

Title: Laser spectroscopy of muonic atoms and the proton radius puzzle

Date: Nov 20, 2018 01:00 PM

URL: <http://pirsa.org/18110037>

Abstract: <p>Laser spectroscopy of muonic hydrogen [1,2] yielded a proton rms charge radius which is 4% (or ~6 sigmas) smaller than the CODATA value [3]. This discrepancy is now called the "proton radius puzzle" [4].<br />

Also the deuteron charge radius from muonic deuterium [5] is 6 sigmas smaller than the<br />

CODATA value, but consistent with the smaller proton inside the deuteron.<br />

These smaller charge radii, when combined with precision measurements of the 1S-2S transitions in regular (electronic) hydrogen [6] and deuterium [7], yield a 6 sigmas smaller value of the Rydberg constant [8], compared to the CODATA value.<br />

In this talk I will report about a new measurement of the Rydberg constant from the 2S-4P transition in regular hydrogen performed in Garching [9], which supports the smaller, "muonic" value. More recently, however, a new measurement of the 1S-3S transition in Paris confirmed the larger proton radius [10].<br />

Several new measurements, such as hydrogen from Toronto, elastic electron scattering at lower  $Q^2$ , and new results from electronic and muonic helium will help understand the proton radius puzzle.</p>

# Laser spectroscopy of muonic atoms: Charge radii and polarizabilities

The Proton Radius Puzzle and beyond

Randolf Pohl

Johannes Gutenberg-Universität Mainz  
Institut für Physik, QUANTUM und Exzellenzcluster PRISMA+

before: Max-Planck Institute of Quantum Optics, Garching



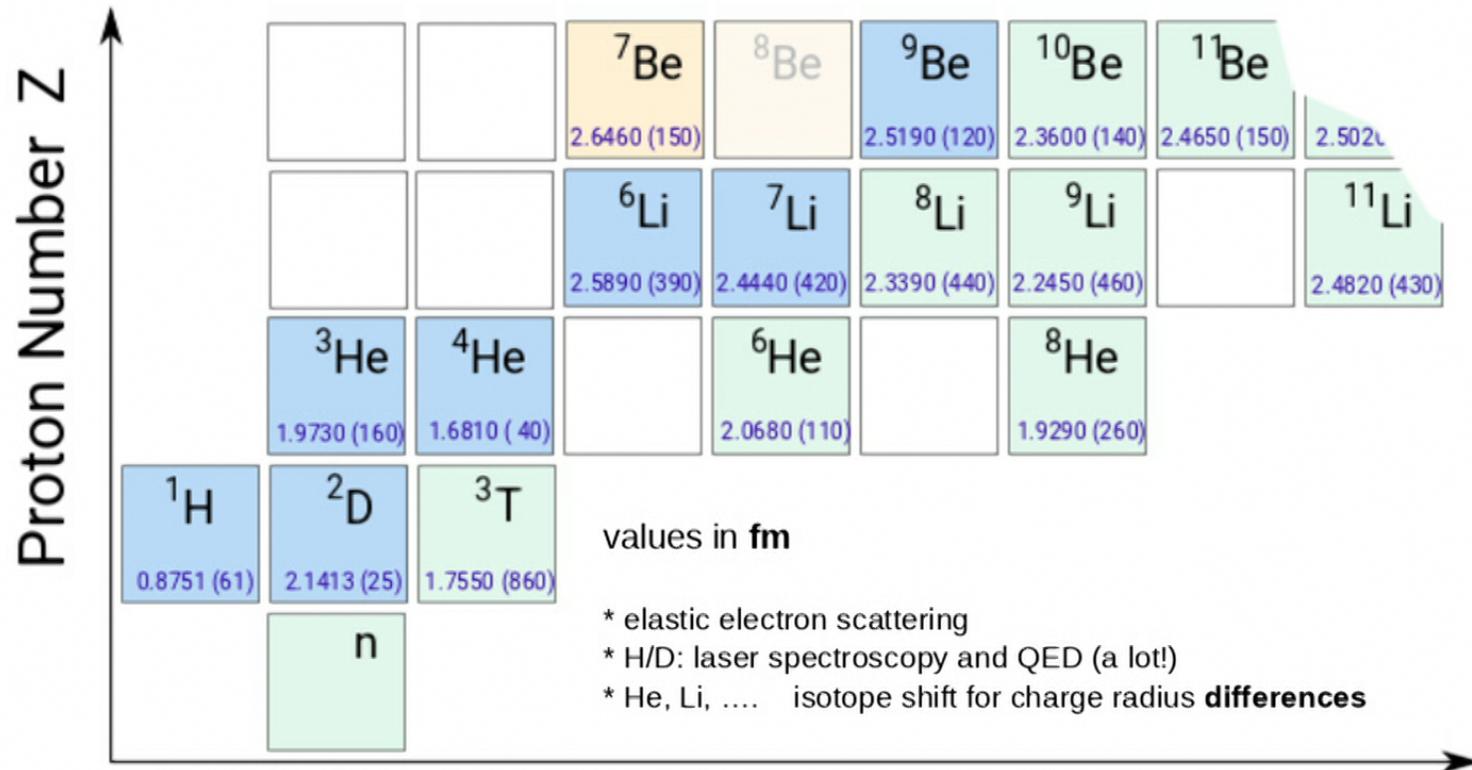
Perimeter Inst.  
Nov. 20, 2018

# Outline

- **Muonic atoms**  
as a probe of nuclear physics (**charge radii**, magnetization radii, polarizabilities, ...)
- The “**Proton Radius Puzzle**”
- **Rydberg constant**  
key parameter to check **atomic physics** part of the discrepancy
- **Muonic helium**
- **Muonic future:** Li, Be, T?

# Nuclear rms charge radii

from measurements with **electrons**

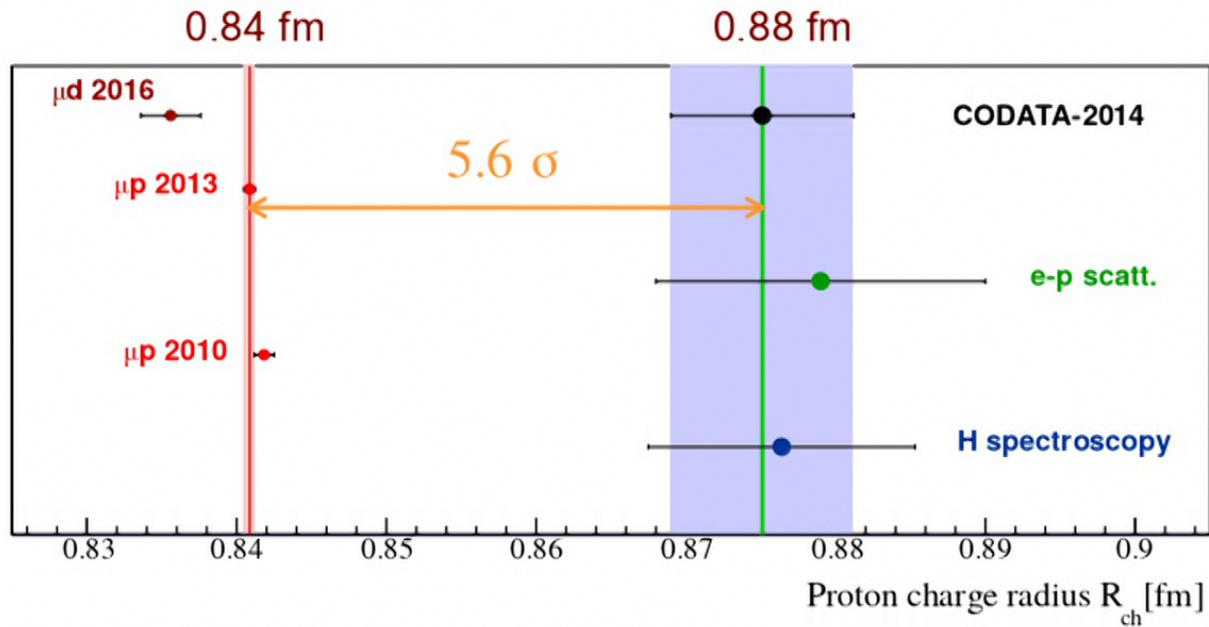


sources: \* p,d: CODATA-2014  
 \* t: Amroun et al. (Saclay), NPA 579, 596 (1994)  
 \*  $^3,^4\text{He}$ : Sick, J.Phys.Chem.Ref Data 44, 031213 (2015)  
 \* Angeli, At. Data Nucl. Data Tab. 99, 69 (2013)

Neutron number N

# The “Proton Radius Puzzle”

Measuring  $R_p$  using **electrons**: 0.88 fm (  $\pm 0.7\%$  )  
using **muons**: 0.84 fm (  $\pm 0.05\%$  )



$\mu d$  2016: RP et al (CREMA Coll.) Science 353, 669 (2016)

$\mu p$  2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

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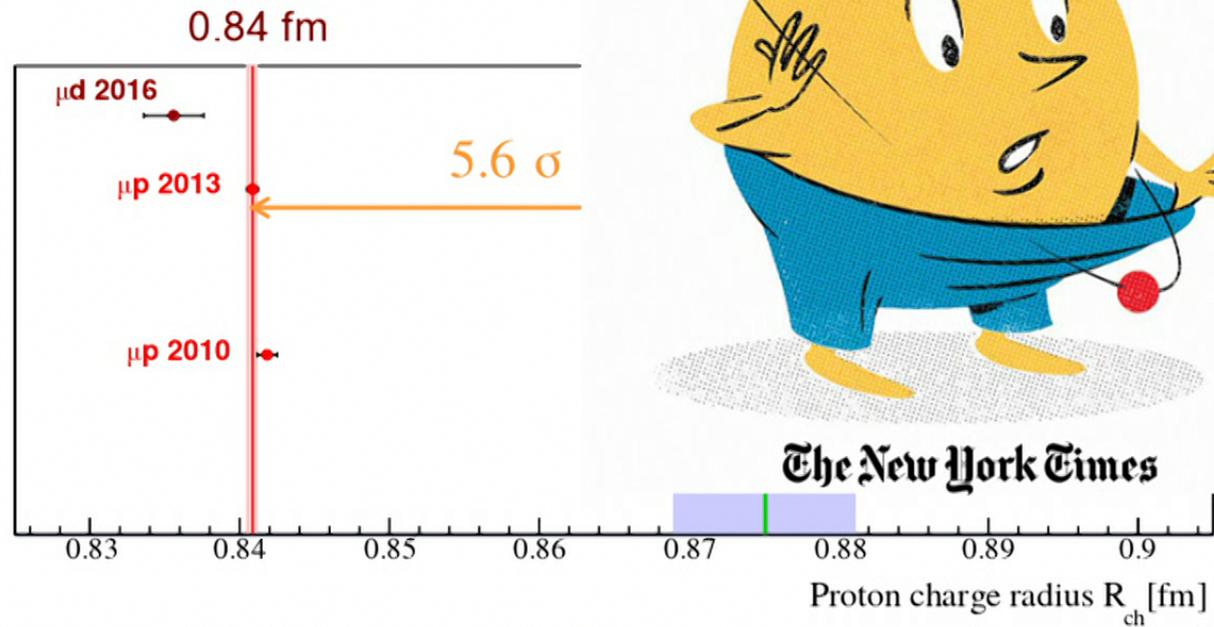


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# The “Proton Radius Puzzle”

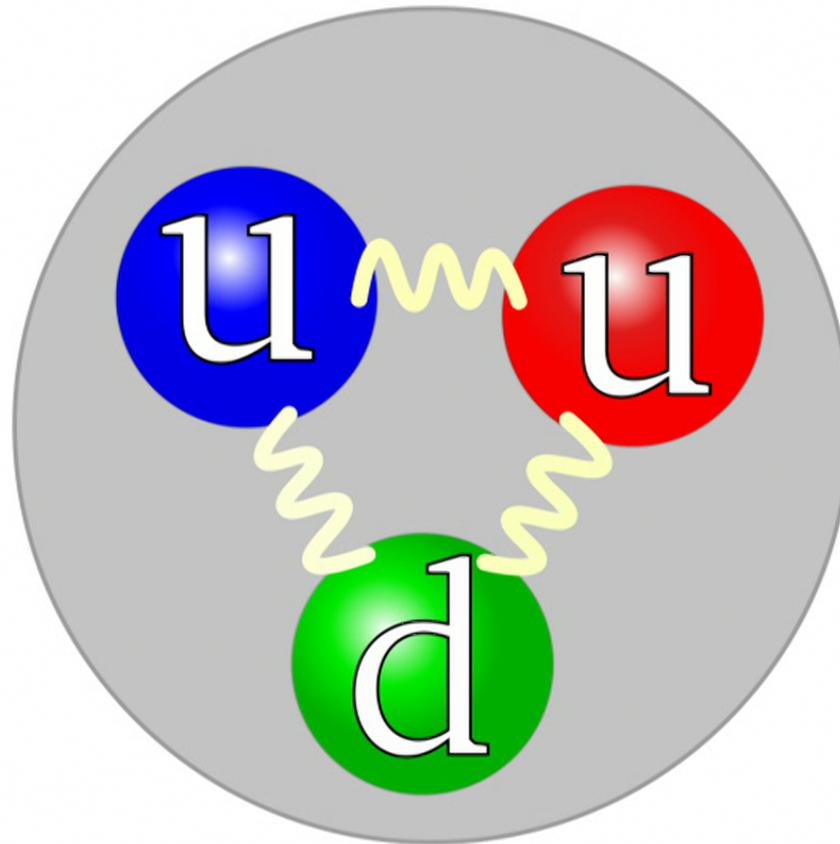
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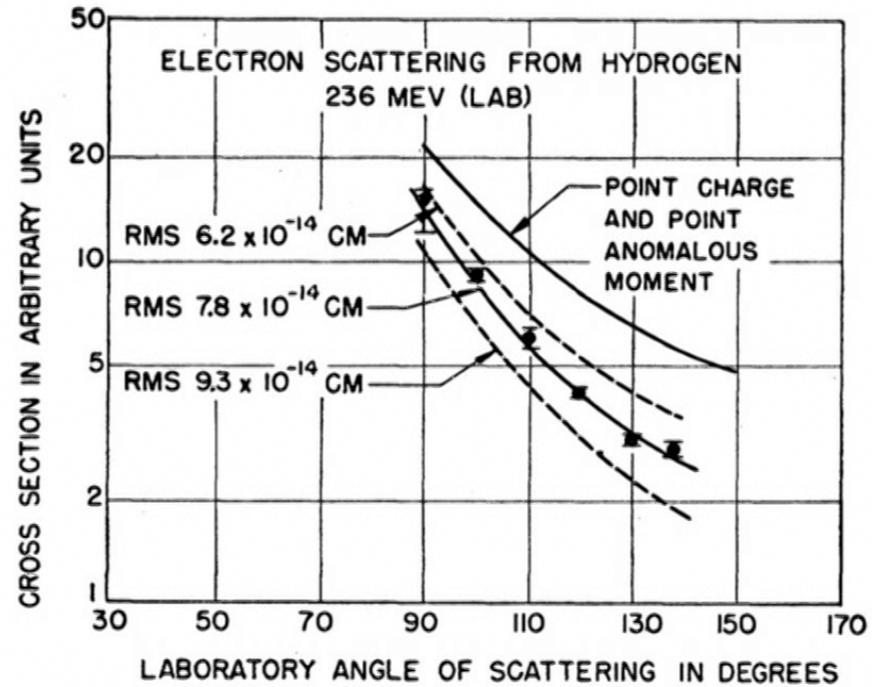
# Proton – 3 Quarks



# Robert Hofstadter – 1955

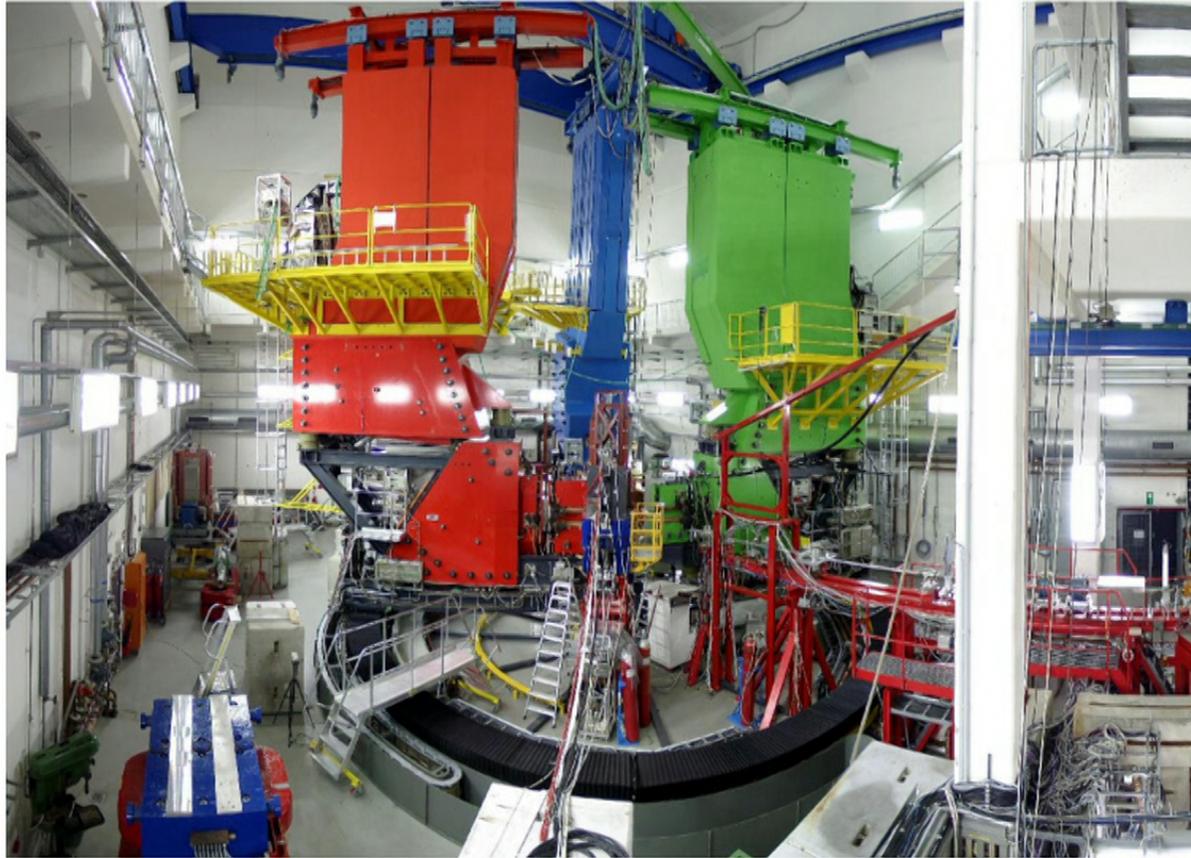


1915 – 1990

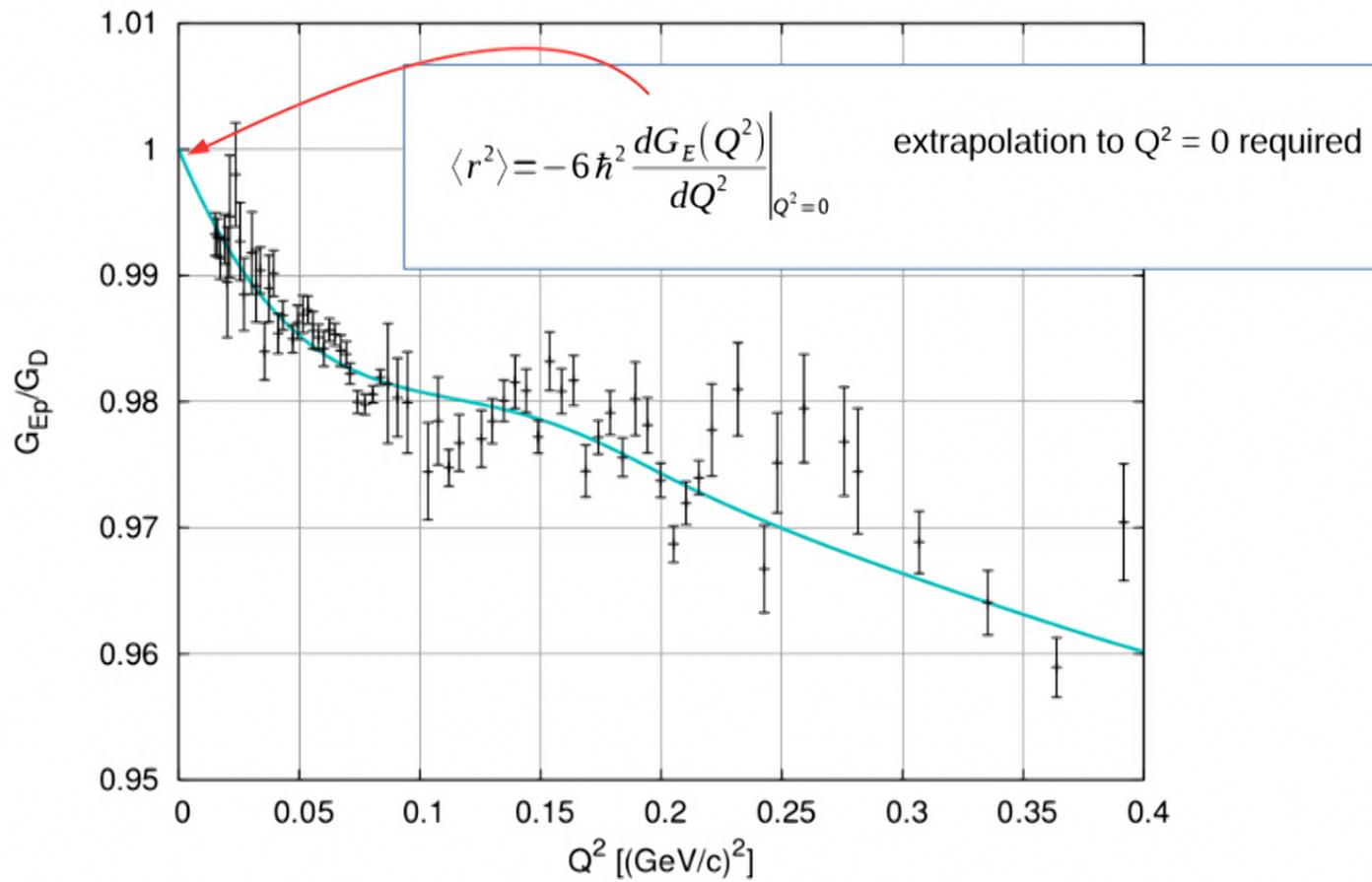


Phys. Rev. 102, 851 (1956)

# Mainzer Microtron MAMI



# Electron scattering - today

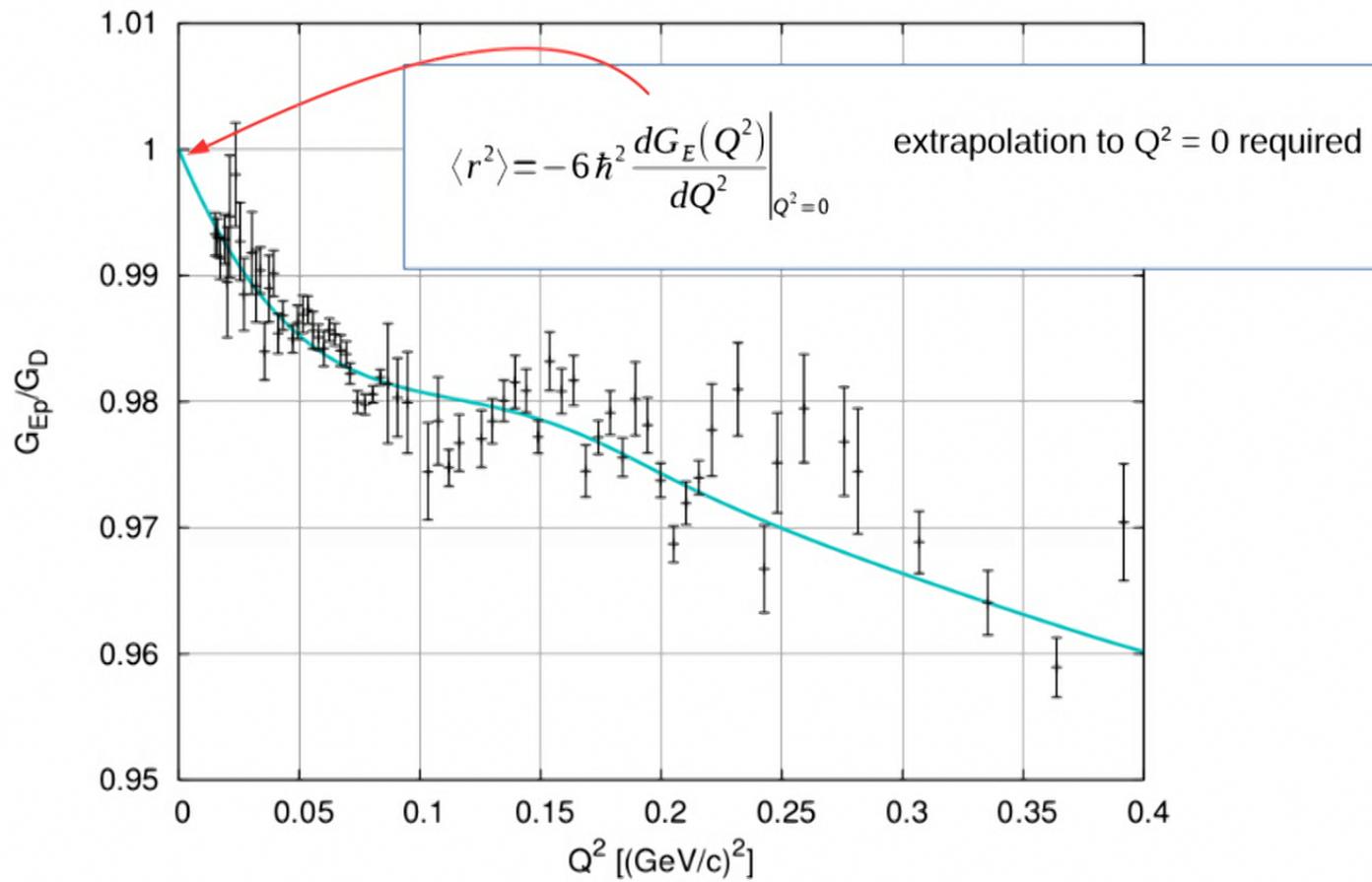


Mainz MAMI data 2010

Vanderhaeghen, Walcher: 1008.4225

# Hydrogen

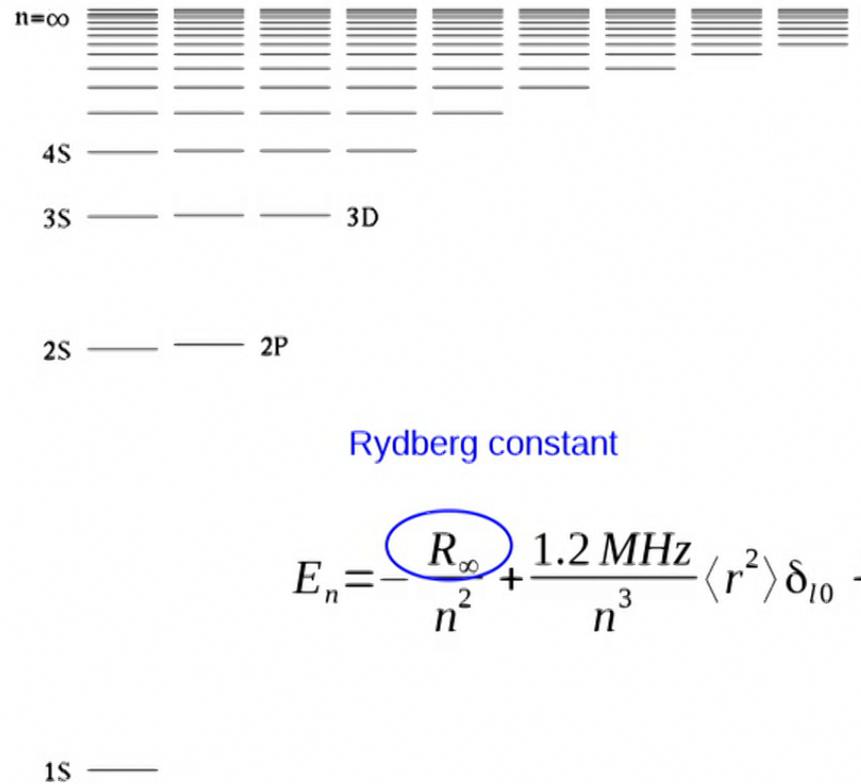
# Electron scattering - today



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# Energy levels of hydrogen

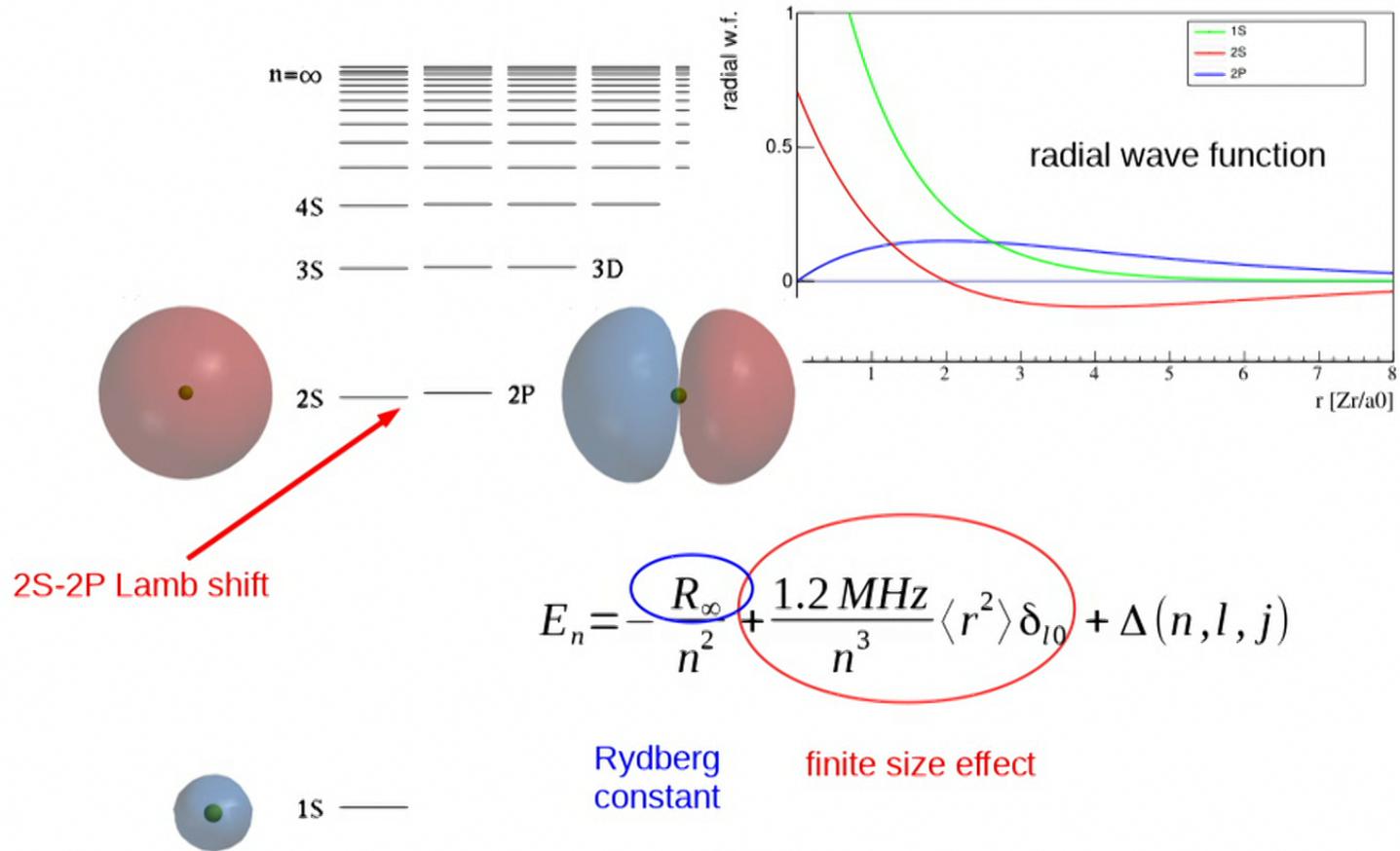


Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2 \text{ MHz}}{n^3} \langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

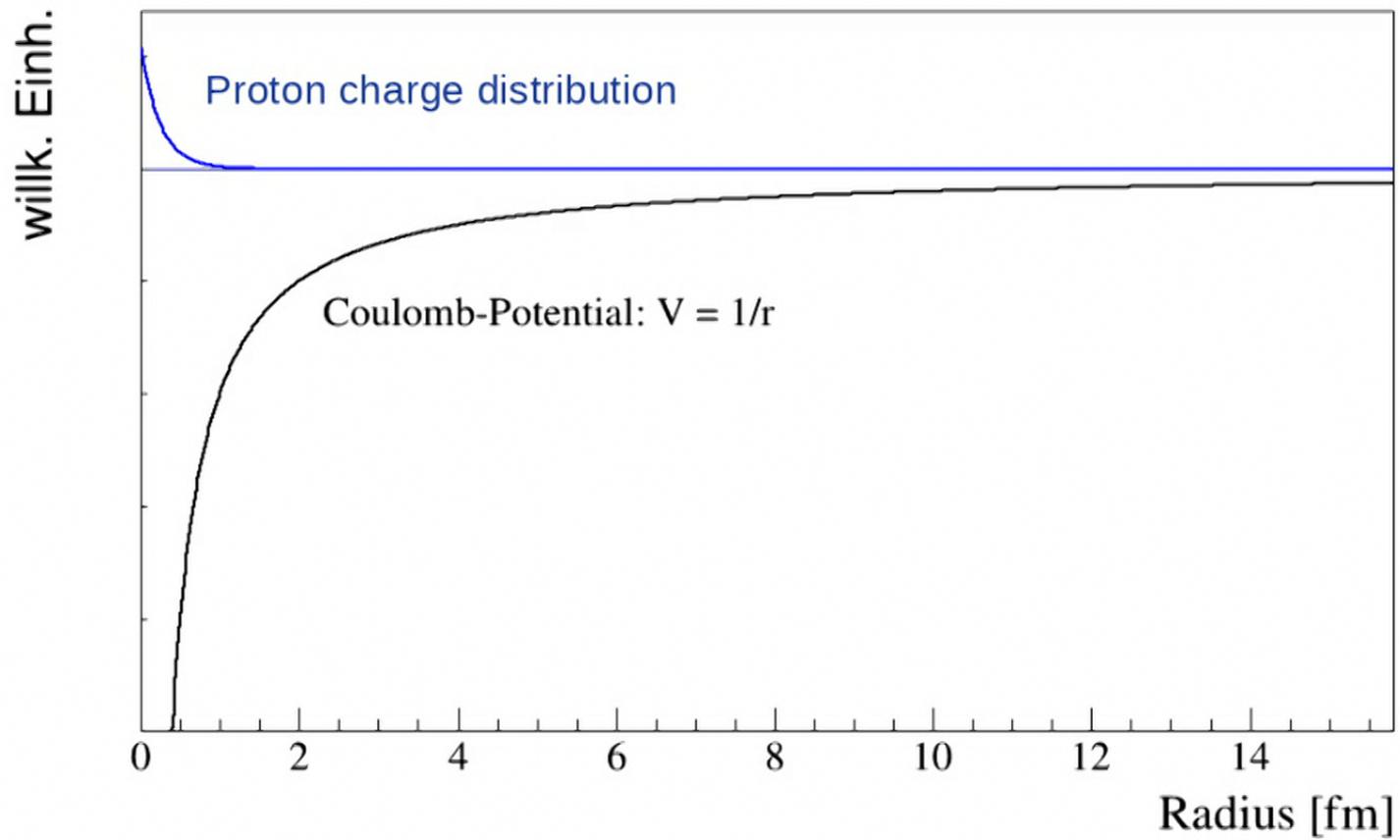
RP et al., Metrologia 54, L1 (2017)

# Energy levels of hydrogen



RP et al., Metrologia 54, L1 (2017)

# Proton radius and hydrogen



# Muonic Hydrogen

A proton, orbited by a **negative muon**.

# What is a Muon?



Carl David Anderson



Seth Neddermeyer

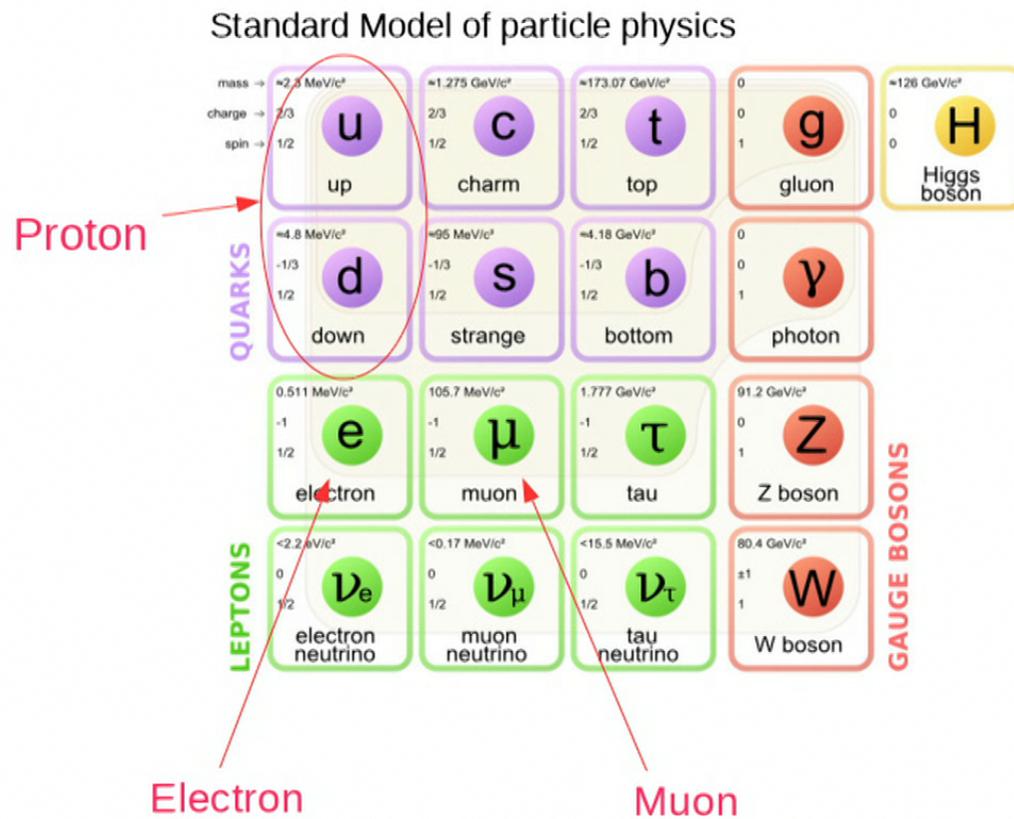
Nobel prize 1936  
(for the Positron!)

Muons are **heavy electrons**:  
**200x the mass** of an electron.

Discovered in 1936 by Carl D. Anderson and  
Seth Neddermeyer as **part of the cosmic  
radiation**.

Muon have a **lifetime of 2 microseconds**.  
(millionths of a second).

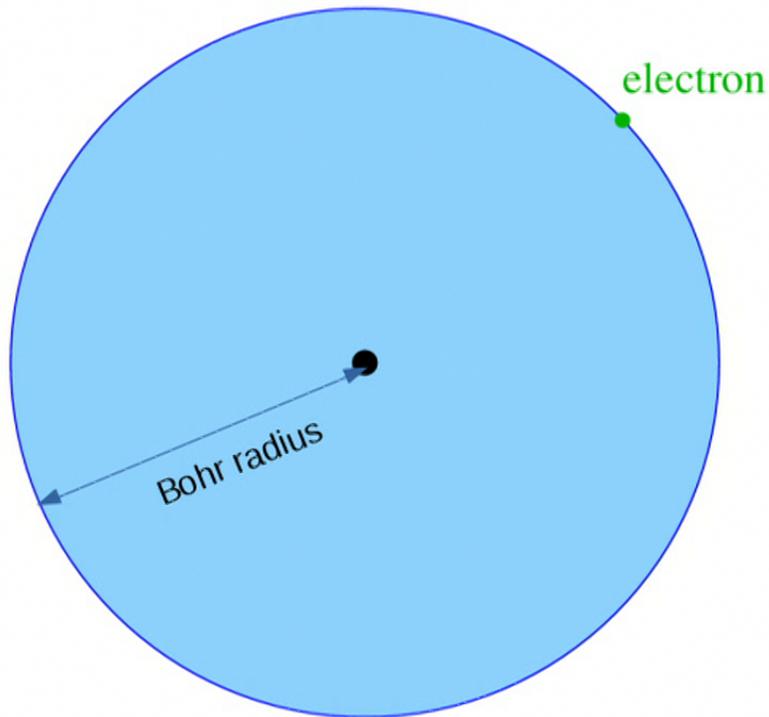
# The Muon and its place in the Universe



# Electronic and muonic atoms

Regular hydrogen:

Proton + Electron



Muonic hydrogen:

Proton + Muon

Muon mass = **200** \* electron mass

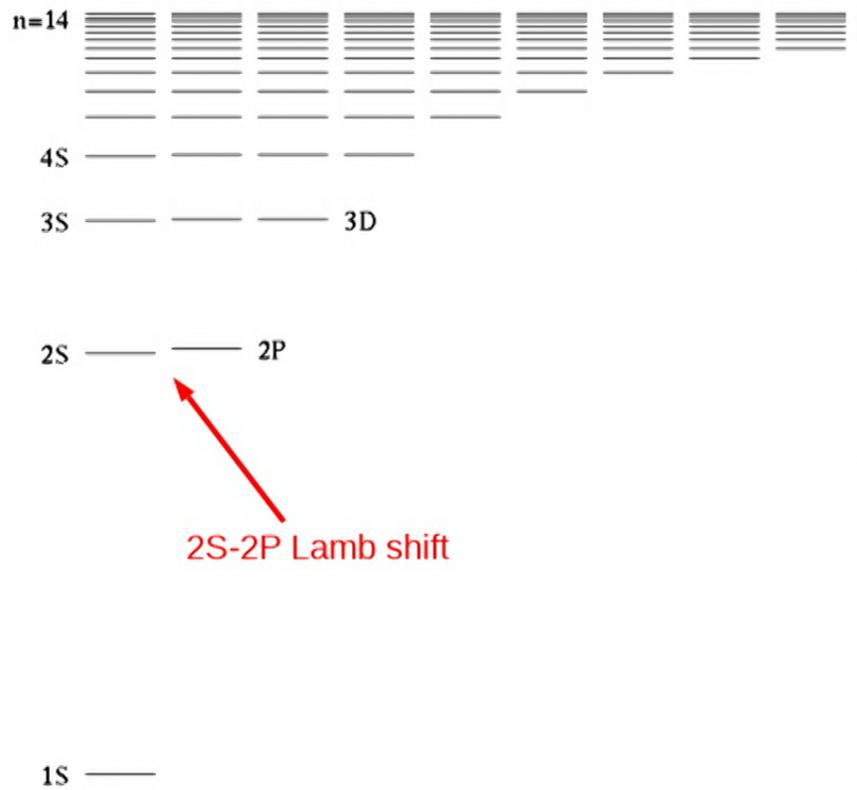
Bohr radius = **1/200** of H

**200<sup>3</sup>** = a **few million times** more sensitive to proton size

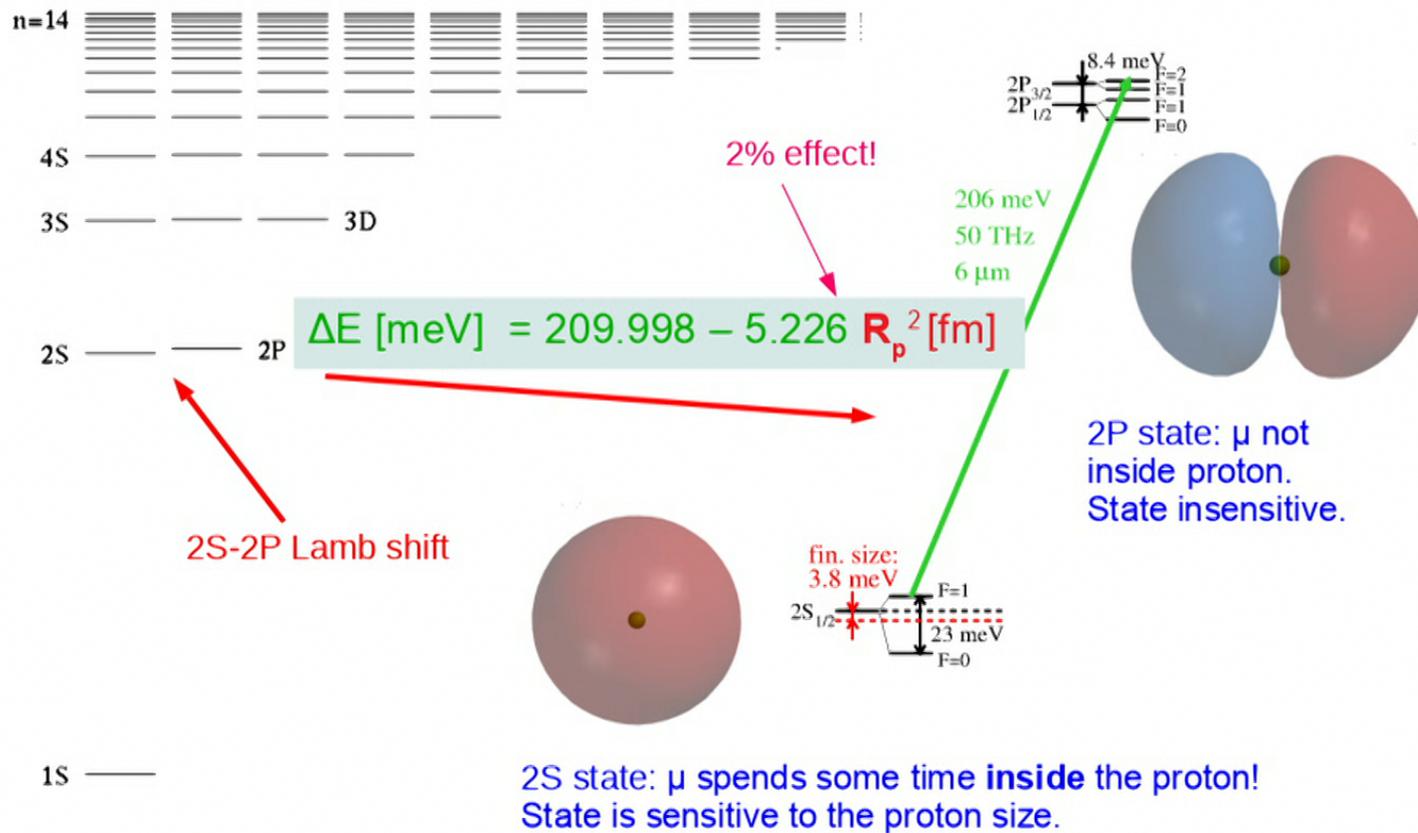


Vastly not to scale!!

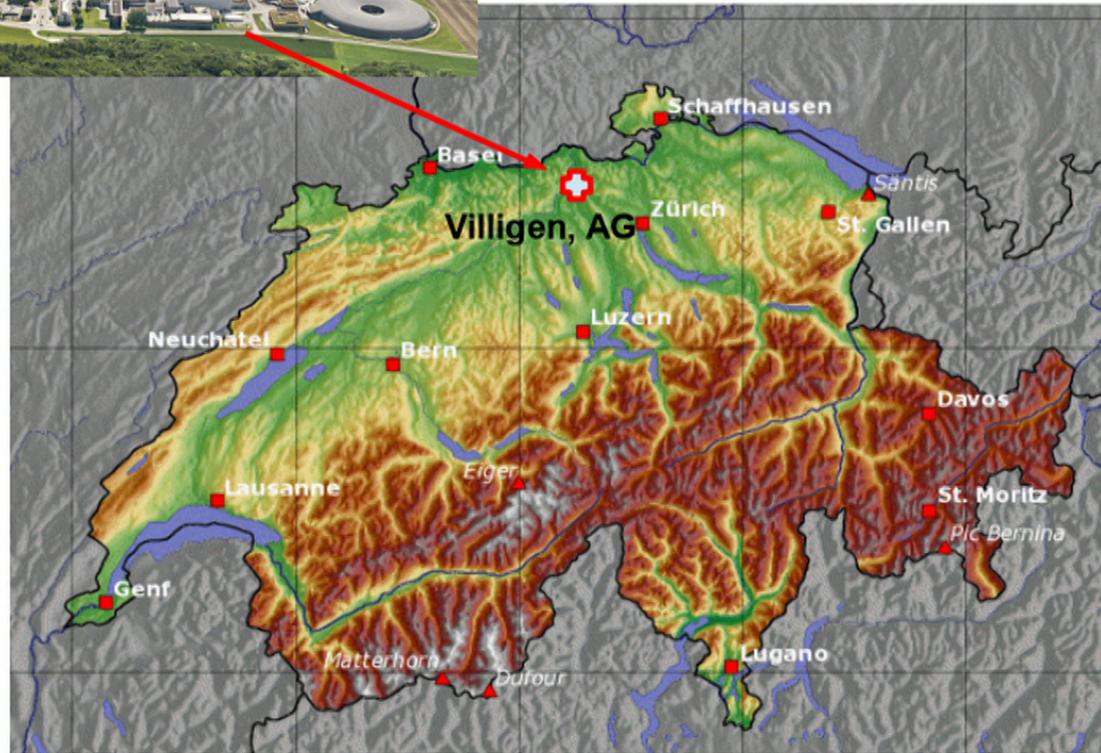
# Muonic Hydrogen



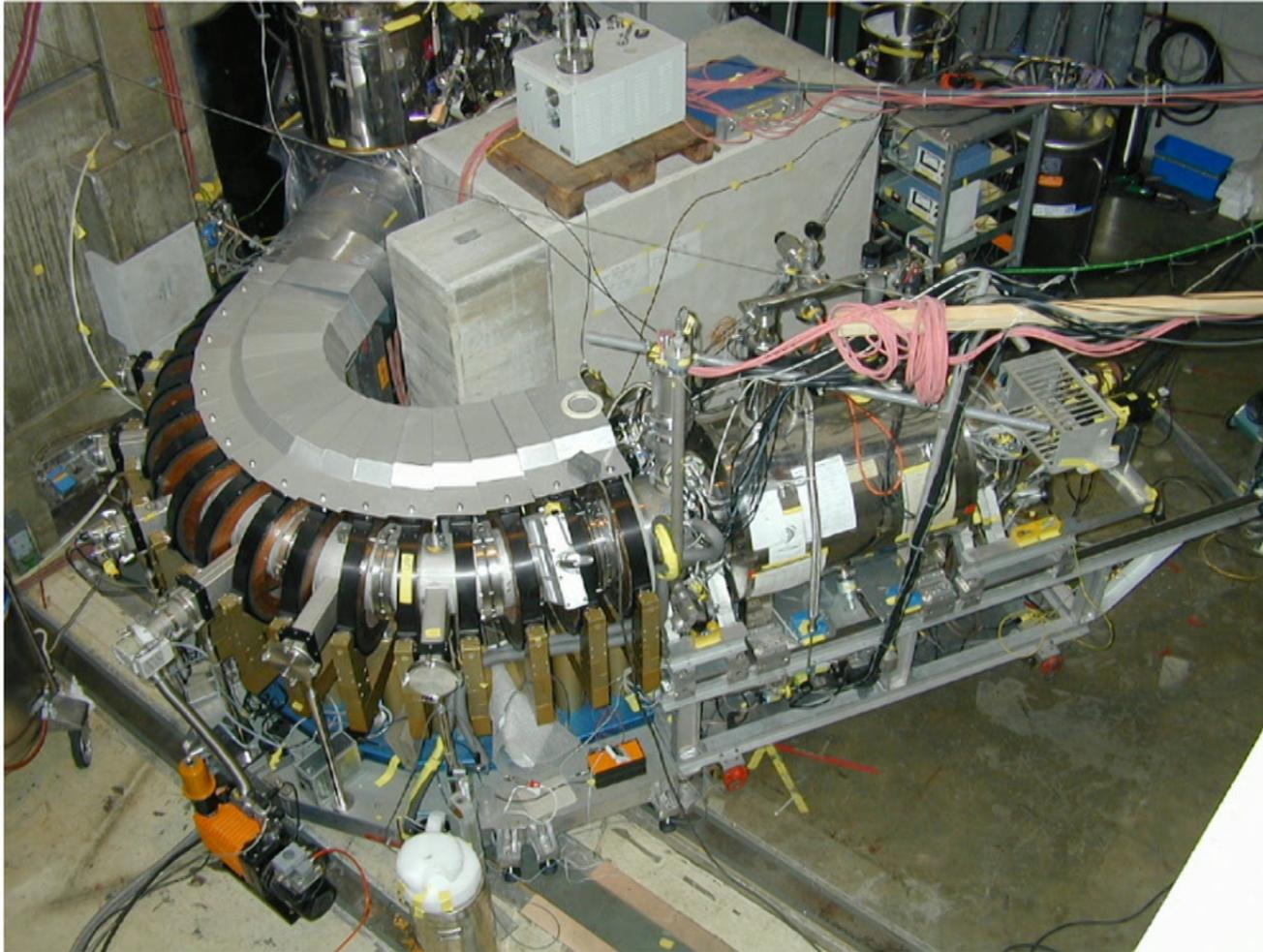
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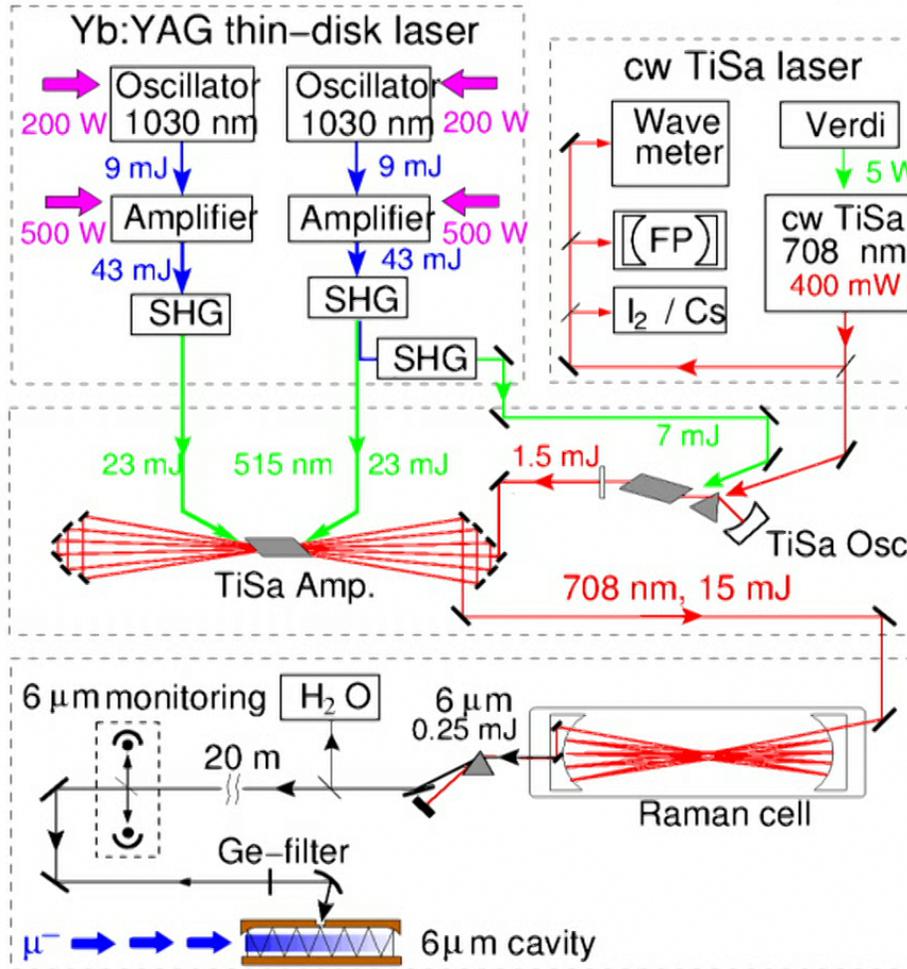
# The accelerator at PSI



# The muon beam line in $\pi E5$

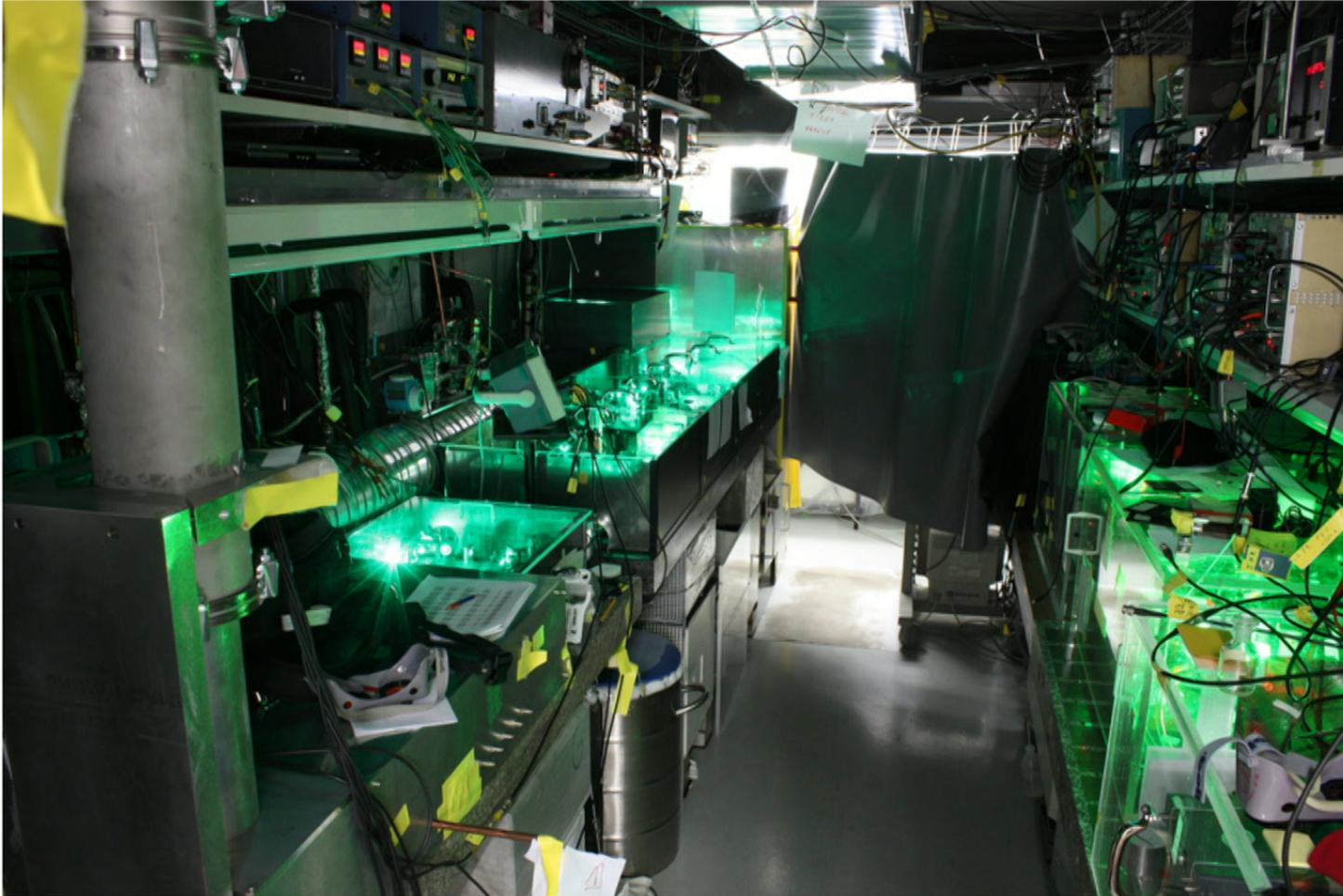


# The laser system

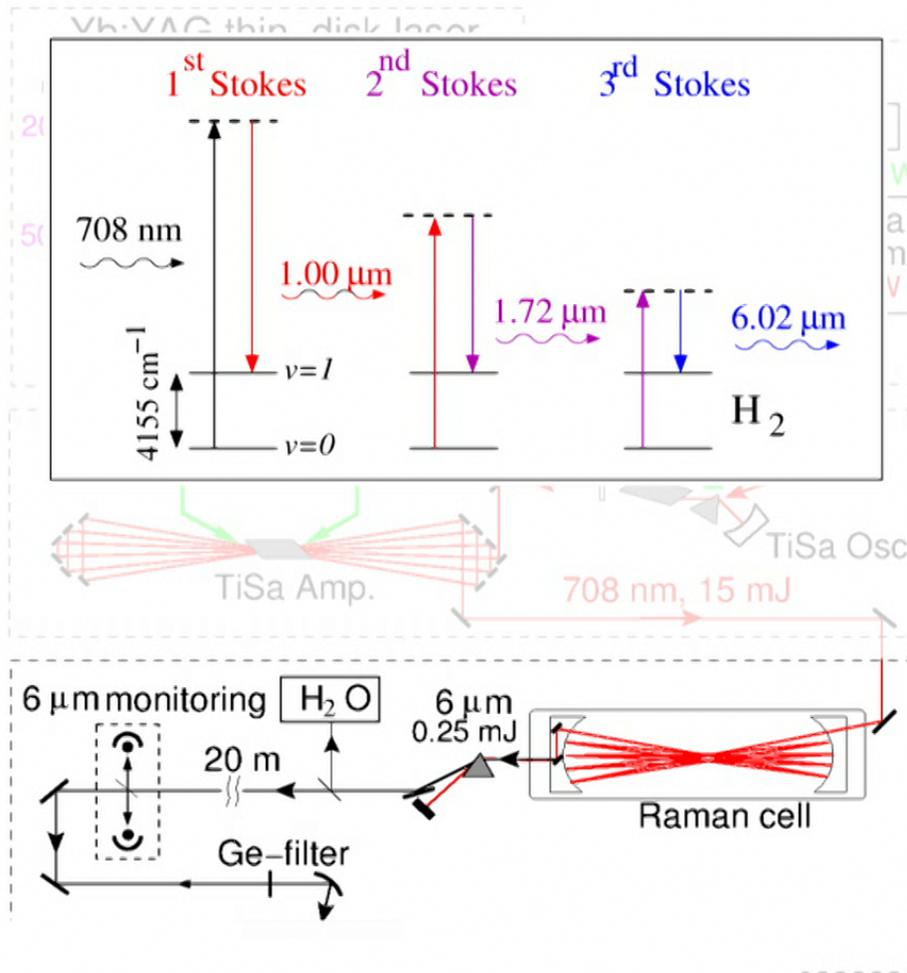


- Yb:YAG Disk laser
  - fast response on  $\mu$
- Frequency doubling (SHG)
  - green light to pump
  - Ti:sapphire laser
- Ti:sapphire cw laser
  - determines laser frequency
- Ti:sapphire MOPA
  - high pulse energy (15 mJ)
- Raman cell
  - 3 sequential stimulated Raman Stokes shifts
  - Laser wave length → 6  $\mu$ m
- Target Cavity
  - Mirror system to fill the muon stop volume (H<sub>2</sub>)

# The laser hut at PSI



# Laser system: Raman cell



Yb:YAG Disk laser  
→ fast response on μ

Frequency doubling (SHG)  
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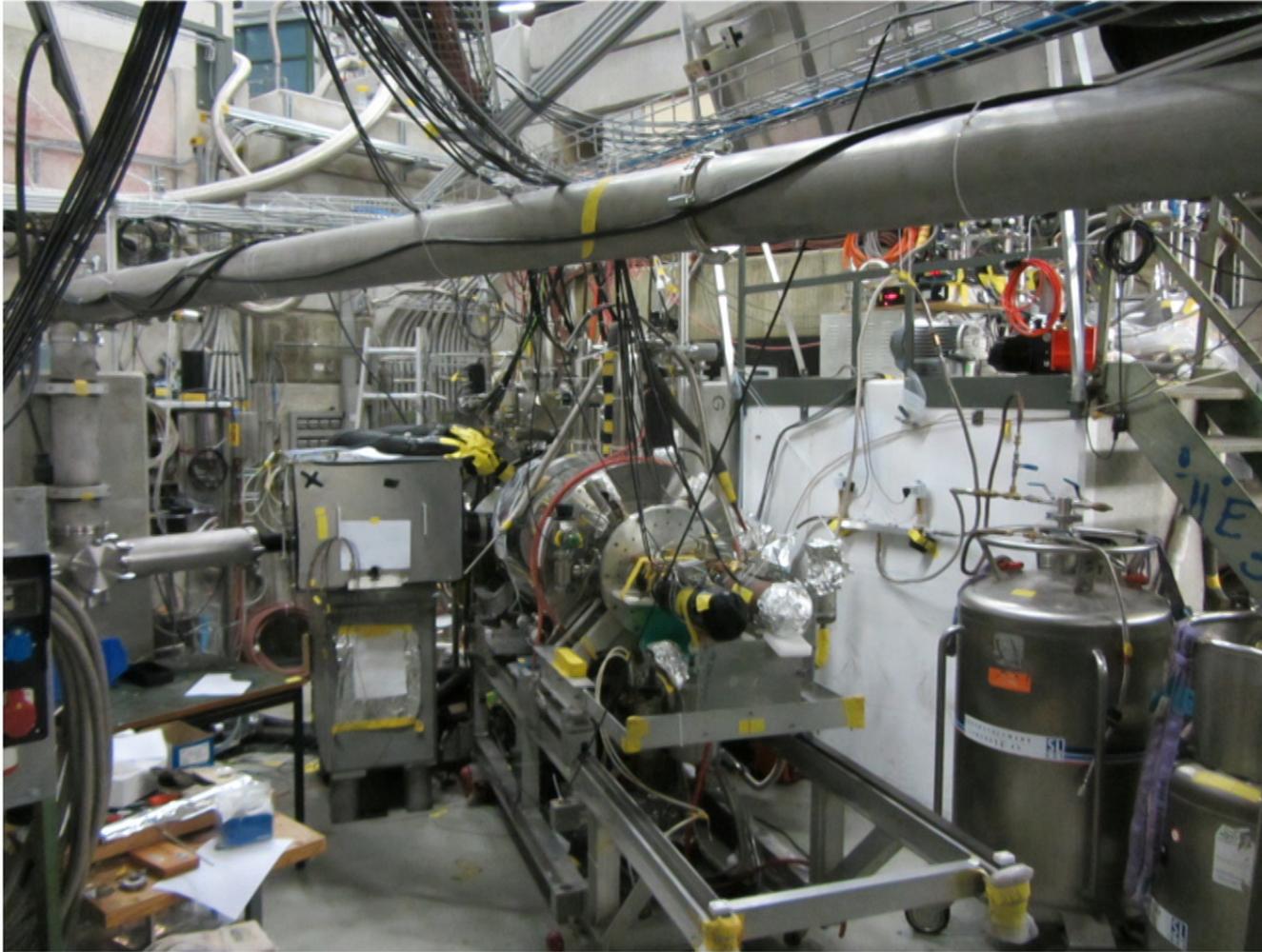
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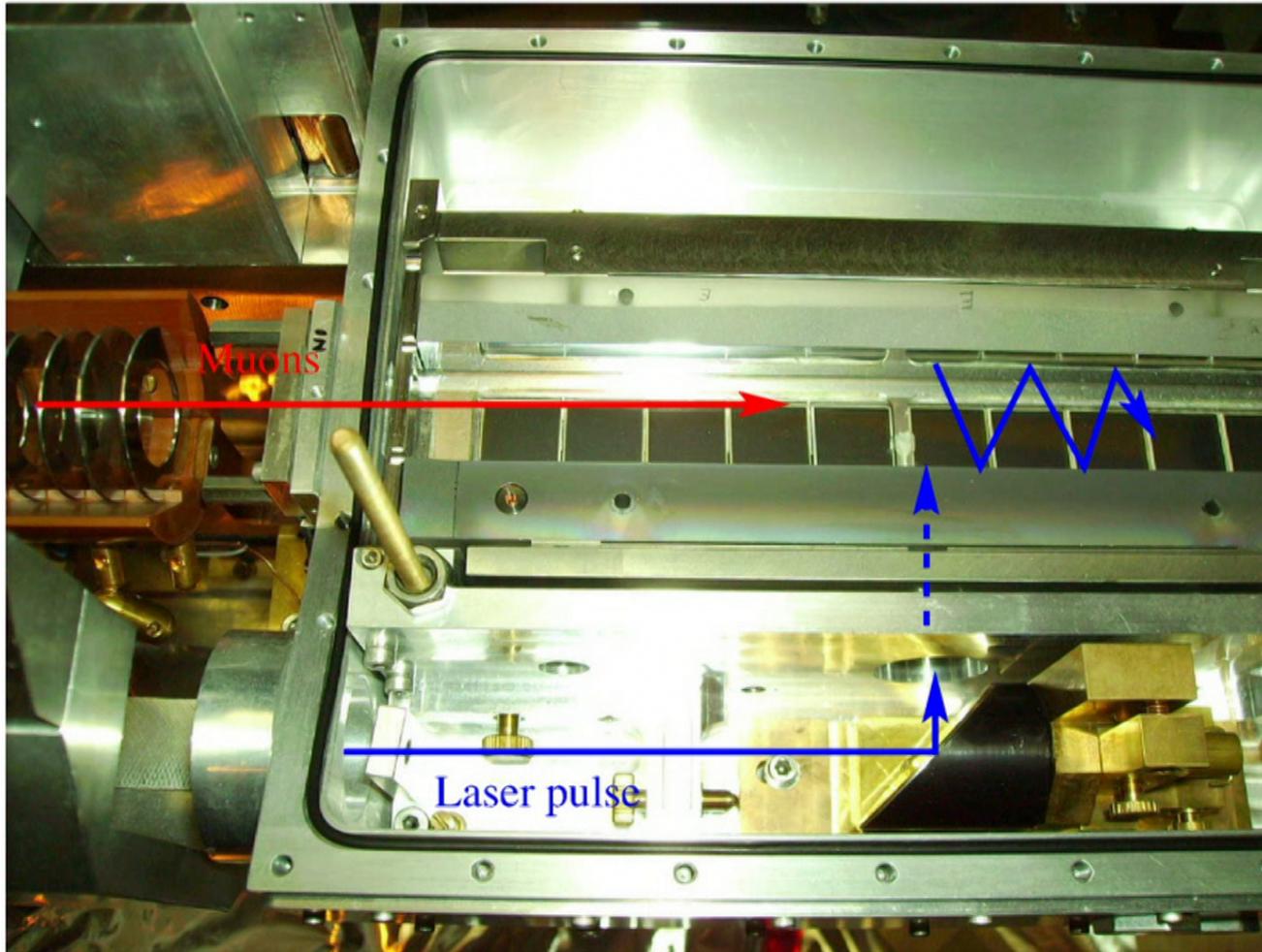
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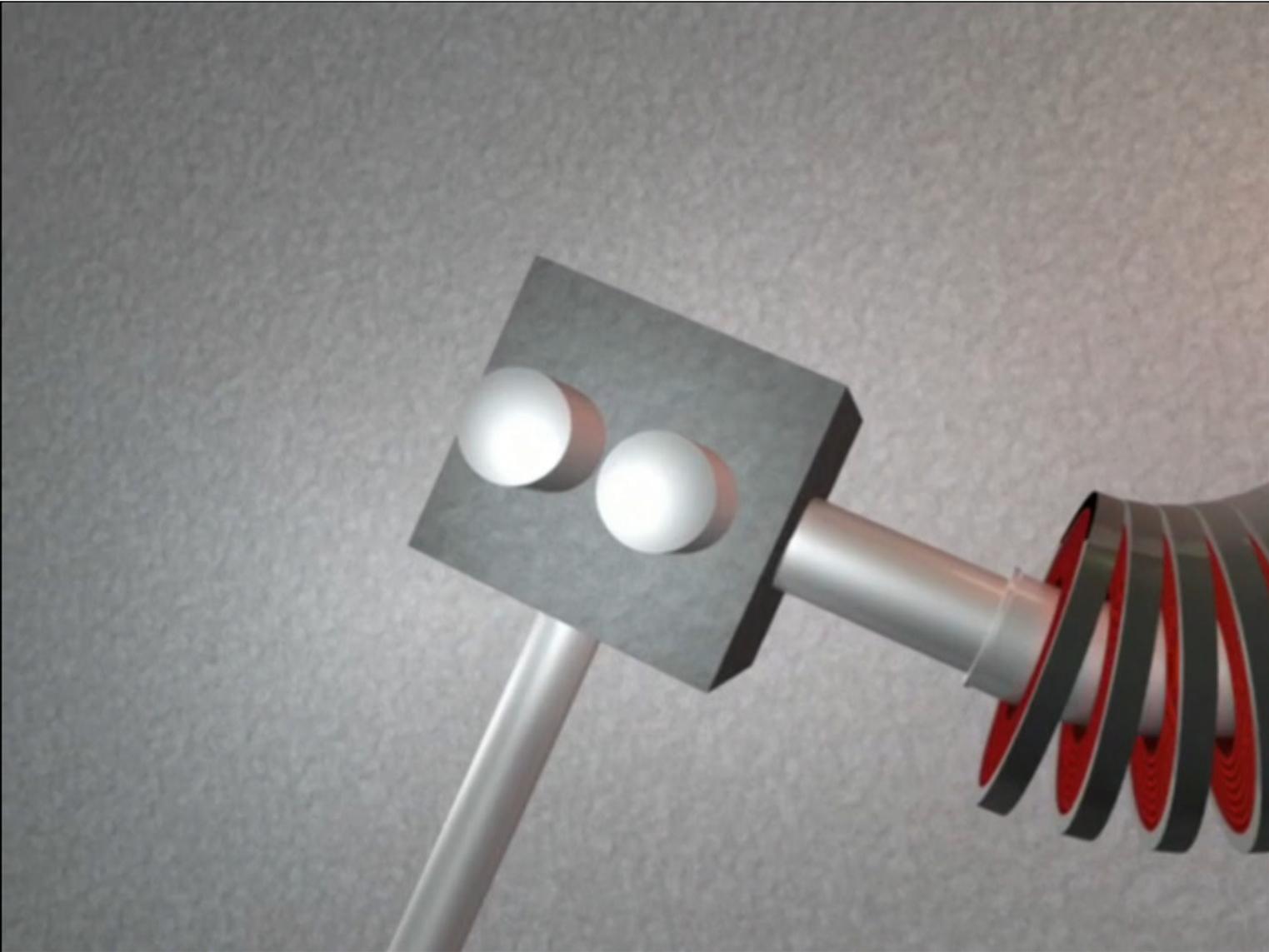


# Laser beam line

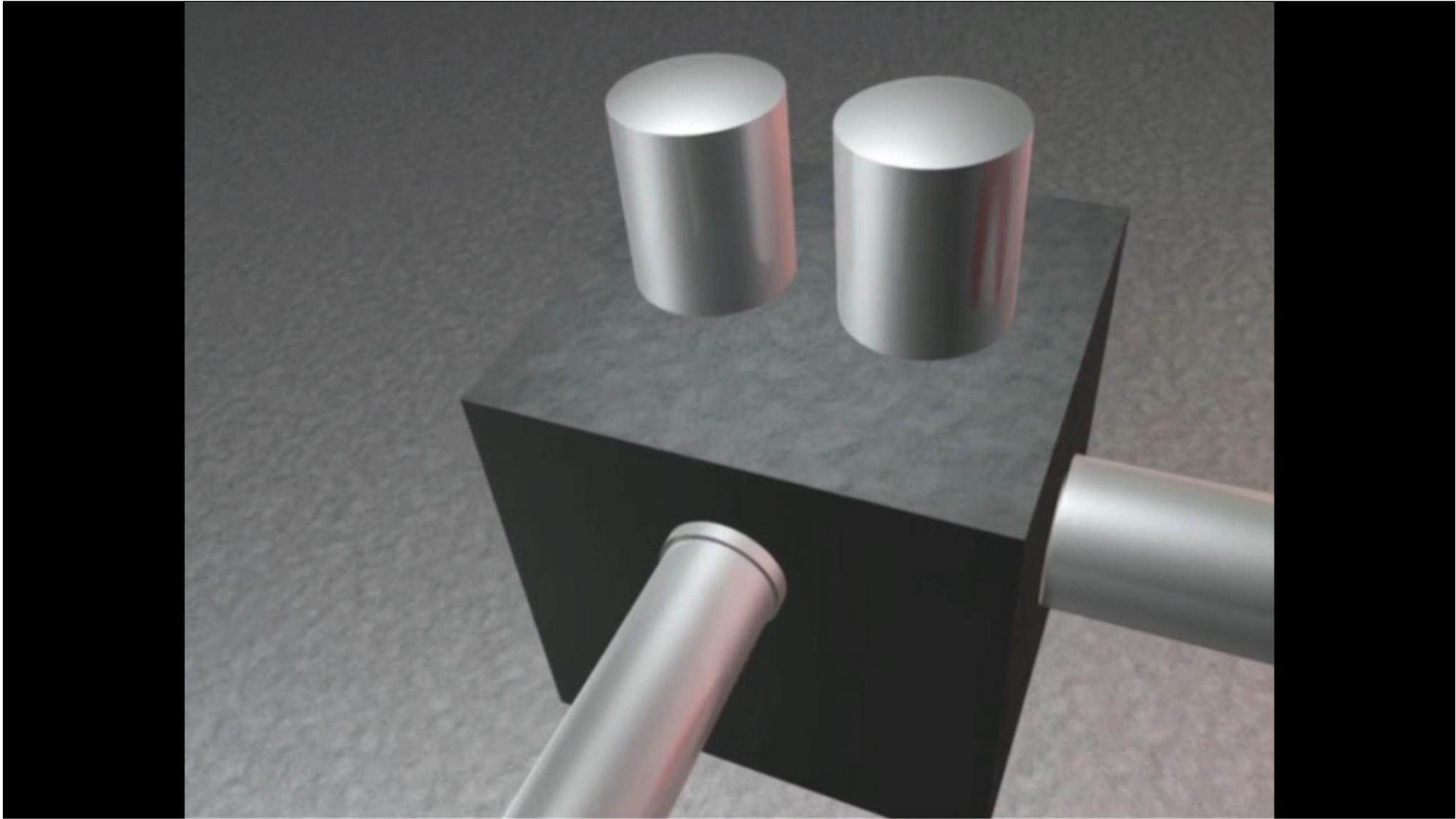


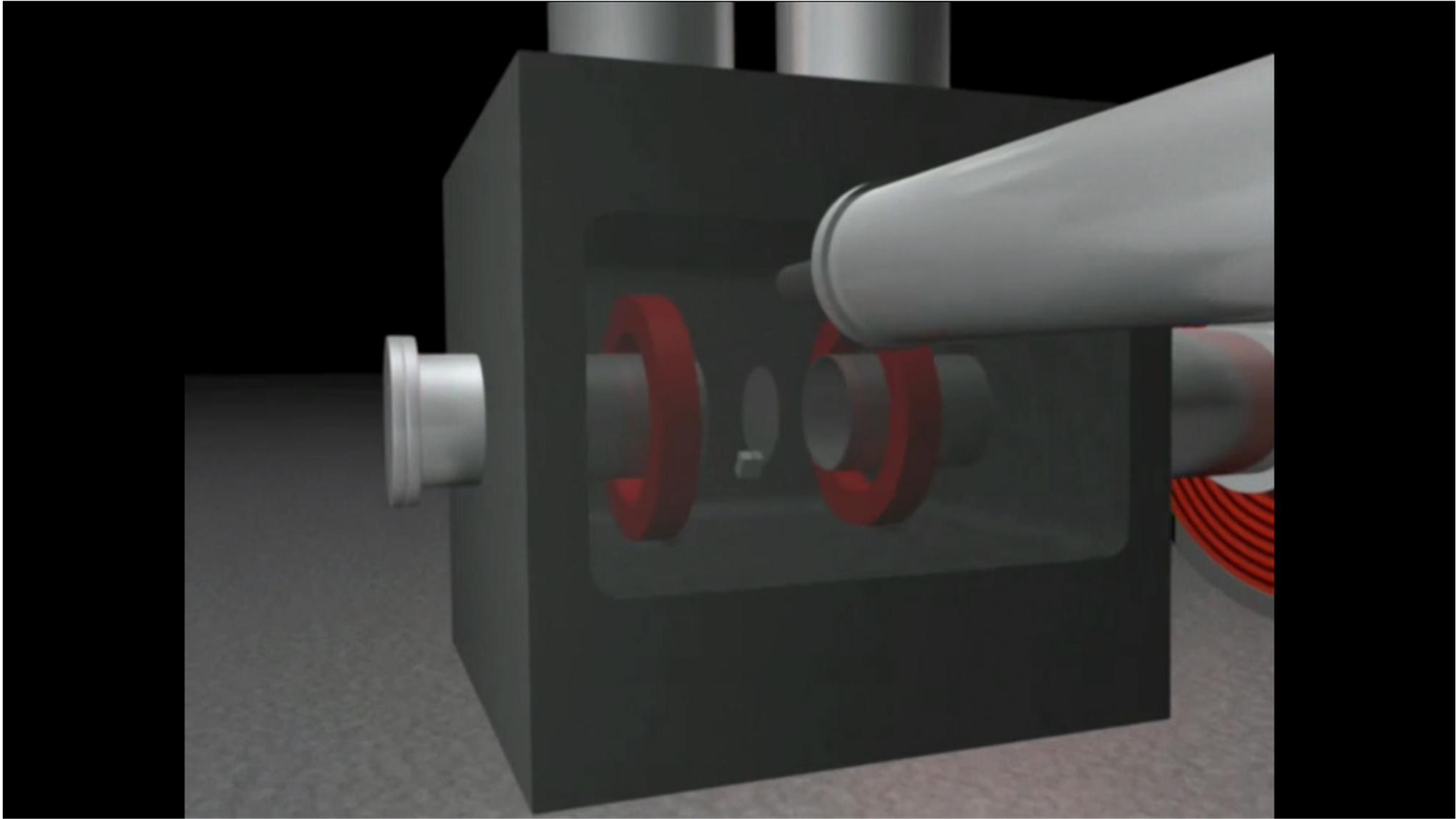
# The hydrogen target

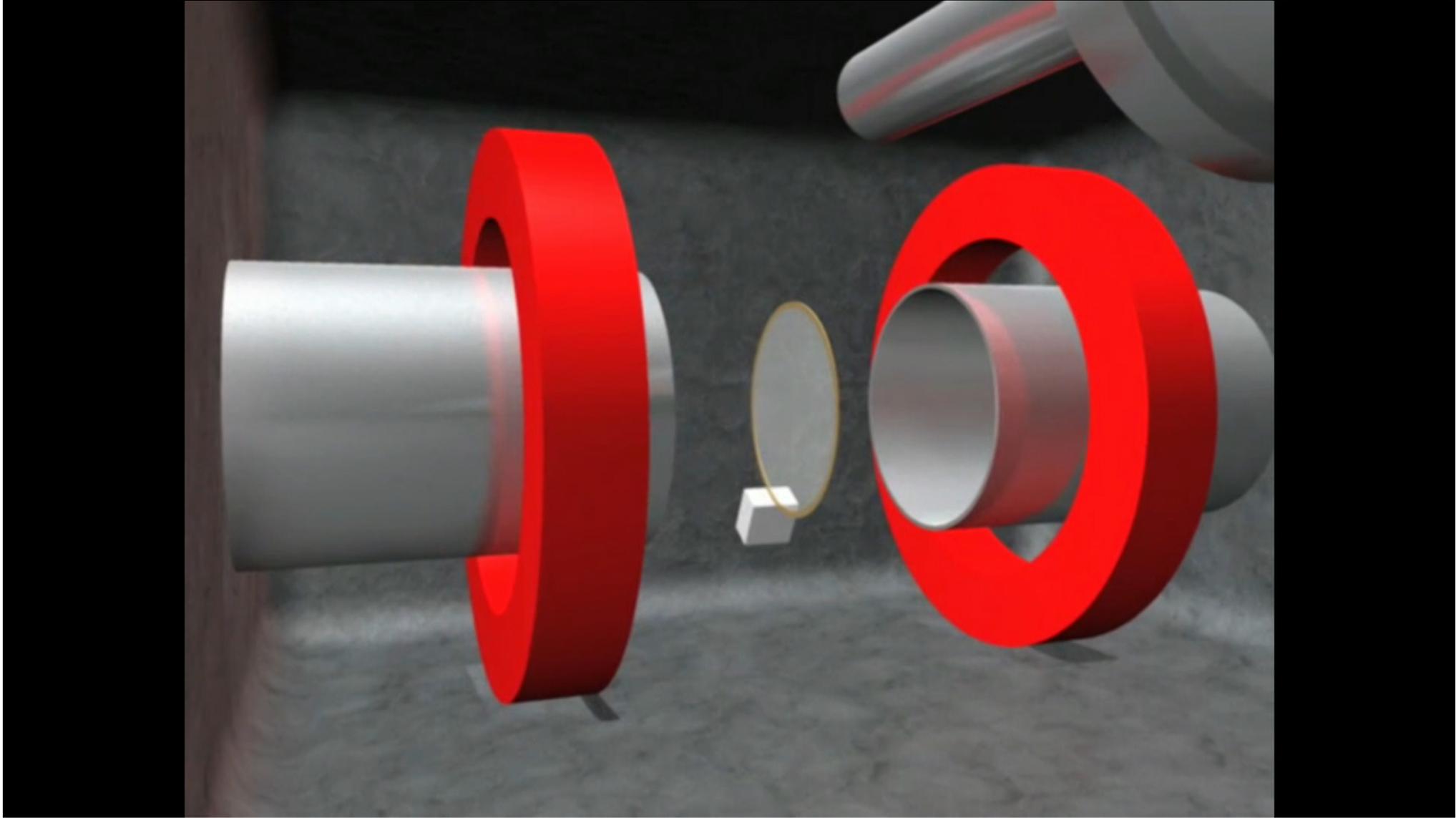


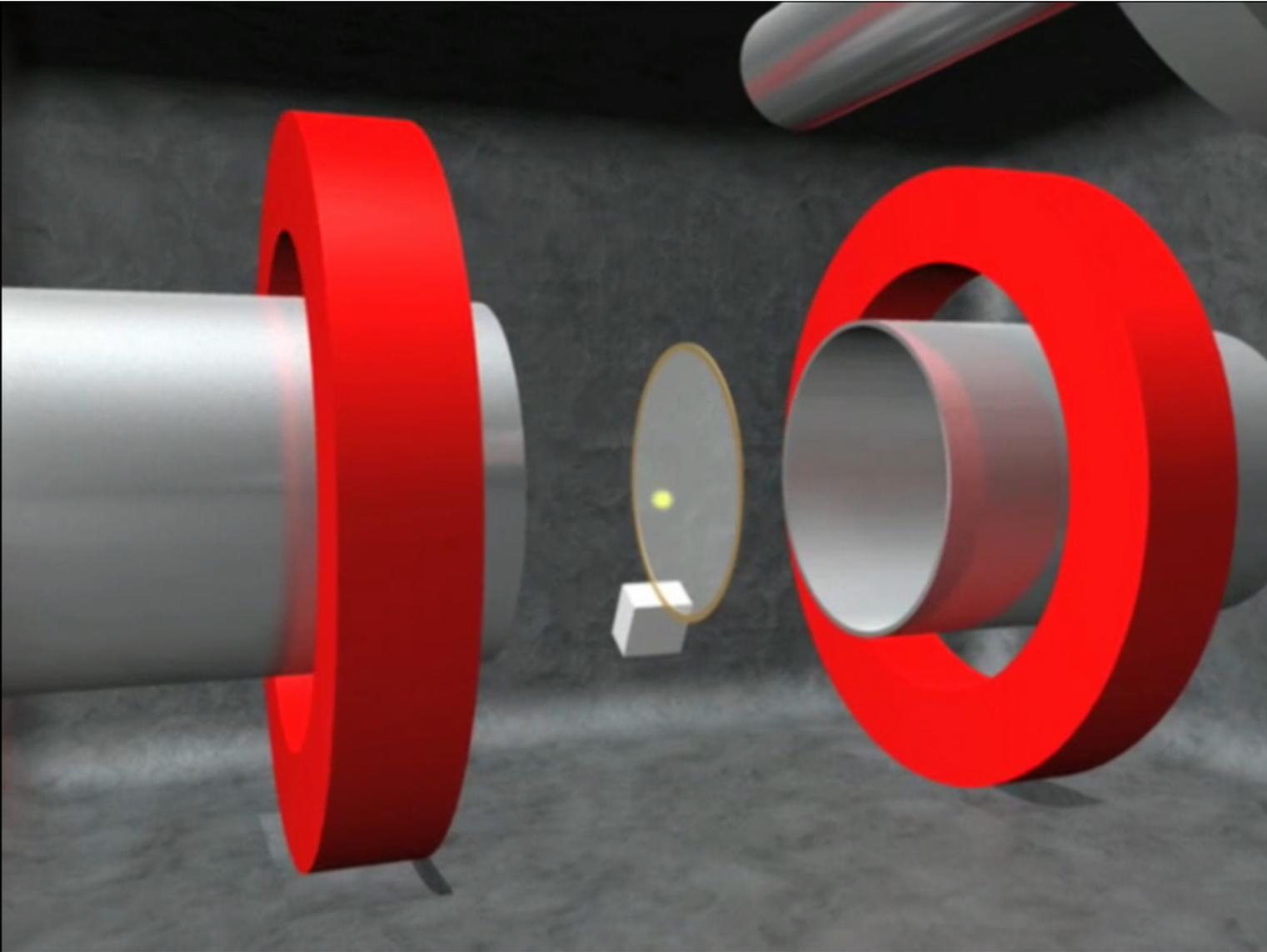


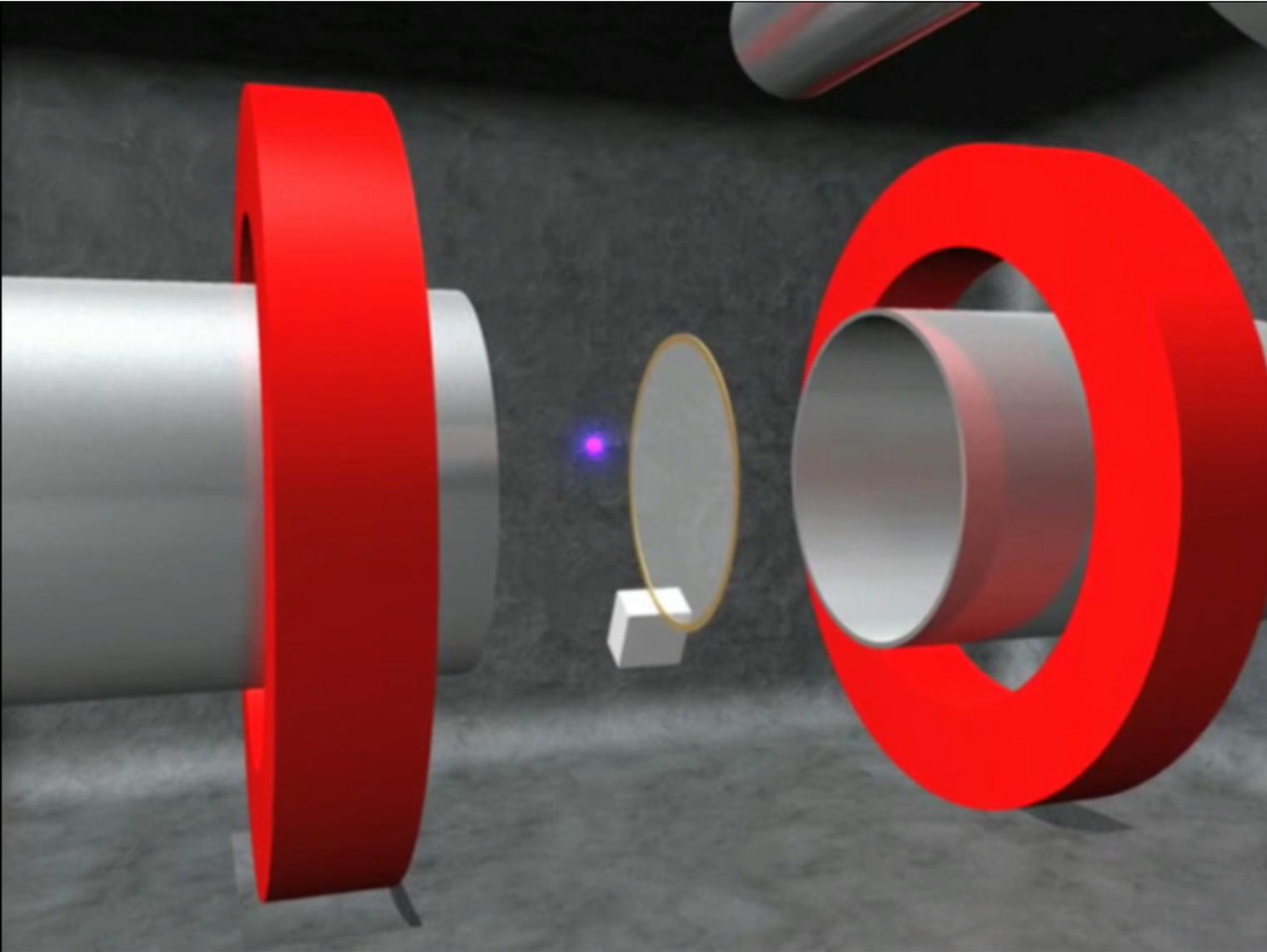
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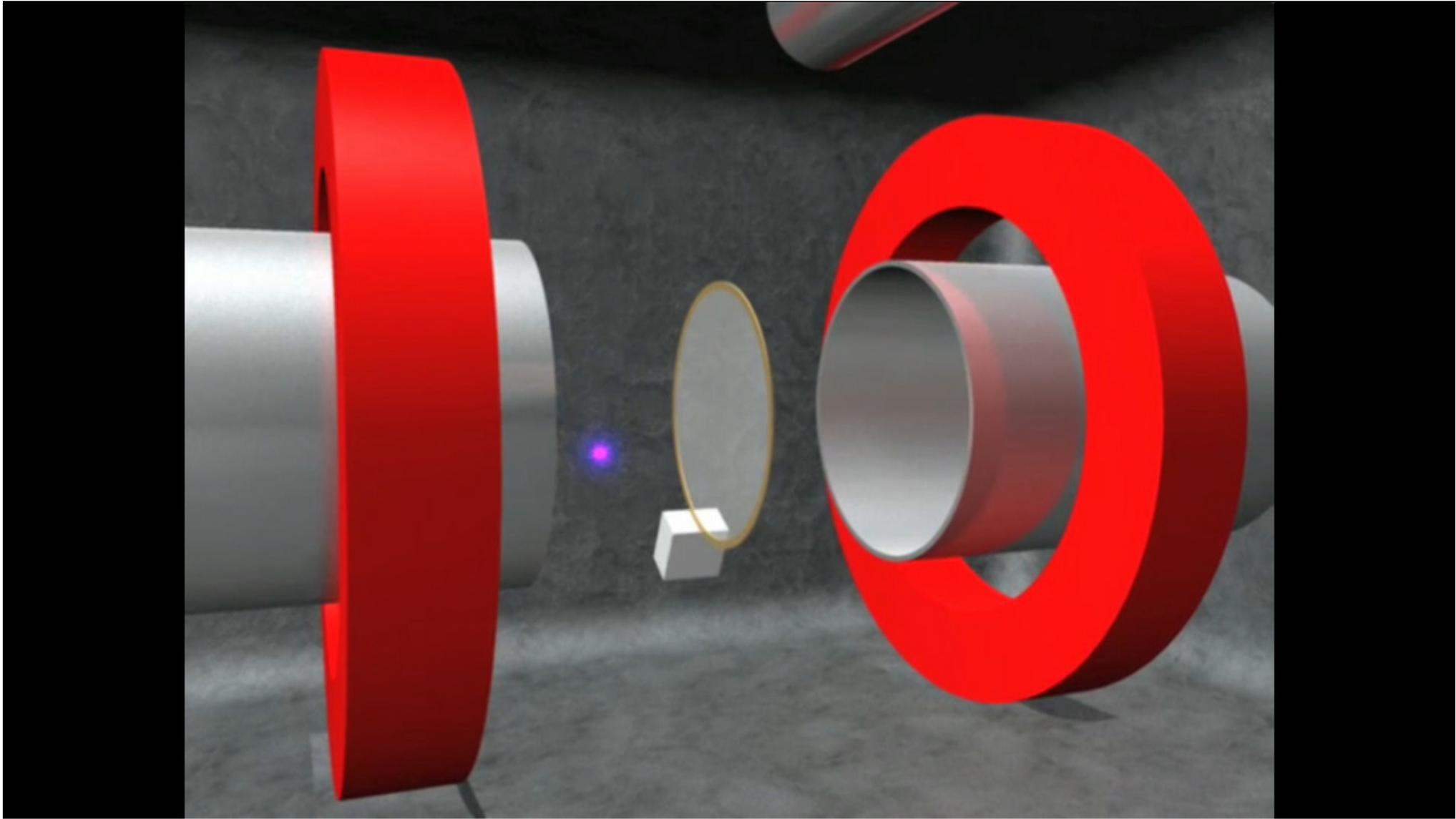


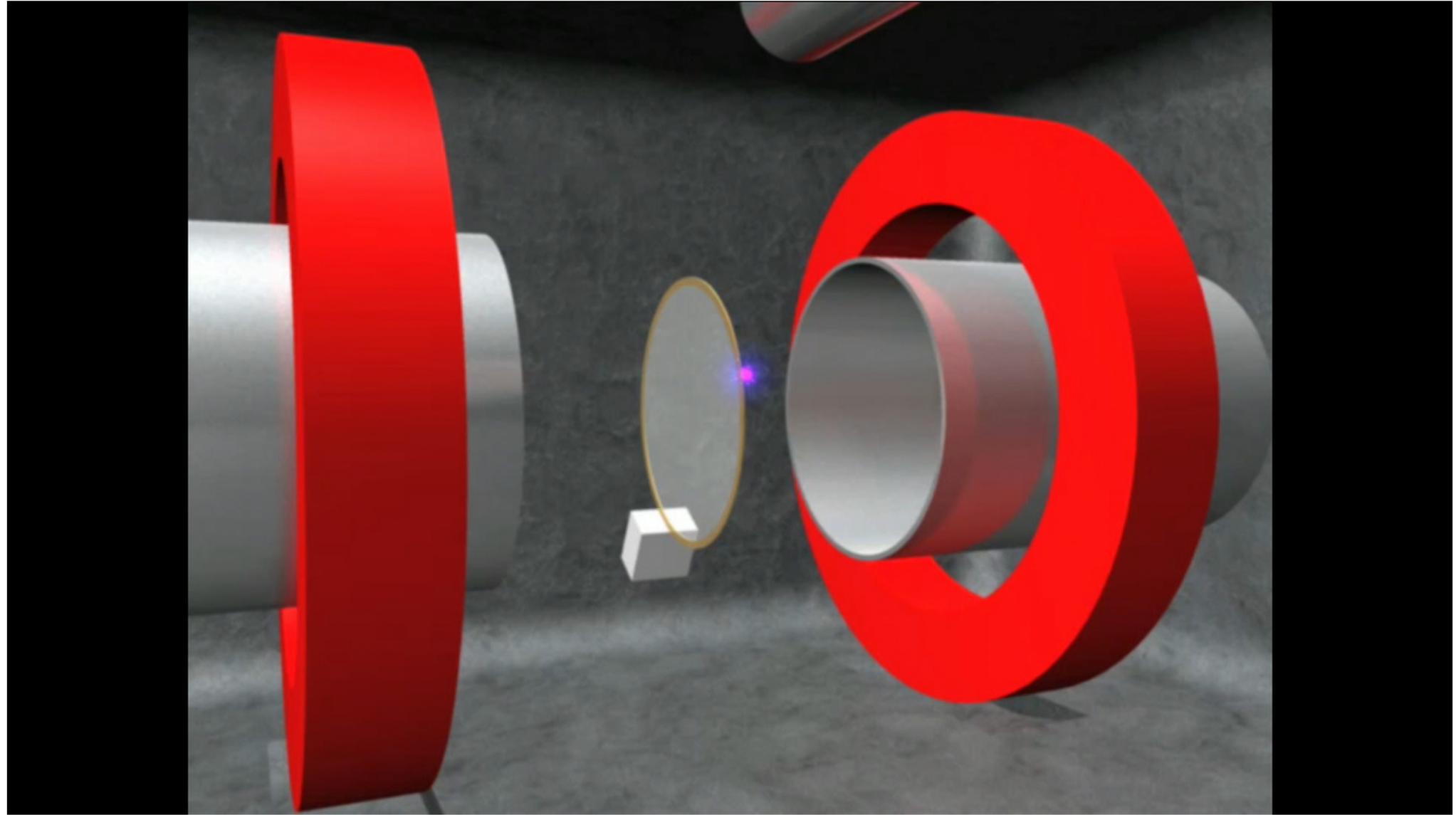


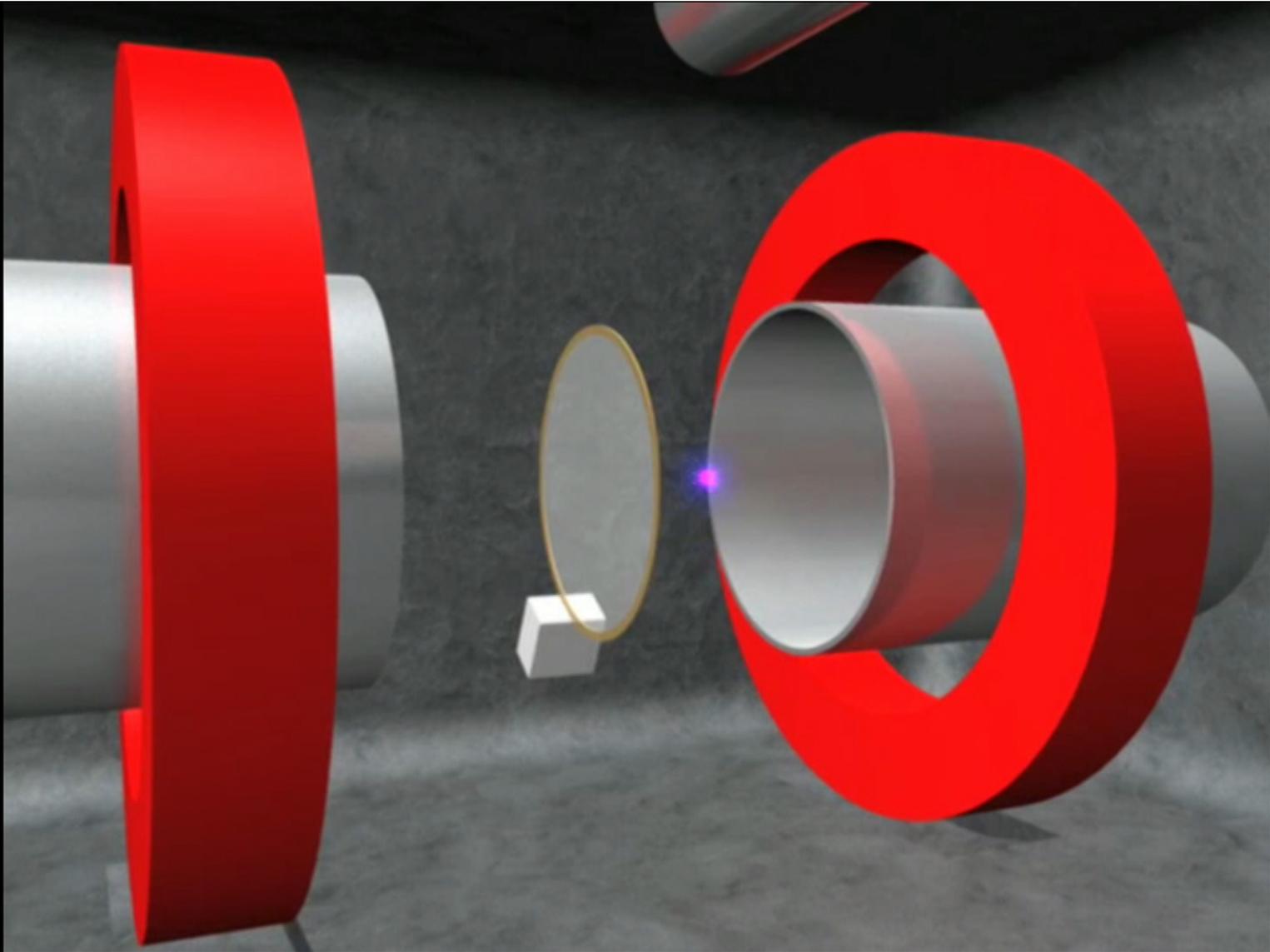


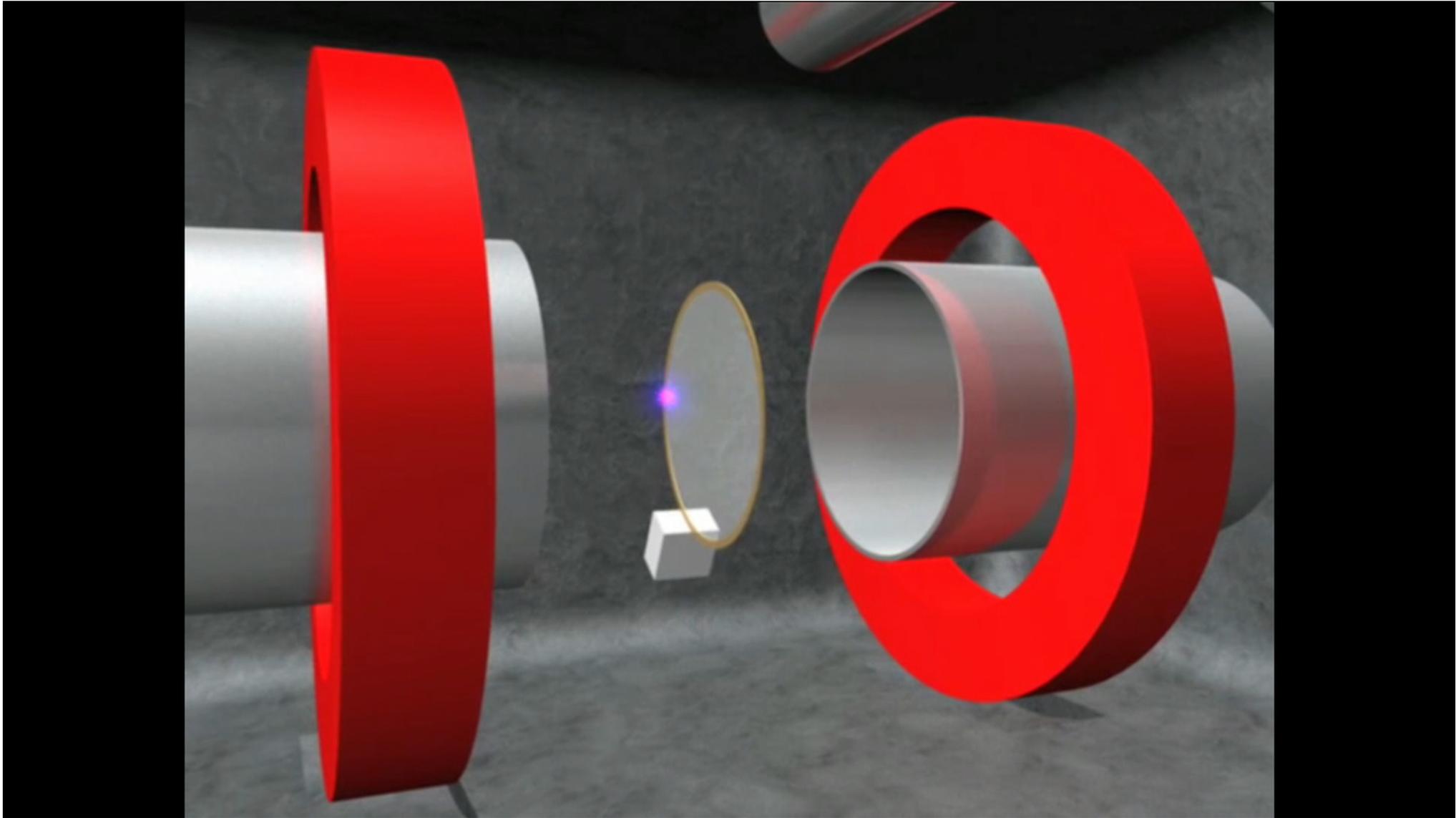


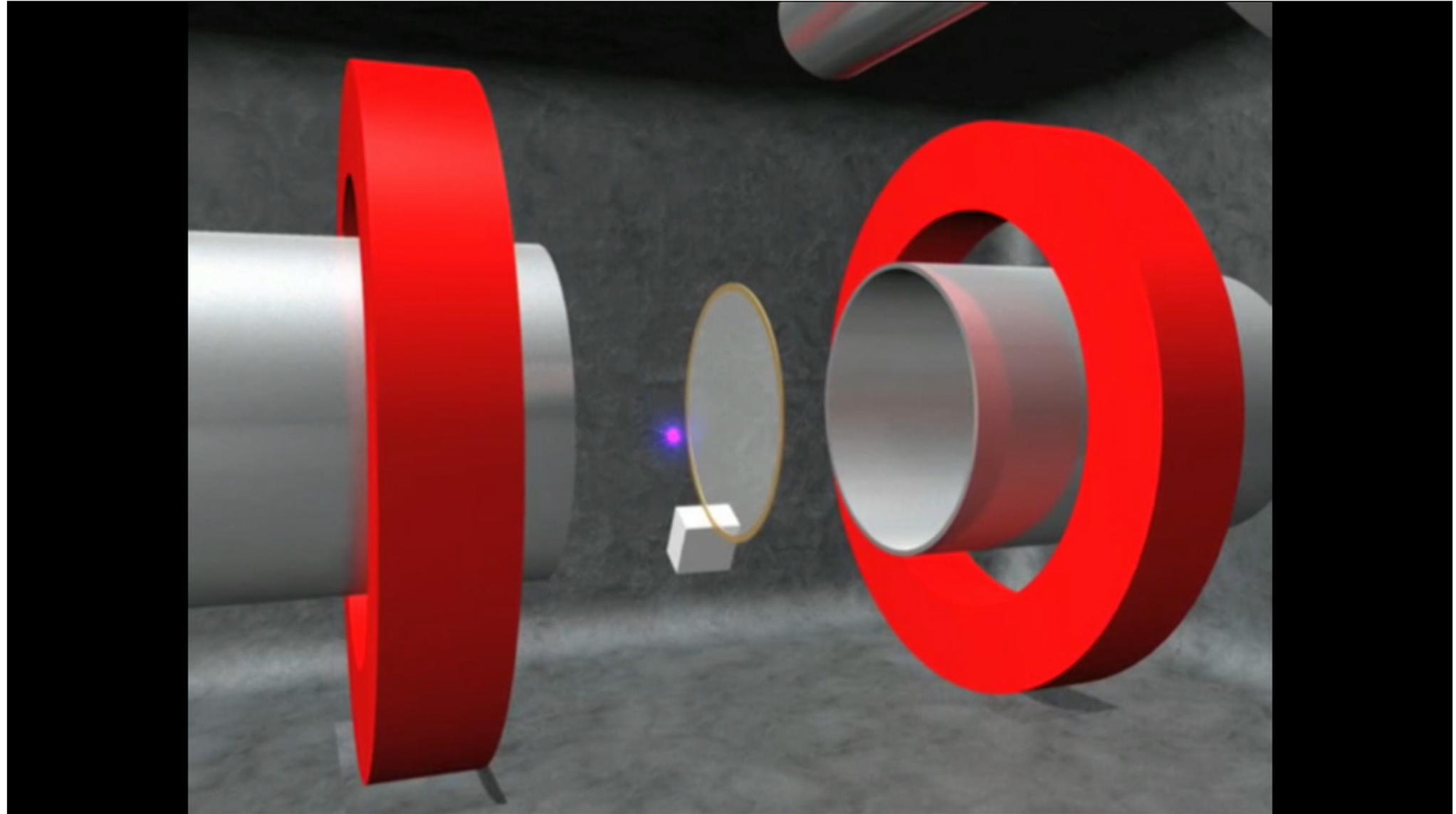


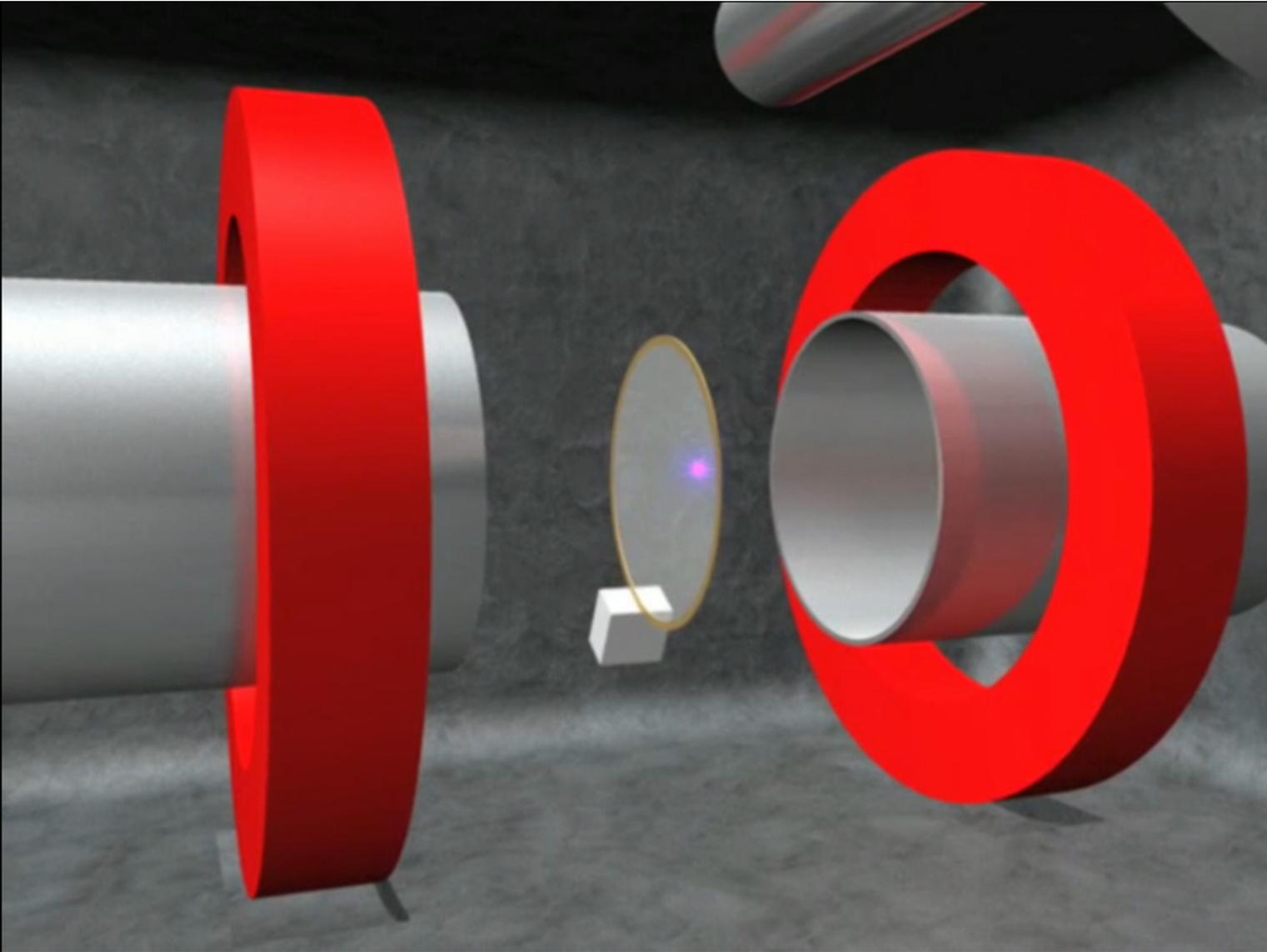


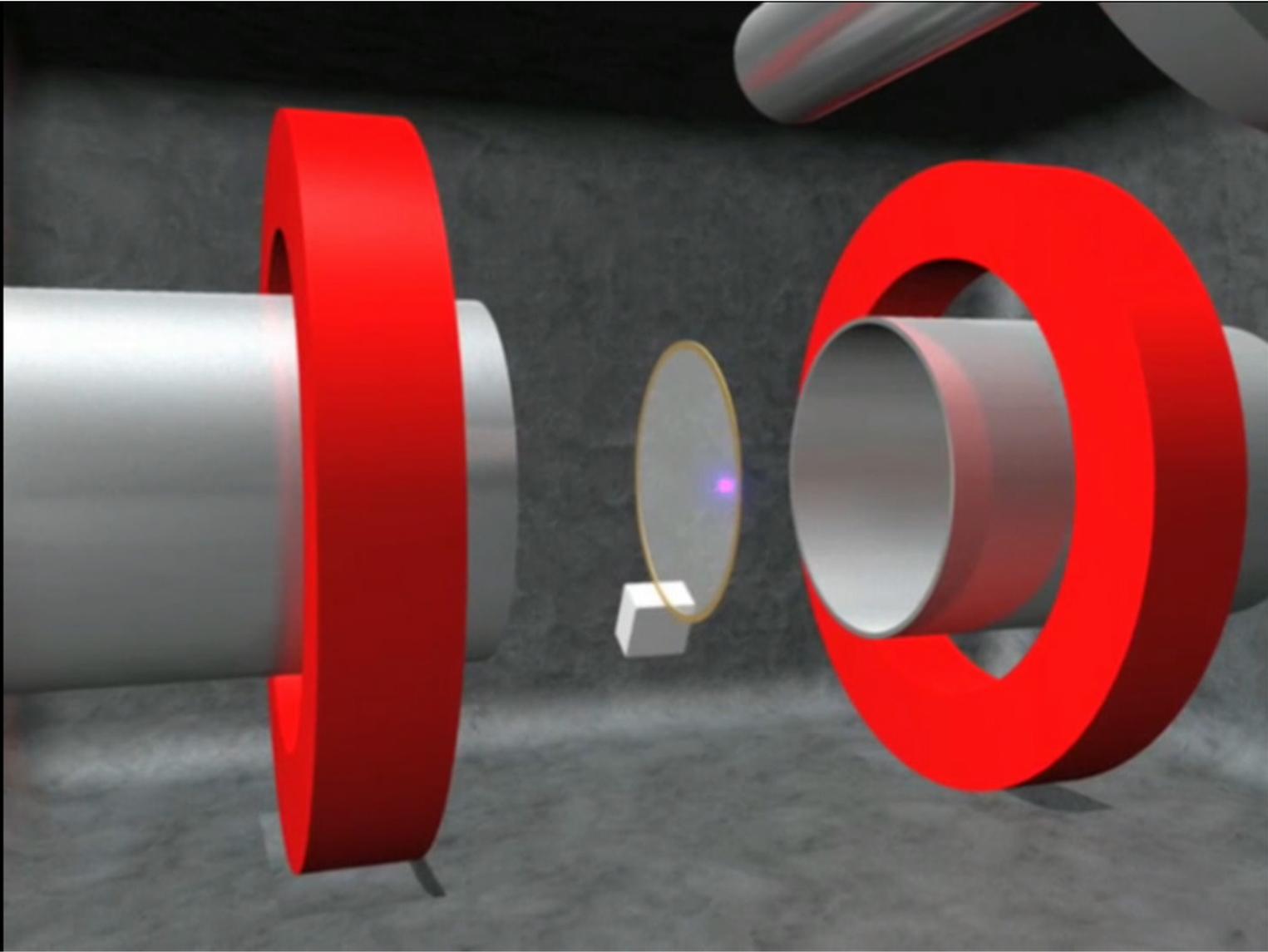


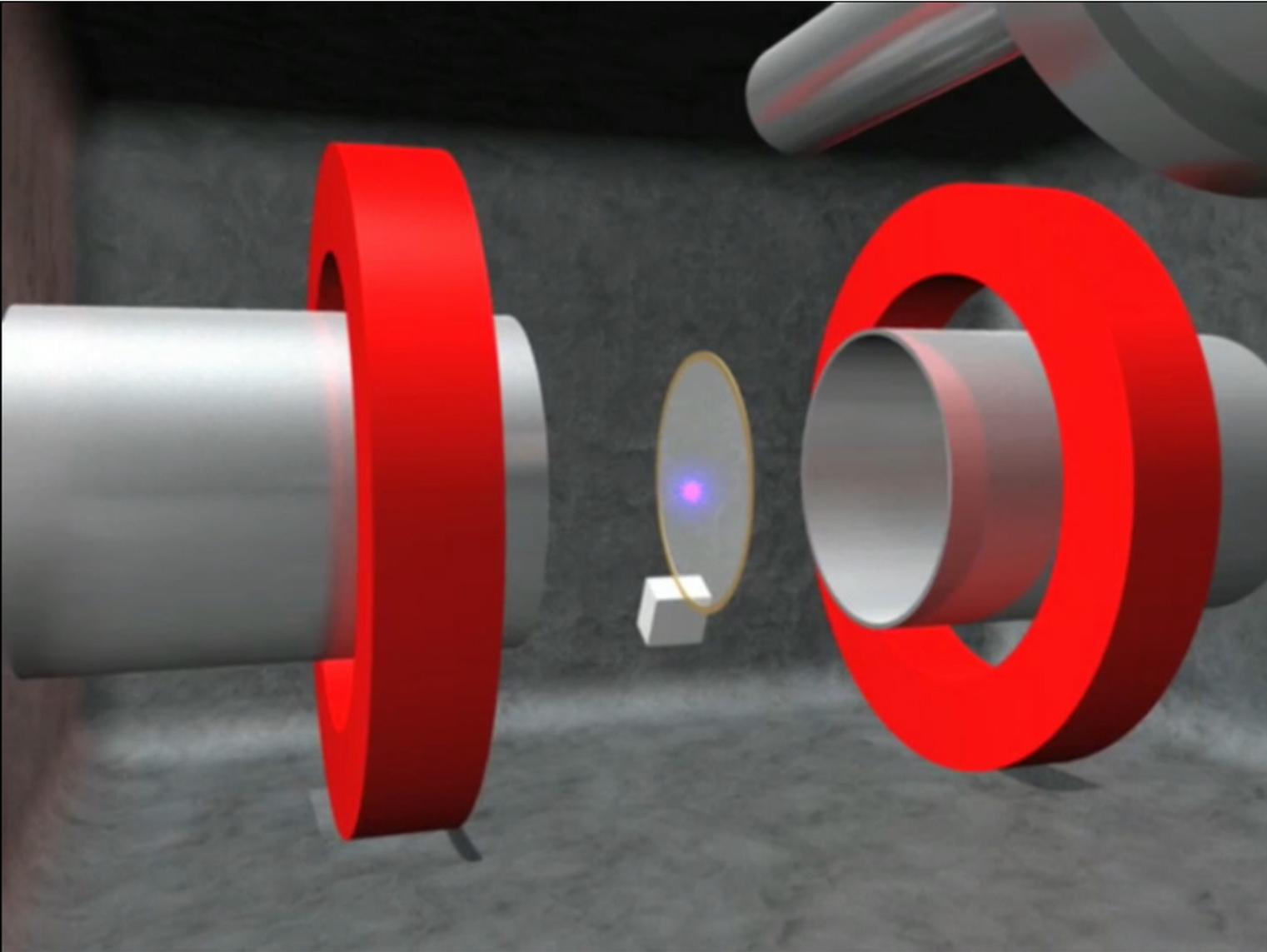


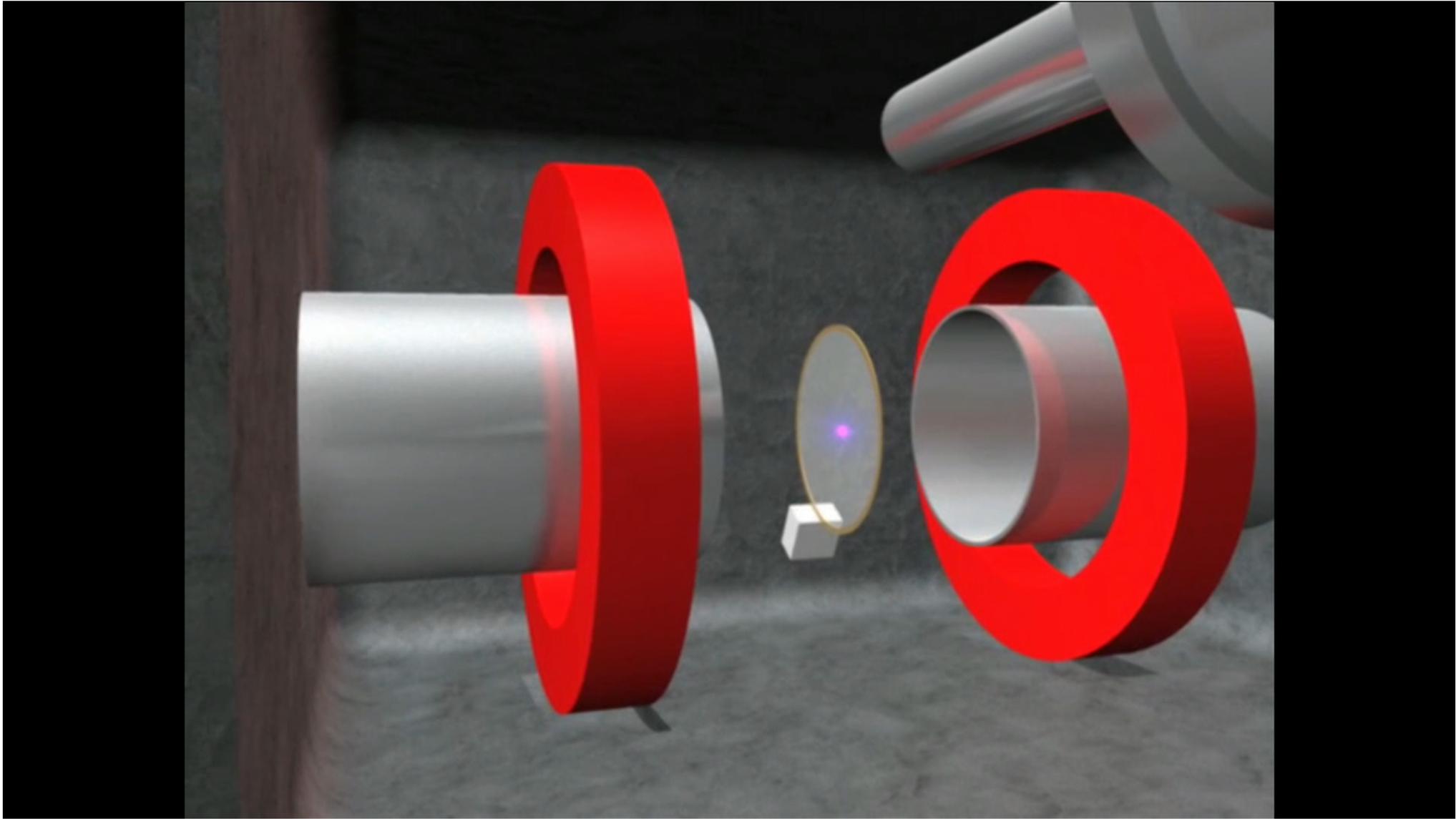


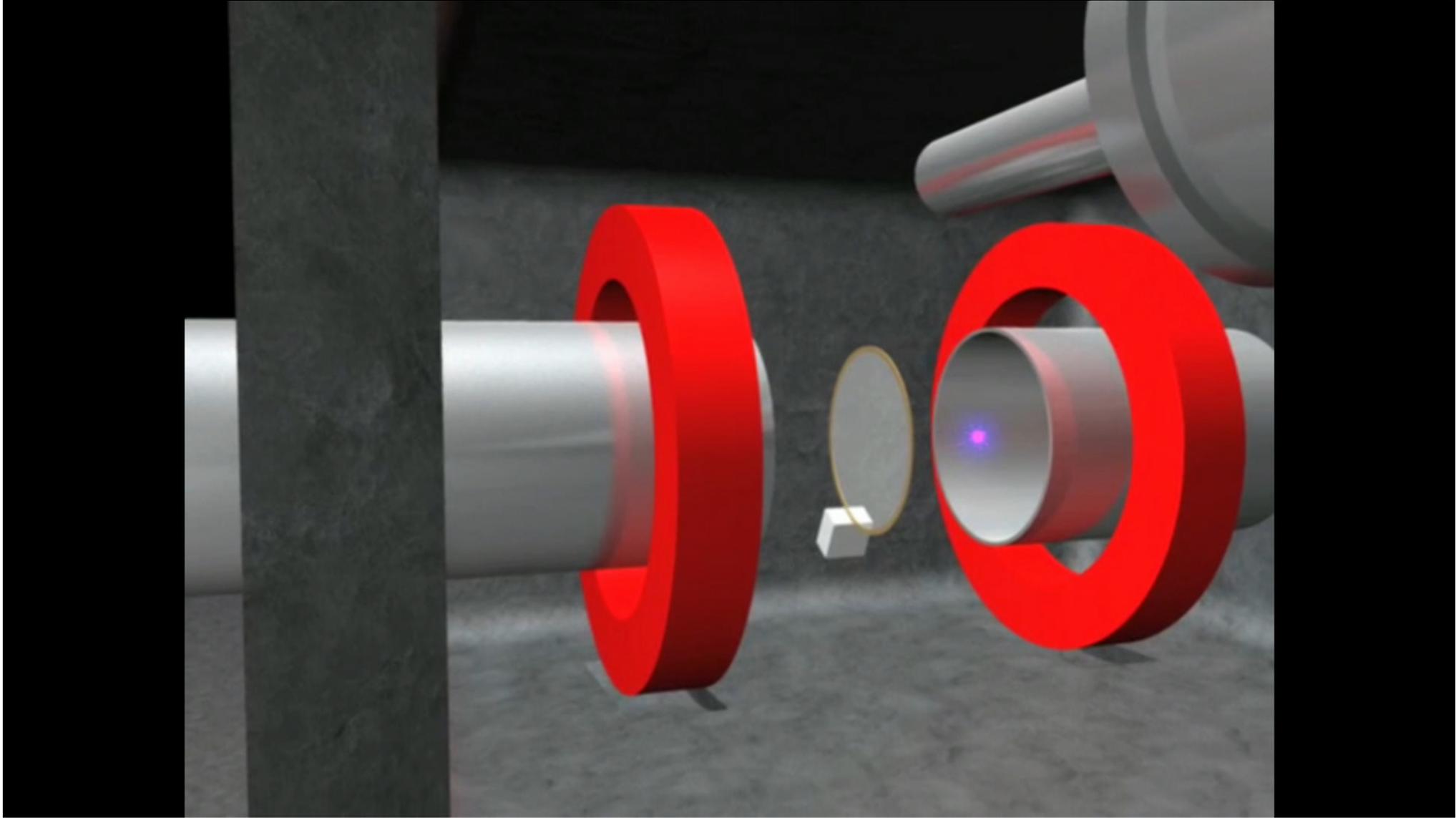


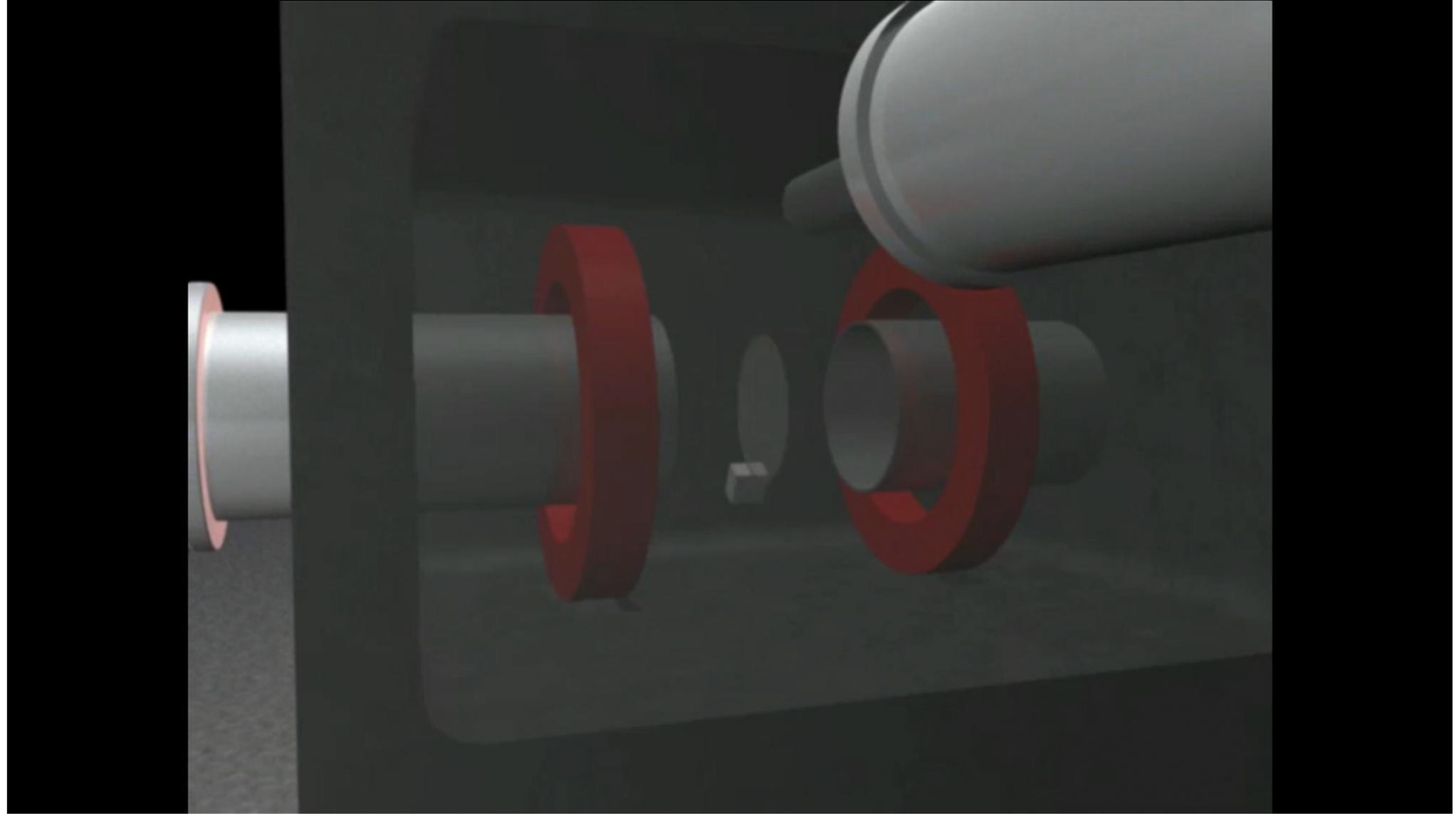


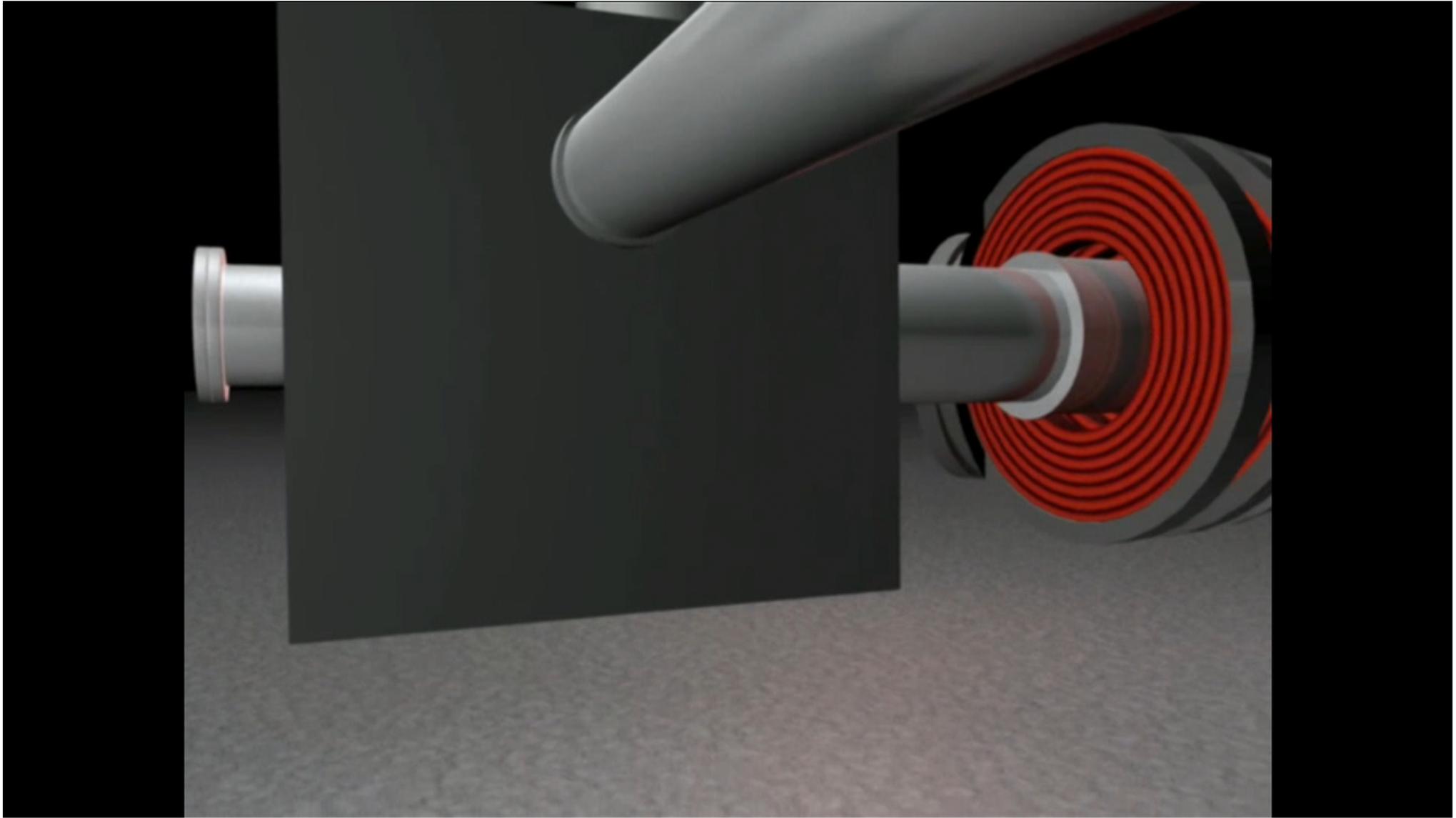


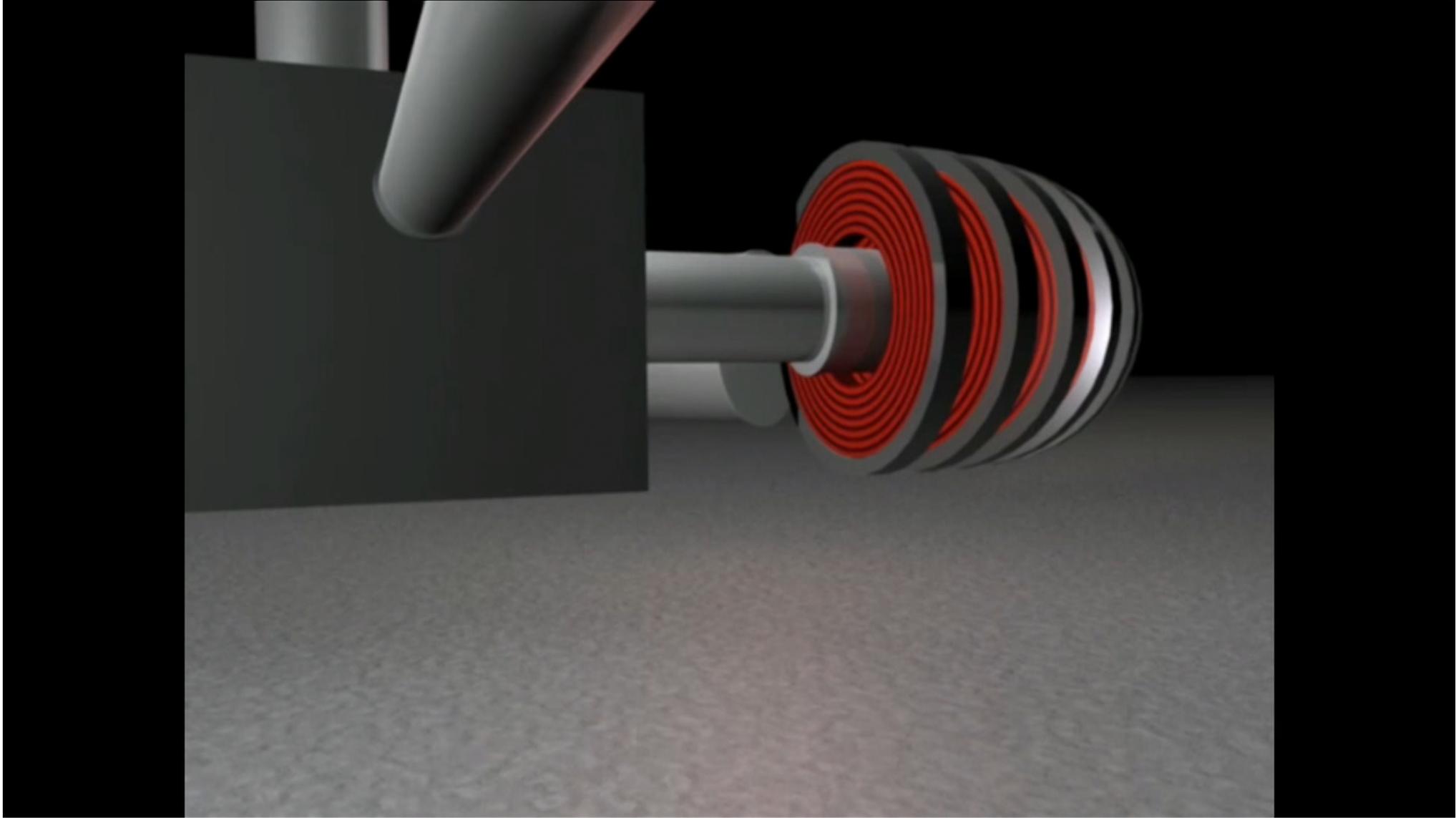


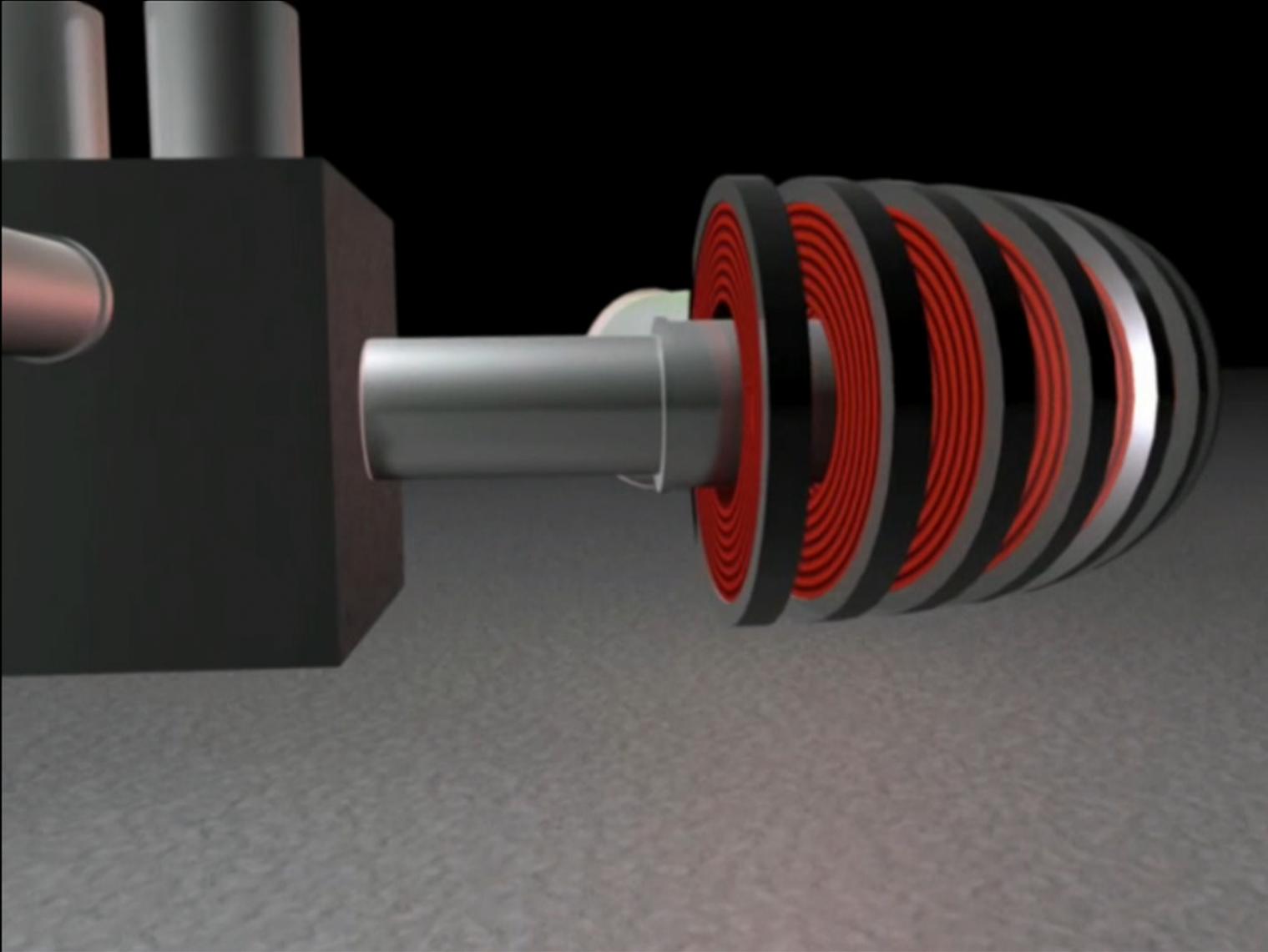


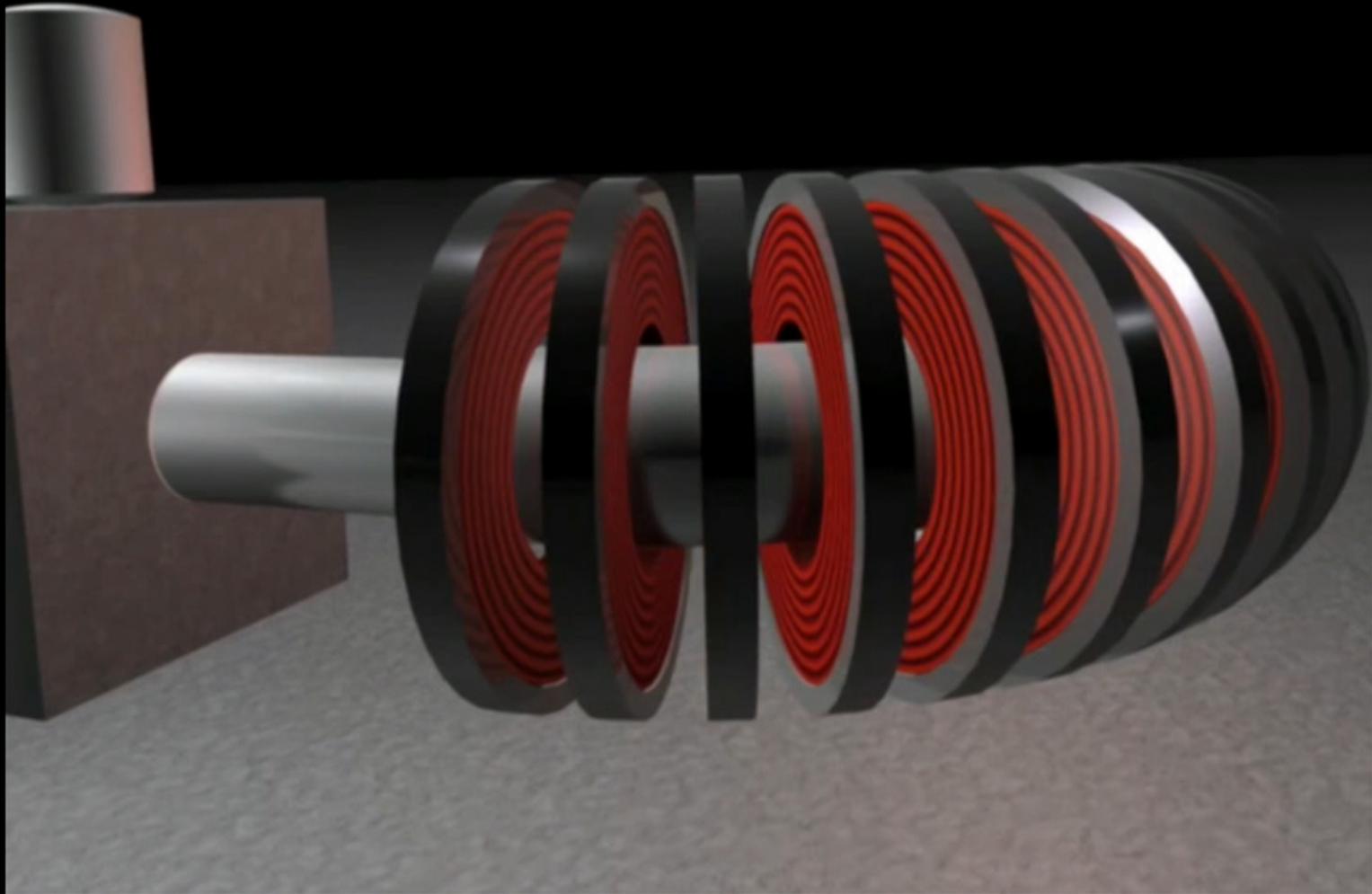


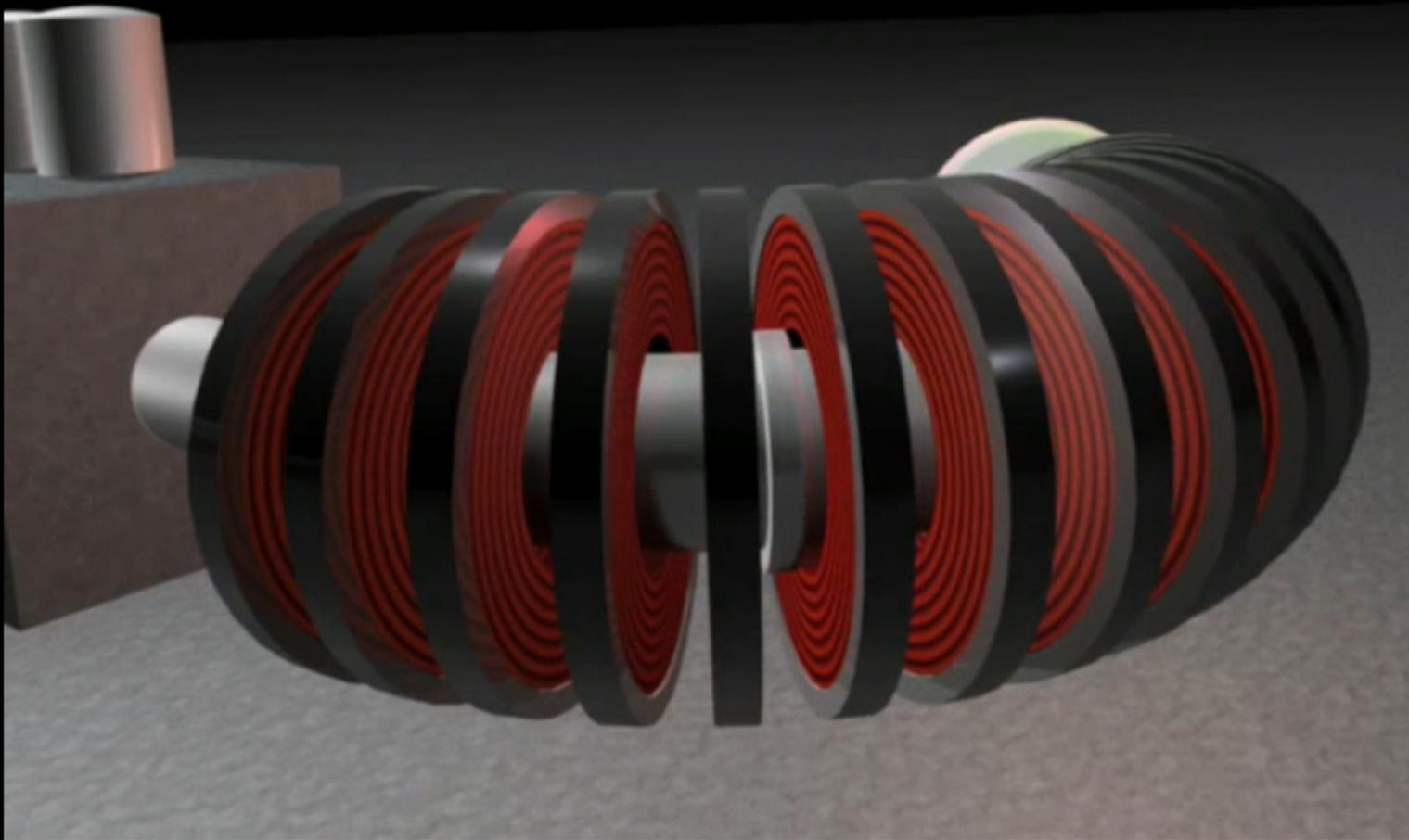


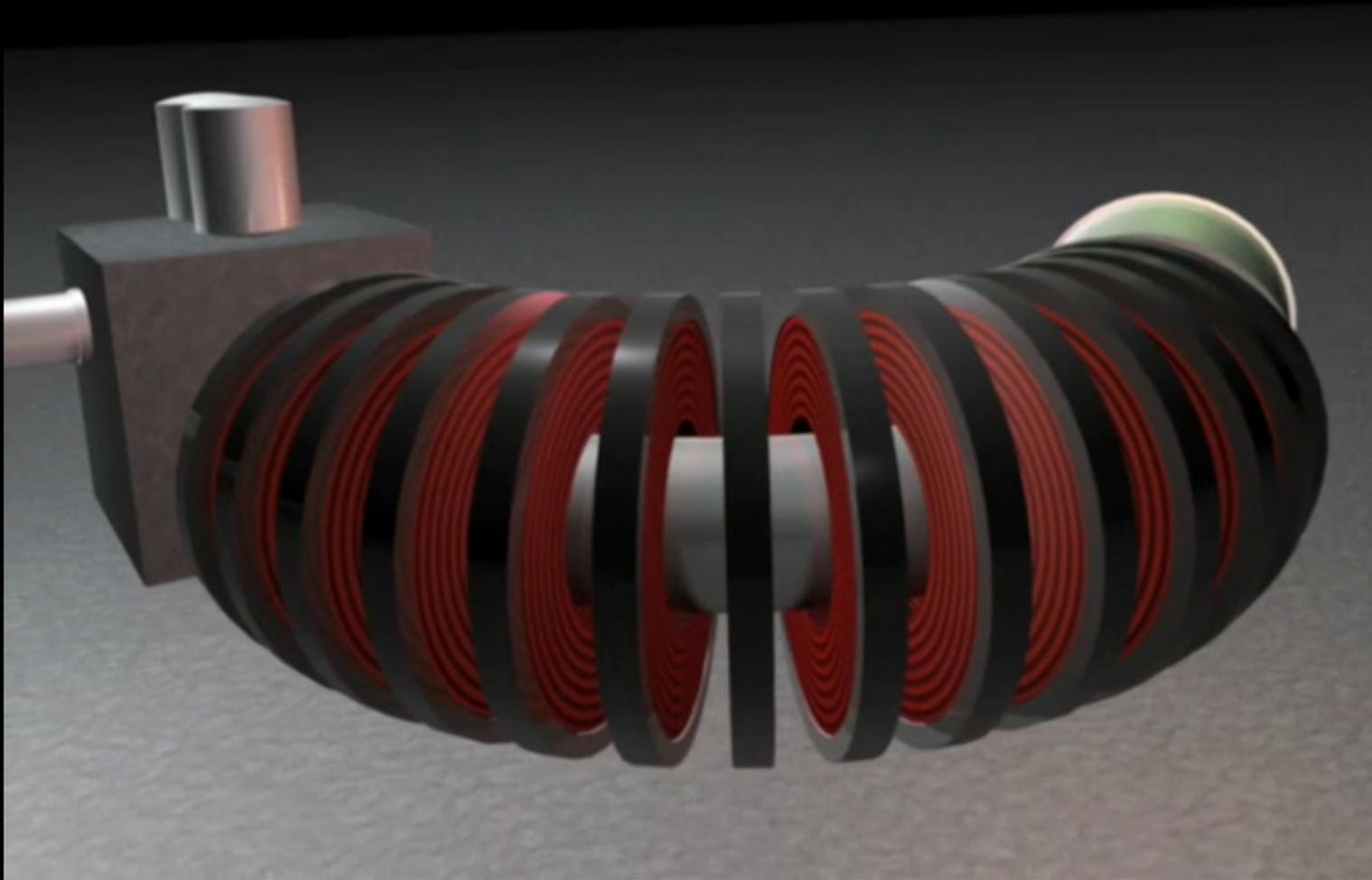


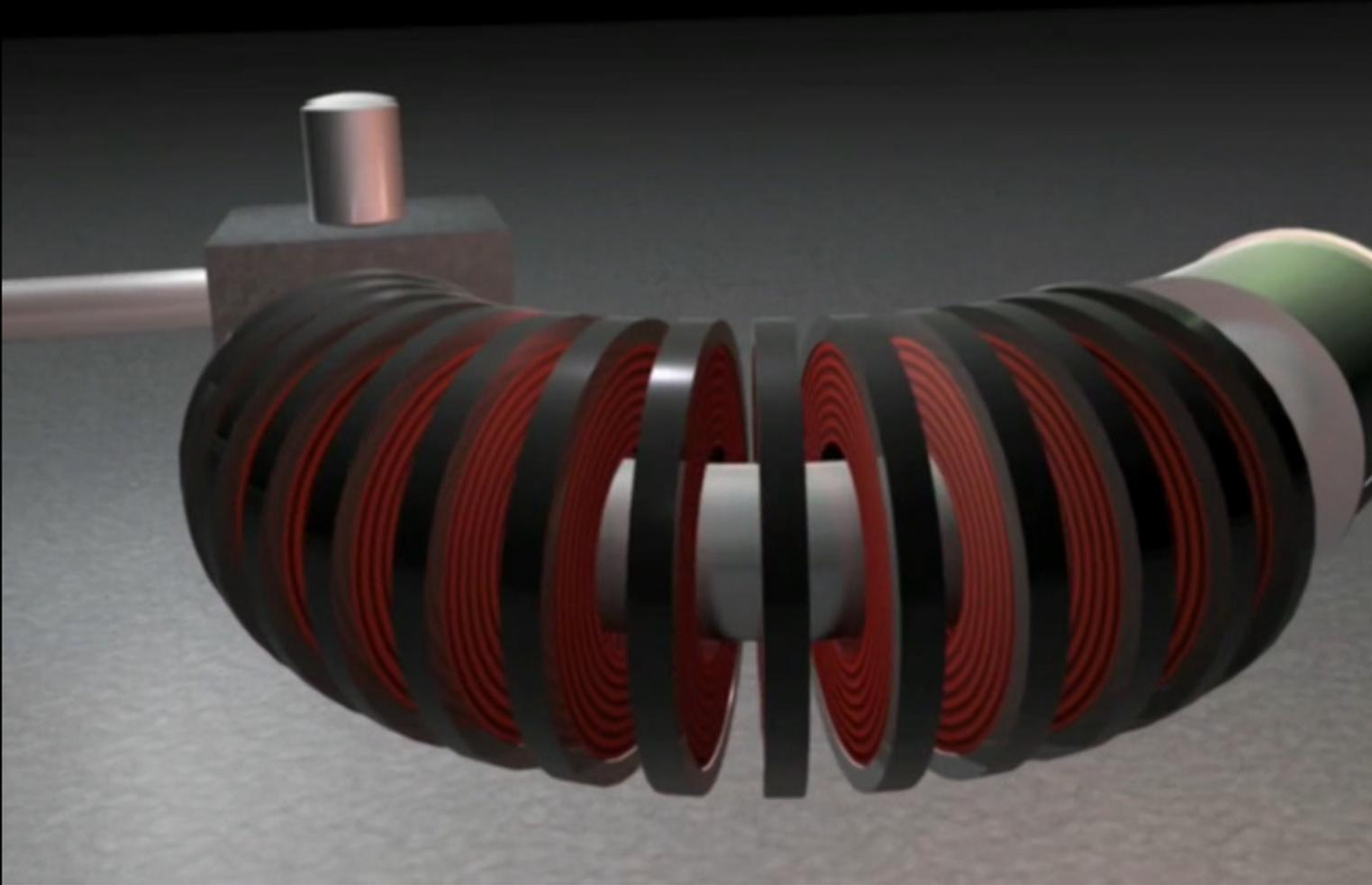


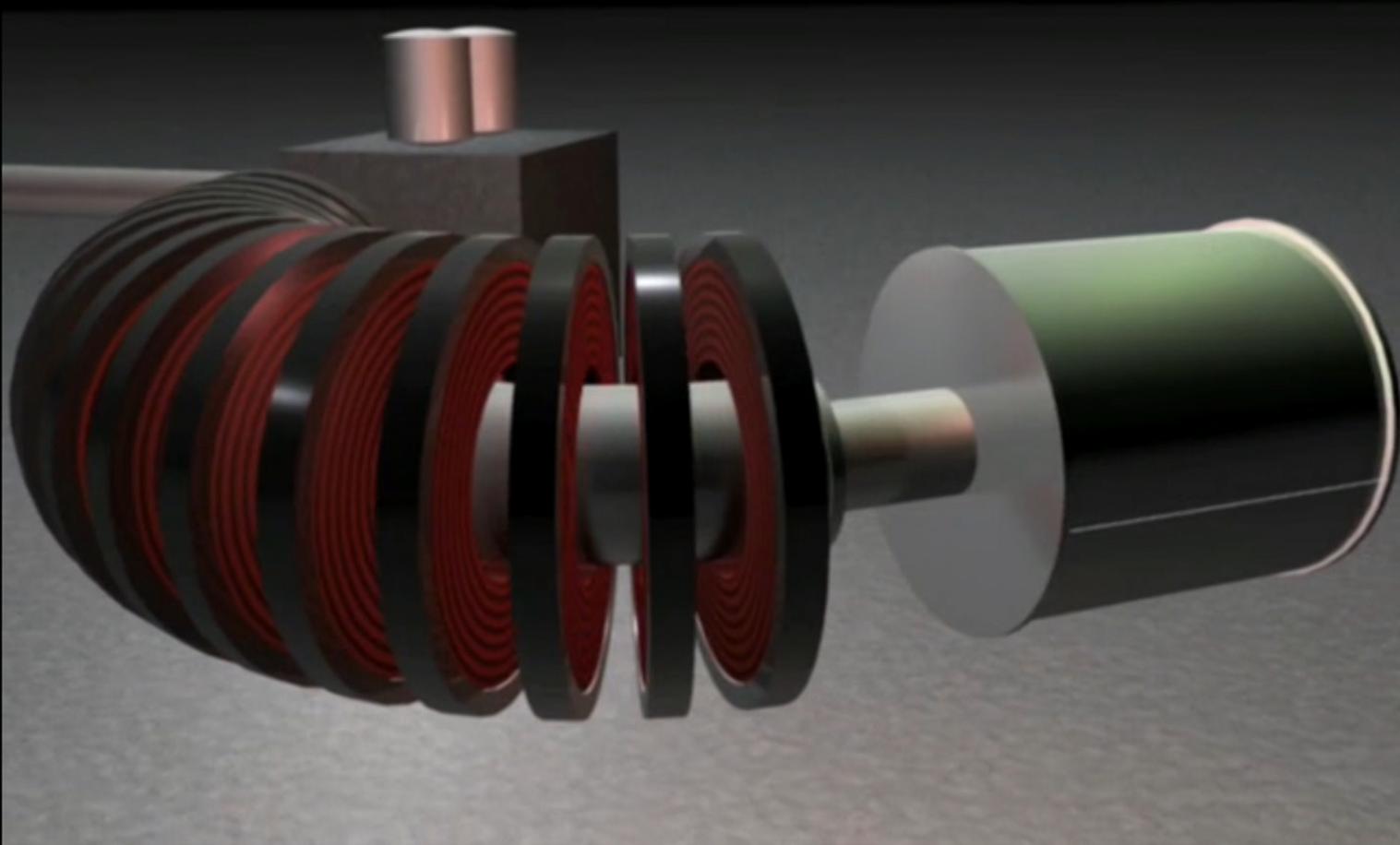


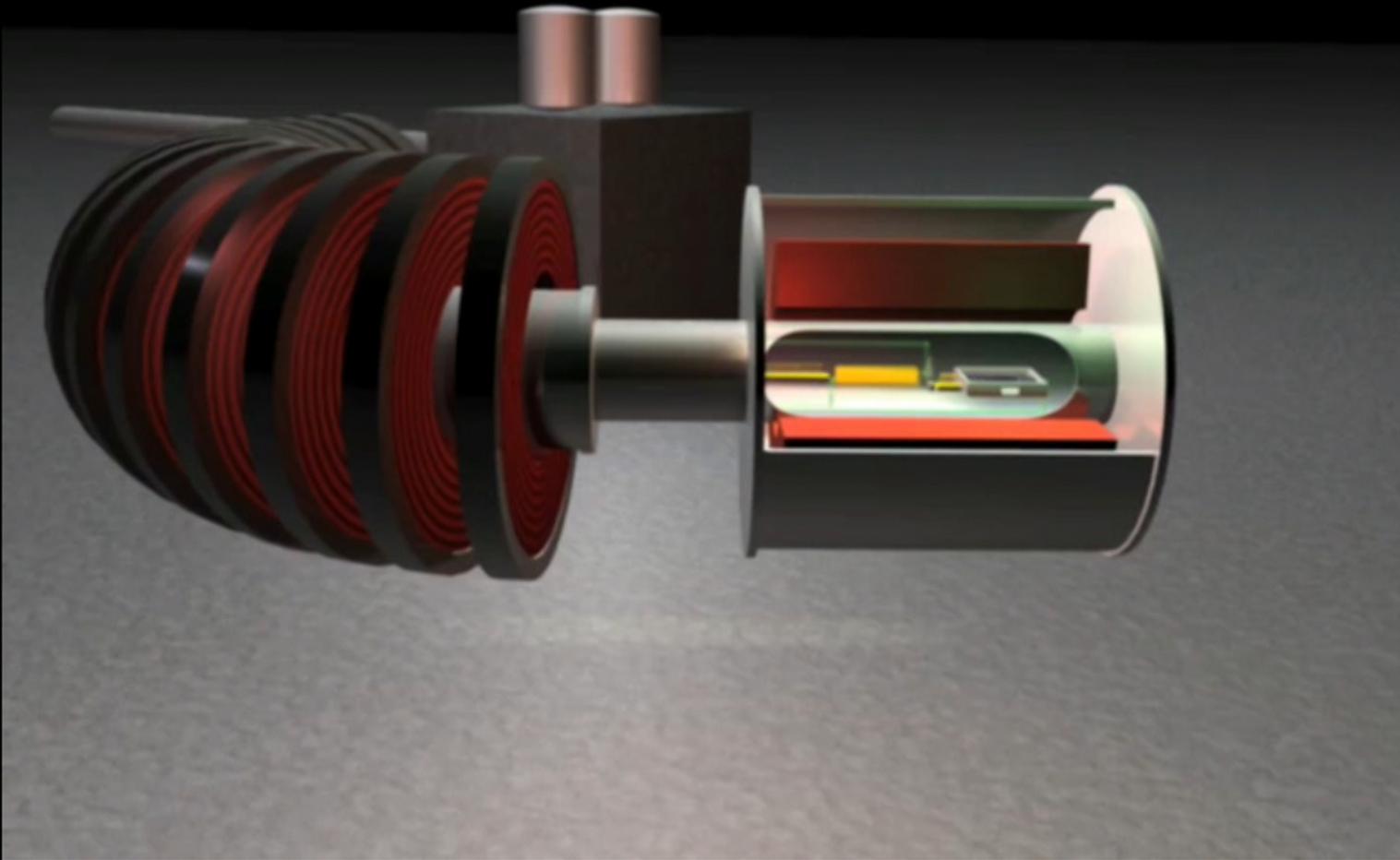


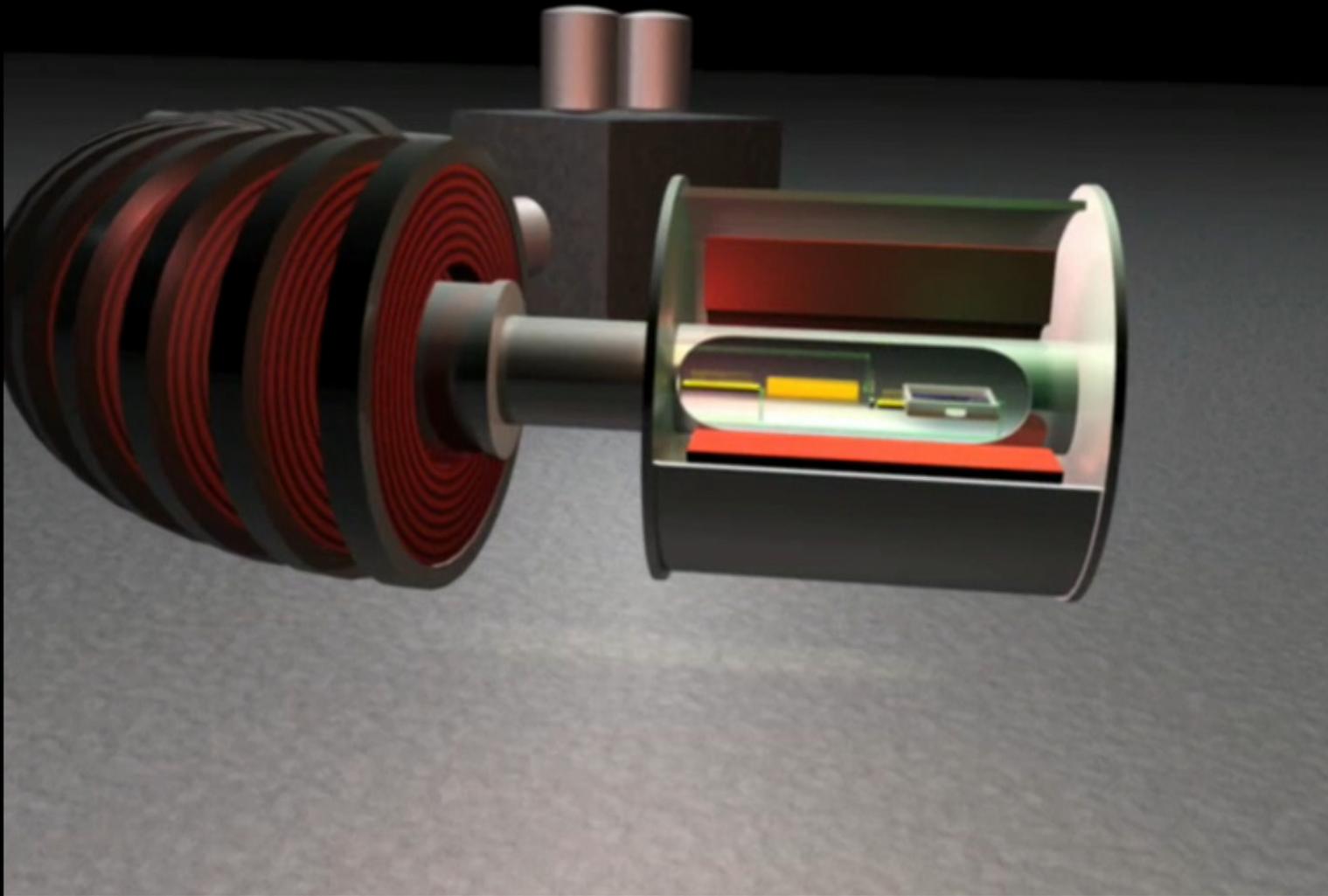


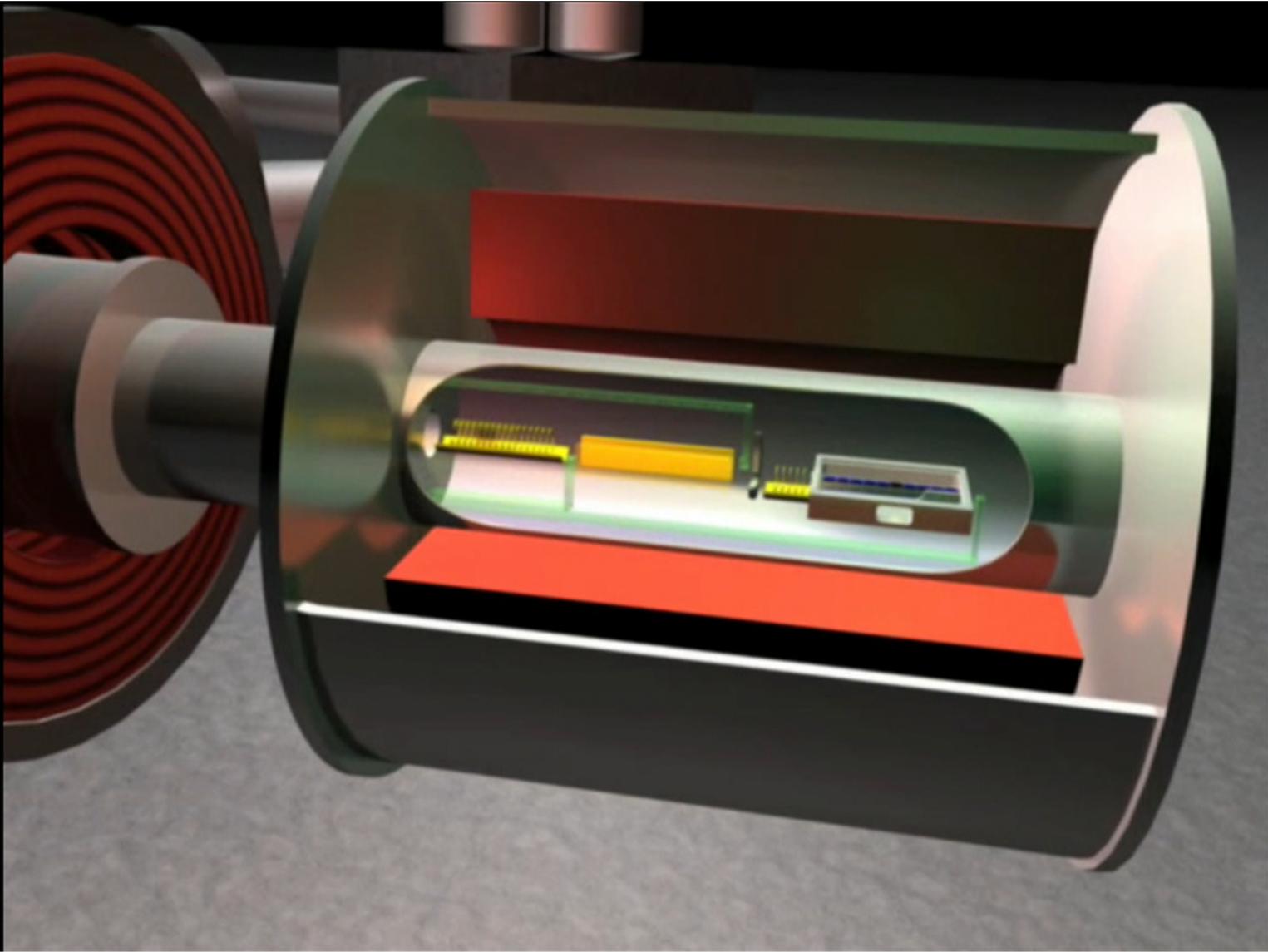


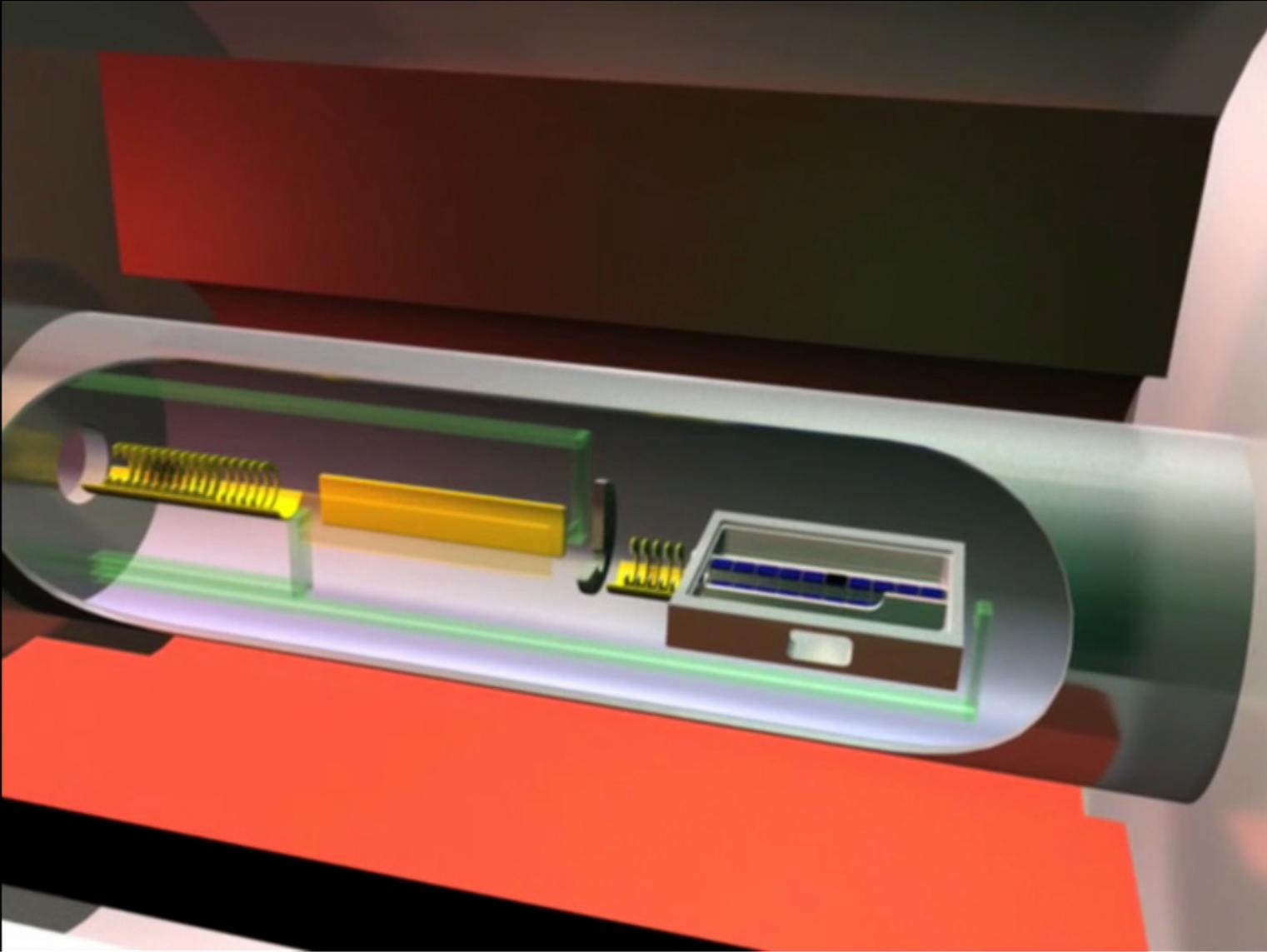


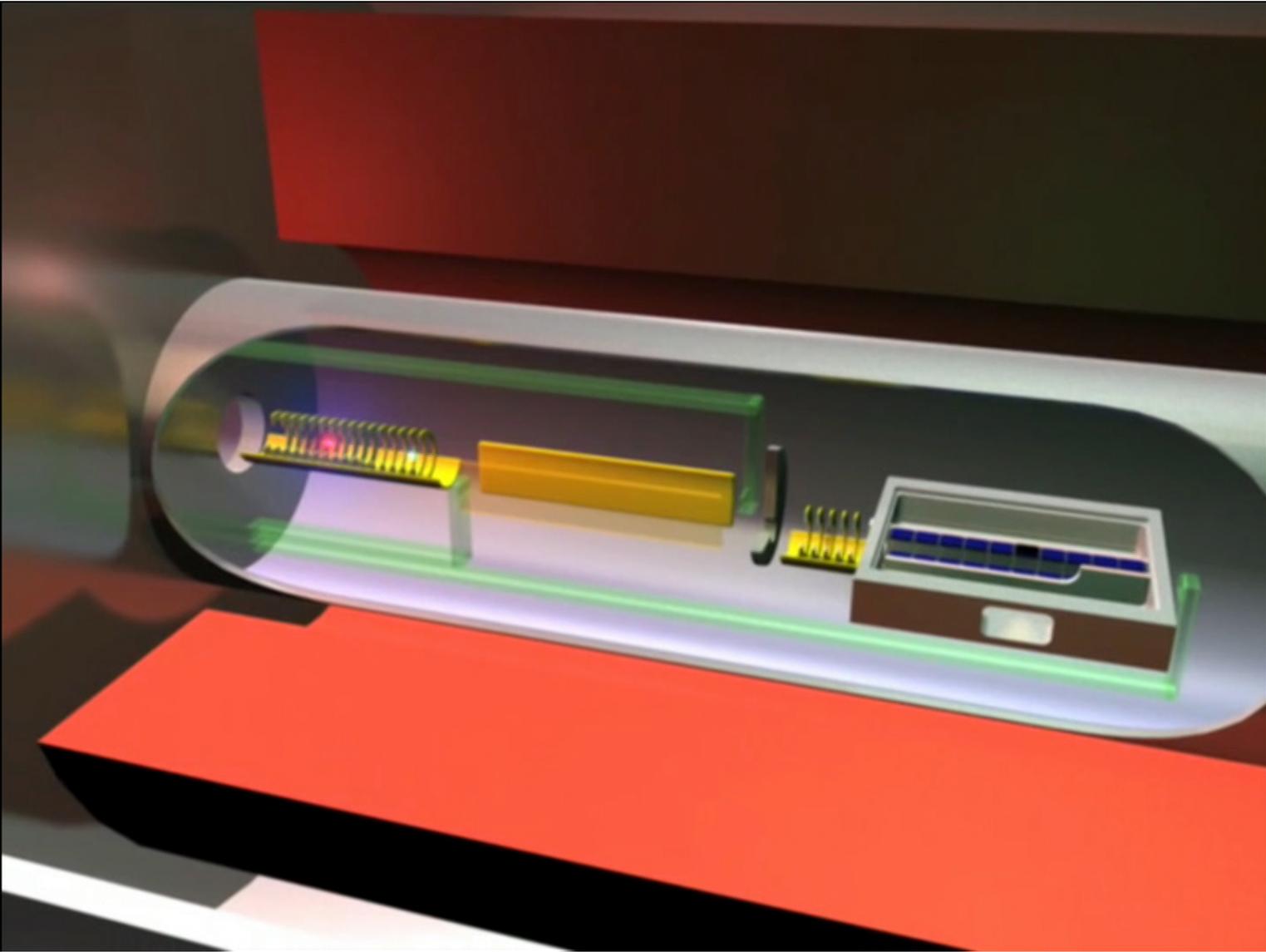


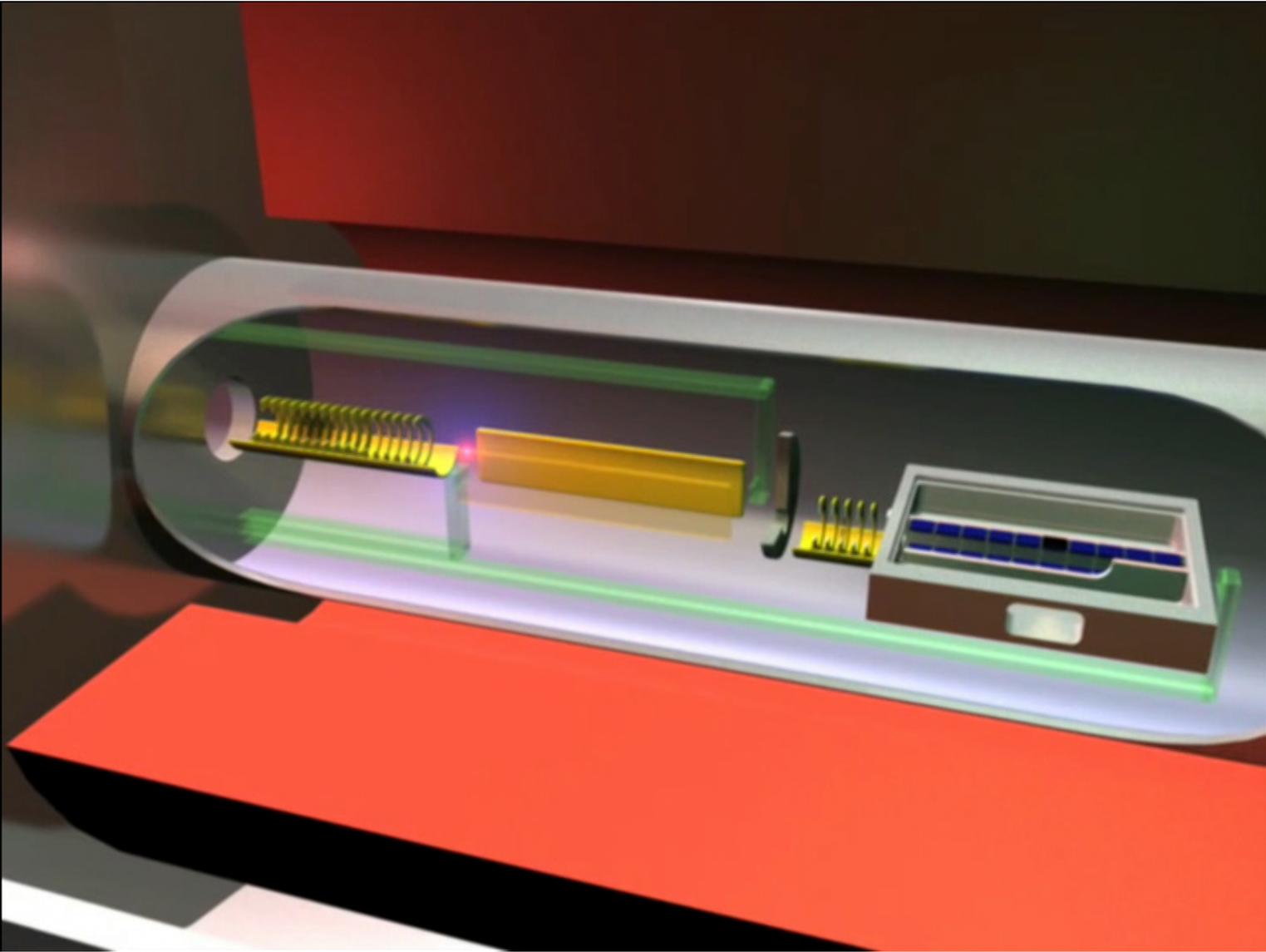


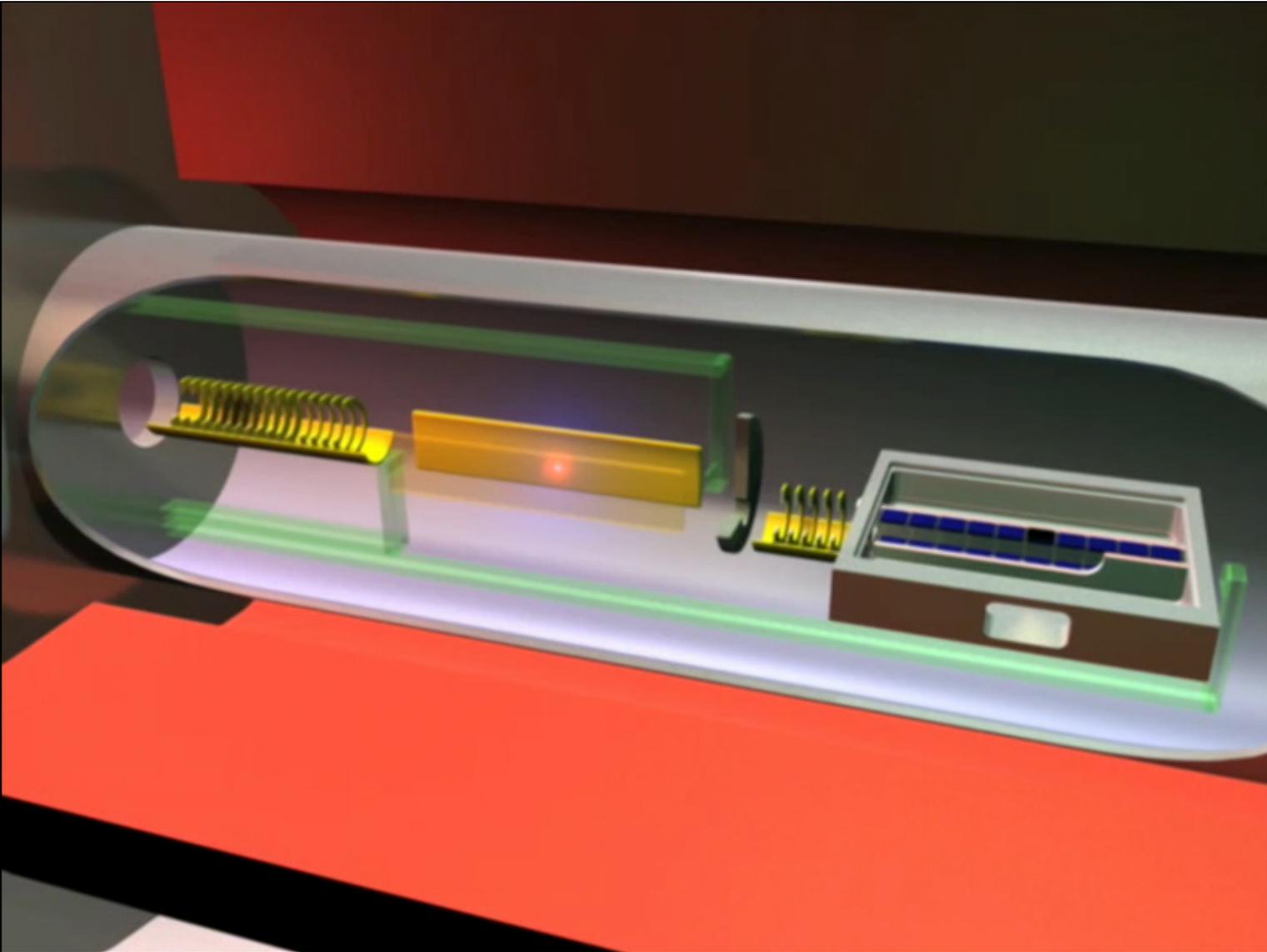


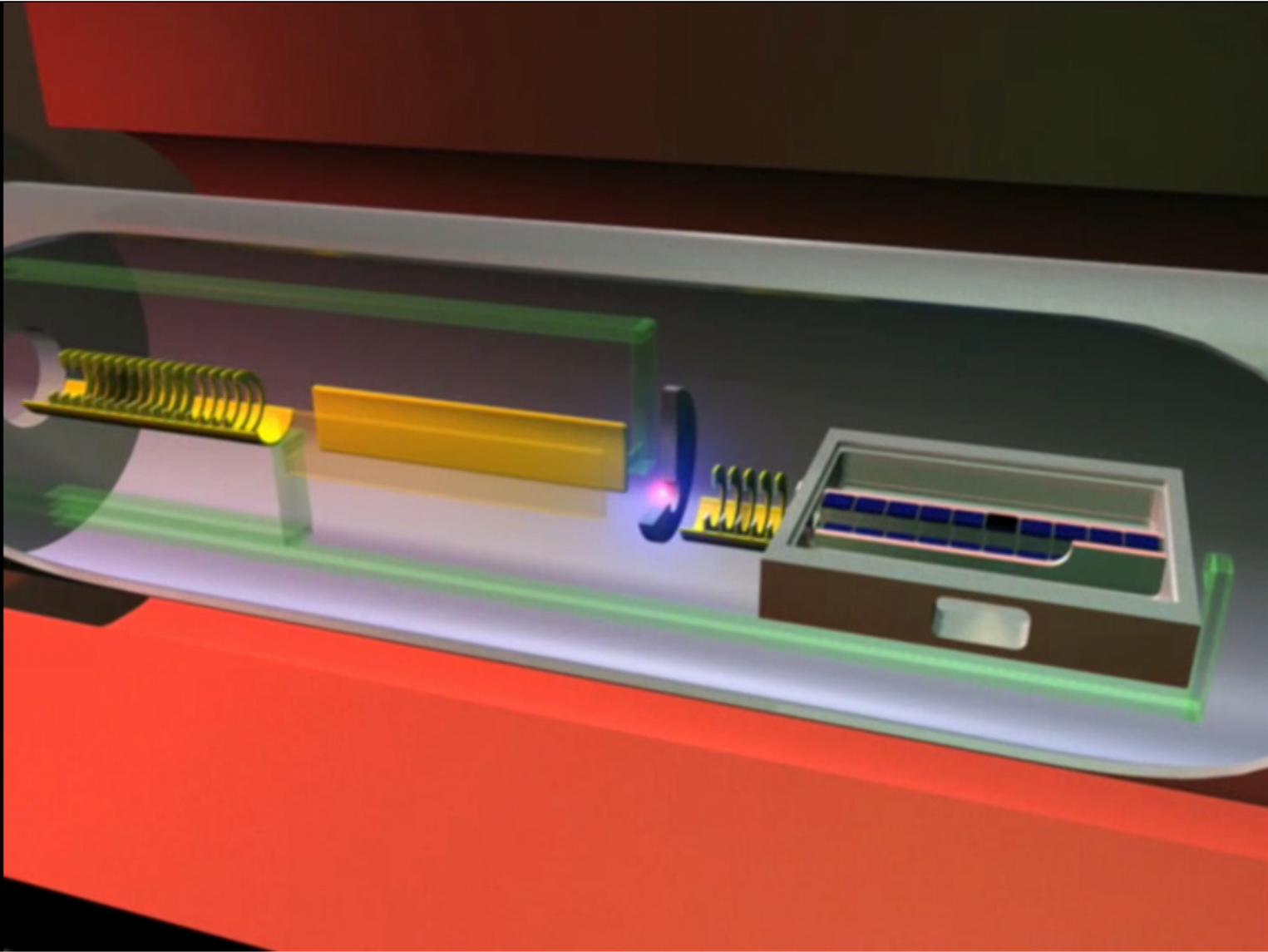


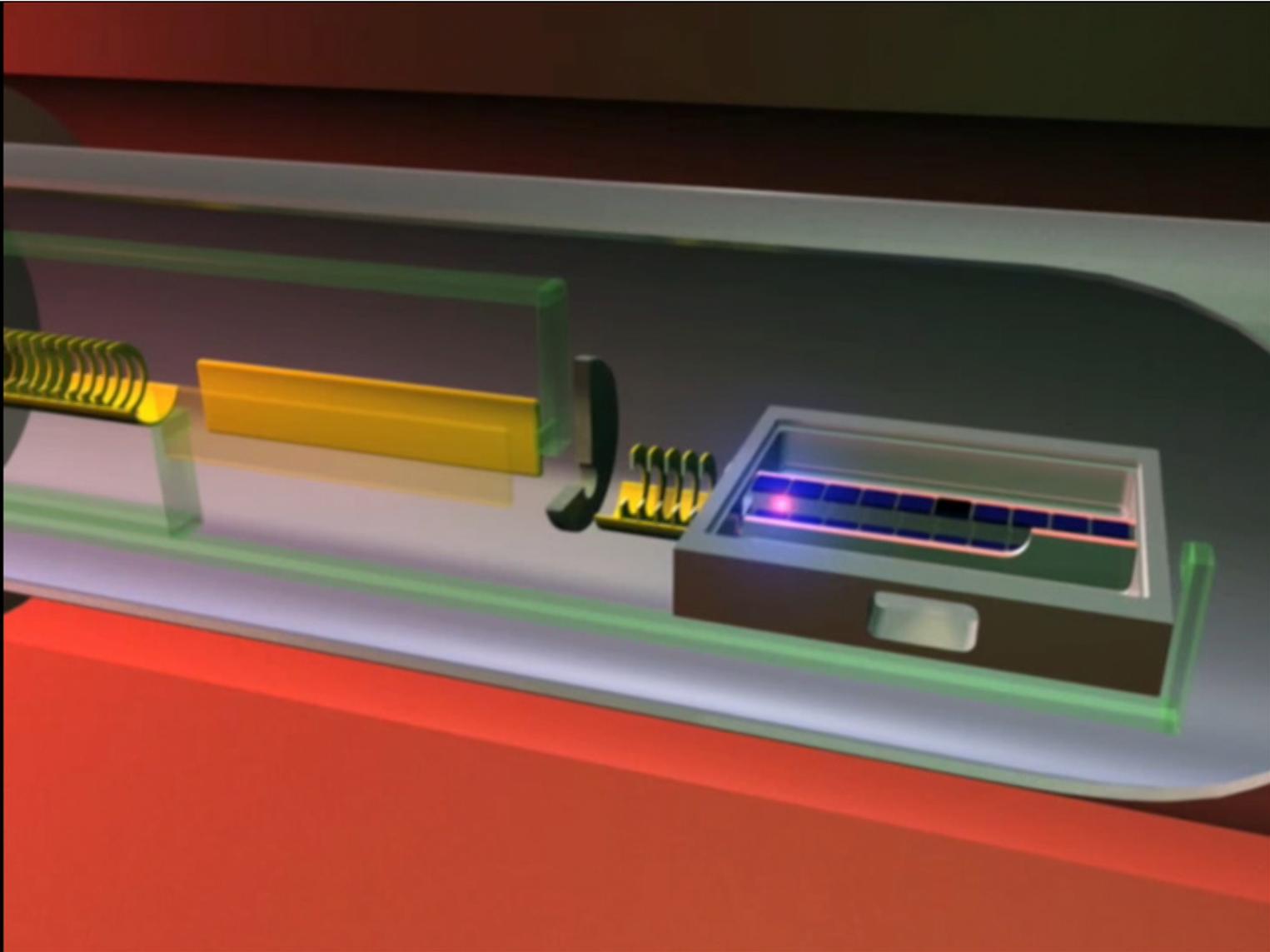


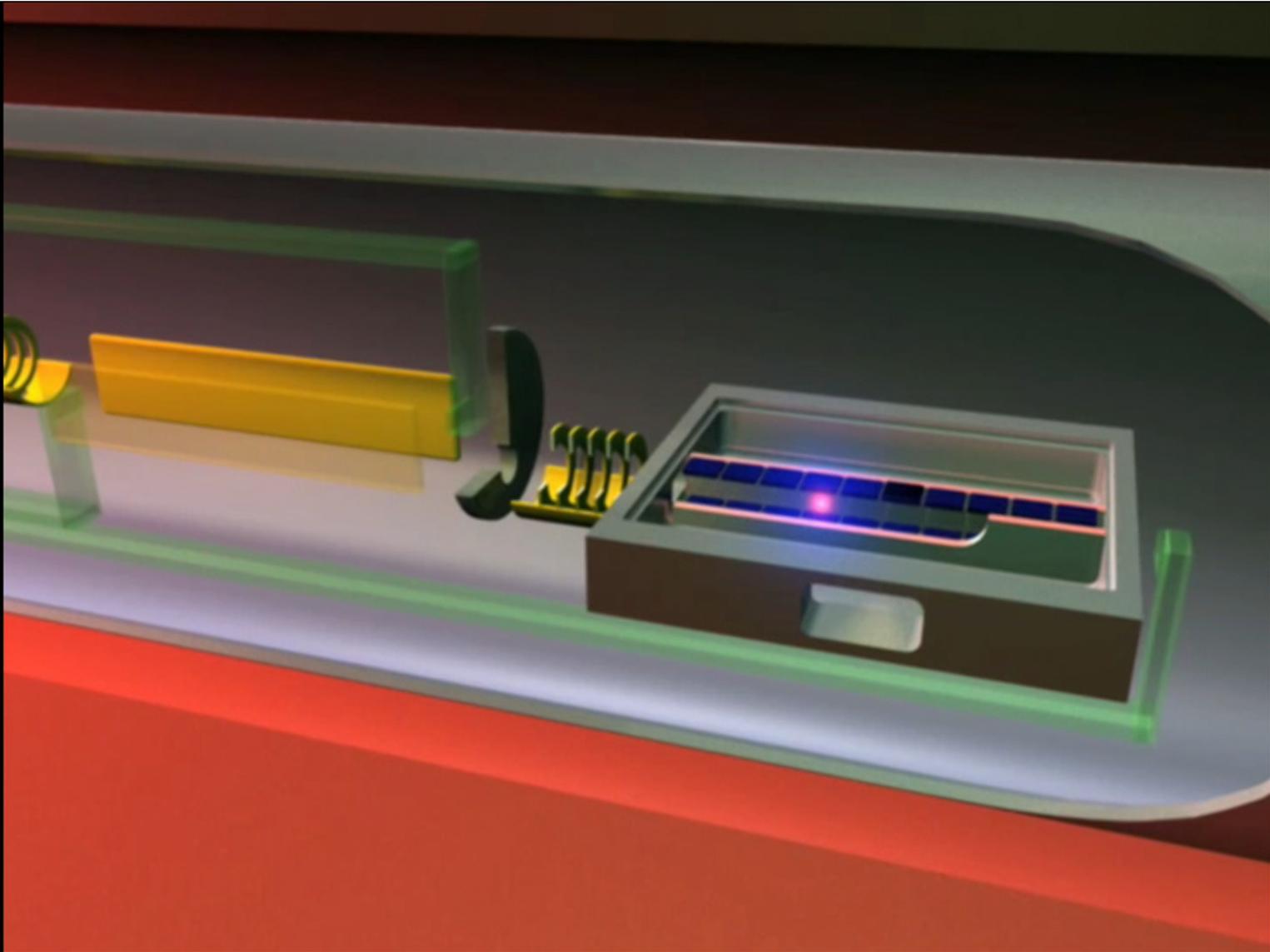


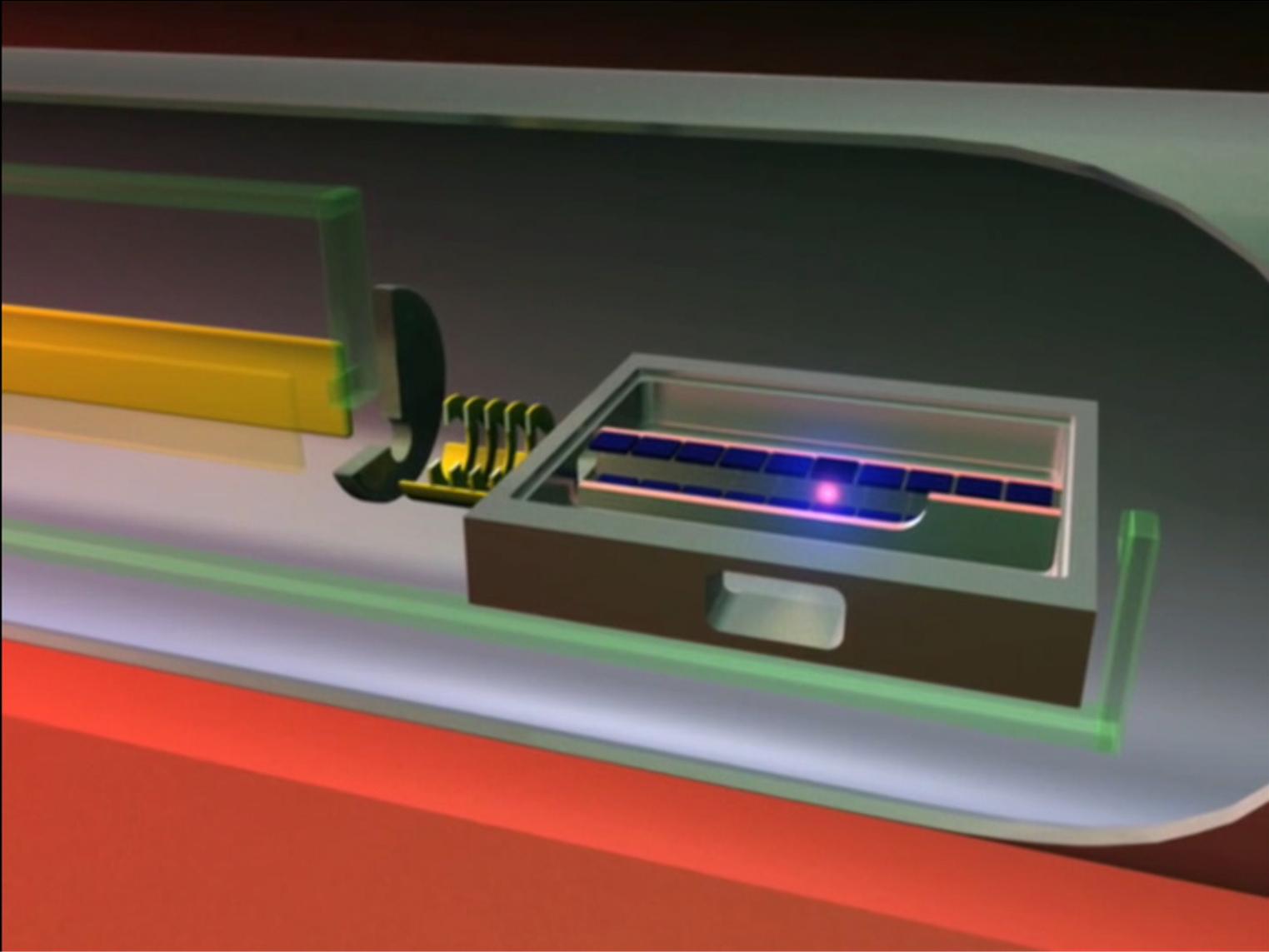


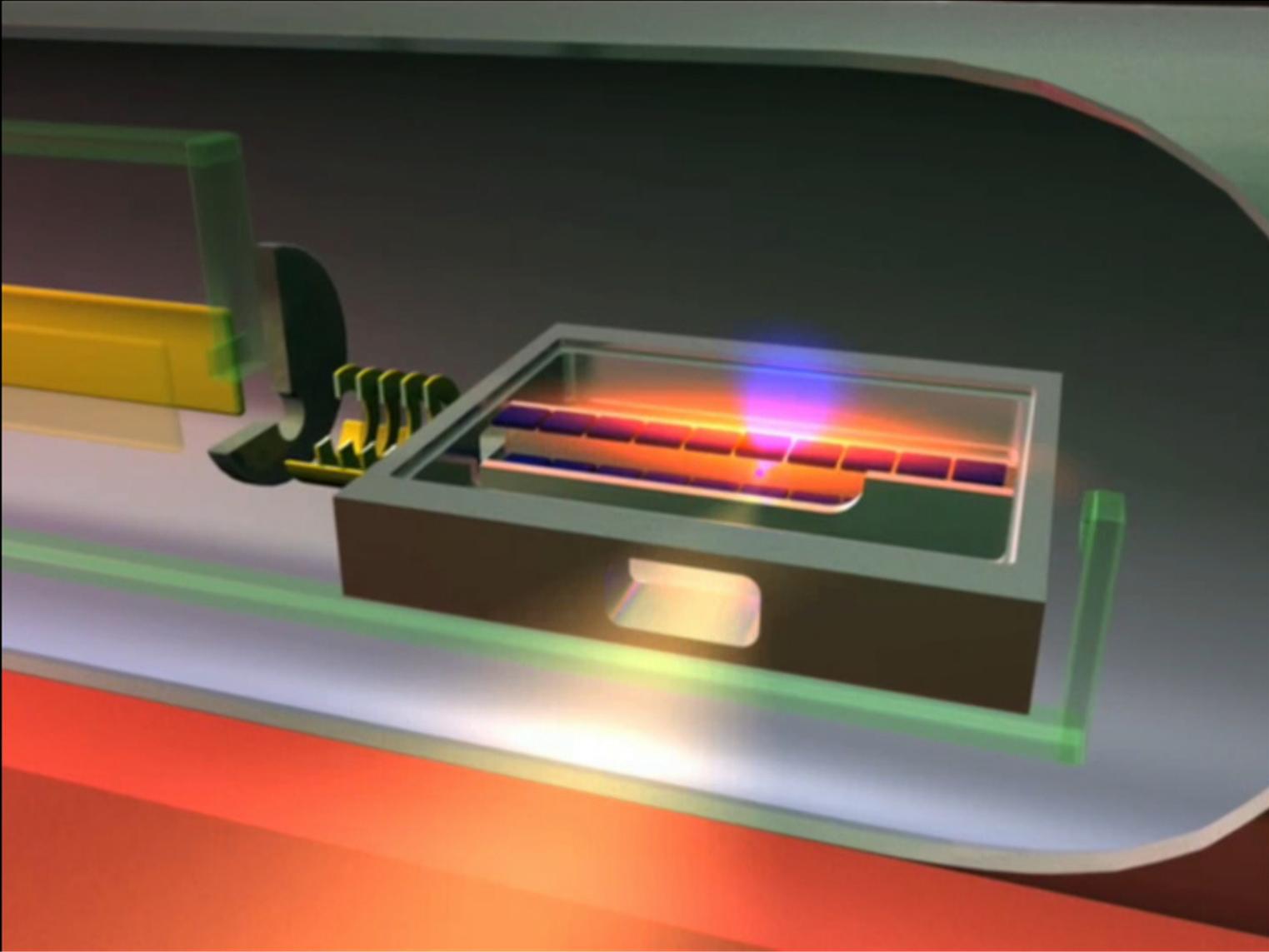


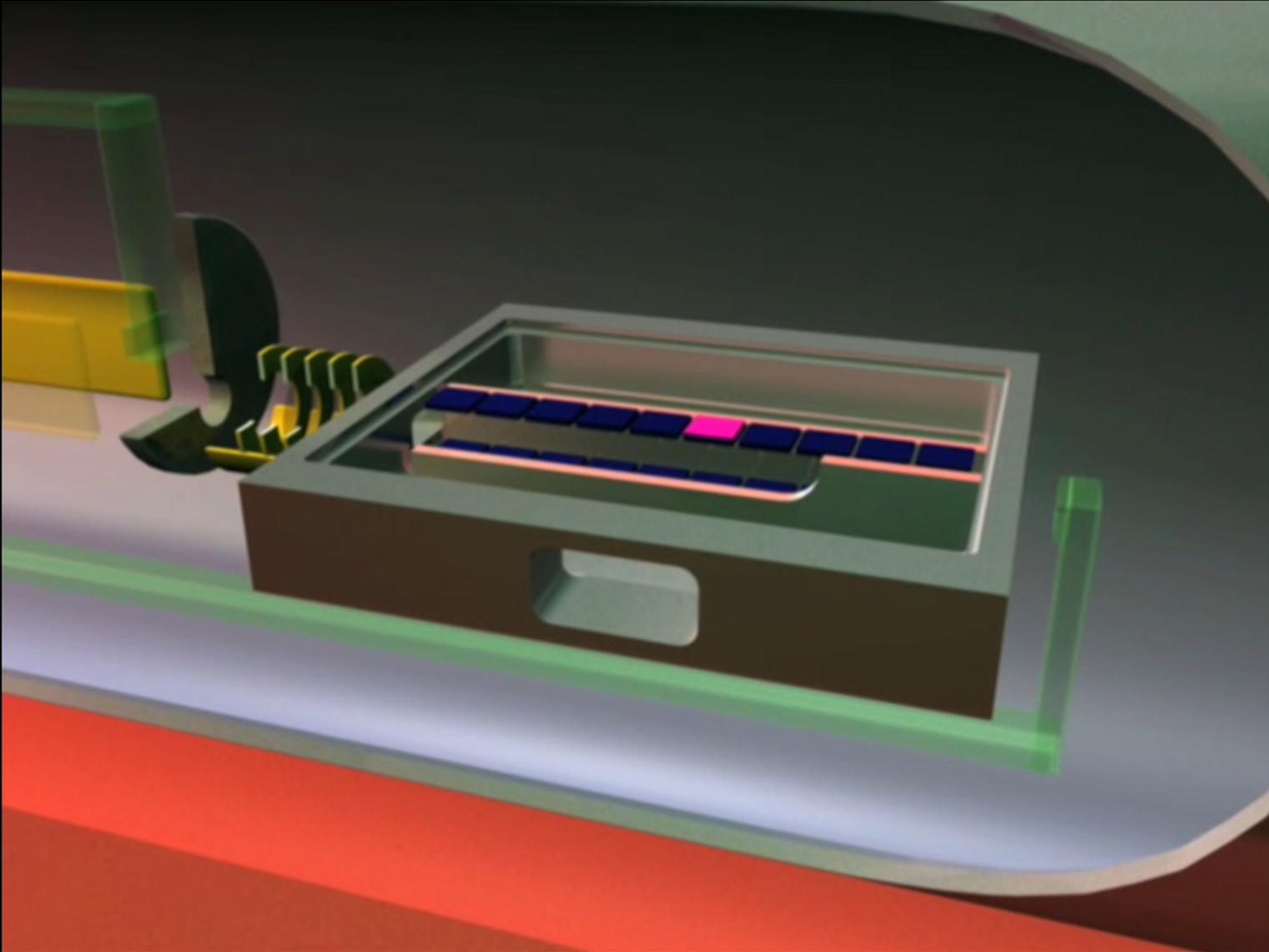






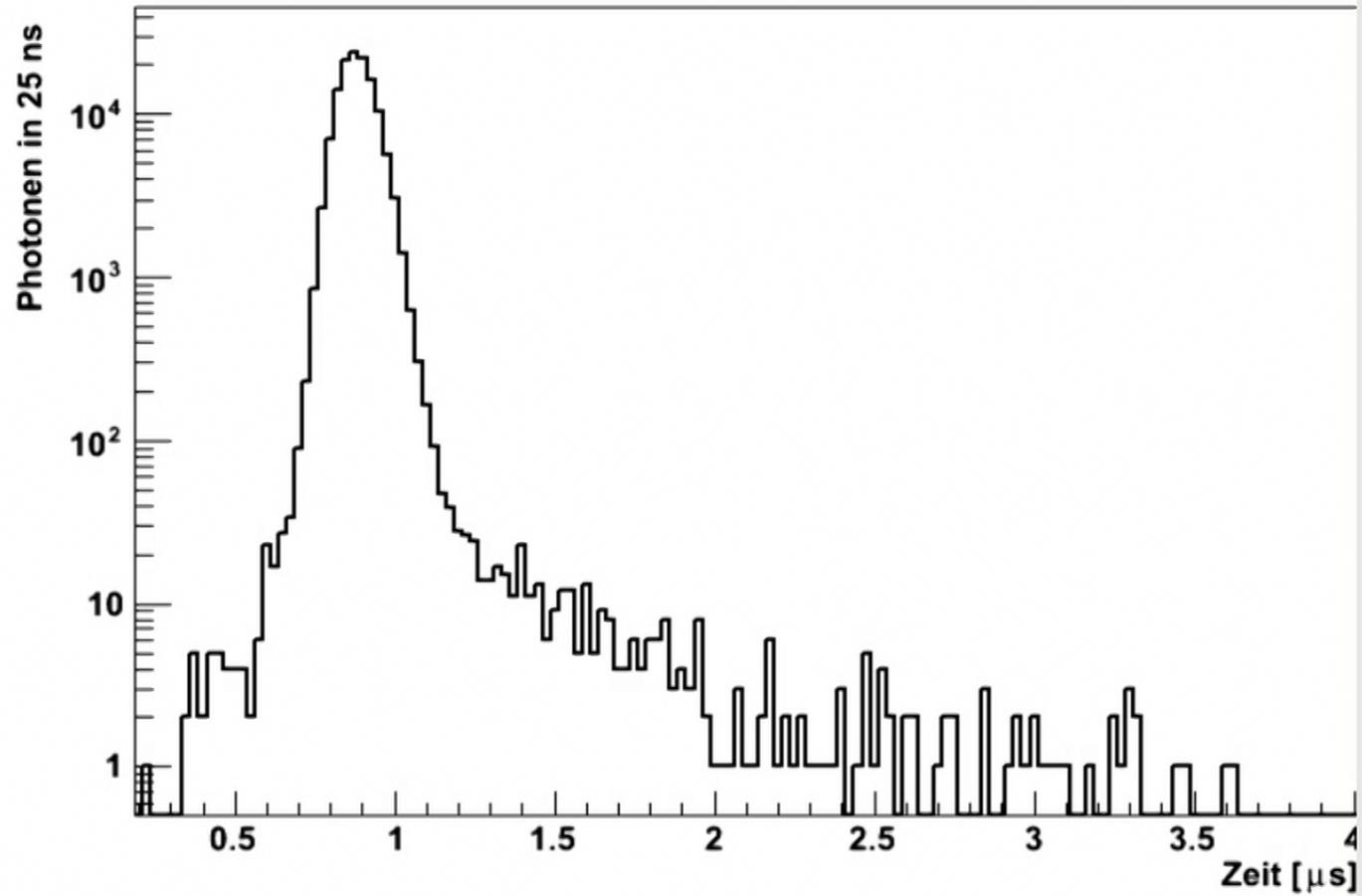






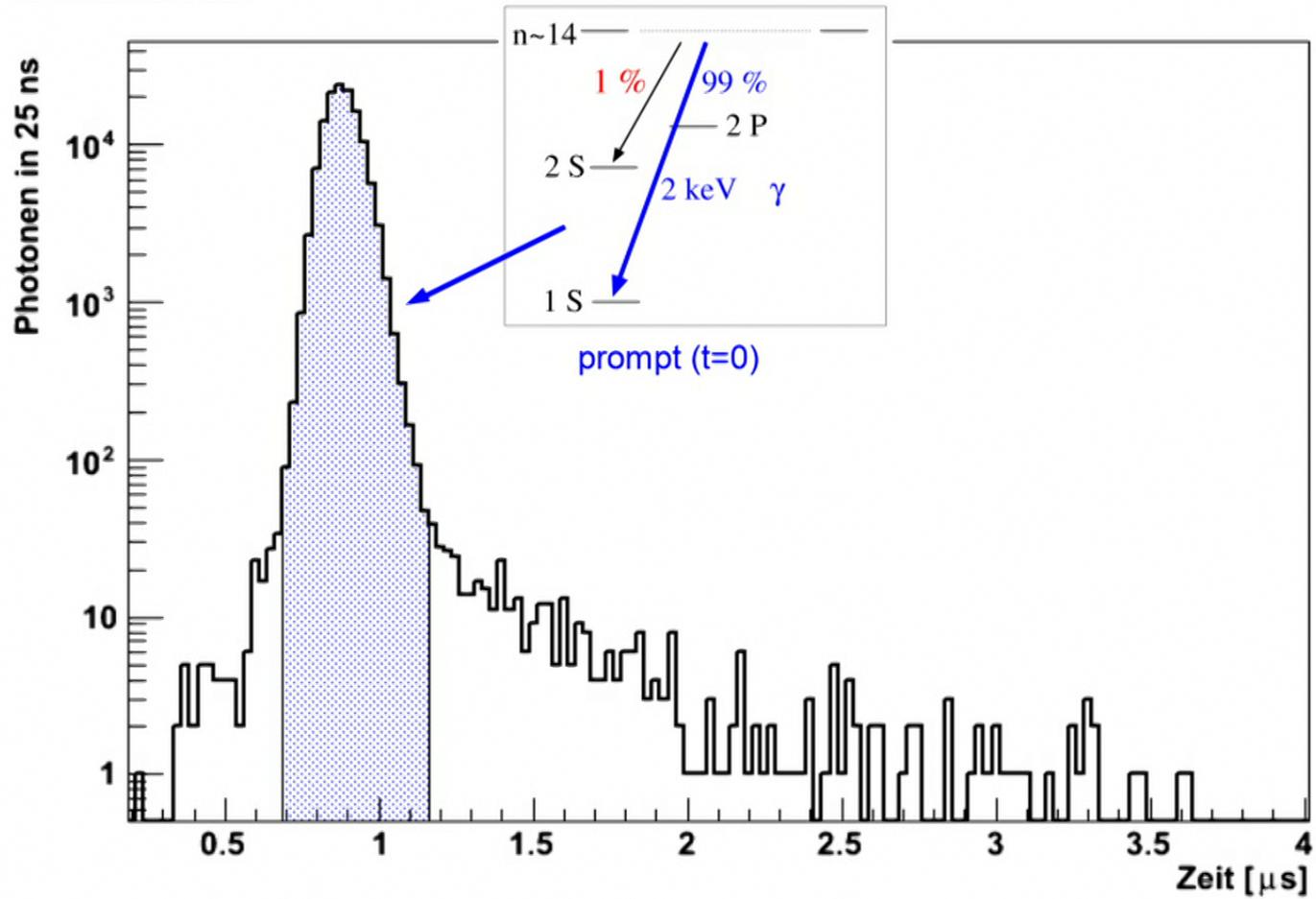
# Time Spectra

13 hours of data

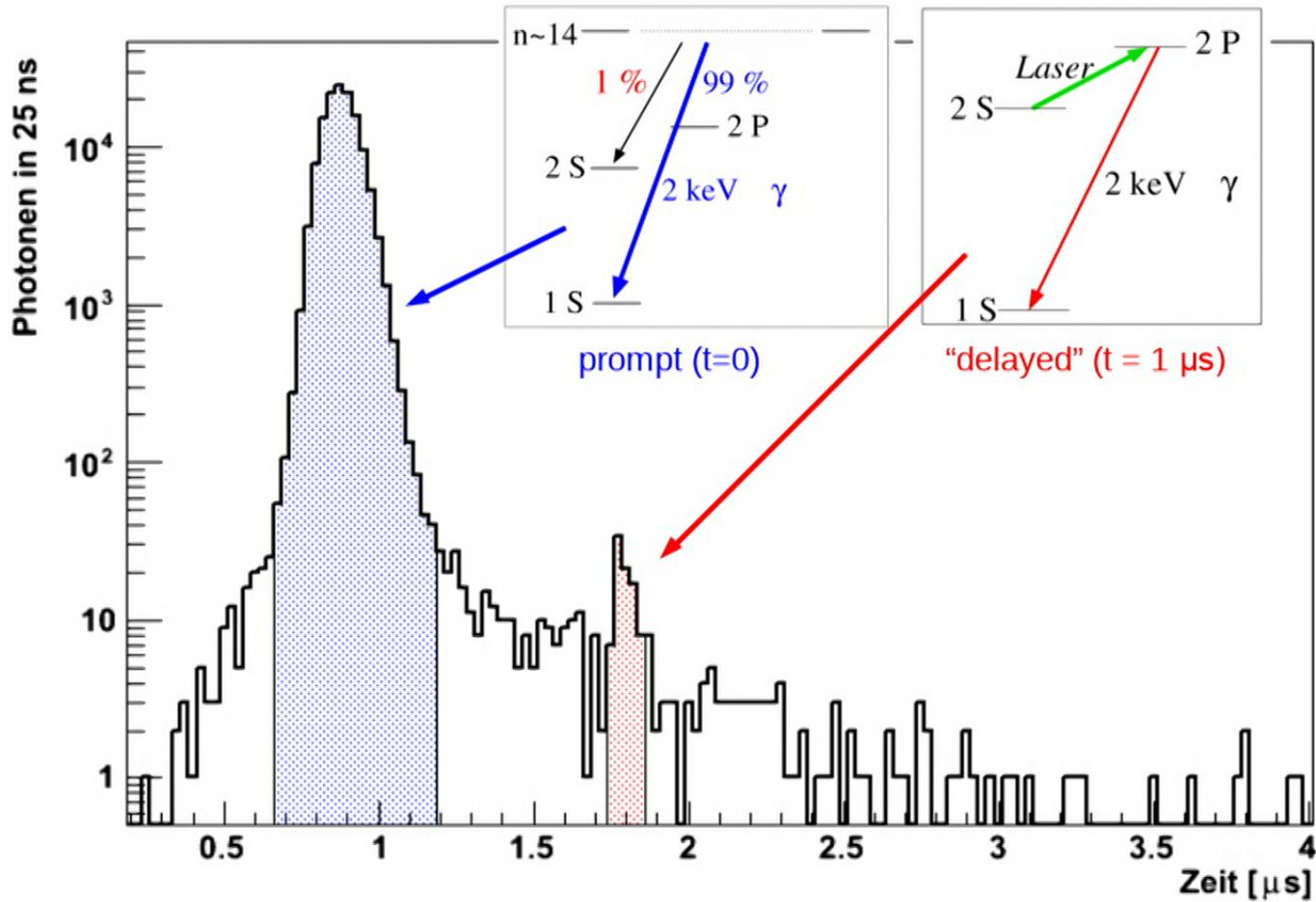


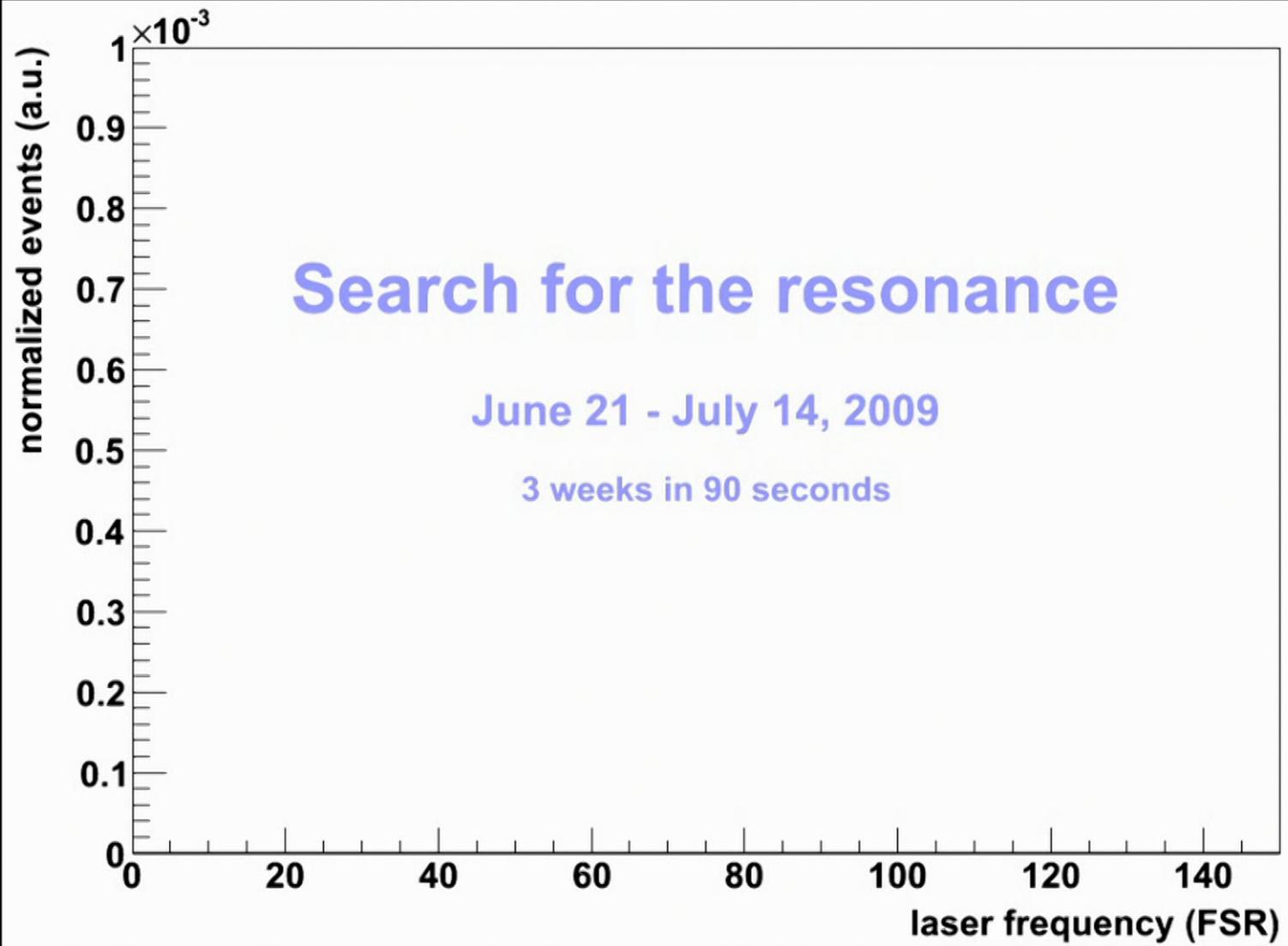
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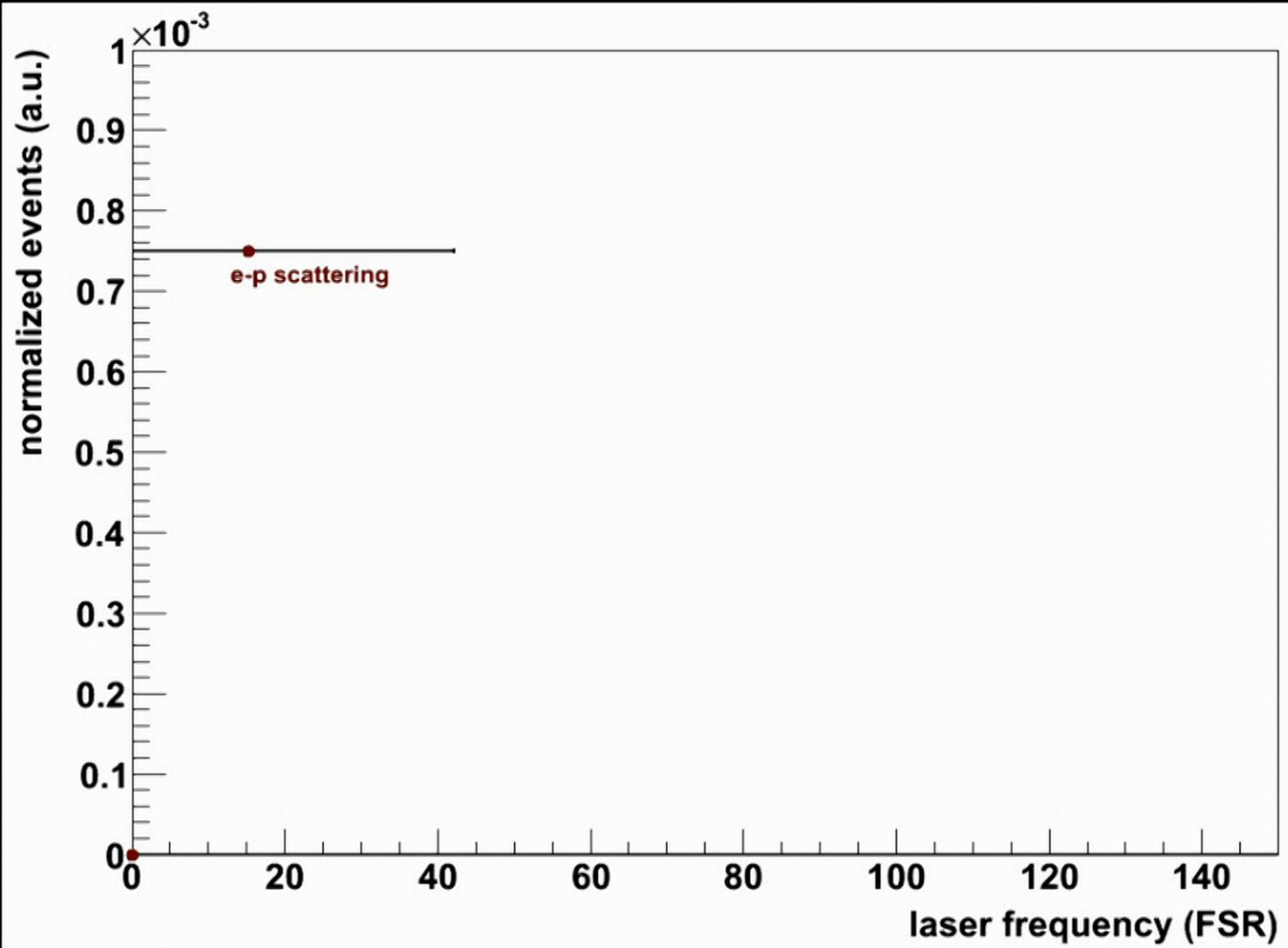
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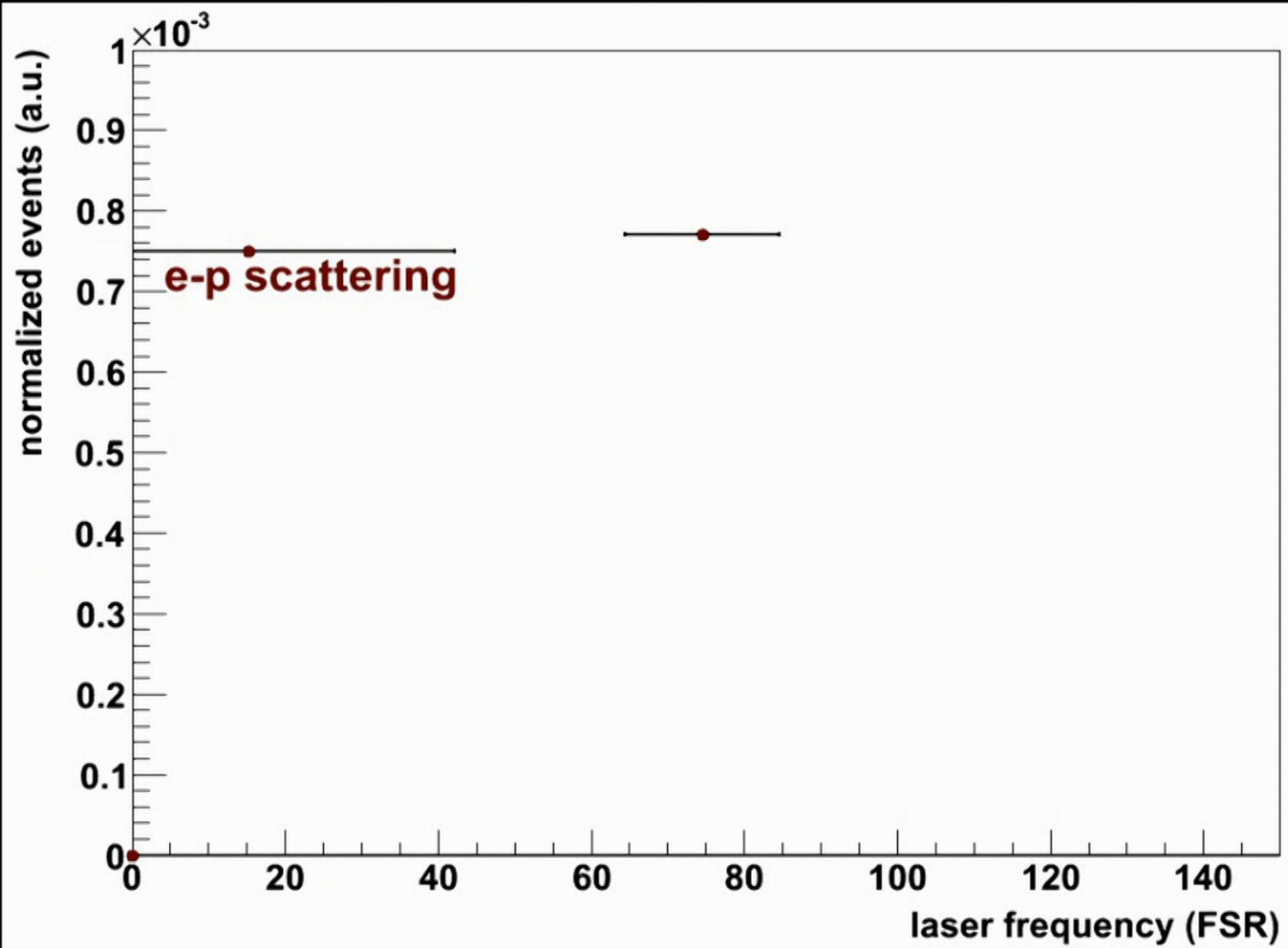


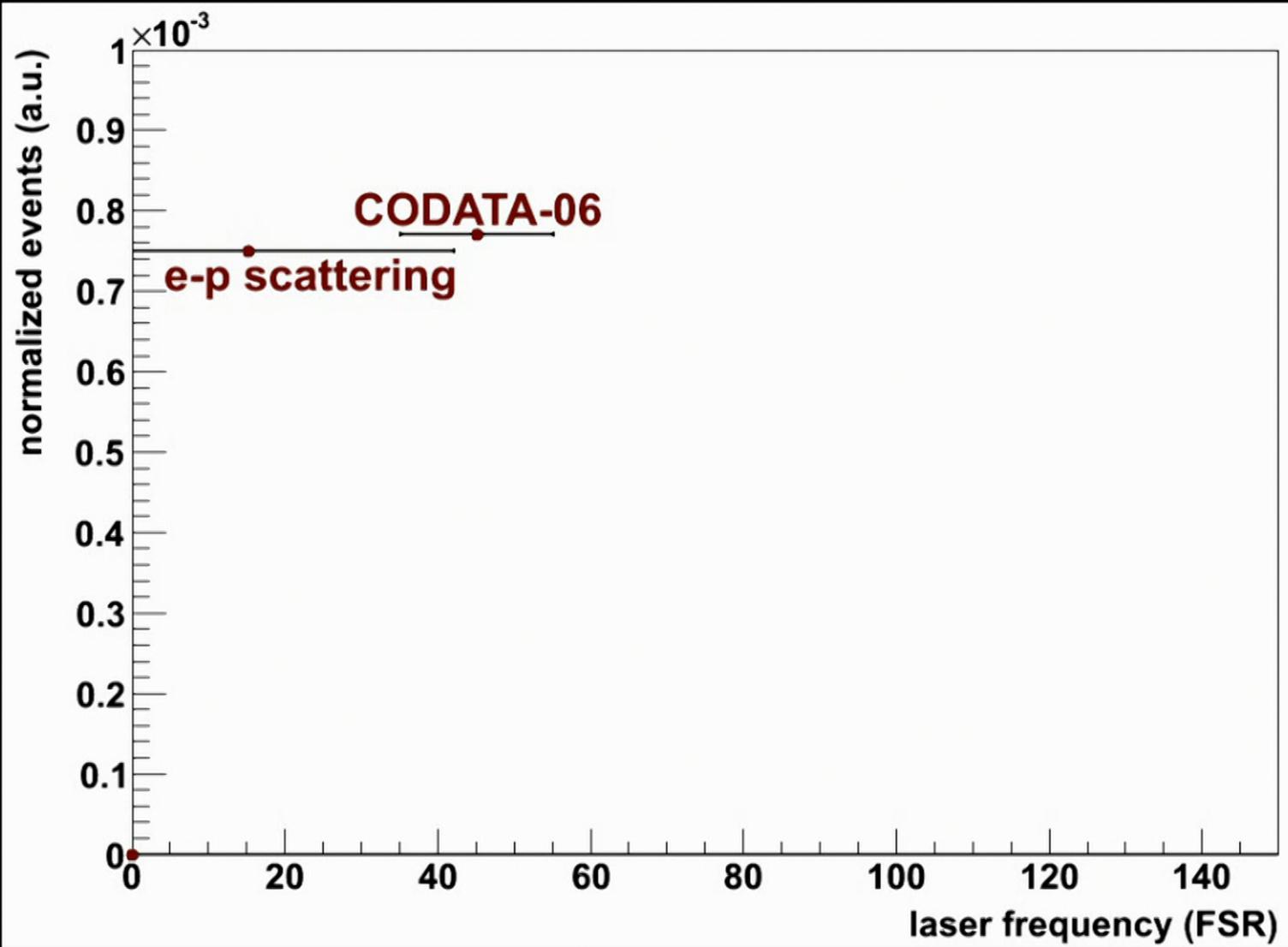
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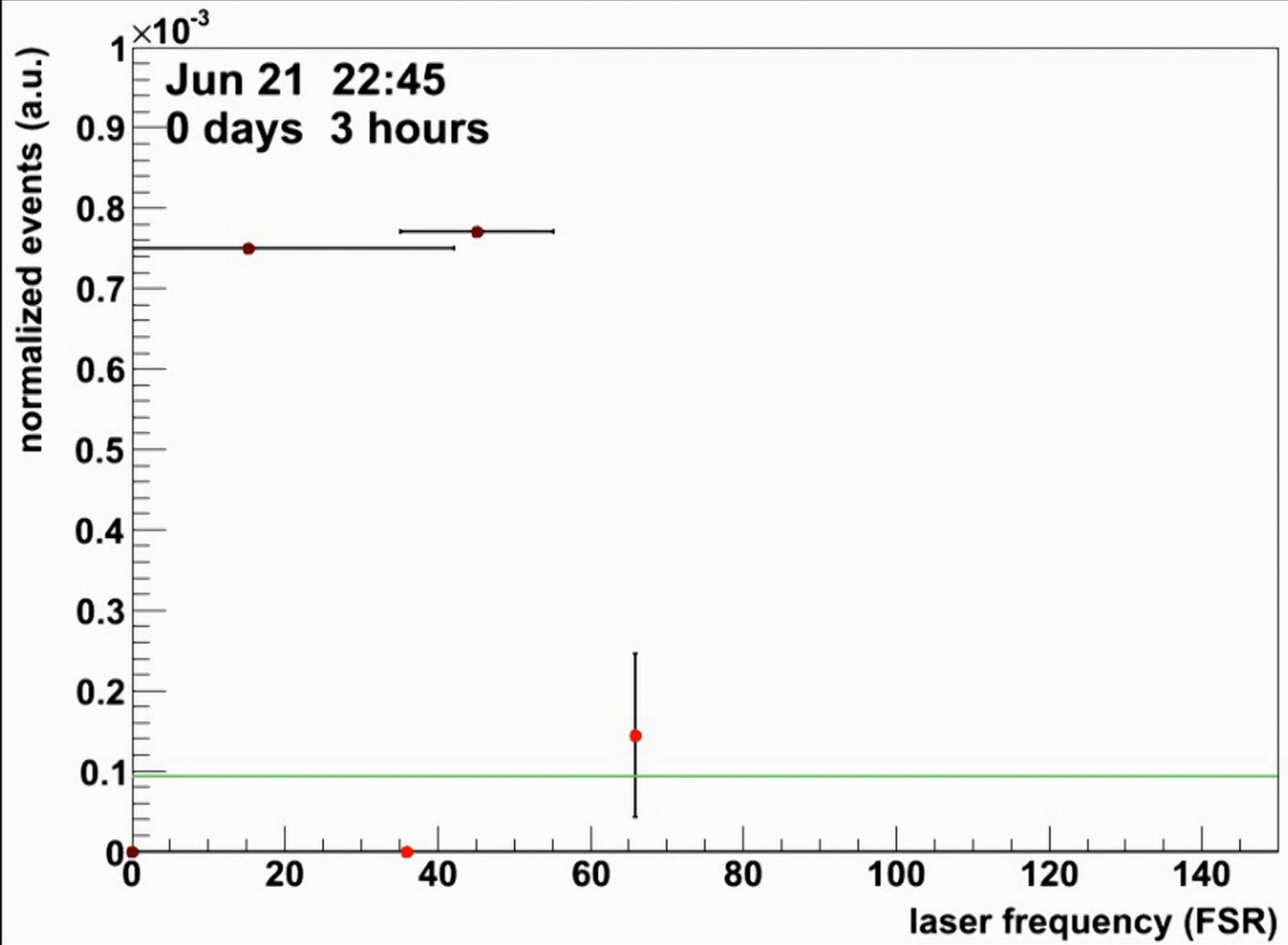


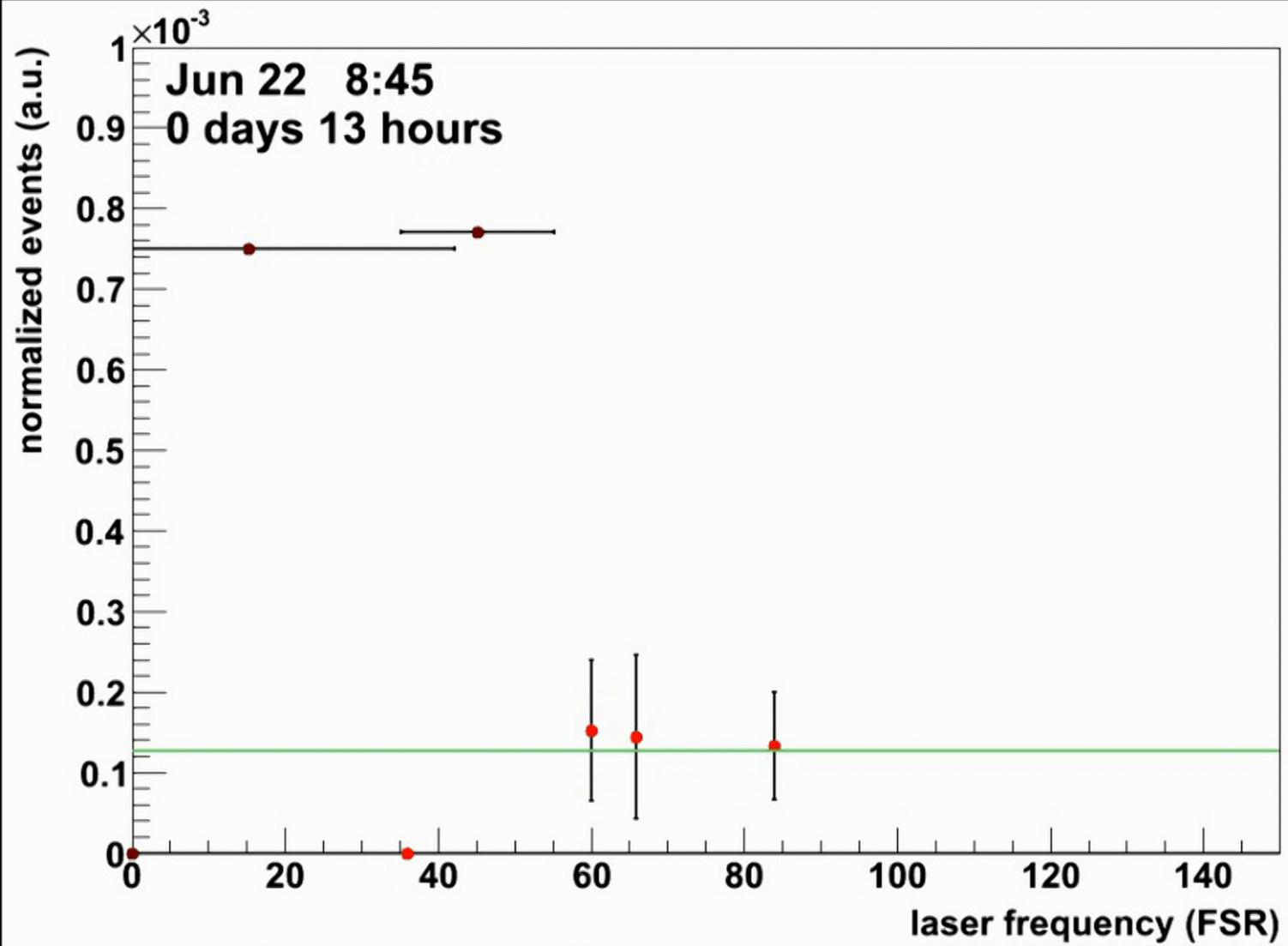


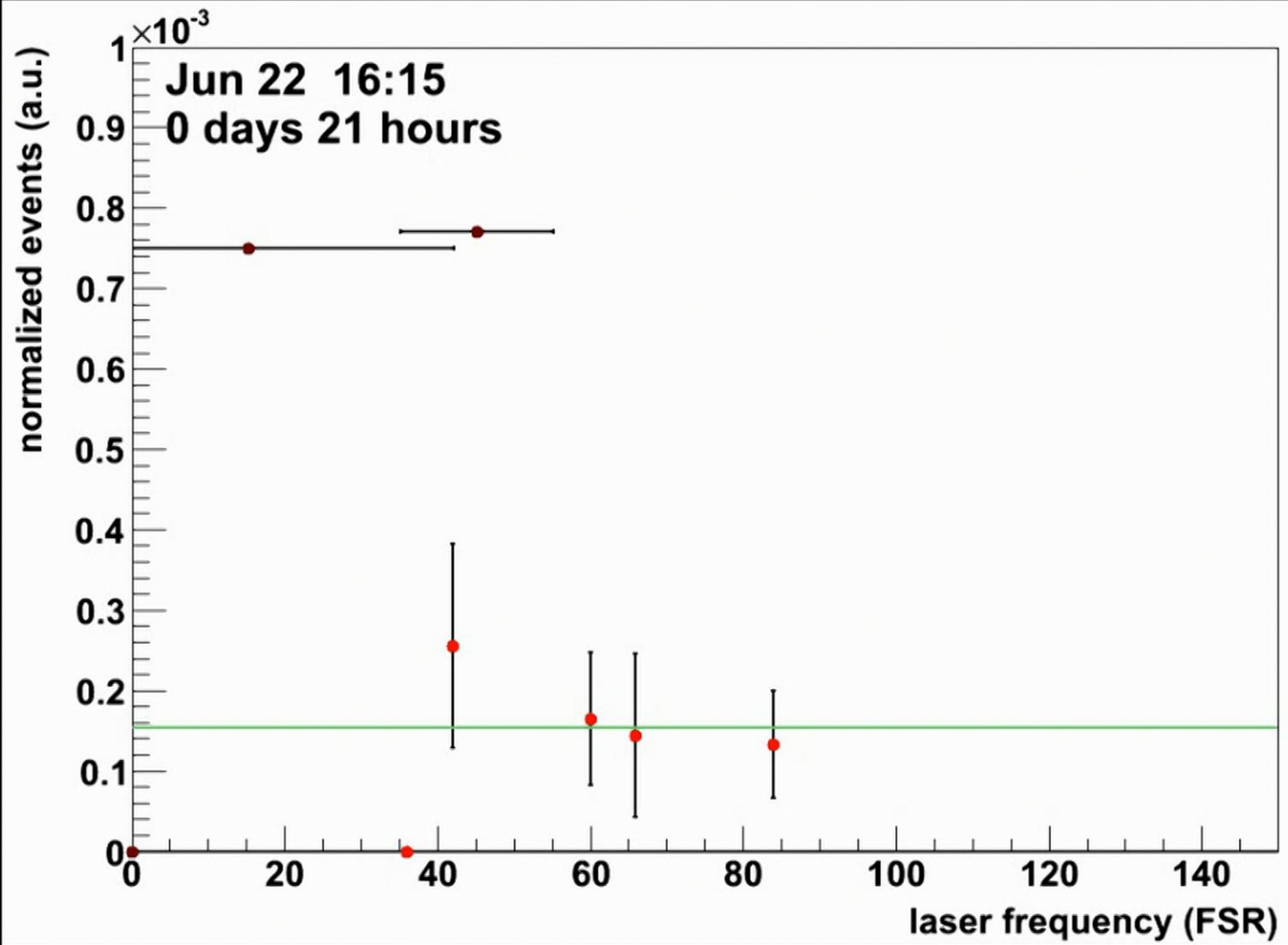


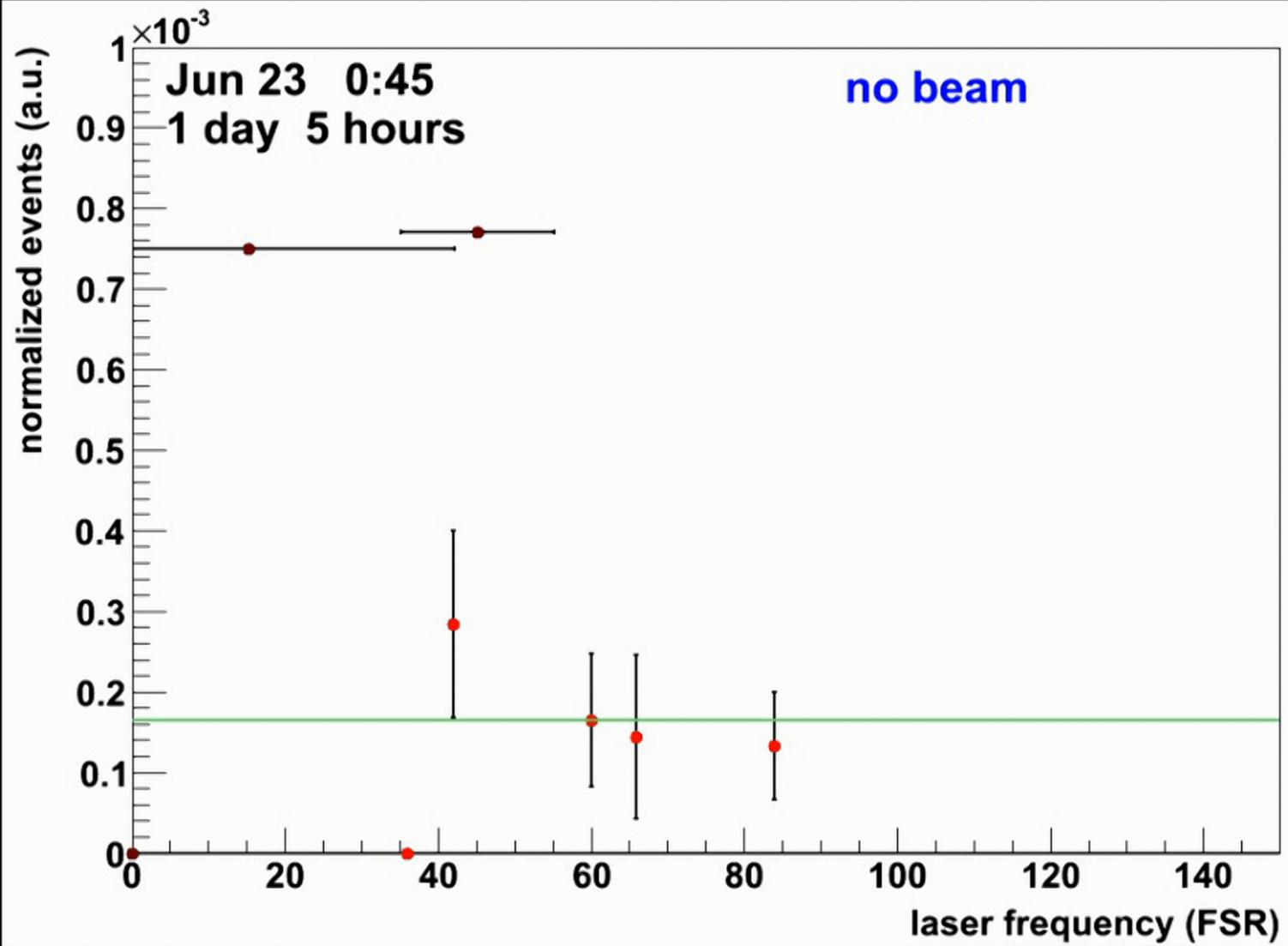


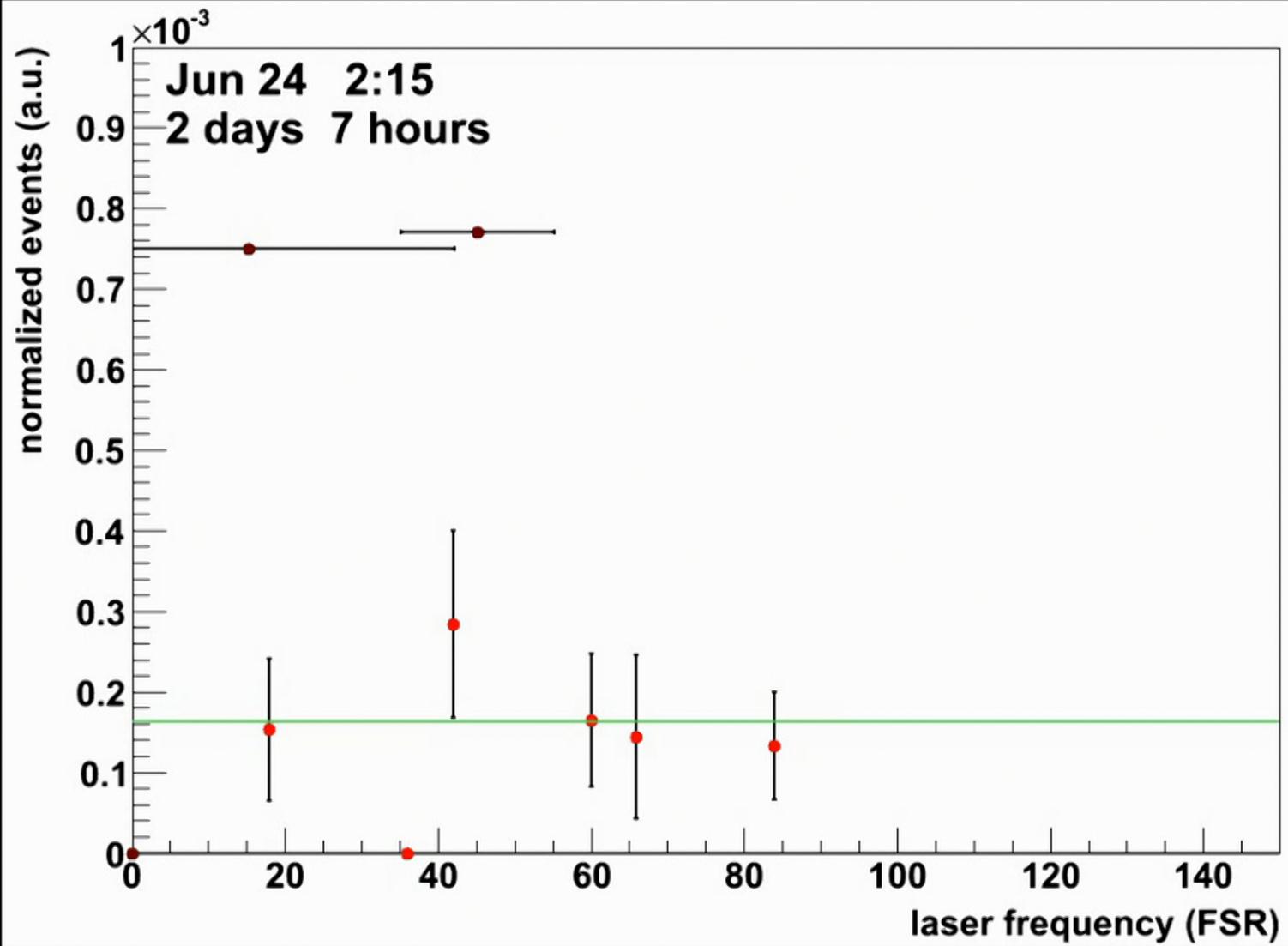


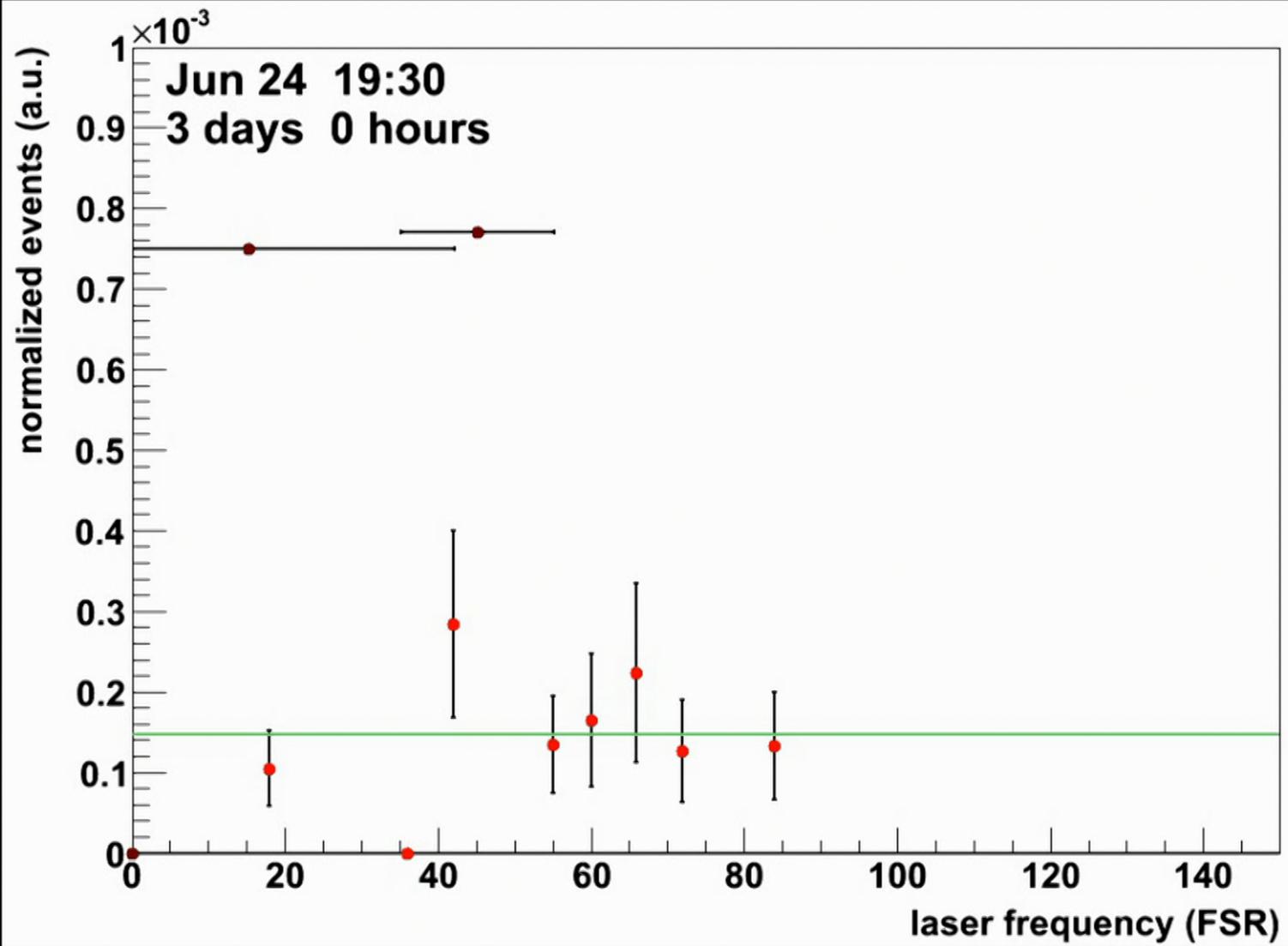


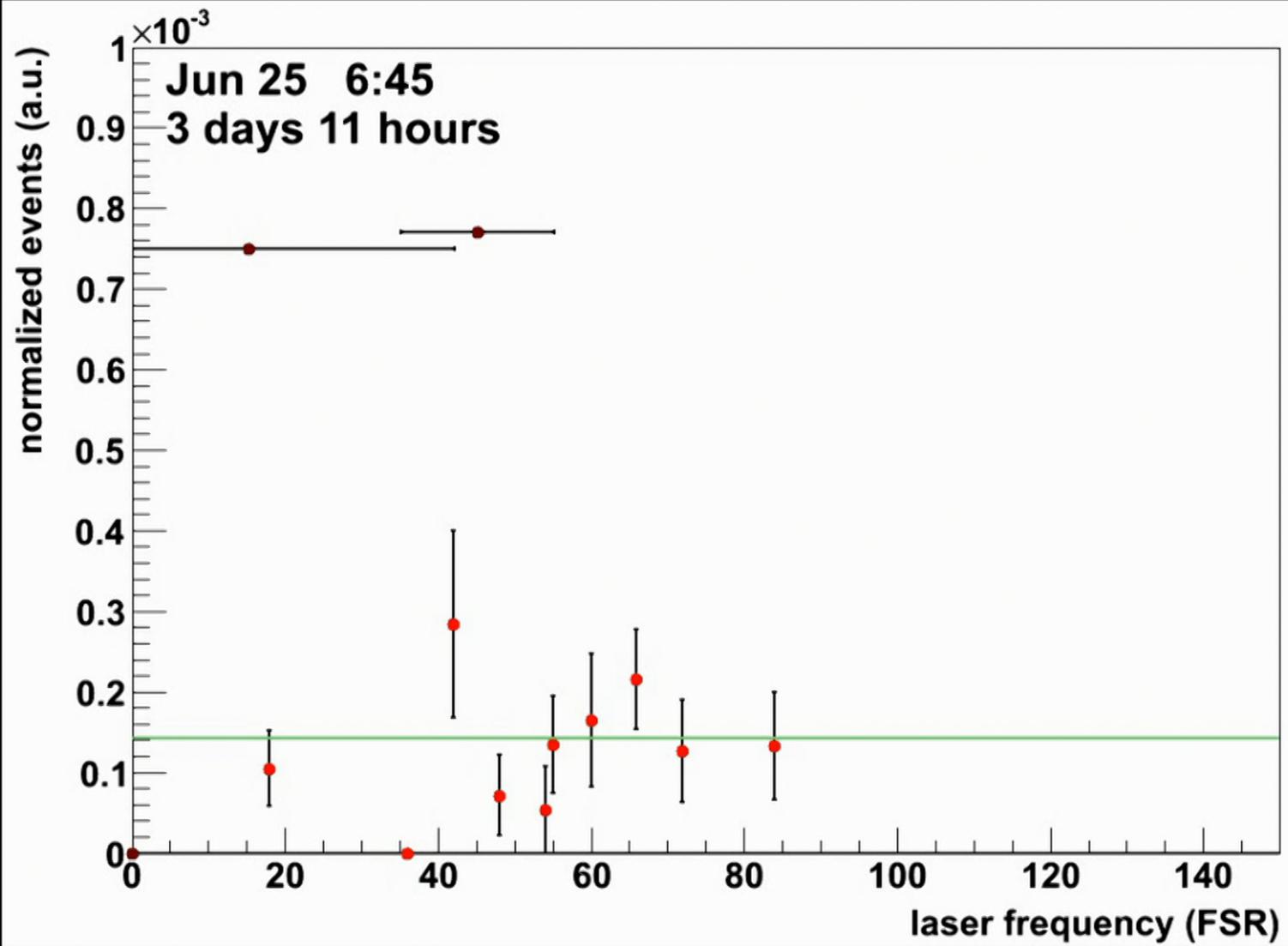


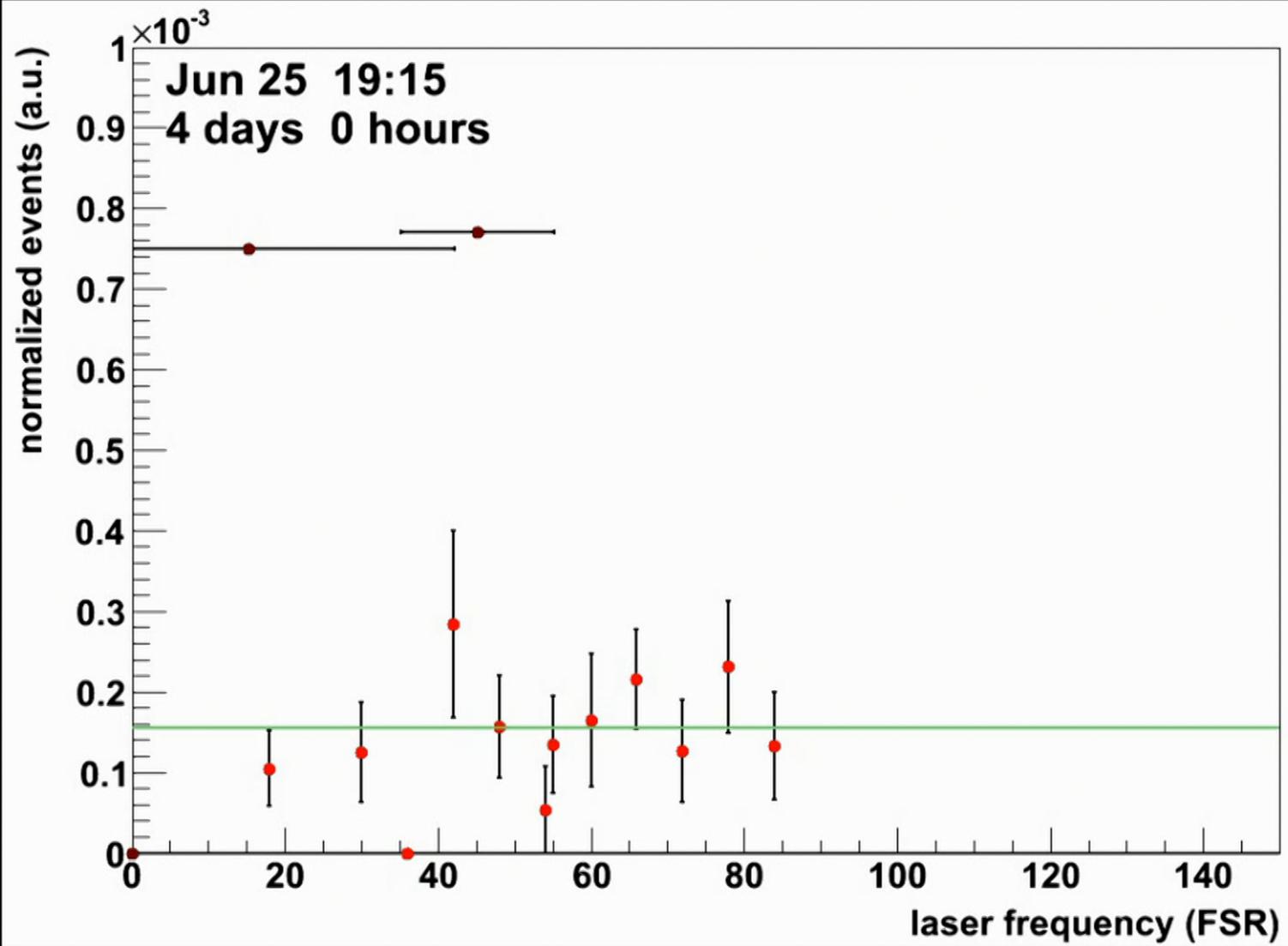


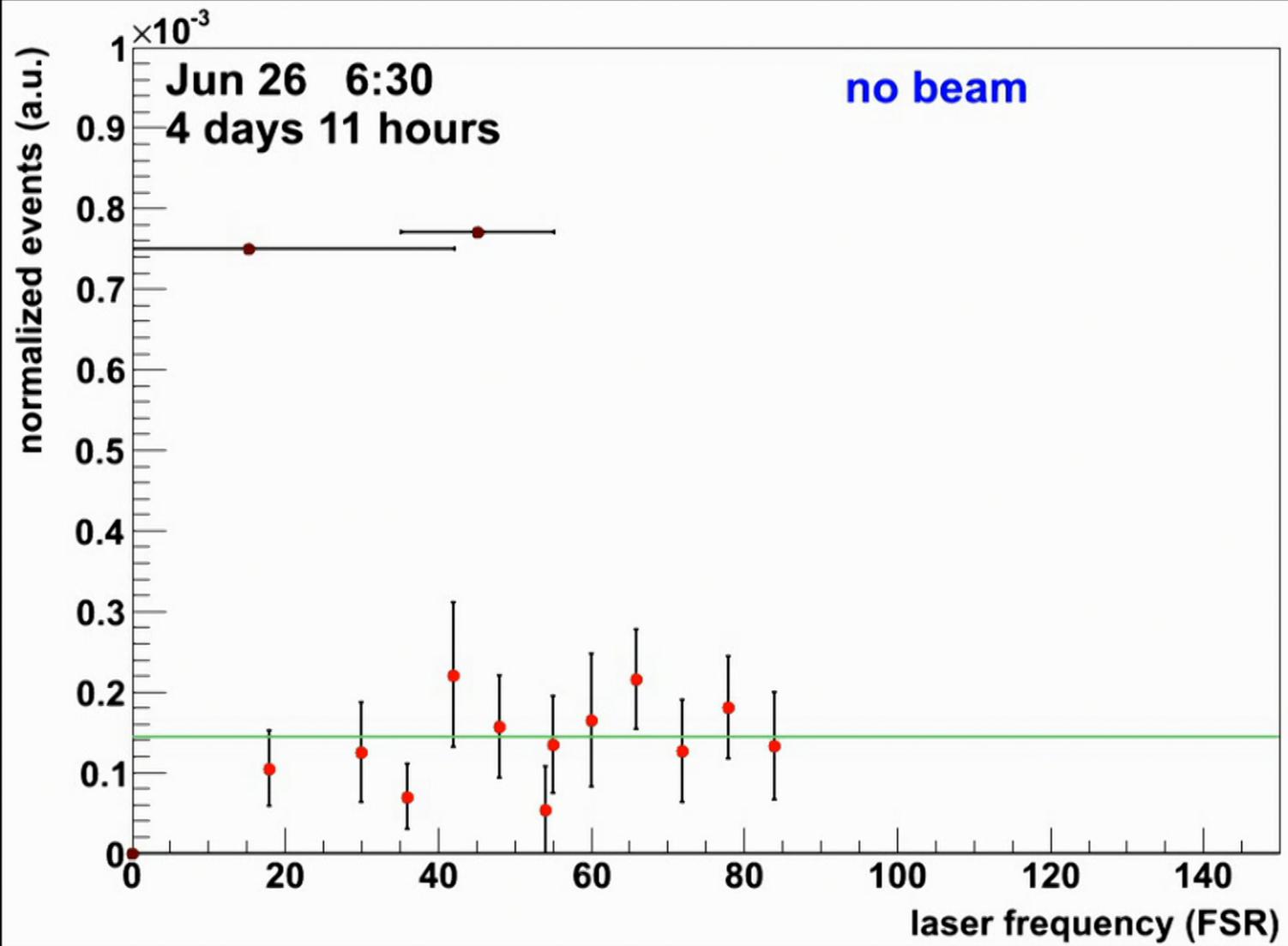


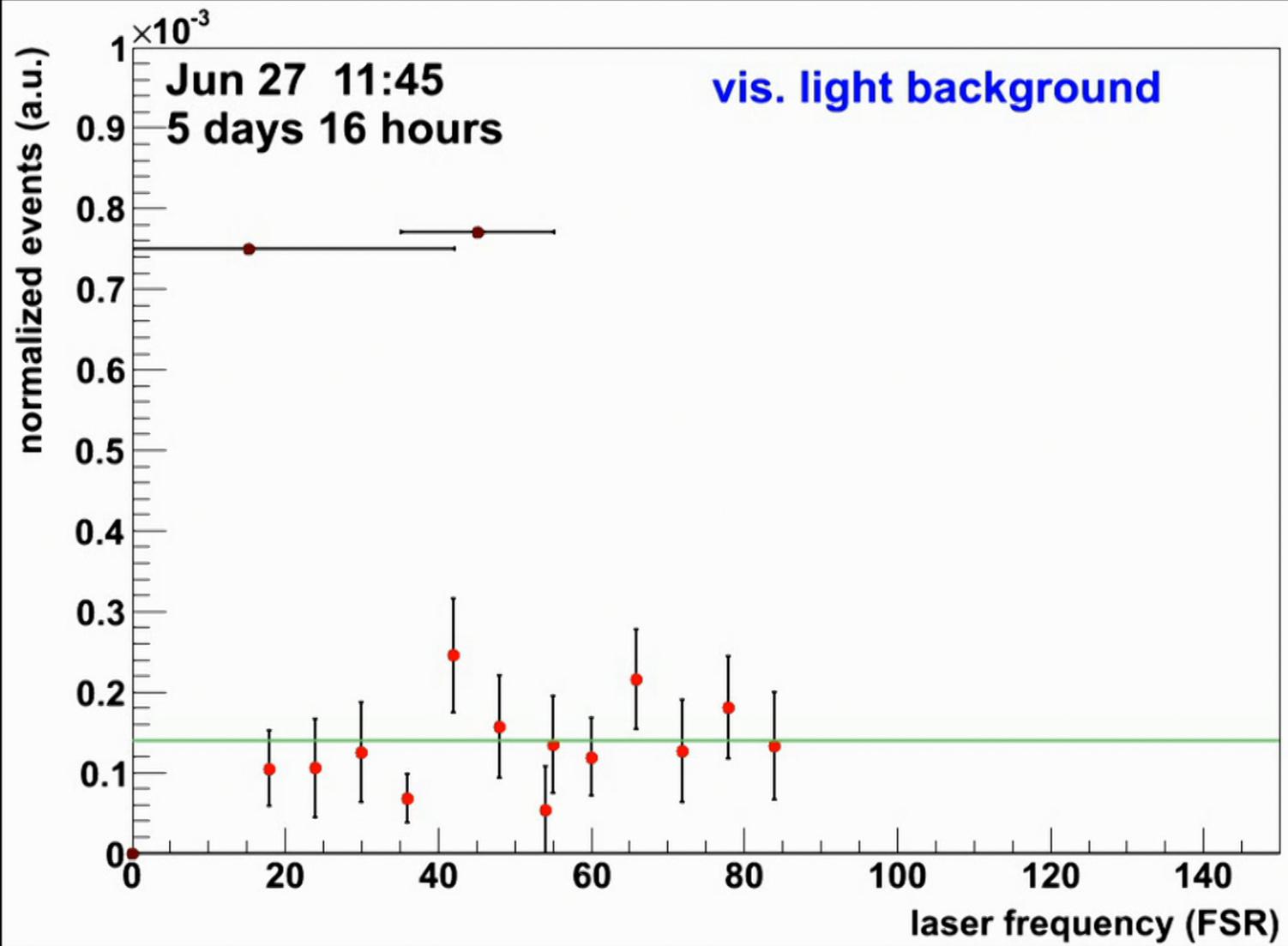


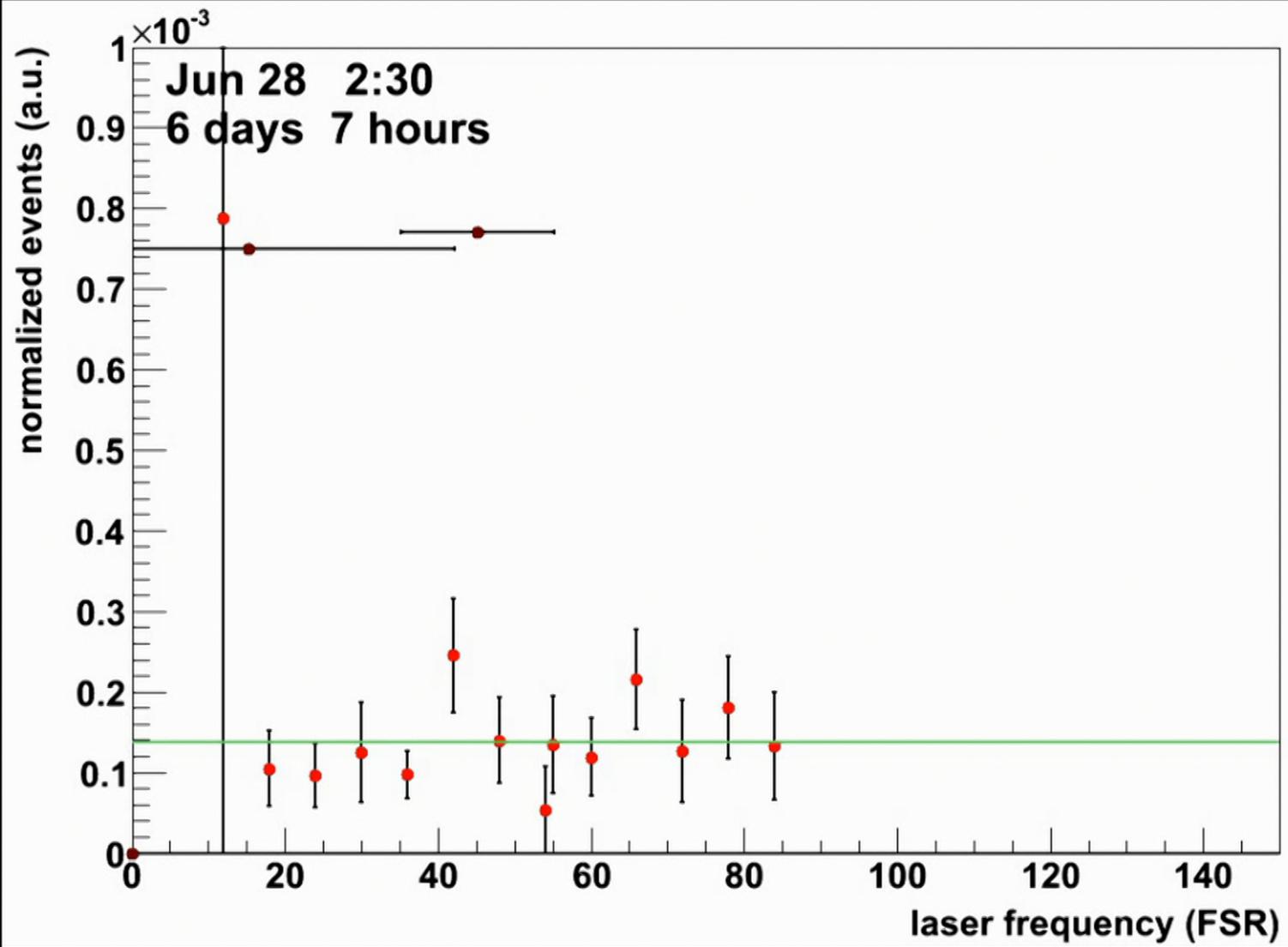


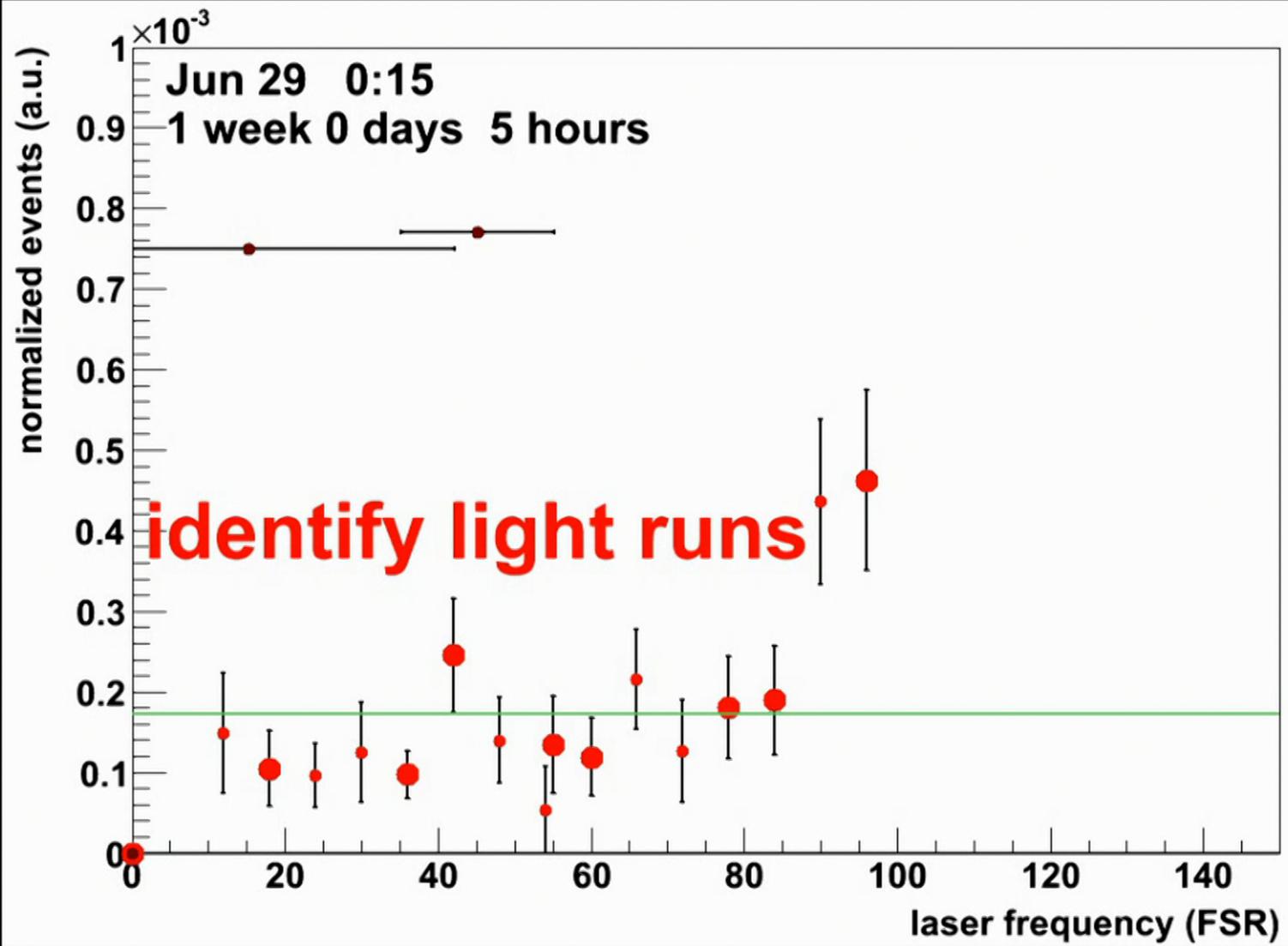


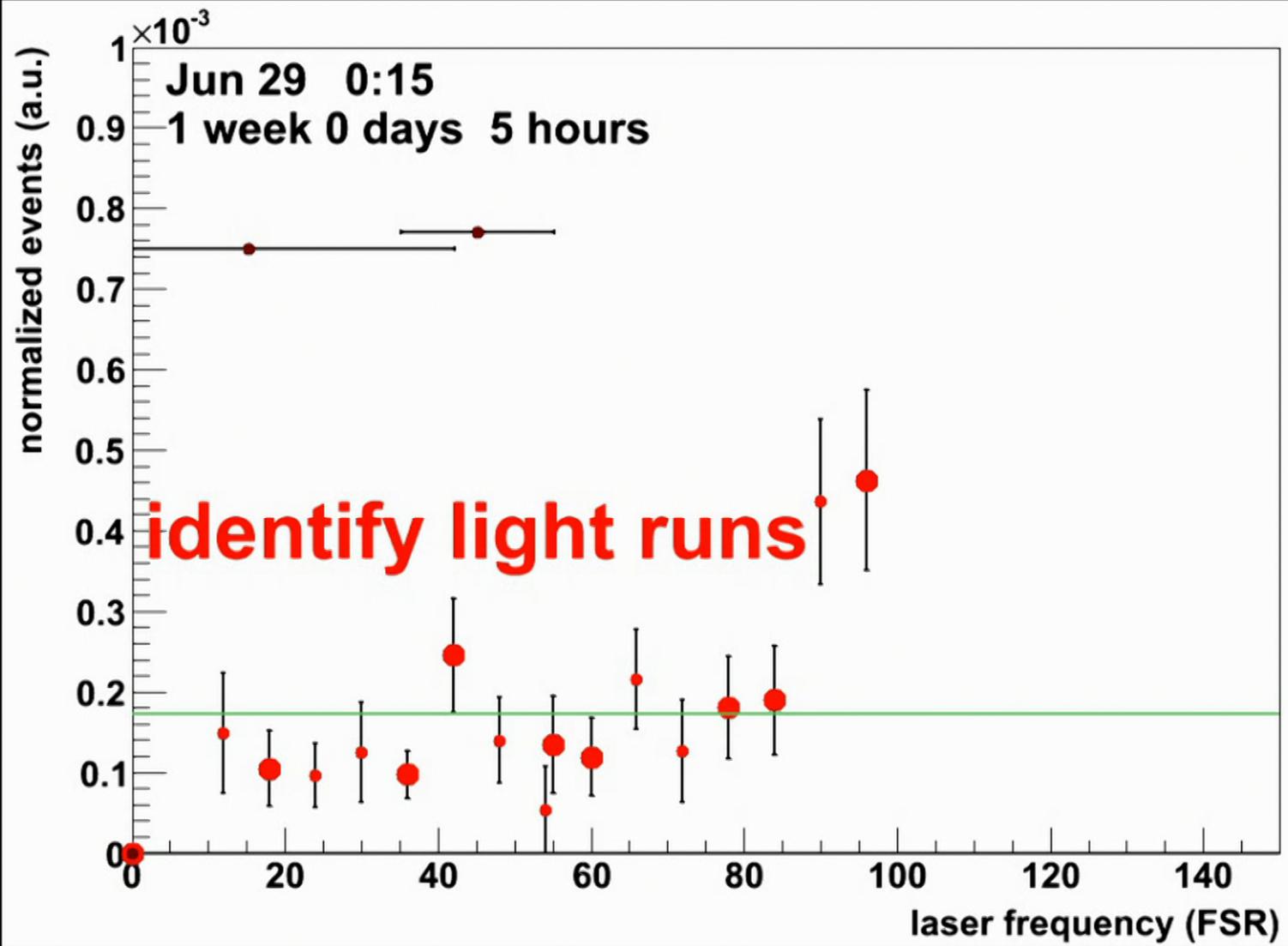


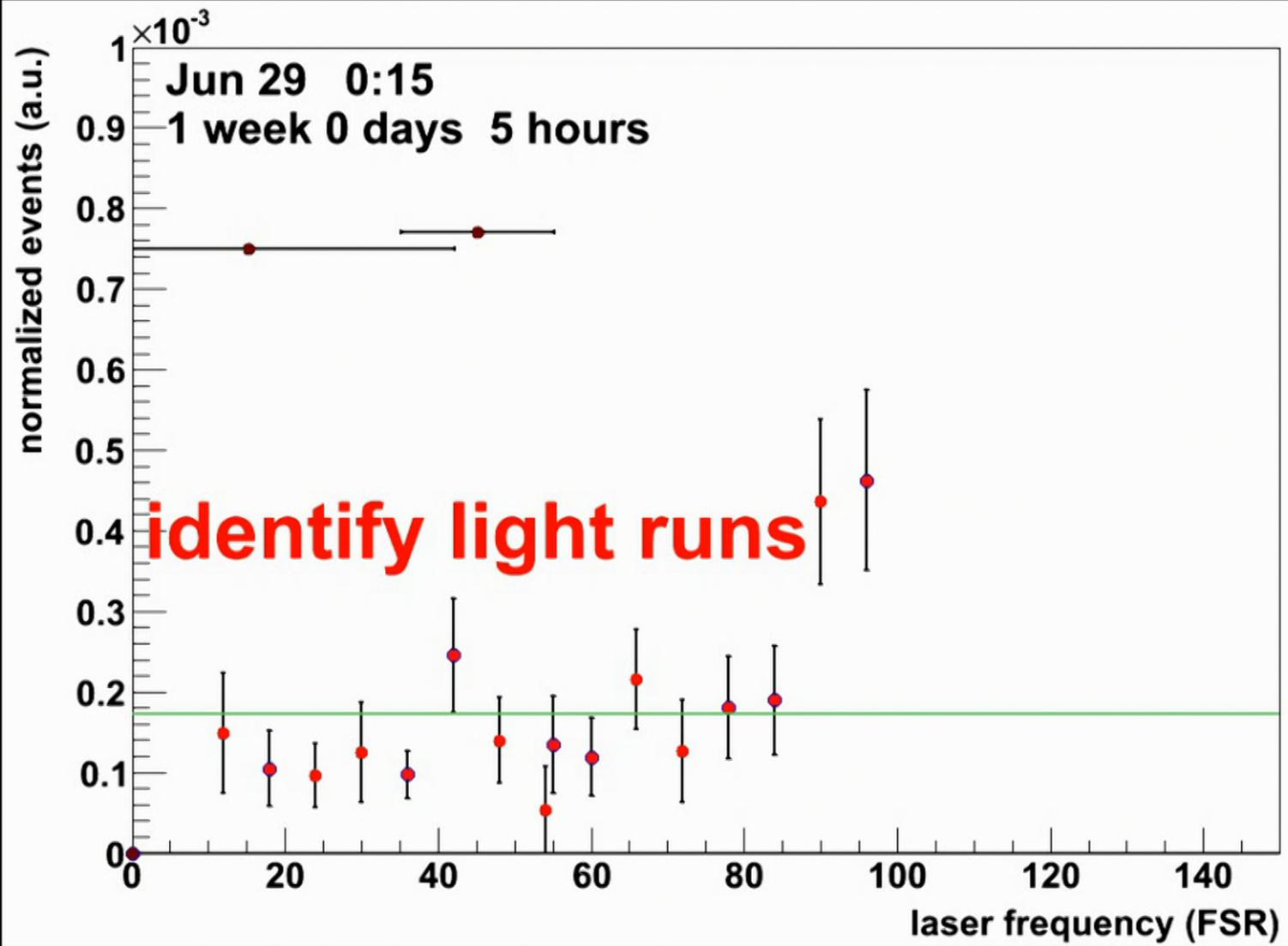


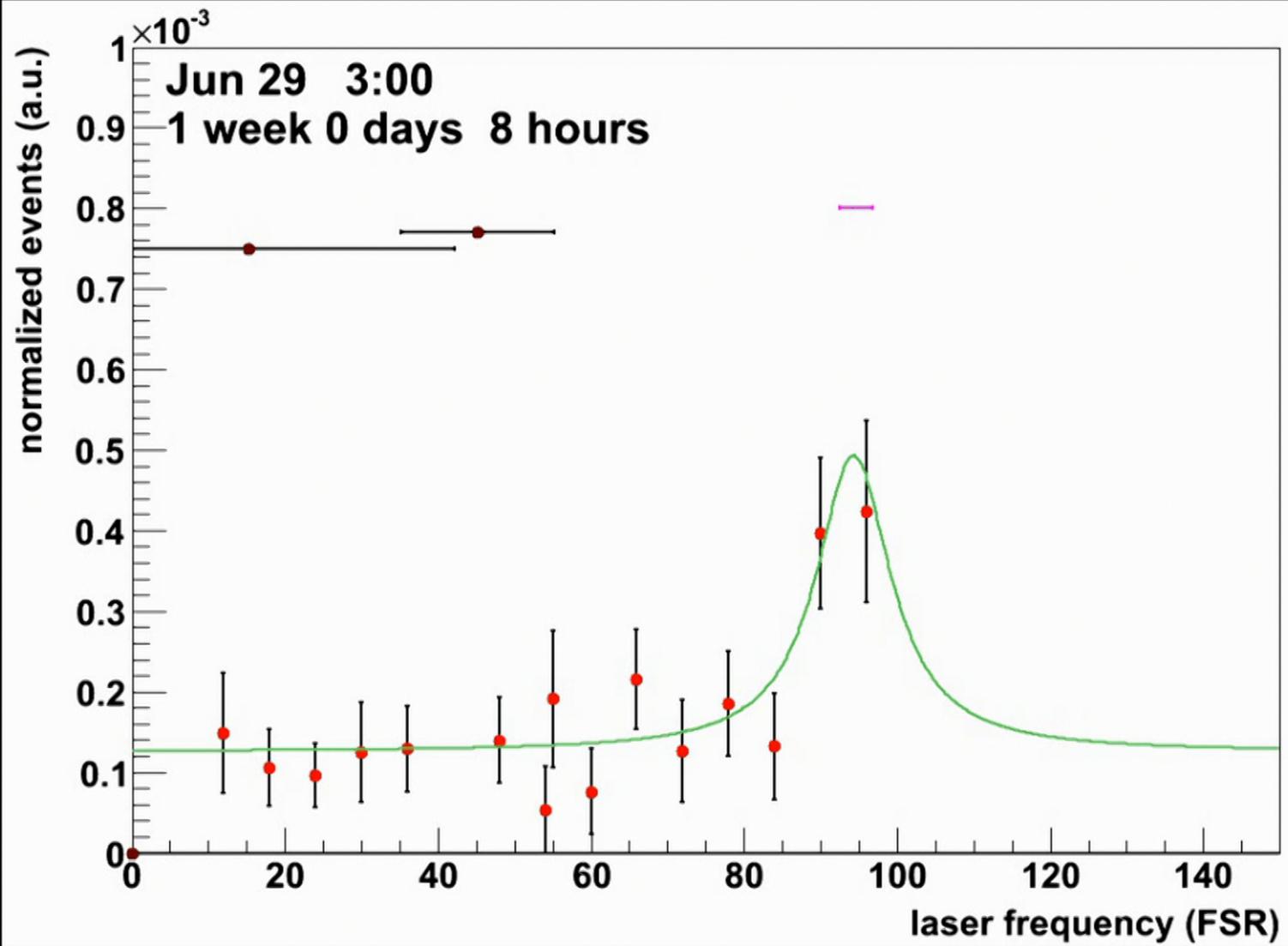


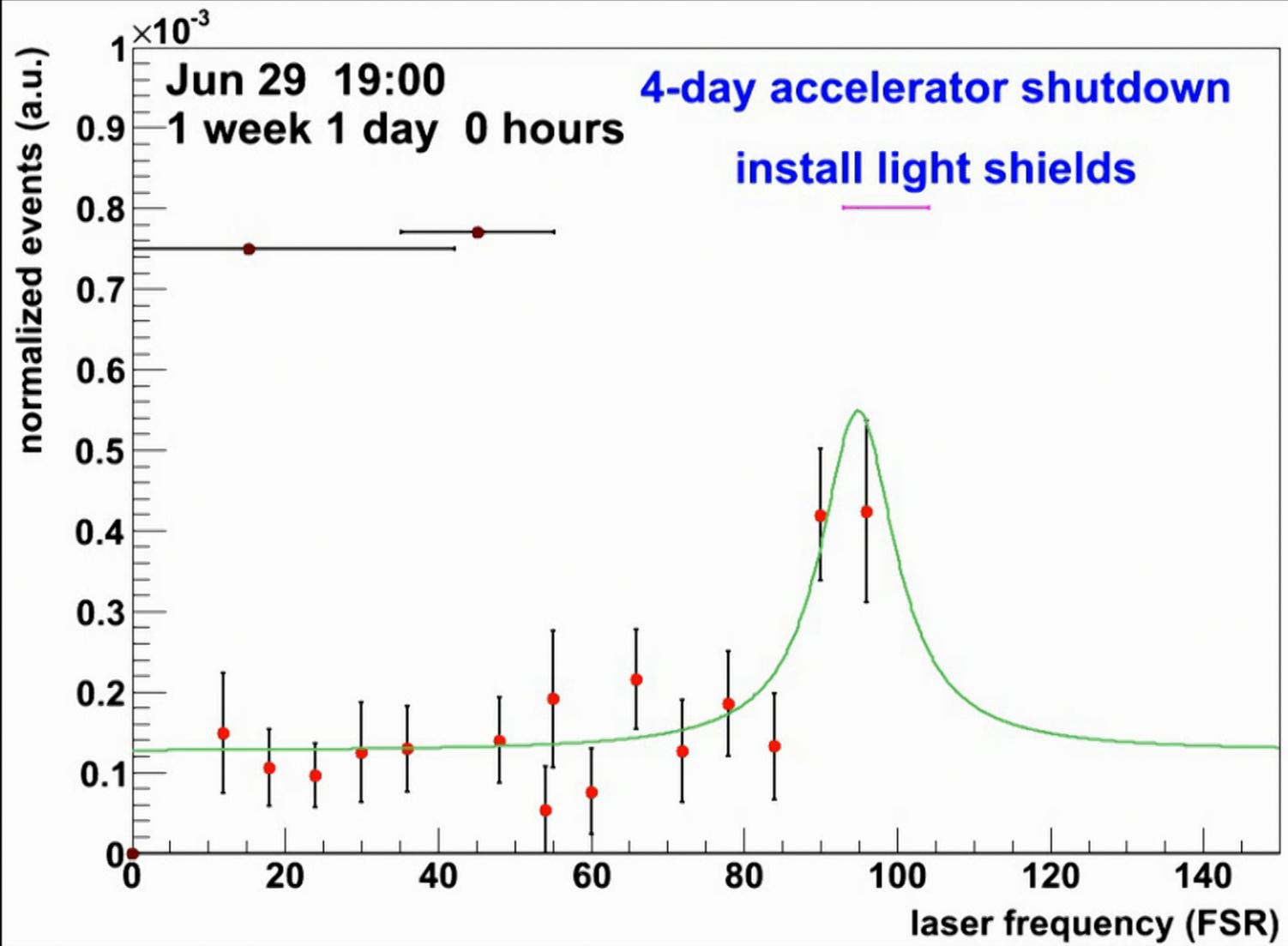


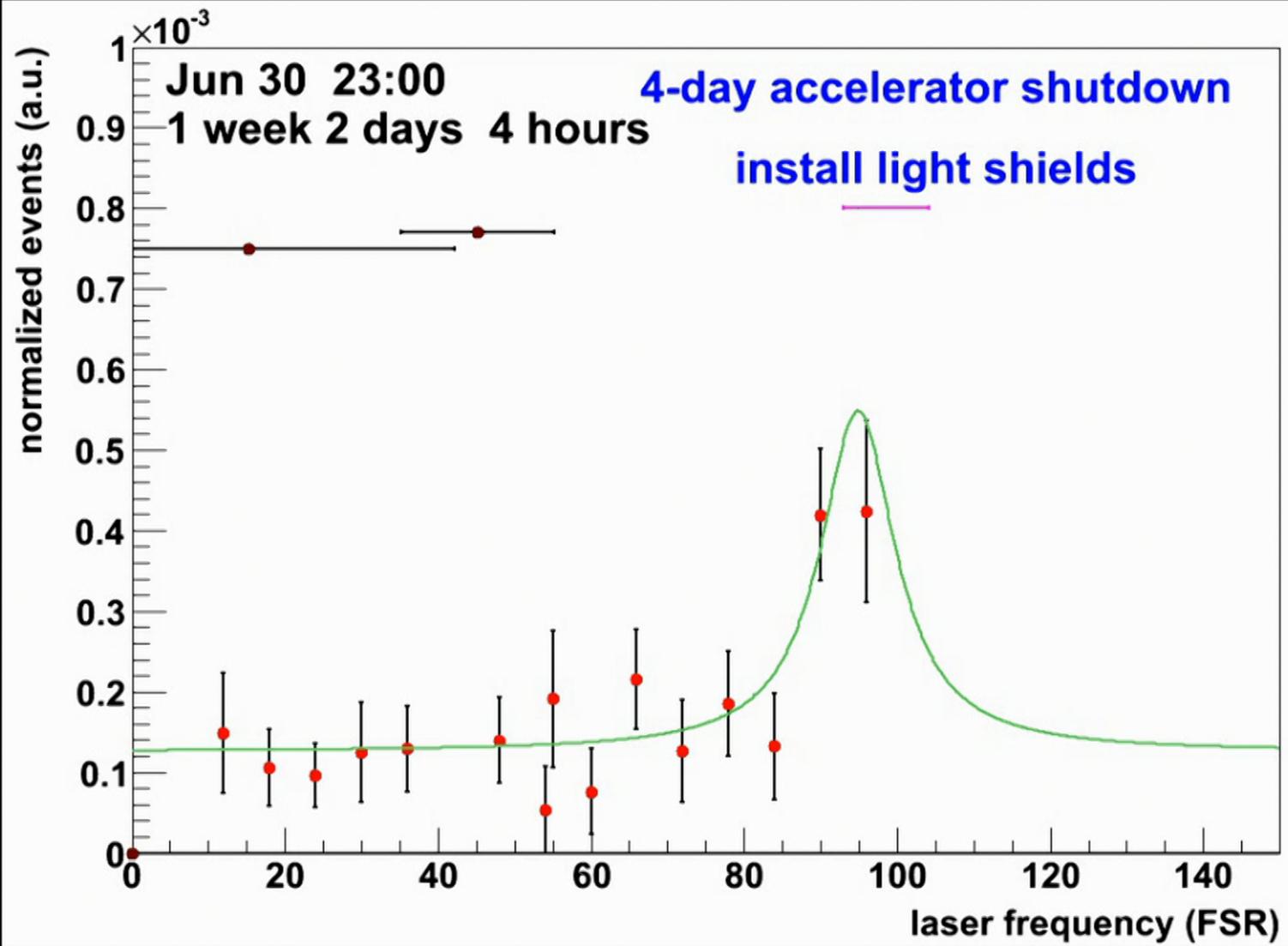


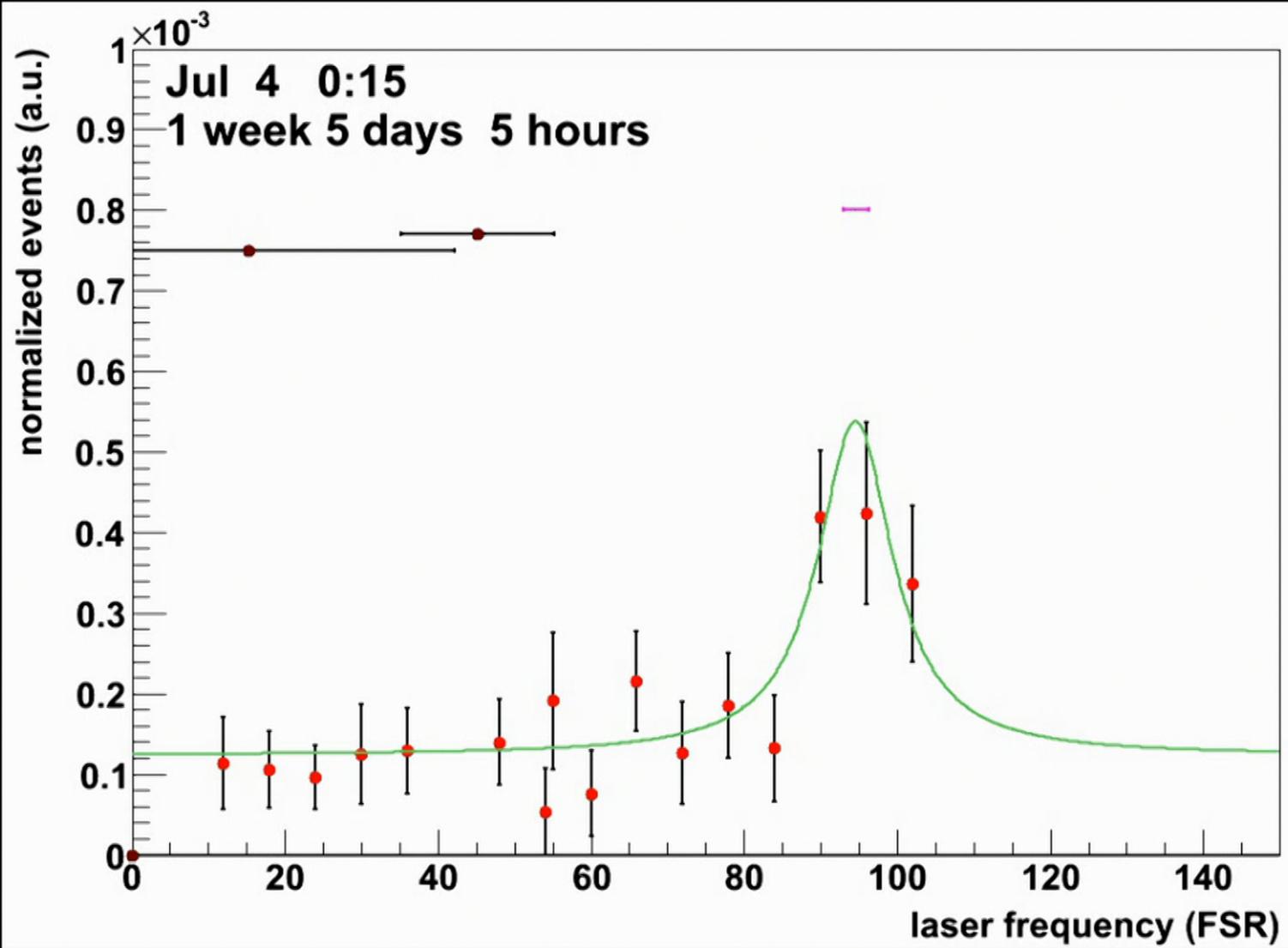


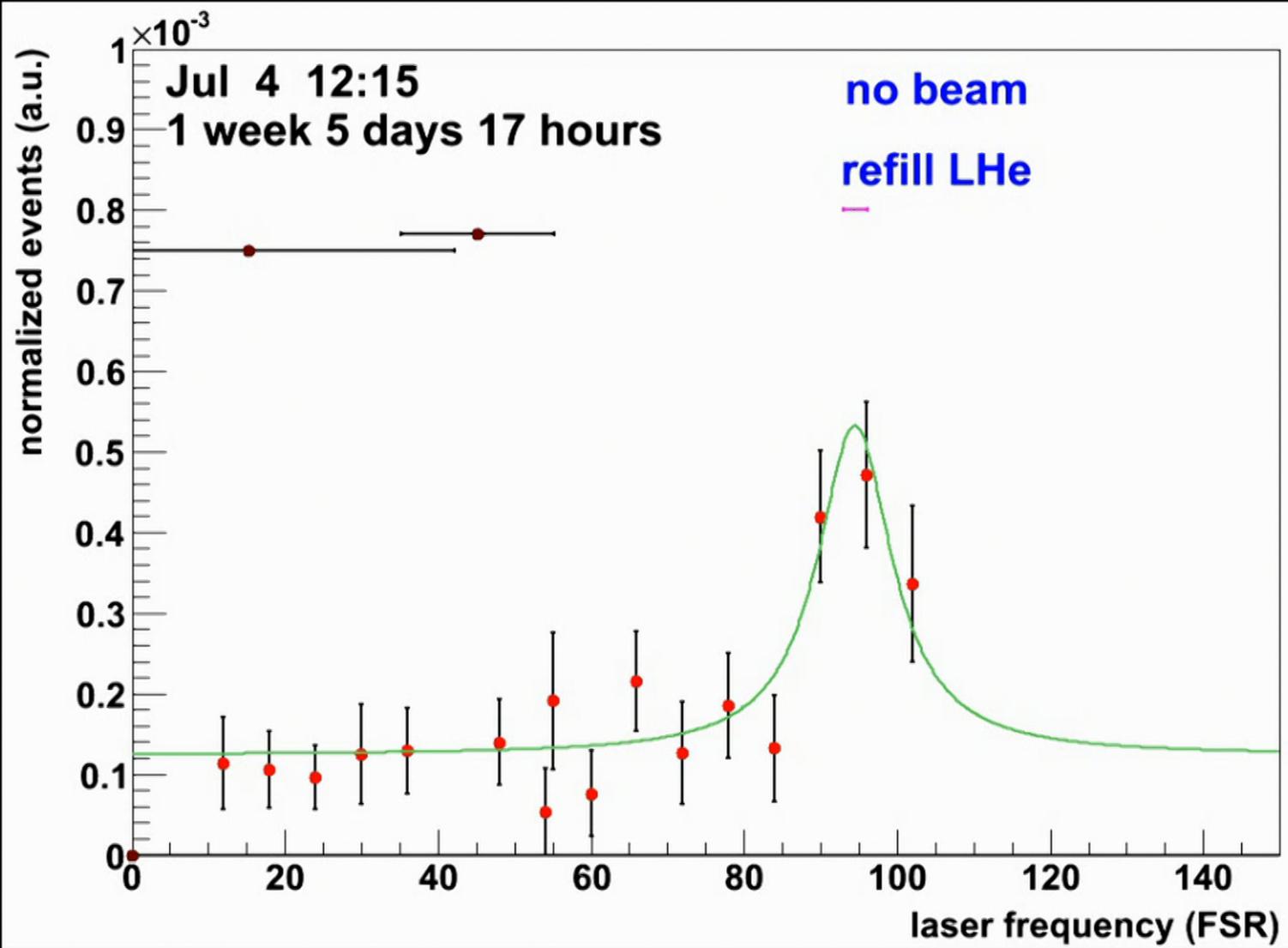


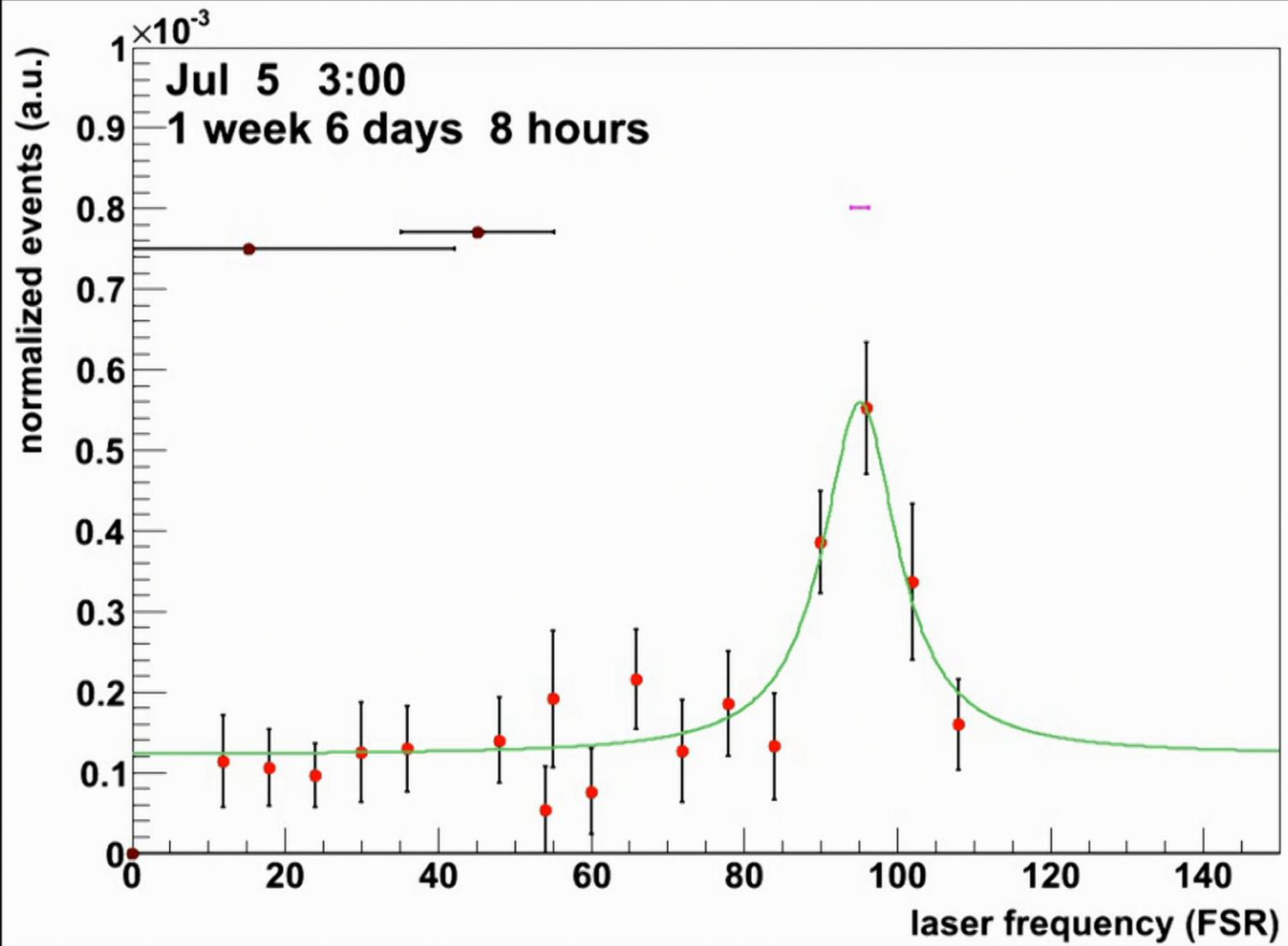


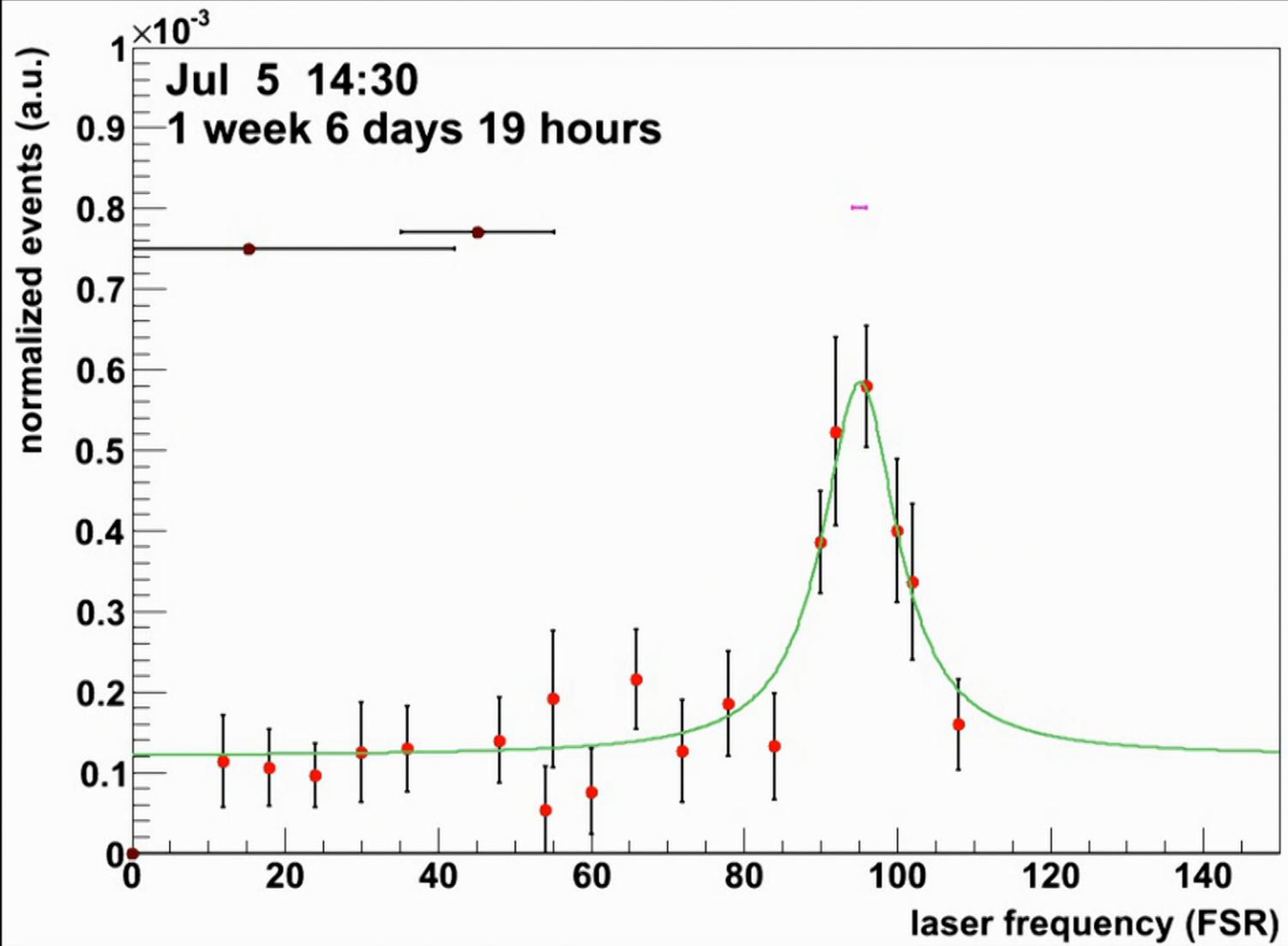


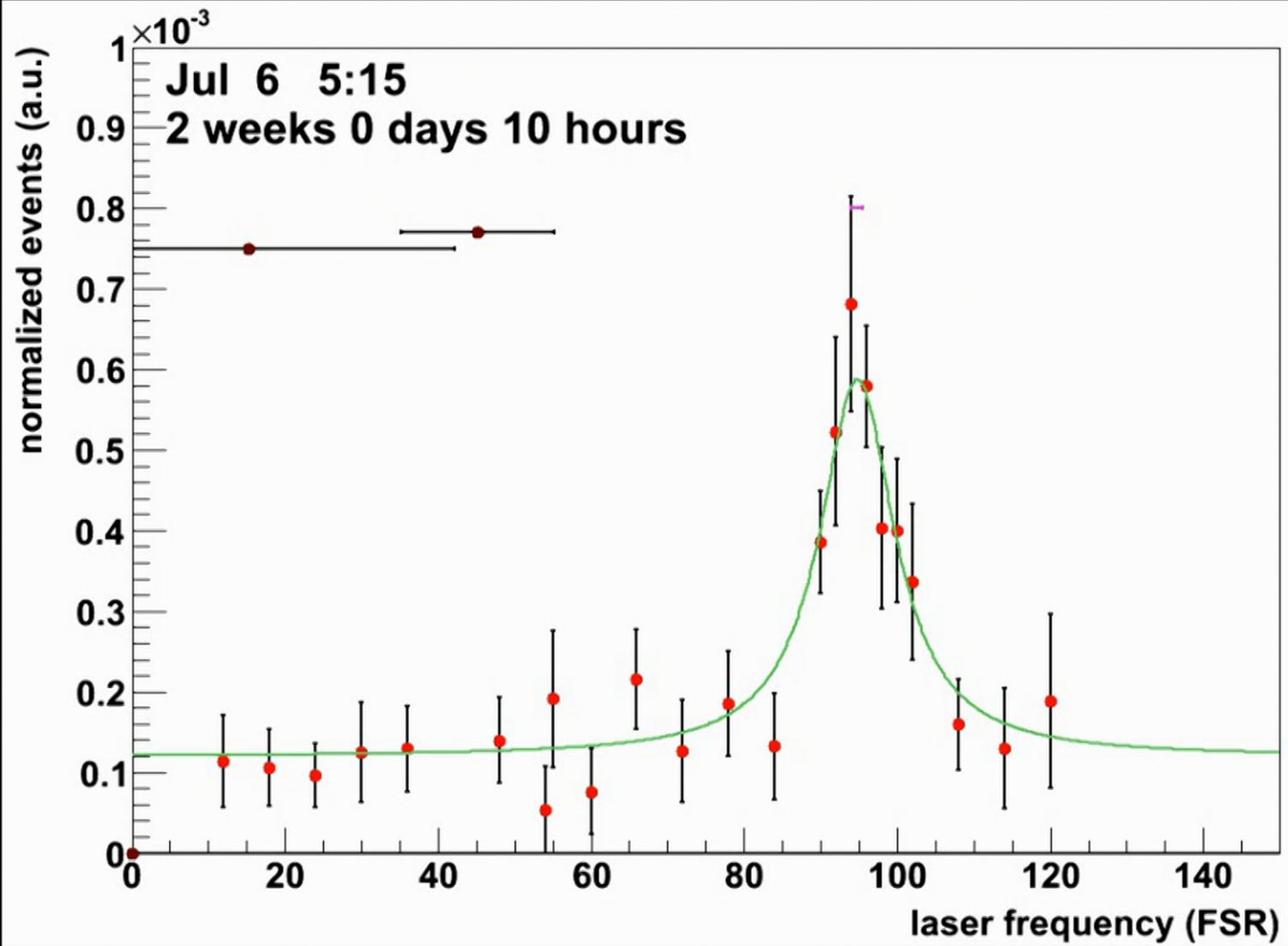


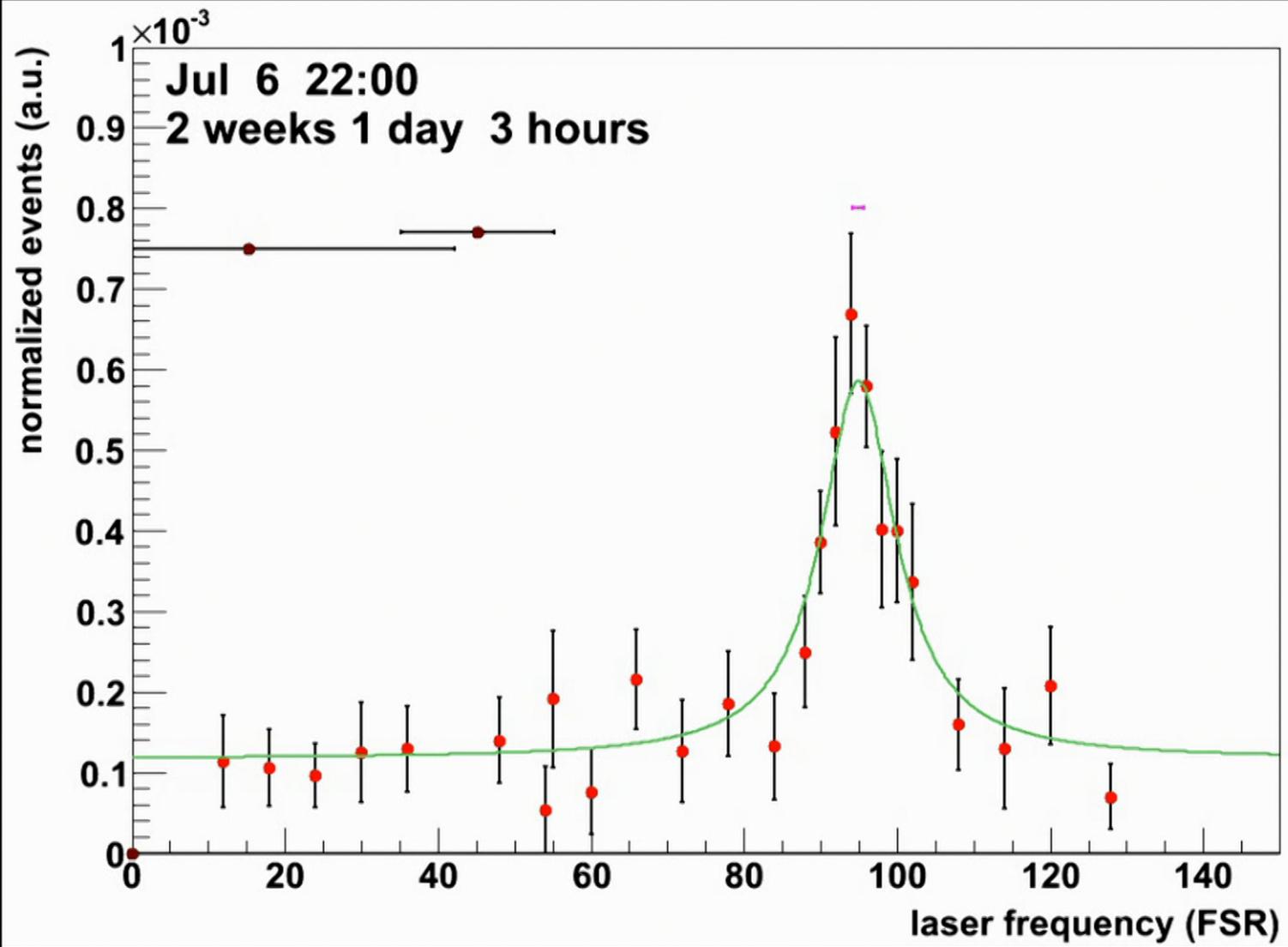


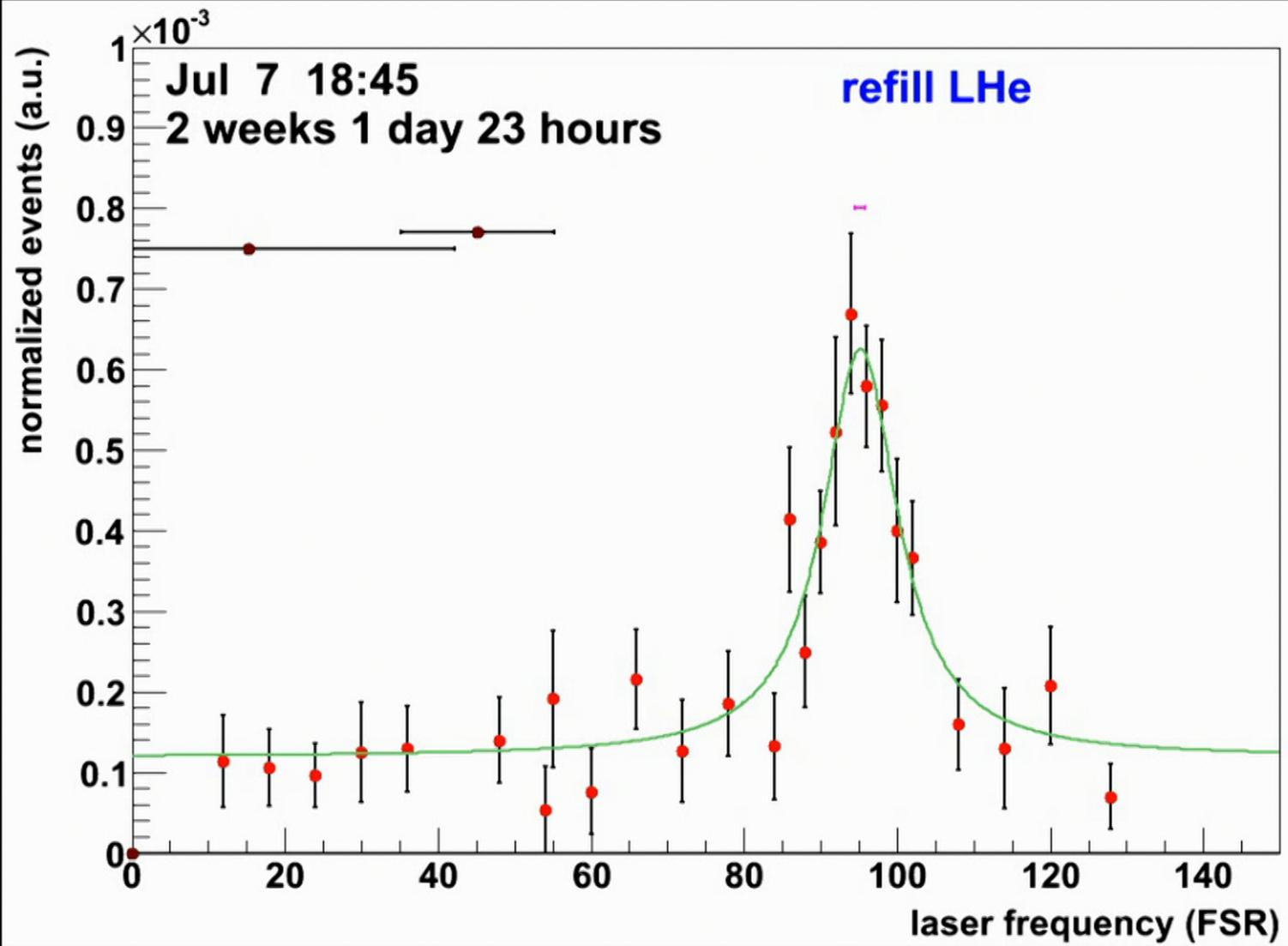


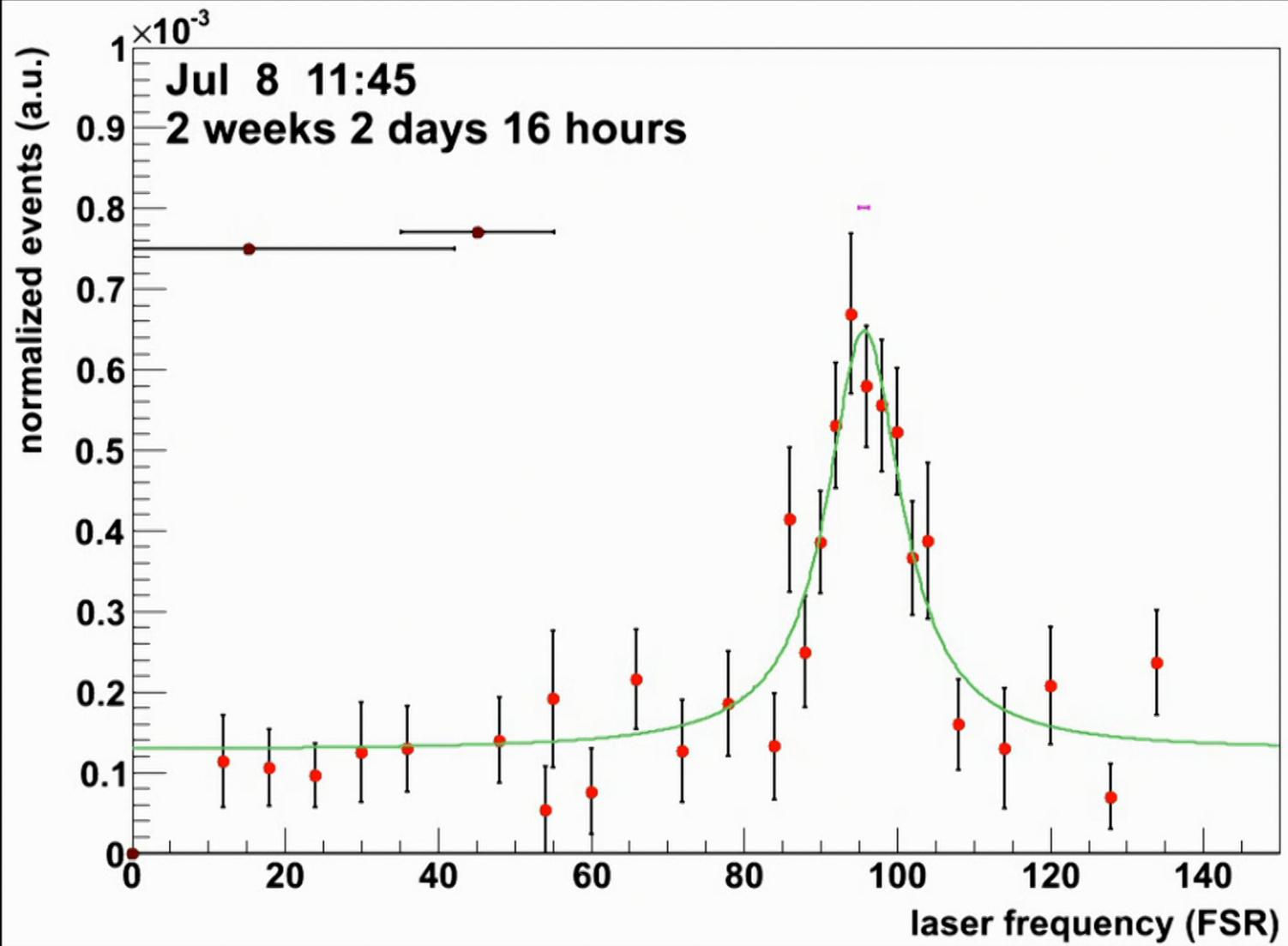


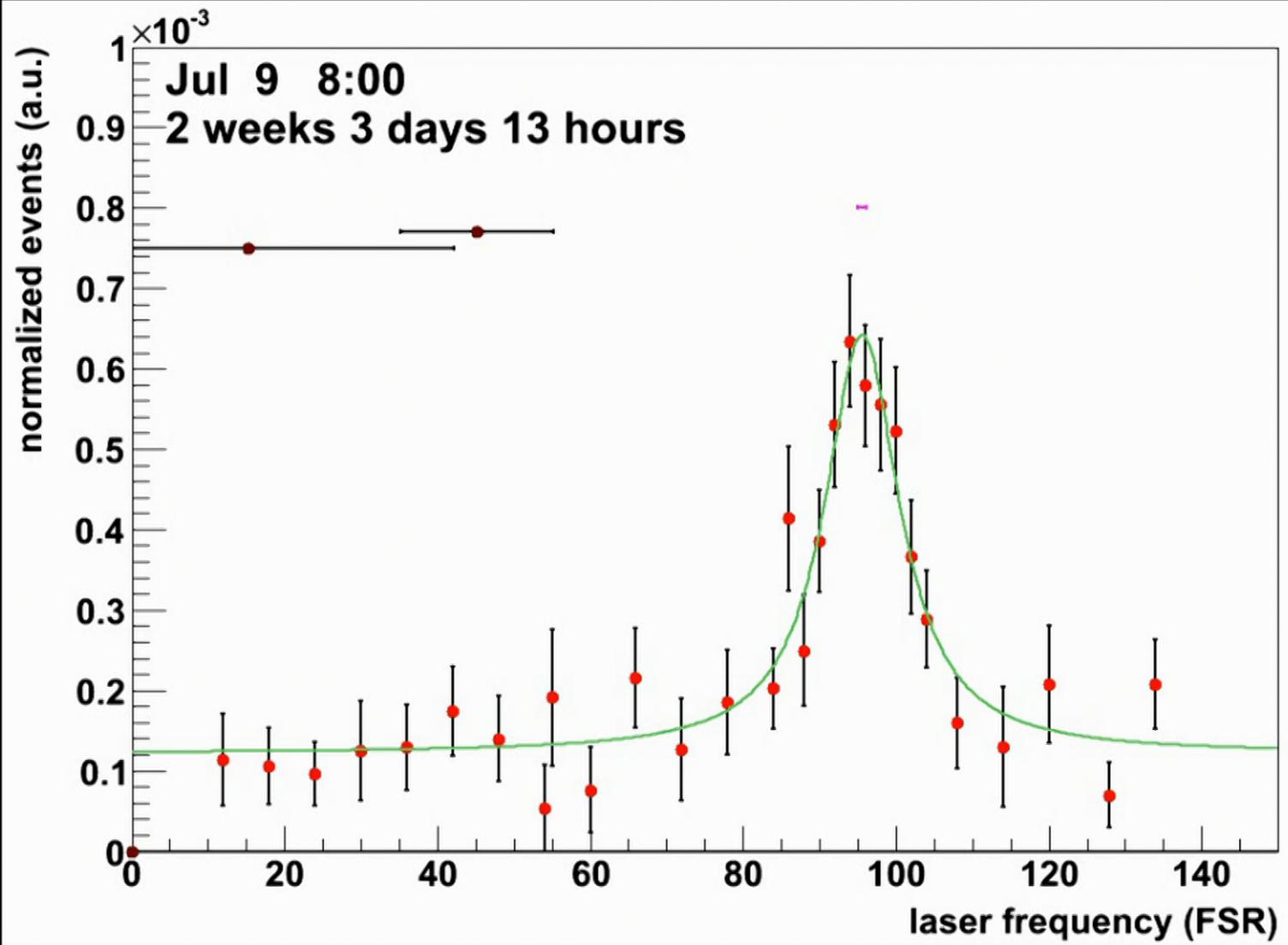


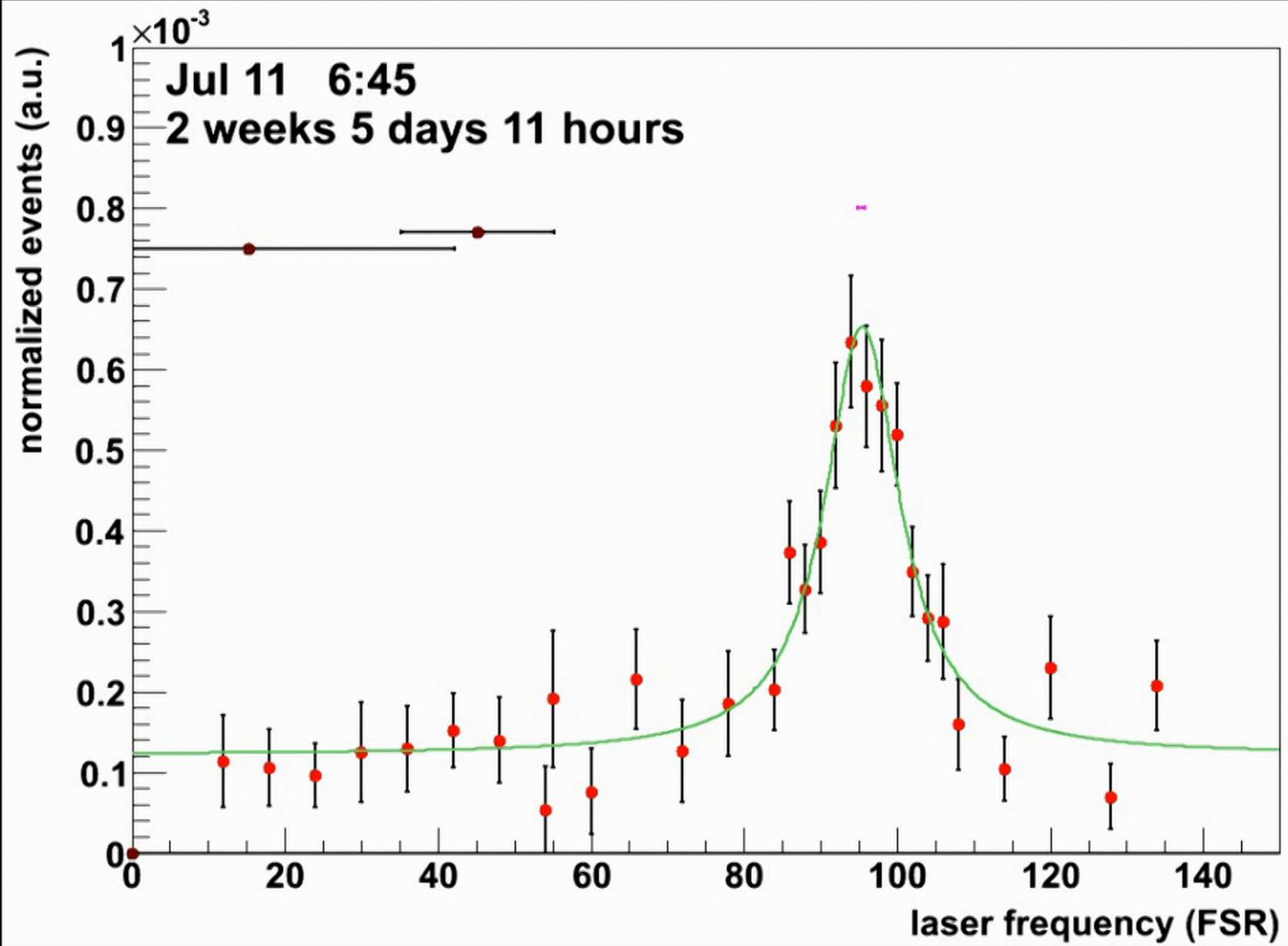


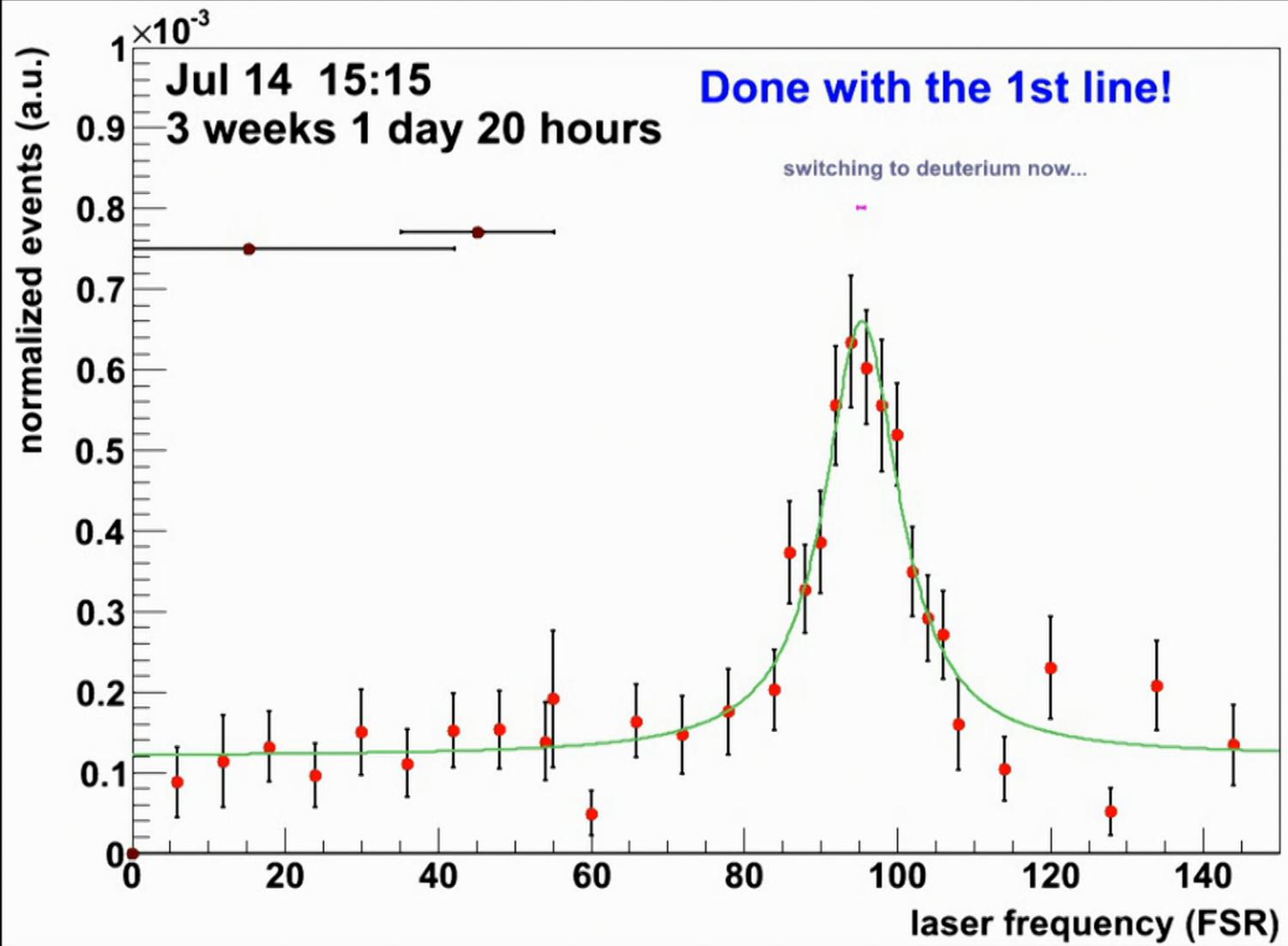




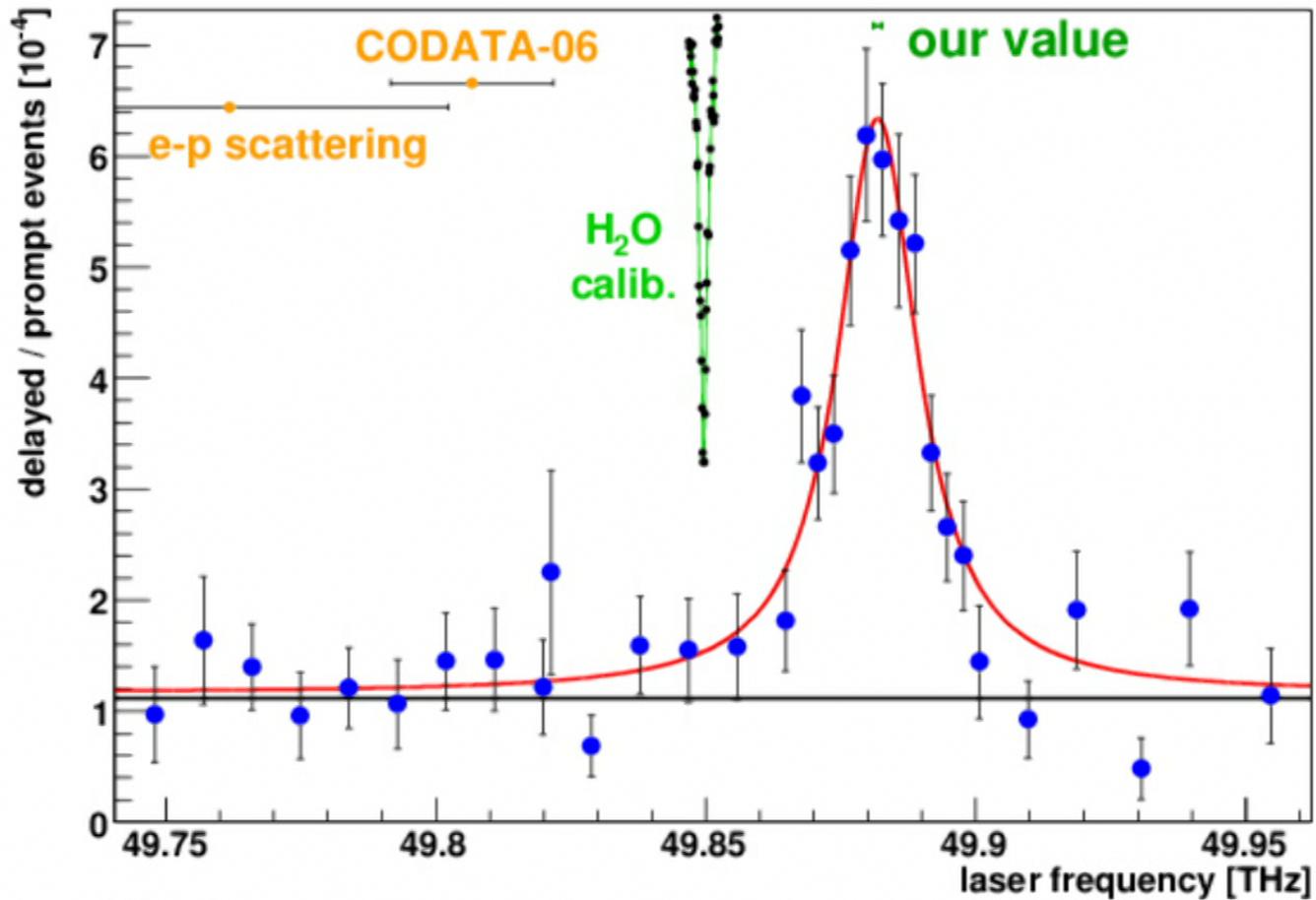






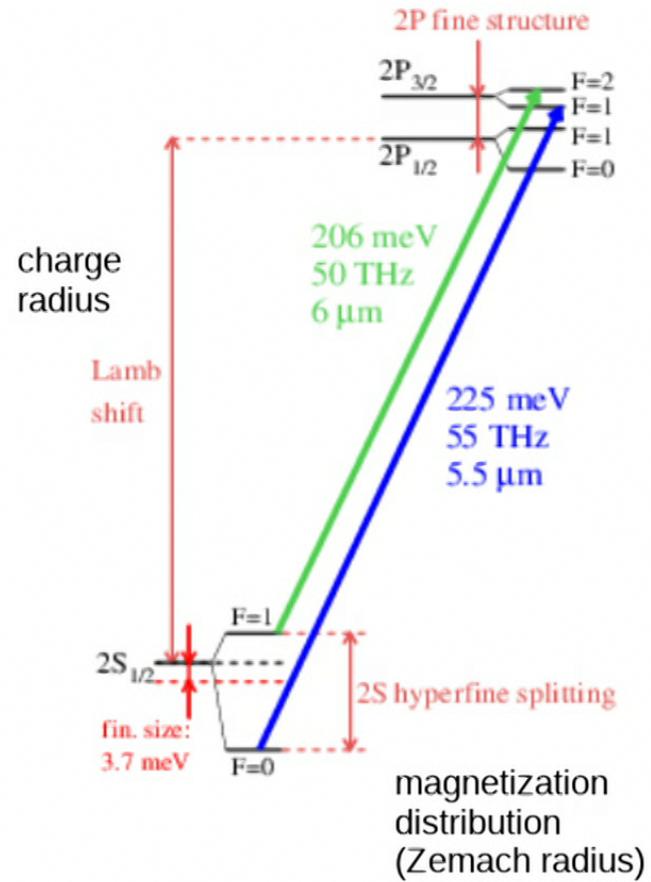
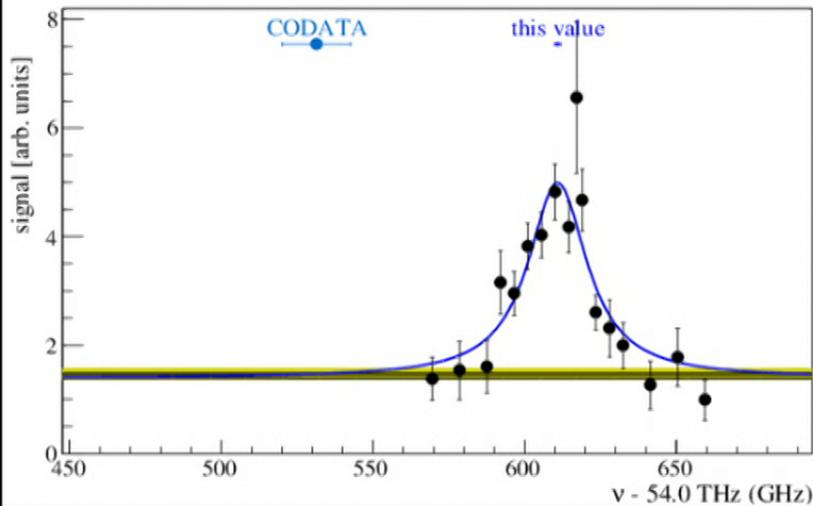
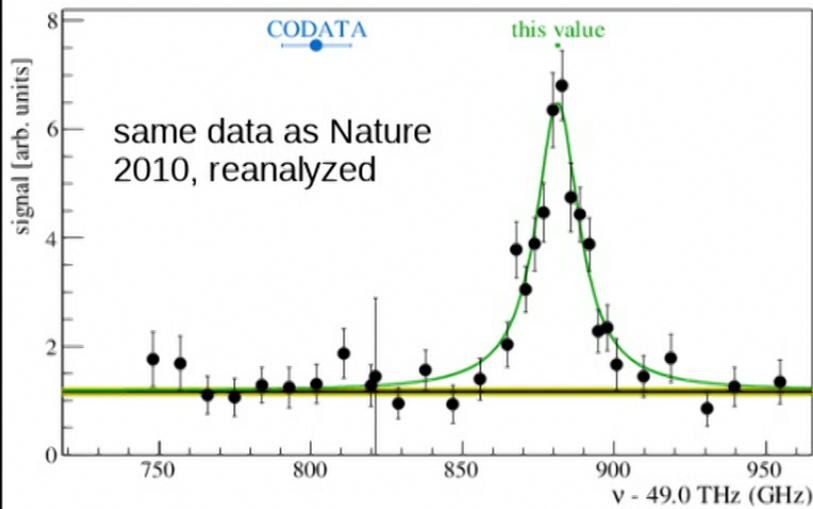


# Resonance in muonic hydrogen



Pohl et al. (CREMA Coll.), Nature 466, 213 (2010)

# 2 transitions in muonic H



Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

# Theory in muonic H

$$\Delta E_{\text{Lamb}} = 206.0336 (15) \text{ meV}_{\text{QED}} + 0.0332 (20) \text{ meV}_{\text{TPE}} - 5.2275 (10) \text{ meV/fm}^2 * R_p^2$$

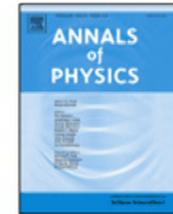
Annals of Physics 331 (2013) 127–145



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## Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen



Aldo Antognini<sup>a,\*</sup>, Franz Kottmann<sup>a</sup>, François Biraben<sup>b</sup>, Paul Indelicato<sup>b</sup>,  
François Nez<sup>b</sup>, Randolph Pohl<sup>c</sup>

<sup>a</sup> Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland

<sup>b</sup> Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France

<sup>c</sup> Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Our attempt to summarize all the original work by many theorists....

# Theory I: “pure” QED

**Table 1**

All known radius-independent contributions to the Lamb shift in  $\mu\text{p}$  from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The “finite-size to relativistic recoil correction” (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

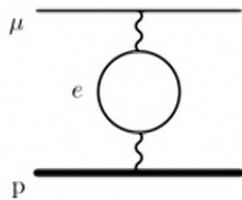
#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit-Pauli)	0.0169 <sup>a</sup>					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2) <sup>b</sup>	-0.0041	-0.0041		-0.00208 <sup>c</sup>	[77,78]
4	Two-loop eVP (Källén-Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén-Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann-Kroll, “1:3” LBL		-0.00103	-0.00102	-0.00102	-0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, “2:2” LBL		0.00135	0.00115		0.00115	[74,89]
New	“3:1” LBL			-0.00102		-0.00102	[89]
20	$\mu\text{SE}$ and $\mu\text{VP}$	-0.6677	-0.66770	-0.66788	-0.66761	-0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	-0.005(1)	-0.00500	-0.004924 <sup>d</sup>		-0.00254	[85] Eq. (29a) <sup>e</sup>
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	-0.001	-0.00150			<sup>f</sup>	[74,90-92]
21	Higher order corr. to $\mu\text{SE}$ and $\mu\text{VP}$		-0.00169	-0.00171 <sup>g</sup>		-0.00171	[86] Eq. (177)
13	Mixed eVP + $\mu\text{VP}$		0.00007	0.00007		0.00007	[74]
New	eVP and $\mu\text{VP}$ in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93-95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		-0.000015			-0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	-0.045	-0.04497	-0.04497	-0.04497	-0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^0$ (proton SE)	-0.0099	-0.00960	-0.0100		-0.01080(100)	[43] <sup>h</sup> [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

Antognini, RP et al., Ann. Phys. (N.Y.) 331, 127 (2013)

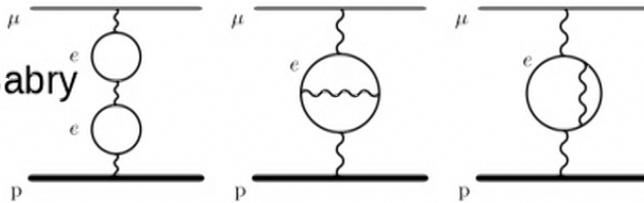
# Theory in muonic H

$$\Delta E_{\text{Lamb}} = 206.0336 (15) \text{ meV}_{\text{QED}} + 0.0332 (20) \text{ meV}_{\text{TPE}} - 5.2275 (10) \text{ meV/fm}^2 * R_p^2$$

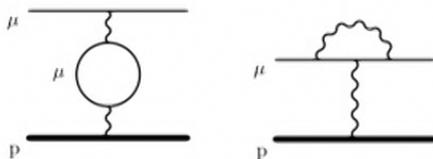
Uehling



Källen-Sabry

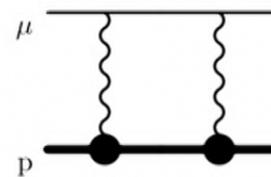
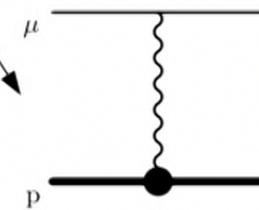


Muon SE+VP

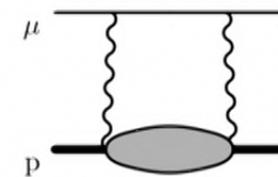


and 20+ more....

Proton form factor



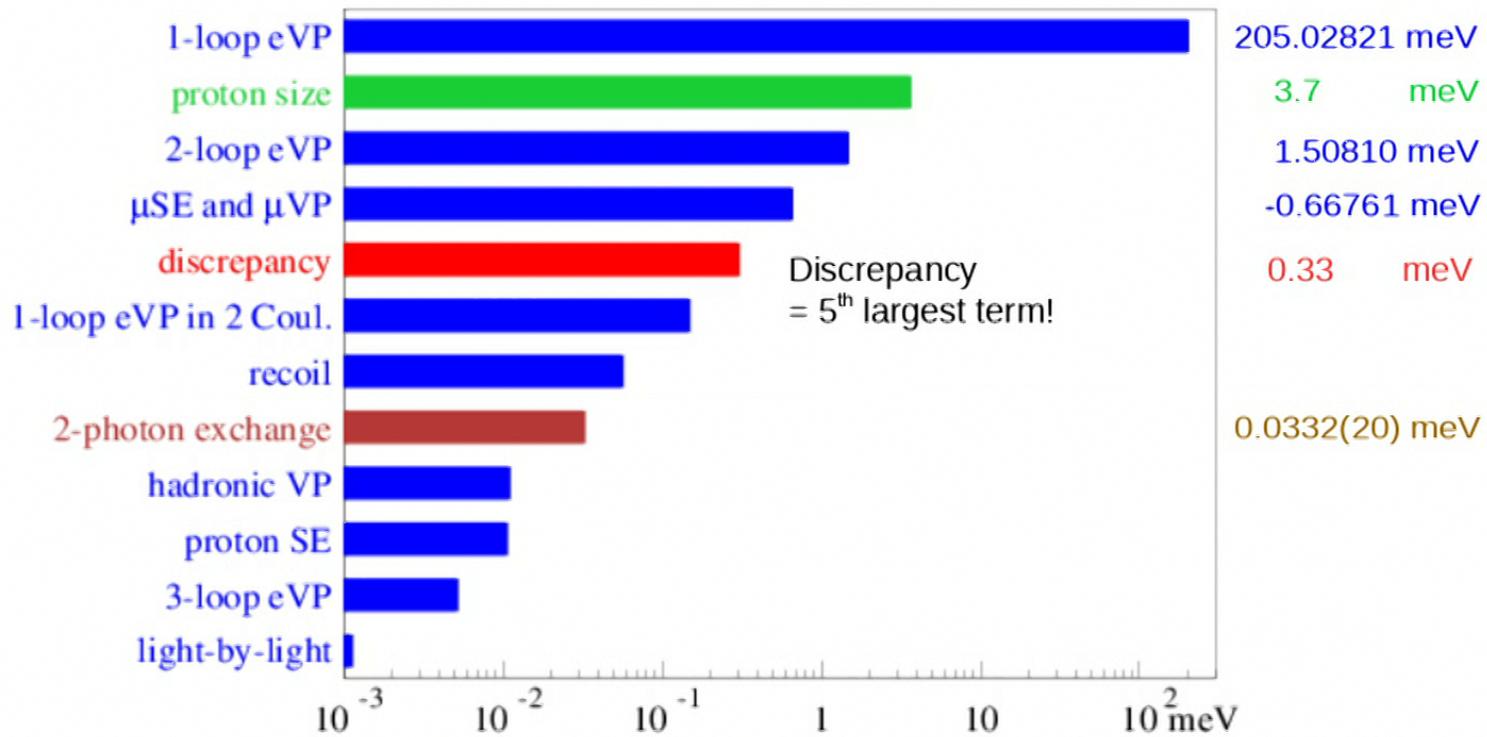
elastic and inelastic two-photon exchange  
(Friar moment and polarizability)



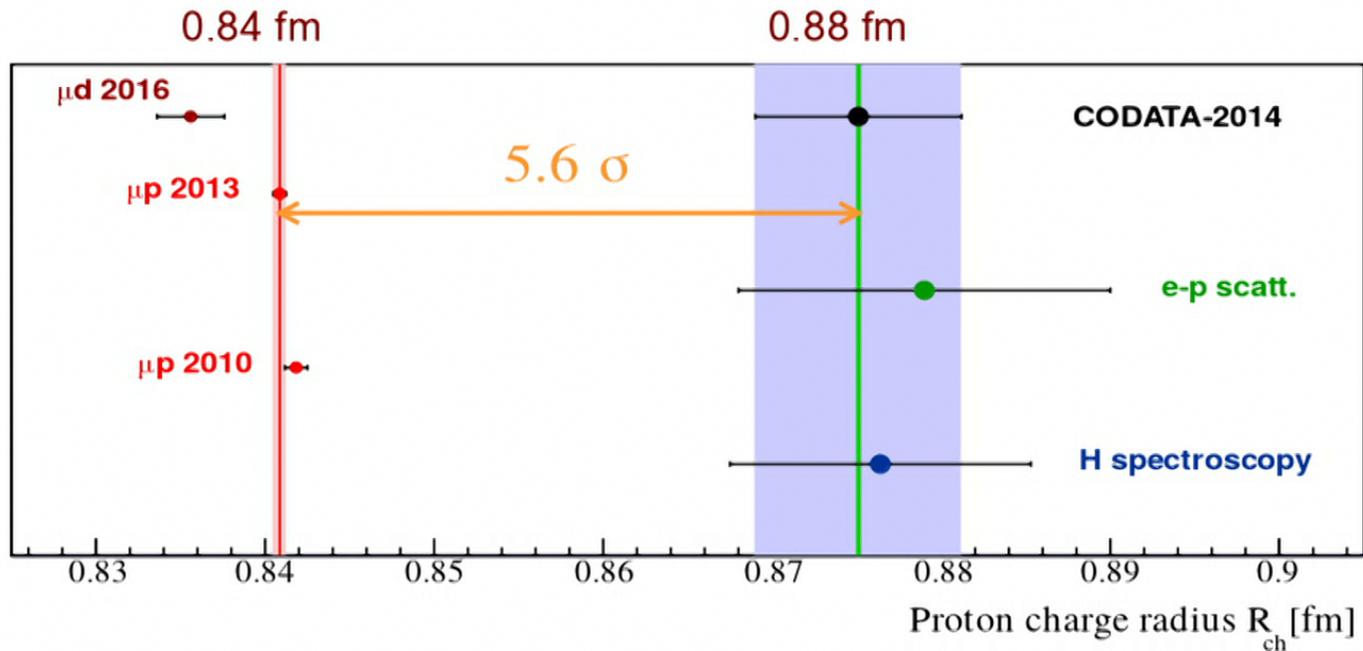
# Theory in muonic H

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Nice hierarchy



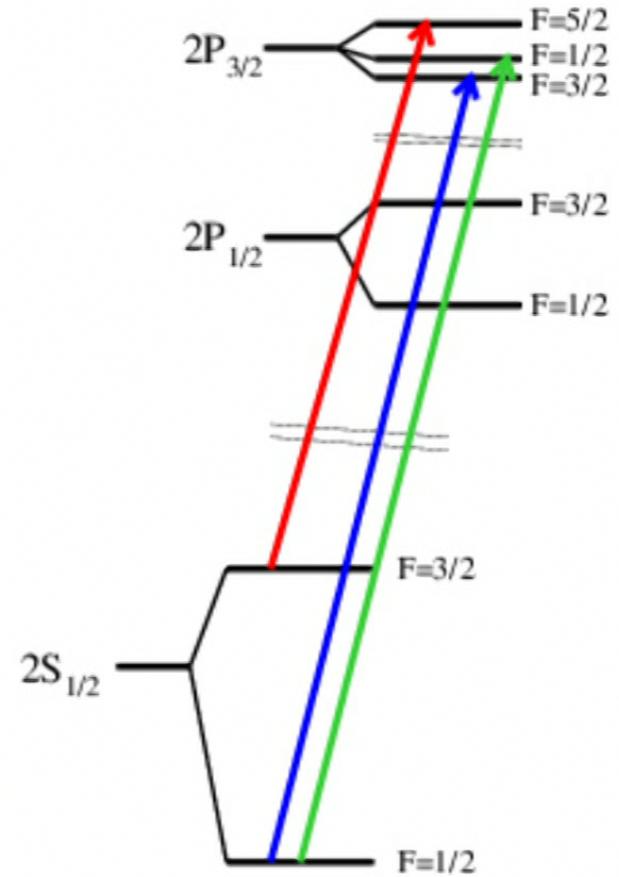
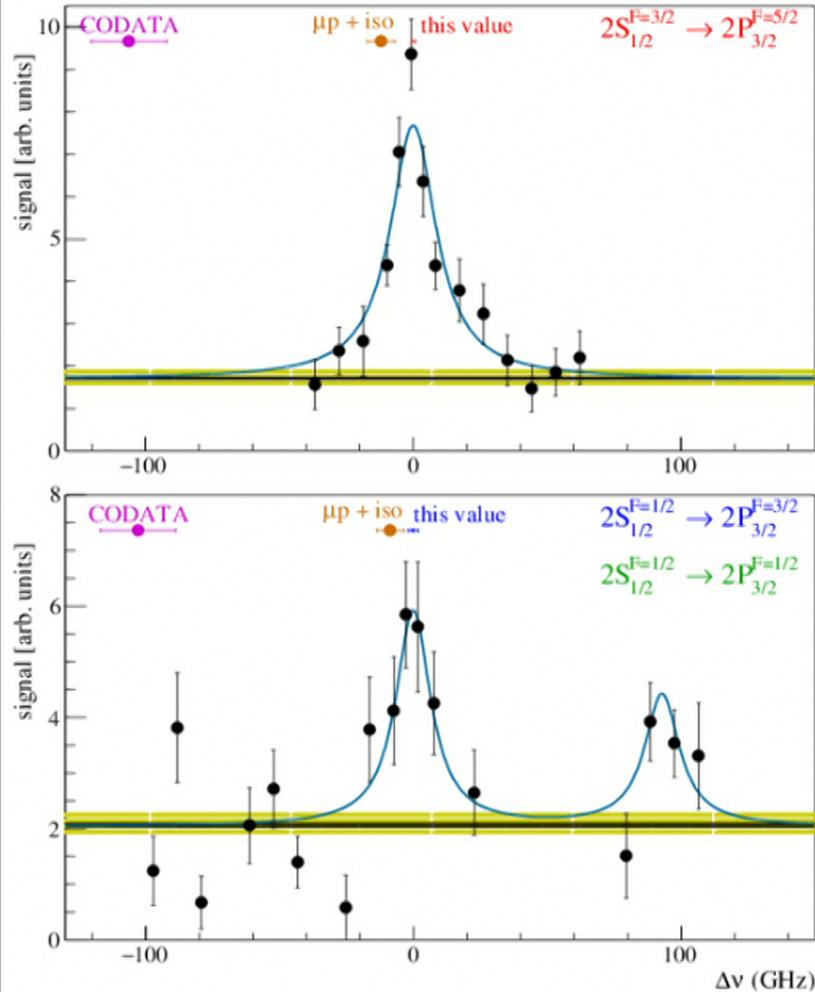
# Muonic Hydrogen



muonic hydrogen:  $0.8409 \pm 0.0004$  fm  
electronic hydrogen:  $0.876 \pm 0.008$  fm  
electron scattering  $0.879 \pm 0.011$  fm

20x more precise

# 2.5 transitions in muonic D



Pohl et al. (CREMA Coll.), Science 353, 669 (2016)

# Theory in muonic D

$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7854 (13) \text{ meV}_{\text{QED}} + 1.7150 (230) \text{ meV}_{\text{TPE}} - 6.1103 (3) \text{ meV/fm}^2 * R_{\text{d}}^2$$

$$\Delta E_{\text{Lamb}}^{\mu\text{H}} = 206.0336 (15) \text{ meV}_{\text{QED}} + 0.0332 (20) \text{ meV}_{\text{TPE}} - 5.2275 (10) \text{ meV/fm}^2 * R_{\text{p}}^2$$

Annals of Physics 331 (2013) 127–145

Annals of Physics 366 (2016) 168–196



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Theory of the 2S–2P Lamb shift and splitting in muonic hydrogen

Aldo Antognini<sup>a,\*</sup>, Franz Kottmann<sup>a</sup>, François François Nez<sup>b</sup>, Randolph Pohl<sup>c</sup>

<sup>a</sup> Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland

<sup>b</sup> Laboratoire Kastler Brassel, École Normale Supérieure, CNRS and Université P.

<sup>c</sup> Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

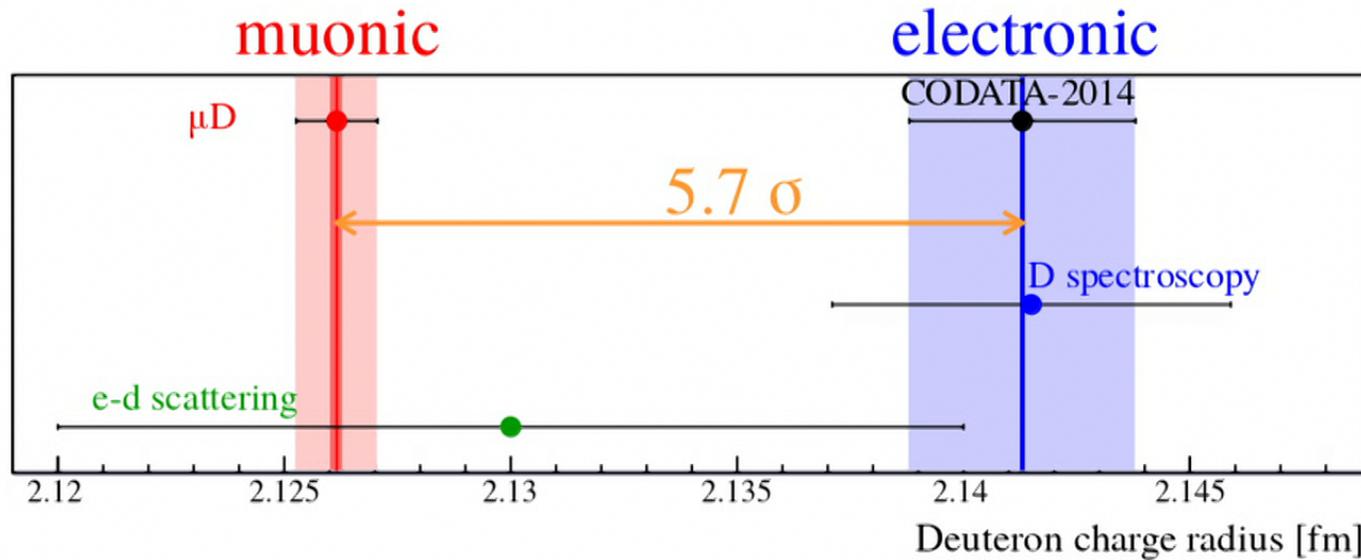
Theory of the  $n = 2$  levels in muonic deuterium

Julian J. Krauth<sup>a,\*</sup>, Marc Diepold<sup>a</sup>, Beatrice Franke<sup>a</sup>,  
Aldo Antognini<sup>b,c</sup>, Franz Kottmann<sup>b</sup>, Randolph Pohl<sup>a</sup>

Summarizes original work by: Bacca, Barnea, Birse, Borie, Carlson, Eides, Faustov, Friar, Gorchtein, Hernandez, Ivanov, Jentschura, Ji, Karshenboim, Korzinin, Krutov, Martynenko, McGovern, Nevo-Dinur, Pachucki, Shelyuto, Sick, Vanderhaeghen, et al.

Newer work: Pachucki et al., PRA 97, 062511 (2018), Hernandez et al., PLB 778, 377 (2018)

# Muonic Deuterium



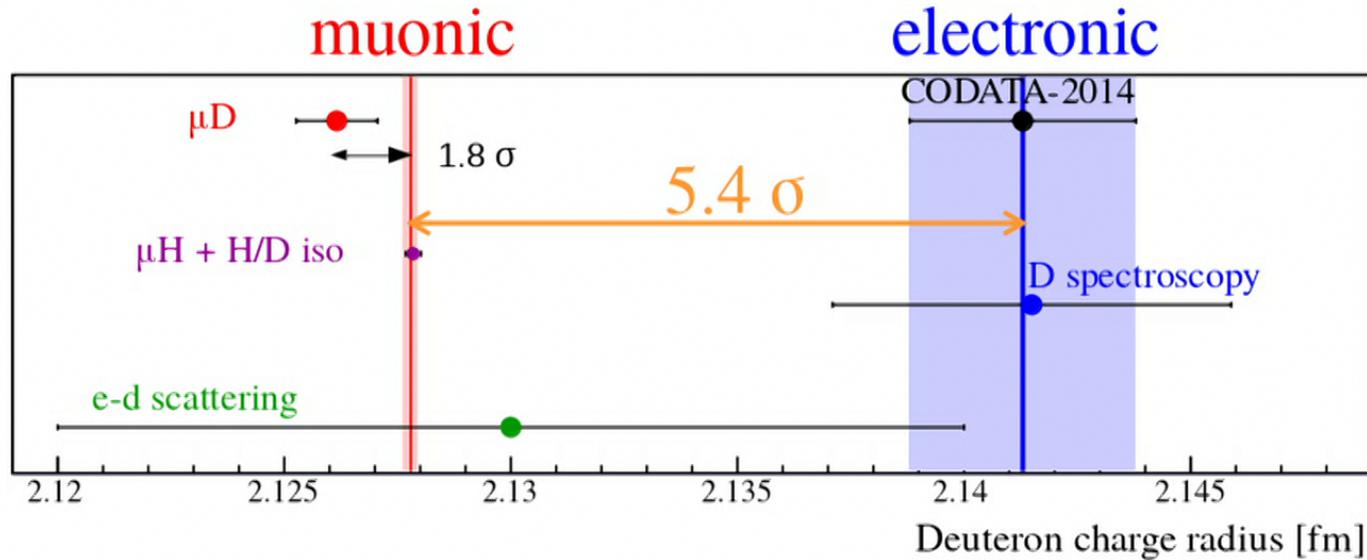
$\mu\text{D}$ :  $2.12616 (13)_{\text{exp}} (89)_{\text{theo}}$  fm (theo = nucl. polarizability)

CODATA-2014:  $2.14130 (250)$  fm

RP et al. (CREMA Coll.), Science 353, 559 (2016)

Krauth, RP et al., Ann. Phys. (N.Y.) 366, 168 (2016)  
 + Pachucki et al., PRA 97, 062511 (2018)  
 + Hernandez et al., PLB 778, 377 (2018)

# Muonic Deuterium



$\mu\text{D}$ : 2.12616 (13)<sub>exp</sub> (89)<sub>theo</sub> fm (theo = nucl. polarizability)

$\mu\text{H} + \text{H/D}(1\text{S}-2\text{S})$ : 2.12785 (17) fm

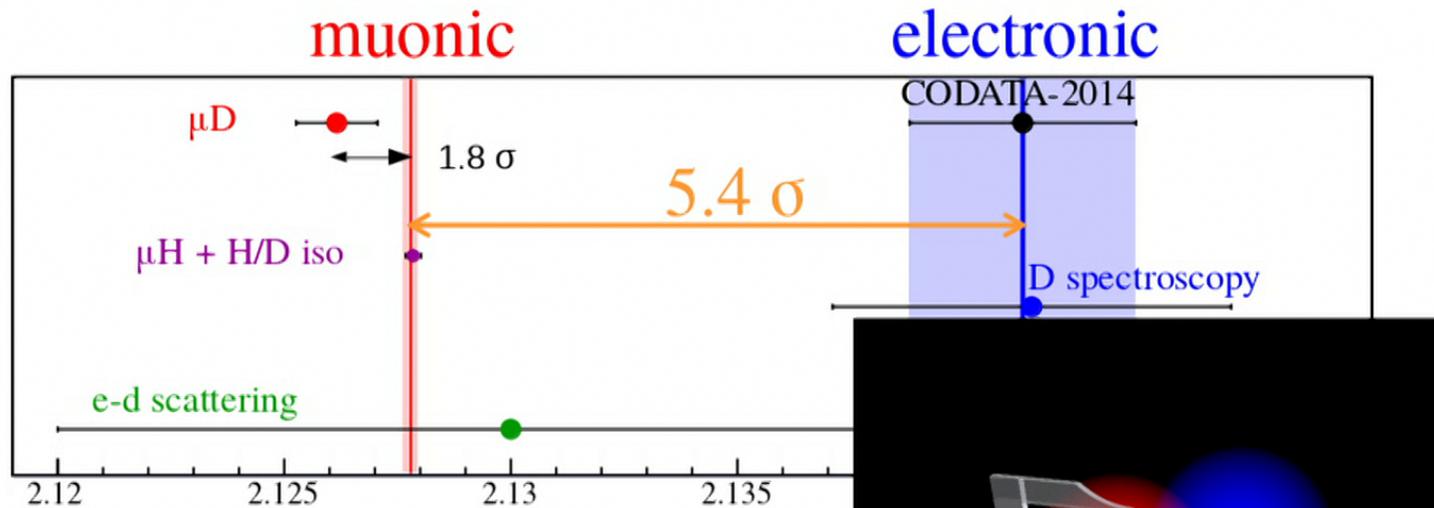
CODATA-2014: 2.14130 (250) fm

H/D 1S-2S isotope shift:  
 $r_d^2 - r_p^2 = 3.82070(31) \text{ fm}^2$

Pachucki et al., PRA 97, 062511 (2018)

H/D 1S-2S. Parthey, RP et al. (MPQ Garching), PRL 104, 233001 (2010)  
 PRL 107, 203001 (2011)

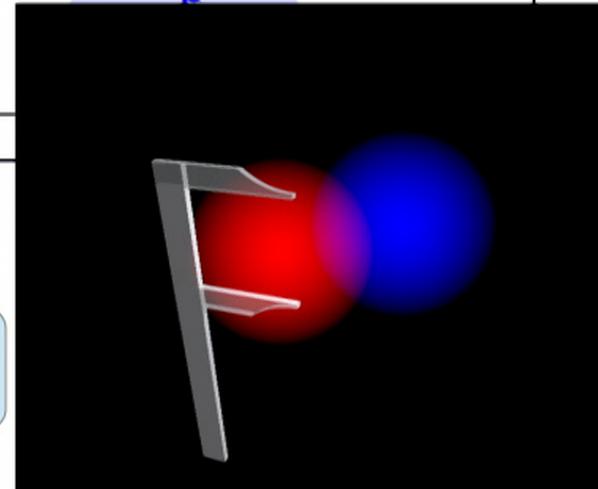
# Muonic Deuterium



Deuteron is CONSISTENTLY smaller!

$$R_d^2 = R_{\text{struct}}^2 + R_p^2 + R_n^2 (+ DF)$$

Pohl et al. (CREMA), Science 353, 669 (2016)

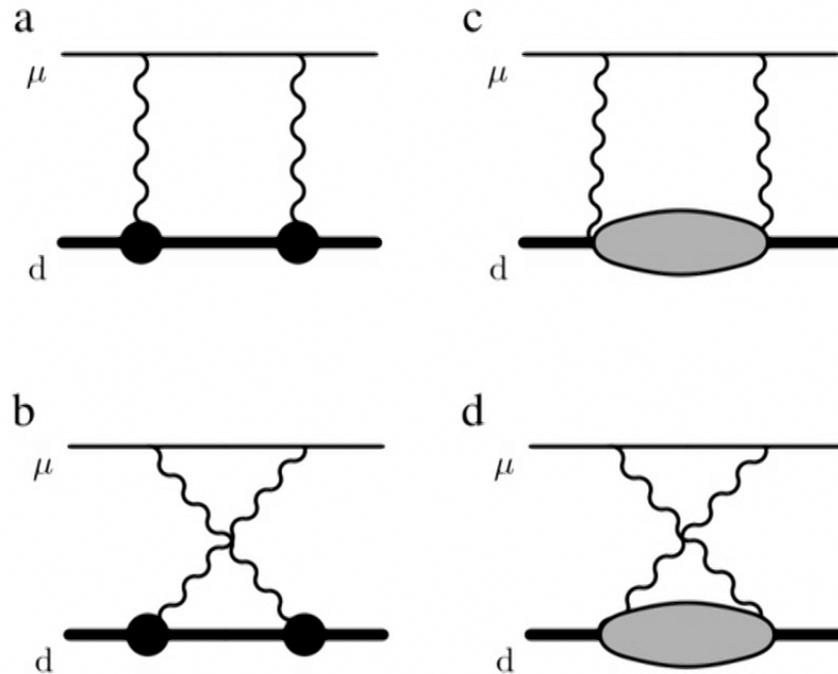


# Theory in muonic D

$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7854 (13) \text{ meV}_{\text{QED}} + 1.7150 (230) \text{ meV}_{\text{TPE}} - 6.1103 (3) \text{ meV}/\text{fm}^2 * R_d^2$$

Krauth, RP et al., Ann. Phys. (N.Y.) 366, 168 (2016)  
 + Pachucki et al., PRA 97, 062511 (2018)  
 + Hernandez et al., PLB 778, 377 (2018)

Two-photon nuclear structure contributions to the Lamb shift in muonic deuterium.



# Theory in muonic D

$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7854 (13) \text{ meV}_{\text{QED}} + 1.7150 (230) \text{ meV}_{\text{TPE}} - 6.1103 (3) \text{ meV/fm}^2 * R_d^2$$

Nuclear structure contributions to the Lamb shift in muonic deuterium.

Item	Contribution	Pachucki [55]		Friar [60]		Hernandez <i>et al.</i> [58]		Pach. & Wienczek [65]		Carlson <i>et al.</i> [64]	Our choice											
		AV18		ZRA		AV18	N <sup>3</sup> LO <sup>†</sup>	AV18		data	value	source										
	Source	1		2		3	4	5		6												
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta_{D1}^{(0)}$	1.910	$\delta_0 E$		1.9165 ± 0.0095	3-5									
p2	Rel. corr. to p1, longitudinal part	-0.035	$\delta_R E$	-0.037	Subleading C1	-0.029	-0.030	$\delta_L^{(0)}$	-0.026	$\delta_R E$												
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta_T^{(0)}$														
p4	Rel. corr. to p1, higher-order								0.004	$\delta_{HO} E$												
sum	Total rel. corr., p2+p3+p4	-0.035		-0.037		-0.017	-0.017		-0.022			-0.0195 ± 0.0025	3-5									
p5	Coulomb distortion, leading	-0.255	$\delta_{C1} E$						-0.255	$\delta_{C1} E$												
p6	Coul. distortion, next order	-0.006	$\delta_{C2} E$						-0.006	$\delta_{C2} E$												
sum	Total Coulomb distortion, p5+p6	-0.261				-0.262	-0.264	$\delta_C^{(0)}$	-0.261			-0.2625 ± 0.0015	3-5									
p7	El. monopole excitation	-0.045	$\delta_{Q0} E$	-0.042	C0	-0.042	-0.041	$\delta_{R2}^{(2)}$	-0.042	$\delta_{Q0} E$												
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta_{D1}^{(2)}$	0.139	$\delta_{Q1} E$												
p9	El. quadrupole excitation	-0.066	$\delta_{Q2} E$	-0.061	C2	-0.061	-0.061	$\delta_Q^{(2)}$	-0.061	$\delta_{Q2} E$												
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036			0.0360 ± 0.0020	2-5									
p10	Magnetic	-0.008 <sup>o</sup>	$\delta_M E$	-0.011	M1	-0.008	-0.007	$\delta_M^{(0)}$	-0.008	$\delta_M E$		-0.0090 ± 0.0020	2-5									
SUM.1	Total nuclear (corrected)	1.646		1.648 <sup>b</sup>		1.656	1.676		1.655			1.6615 ± 0.0103										
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 <sup>o</sup>	0.021 <sup>o</sup>	$\delta_{NS}^{(3)}$	0.020	$\delta_{FS} E$												
p12	n p charge correlation			-0.023	pn correl. f.s.	-0.017	-0.017	$\delta_{np}^{(1)}$	-0.018	$\delta_{FZ} E$												
sum	p11+p12			-0.002		0.003	0.004		0.002			0.0010 ± 0.0030	2-5									
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^{-3} \rangle_{(2)}^{pp}$	} 0.027(2)	$\delta_{pol}^N$ [64]	} 0.043(3)	$\delta_P E$	} 0.028(2)	$\Delta E^{hadr}$	} 0.0289 ± 0.0015	Eq.(13) <sup>†</sup>									
p14	Proton inelastic polarizab.																					
p15	Neutron inelastic polarizab.																	0.016(8)	$\delta_N E$		0.0280 ± 0.0020	6
p16	Proton & neutron subtraction term											-0.0098 ± 0.0098	Eq.(15) <sup>†</sup>									
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)		0.059(9)				0.0471 ± 0.0101	<sup>j</sup>									
SUM.2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)		0.061(9)				0.0476 ± 0.0105										
	Sum, published	1.680(16)		1.941(19)		1.690(20)		1.717(20)		2.011(740)												
	Sum, corrected			1.697(19) <sup>g</sup>		1.714(20) <sup>h</sup>		1.707(20) <sup>i</sup>		1.748(740) <sup>j</sup>		<b>1.7096 ± 0.0147</b>										

Krauth, RP *et al.*, Ann. Phys. (N.Y.) 366, 168 (2016)

+ Pachucki *et al.*, PRA 97, 062511 (2018)  
 + Hernandez *et al.*, PLB 778, 377 (2018)

# Theory in muonic D

$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7854 (13) \text{ meV}_{\text{QED}} + 1.7150 (230) \text{ meV}_{\text{TPE}} - 6.1103 (3) \text{ meV/fm}^2 * R_d^2$$



$$\Delta E_{\text{TPE}} (\text{theo}) = 1.7150 \pm 0.0230 \text{ meV} \quad (\text{Hernandez et al., 2018})$$

vs.  $\pm 0.0034 \text{ meV}$  experimental uncertainty

(1) **charge radius**, using **calculated TPE**

$$r_d (\mu\text{D}) = 2.12616 (13)_{\text{exp}} (89)_{\text{theo}} \text{ fm} \quad \text{vs.}$$

$$r_d (\text{CODATA-14}) = 2.14130 (250) \text{ fm}$$

(2) **polarizability**, using **charge radius from isotope shift**

$$\Delta E_{\text{TPE}} (\text{theo}) = 1.7150 (230) \text{ meV} \quad \text{vs.}$$

$$\Delta E_{\text{TPE}} (\text{exp}) = 1.7591 (59) \text{ meV} \quad 4x \text{ more accurate, } 1.8\sigma$$

Krauth et al. (2016) + Pachucki et al. (2018) + Hernandez et al. (2018)

# Theory in muonic He-3

$$\Delta E_{\text{Lamb}}^{\mu^3\text{He}} = 1644.4820(149)_{\text{QED}} + 15.3000(5200)_{\text{TPE}} - 103.5184(10) * R_h^2 / \text{fm}^2 \quad [\text{meV}]$$

$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7854(13)_{\text{QED}} + 1.7150(230)_{\text{TPE}} - 6.1103(3) * R_d^2 / \text{fm}^2 \quad [\text{meV}]$$

$$\Delta E_{\text{Lamb}}^{\mu\text{H}} = 206.0336(15)_{\text{QED}} + 0.0332(20)_{\text{TPE}} - 5.2275(10) * R_p^2 / \text{fm}^2 \quad [\text{meV}]$$

Annals of Physics 366 (2016) 168–196



Eur. Phys. J. D (2017) 71: 341  
DOI: 10.1140/epjd/e2017-80296-1

THE EUROPEAN  
PHYSICAL JOURNAL D

Topical Review

ELSEVIER

## Theory of the $n = 2$ levels in muonic helium-3 ions

Beatrice Franke<sup>1,2,a</sup>, Julian J. Krauth<sup>1,3,b</sup>, Aldo Antognini<sup>4,5</sup>, Marc Diepold<sup>1</sup>, Franz Kottmann<sup>4</sup>,  
and Randolph Pohl<sup>3,1,c</sup>

Theory of

<sup>1</sup> Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

<sup>2</sup> TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

<sup>3</sup> Johannes Gutenberg-Universität Mainz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA,  
55099 Mainz, Germany

Julian J. Krau

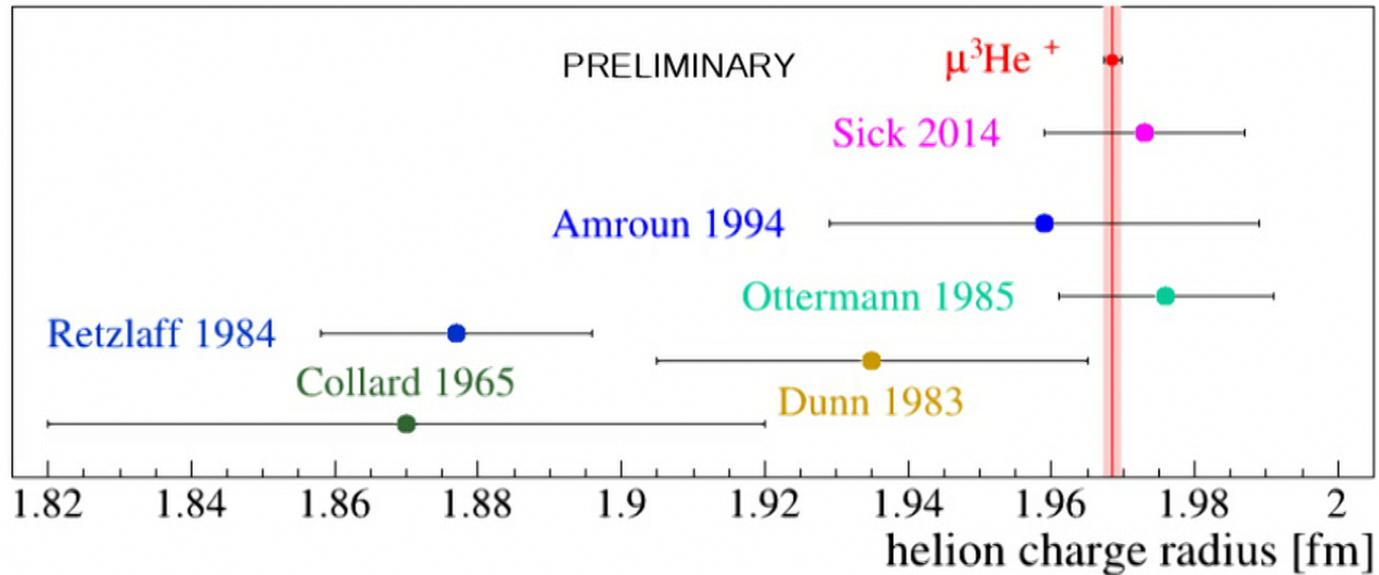
<sup>4</sup> Institute for Particle Physics and Astrophysics, ETH Zurich, 8093 Zurich, Switzerland

Aldo Antogni

<sup>5</sup> Paul Scherrer Institute, 5232 Villigen, Switzerland

Three-photon contribution still missing (Pachucki et al., PRA 97, 052511 (2018))

# Muonic Helium-3



prel. accuracy: exp  $\pm$  0.00012 fm, theo  $\pm$  0.00128 fm (nucl. polarizability)

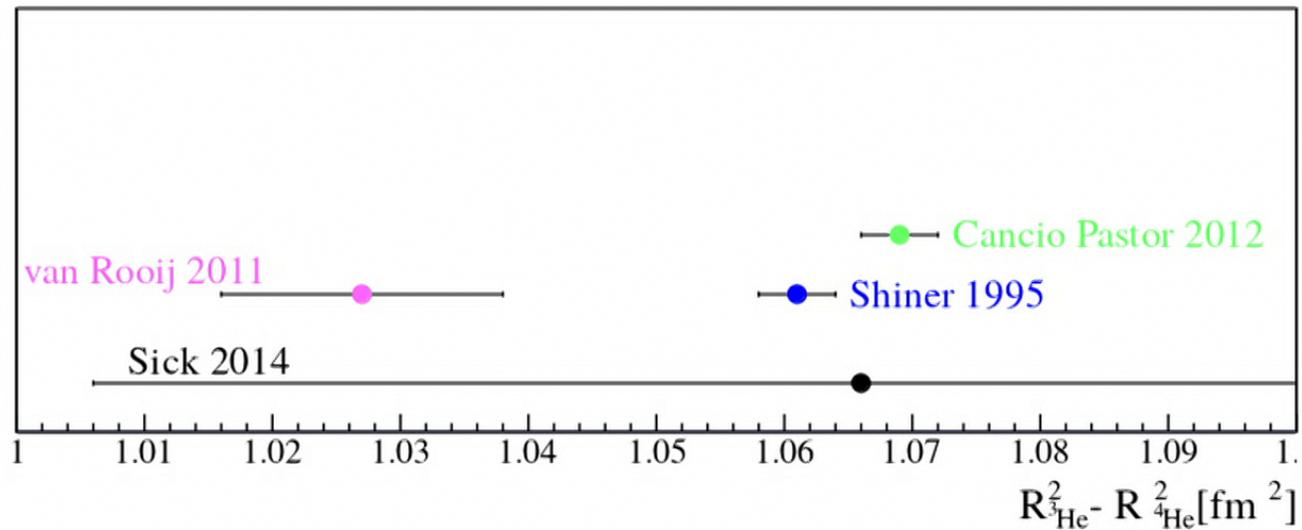
Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

# Muonic conclusions

- The **proton** radius is  $0.84087 (26)_{\text{exp}} (29)_{\text{theo}}$  fm
- The **deuteron** radius is  $2.12771 (22)$  fm
- both are **>5 $\sigma$  smaller** than CODATA values
- No discrepancy for the **absolute radii** of the **helion** and **alpha** particle  
(limited by e-scattering accuracy)
- **BUT: The helium isotope shift!!!**
- **(caveat: 3-photon, maybe more missing?)**

# The $^3\text{He}$ – $^4\text{He}$ isotope shift

$^3\text{He} / ^4\text{He}$  (squared) charge radius difference



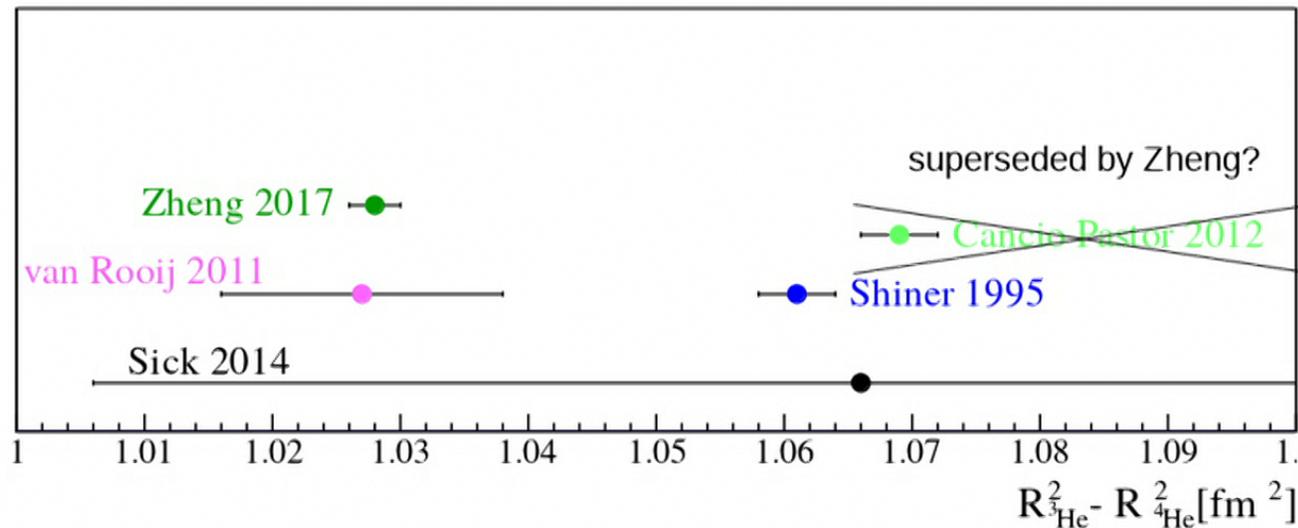
Shiner et al., PRL 74, 3553 (1995)  
vanRooij, Science 333, 196 (2011)  
Cancio Pastor et al., PRL 108, 143001 (2012)

**all evaluated with recent theory by  
Pachucki et al.**

Sick, PRC 90, 064002 (2014)

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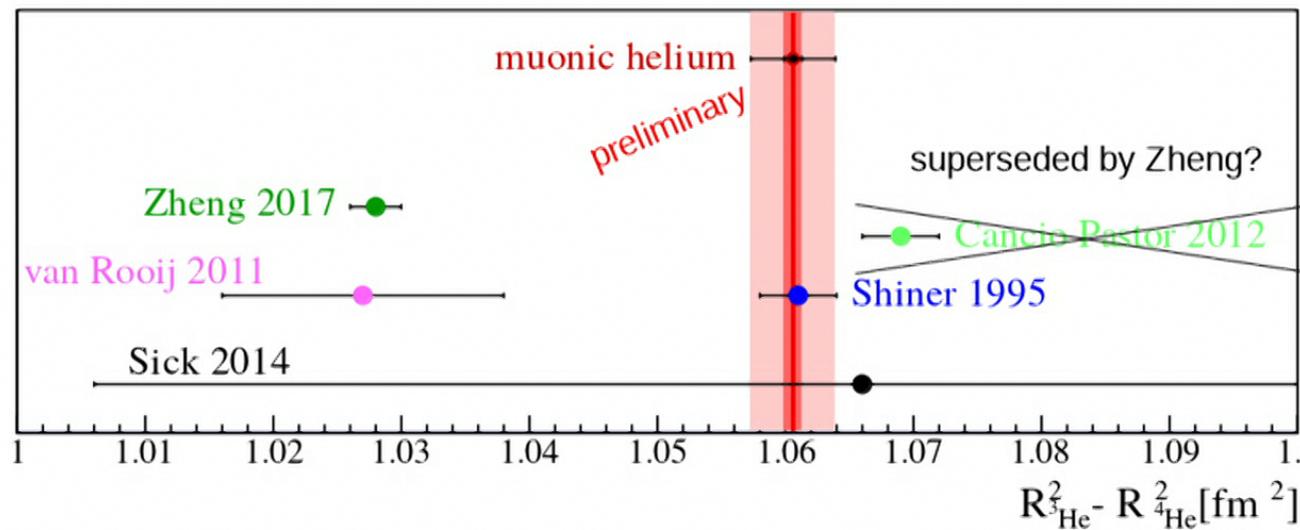
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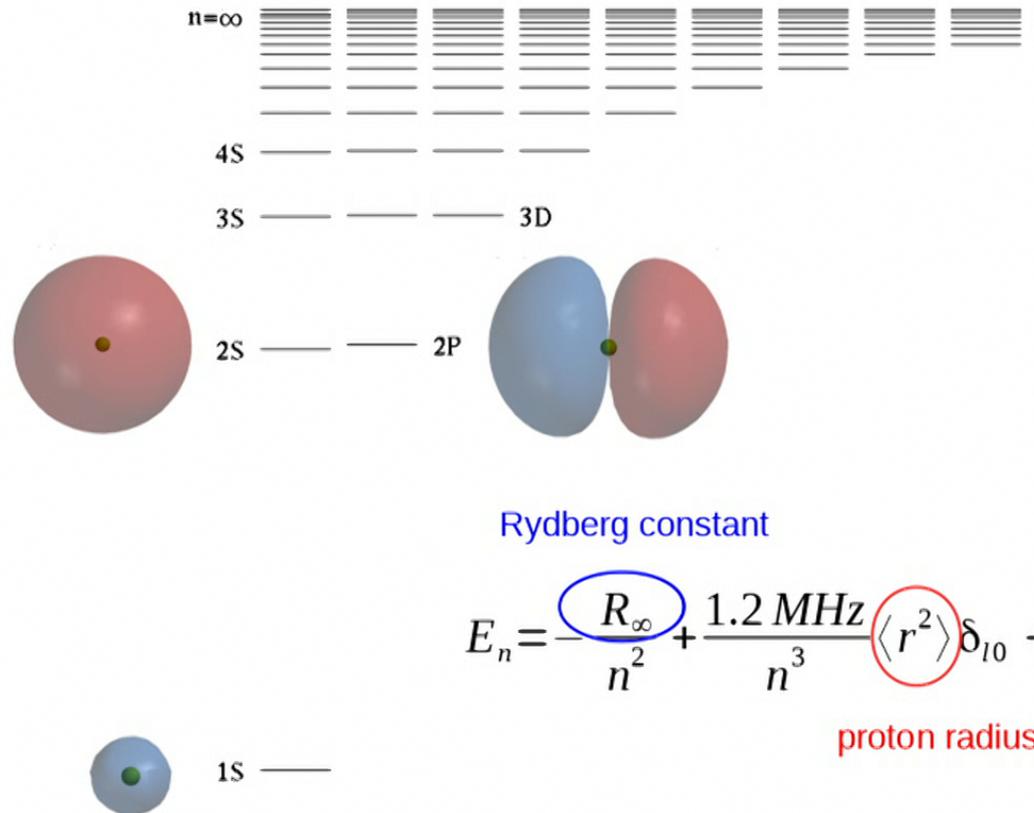
Another  $>5\sigma$  discrepancy?!

## Part 2: The Rydberg constant

$$R_{\infty} = \frac{\alpha^2 m_e c}{2h}$$

- **most accurately determined** fundamental constant  $u_r = 5.9 * 10^{-12}$
- corner stone of the CODATA LSA of fundamental constants  
links **fine structure constant  $\alpha$** , **electron mass  $m_e$** , **velocity of light  $c$**   
and **Planck's constant  $h$**
- correlation coefficient with **proton radius**: 0.9891  
→ The “proton radius puzzle” could be a “Rydberg puzzle”
- $R_{\infty}$  is a “unit converter”: atomic units → SI (Hertz)

# Energy levels of hydrogen

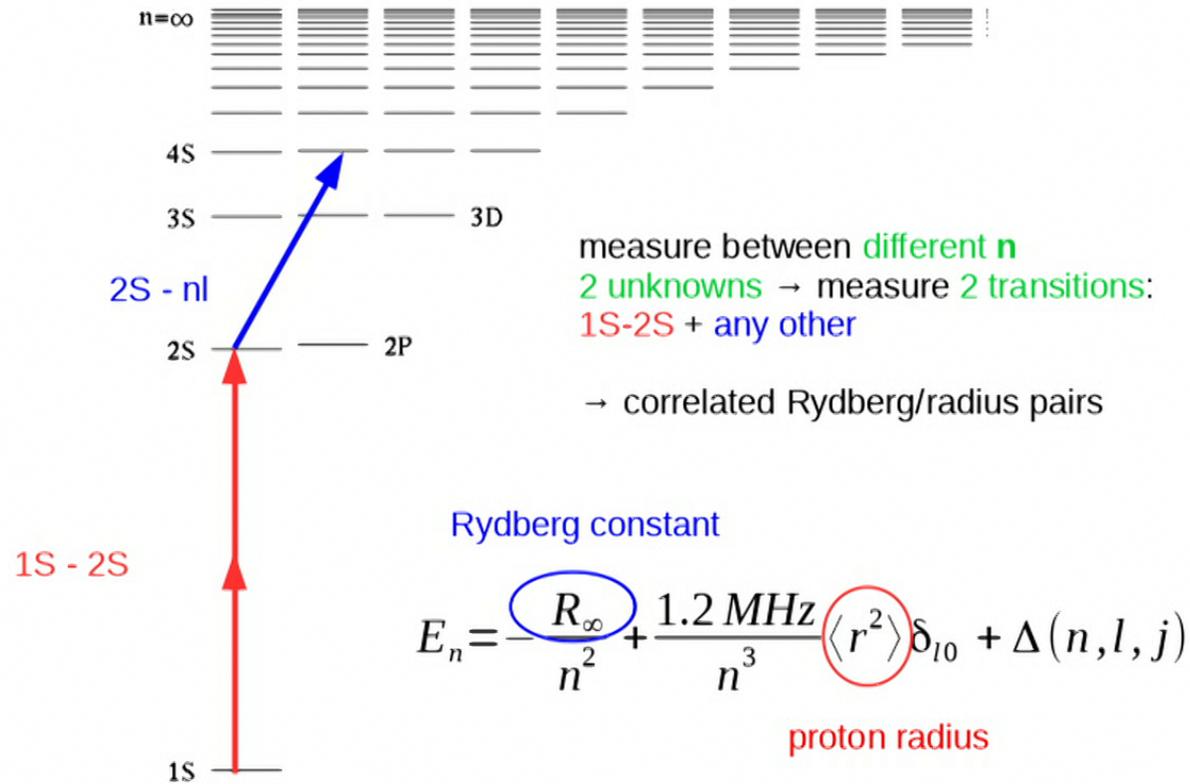


Rydberg constant

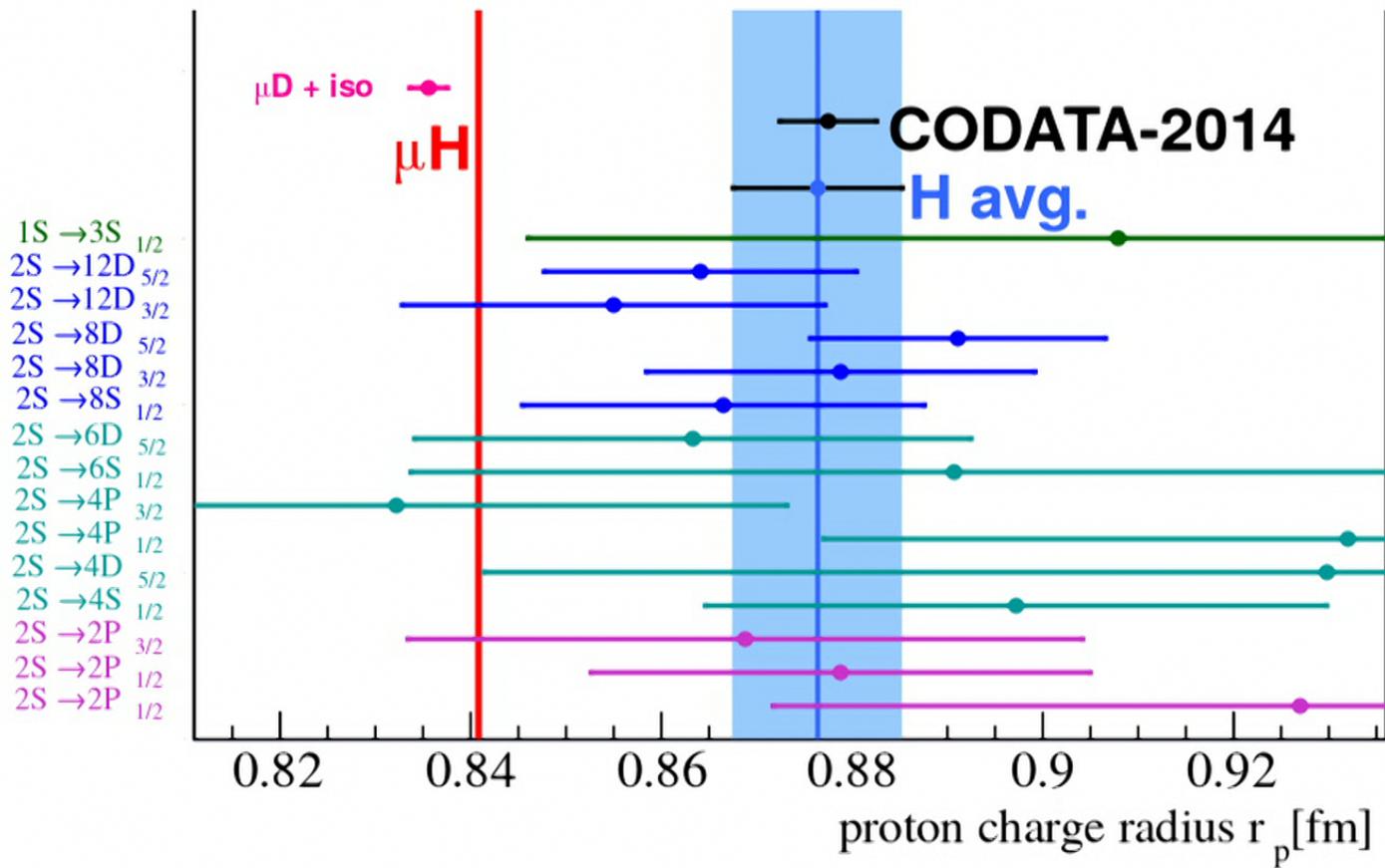
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2 \text{ MHz}}{n^3} \langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

proton radius

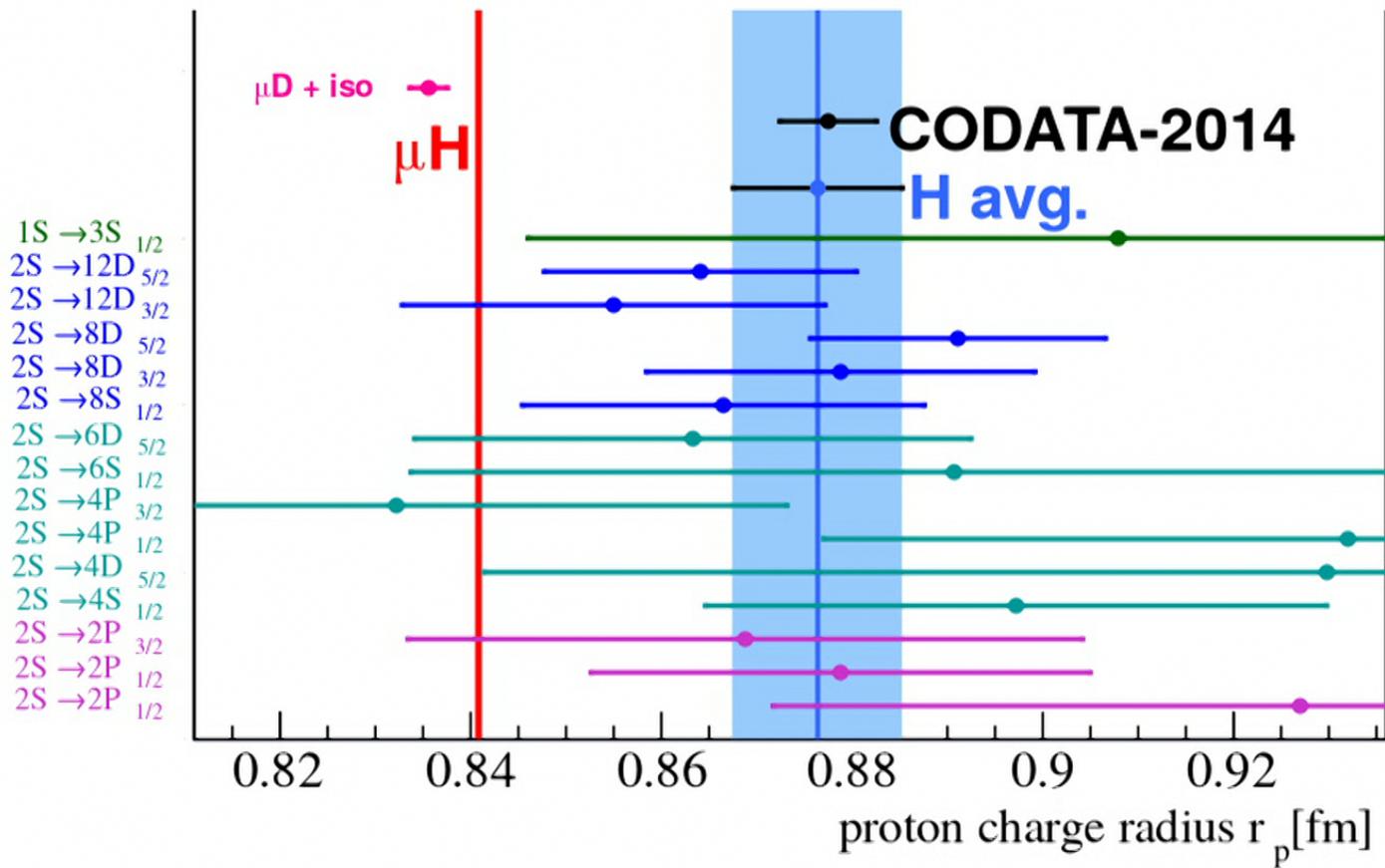
# Energy levels of hydrogen



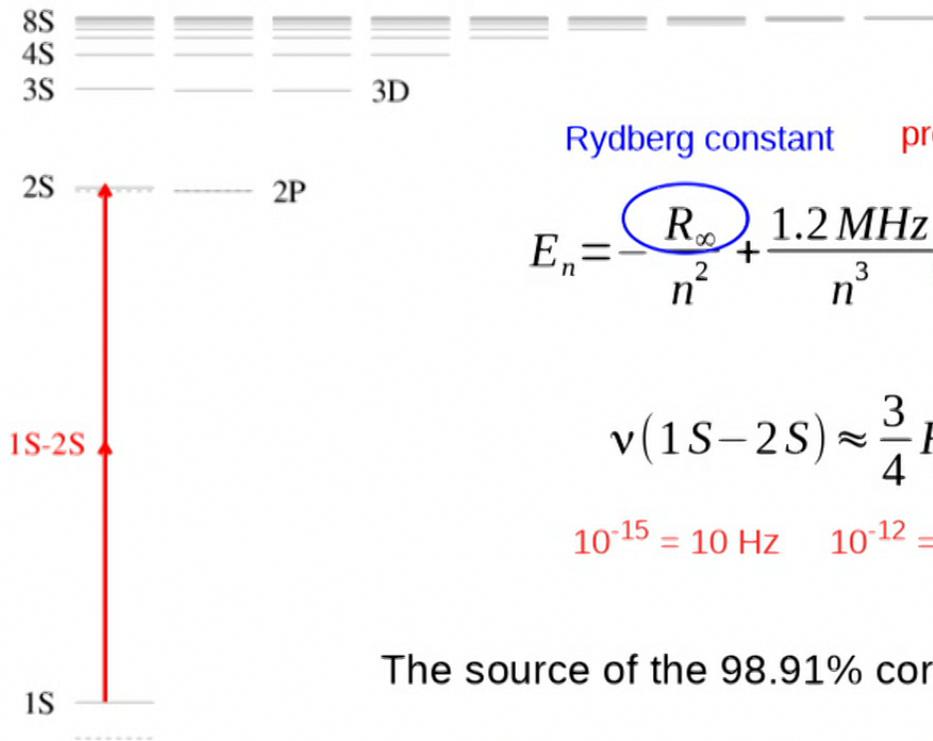
# Rp from H spectroscopy



# Rp from H spectroscopy



# Correlation between $R_\infty$ and $R_p / R_d$



Rydberg constant      proton radius

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2 \text{ MHz}}{n^3} \langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

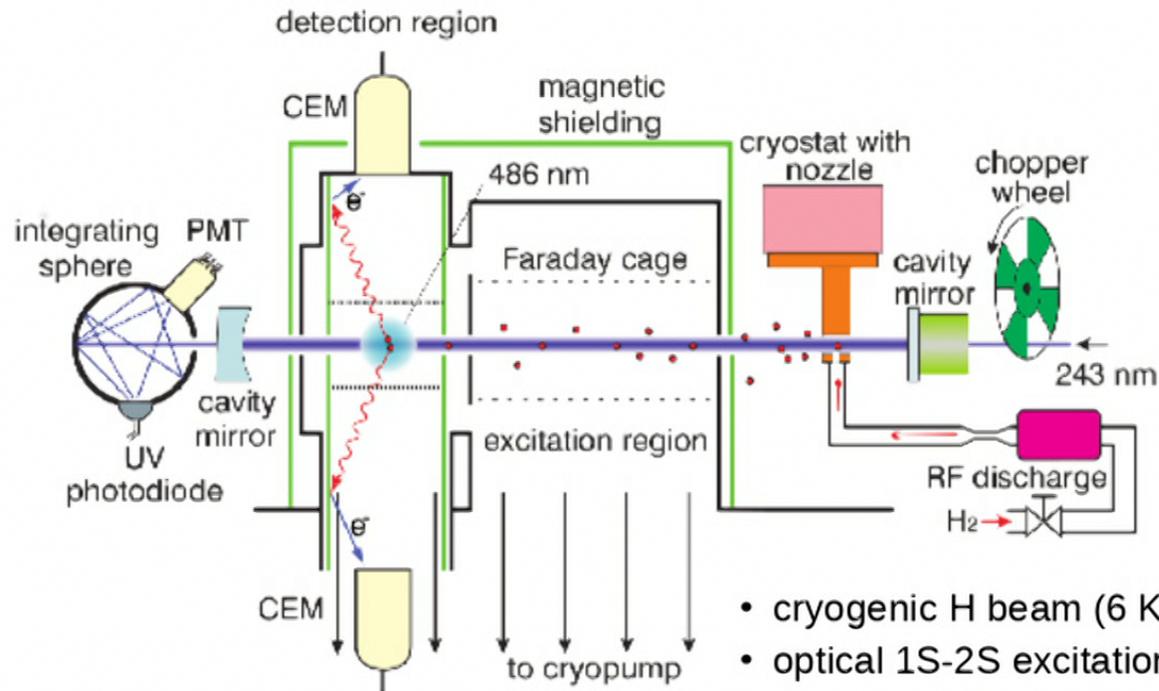
$$\nu(1S-2S) \approx \frac{3}{4} R_\infty - \frac{7}{8} E_{NS}$$

$$10^{-15} = 10 \text{ Hz} \quad 10^{-12} = 20 \text{ kHz}$$

The source of the 98.91% correlation of  $R_\infty$  and  $R_p$

1S-2S: Parthey, RP et al., PRL 107, 203001 (2011)

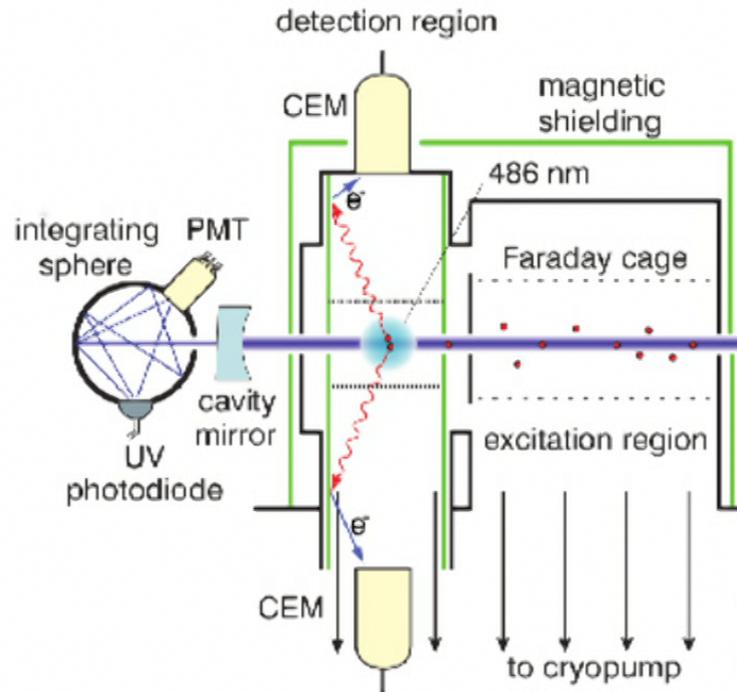
# Garching H(2S-4P)



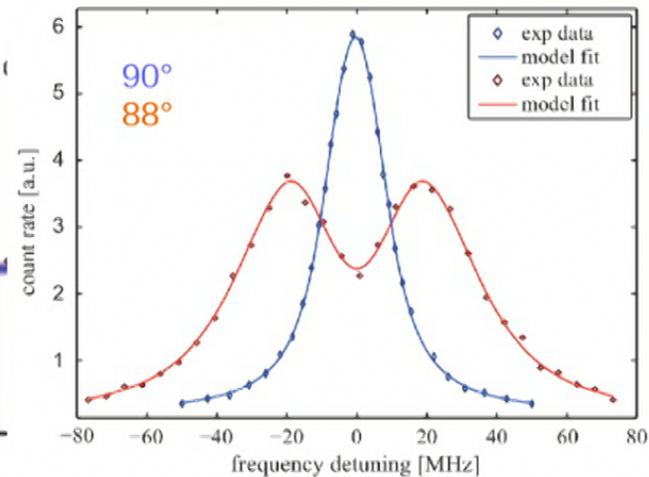
- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to  $10^{-4}$ !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

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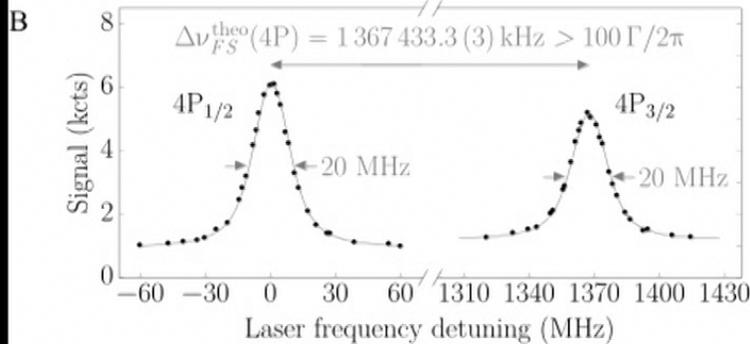
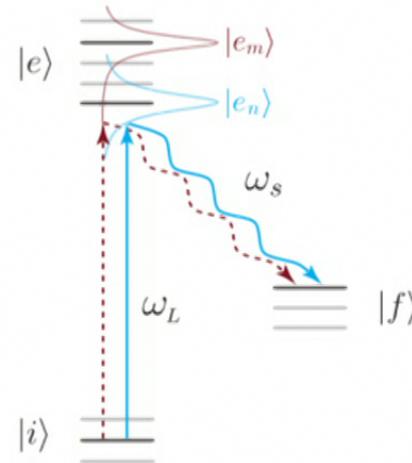
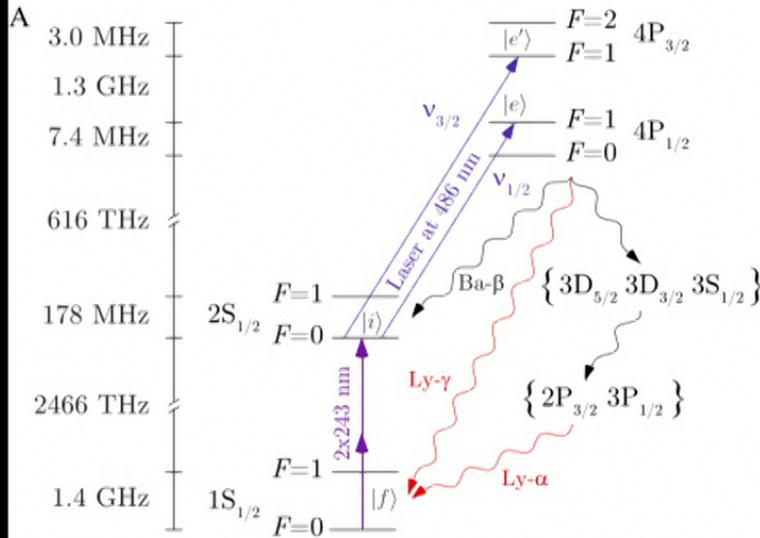
1<sup>st</sup> order Doppler cancellation



- cryogenic H beam (6 K)
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- split line to  $10^{-4}$ !!!
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Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

# Quantum interference shifts



$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0) \vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0) \vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

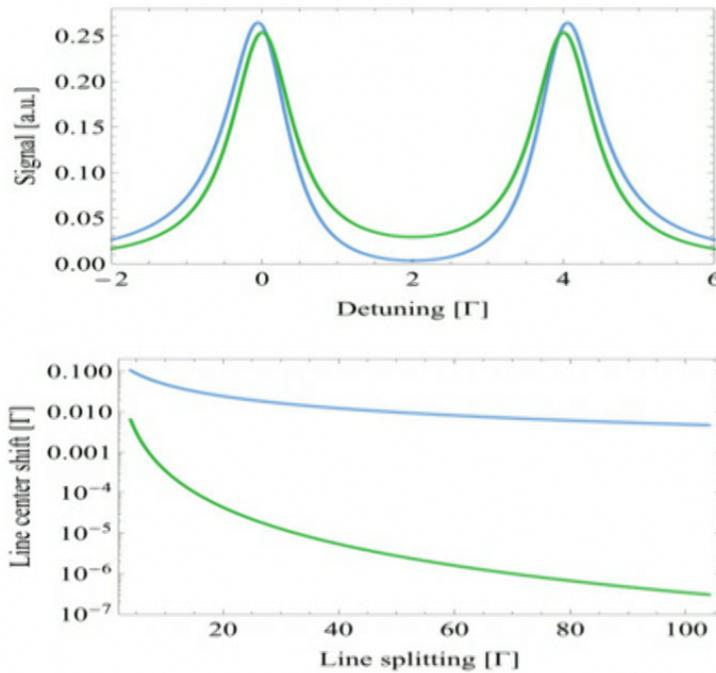
= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

see

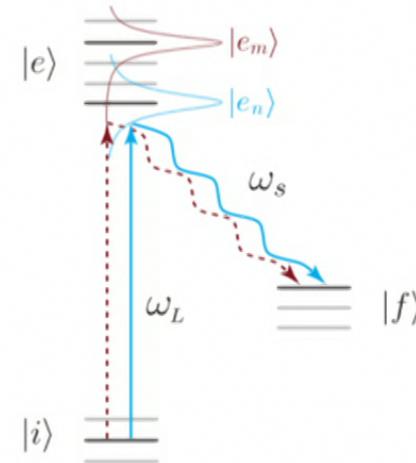
Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)  
 Sansonetti et al., PRL 107, 021001 (2011)  
 Brown et al., PRA 87, 032504 (2013)

# Quantum interference shifts



Fitting this with 2 Lorentzians creates

**line shifts**



$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0) \vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0) \vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

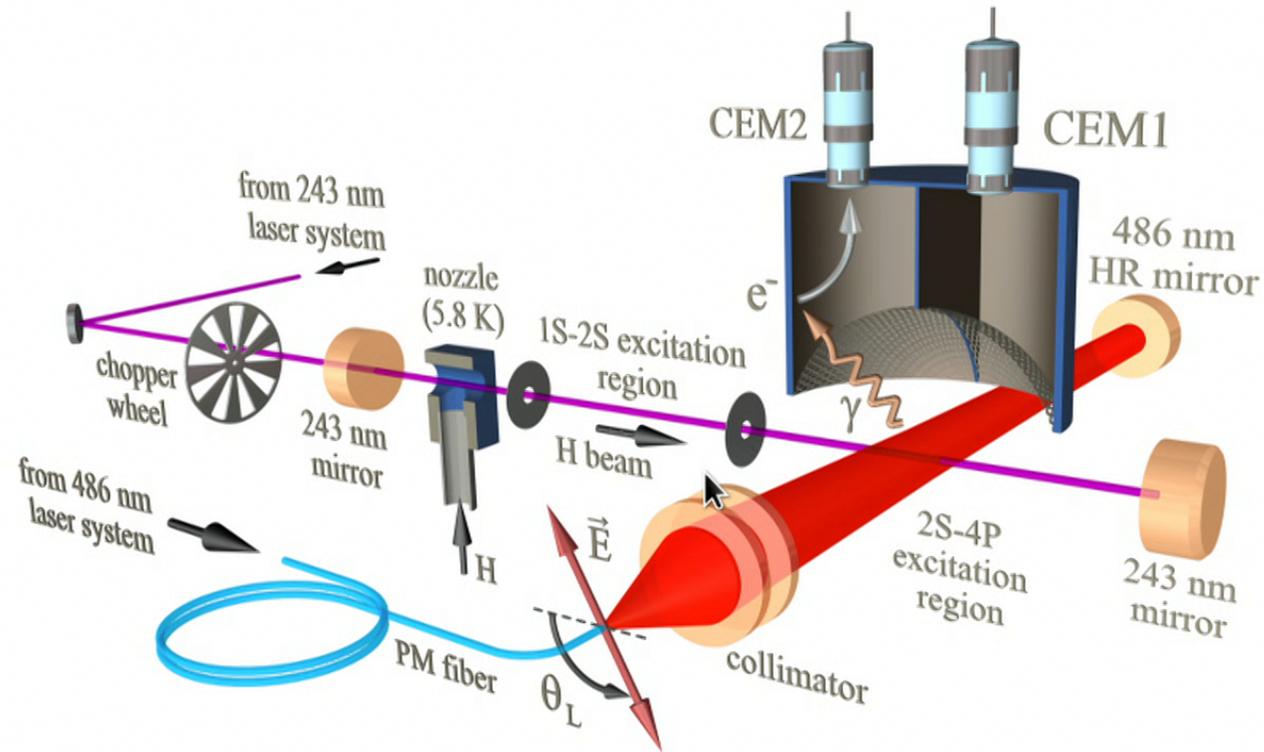
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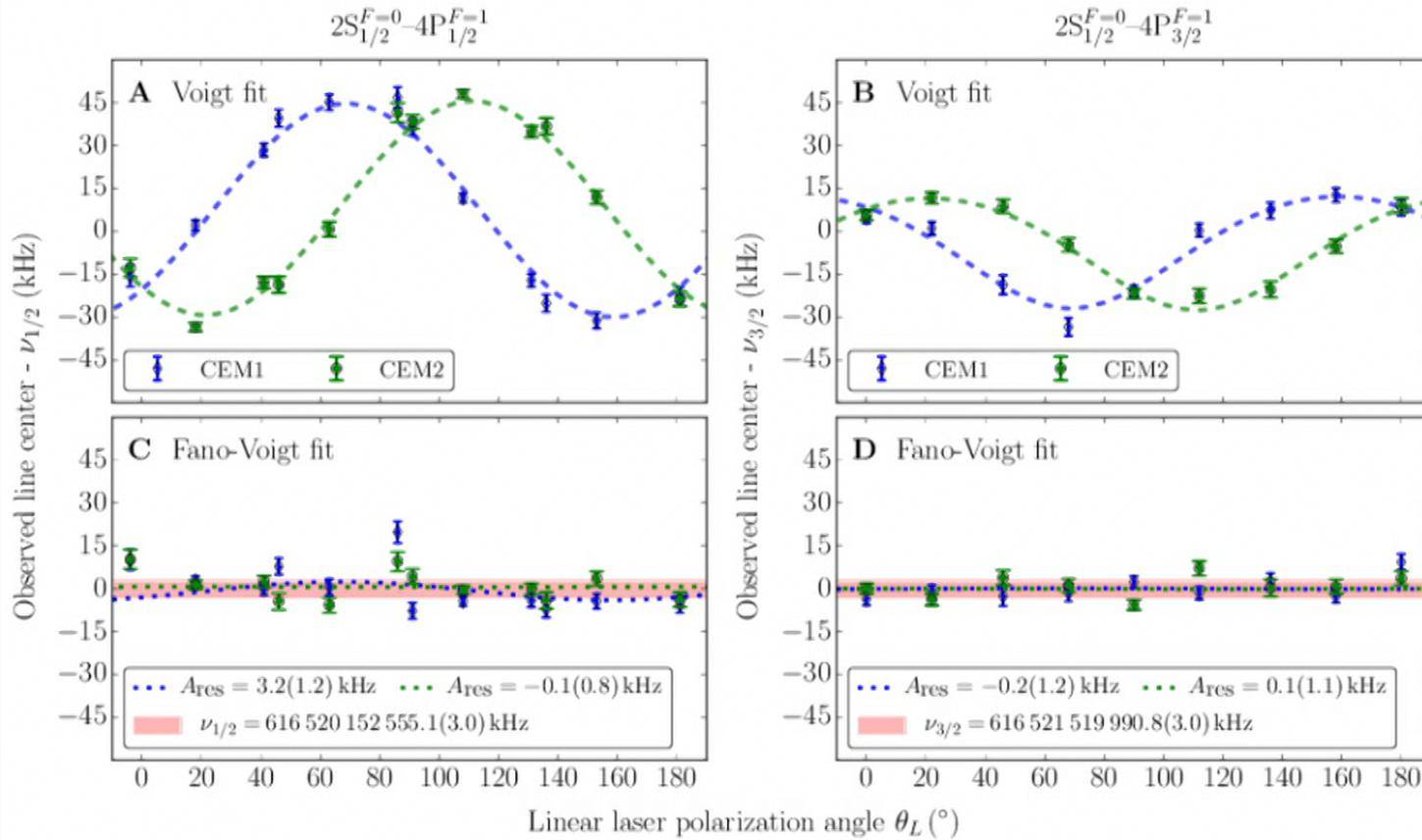
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 Brown et al., PRA 87, 032504 (2013)

# Studying QI in 2S-4P



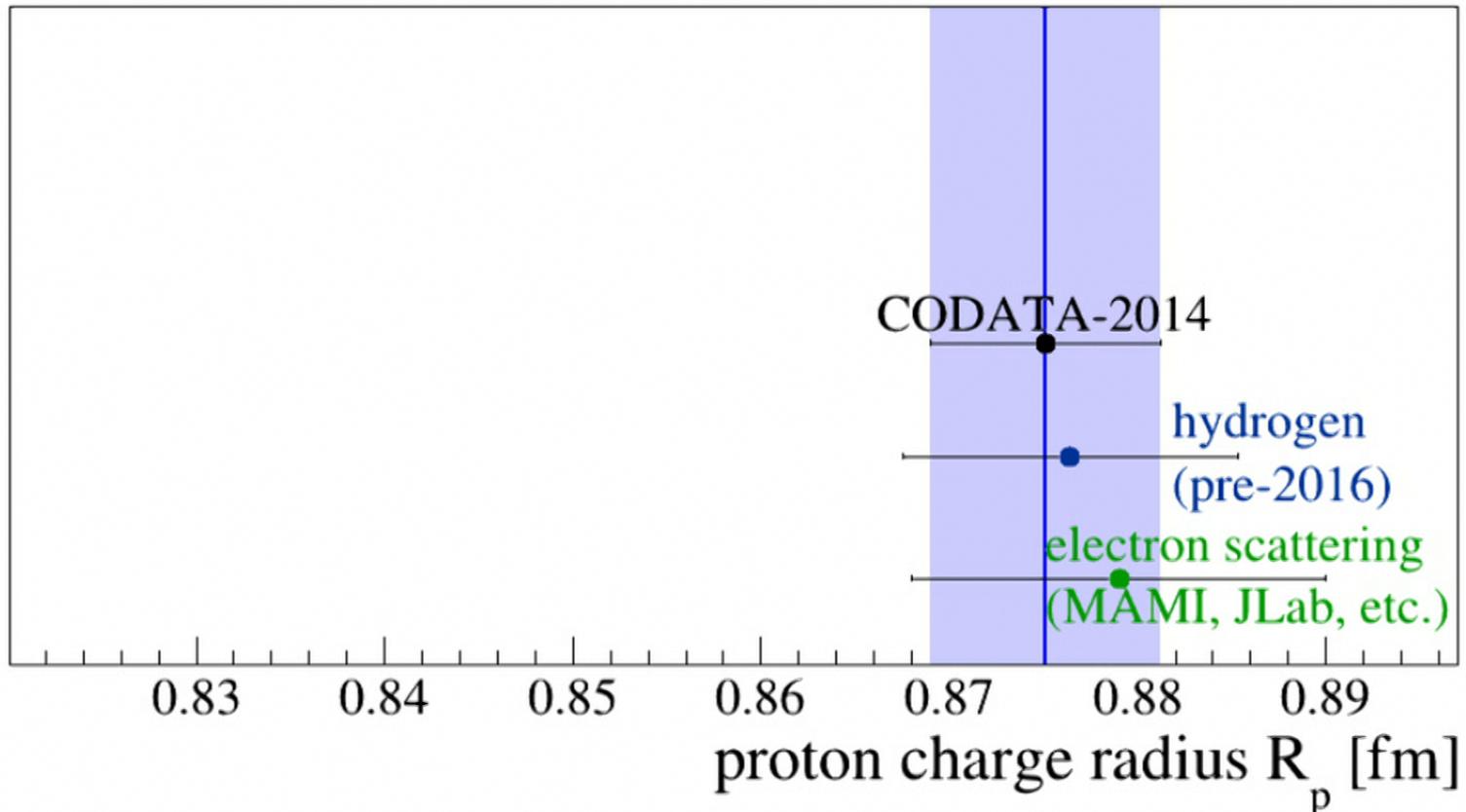
# QI in hydrogen ( $\Delta = 100 \Gamma$ )



Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

# The “old” measurements

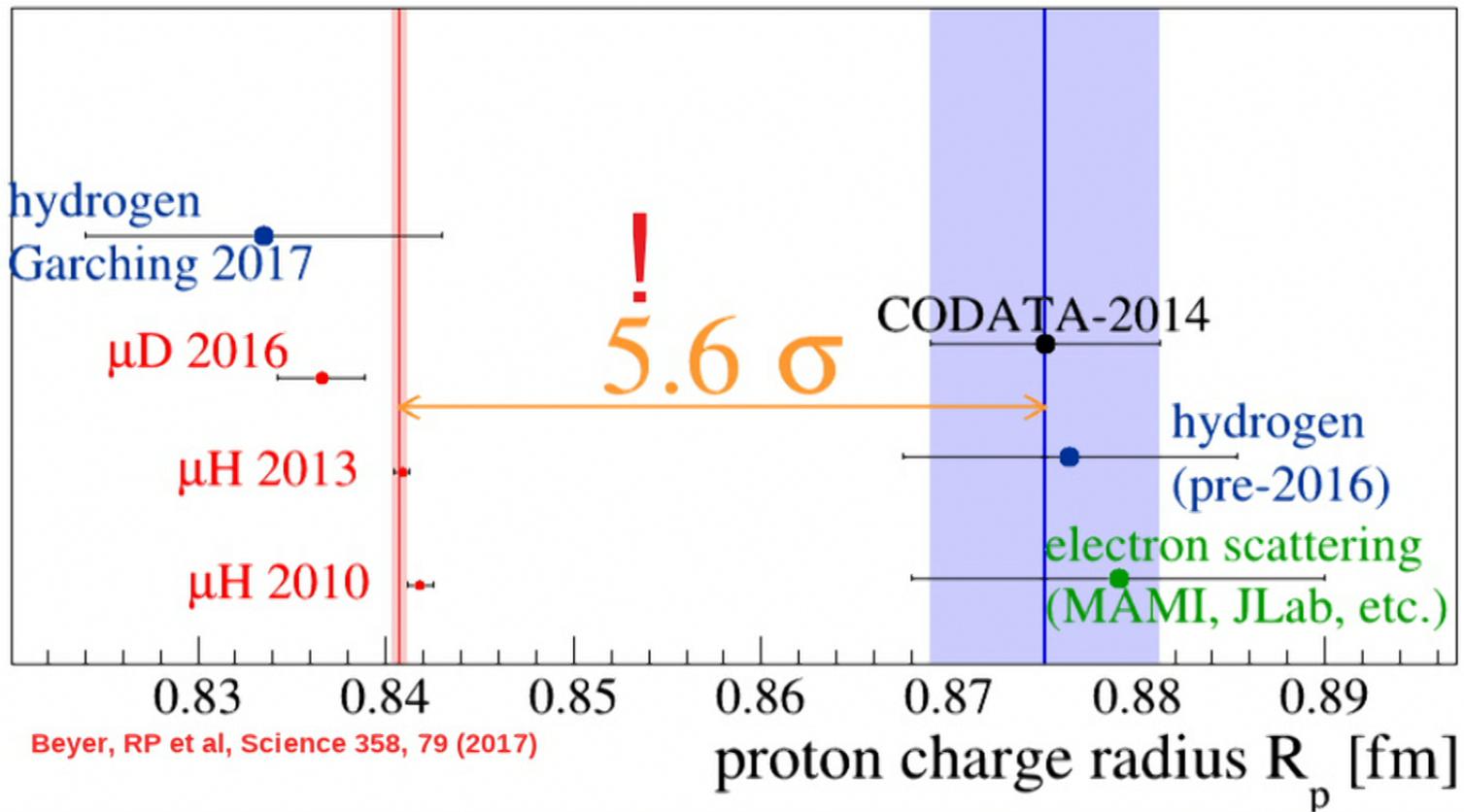
## Electrons



# New Measurements

## Muons

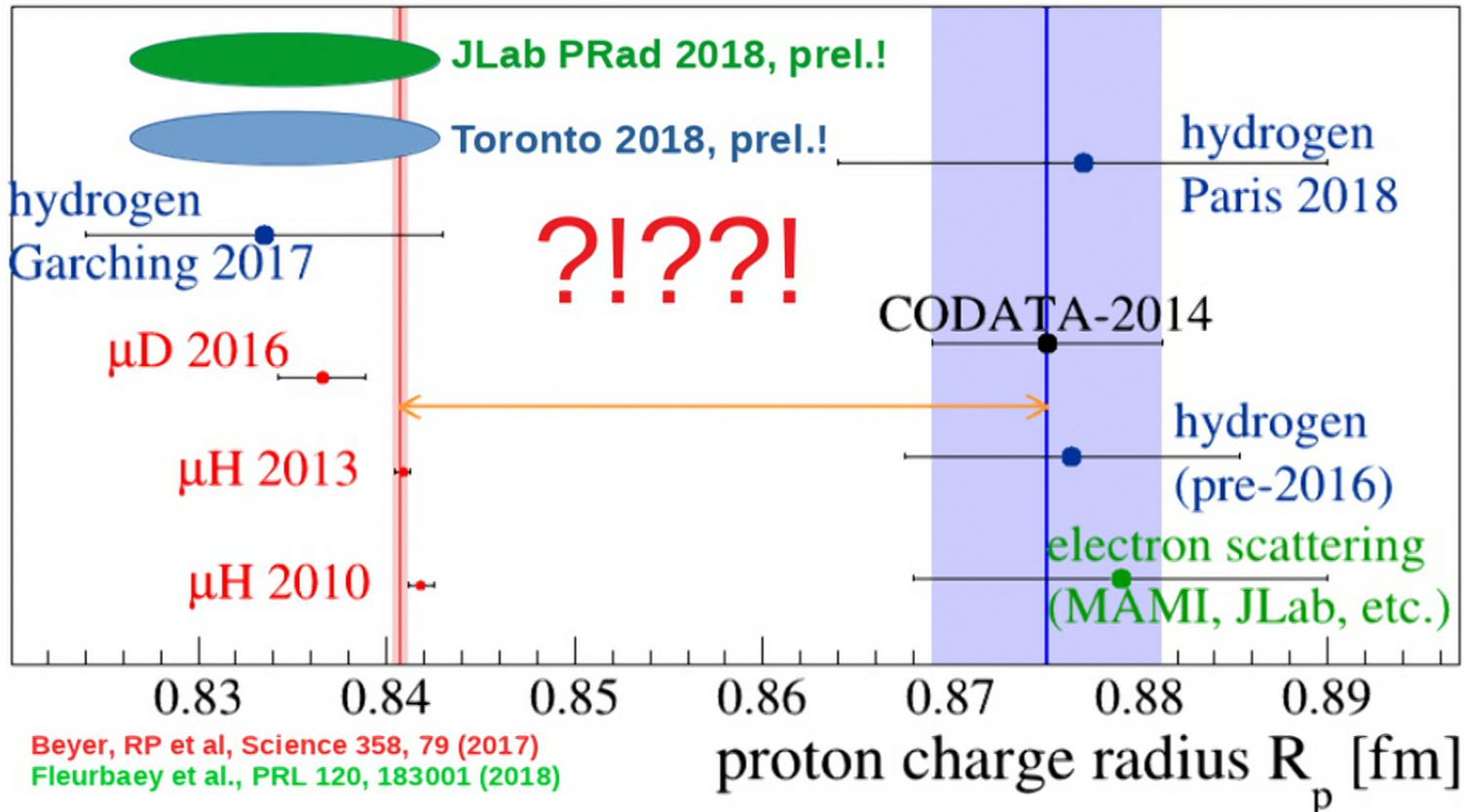
## Electrons



# New Measurements

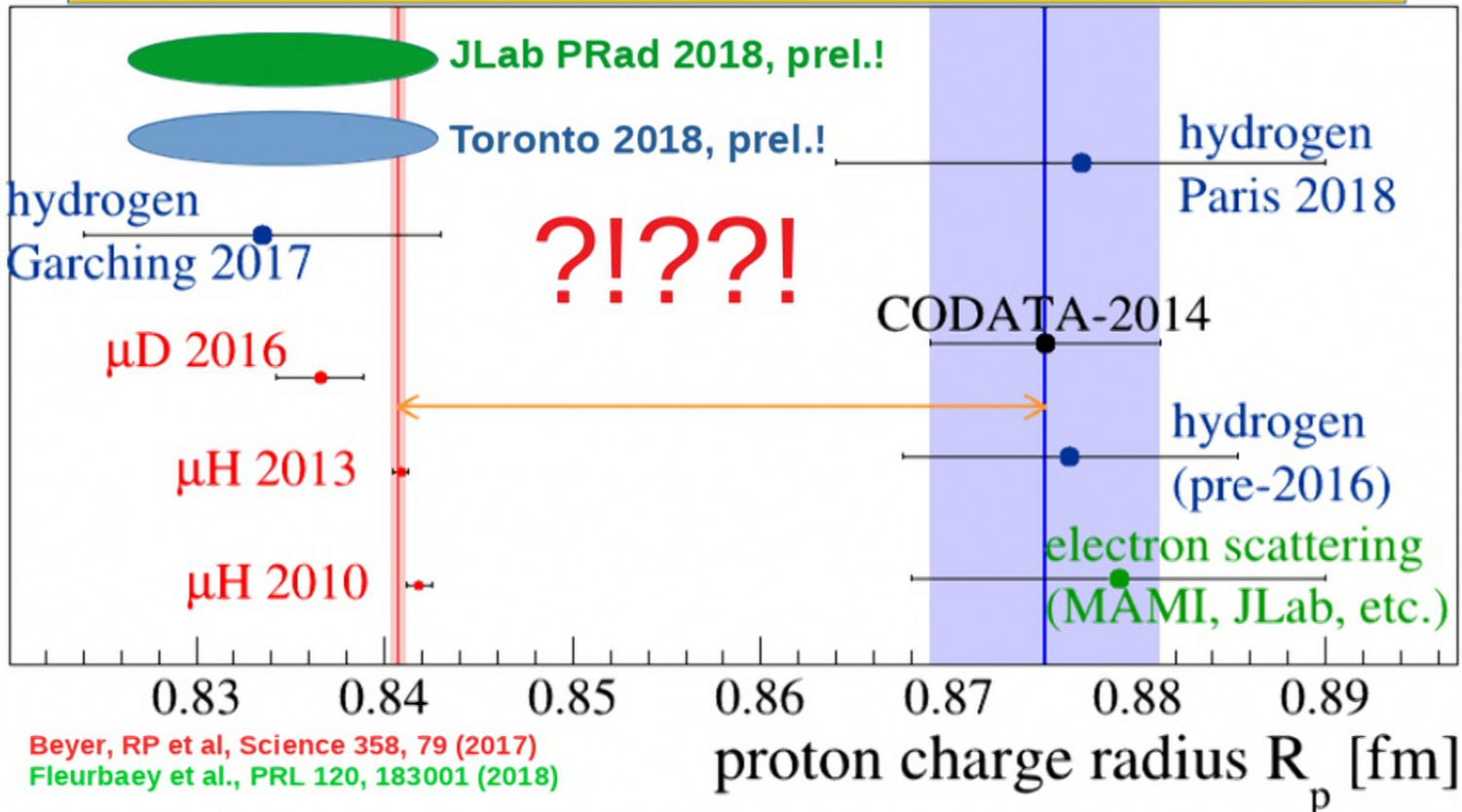
## Muons

## Electrons



We have a "Rydberg problem" AND a scattering problem?!?!?!?

→ need even more data!

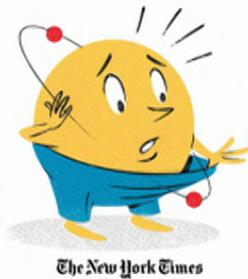
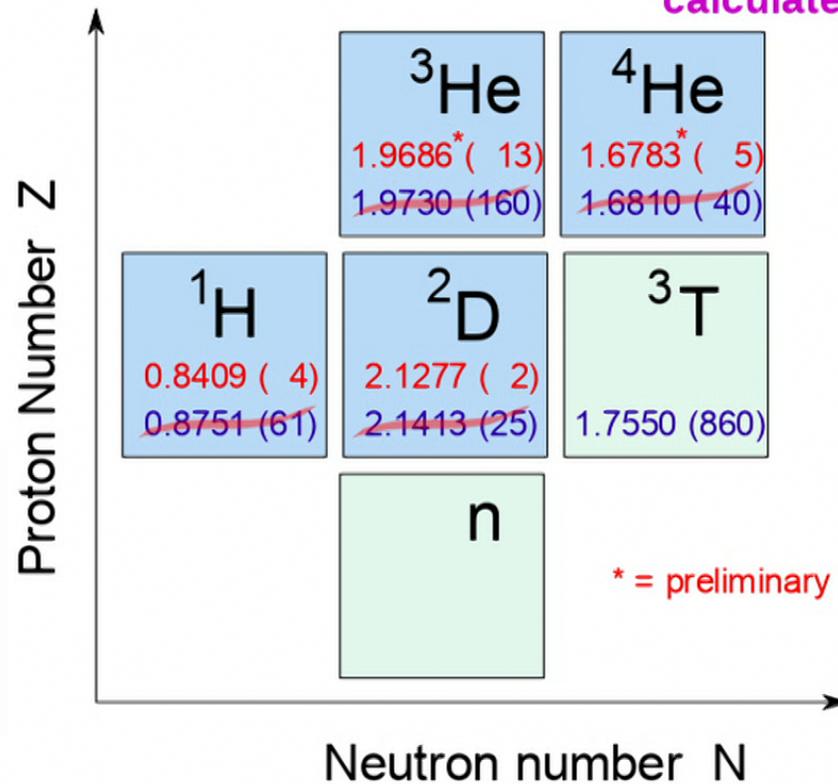


# Conclusions

Muonic atoms / ions provide:

- **10x more accurate charge radii**, when combined with

calculated polarizability



# Conclusions

Muonic atoms / ions provide:

- **10x more accurate charge radii**, when combined with  
calculated polarizability
- few times more accurate **nuclear polarizability**,  
when combined with **charge radius from regular atoms**

**Muonic atoms are a novel tool for proton and new-nucleon properties!**

# Conclusions

Proton radius situation:

- smaller radii from **muonic hydrogen** and **deuterium** imply a **smaller Rydberg** constant
- new H(2S-4P) gives a **smaller proton radius**
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## More data needed:

- H(2S – 6P, 8P, **9P**, ...) and D(2S-nl) underway in Garching and Colorado
- H(1S – 3S, 4S, ..) underway in Paris and Garching
- H(2S-2P) (Hessels @ Toronto)
- Muonium at PSI, J-PARC
- Positronium (Cassidy @ UCL, Crivelli @ ETH)
- He<sup>+</sup>(1S-2S) underway in Garching (Udem) and Amsterdam (Eikema)
- HD<sup>+</sup>, H<sub>2</sub>, etc. in Amsterdam (Ubachs @ Amsterdam) and Paris (Hilico, Karr @ Paris)
- He (Vassen @ Amsterdam), Li<sup>+</sup> (Udem @ Garching)
- HCl, e.g. H-like Ne (Tan @ NIST)
- Rydberg-atoms, e.g. Rb (Raithel @ Ann Arbor)
- new low-Q<sup>2</sup> electron scattering at MAMI, JLab, MESA
- muon scattering: MUSE @ PSI, COMPASS @ CERN

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**Compare Rydberg values  
to test QED and SM**

# Up next: Hyperfine structure in $\mu\text{p}$

The **21 cm line** in hydrogen (1S hyperfine splitting) has been **measured to 12 digits** (0.001 Hz) in **1971**:

$$\nu_{\text{exp}} = 1\,420\,405.751\,766\,7 \pm 0.000\,001 \text{ kHz}$$

Essen et al., Nature 229, 110 (1971)

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QED test is limited to **6 digits** (800 Hz) because of **proton structure** effects:

$$\nu_{\text{theo}} = 1\,420\,403.1 \pm 0.6_{\text{proton size}} \pm 0.4_{\text{polarizability}} \text{ kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

# Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha)m\langle r \rangle_{(2)} E_F$$

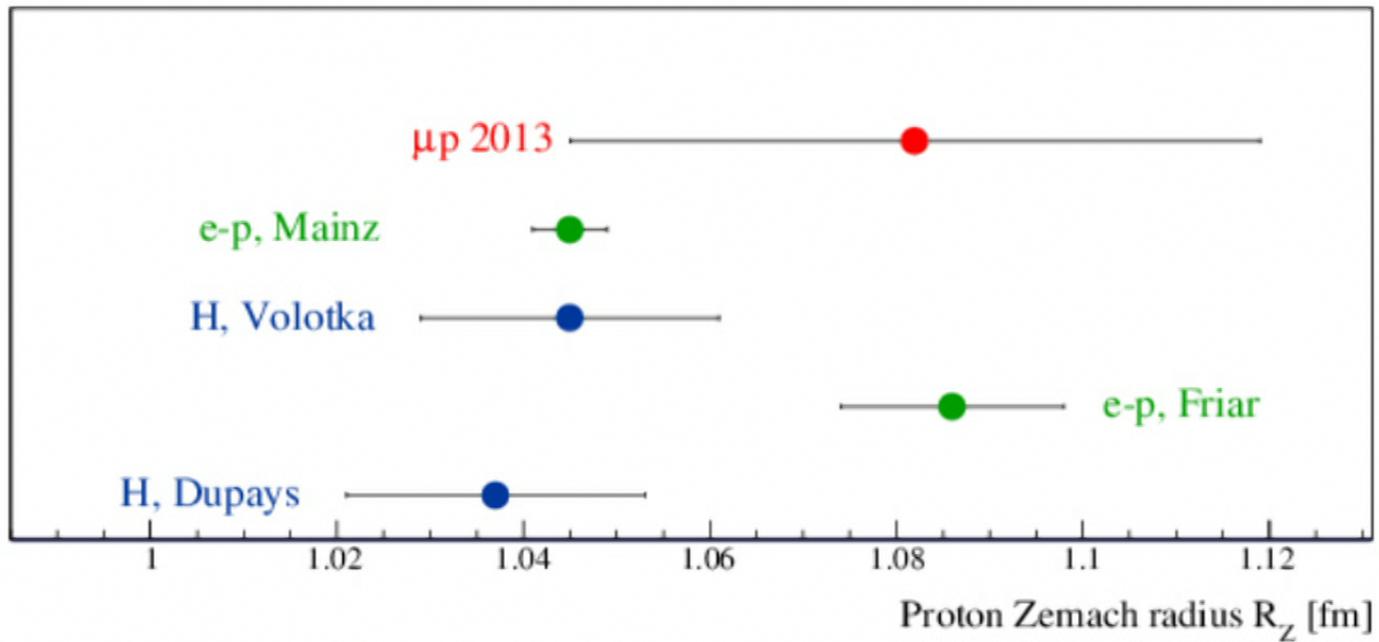
$$\langle r \rangle_{(2)} = \int d^3r d^3r' \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

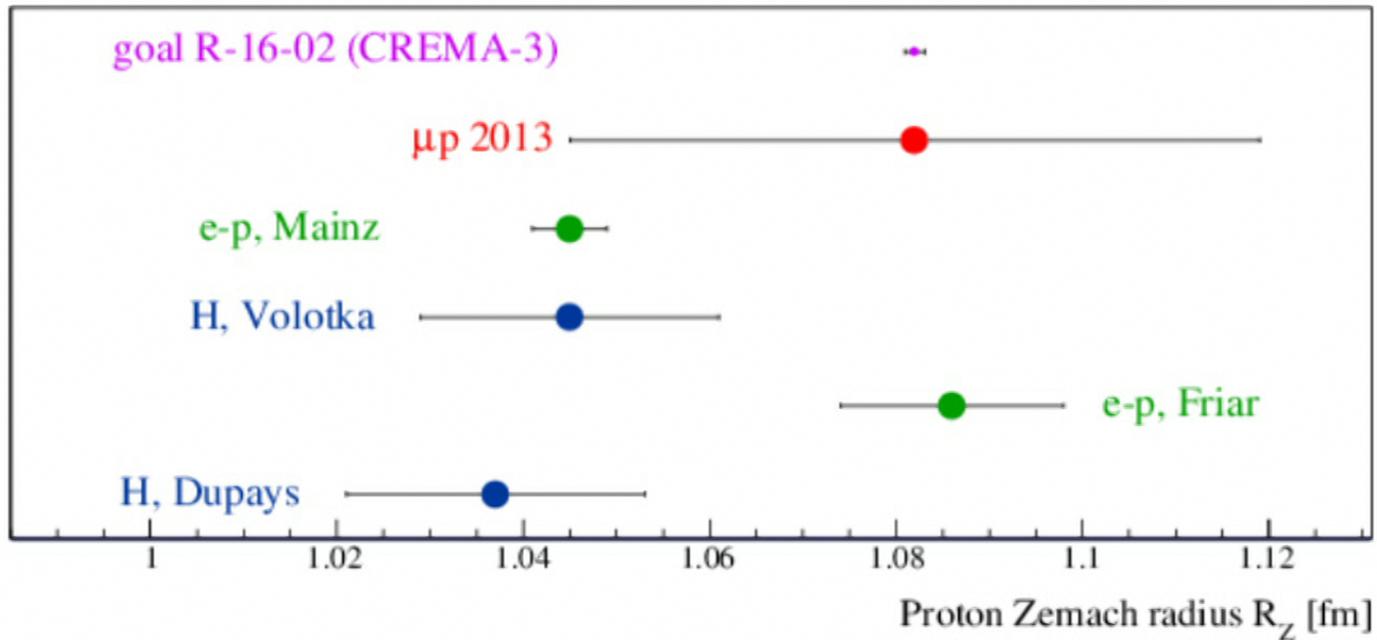
Form factors and momentum space

$$\Delta E = \frac{8(Z\alpha)m}{\pi n^3} E_F \int_0^\infty \frac{dk}{k^2} \left[ \frac{G_E(-k^2) G_M(-k^2)}{1 + \kappa} \right]$$

# Proton Zemach radius from $\mu p$



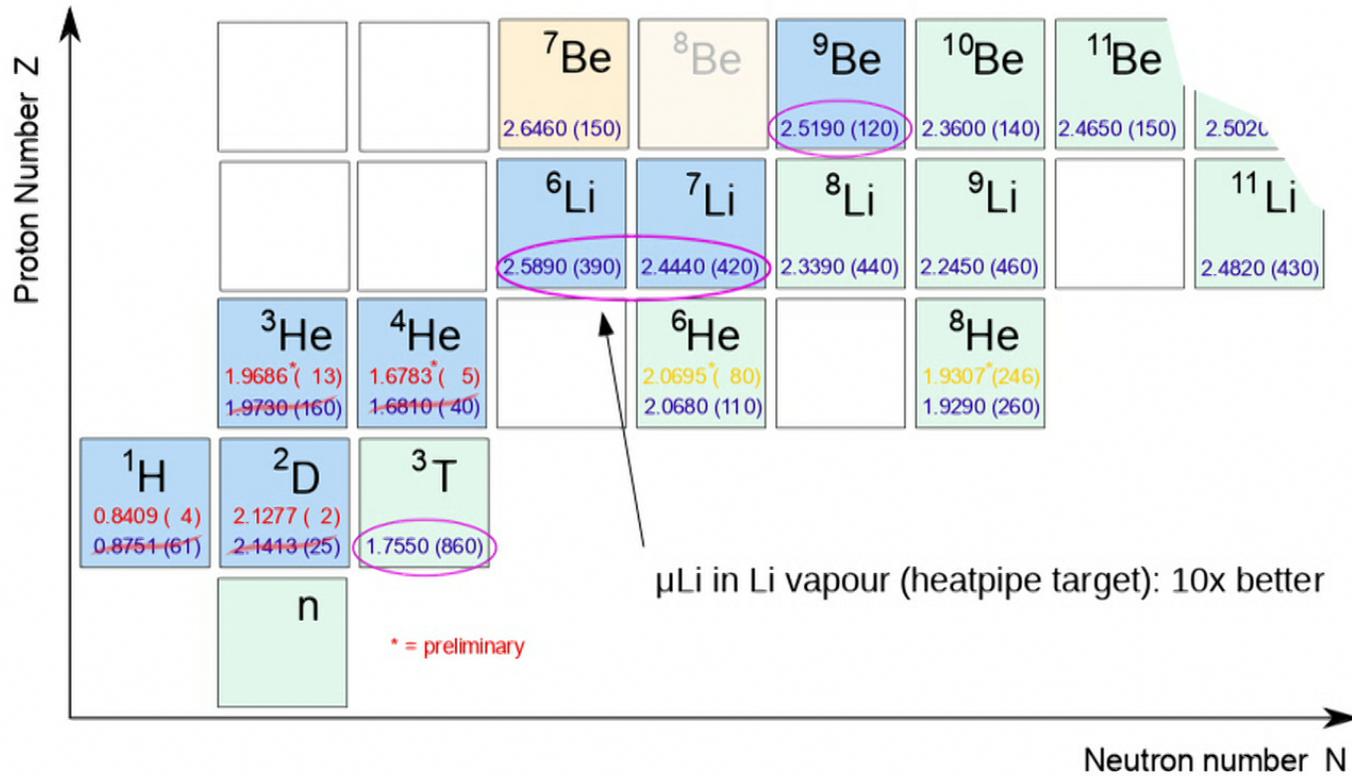
# Proton Zemach radius from $\mu p$



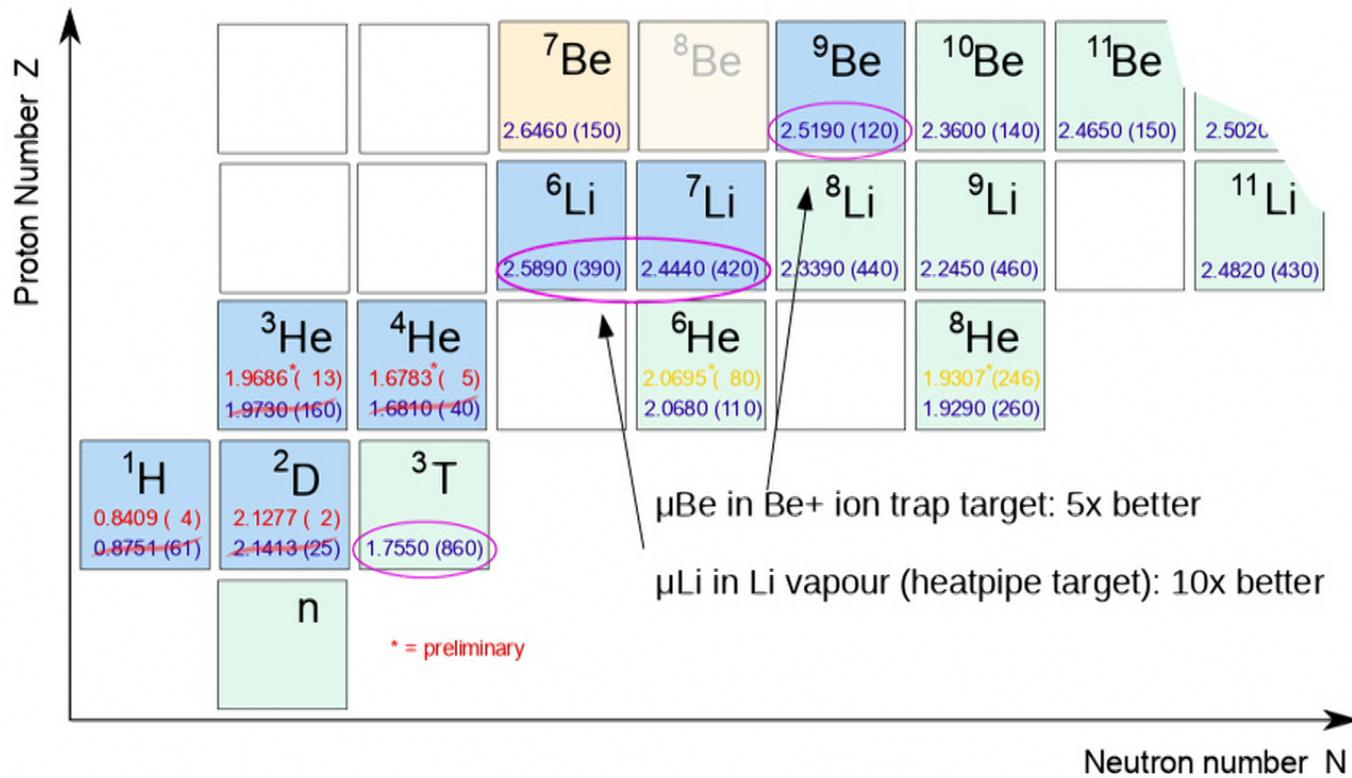
PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

see e.g. Schmidt, RP et al., arXiv 1808.07240

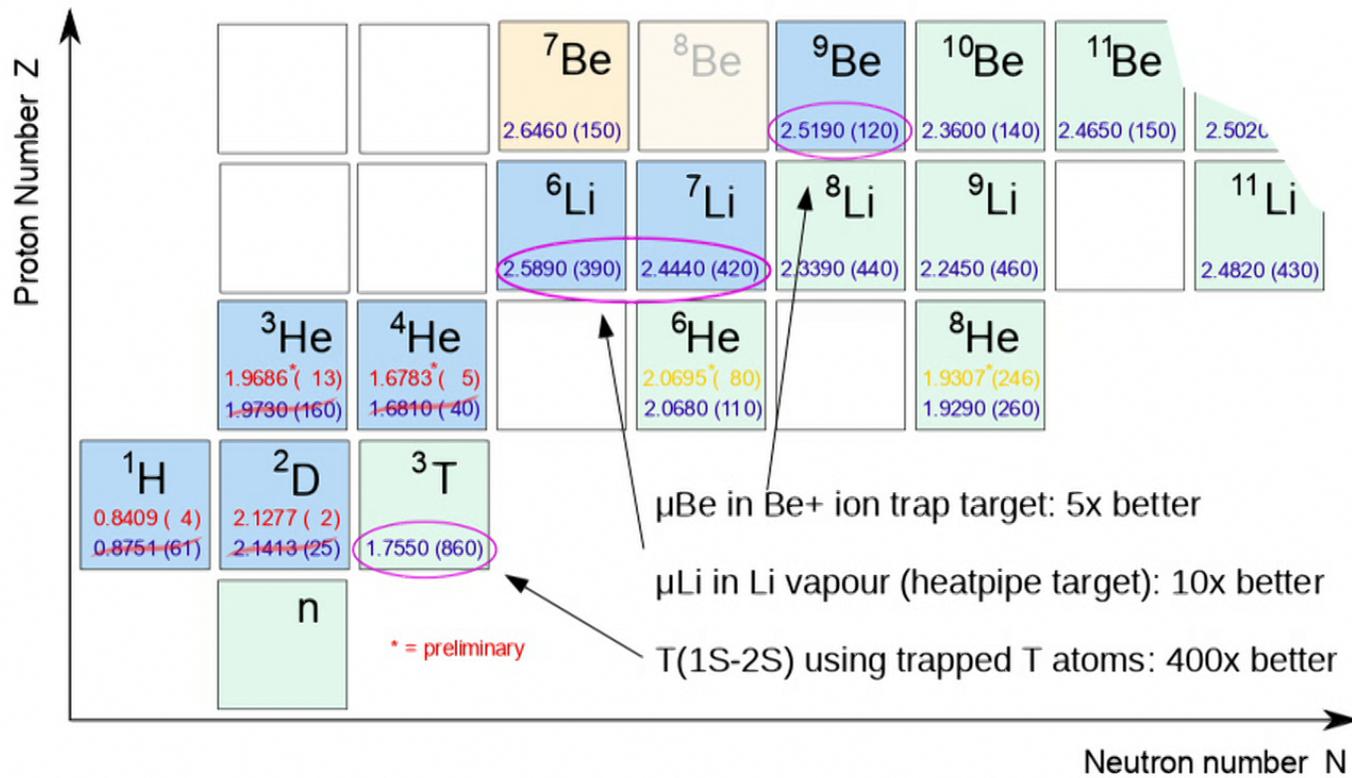
# Charge radii: The future

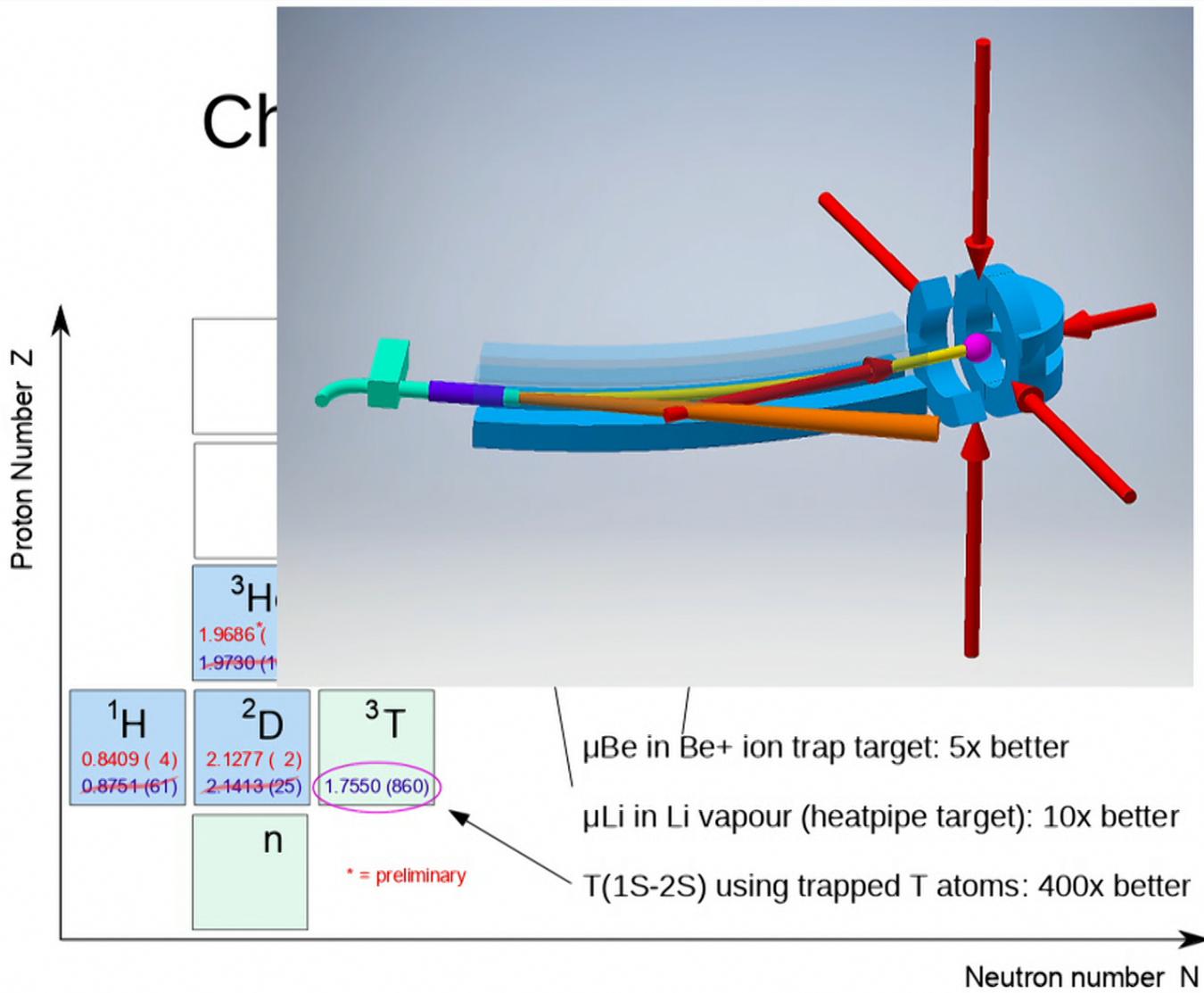


# Charge radii: The future



# Charge radii: The future





# Thanks a lot for your attention

## The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP,  
Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost,  
Theodor W. Hänsch, Nikolai Kolachevsky, Thomas Udem

## The CREMA Collaboration:

Aldo Antognini, Fernando D. Amaro, François Biraben, João M. R. Cardoso,  
Daniel S. Covita, Andreas Dax, Satish Dhawan, Marc Diepold, Luis M. P.  
Fernandes, Adolf Giesen, Andrea L. Gouvea, Thomas Graf, Theodor W.  
Hänsch, Paul Indelicato, Lucile Julien, Paul Knowles, Franz Kottmann, Juilian  
J. Krauth, Eric-Olivier Le Bigot, Yi-Wei Liu, José A. M. Lopes, Livia Ludhova,  
Cristina M. B. Monteiro, Françoise Mulhauser, Tobias Nebel, François Nez,  
Paul Rabinowitz, Joaquim M. F. dos Santos, Lukas A. Schaller, Karsten  
Schuhmann, Catherine Schwob, David Taqqu, João F. C. A. Veloso, RP

# Thanks a lot for your attention

My new Mainz group:

Jan Haack, Merten Heppener, Rishi Horn, Ahmed Ouf, Stefan Schmidt, Gregor Schwendler, Lukas Schumacher, Andreas Wieltsch, Marcel Willig

The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP, Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost, Theodor W. Hänsch, Nikolai Kolachevsky, Thomas Udem

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# Group at JGU Mainz



"old" picture from summer 2018, now + 1 PhD, +1 MSc, +2 BSc

# Charge radii: The future

