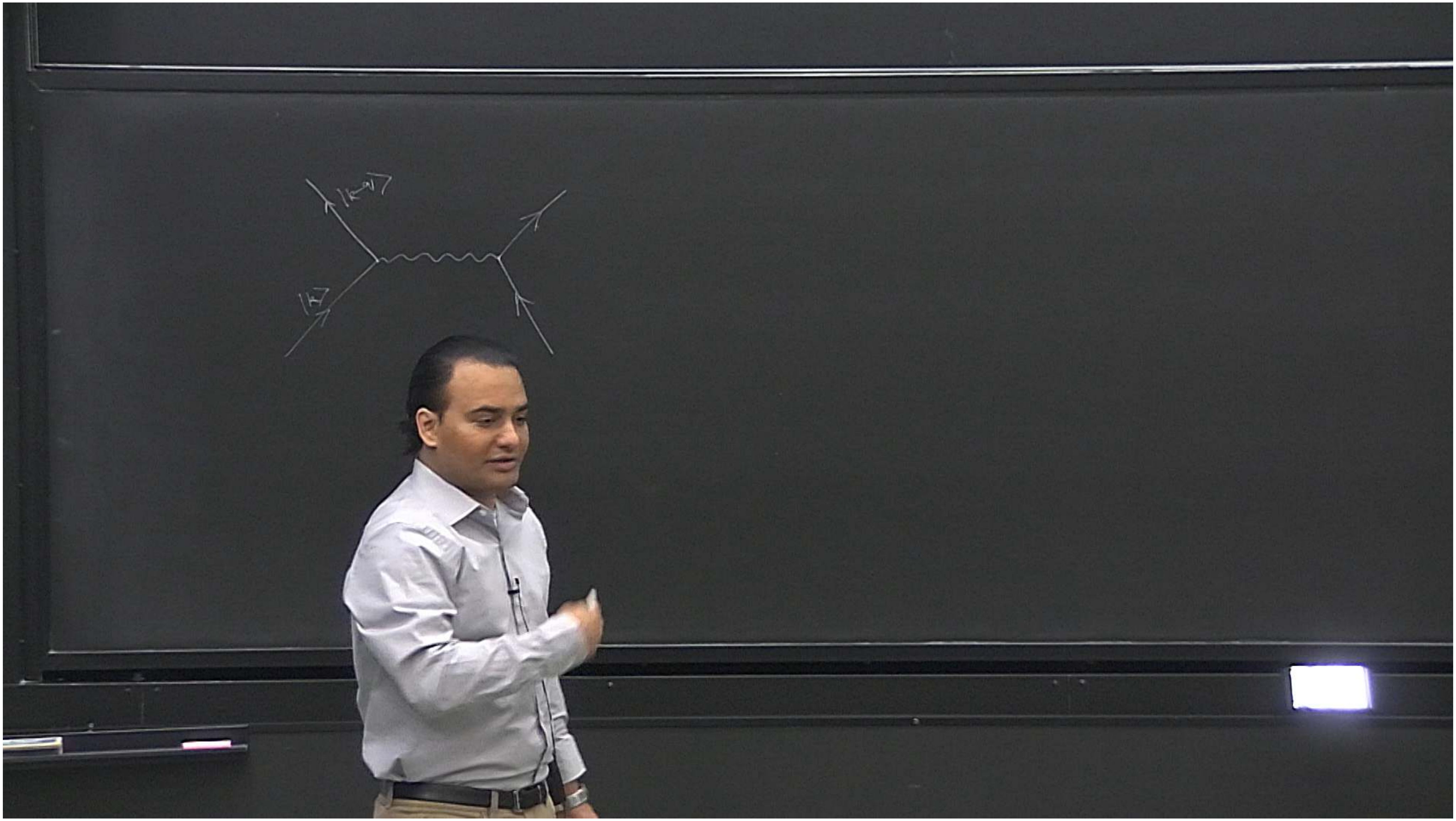
Title: PSI 2018/2019 - Condensed Matter - Lecture 13

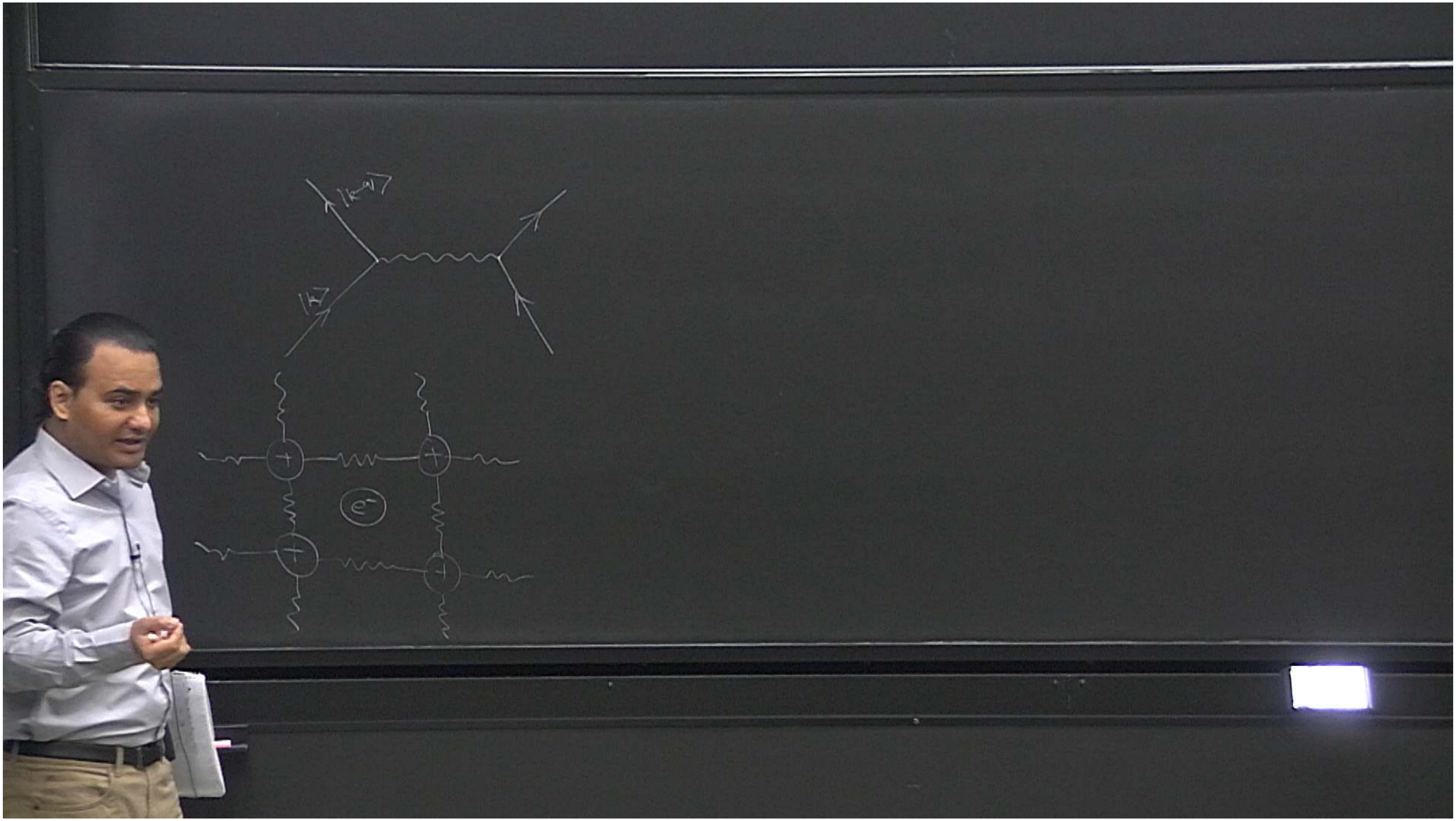
Date: Nov 28, 2018 10:45 AM

URL: http://pirsa.org/18110031

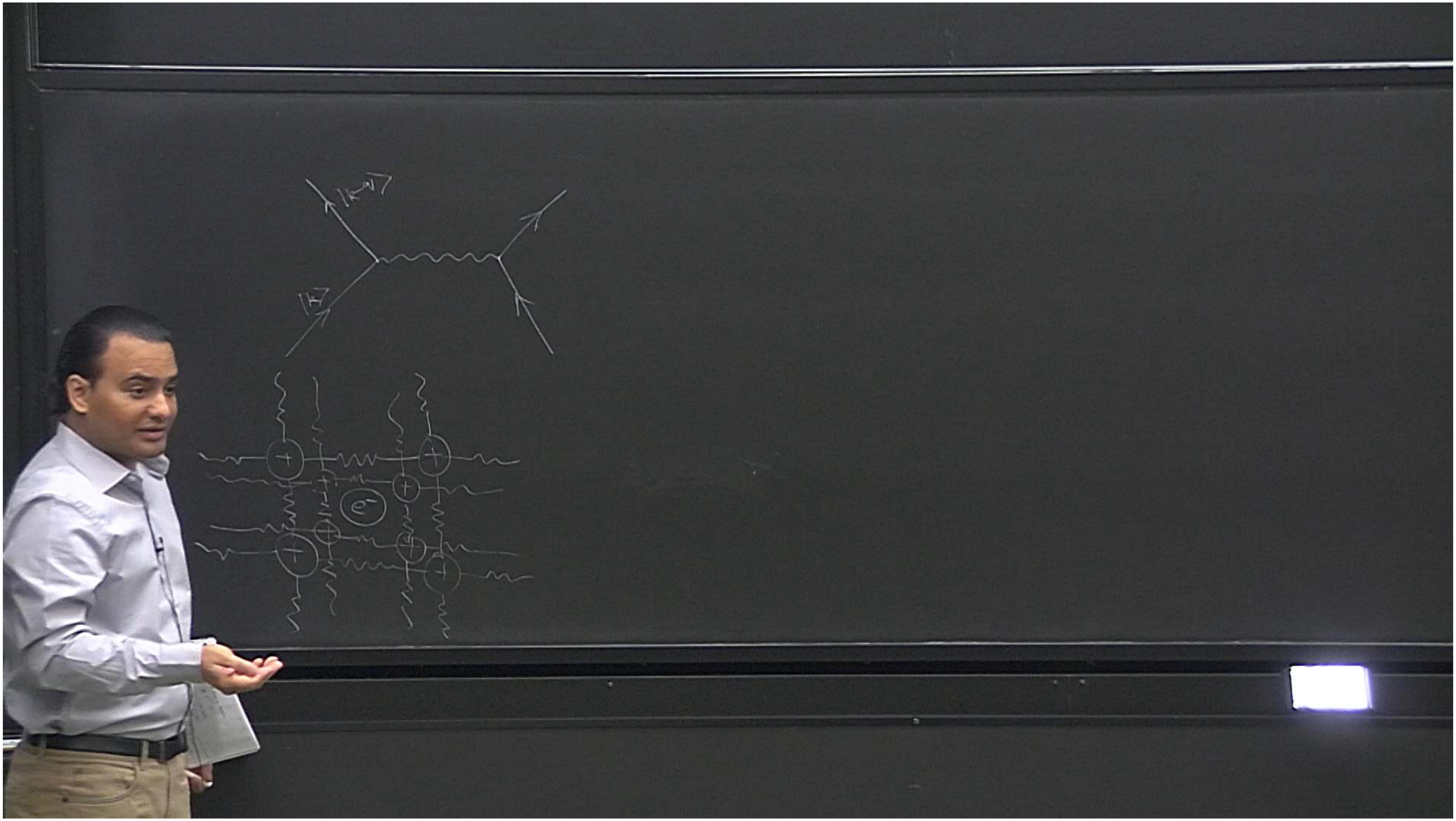
Abstract:

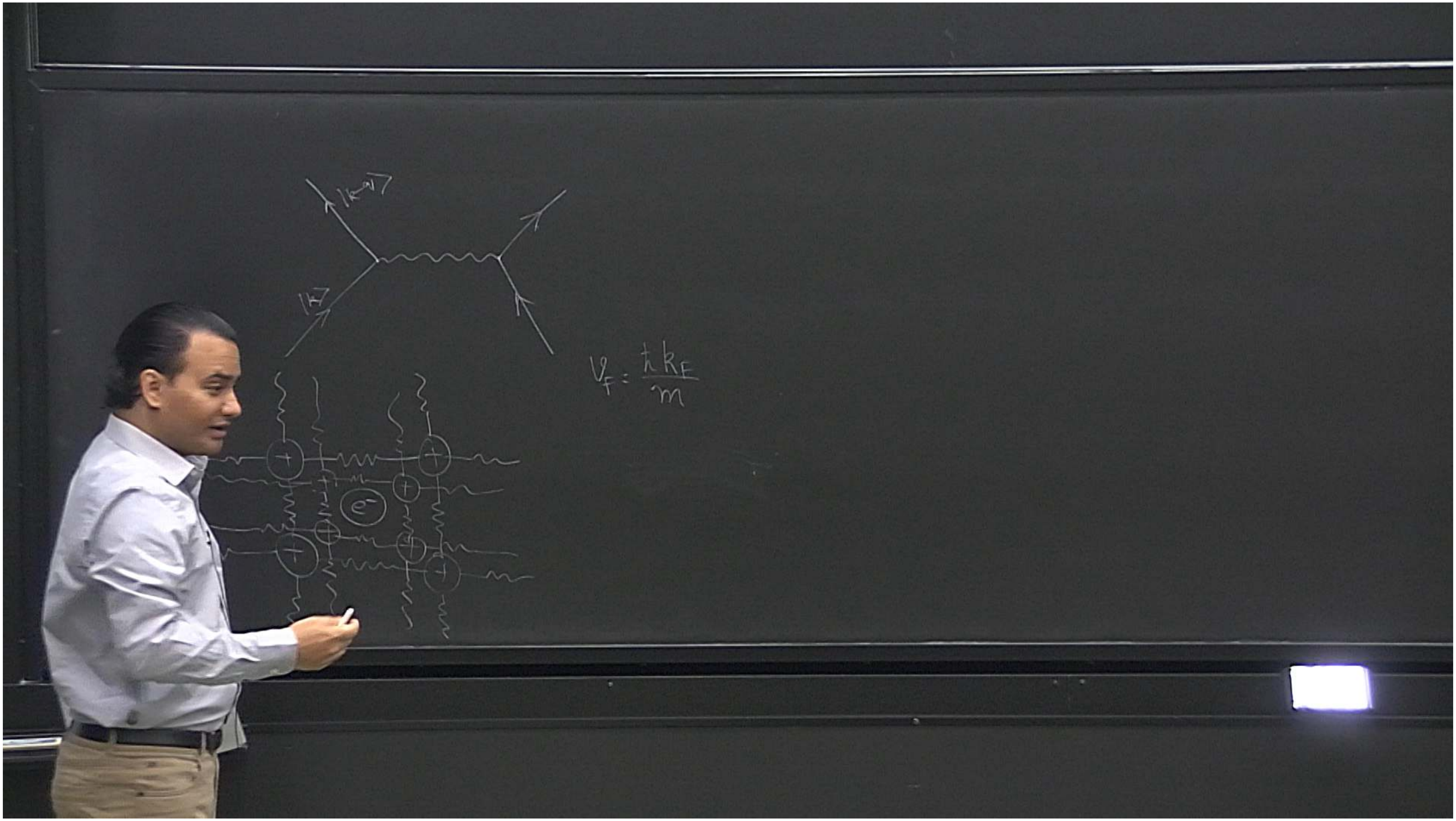


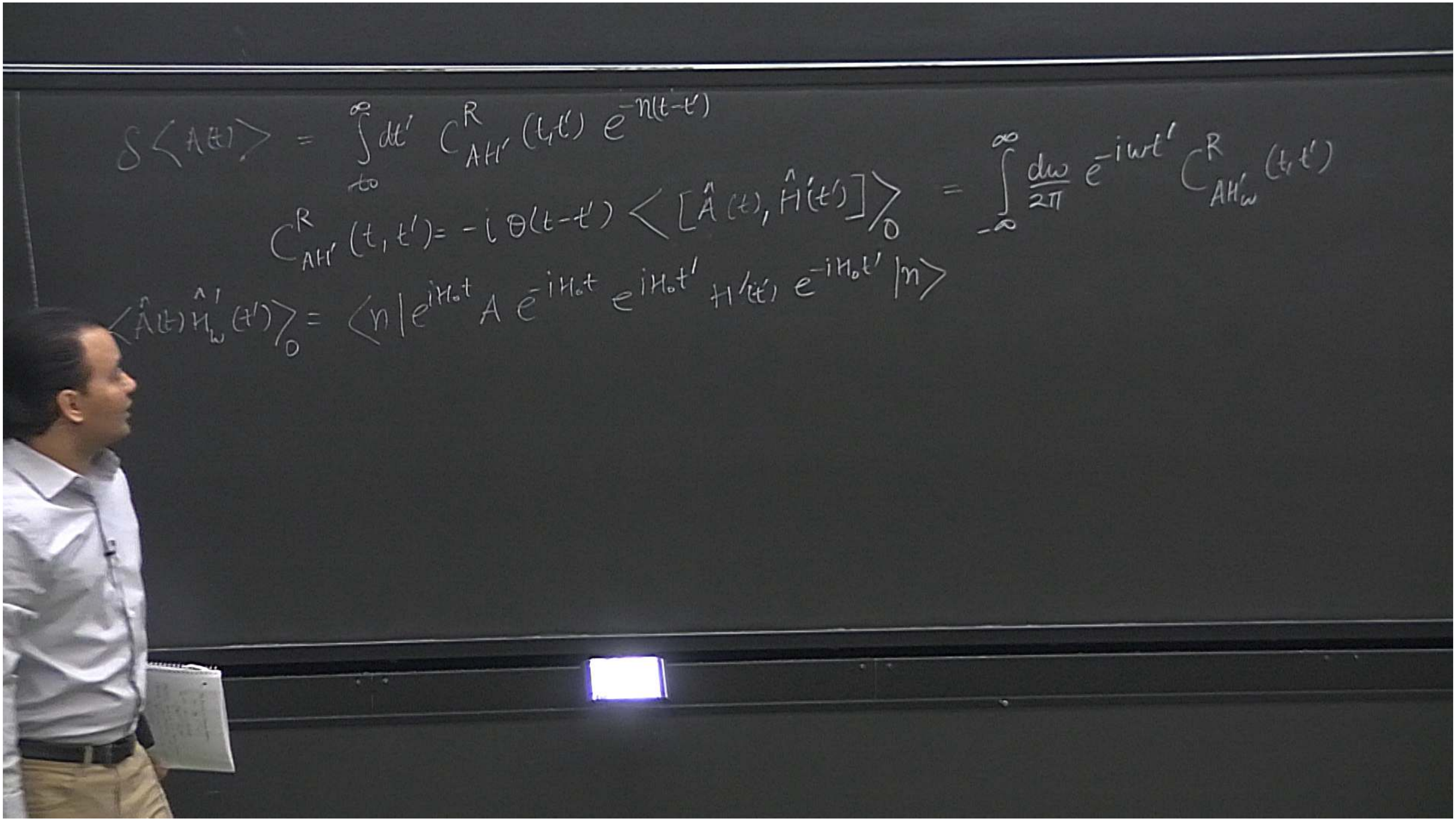
$|k\rangle$
$|k-q\rangle$



|k-q⟩
|k⟩
e⁻


$|k\rangle$
e^-


|k⟩
|k−q⟩
e⁻
v_F = ħk_F / m


$\delta\langle A(t)\rangle = \int_{t_0}^{\infty} dt' \, C^R_{AH'}(t,t') \, e^{-\eta(t-t')}$
$C^R_{AH'}(t,t') = -i\,\theta(t-t') \langle [\hat{A}(t), \hat{H}'(t')] \rangle_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t'} C^R_{AH'_\omega}(t,t')$
$\langle \hat{A}(t) \hat{H}'_\omega(t') \rangle_0 = \langle n | e^{iH_0 t} A e^{-iH_0 t} e^{iH_0 t'} H'(t') e^{-iH_0 t'} | n \rangle$

$$\delta\langle A(t)\rangle = \int_{t_0}^{\infty} dt'\, C^R_{AH'}(t,t')\, e^{-\eta(t-t')}$$

$$C^R_{AH'}(t,t') = -i\,\theta(t-t')\,\langle[\hat{A}(t), \hat{H}'(t')]\rangle_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t'}\, C^R_{AH'_\omega}(t,t')$$

$$\langle \hat{A}(t)\, \hat{H}'_\omega(t')\rangle_0 = \langle n|\, e^{iH_0 t} A\, e^{-iH_0 t}\, e^{iH_0 t'} H'_\omega\, e^{-iH_0 t'}\,|n\rangle$$

$$C^R_{AH'}(t,t') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t'}\, C^R_{AH'_\omega}(t-t')$$

$$\delta\langle A(t)\rangle = \int_{t_0}^{\infty} dt'\, C^R_{AH'}(t,t')\, e^{-\eta(t-t')}$$

$$C^R_{AH'}(t,t') = -i\,\theta(t-t')\langle[\hat{A}(t),\hat{H}(t')]\rangle_0 = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\, e^{-i\omega t'}\, C^R_{AH'_\omega}(t,t')$$

$$\langle \hat{A}(t)\hat{H}'_\omega(t')\rangle_0 = \langle n| e^{iH_0 t} A\, e^{-iH_0 t}\, e^{iH_0 t'}\, H'_\omega\, e^{-iH_0 t'}|n\rangle$$

$$C^R_{AH'}(t,t') = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\, e^{-i\omega t'}\, C^R_{AH'_\omega}(t-t')$$

$$\delta\langle A(t)\rangle = \int\frac{d\omega}{2\pi}\, e^{-i\omega t}\int_{t_0}^{\infty} dt'\, e^{-i(\omega+i\eta)(t'-t)}\, C^R_{AH'_\omega}(t-t')$$

$$\delta\langle A(t)\rangle = \int_{t_0}^{\infty} dt' \, C^R_{AH'}(t,t') \, e^{-\eta(t-t')}$$

$$C^R_{AH'}(t,t') = -i\,\theta(t-t') \left\langle [\hat{A}(t), \hat{H}(t')] \right\rangle_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t'} \, C^R_{AH'_\omega}(t,t')$$

$$\langle \hat{A}(t) \hat{H}'_\omega(t') \rangle_0 = \langle n | e^{iH_0 t} A \, e^{-iH_0 t} \, e^{iH_0 t'} H'_\omega \, e^{-iH_0 t'} | n \rangle$$

$$C^R_{AH'}(t,t') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t'} \, C^R_{AH'_\omega}(t-t')$$

$$\delta\langle A(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t} \int_{t_0}^{\infty} dt' \, e^{-i(\omega+i\eta)(t'-t)} \, C^R_{AH'_\omega}(t-t')$$

$') \, e^{-\eta(t-t')}$

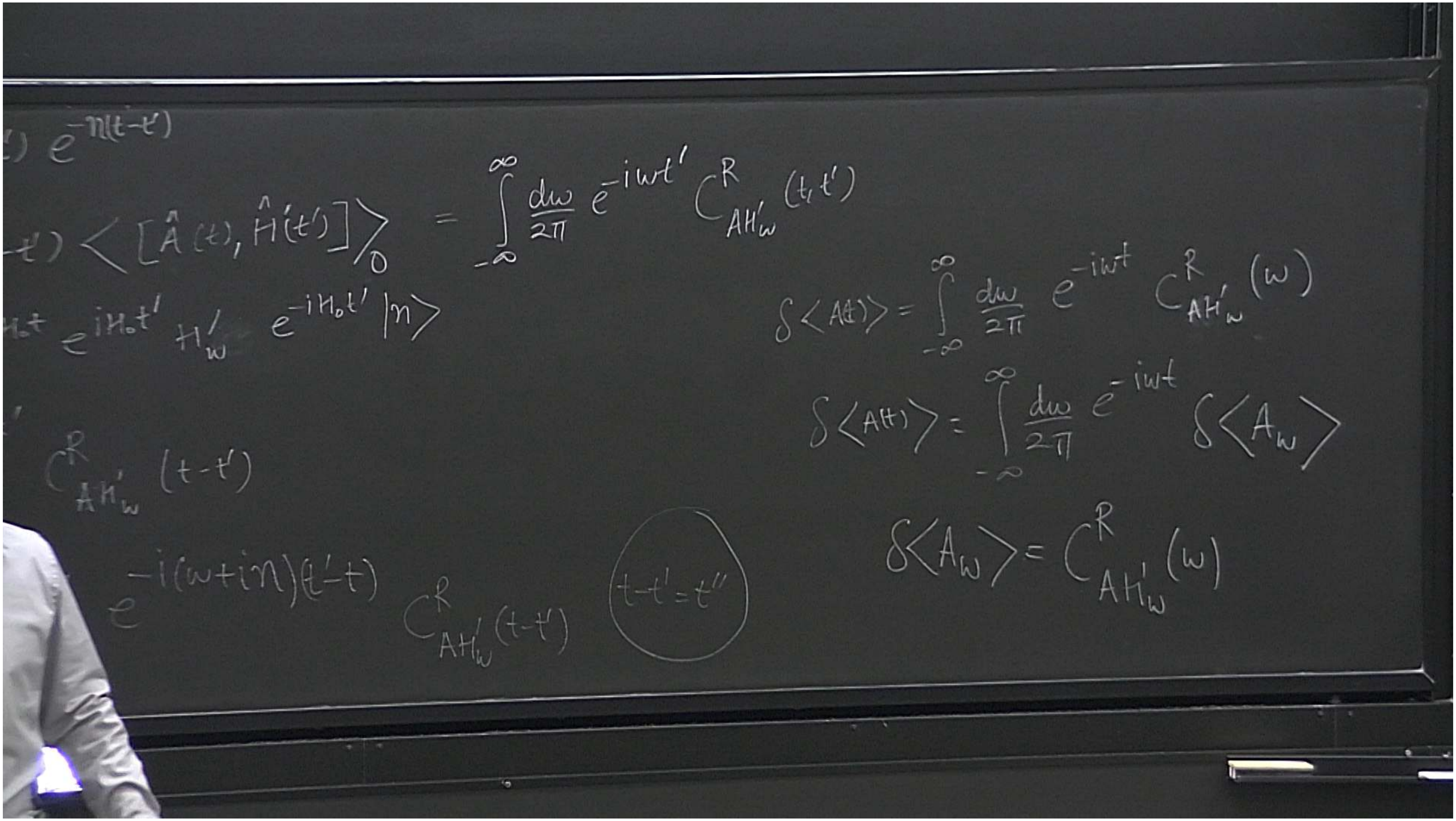
$-t') \left\langle [\hat{A}(t), \hat{H}'(t')] \right\rangle_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t'} C^R_{AH'_\omega}(t,t')$

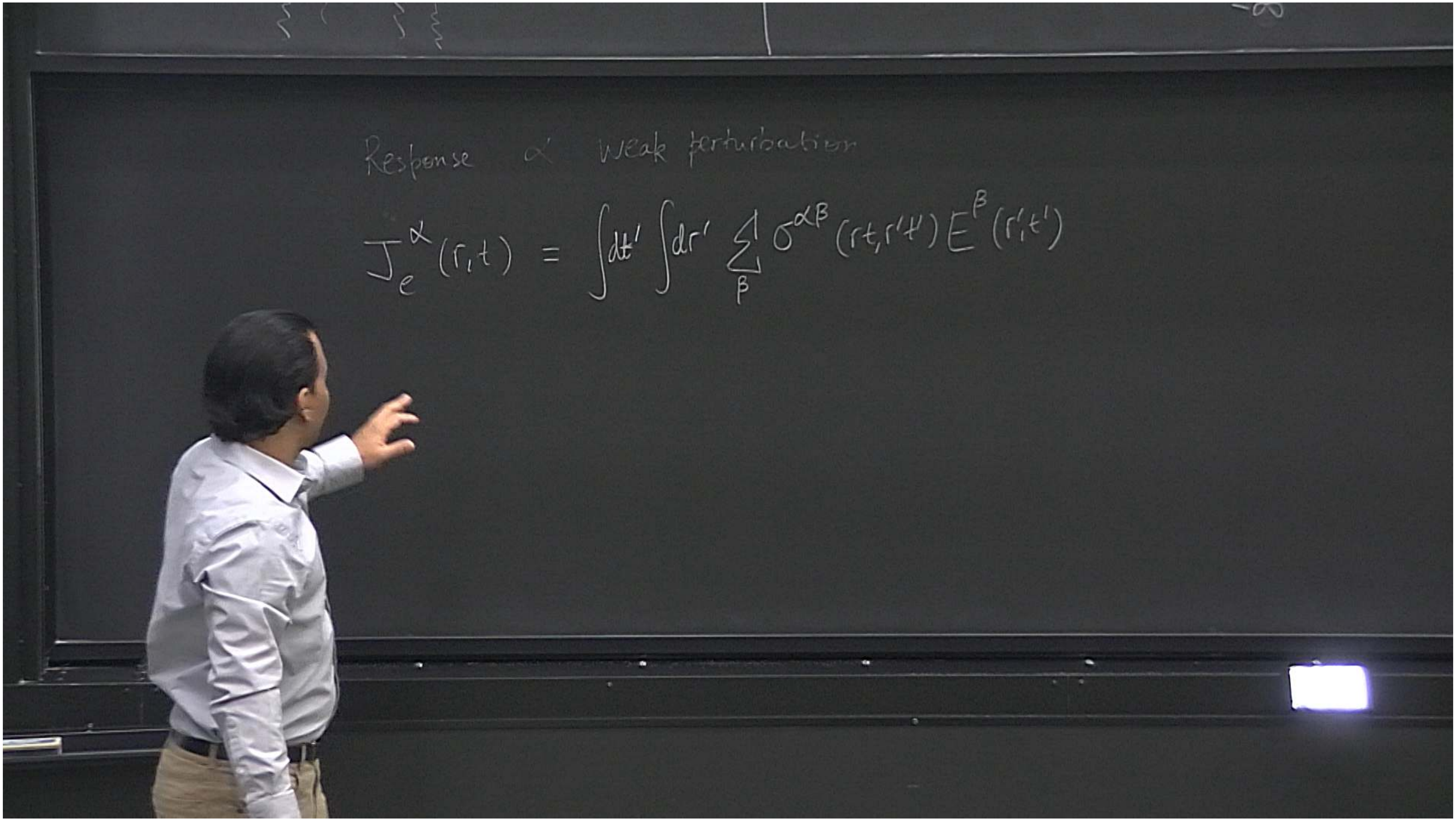
$H_0 t} \, e^{iH_0 t'} H'_\omega \, e^{-iH_0 t'} |n\rangle$

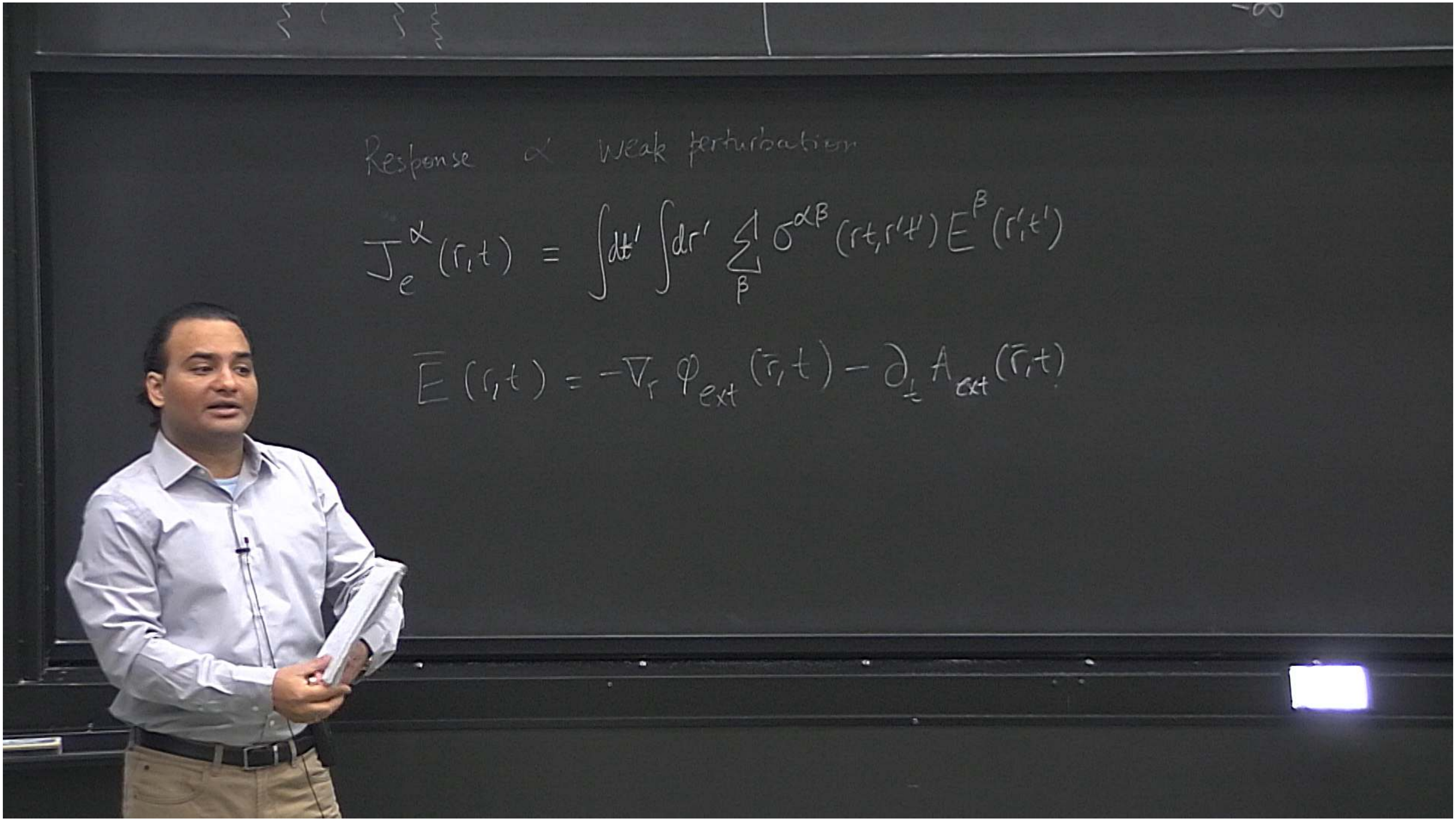
$$\delta\langle A(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} C^R_{AH'_\omega}(\omega)$$

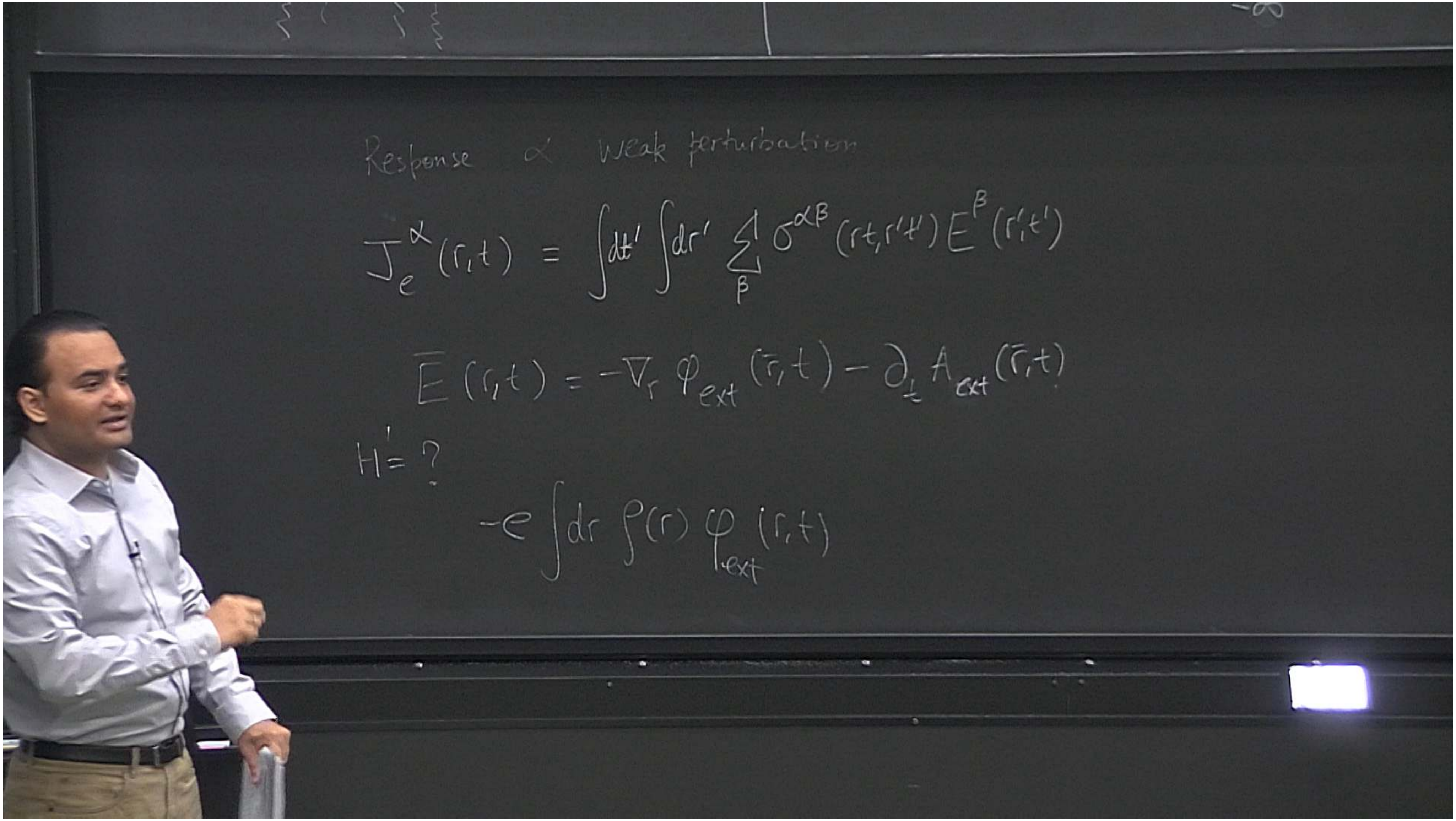
$C^R_{AH'_\omega}(t-t')$

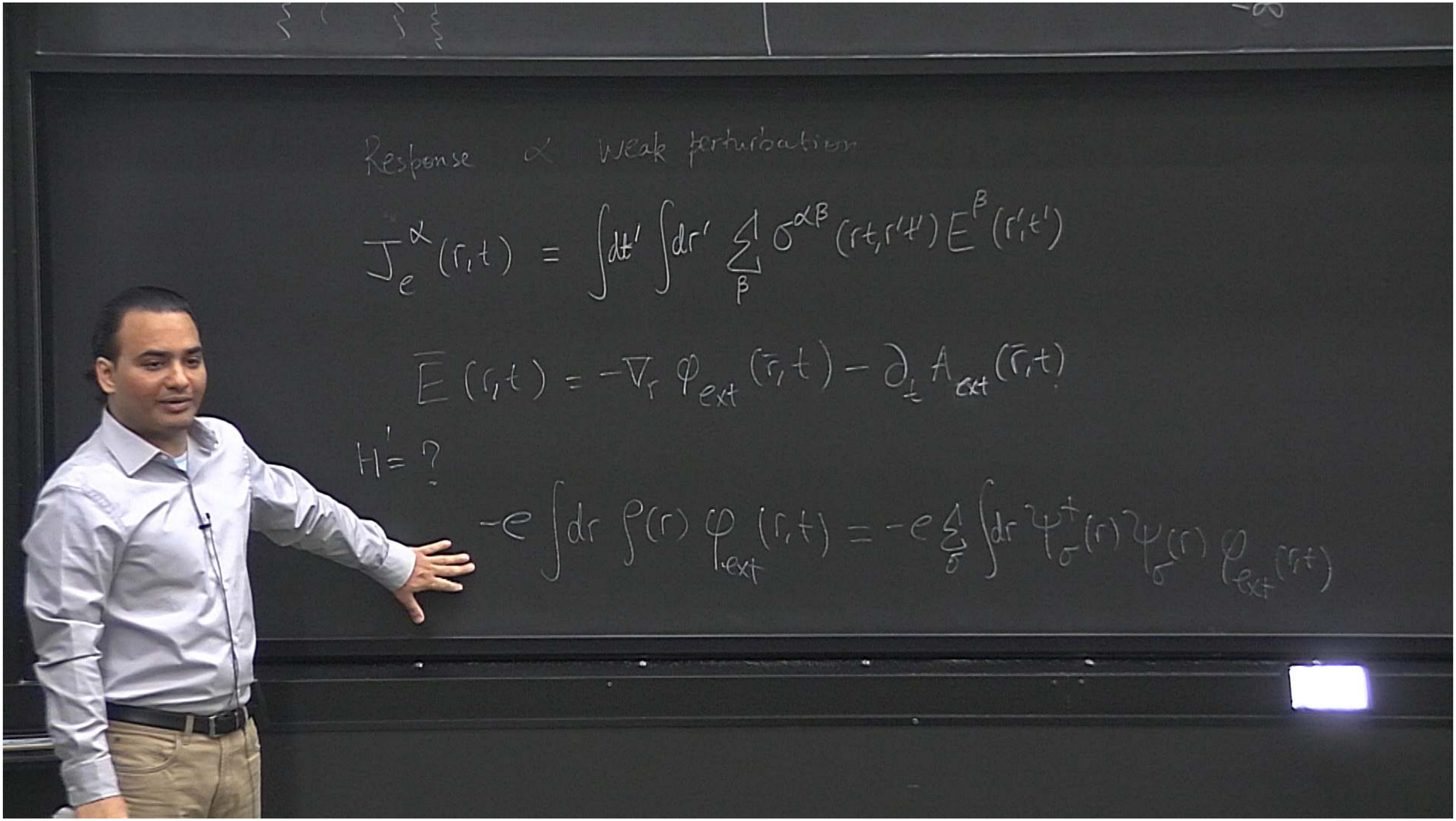
$dt' \, e^{-i(\omega+i\eta)(t'-t)} \, C^R_{AH'_\omega}(t-t')$ $\quad t-t'=t''$


$e^{-\eta(t-t')}$
$\langle [\hat{A}(t), \hat{H}'(t')] \rangle_0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t'} C^R_{AH'_\omega}(t,t')$
$e^{iH_0 t'} H'_\omega e^{-iH_0 t'} |n\rangle$
$C^R_{AH'_\omega}(t-t')$
$e^{-i(\omega+i\eta)(t'-t)} C^R_{AH'_\omega}(t-t')$
$t-t'=t''$
$\delta\langle A(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} C^R_{AH'_\omega}(\omega)$
$\delta\langle A(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \delta\langle A_\omega\rangle$
$\delta\langle A_\omega\rangle = C^R_{AH'_\omega}(\omega)$


Response $\propto$ weak perturbation
$J_e^{\alpha}(r,t) = \int dt' \int dr' \sum_{\beta} \sigma^{\alpha\beta}(rt, r't') E^{\beta}(r',t')$


Response ∝ weak perturbation
$J_e^{\alpha}(r,t) = \int dt' \int dr' \sum_{\beta} \sigma^{\alpha\beta}(rt, r't') E^{\beta}(r',t')$
$\bar{E}(r,t) = -\nabla_r \varphi_{ext}(\bar{r},t) - \partial_t A_{ext}(\bar{r},t)$


Response $\propto$ weak perturbation
$J_e^{\alpha}(r,t) = \int dt' \int dr' \sum_{\beta} \sigma^{\alpha\beta}(rt, r't') E^{\beta}(r',t')$
$\bar{E}(r,t) = -\nabla_r \varphi_{ext}(\bar{r},t) - \partial_t A_{ext}(\bar{r},t)$
$H' = ?$
$-e \int dr \, \rho(r) \, \varphi_{ext}(r,t)$


Response $\propto$ weak perturbation
$J_e^{\alpha}(r,t) = \int dt' \int dr' \sum_{\beta} \sigma^{\alpha\beta}(rt, r't') E^{\beta}(r',t')$
$\bar{E}(r,t) = -\nabla_r \varphi_{ext}(\bar{r},t) - \partial_t A_{ext}(\bar{r},t)$
$H' = ?$
$-e \int dr\, \rho(r)\, \varphi_{ext}(r,t) = -e \sum_{\sigma} \int dr\, \psi^{\dagger}_{\sigma}(r) \psi_{\sigma}(r)\, \varphi_{ext}(r,t)$

$$\text{Kinetic Energy} = \frac{1}{2m} \sum_{\sigma} \int dr\, \psi_{\sigma}^{\dagger}(r) \left(\frac{\hbar}{i} \nabla_r - q\bar{A} \right)^2 \psi_{\sigma}(r)$$

$$H' = \sum_{\sigma} \left(\frac{q\hbar}{2mi} \int dr\, A \cdot \left[\left(\nabla \psi_{\sigma}^{\dagger}(r)\right) \psi_{\sigma}(r) - \psi_{\sigma}^{\dagger}(r) \left(\nabla \psi_{\sigma}(r)\right) \right] + \frac{q^2 A^2}{2m} \psi_{\sigma}^{\dagger}(r) \psi_{\sigma}(r) \right)$$

$$\text{Kinetic Energy} = \frac{1}{2m}\sum_{\sigma}\int dr\, \psi_\sigma^{\dagger}(r)\left(\frac{\hbar}{i}\nabla_r - q\bar{A}\right)^2 \psi_\sigma(r)$$

$$H' = \sum_{\sigma}\left(\frac{q\hbar}{2mi}\int dr\; A\cdot\left[\left(\nabla\psi_\sigma^{\dagger}(r)\right)\psi_\sigma(r) - \psi_\sigma^{\dagger}(r)\left(\nabla\psi_\sigma(r)\right)\right] + \frac{q^2A^2}{2m}\psi_\sigma^{\dagger}(r)\psi_\sigma(r)\right)$$

$$\delta H = -q\int dr\; J\cdot\delta A$$

$$\text{Kinetic Energy} = \frac{1}{2m}\sum_\sigma \int dr\, \psi_\sigma^\dagger(r)\left(\frac{\hbar}{i}\nabla_r - q\bar{A}\right)^2 \psi_\sigma(r)$$

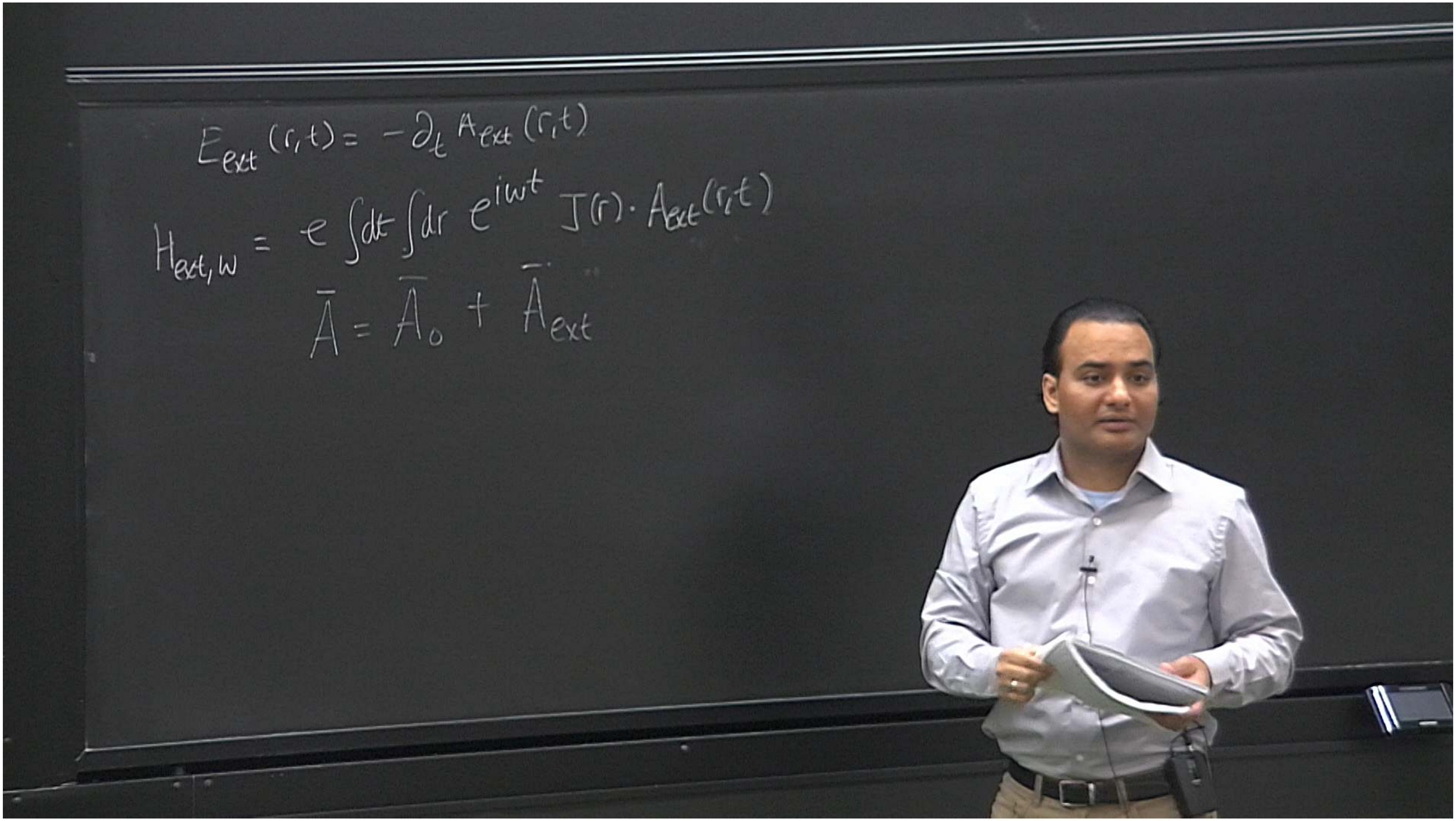
$$H' = \sum_\sigma \left(\frac{q\hbar}{2mi} \int dr\, A \cdot \left[\left(\nabla \psi_\sigma^\dagger(r)\right)\psi_\sigma(r) - \psi_\sigma^\dagger(r)\left(\nabla \psi_\sigma(r)\right) \right] + \frac{q^2 A^2}{2m}\psi_\sigma^\dagger(r)\psi_\sigma(r) \right)$$

$$\delta H = -q \int dr\, J \cdot \delta A$$

$$J(r) = J^\nabla(r) + \frac{e}{m} A(r)\rho(r)$$

$$J_\sigma^\nabla(r) = \frac{\hbar}{2mi}\left[\psi_\sigma^\dagger(\nabla\psi_\sigma) - (\nabla\psi_\sigma^\dagger)\psi \right]$$

$$J_\sigma^A(r) = -\frac{q}{m} A(r)\, \psi_\sigma^\dagger \psi_\sigma$$


$E_{ext}(r,t) = -\partial_t A_{ext}(r,t)$
$H_{ext,w} = e \int dt \int dr\, e^{iwt} J(r) \cdot A_{ext}(r,t)$
$\bar{A} = \bar{A}_0 + \bar{A}_{ext}$

$$E_{ext}(r,t) = -\partial_t A_{ext}(r,t)$$

$$H_{ext,\omega} = e \int dt \int dr\, e^{i\omega t} J(r) \cdot A_{ext}(r,t)$$

$$\bar{A} = \bar{A}_0 + \bar{A}_{ext}$$

$$\langle J(r,\omega) \rangle = C^R_{J_0(r) H_{ext,\omega}}(\omega) + \frac{e}{i\omega m} E_{ext}(r,\omega) \langle \rho(r) \rangle_0$$

$$E_{ext}(r,t) = -\partial_t A_{ext}(r,t)$$

$$H_{ext,\omega} = e\int dt \int dr\, e^{i\omega t}\, J(r)\cdot A_{ext}(r,t)$$

$$\bar{A} = \bar{A}_0 + \bar{A}_{ext}$$

$$\langle J(r,\omega)\rangle = C^R_{J_0(r)\,H_{ext,\omega}}(\omega) + \frac{e}{i\omega m} E_{ext}(r,\omega)\langle \rho(r)\rangle_0$$

$$C^R_{J_0(r)\,H_{ext,\omega}}(\omega) = \int dr'\, C^R_{J_0(r)\,J_0^\beta(r')}(\omega)\,\frac{e}{i\omega}\,E^\beta_{ext}(r',\omega)$$

$$J_e^{\alpha}(r,\omega) = \int dr' \sum_{\beta} \sigma^{\alpha\beta}(r,r',\omega)\, E^{\beta}(r',\omega)$$

$$\sigma^{\alpha\beta} = \frac{ie^2}{\omega} \Pi^{R}_{\alpha\beta}(r,r',\omega) + \frac{ie^2 n(r)}{\omega m} \delta(r-r')\, \delta_{\alpha\beta}$$

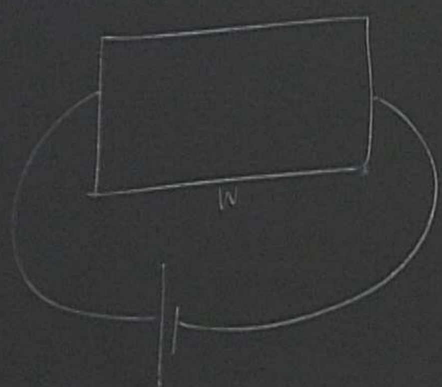
$$J_e^{\alpha}(r,\omega) = \int dr' \sum_{\beta} \sigma^{\alpha\beta}(r,r',\omega)\, E^{\beta}(r',\omega)$$

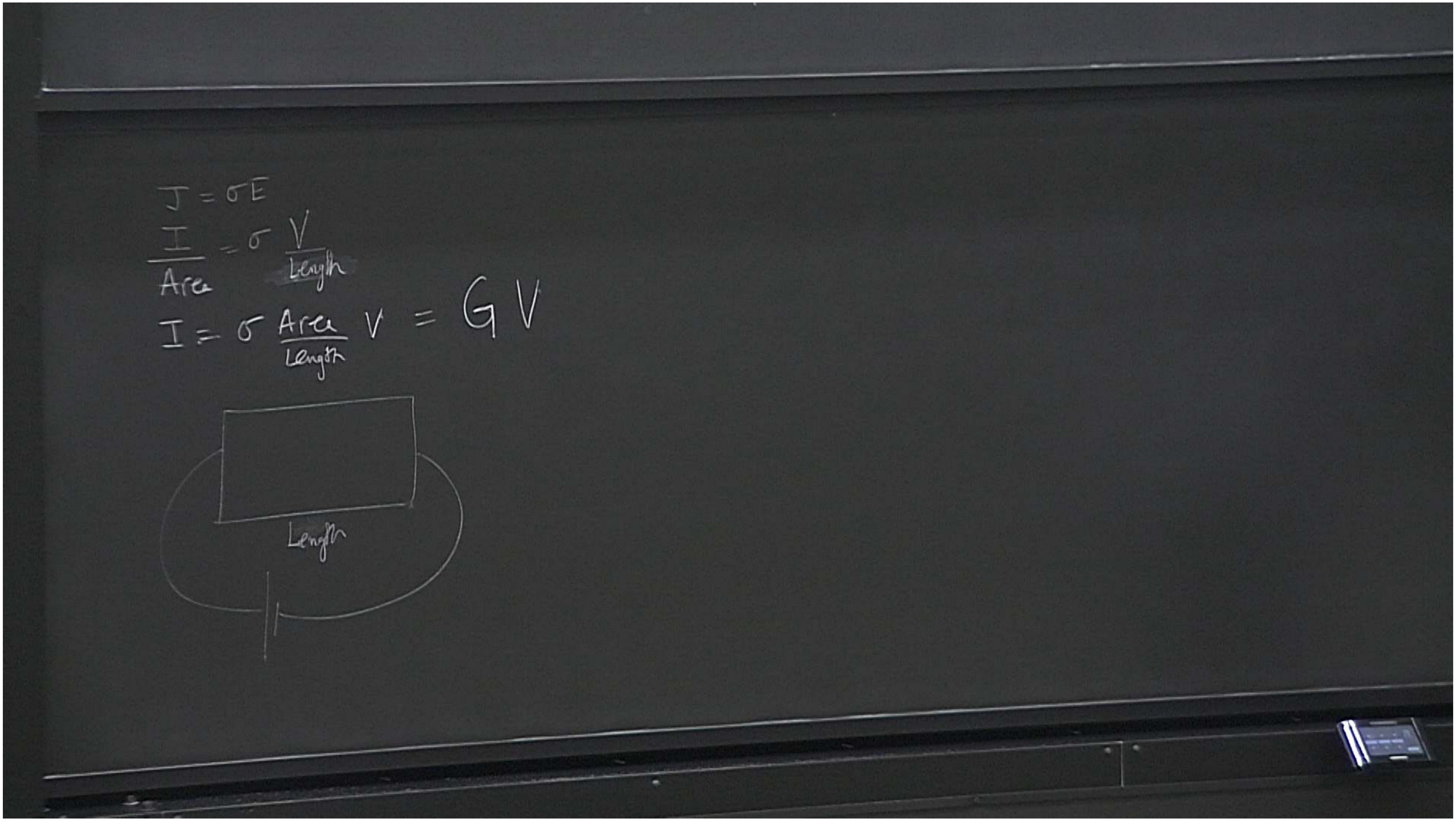
$$\sigma^{\alpha\beta} = \frac{ie^2}{\omega}\, \Pi^R_{\alpha\beta}(r,r',\omega) + \frac{ie^2 n(r)}{\omega m}\, \delta(r-r')\, \delta_{\alpha\beta}$$

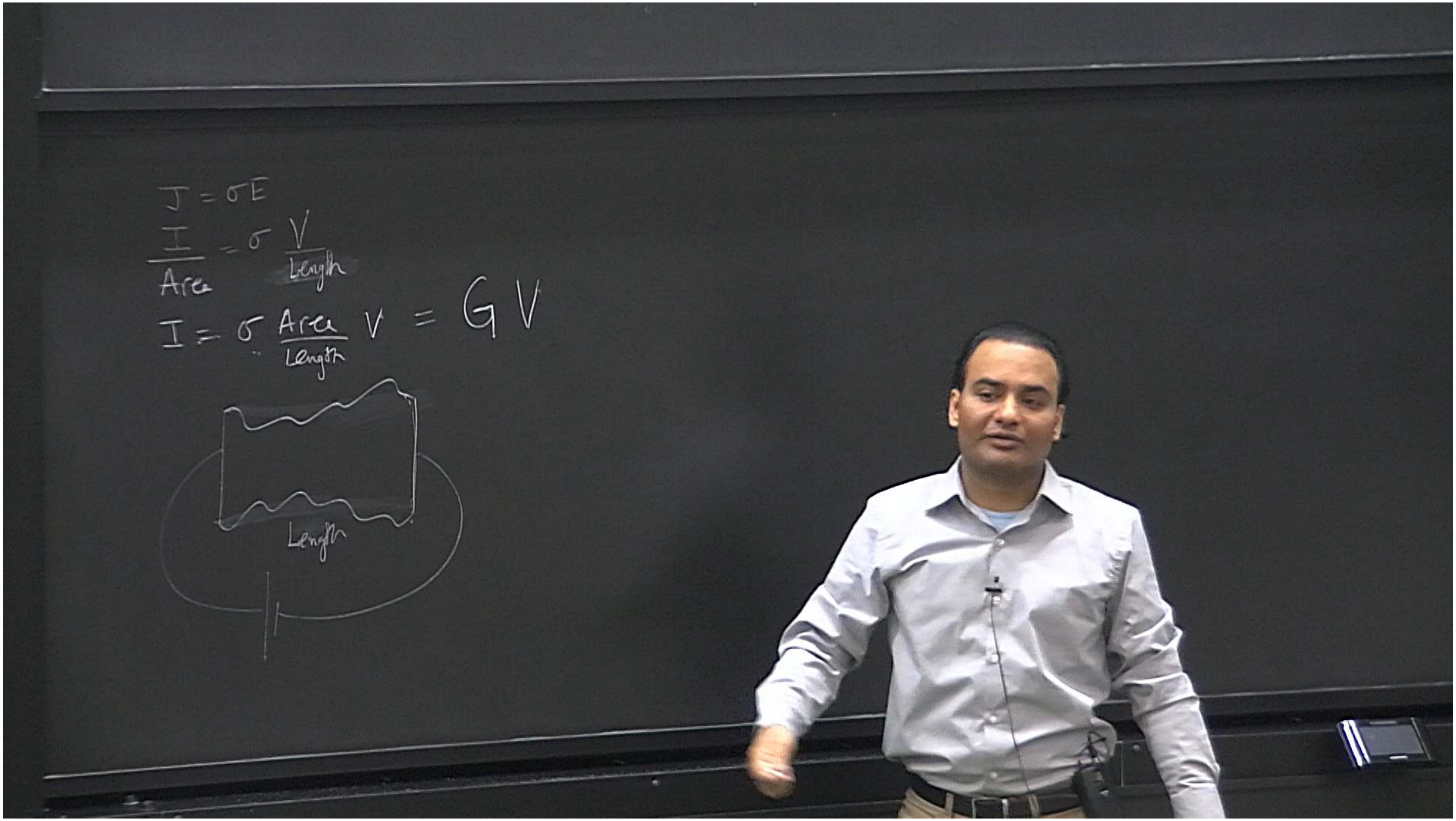
$$\Pi^R_{\alpha\beta}(r,r',t-t') = C^R_{J_0^{\alpha}(r)\, J_0^{\beta}(r')}(t-t') = -i\theta(t-t') \left\langle \left[\hat{J}_0^{\alpha}(r,t), \hat{J}_0^{\beta}(r',t') \right] \right\rangle_0$$

$$J = \sigma E$$

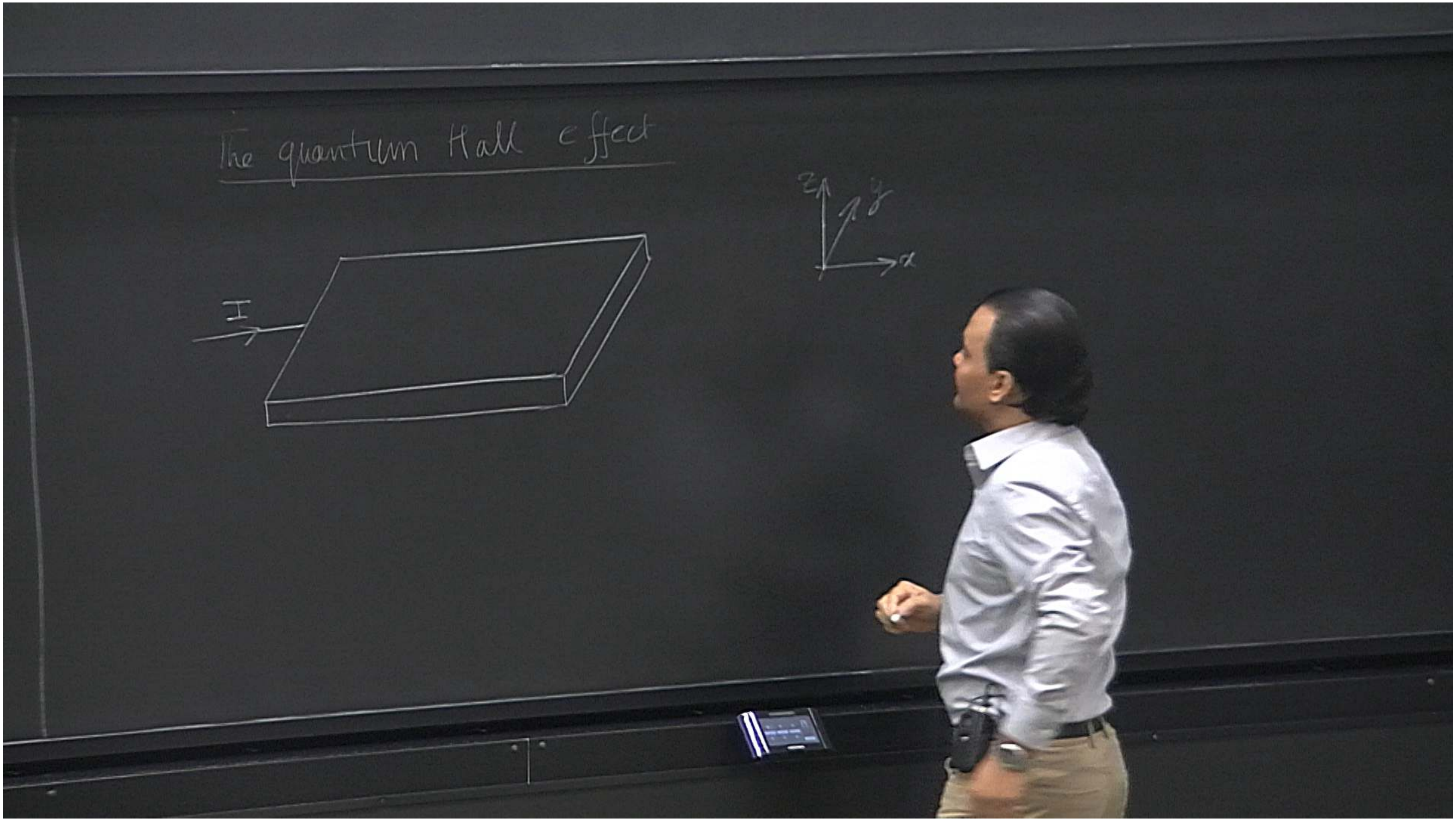
$$\frac{I}{\text{Area}} = \sigma \frac{V}{\text{width}}$$

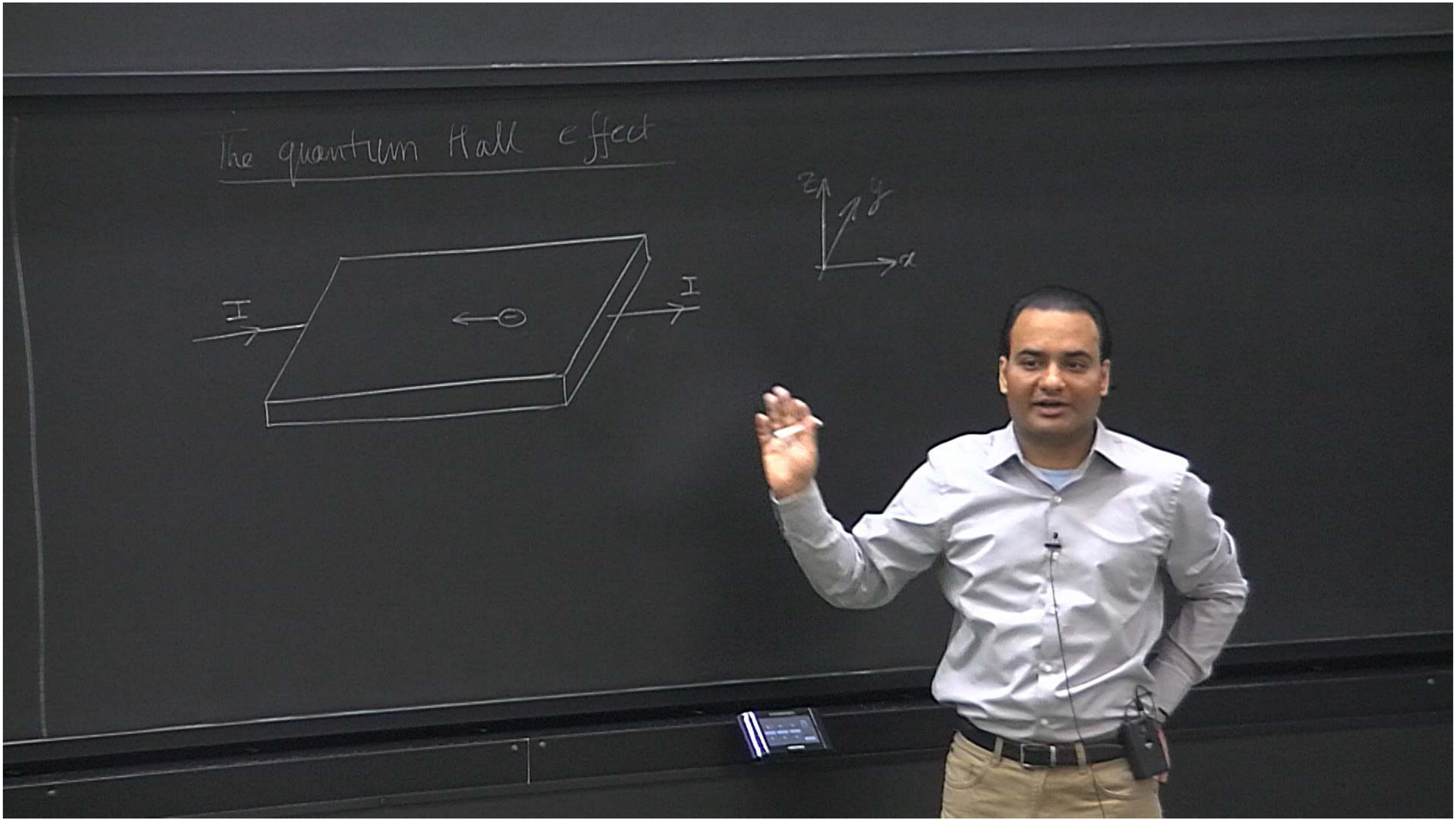


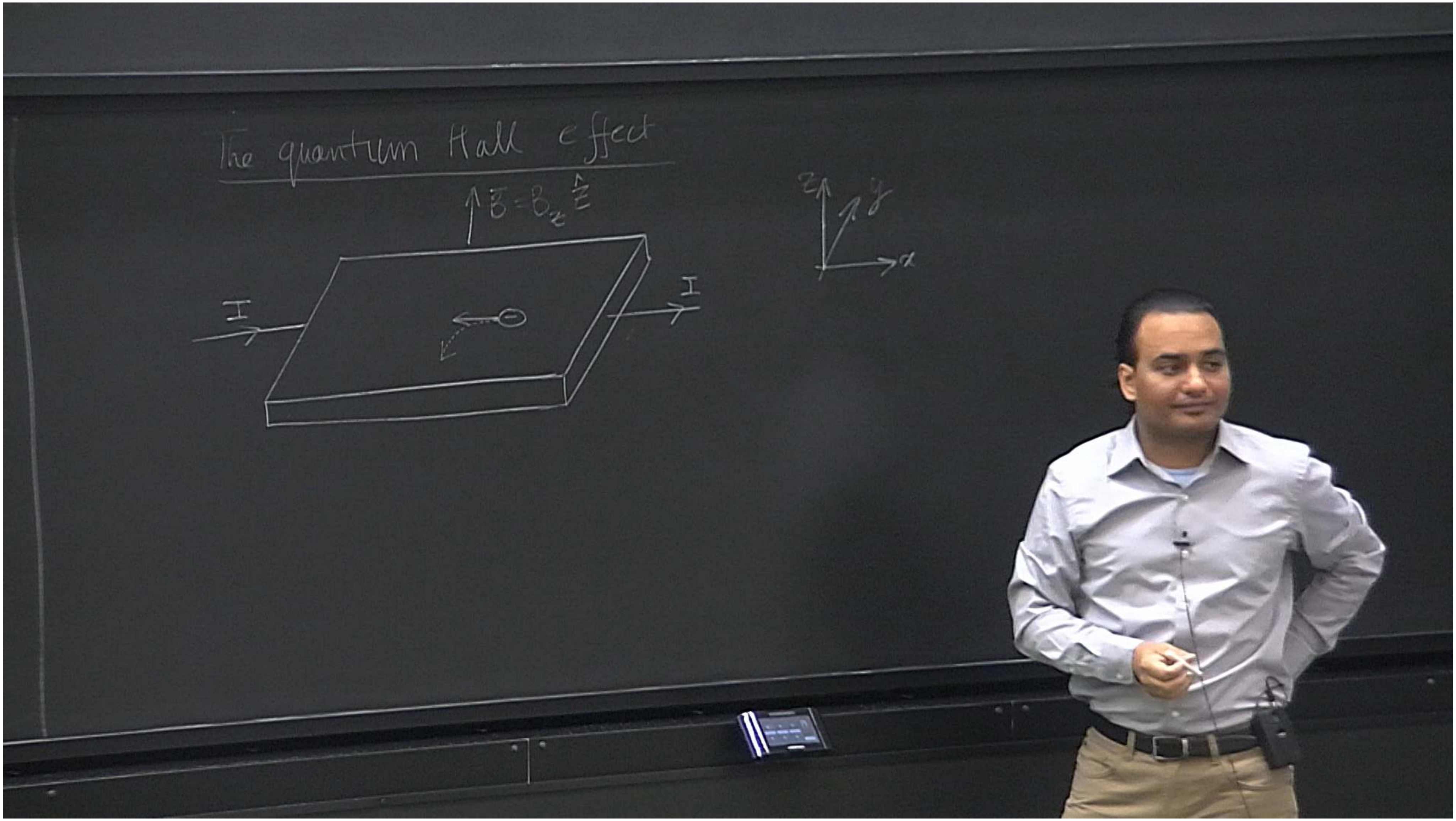

$J = \sigma E$
$\frac{I}{\text{Area}} = \sigma \frac{V}{\text{Length}}$
$I = \sigma \frac{\text{Area}}{\text{Length}} V = G V$
Length

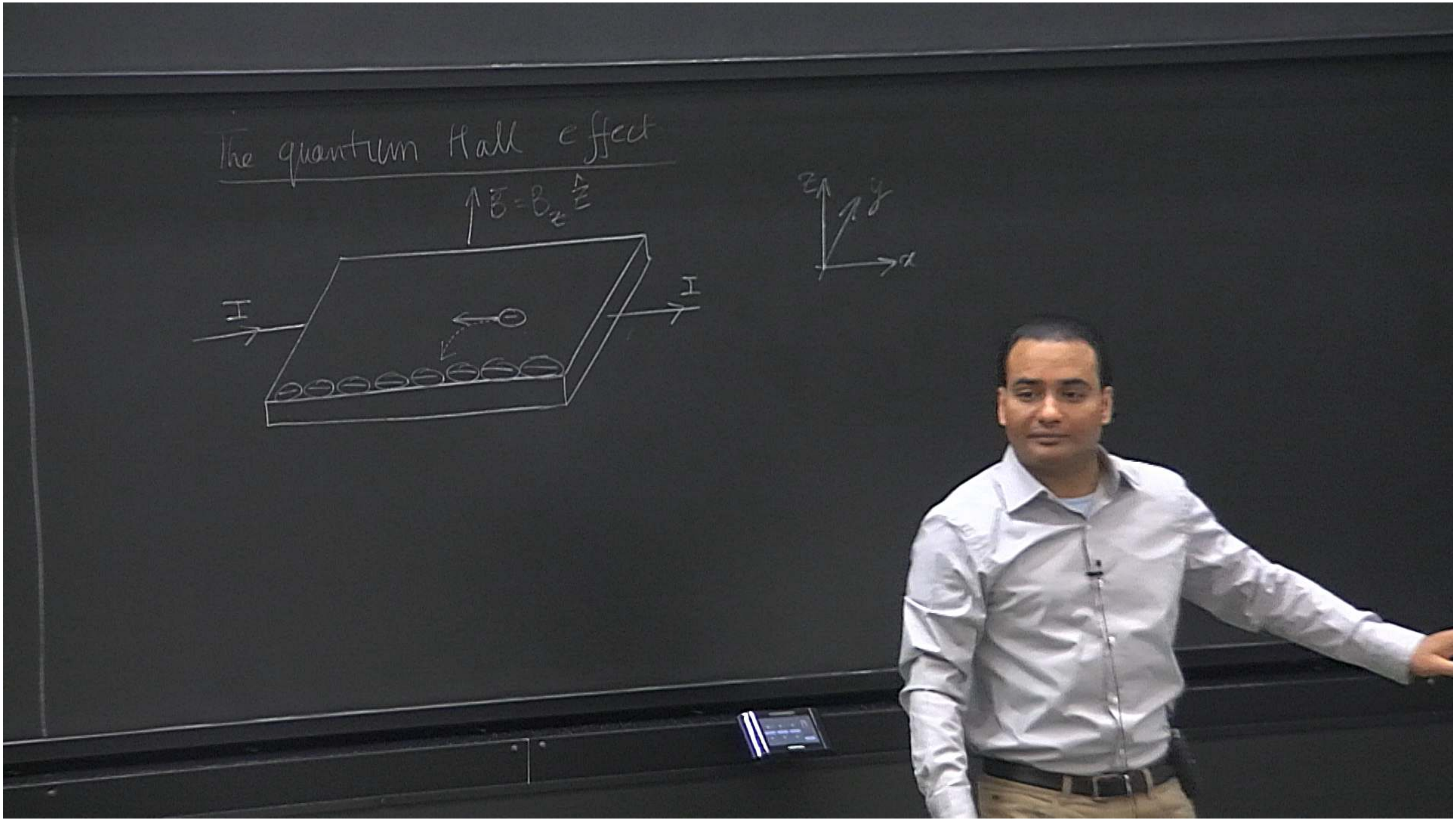


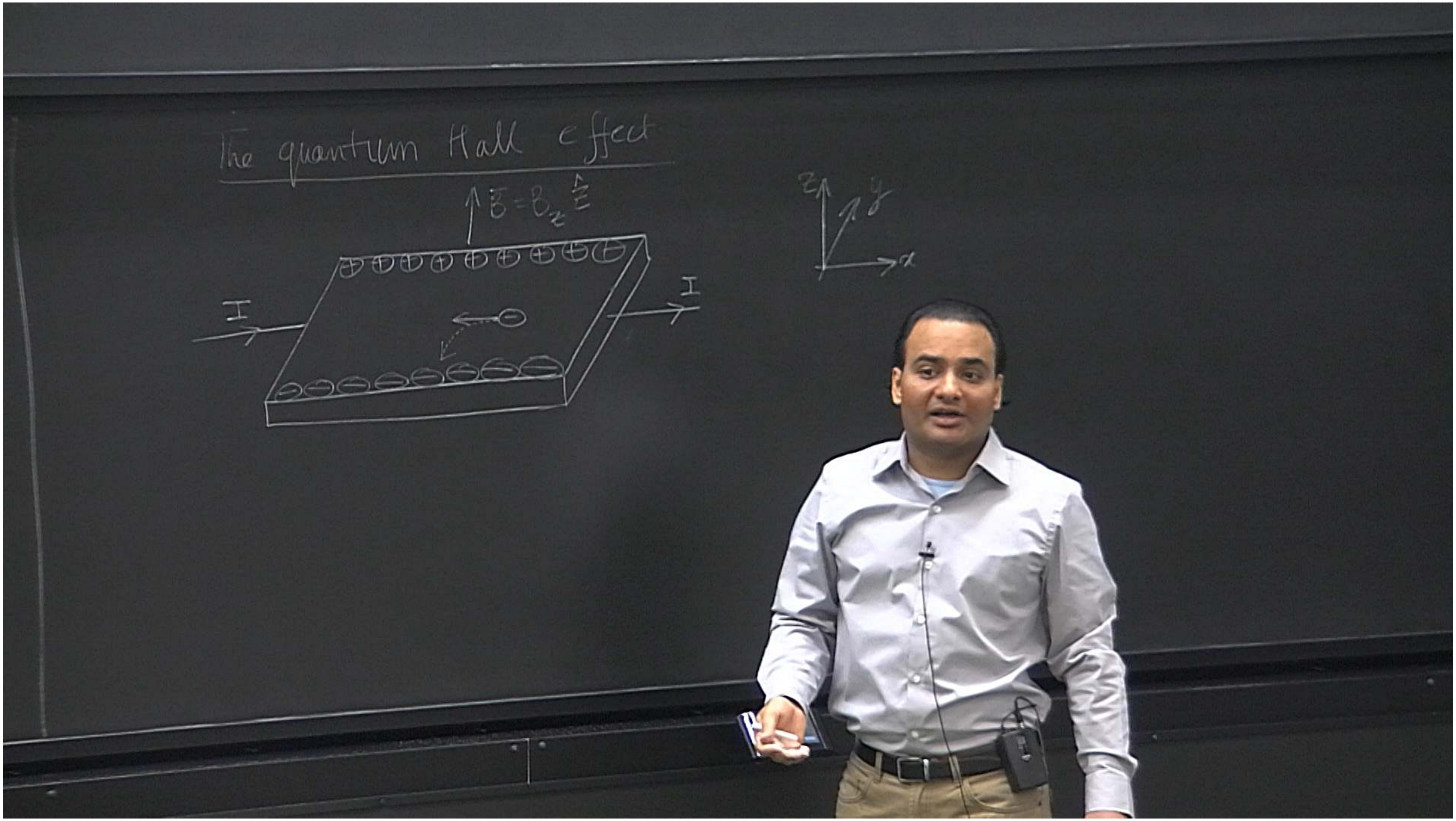
$J = \sigma E$
$\frac{I}{\text{Area}} = \sigma \frac{V}{\text{Length}}$
$I = \sigma \frac{\text{Area}}{\text{Length}} V = G V$
Length

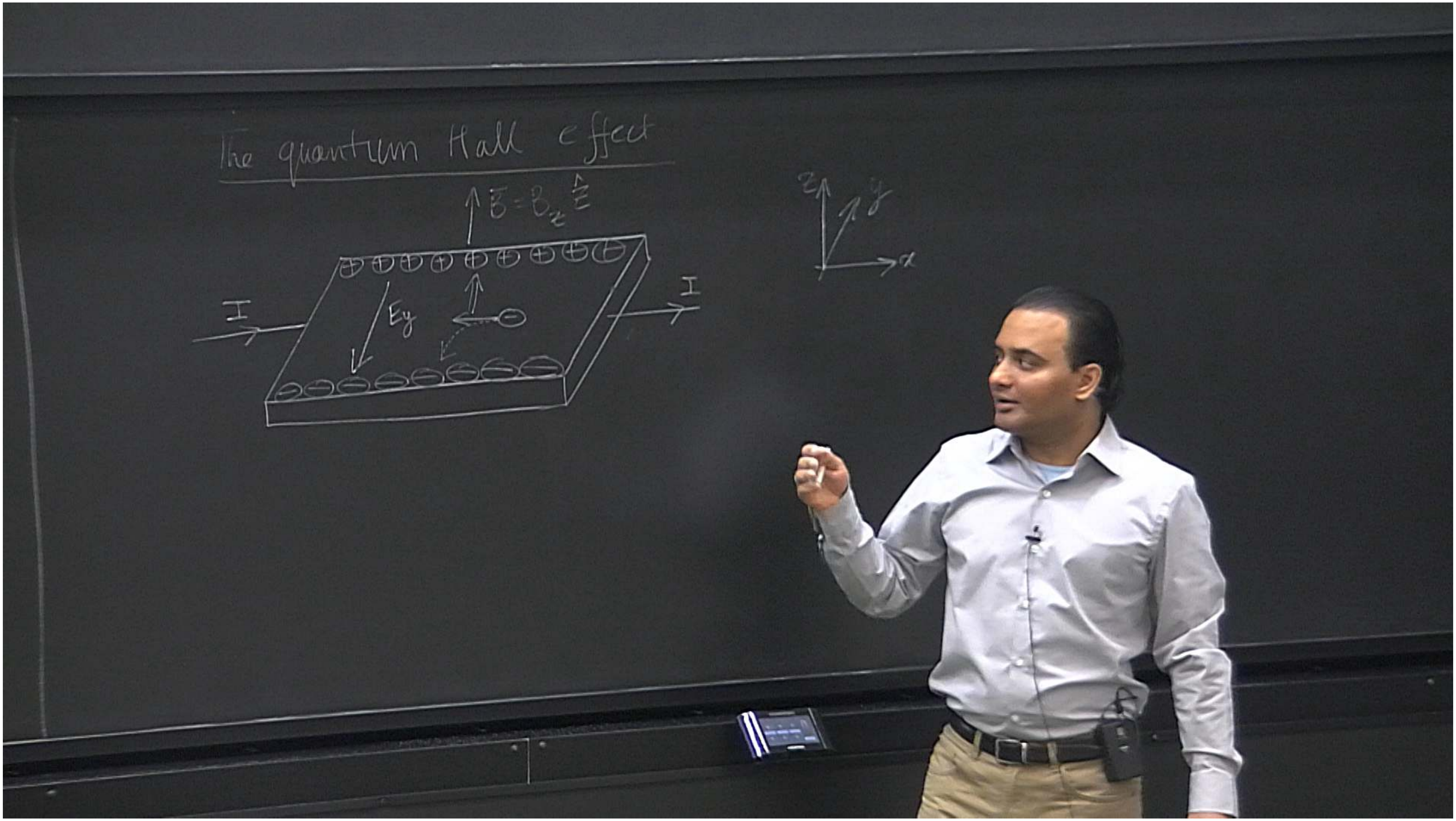

The quantum Hall effect
I
z
y
x

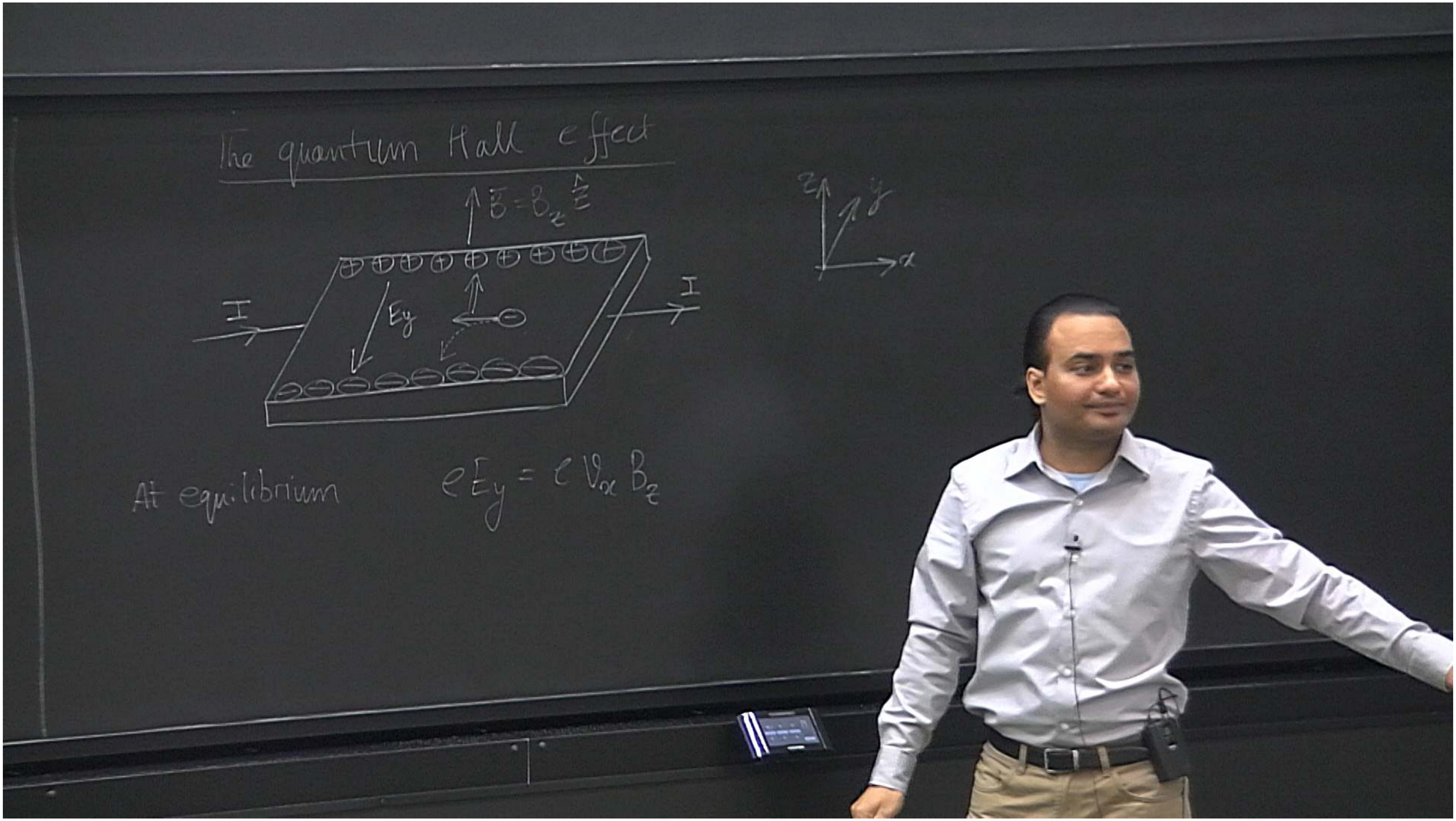

The quantum Hall effect
I
I
z
y
x


The quantum Hall effect
$\vec{B} = B_z \hat{z}$
I
I
z
y
x

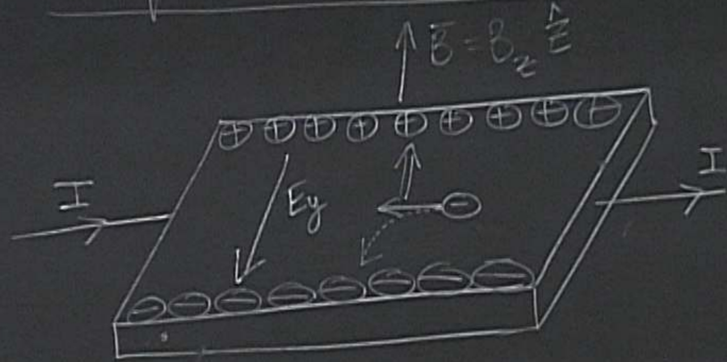

The quantum Hall effect
$\vec{B} = B_z \hat{z}$
I
I
z
y
x

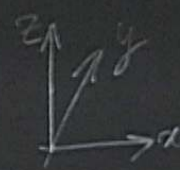

The quantum Hall effect
$\vec{B} = B_z \hat{z}$
I
I
z
y
x


The quantum Hall effect
$\vec{B} = B_z \hat{z}$
E_y
I
I
z
y
x

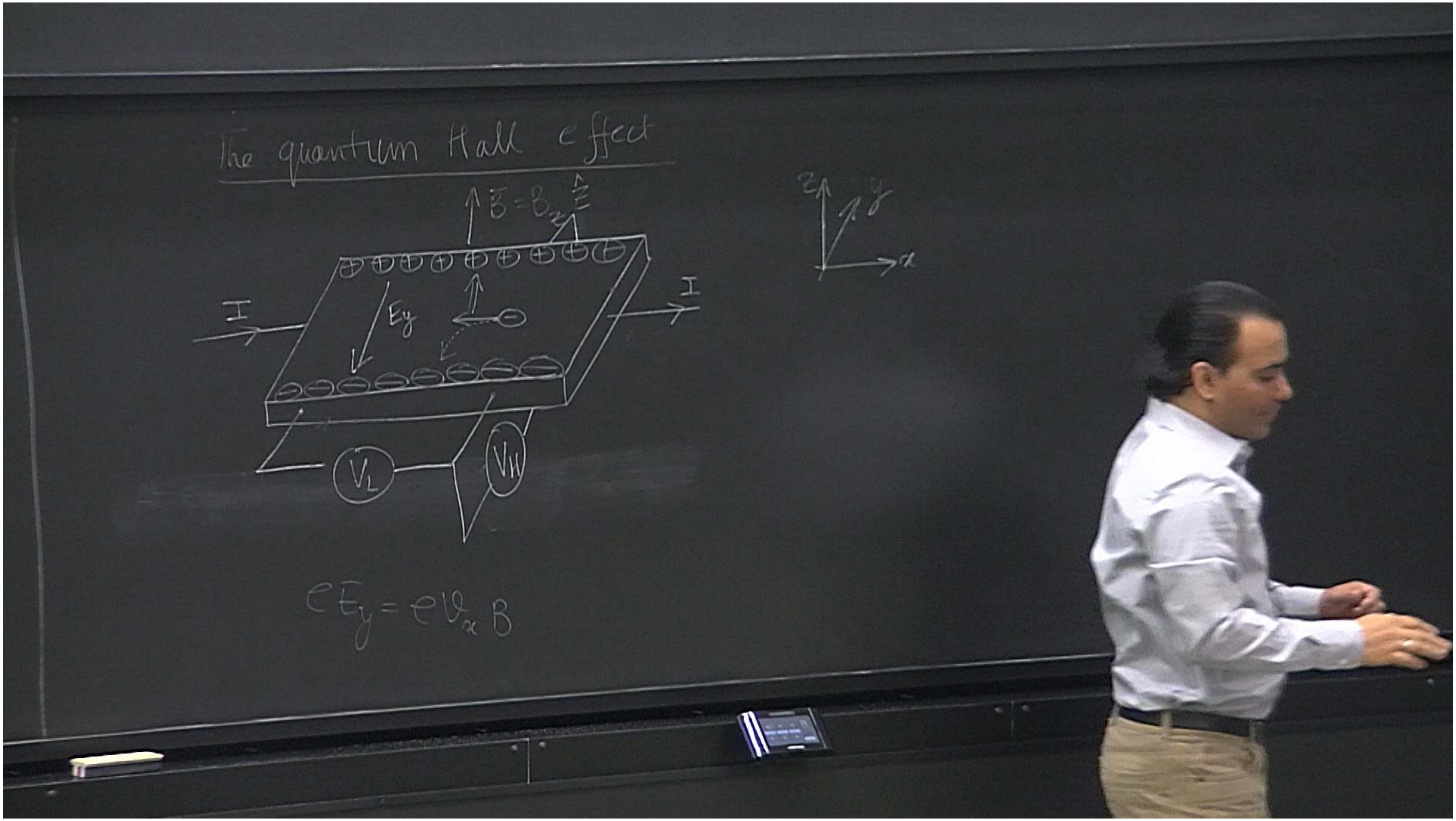

The quantum Hall effect
$\vec{B} = B_z \hat{z}$
I
E_y
I
z
y
x
At equilibrium
$eE_y = e v_x B_z$

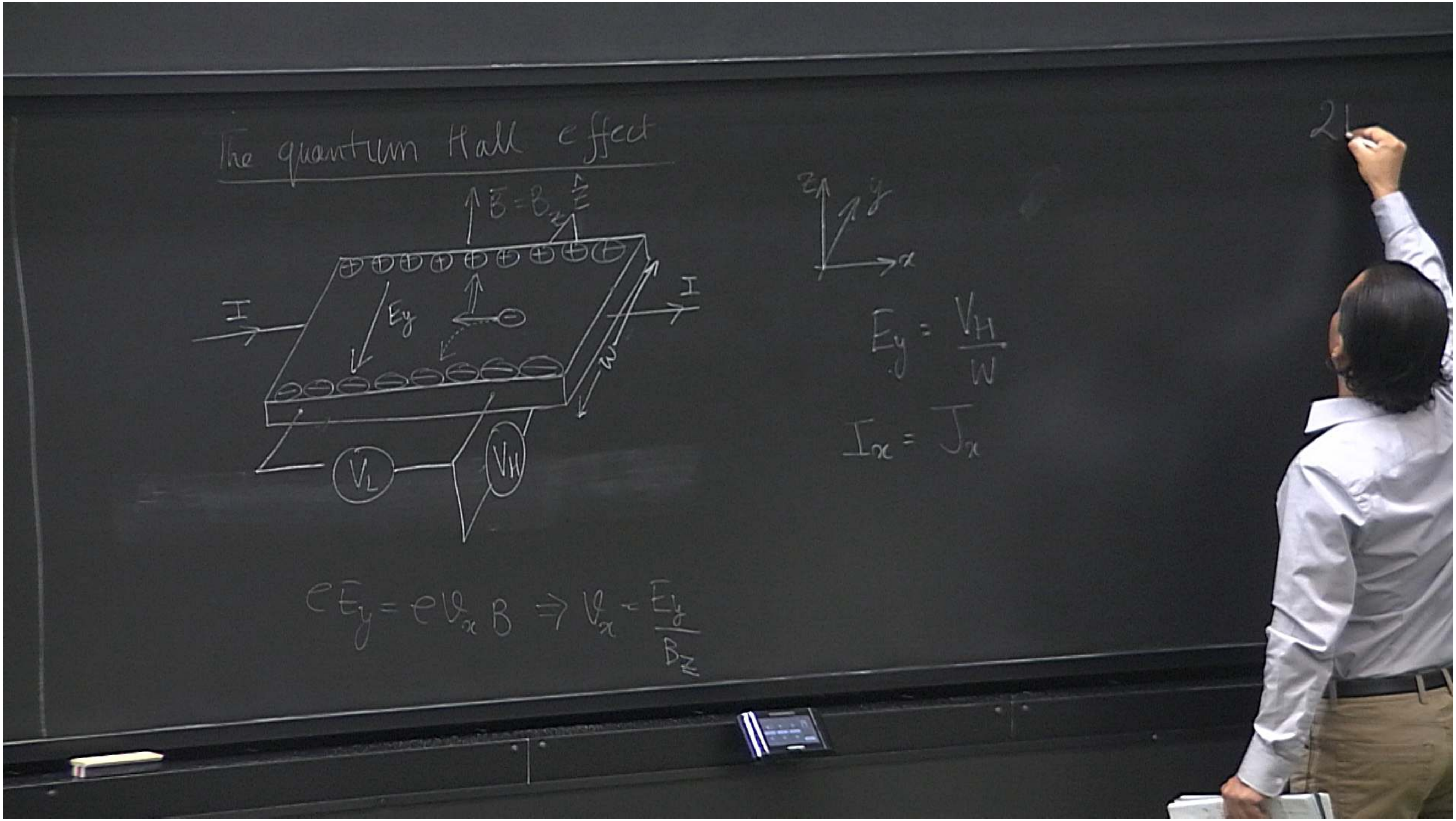
The quantum Hall effect

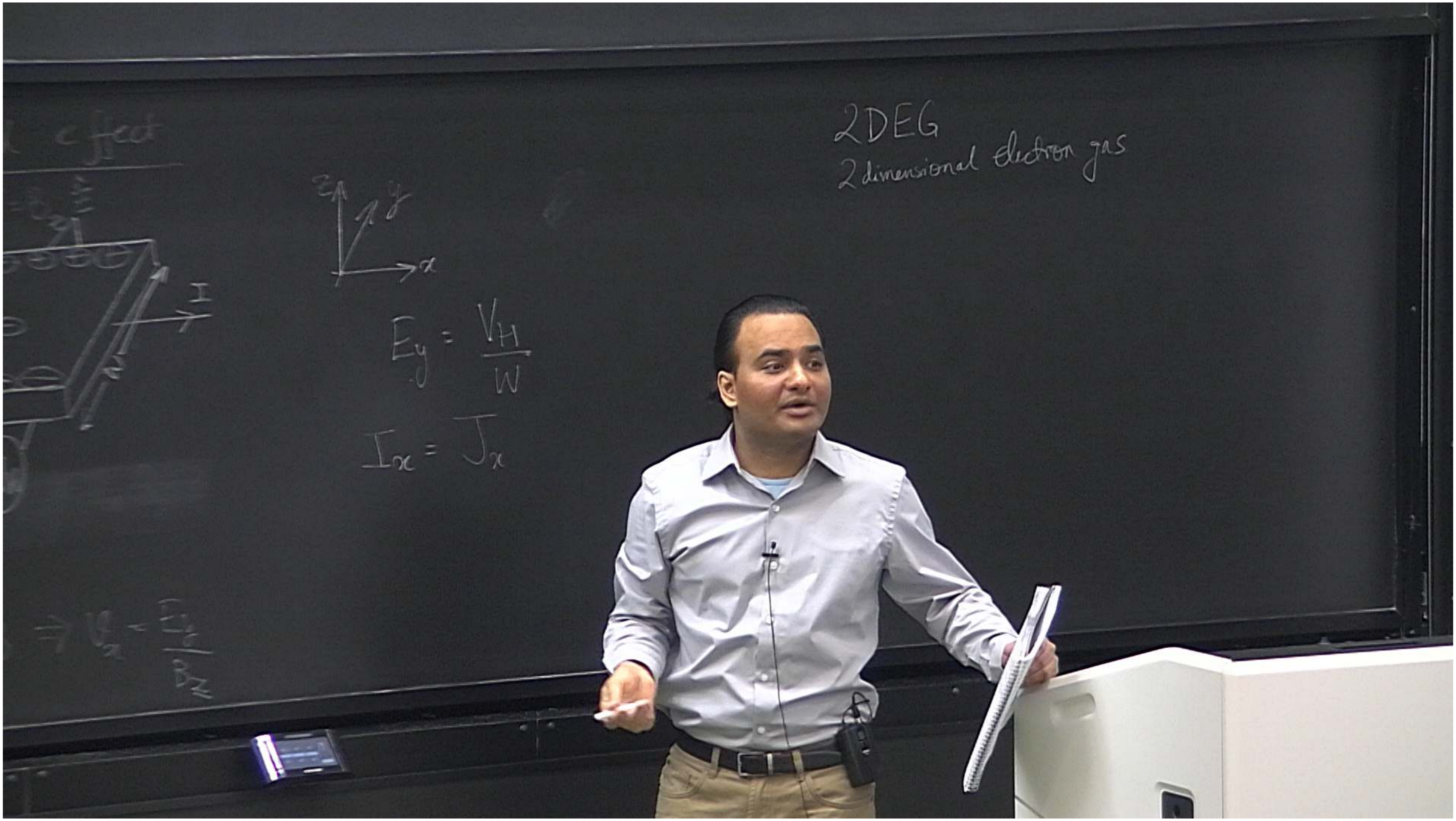


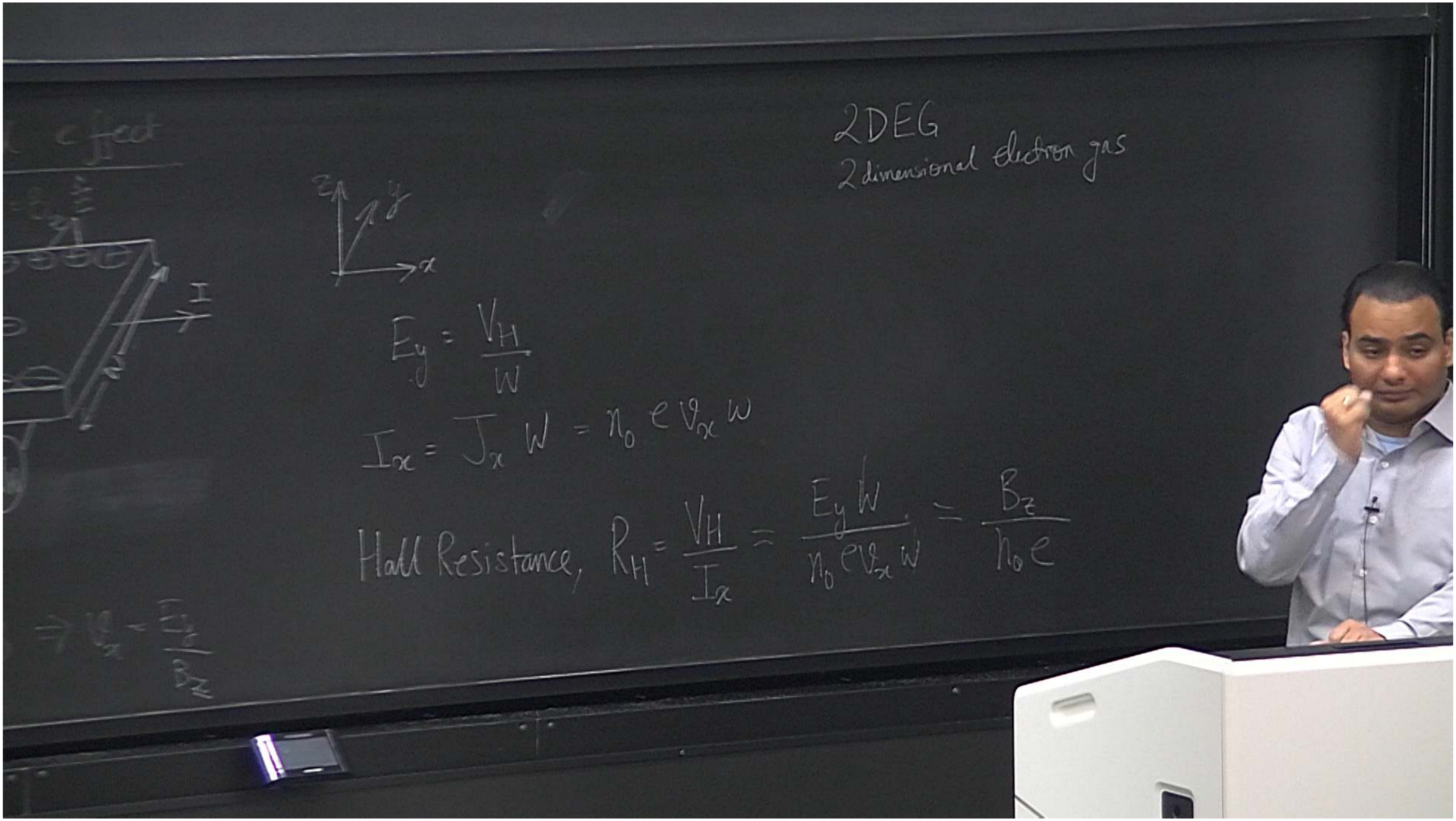


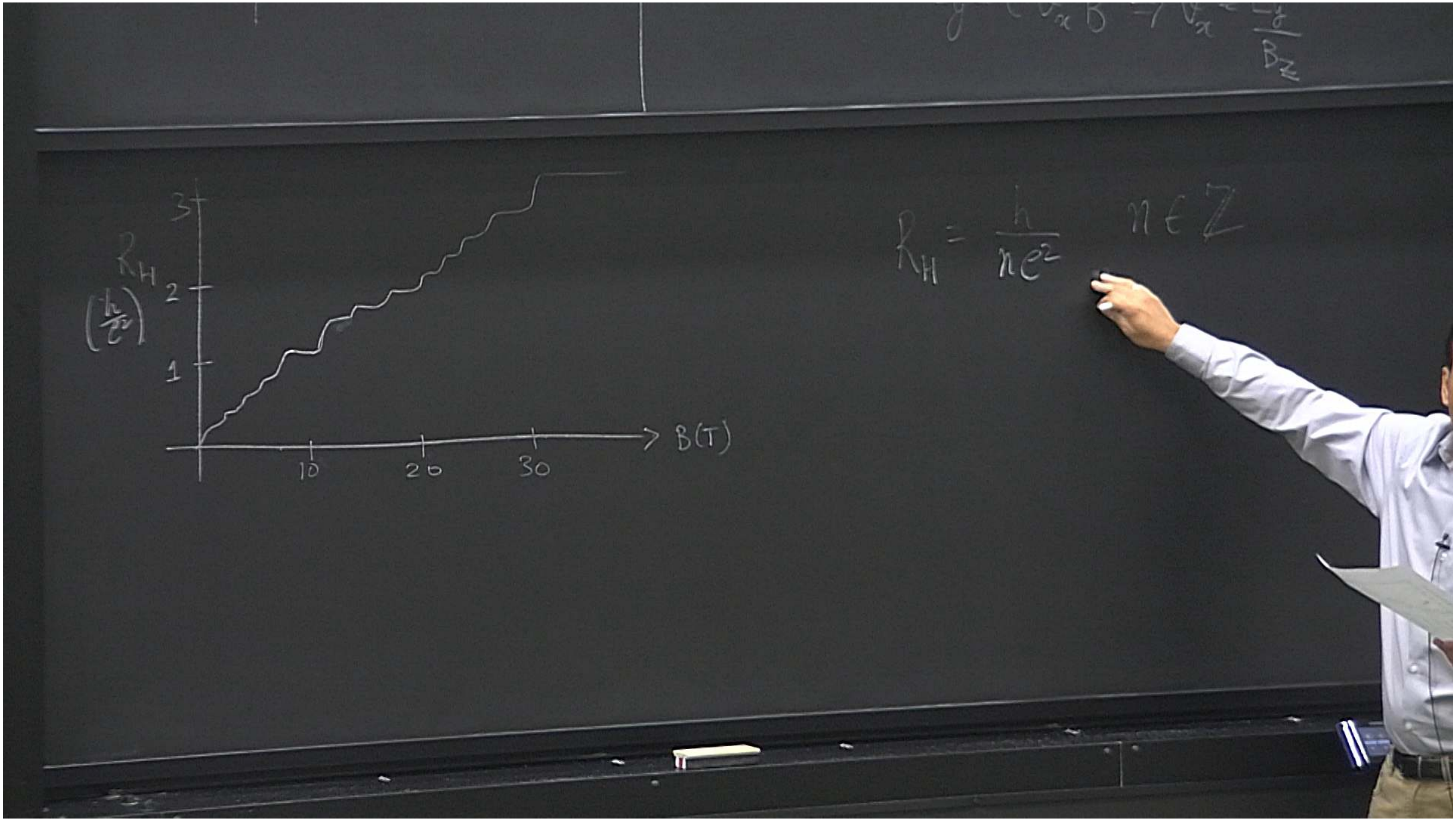
At equilibrium $\quad eE_y = e v_x B_z$

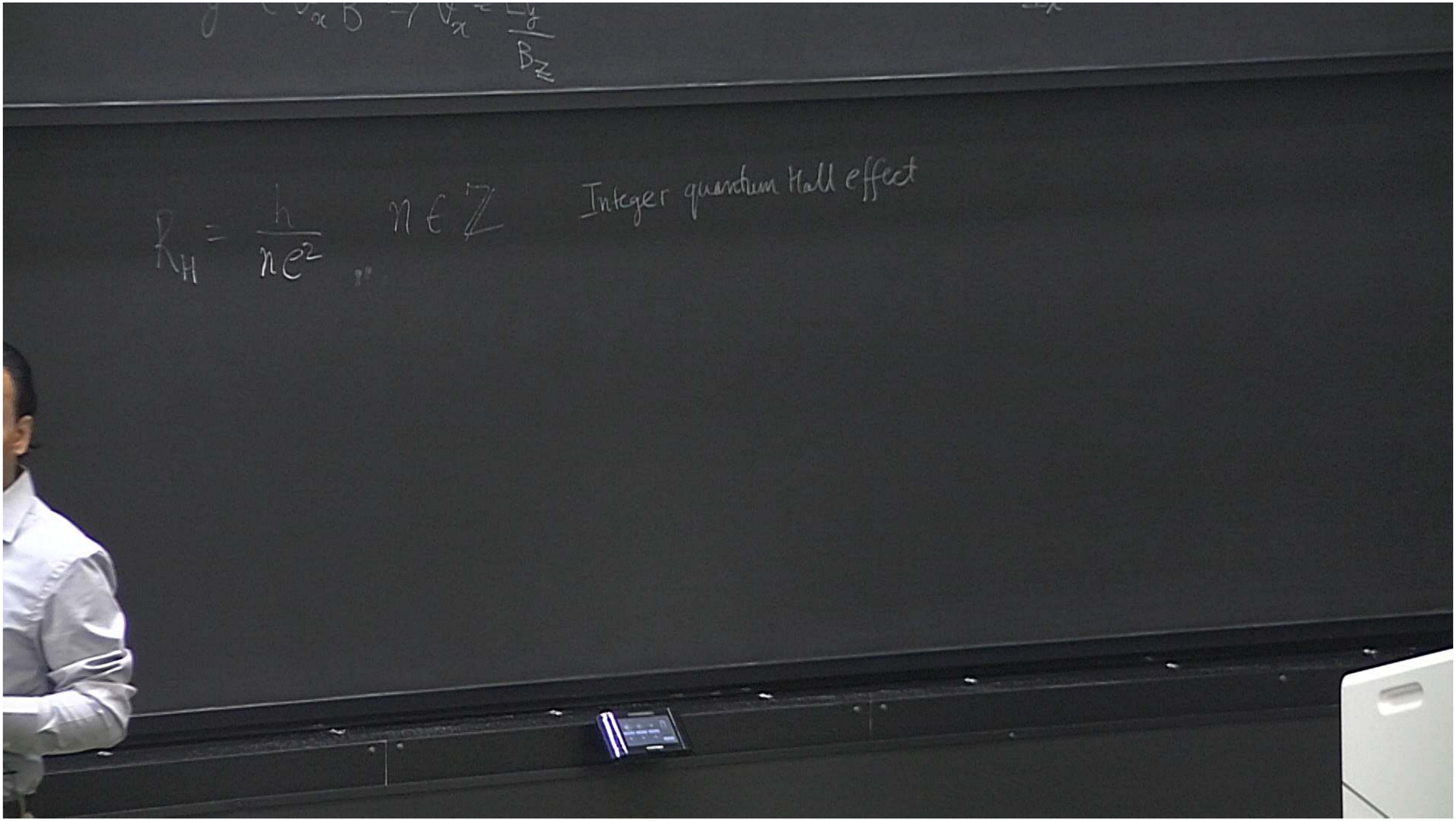

The quantum Hall effect
$\vec{B} = B_z \hat{z}$
I
E_y
I
V_L
V_H
z
y
x
$eE_y = e v_x B$

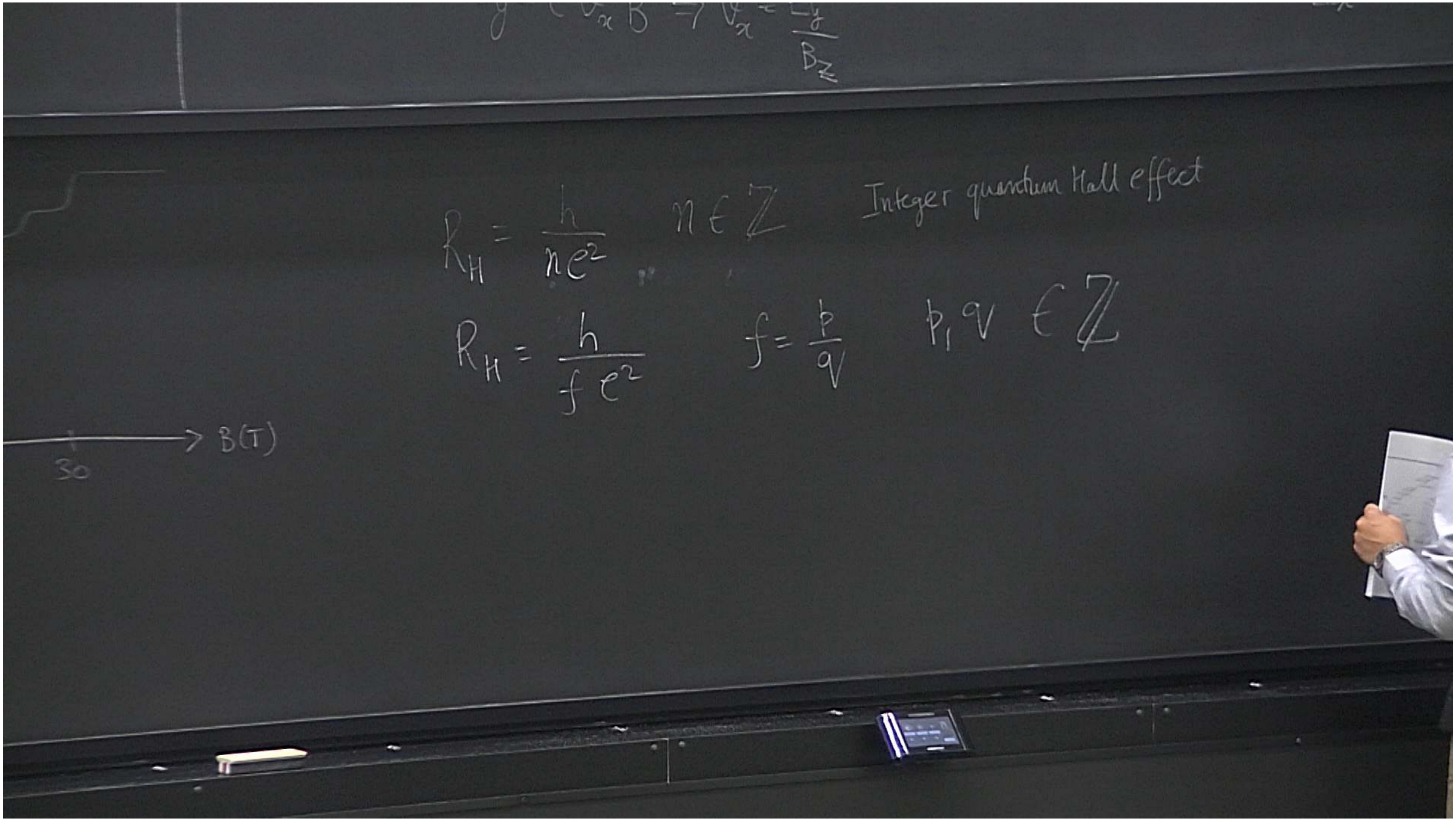

The quantum Hall effect
B = B_z ẑ
I
E_y
I
W
V_L
V_H
z
y
x
E_y = V_H / W
I_x = J_x
eE_y = e v_x B ⇒ v_x = E_y / B_z

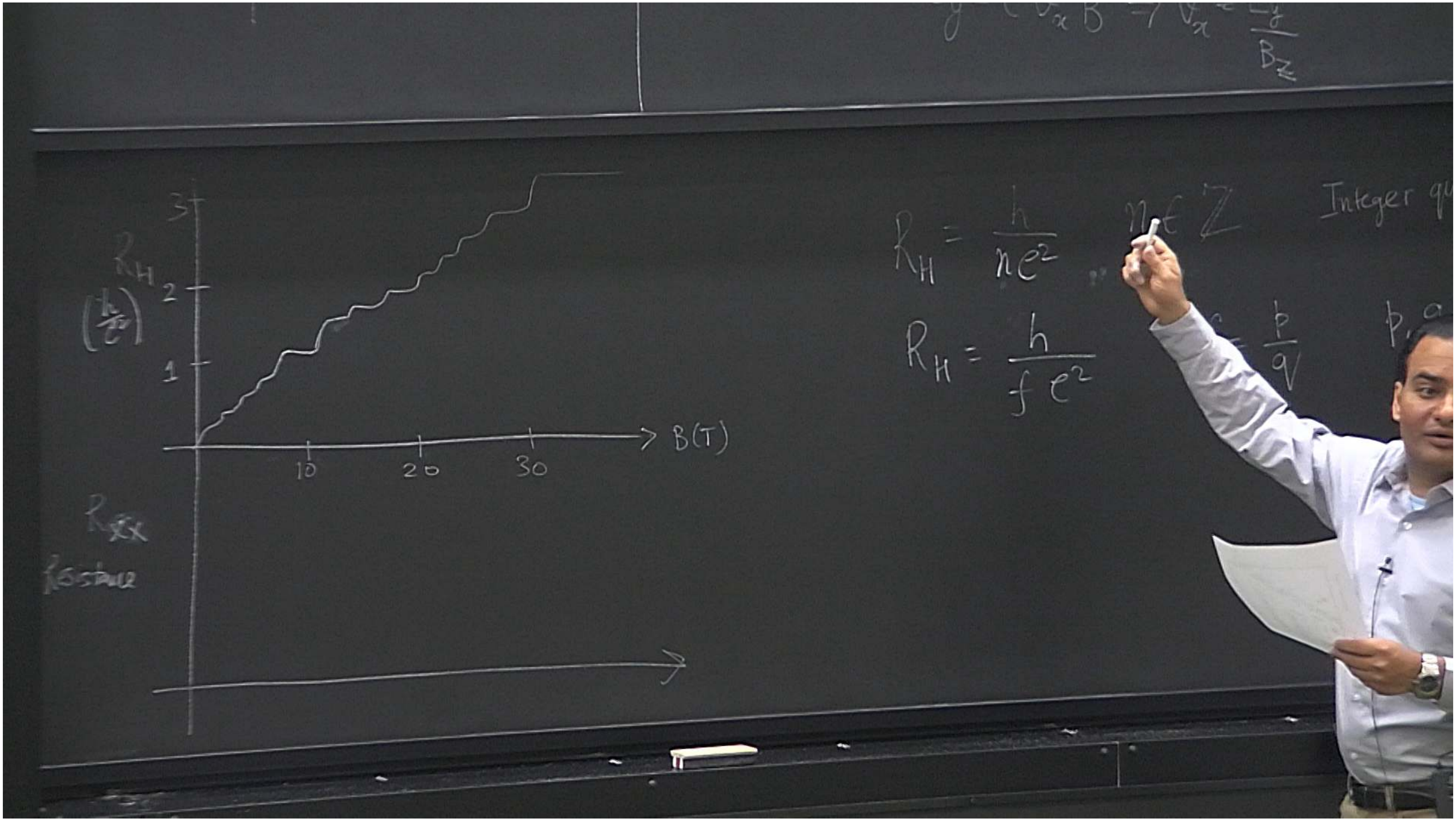

2DEG
2 dimensional electron gas
Ey = VH / W
Ix = Jx


effect
2DEG
2 dimensional electron gas
$E_y = \frac{V_H}{W}$
$I_x = J_x W = n_0 e v_x W$
Hall Resistance, $R_H = \frac{V_H}{I_x} = \frac{E_y W}{n_0 e v_x W} = \frac{B_z}{n_0 e}$

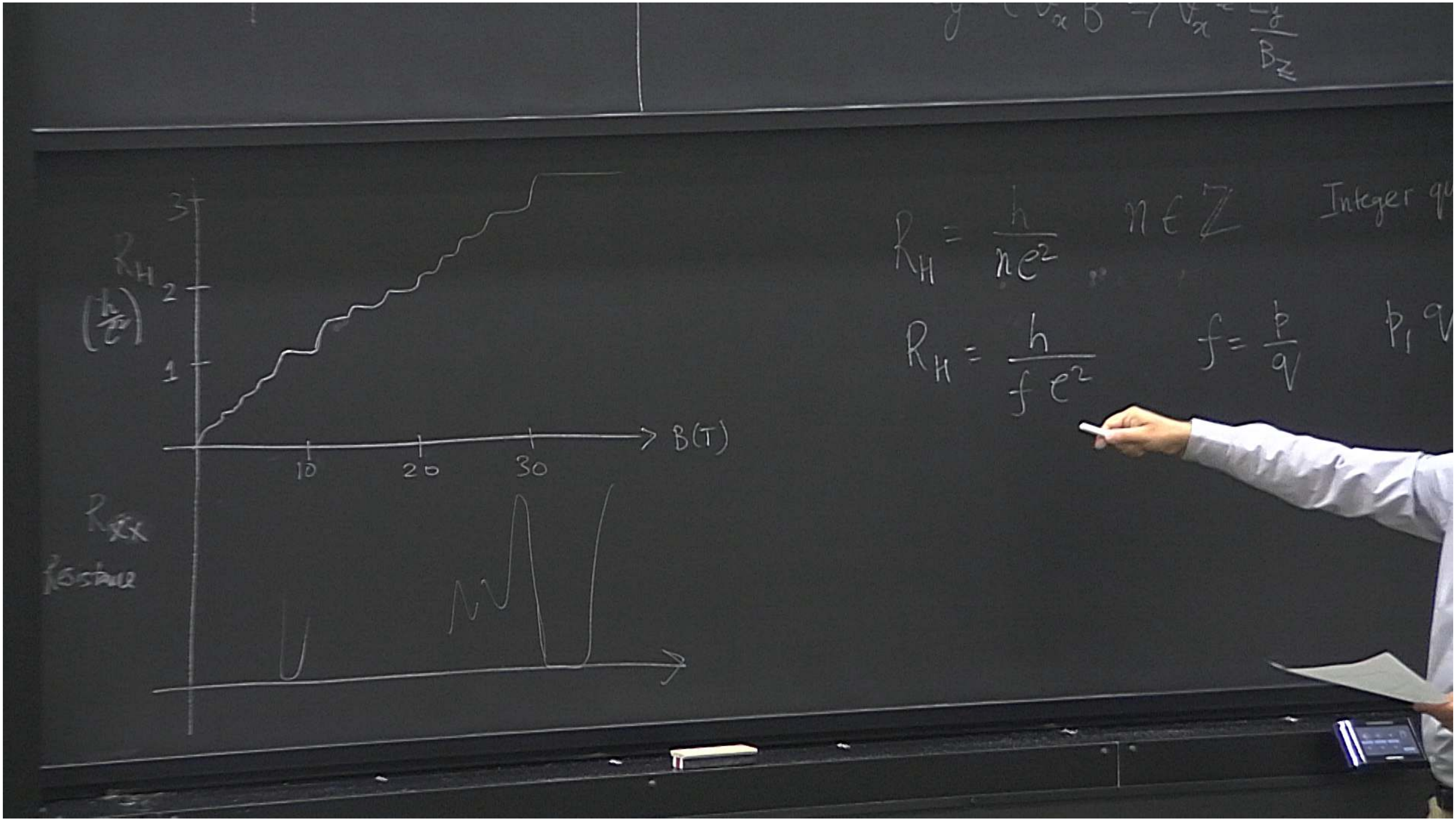

R_H
$\left(\frac{h}{e^2}\right)$
3
2
1
10
20
30
B(T)
$R_H = \frac{h}{ne^2}$
$n \in \mathbb{Z}$


$R_H = \frac{h}{ne^2}$, $n \in \mathbb{Z}$
Integer quantum Hall effect

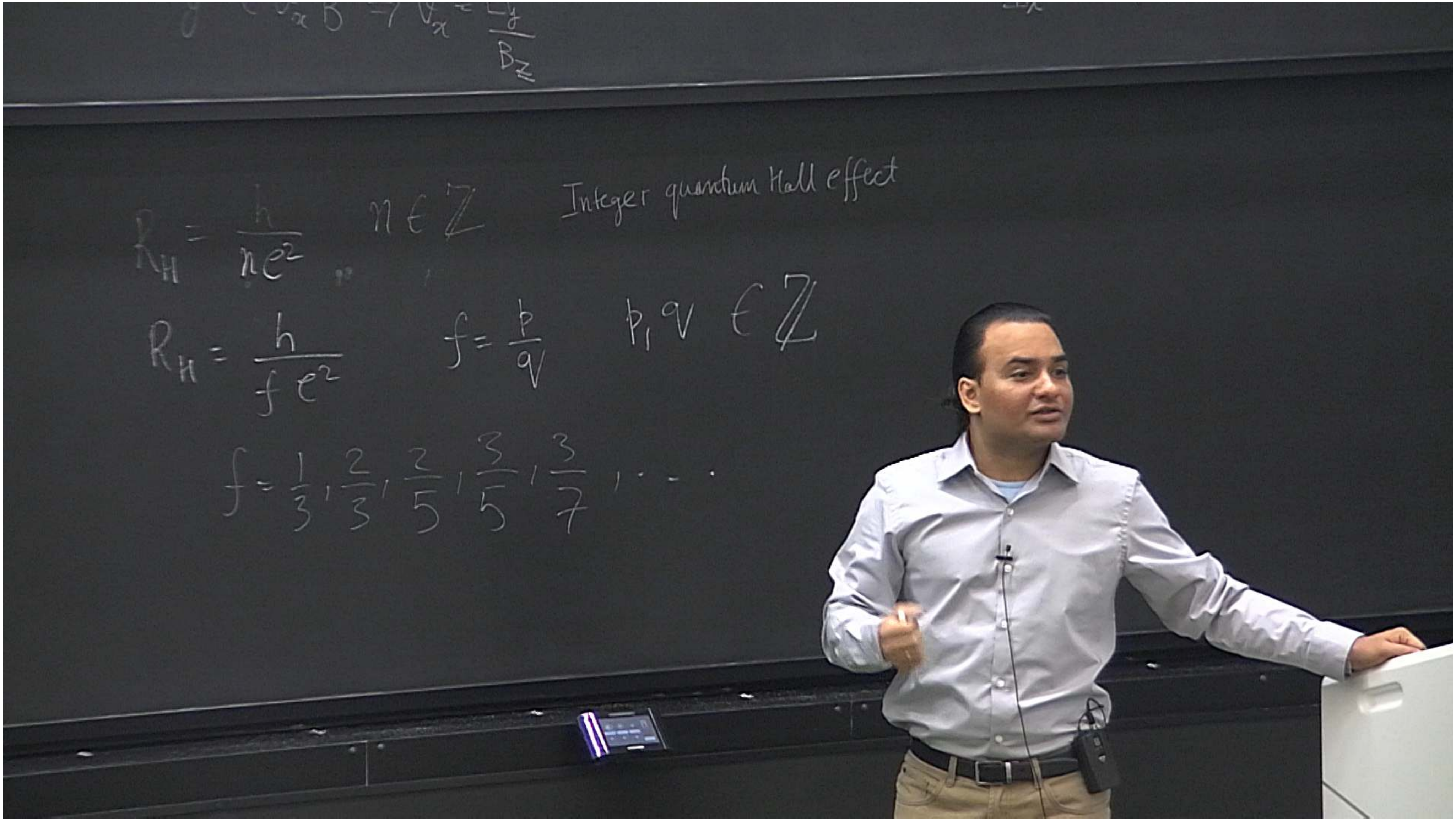

$R_H = \frac{h}{n e^2}$ $n \in \mathbb{Z}$
Integer quantum Hall effect
$R_H = \frac{h}{f e^2}$ $f = \frac{p}{q}$ $p, q \in \mathbb{Z}$
$B(T)$
30



$R_H = \frac{h}{ne^2}$ $n \in \mathbb{Z}$
Integer qu
$R_H = \frac{h}{f e^2}$
$\frac{p}{q}$
R_H $\left(\frac{h}{e^2}\right)$
B(T)
10
20
30
R_{xx}
Resistance



R_H = h / ne^2
n ∈ Z
R_H = h / f e^2
f = p / q
R_H
(h/e^2)
1
2
3
B(T)
10
20
30
R_xx
Resistance


$R_H = \frac{h}{n e^2}$, $n \in \mathbb{Z}$
Integer quantum Hall effect
$R_H = \frac{h}{f e^2}$
$f = \frac{p}{q}$
$p, q \in \mathbb{Z}$
$f = \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{3}{7}, \ldots$
B_z