

Title: Computational Physics - Lecture 17

Date: Nov 07, 2018 01:00 PM

URL: <http://pirsa.org/18110043>

Abstract:

# Finite Differencing

## Setup

In [1]: `versioninfo()`

```
Julia Version 1.0.1
Commit 0d713926f8 (2018-09-29 19:05 UTC)
Platform Info:
  OS: macOS (x86_64-apple-darwin14.5.0)
  CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.0 (ORCJIT, haswell)
Environment:
  JULIA_ERROR_COLOR = red
  JULIA_WARN_COLOR = magenta
  JULIA_INFO_COLOR = blue
  JULIA_DEBUG_COLOR = blue
  JULIA_INPUT_COLOR = normal
  JULIA_ANSWER_COLOR = normal
  JULIA_STACKFRAME_LINEINFO_COLOR = bold
  JULIA_STACKFRAME_FUNCTION_COLOR = bold
```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`



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Trusted Julia 1.0.1

In [1]: `versioninfo()`

```
Julia Version 1.0.1
Commit 0d713926f8 (2018-09-29 19:05 UTC)
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```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

In [ ]:

The image shows a screenshot of a Julia REPL interface. At the top, there is a browser address bar with the URL `localhost:8888/notebooks/txt/Courses/CompPhys2018/jl/FiniteDifferencing/FiniteDifferencing.ipynb`. Below the browser bar is a menu bar with options: File, Edit, View, Insert, Cell, Kernel, Widgets, Help. On the right side of the menu bar, there is a "Trusted" button, a pencil icon, and the text "Julia 1.0.1".

The main content area displays the following text:

```
Commit 0d713926f8 (2018-09-29 19:05 UTC)
Platform Info:
  OS: macOS (x86_64-apple-darwin14.5.0)
  CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
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  JULIA_STACKFRAME_LINEINFO_COLOR = bold
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```

Below this, there are three input/output pairs:

```
In [2]: pwd()
Out[2]: "/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"

In [3]: ]activate .
```

At the bottom of the screenshot, there is a section header:

## Piecewise Linear Continuous Functions

Below the header, there is a code input field with the text:

```
In [ ]: struct PLCFun|
```



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```
WORD_SIZE: 64
LIBM: libopenlibm
LLVM: libLLVM-6.0.0 (ORCJIT, haswell)
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JULIA_STACKFRAME_FUNCTION_COLOR = bold
```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

## Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T <: Real ,U <: Num}
 points::Vector{U}
end`

UndefVarError: Num not defined

Stacktrace:

[1] top-level scope at none:0

In [ ]:

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Julia 1.0.1

```
WORD_SIZE: 64
LIBM: libopenlibm
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```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

## Piecewise Linear Continuous Functions

In [5]: `struct PLCFun{T <: Real ,U <: Number}
 points::Vector{U}
end`

In [ ]:



File Edit View Insert Cell Kernel Widgets Help

Trusted Julia 1.0.1

```
LLVM: libLLVM-6.0.0 (ORCJIT, haswell)
Environment:
JULIA_ERROR_COLOR = red
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```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

## Piecewise Linear Continuous Functions

```
In [5]: struct PLCFun{T <: Real ,U <: Number}
        # Domain: [0; 1]
        points::Vector{U}
        end
```

```
In [ ]: evaluate(f::PLCFun{T,U}, |)
```



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```
LLVM: libLLVM-6.0.0 (ORCJIT, haswell)
Environment:
JULIA_ERROR_COLOR = red
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```

```
In [2]: pwd()
```

```
Out[2]: "/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"
```

```
In [3]: ]activate .
```

## Piecewise Linear Continuous Functions

```
In [5]: struct PLCFun{T <: Real ,U <: Number}
        # Domain: [0, 1]
        points::Vector{U}
        end
```

```
In [ ]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
```

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In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

### Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T,U}  
 # Domain: [0; 1]  
 points::Vector{U}  
end`

In [ ]: `function xcoord(f::PLCFun{T, U}, i::Int)::T where{T, U}  
 nlines = length(f.points) - 1  
  
 @assert 1 <= i <= nlines  
 dx = 1 / (length)`

In [5]: `function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}  
 @assert 0 <= x <= 1  
 ...  
end`

syntax: invalid identifier name "..."

In [ ]:

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Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

## Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T,U}  
 # Domain: [0; 1]  
 points::Vector{U}  
end`

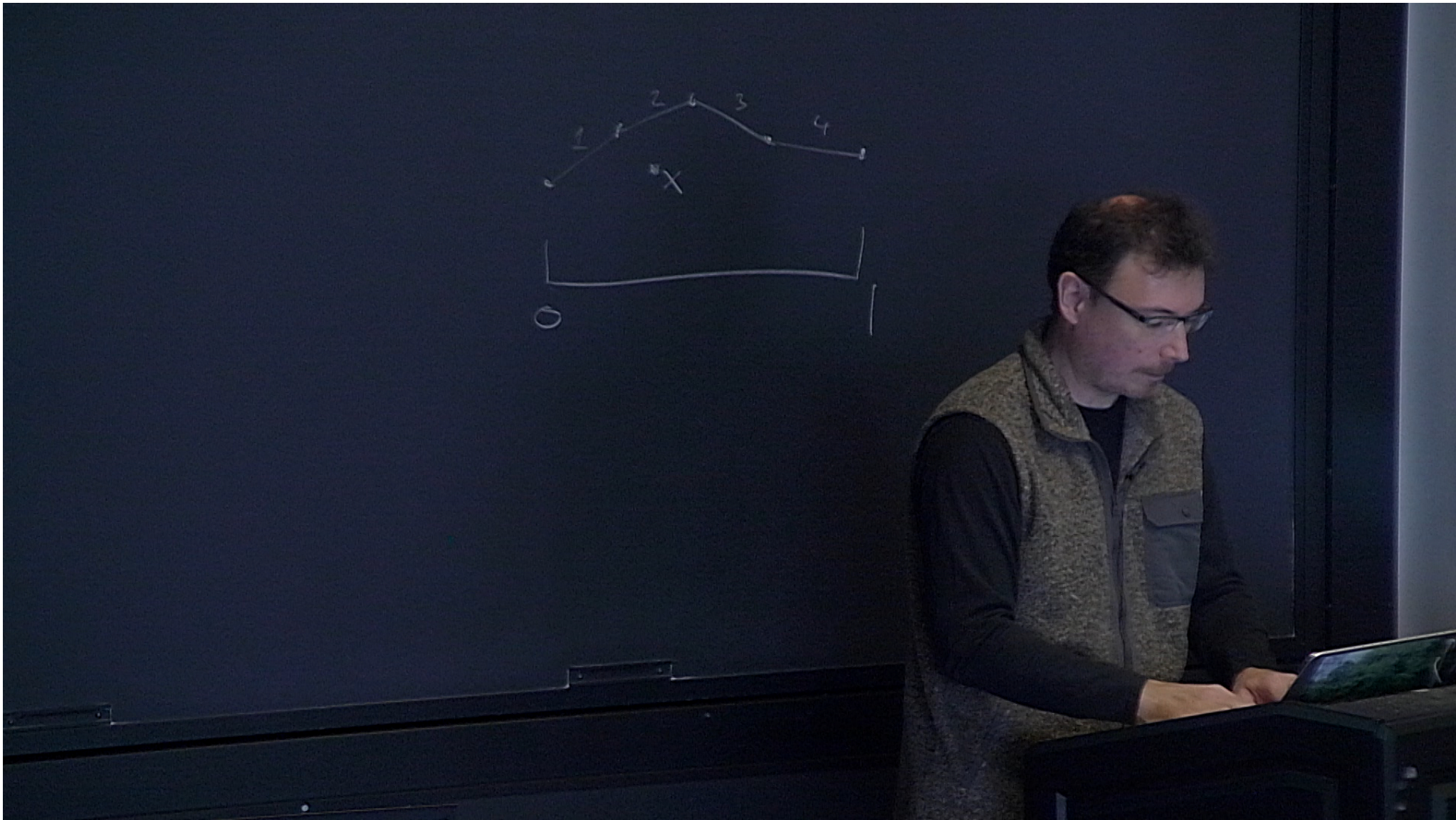
In [ ]: `function xcoord(f::PLCFun{T, U}, i::Int)::T where{T, U}  
 nlines = length(f.points) - 1  
 @assert 1 <= i <= nlines + 1  
 dx = 1 / nlines  
 x = (i-1) * dx  
 x  
end`

In [5]: `function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}  
 @assert 0 <= x <= 1  
 ...  
end`

syntax: invalid identifier name "..."

In [ ]:





Out[2]: "/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"

In [3]: ]activate .

### Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T,U}  
 # Domain: [0; 1]  
 points::Vector{U}  
end`

In [7]: `function xcoord(f::PLCFun{T, U}, i::Int)::T where {T, U}  
 nlines = length(f.points) - 1  
 @assert 1 <= i <= nlines + 1  
 dx = 1 / nlines  
 x = (i-1) * dx  
 x  
end`

Out[7]: xcoord (generic function with 1 method)

In [ ]: `function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}  
 x = (i-1) * dx  
end`

In [5]: `function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}  
 @assert 0 <= x <= 1  
 ...  
end`



```
Out[2]: "/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"
```

```
In [3]: ]activate .
```

## Piecewise Linear Continuous Functions

```
In [4]: struct PLCFun{T,U}
        # Domain: [0; 1]
        points::Vector{U}
        end
```

```
In [7]: function xcoord(f::PLCFun{T, U}, i::Int)::T where {T, U}
        nlines = length(f.points) - 1
        @assert 1 <= i <= nlines + 1
        dx = 1 / nlines
        x = (i-1) * dx
        x
        end
```

```
Out[7]: xcoord (generic function with 1 method)
```

```
In [ ]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
        @assert 0 <= x <= 1
        dx = 1 / nlines
        i = x / dx + 1
        end
```

```
In [5]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
        @assert 0 <= x <= 1
```

```
In [3]: ]activate .
```

## Piecewise Linear Continuous Functions

```
In [4]: struct PLCFun{T,U}
        # Domain: [0; 1]
        points::Vector{U}
    end
```

```
In [7]: function xcoord(f::PLCFun{T, U}, i::Int)::T where {T, U}
        nlines = length(f.points) - 1
        @assert 1 <= i <= nlines + 1
        dx = 1 / nlines
        x = (i-1) * dx
        x
    end
```

```
Out[7]: xcoord (generic function with 1 method)
```

```
In [ ]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
        @assert 0 <= x <= 1
        dx = 1 / nlines
        i = floor(Int, x / dx) + 1
        i
    end
```

```
In [5]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
        @assert 0 <= x <= 1
        ...
    end
```

localhost:8888/notebooks/txt/Courses/CompPhys2018//FiniteDifferencing/FiniteDifferencing.ipynb

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Run Code

```
In [7]: function xcoord(f::PLCFun{T, U}, i::Int)::T where {T, U}
        nlines = length(f.points) - 1
        @assert 1 <= i <= nlines + 1
        dx = 1 / nlines
        x = (i-1) * dx
        x
    end
```

Out[7]: xcoord (generic function with 1 method)

```
In [9]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
        @assert 0 <= x <= 1
        nlines = length(f.points) - 1
        dx = 1 / nlines
        i = floor(Int, x / dx) + 1
        i = max(1, i)
        i = min(nlines, i)
        i
    end
```

Out[9]: lineidx (generic function with 1 method)

```
In [5]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
        @assert 0 <= x <= 1
        i = lineidx(f, x)
        ...
    end
```

syntax: invalid identifier name "..."



localhost:8888/notebooks/txt/Courses/CompPhys2018//FiniteDifferencing/FiniteDifferencing.ipynb

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```
x = (i-1) * dx
x
end
```

Out[7]: xcoord (generic function with 1 method)

```
In [10]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
    @assert 0 <= x <= 1
    nlines = length(f.points) - 1
    dx = 1 / nlines
    i = floor(Int, x / dx) + 1
    i = max(1, i)
    i = min(nlines, i)
    i
end
```

Out[10]: lineidx (generic function with 1 method)

```
In [ ]: function linterp
```

```
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
    @assert 0 <= x <= 1
    i = lineidx(f, x)
    y = linterp(x1, y1, x2, y2, x)
    y
end
```

Out[11]: evaluate (generic function with 1 method)

```
In [ ]:
```

```
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x = (i-1) * dx
x
end

Out[7]: xcoord (generic function with 1 method)

In [10]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
    @assert 0 <= x <= 1
    nlines = length(f.points) - 1
    dx = 1 / nlines
    i = floor(Int, x / dx) + 1
    i = max(1, i)
    i = min(nlines, i)
    i
end

Out[10]: lineidx (generic function with 1 method)

In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
    y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
end

Out[12]: linterp (generic function with 1 method)

In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
    @assert 0 <= x <= 1
    i = lineidx(f, x)
    y = linterp(x1, y1, x2, y2, x)
    y
end

Out[11]: evaluate (generic function with 1 method)

In [1]:
```



localhost:8888/notebooks/txt/Courses/CompPhys2018//FiniteDifferencing/FiniteDifferencing.ipynb

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## Piecewise Linear Continuous Functions

```
In [4]: struct PLCFun{T,U}
        # Domain: [0; 1]
        points::Vector{U}
      end
```

```
In [18]: function Base.*(a::U, f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          PLCFun{T, U}(a .* f.points)
        end
function Base.+(f::PLCFun{T, U}, g::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          PLCFun{T, U}(f.point .+ g.points)
        end
```

```
In [7]: function xcoord(f::PLCFun{T, U}, i::Int)::T where {T, U}
          nlines = length(f.points) - 1
          @assert 1 <= i <= nlines + 1
          dx = 1 / nlines
          x = (i-1) * dx
          x
        end
```

Out[7]: xcoord (generic function with 1 method)

```
In [10]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
          @assert 0 <= x <= 1
          nlines = length(f.points) - 1
          dx = 1 / nlines
          i = floor(Int, x / dx) + 1
          i = max(1, i)
        end
```

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```
nlines = length(f.points) - 1
dx = 1 / nlines
i = floor(Int, x / dx) + 1
i = max(1, i)
i = min(nlines, i)
i
end
```

Out[10]: lineidx (generic function with 1 method)

```
In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
          y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
        end
```

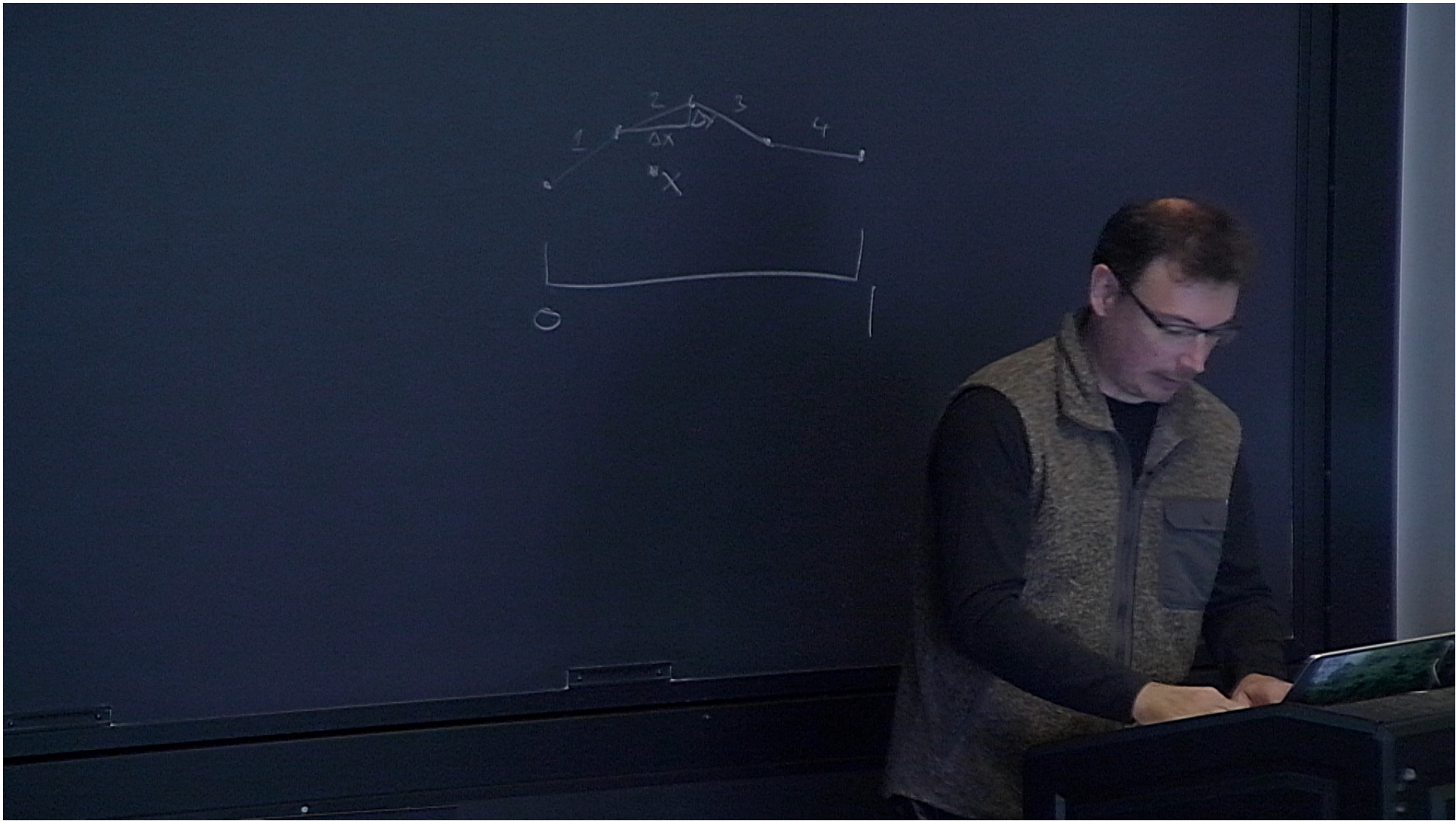
Out[12]: linterp (generic function with 1 method)

```
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
          @assert 0 <= x <= 1
          i = lineidx(f, x)
          y = linterp(x1, y1, x2, y2, x)
          y
        end
```

Out[11]: evaluate (generic function with 1 method)

```
In [ ]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          end
```





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Out[10]: lineidx (generic function with 1 method)

```
In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
          y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
        end
```

Out[12]: linterp (generic function with 1 method)

```
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
          @assert 0 <= x <= 1
          i = lineidx(f, x)
          y = linterp(x1, y1, x2, y2, x)
          y
        end
```

Out[11]: evaluate (generic function with 1 method)

```
In [20]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          nlines = length(f.points) - 1
          dx = 1 / nlines
          ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines+1]
          PLCFun{T, U}(ys)
        end
```

Out[20]: derivRight (generic function with 1 method)

```
In [ ]: |
```



localhost:8888/notebooks/txt/Courses/CompPhys2018//FiniteDifferencing/FiniteDifferencing.ipynb

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Out[10]: lineidx (generic function with 1 method)

In [12]: `function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
 y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
end`

Out[12]: linterp (generic function with 1 method)

In [11]: `function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
 @assert 0 <= x <= 1
 i = lineidx(f, x)
 y = linterp(x1, y1, x2, y2, x)
 y
end`

Out[11]: evaluate (generic function with 1 method)

In [20]: `function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
 nlines = length(f.points) - 1
 dx = 1 / nlines
 ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
 (f.points[end] - f.points[end-1]) / dx]
 |
 PLCFun{T, U}(ys)
end`

Out[20]: derivRight (generic function with 1 method)

In [ ]:



localhost:8888/notebooks/txt/Courses/CompPhys2018//FiniteDifferencing/FiniteDifferencing.ipynb

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Out[10]: lineidx (generic function with 1 method)

```
In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
          y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
        end
```

Out[12]: linterp (generic function with 1 method)

```
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
          @assert 0 <= x <= 1
          i = lineidx(f, x)
          y = linterp(x1, y1, x2, y2, x)
          y
        end
```

Out[11]: evaluate (generic function with 1 method)

```
In [21]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          nlines = length(f.points) - 1
          dx = 1 / nlines
          ys = U[(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
              (f.points[end] - f.points[end-1]) / dx]
          PLCFun{T, U}(ys)
        end
```

Out[21]: derivRight (generic function with 1 method)

```
In [ ]:
```

```
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txt/Courses/CompPhys2018//FiniteDifferencing/ FiniteDifferencing
File Edit View Insert Cell Kernel Widgets Help Trusted Julia 1.0.1
Code
Out[7]: xcoord (generic function with 1 method)
In [10]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
          @assert 0 <= x <= 1
          nlines = length(f.points) - 1
          dx = 1 / nlines
          i = floor(Int, x / dx) + 1
          i = max(1, i)
          i = min(nlines, i)
          i
        end
Out[10]: lineidx (generic function with 1 method)
In [ ]: function samplePLC(::Type{T}, ::Type{U}, nlines::Int, f::Function)::PLCFun{T, U} where {T, U}
          |
          end
In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
          y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
        end
Out[12]: linterp (generic function with 1 method)
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
          @assert 0 <= x <= 1
          i = lineidx(f, x)
          y = linterp(x1, y1, x2, y2, x)
          y
        end
Out[11]: evaluate (generic function with 1 method)
```



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Out[7]: xcoord (generic function with 1 method)

```
In [10]: function lineidx(f::PLCFun{T, U}, x::T)::Int where {T, U}
    @assert 0 <= x <= 1
    nlines = length(f.points) - 1
    dx = 1 / nlines
    i = floor(Int, x / dx) + 1
    i = max(1, i)
    i = min(nlines, i)
    i
end
```

Out[10]: lineidx (generic function with 1 method)

```
In [ ]: function samplePLC(::Type{T}, ::Type{U}, nlines::Int, f::Function)::PLCFun{T, U} where {T, U}
    ys = U[f(xcoord(i)) for i in 1:nlines+1]
    PLCFun{T, U}(ys)
end
```

```
In [12]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
    y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
end
```

Out[12]: linterp (generic function with 1 method)

```
In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
    @assert 0 <= x <= 1
    i = lineidx(f, x)
    y = linterp(x1, y1, x2, y2, x)
    y
end
```



```
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txt/Courses/CompPhys2018//FiniteDifferencing/ FiniteDifferencing
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Run Code

end

Out[22]: samplePLC (generic function with 1 method)

In [23]: samplePLC(Float64, Float64, 10, sin)

i = 1
i = 2
i = 3
i = 4
i = 5
i = 6
i = 7
i = 8
i = 9
i = 10
i = 11

Out[23]: PLCFun{Float64,Float64}([0.0, 0.0998334, 0.198669, 0.29552, 0.389418, 0.479426, 0.564642, 0.644218, 0.717356, 0.783327, 0.841471])

In [10]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
          y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
        end

Out[10]: linterp (generic function with 1 method)

In [11]: function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
          @assert 0 <= x <= 1
          i = lineidx(f, x)
          y = linterp(x1, y1, x2, y2, x)
          y
        end
```

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end

Out[22]: samplePLC (generic function with 1 method)

In [25]: `fsin = samplePLC(Float64, Float64, 10, sin)`

Out[25]: `PLCFun{Float64,Float64}([0.0, 0.0998334, 0.198669, 0.29552, 0.389418, 0.479426, 0.564642, 0.644218, 0.717356, 0.783327, 0.841471])`

In [30]: `(evaluate(fsin, 0.3333), sin(0.3333))`

Out[30]: `(0.32678828583189407, 0.32716319804950605)`

In [10]: `function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
 y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
end`

Out[10]: linterp (generic function with 1 method)

In [28]: `function evaluate(f::PLCFun{T,U}, x::T)::U where {T, U}
 @assert 0 <= x <= 1
 nlines = length(f.points) - 1
 i = lineidx(f, x)
 x1 = xcoord(T, nlines, i)
 x2 = xcoord(T, nlines, i+1)
 y1 = f.points[i]
 y2 = f.points[i+1]
 y = linterp(x1, y1, x2, y2, x)
 y
end`

Out[28]: evaluate (generic function with 1 method)



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Run Markdown

```
CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
WORD_SIZE: 64
LIBM: libopenlibm
LLVM: libLLVM-6.0.0 (ORCJIT, haswell)
Environment:
JULIA_ERROR_COLOR = red
JULIA_WARN_COLOR = magenta
JULIA_INFO_COLOR = blue
JULIA_DEBUG_COLOR = blue
JULIA_INPUT_COLOR = normal
JULIA_ANSWER_COLOR = normal
JULIA_STACKFRAME_LINEINFO_COLOR = bold
JULIA_STACKFRAME_FUNCTION_COLOR = bold
```

In [2]: `pwd()`

Out[2]: `"/Users/eschnett/txt/Courses/CompPhys2018/jl/FiniteDifferencing"`

In [3]: `]activate .`

In [\*]: `]add Plots`

```
Updating registry at `~/..julia/registries/General`
Updating git-repo `https://github.com/JuliaRegistries/General.git`
```

## Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T,U}
 # Domain: [0; 1]
 points::Vector{U}
end`

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Run

```
[9a3f8284] + Random
[ea8e919c] + SHA
[9e88b42a] + Serialization
[1a1011a3] + SharedArrays
[6462fe0b] + Sockets
[2f01184e] + SparseArrays
[10745b16] + Statistics
[8dfed614] + Test
[cf7118a7] + UUIDs
[4ec0a83e] + Unicode
```

In [\*]: `using Plots`

## Piecewise Linear Continuous Functions

In [4]: `struct PLCFun{T,U}
# Domain: [0; 1]
points::Vector{U}
end`

In [5]: `function Base.*(a::U, f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
PLCFun{T, U}(a .* f.points)
end
function Base.+(f::PLCFun{T, U}, g::PLCFun{T, U})::PLCFun{T, U} where {T, U}
@assert length(f.points) == length(g.points)
PLCFun{T, U}(f.points .+ g.points)
end`

In [24]: `function xcoord(::Type{T}, nlines::Int, i::Int)::T where {T}
@assert 1 <= i <= nlines + 1
dx = 1 / nlines`



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```

+ = f(xcoord(T, nlines, i))
i = max(1, i)
i = min(nlines, i)
i
end

```

Out[21]: lineidx (generic function with 1 method)

```

In [22]: function samplePLC(::Type{T}, ::Type{U}, nlines::Int, f::Function)::PLCFun{T, U} where {T, U}
ys = U[f(xcoord(T, nlines, i)) for i in 1:nlines+1]
PLCFun{T, U}(ys)
end

```

Out[22]: samplePLC (generic function with 1 method)

```

In [25]: fsin = samplePLC(Float64, Float64, 10, sin)

```

Out[25]: PLCFun{Float64,Float64}([0.0, 0.0998334, 0.198669, 0.29552, 0.389418, 0.479426, 0.564642, 0.644218, 0.717356, 0.783327, 0.841471])

```

In [30]: (evaluate(fsin, 0.3333), sin(0.3333))

```

Out[30]: (0.32678828583189407, 0.32716319804950605)

```

In [*]: xs = collect(range(start=0, stop=1, length=100))
plot(sin.(xs))

```

```

In [10]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)
end

```

Out[10]: linterp (generic function with 1 method)

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```
ys = U[f(xcoord(T, nlines, i)) for i in 1:nlines+1]
PLCFun{T, U}(ys)
end
```

Out[22]: samplePLC (generic function with 1 method)

In [25]: `fsin = samplePLC(Float64, Float64, 10, sin)`

Out[25]: `PLCFun{Float64,Float64}([0.0, 0.0998334, 0.198669, 0.29552, 0.389418, 0.479426, 0.564642, 0.644218, 0.717356, 0.783327, 0.841471])`

In [30]: `(evaluate(fsin, 0.3333), sin(0.3333))`

Out[30]: `(0.32678828583189407, 0.32716319804950605)`

In [33]: `xs = collect(range(start=0, stop=1, length=100))`  
`plot(sin.(xs))`

MethodError: no method matching range(; start=0, stop=1, length=100)  
Closest candidates are:  
range(!Matched::T<:Colorant; stop, length) where T<:Colorant at /Users/eschnett/.julia/packages/Colors/4hvzi/src/utilities.jl:58 got unsupported keyword argument "start"  
range(!Matched::Any; length, stop, step) at range.jl:76 got unsupported keyword argument "start"  
range(!Matched::Any, !Matched::Any; kwargs...) at /Users/eschnett/.julia/packages/Compat/XoxMM/src/Compat.jl:1846

Stacktrace:  
[1] top-level scope at In[33]:1

In [10]: `function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}`  
`y1 * (x - x2) / (x1 - x2) + y2 * (x - x1) / (x2 - x1)`



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Out[21]: lineidx (generic function with 1 method)

```
In [22]: function samplePLC(::Type{T}, ::Type{U}, nlines::Int, f::Function)::PLCFun{T, U} where {T, U}
          ys = U[f(xcoord(T, nlines, i)) for i in 1:nlines+1]
          PLCFun{T, U}(ys)
        end
```

Out[22]: samplePLC (generic function with 1 method)

```
In [25]: fsin = samplePLC(Float64, Float64, 10, sin)
```

Out[25]: PLCFun{Float64,Float64}([0.0, 0.0998334, 0.198669, 0.29552, 0.389418, 0.479426, 0.564642, 0.644218, 0.717356, 0.783327, 0.841471])

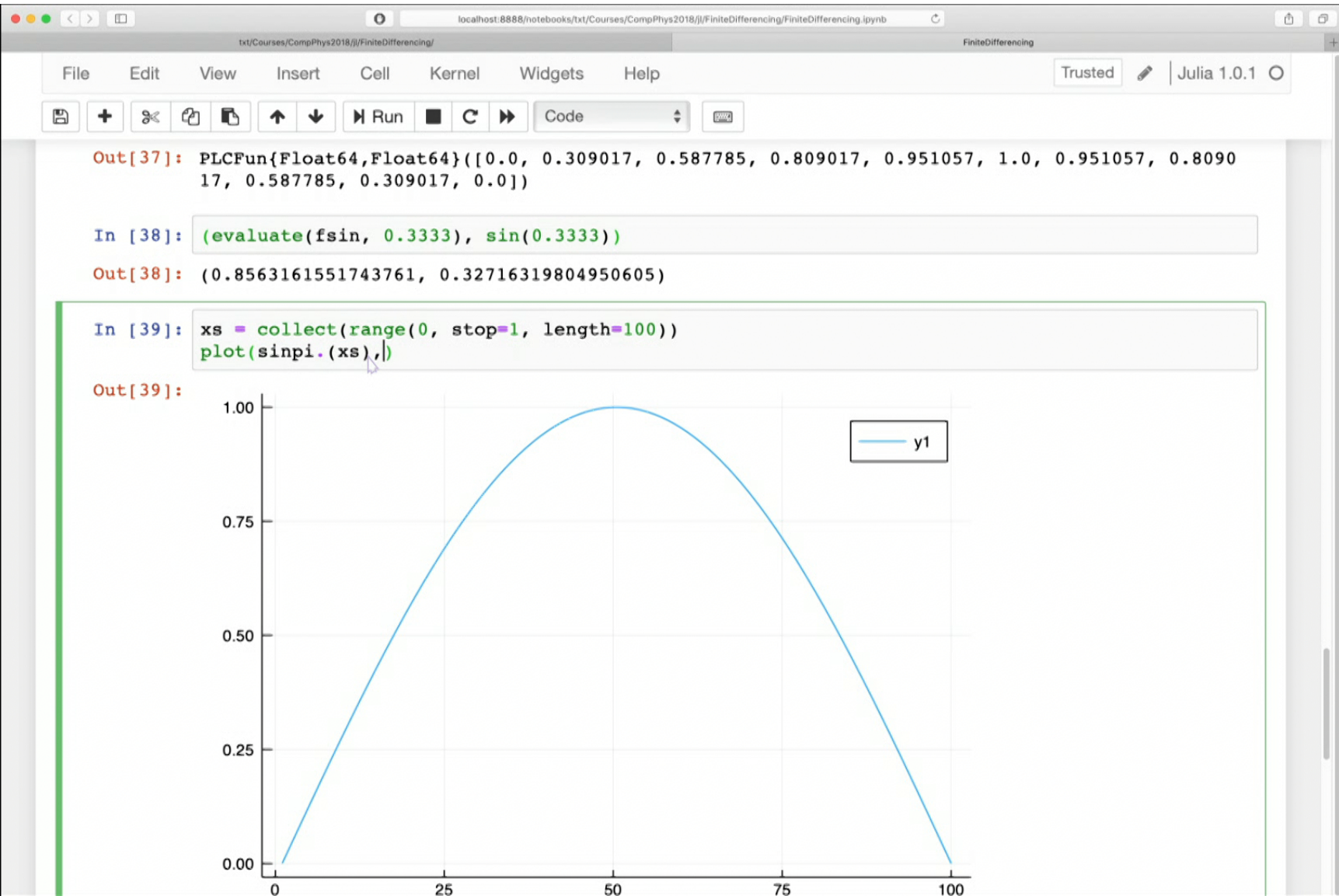
```
In [30]: (evaluate(fsin, 0.3333), sin(0.3333))
```

Out[30]: (0.32678828583189407, 0.32716319804950605)

```
In [36]: xs = collect(range(0, stop=1, length=100))
          plot(sin.(xs))
```

Out[36]:

The plot displays a blue line representing the sine function, labeled 'y1'. The x-axis ranges from 0 to 1, and the y-axis ranges from 0 to 1. The plot shows the sine wave starting at (0,0) and ending at (1,1).





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```
In [40]: xs = collect(range(0, stop=1, length=100))
plot(sinpi.(xs), [evaluate(fsin, x) for x in xs])
```

Out[40]:

In [10]: function linterp(x1::T, y1::U, x2::T, y2::U, x::T)::U where {T, U}
y1 \* (x - x2) / (x1 - x2) + y2 \* (x - x1) / (x2 - x1)
end

Out[10]: linterp (generic function with 1 method)

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```
nlines = length(f.points) - 1
i = lineidx(f, x)
x1 = xcoord(T, nlines, i)
x2 = xcoord(T, nlines, i+1)
y1 = f.points[i]
y2 = f.points[i+1]
y = linterp(x1, y1, x2, y2, x)
y
end
```

Out[28]: evaluate (generic function with 1 method)

```
In [12]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
nlines = length(f.points) - 1
dx = 1 / nlines
ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
      (f.points[end] - f.points[end-1]) / dx]
PLCFun{T, U}(ys)
end
```

Out[12]: derivRight (generic function with 1 method)

## Advection Equation

In [ ]:



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```
y = linterp(x1, y1, x2, y2, x)
y
end
```

Out[28]: evaluate (generic function with 1 method)

```
In [12]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          nlines = length(f.points) - 1
          dx = 1 / nlines
          ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
               (f.points[end] - f.points[end-1]) / dx
          PLCFun{T, U}(ys)
        end
```

Out[12]: derivRight (generic function with 1 method)

## Advection Equation

```
In [42]: struct AdvectionState{T}
          time::T
          u::PLCFun{T, T}
        end
```

In [ ]:

```
In [12]: function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
          nlines = length(f.points) - 1
          dx = 1 / nlines
          ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
              (f.points[end] - f.points[end-1]) / dx]
          PLCFun{T, U}(ys)
        end
```

Out[12]: derivRight (generic function with 1 method)

## Advection Equation

```
In [42]: struct AdvectionState{T}
          time::T
          u::PLCFun{T, T}
        end
```

```
In [ ]: function gaussian(x::T)::T where {T}
        end
```

```
In [43]: function initialGaussian(nlines::Int)::AdvectionState{Float64}
          t = 0
          u = samplePLC(Float64, Float64, nlines, gaussian)
          AdvectionState{Float64}(t, u)
        end
```

Out[43]: initialGaussian (generic function with 1 method)

```
In [ ]:
```



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In [12]:

```
function derivRight(f::PLCFun{T, U})::PLCFun{T, U} where {T, U}
    nlines = length(f.points) - 1
    dx = 1 / nlines
    ys = [(f.points[i+1] - f.points[i]) / dx for i in 1:nlines];
        (f.points[end] - f.points[end-1]) / dx]
    PLCFun{T, U}(ys)
end
```

Out[12]: derivRight (generic function with 1 method)

## Advection Equation

In [42]:

```
struct AdvectionState{T}
    time::T
    u::PLCFun{T, T}
end
```

In [ ]:

```
function gaussian(x::T)::T where {T}
    exp(-(x - 5)^2)
end
```

In [43]:

```
function initialGaussian(nlines::Int)::AdvectionState{Float64}
    t = 0
    u = samplePLC(Float64, Float64, nlines, gaussian)
    AdvectionState{Float64}(t, u)
end
```

Out[43]: initialGaussian (generic function with 1 method)

In [ ]:

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Run Code

## Advection Equation

```
In [42]: struct AdvectionState{T}
         time::T
         u::PLCFun{T, T}
       end
```

```
In [44]: function gaussian(x::T)::T where {T}
         exp(- ((x - 0.5) / 10)^2)
       end
```

Out[44]: gaussian (generic function with 1 method)

```
In [45]: function initialGaussian(nlines::Int)::AdvectionState{Float64}
         t = 0
         u = samplePLC(Float64, Float64, nlines, gaussian)
         AdvectionState{Float64}(t, u)
       end
```

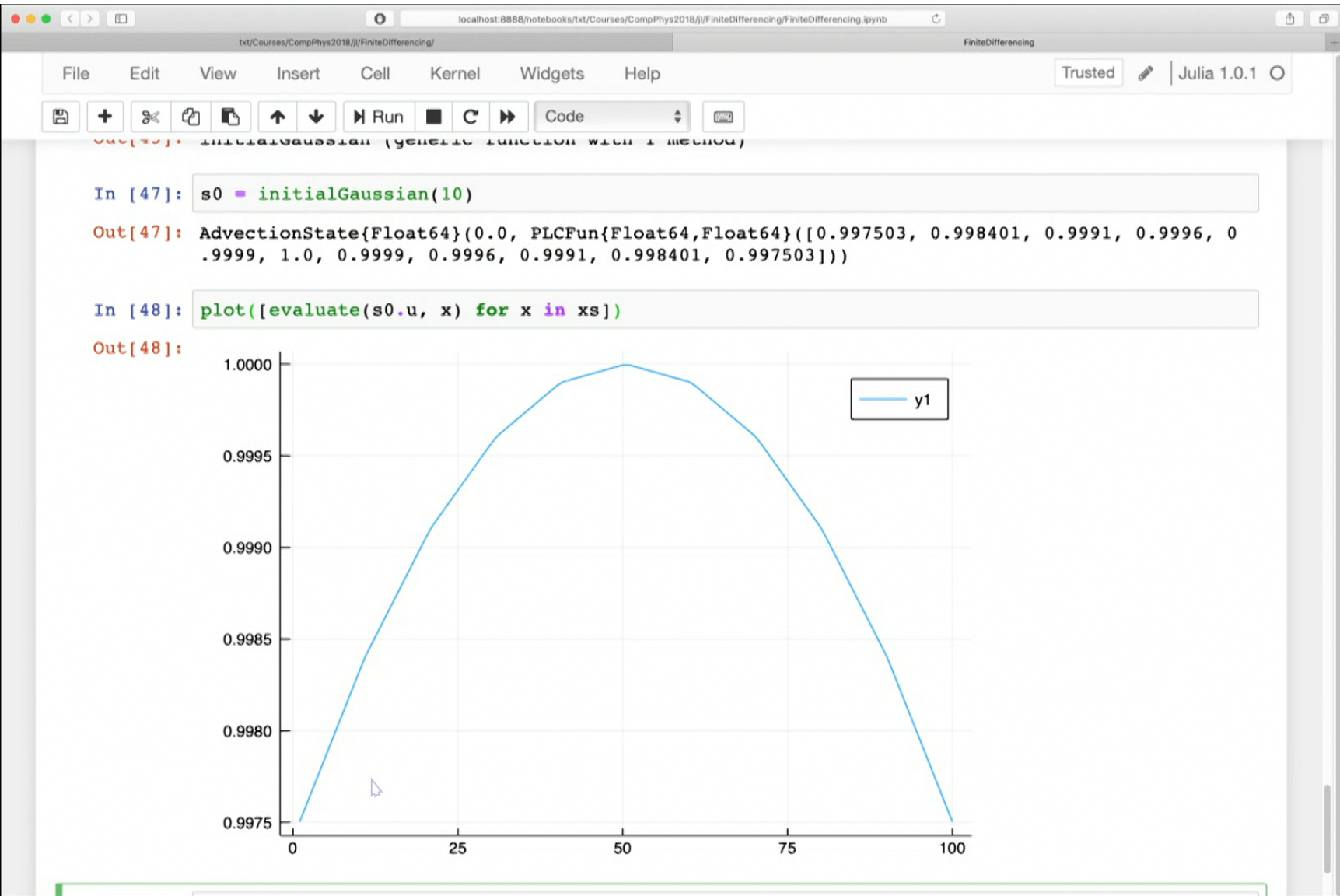
Out[45]: initialGaussian (generic function with 1 method)

```
In [46]: initialGaussian(10)
```

Out[46]: AdvectionState{Float64}(0.0, PLCFun{Float64,Float64}([0.997503, 0.998401, 0.9991, 0.9996, 0.9999, 1.0, 0.9999, 0.9996, 0.9991, 0.998401, 0.997503]))

```
In [ ]: |
```





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In [42]: 

```
struct AdvectionState{T}
    time::T
    u::PLCFun{T, T}
end
```

In [44]: 

```
function gaussian(x::T)::T where {T}
    exp(- ((x - 0.5) / 10)^2)
end
```

Out[44]: gaussian (generic function with 1 method)

In [45]: 

```
function initialGaussian(nlines::Int)::AdvectionState{Float64}
    t = 0
    u = samplePLC(Float64, Float64, nlines, gaussian)
    AdvectionState{Float64}(t, u)
end
```

Out[45]: initialGaussian (generic function with 1 method)

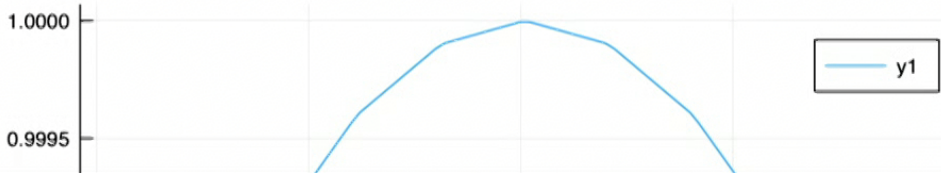
In [47]: 

```
s0 = initialGaussian(10)
```

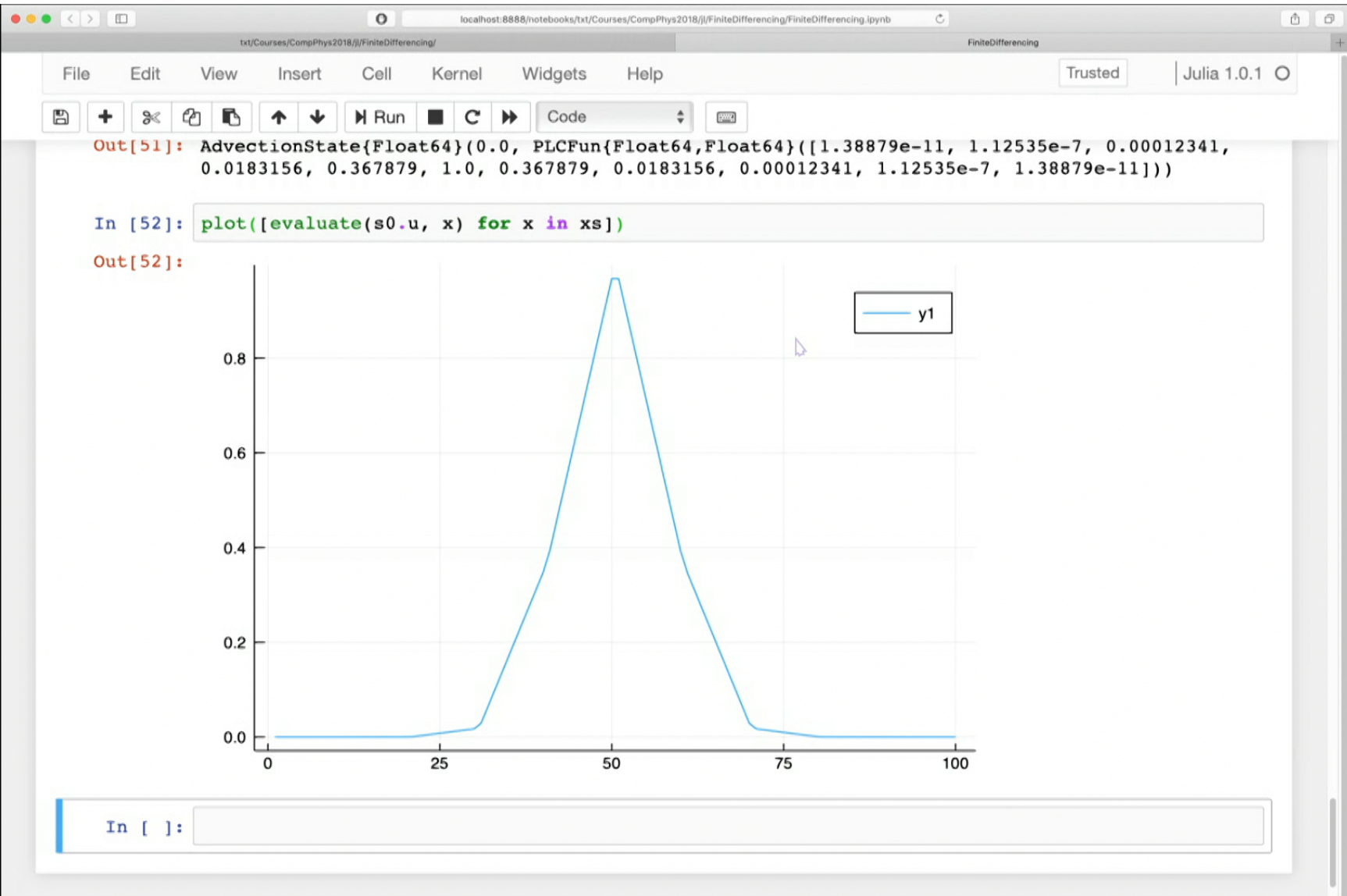
Out[47]: AdvectionState{Float64}(0.0, PLCFun{Float64,Float64}([0.997503, 0.998401, 0.9991, 0.9996, 0.9999, 1.0, 0.9999, 0.9996, 0.9991, 0.998401, 0.997503]))

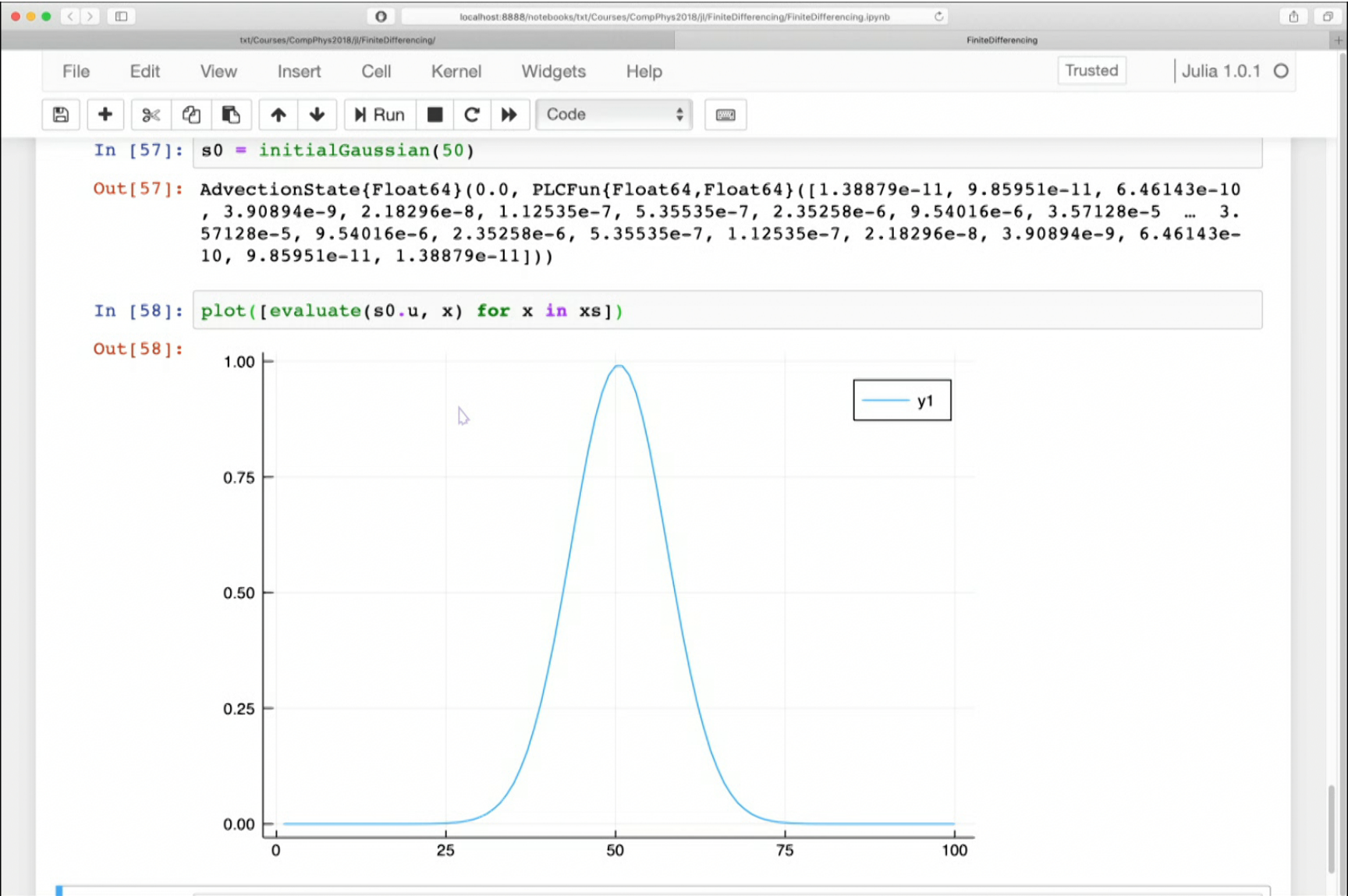
In [48]: 

```
plot([evaluate(s0.u, x) for x in xs])
```

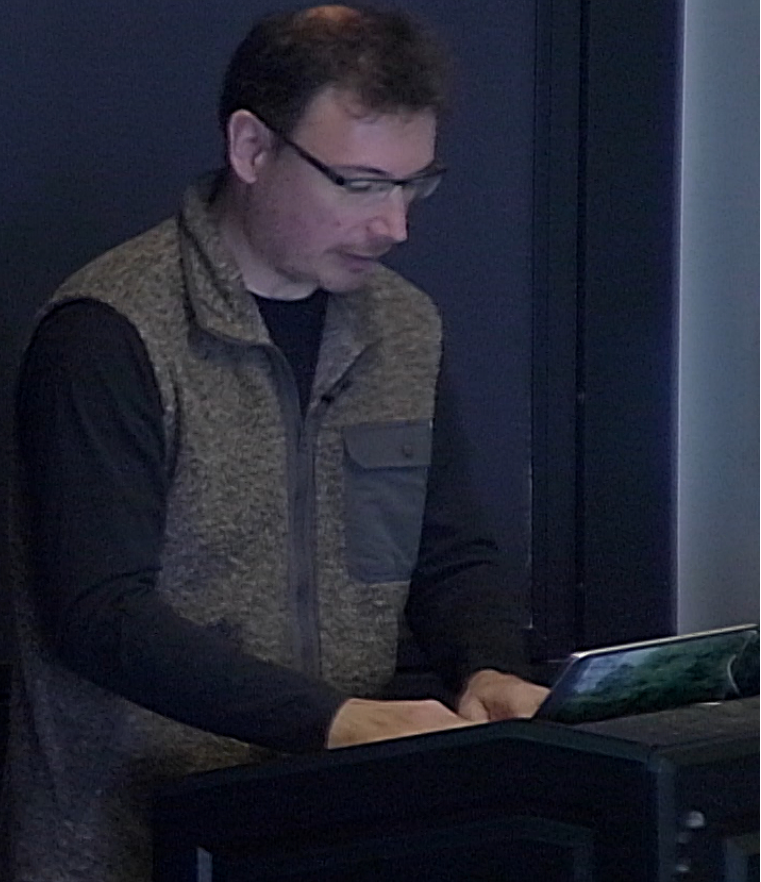
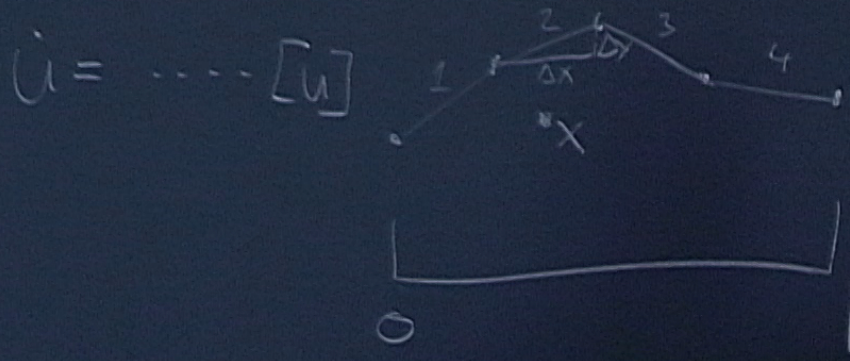
Out[48]: 







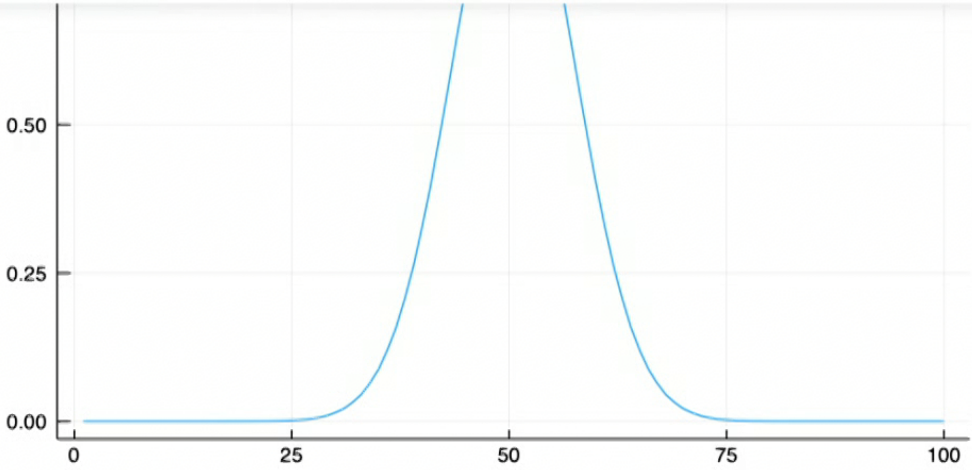




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Code

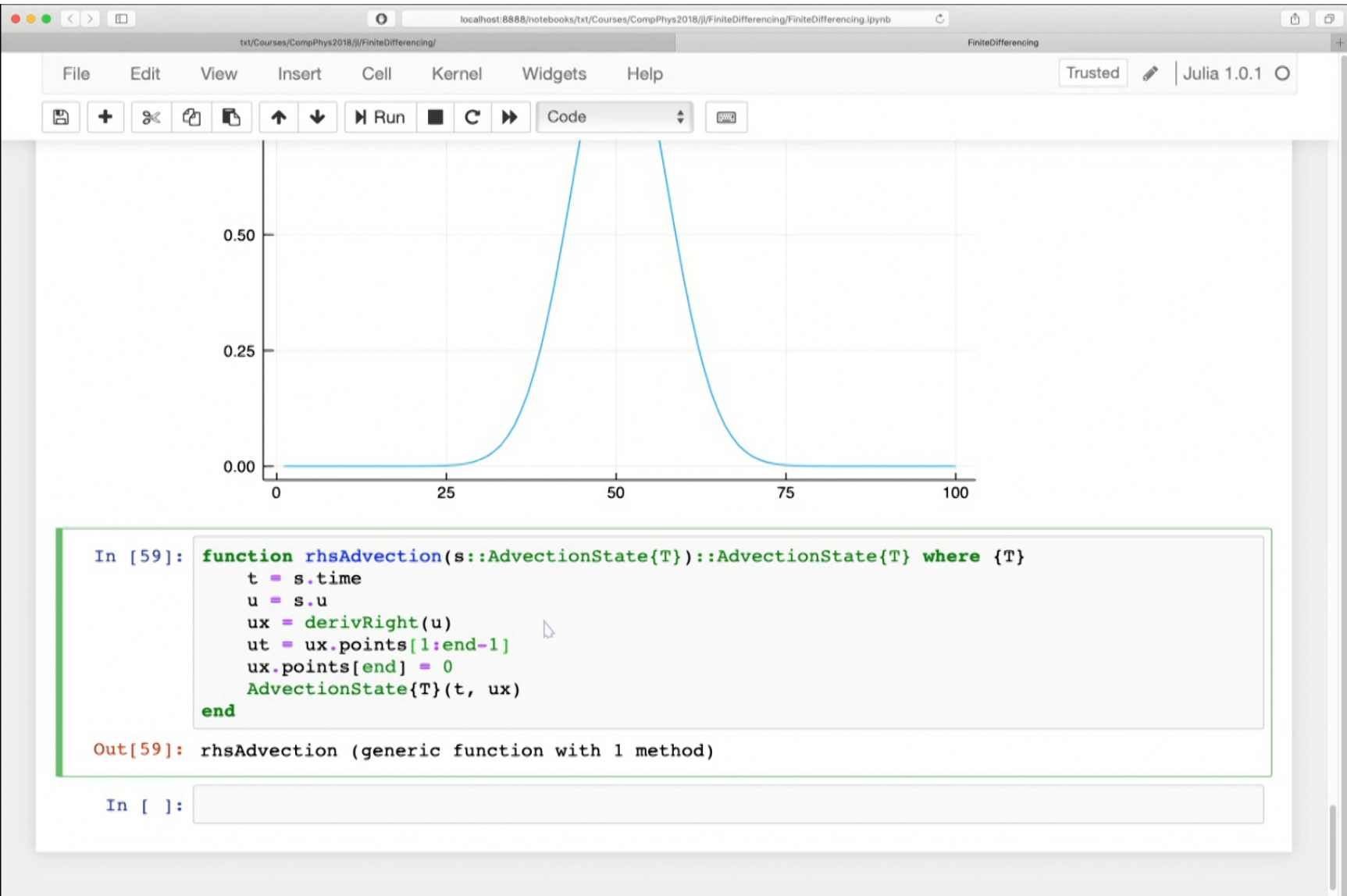


```
In [59]: function rhsAdvection(s::AdvectionState{T})::AdvectionState{T} where {T}
          t = s.time
          u = s.u
          ux = derivRight(u)
          AdvectionState{T}(t, ux)
        end
```

Out[59]: rhsAdvection (generic function with 1 method)

```
In [ ]: |
```

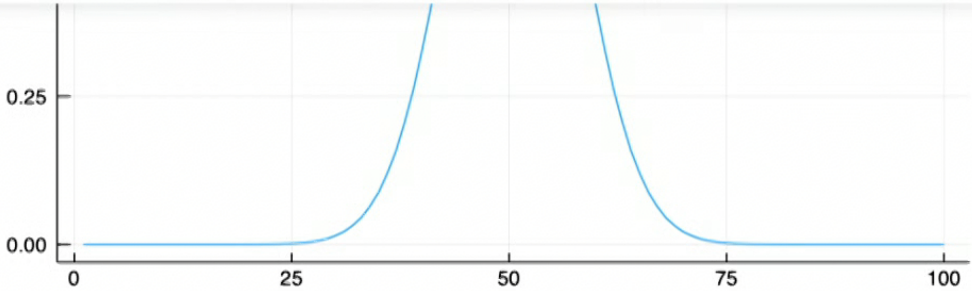




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Code



```
In [60]: function rhsAdvection(s::AdvectionState{T})::AdvectionState{T} where {T}
    t = s.time
    u = s.u
    nlines = length(u.points)
    ux = derivRight(u)
    ut = T[[ux.points[i] for i in 1:nlines]; 0]
    AdvectionState{T}(t, ut)
end
```

```
Out[60]: rhsAdvection (generic function with 1 method)
```

```
In [61]: rhsAdvection(s0)
```

```
MethodError: Cannot `convert` an object of type Array{Float64,1} to an object of type PLCFun{Float64,Float64}
Closest candidates are:
  convert(::Type{T}, !Matched::T) where T at essentials.jl:154
  PLCFun{Float64,Float64}(::Any) where {T, U} at In[4]:3

Stacktrace:
 [1] AdvectionState{Float64}(::Float64, ::Array{Float64,1}) at ./In[42]:2
 [2] rhsAdvection(::AdvectionState{Float64}) at ./In[60]:7
```



