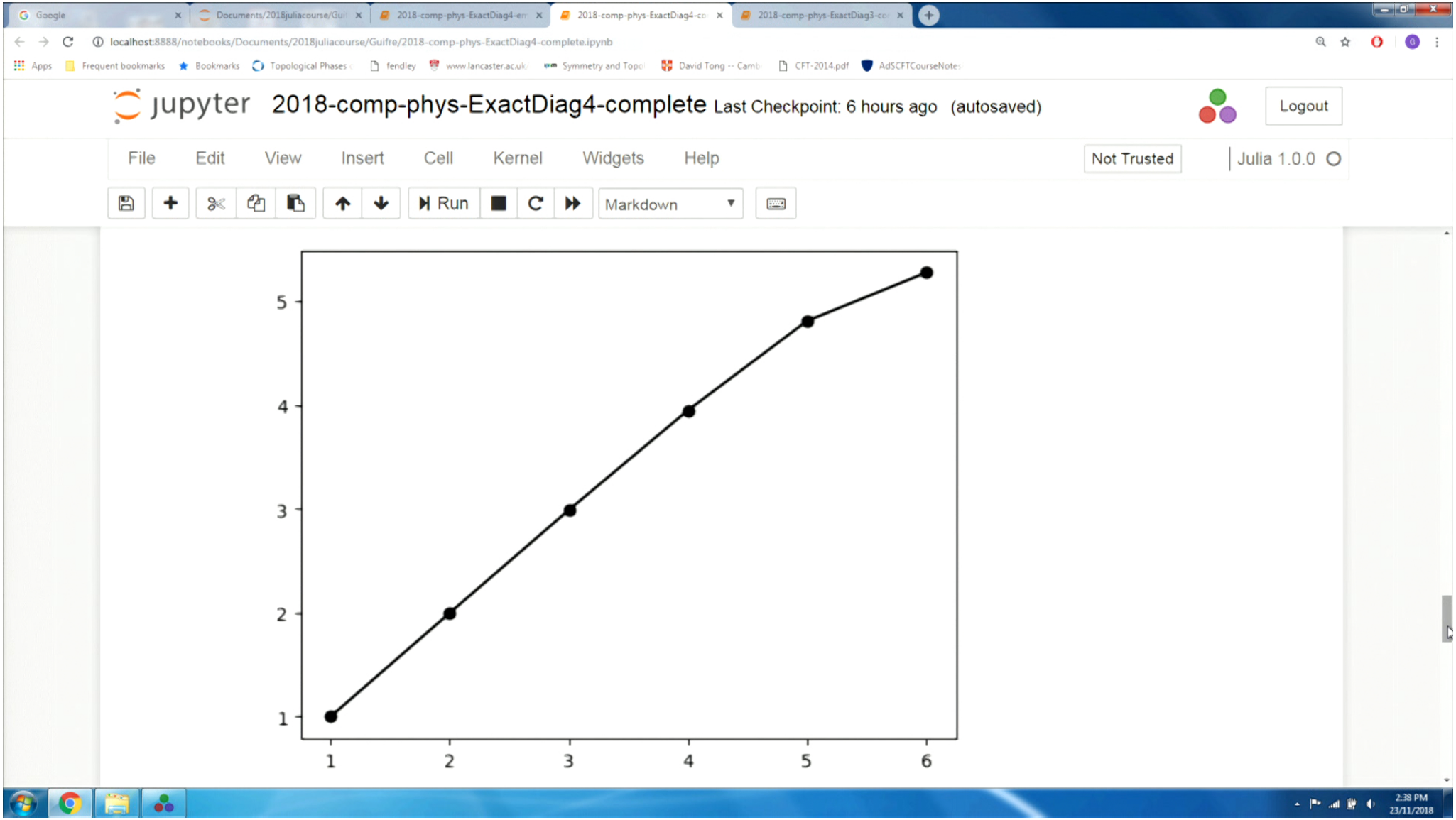
Markdown

In [20]:
```
entropiesRand = compute_entropies(PsiRand)
entropiesIsingCrit = compute_entropies(PsiIsingCrit)
entropiesIsing2 = compute_entropies(PsiIsing2)
# Rand entropies
plot(collect(1:size(entropiesRand,1)),entropiesRand, marker = "o", color = "k");
```

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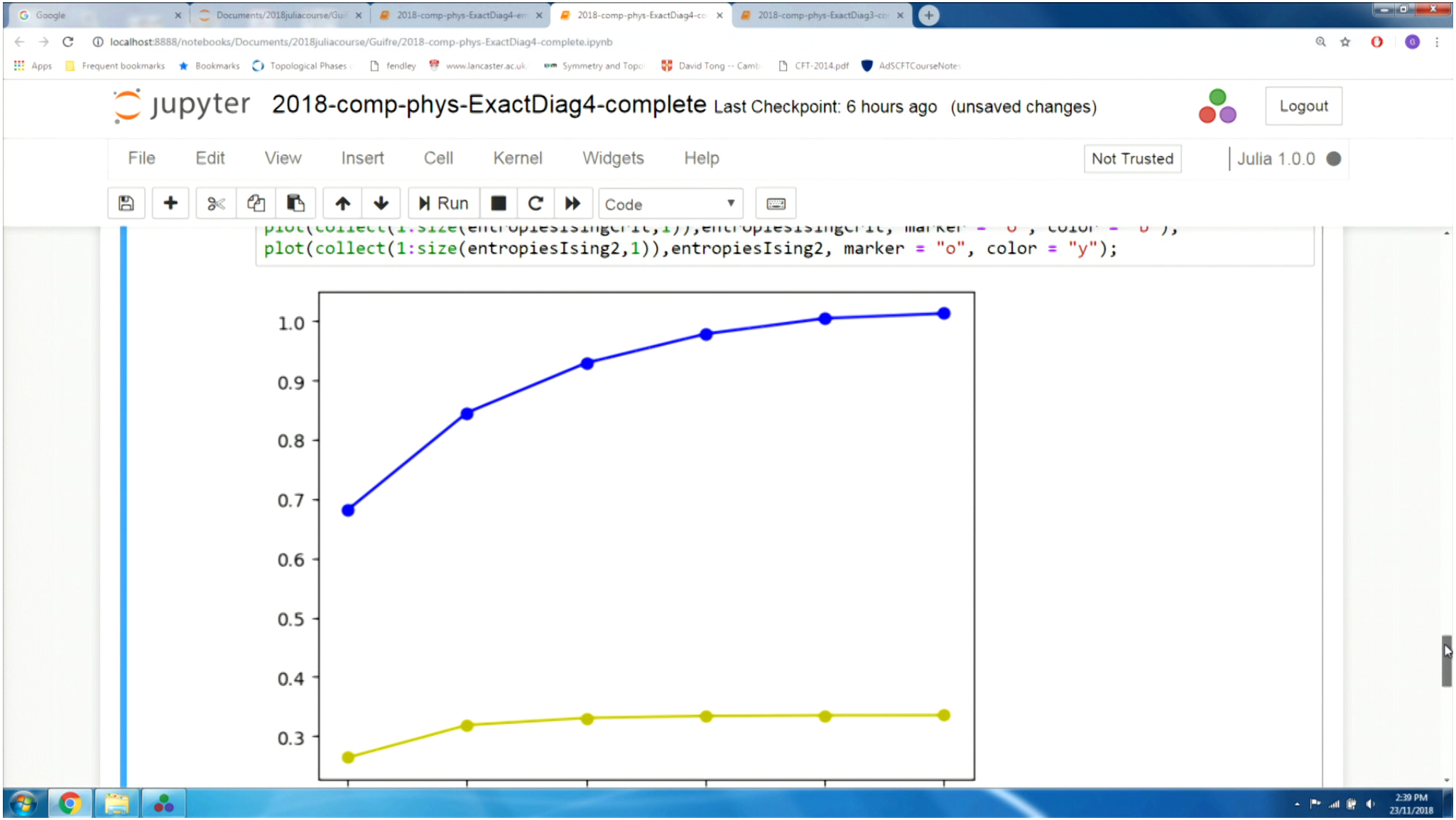
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Julia 1.0.0
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Code
plot(collect(1:size(entropiesIsing2,1)),entropiesIsing2, marker = "o", color = "y");
1.0
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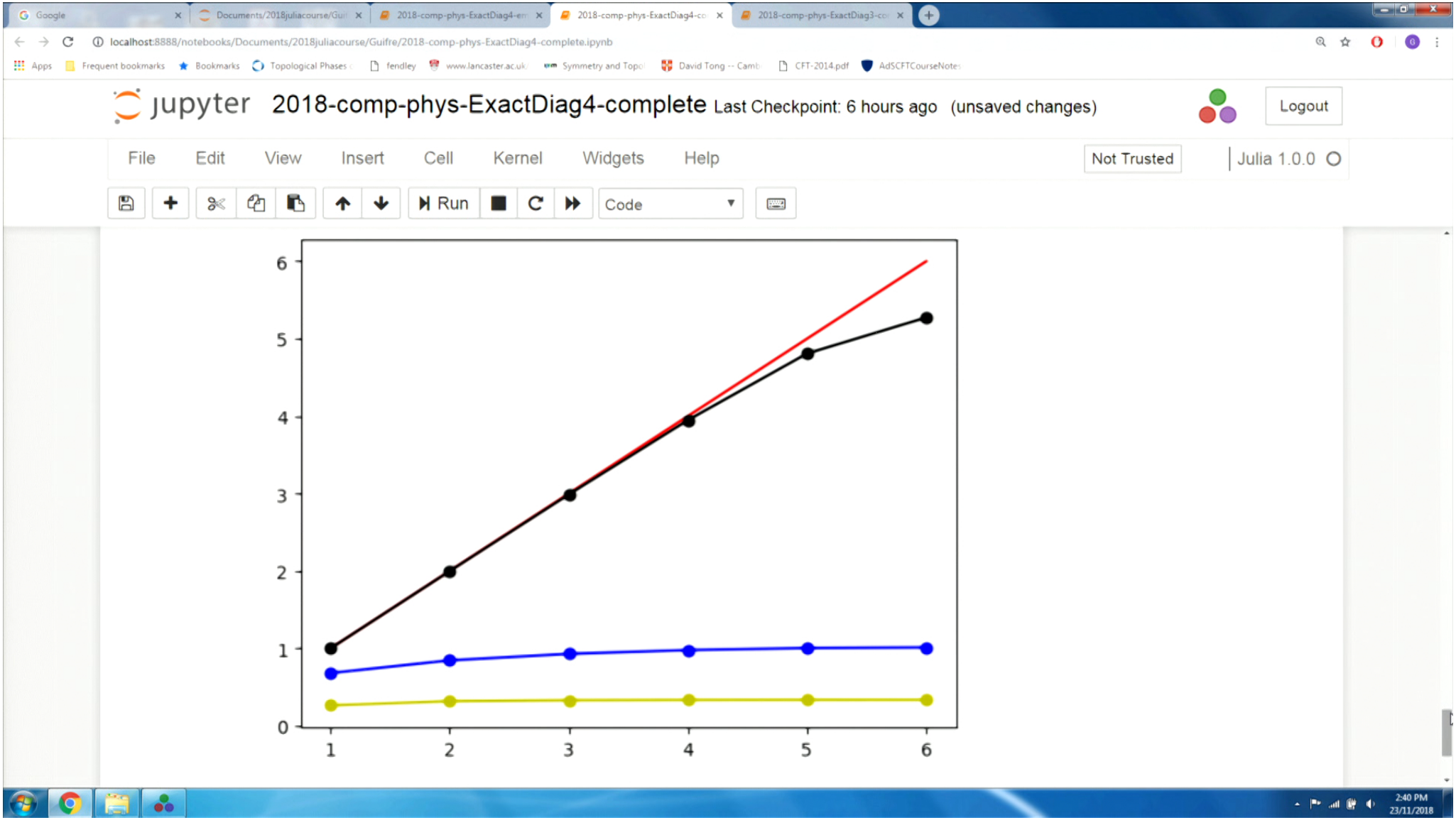
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Julia 1.0.0
Run
Code
0 1 2 3 4 5 6
1 2 3 4 5 6