Title: PSI 2018/2019 - Condensed Matter - Lecture 9

Date: Nov 22, 2018 10:45 AM

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Abstract:

Electron–phonon interactions

$$H_{el\text{-}ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}} S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_{\lambda} \sqrt{\frac{\hbar}{2M\omega_{\lambda}(\bar{q})}} \left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$$

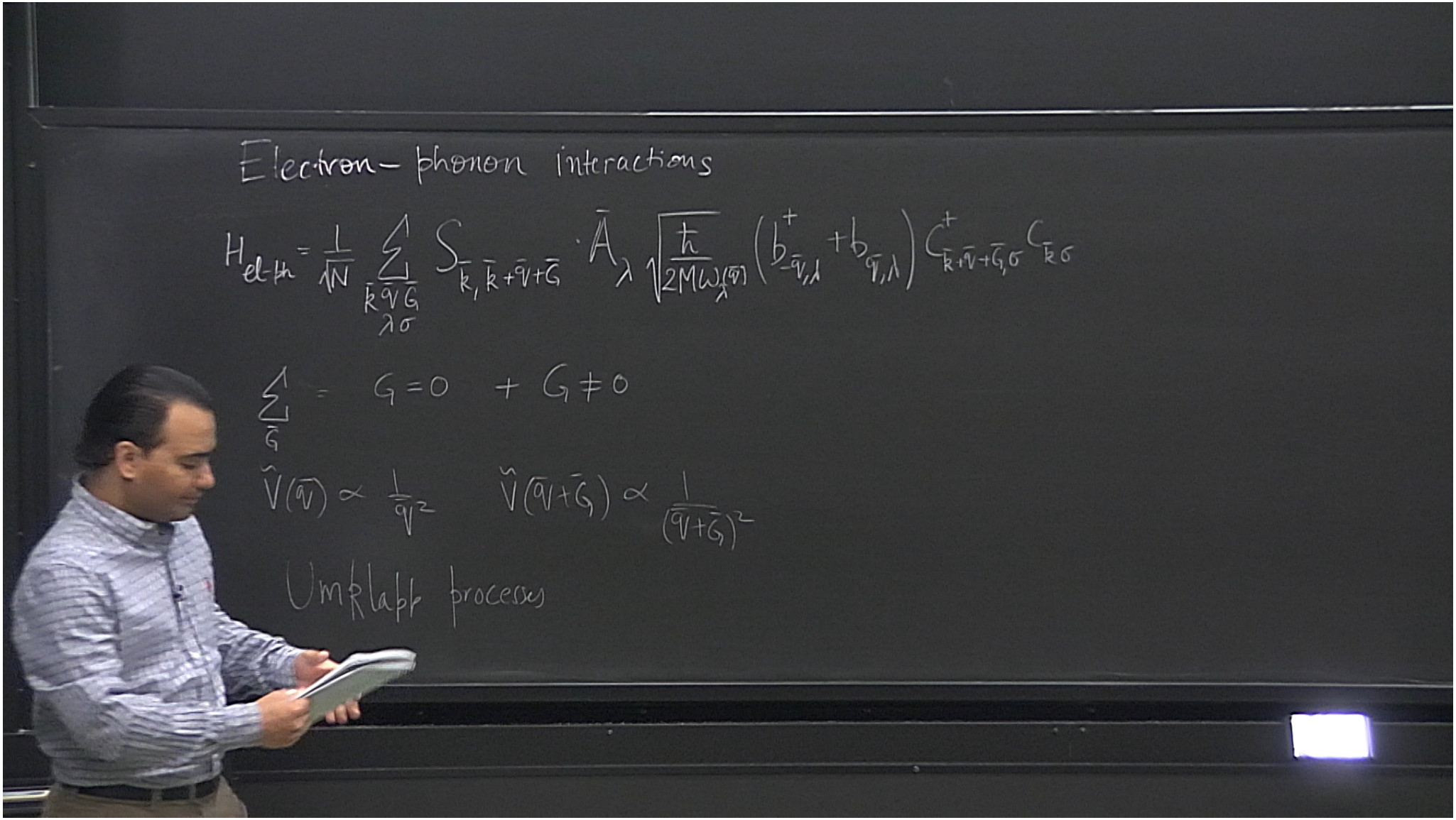
Electron–phonon interactions

$$H_{el\text{-}ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}} S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_\lambda \sqrt{\frac{\hbar}{2M\omega_\lambda(\bar{q})}} \left(b^+_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^+_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$$

$$\sum_{\bar{G}} = G=0 \quad + \quad G \neq 0$$

$$\tilde{V}(\bar{q}) \propto \frac{1}{\bar{q}^2} \qquad \tilde{V}(\bar{q}+\bar{G}) \propto \frac{1}{(\bar{q}+\bar{G})^2}$$




Electron – phonon interactions
$H_{el-ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}} S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_\lambda \sqrt{\frac{\hbar}{2M\omega_\lambda(\bar{q})}} \left(b^+_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^+_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$
$\sum_{\bar{G}} = G=0 + G\neq 0$
$\tilde{V}(\bar{q}) \propto \frac{1}{\bar{q}^2}$ $\tilde{V}(\bar{q}+\bar{G}) \propto \frac{1}{(\bar{q}+\bar{G})^2}$
Umklapp processes

$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma} c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}} c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \frac{1}{2}\right)$$

$$H_{el\text{-}ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right)$$

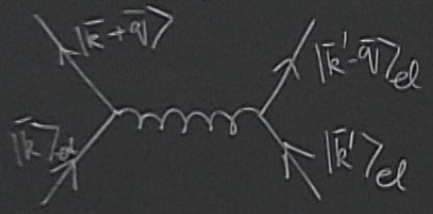


$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma} c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el-ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}} c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \frac{1}{2}\right)$$

$$H_{el-ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{+}_{-\bar{q}} + b_{\bar{q}}\right)$$

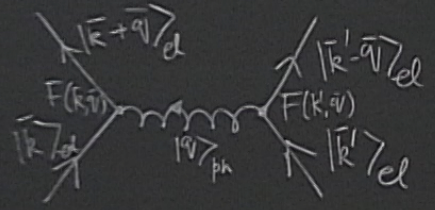


$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma}\, c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}}\, c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \frac{1}{2}\right)$$

$$H_{el\text{-}ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{+}_{-\bar{q}} + b_{\bar{q}}\right)$$

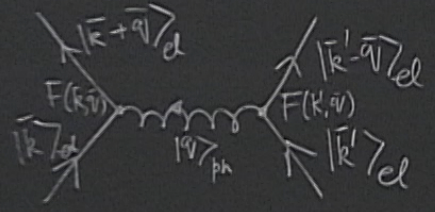


$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma}\, c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}}\, c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\,\omega(\bar{q})\left(b^{+}_{\bar{q}} b_{\bar{q}} + \tfrac{1}{2}\right)$$

$$H_{el\text{-}ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}}\left(b^{+}_{-\bar{q}} + b_{\bar{q}}\right)$$



Schrieffer Wo… …nsform…

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

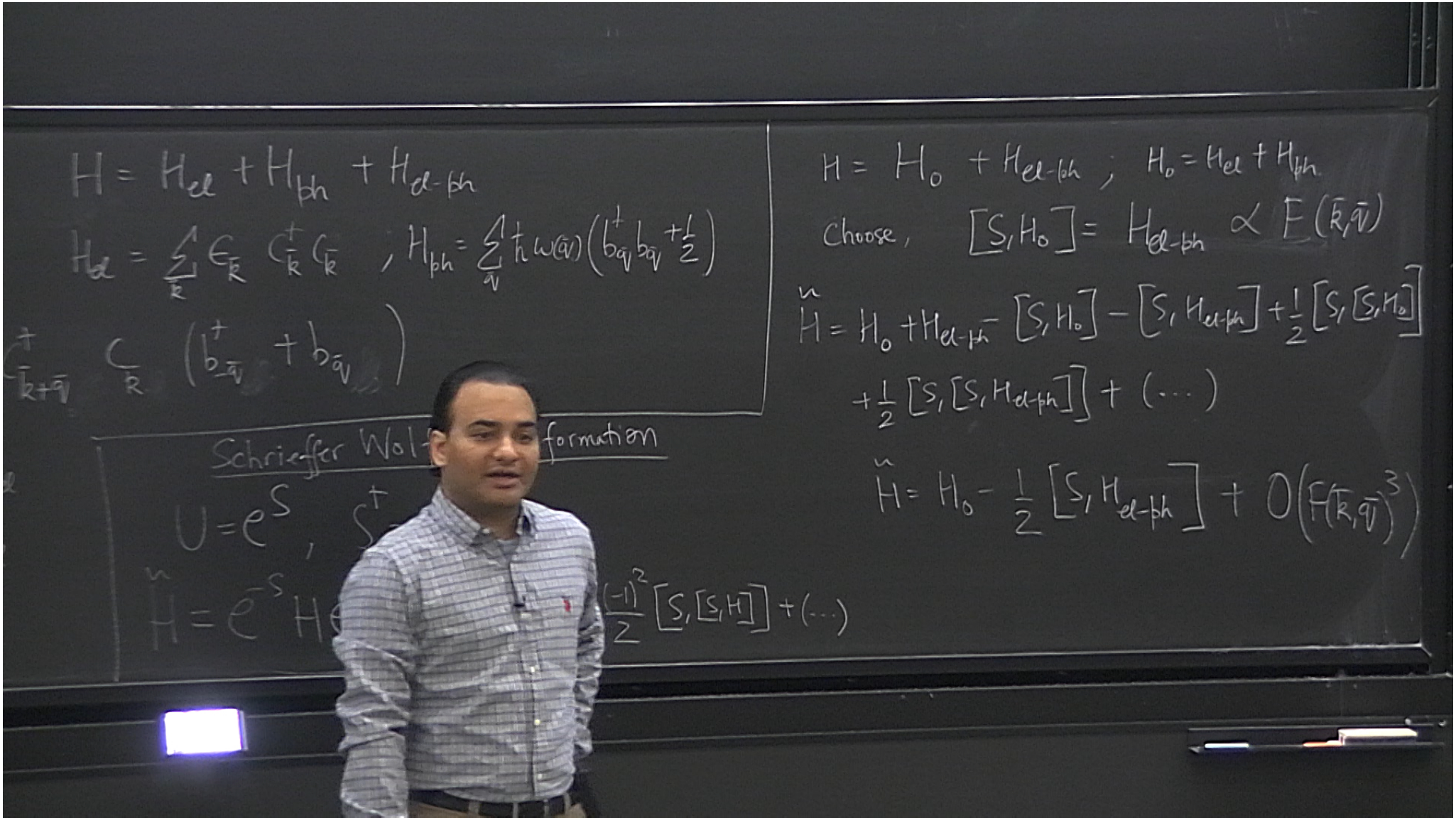
$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{\dagger}_{\bar{k}} c_{\bar{k}} \quad ; \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{\dagger}_{\bar{q}} b_{\bar{q}} + \frac{1}{2} \right)$$

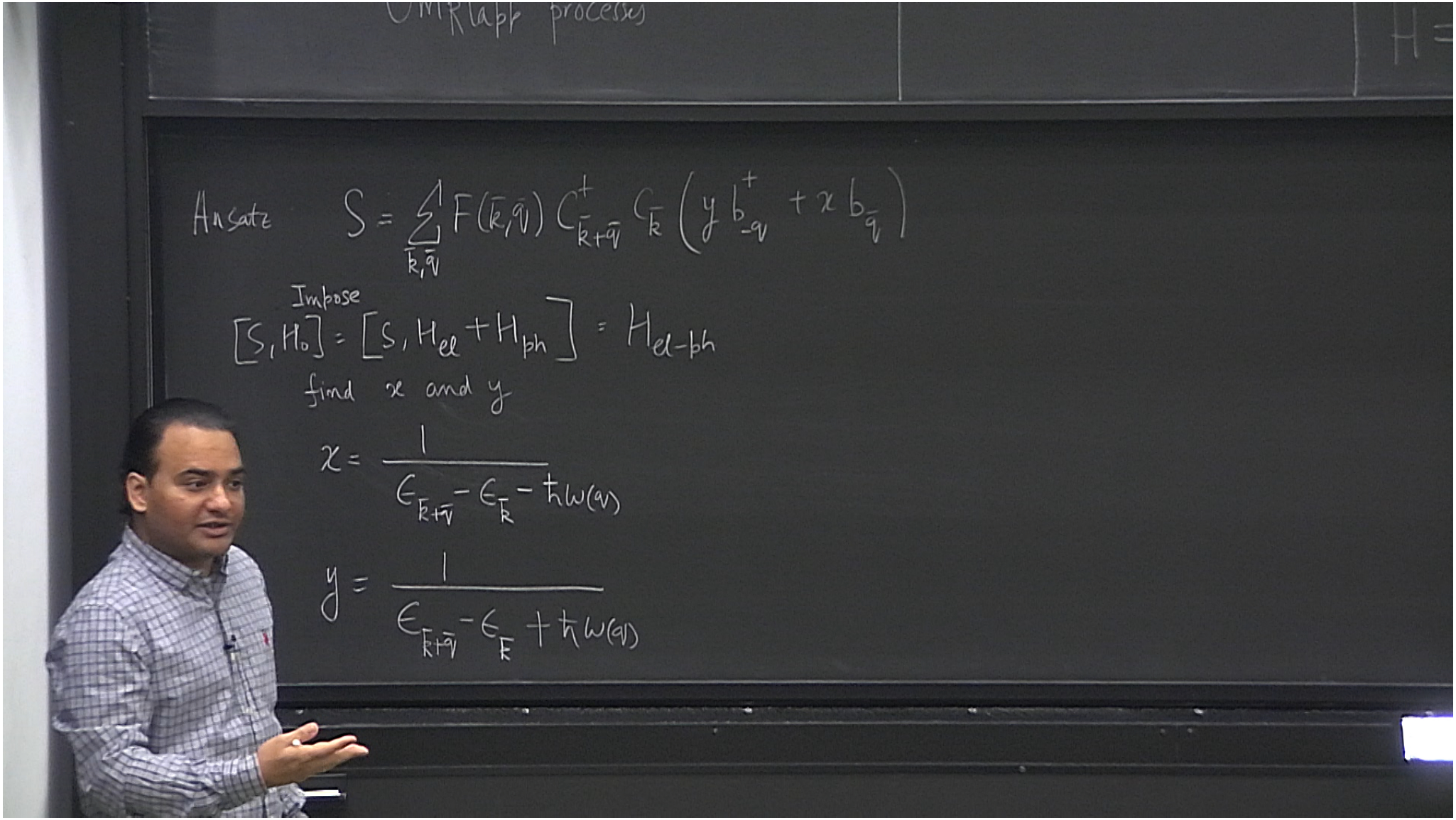
$$c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{\dagger}_{-\bar{q}} + b_{\bar{q}} \right)$$

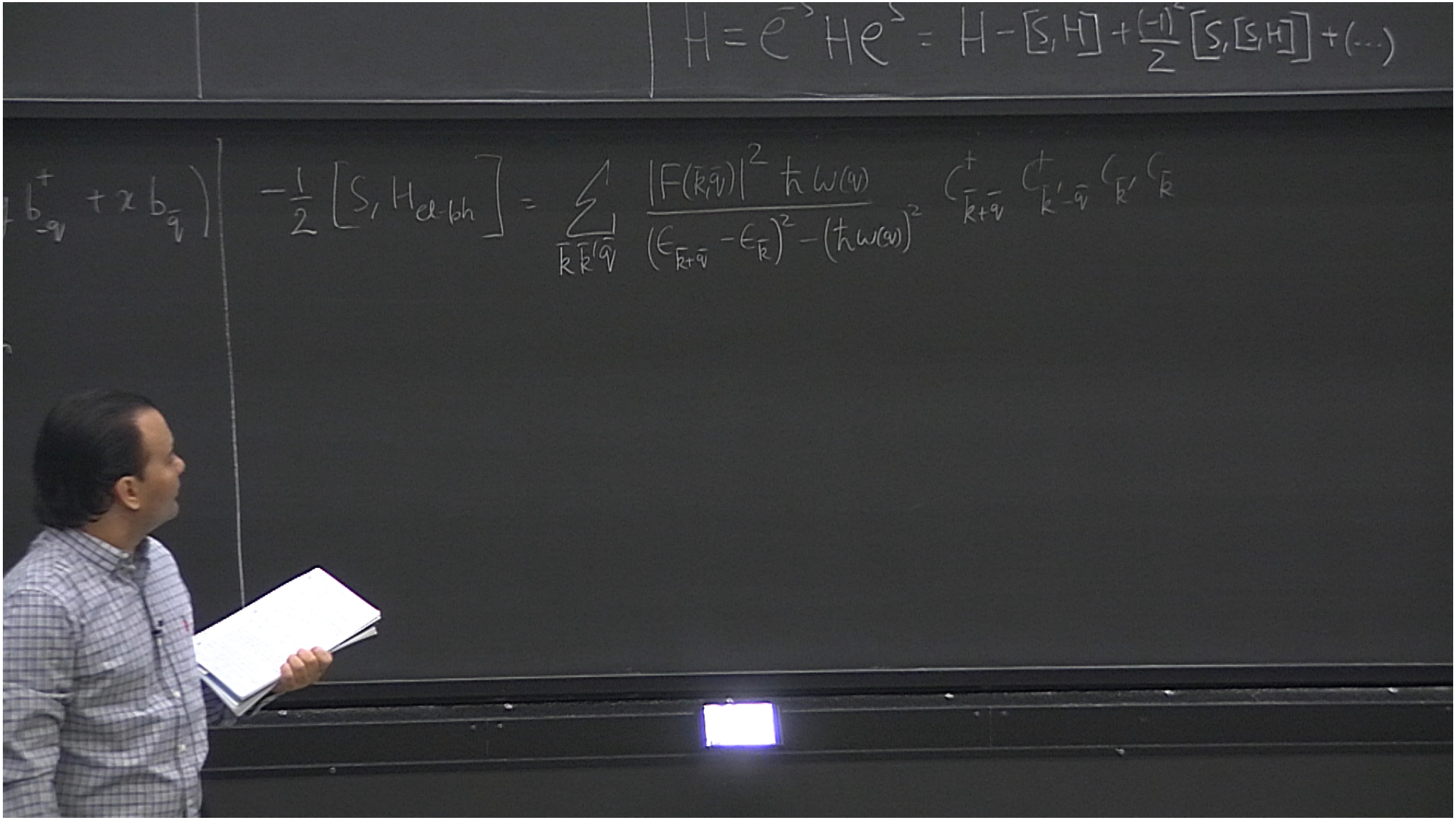
Schrieffer Wolff transformation

$$U = e^{S}, \quad S^{\dagger} = -S$$

$$\tilde{H} = e^{-S} H e^{S} = H - [S, H] + \frac{(-1)^2}{2} [S, [S, H]] + (\dots)$$


$H = H_{el} + H_{ph} + H_{el-ph}$
$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}} c^{\dagger}_{\bar{k}} c_{\bar{k}}$, $H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{\dagger}_{\bar{q}} b_{\bar{q}} + \frac{1}{2} \right)$
$c^{\dagger}_{\bar{k}+\bar{q}} \; c_{\bar{k}} \left(b^{\dagger}_{-\bar{q}} + b_{\bar{q}} \right)$
$H = H_0 + H_{el-ph}$; $H_0 = H_{el} + H_{ph}$
Choose, $[S, H_0] = H_{el-ph} \propto F(\bar{k}, \bar{q})$
$\tilde{H} = H_0 + H_{el-ph} - [S, H_0] - [S, H_{el-ph}] + \frac{1}{2}[S, [S, H_0]]$
$+ \frac{1}{2}[S, [S, H_{el-ph}]] + (\cdots)$
$\tilde{H} = H_0 - \frac{1}{2}[S, H_{el-ph}] + O\left(F(\bar{k}, \bar{q})^3\right)$
$U = e^{S}$,
$\tilde{H} = e^{-S} H$
$\frac{(-1)^2}{2}[S, [S, H]] + (\cdots)$


Klapp processes
H=
Ansatz $S = \sum_{\bar{k},\bar{q}} F(\bar{k},\bar{q})\, C^{+}_{\bar{k}+\bar{q}}\, C_{\bar{k}} \left(y\, b^{+}_{-q} + x\, b_{\bar{q}} \right)$
Impose
$[S, H_0] = [S, H_{el} + H_{ph}] = H_{el-ph}$
find x and y
$x = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} - \hbar\omega(q)}$
$y = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} + \hbar\omega(q)}$


$H = e^{S} H e^{S} = H - [S,H] + \frac{(-1)^2}{2}[S,[S,H]] + (\dots)$
$-\frac{1}{2}[S, H_{el-ph}] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2 \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2} c^{\dagger}_{\bar{k}+\bar{q}} c^{\dagger}_{\bar{k}'-\bar{q}} c_{\bar{k}'} c_{\bar{k}}$

$$H = e^{S} H e^{S} = H - [S,H] + \frac{(-1)^2}{2}[S,[S,H]] + (\dots)$$

$$\left(b^{\dagger}_{-q} + x\, b_{\bar{q}} \right)$$

$$-\frac{1}{2}\left[S, H_{el-ph}\right] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2 \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2}\, c^{\dagger}_{\bar{k}+\bar{q}}\, c^{\dagger}_{\bar{k}'-\bar{q}}\, c_{\bar{k}'}\, c_{\bar{k}}$$

If $(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 > (\hbar\omega(q))^2$: Repulsive

$(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 < (\hbar\omega(q))^2$: Attractive



$$H = e^{S} H e^{S} = H - [S, H] + \frac{(-1)^2}{2}[S,[S,H]] + (\dots)$$

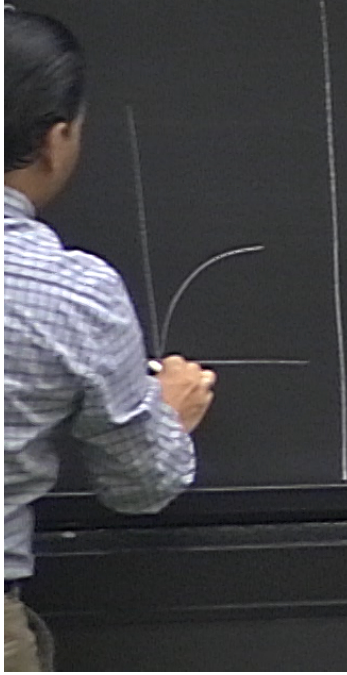
$$\left(b_{-\bar{q}}^{\dagger} + \alpha\, b_{\bar{q}} \right)$$

$$-\frac{1}{2}\left[S, H_{el\text{-}ph}\right] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2\, \hbar\omega(q)}{\left(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}}\right)^2 - \left(\hbar\omega(q)\right)^2}\, c^{\dagger}_{\bar{k}+\bar{q}}\, c^{\dagger}_{\bar{k}'-\bar{q}}\, c_{\bar{k}'}\, c_{\bar{k}}$$

If $\left(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}}\right)^2 > \left(\hbar\omega(q)\right)^2$: Repulsive

$\left(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}}\right)^2 < \left(\hbar\omega(q)\right)^2$: Attractive

Optical phonons $\hbar\omega(q) \xrightarrow[q\to 0]{}$ Constant



$$S = \sum_{\bar{k},\bar{q}} F(\bar{k},\bar{q})\, c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b_{-\bar{q}} + b^{\dagger}_{\bar{q}} \right)$$

Impose

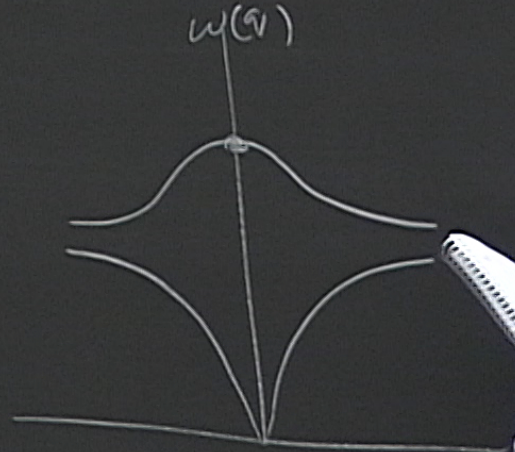
$$[\,\cdot\,, H_{el} + H_{ph}] = H_{el-ph}$$

find x and y

$$x = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} - \hbar\omega(q)}$$

$$y = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} + \hbar\omega(q)}$$

If



$$H = e^{S} H e^{S} = H - [S, H] + \frac{(-1)}{2}[S,[S,H]] + (\dots)$$

$$-\frac{1}{2}[S, H_{el\text{-}bh}] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k}\bar{q})|^2 \hbar\omega(\bar{q})}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2} c^{\dagger}_{\bar{k}+\bar{q}} c^{\dagger}_{\bar{k}'-\bar{q}} c_{\bar{k}'} c_{\bar{k}}$$

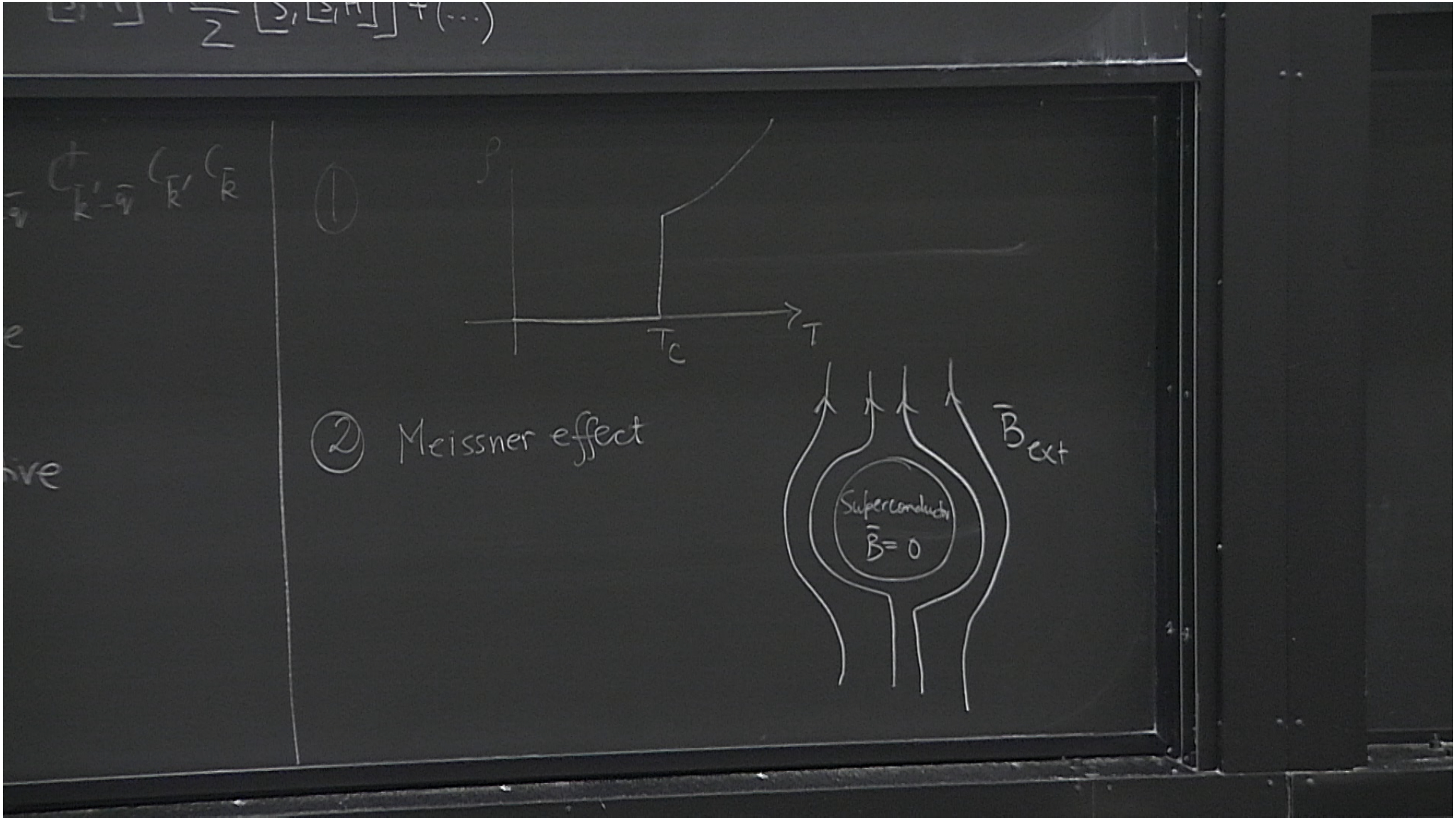
If $(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 > (\hbar\omega(q))^2$: Repulsive

$(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 < (\hbar\omega(q))^2$: Attractive

Optical phonons $\hbar\omega(q) \xrightarrow[q\to 0]{}$ Constant

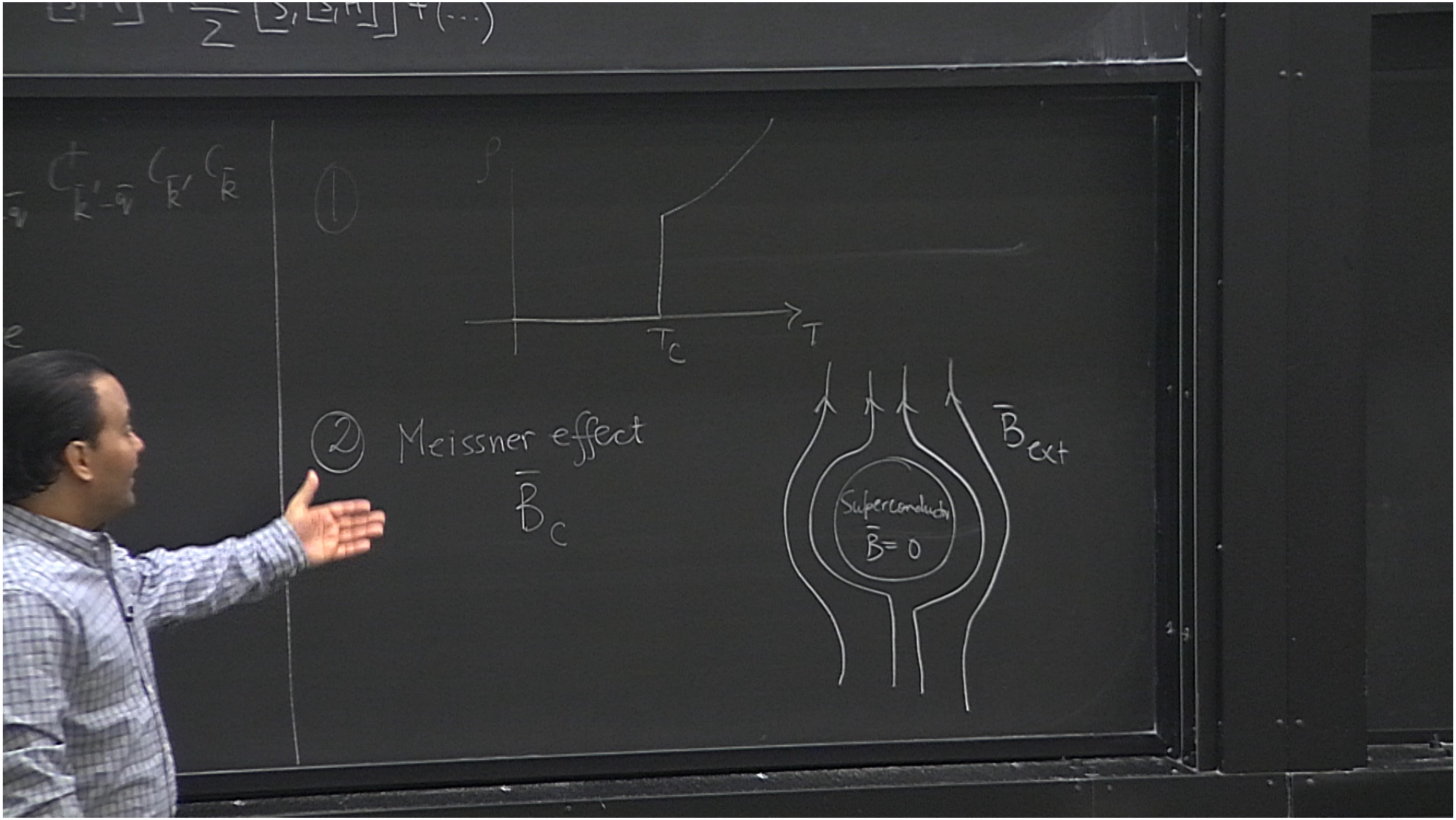
TWO electrons close to Fermi sea

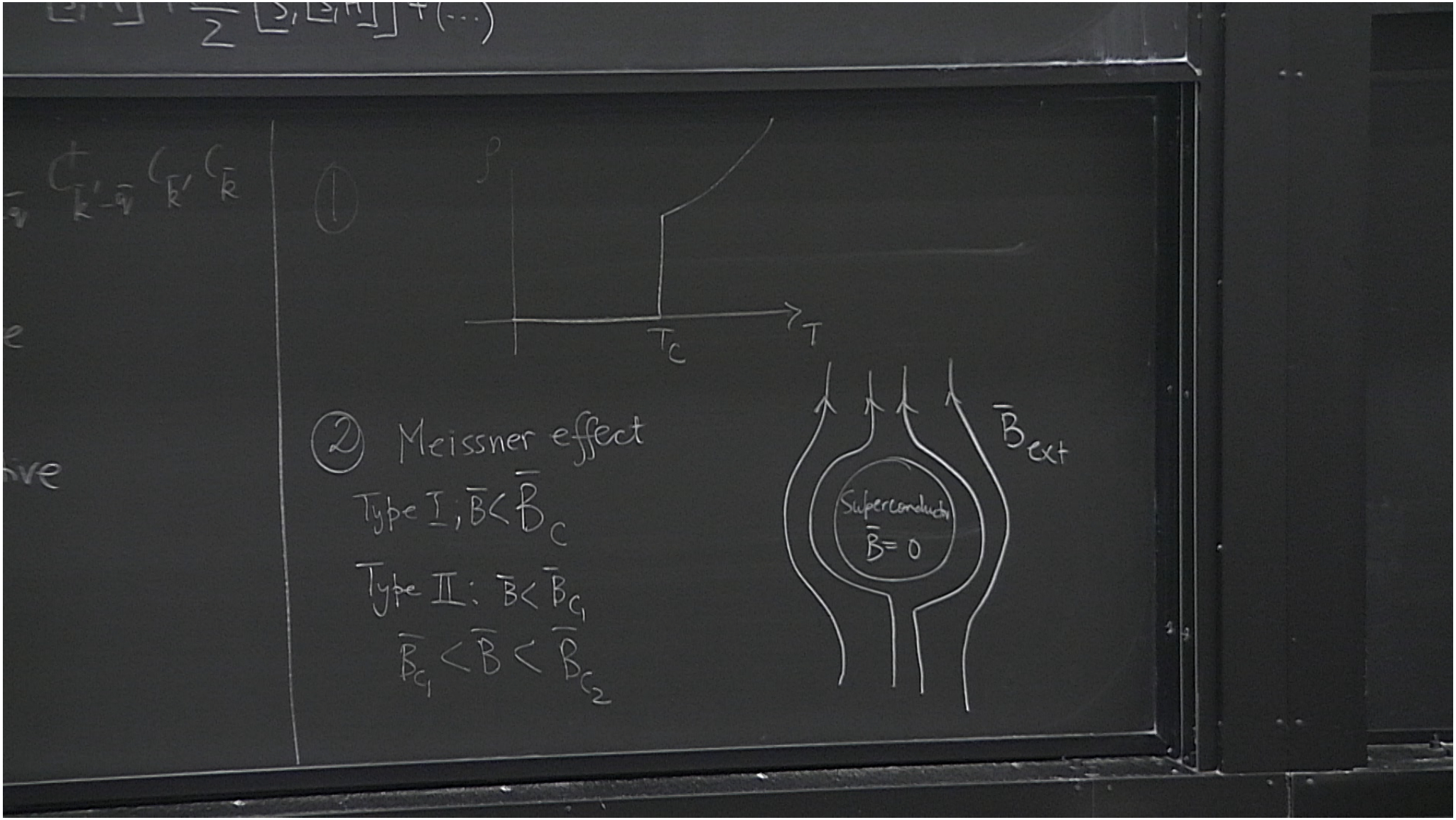
$$\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} = \frac{\hbar^2(k_F+q)^2}{2m} - \frac{\hbar^2 k_F^2}{2m} \xrightarrow{q\to 0} 0$$



① Meissner effect

② Meissner effect


①
ρ
T_c
T
② Meissner effect
$\bar{B}_c$
$\bar{B}_{ext}$
Superconductor
$\bar{B}=0$



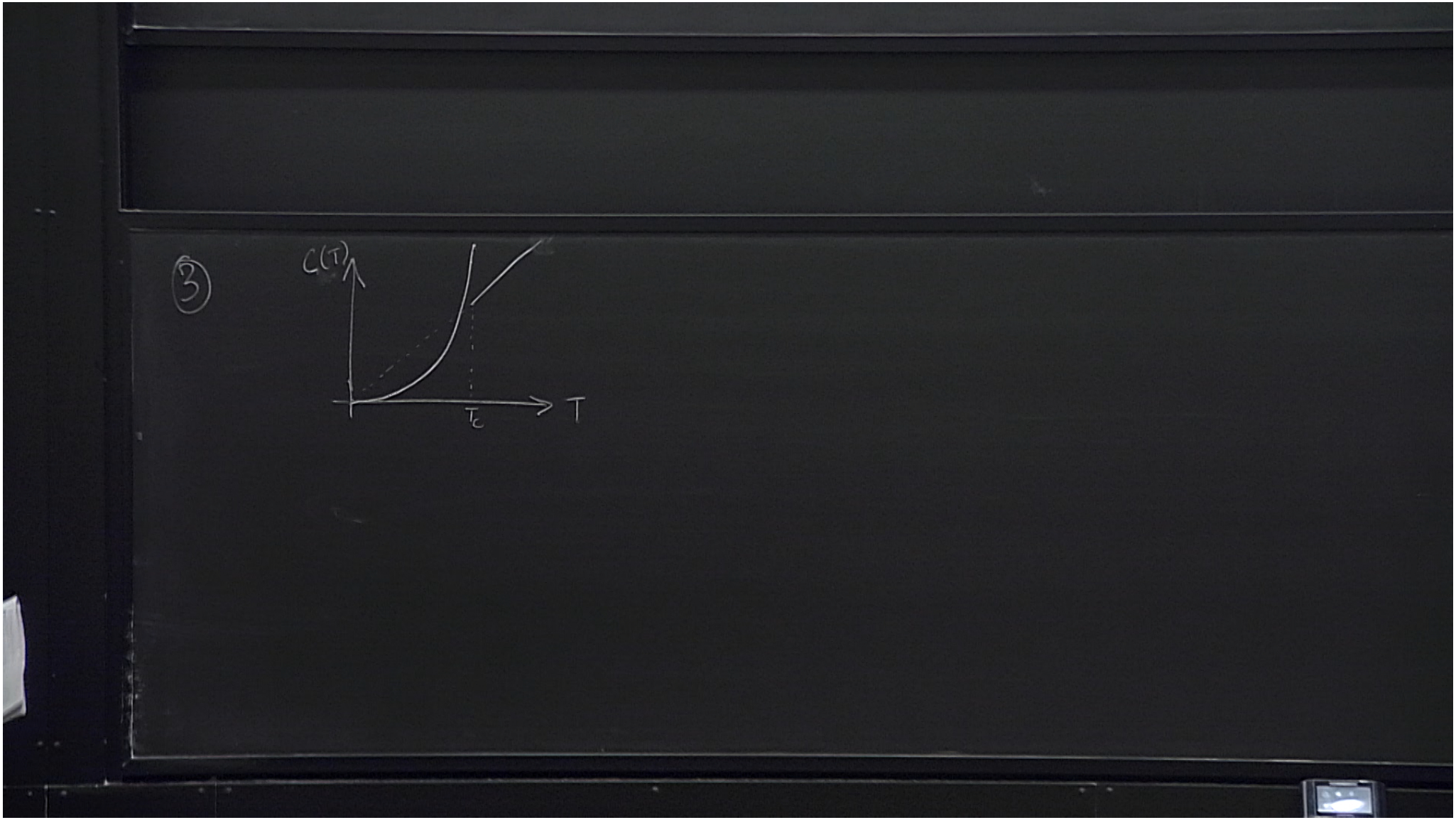
① ρ vs T, T_c

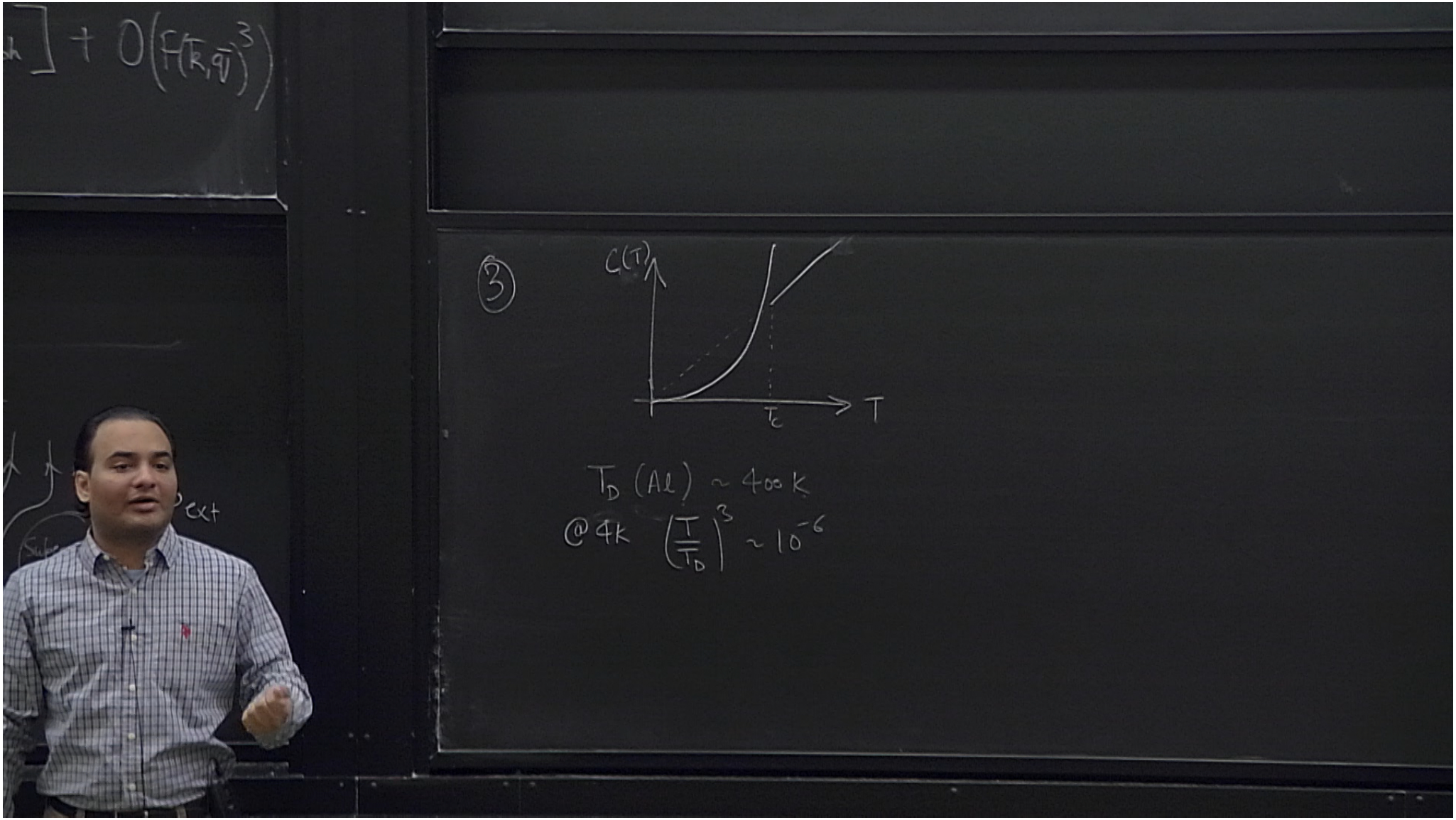
② Meissner effect

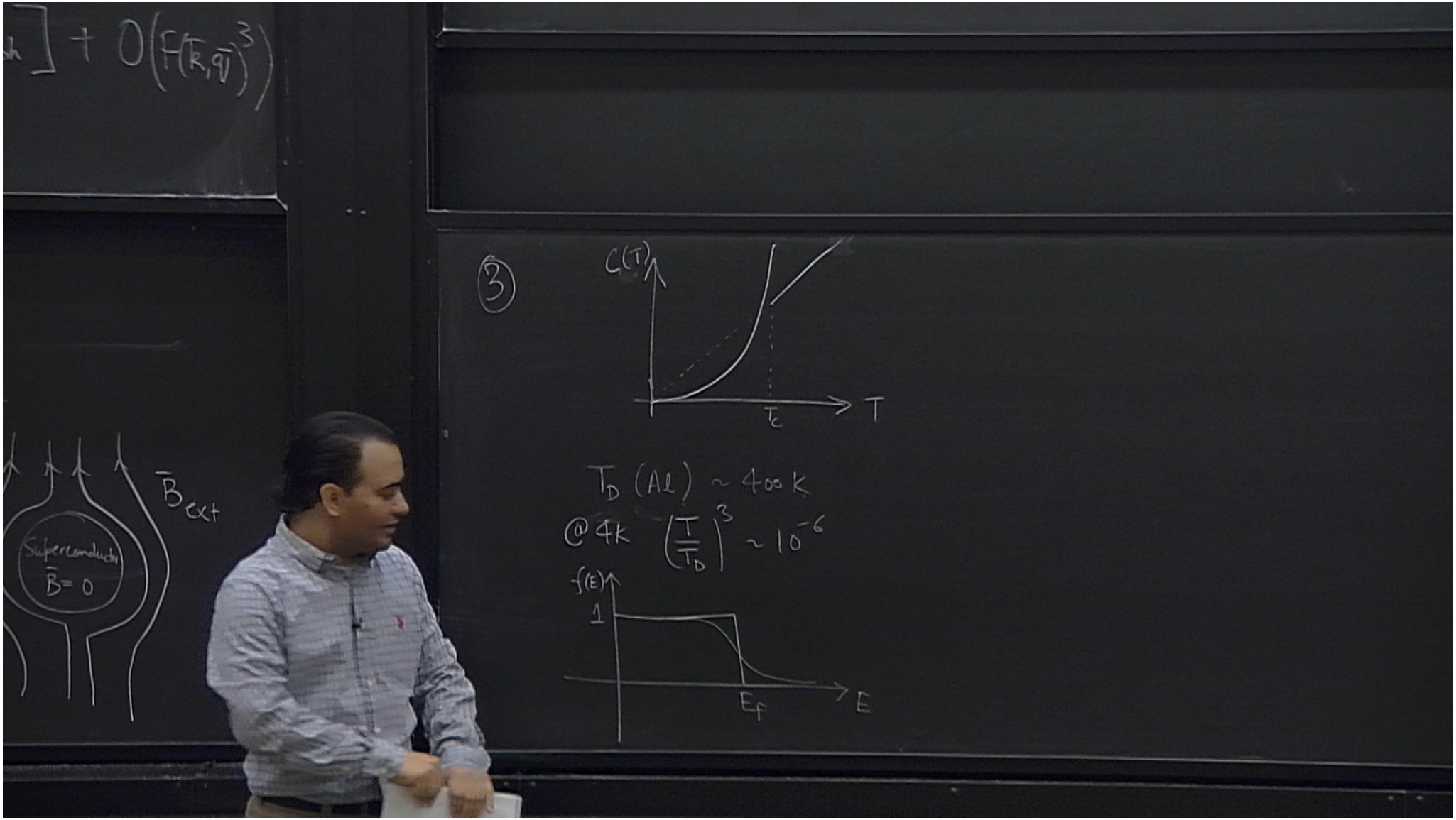
Type I; $\bar{B} < \bar{B}_c$

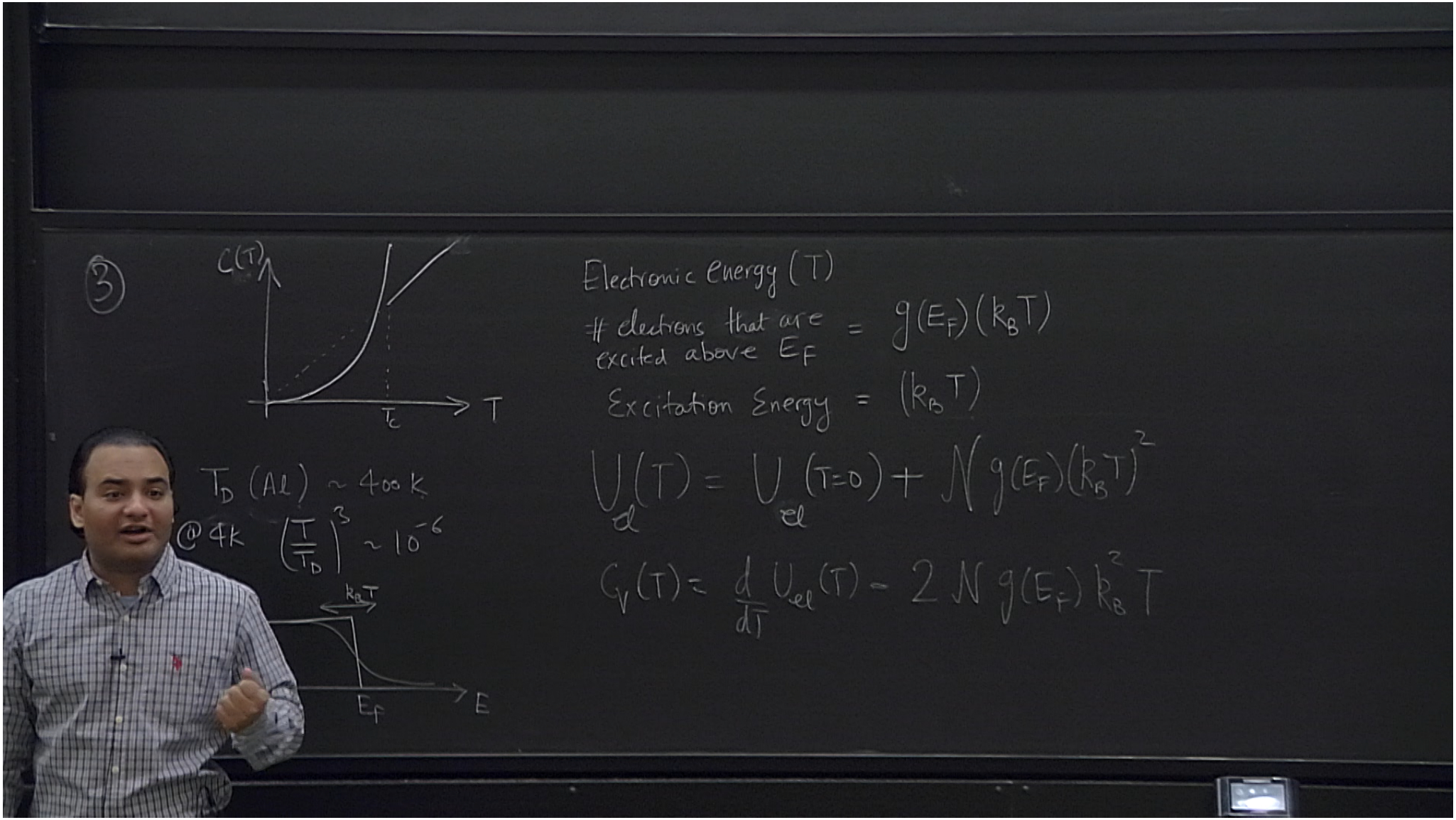
Type II: $\bar{B} < \bar{B}_{c_1}$

$\bar{B}_{c_1} < \bar{B} < \bar{B}_{c_2}$

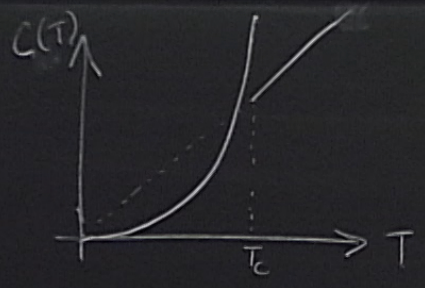

3
C(T)
T
T_c


$+ O(F(\vec{k},\vec{q})^3)$
③
C(T)
T
T_c
T_D (Al) ~ 400 K
@4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$


+ O(F(R,q)^3)
3
C(T)
T
T_c
T_D (Al) ~ 400 K
@ 4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$
f(E)
1
E_f
E
$\bar{B}_{ext}$
Superconductor
$\bar{B}=0$

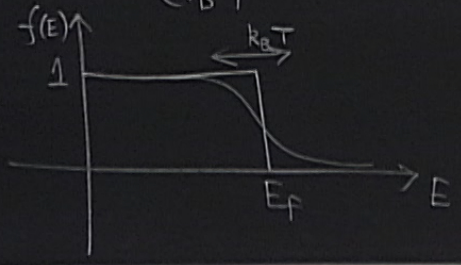

3
C(T)
T
T_c
Electronic Energy (T)
electrons that are excited above E_F = $g(E_F)(k_B T)$
Excitation Energy = $(k_B T)$
T_D (Al) ~ 400 K
@ 4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$
$U_{el}(T) = U_{el}(T=0) + N\,g(E_F)(k_B T)^2$
$C_V(T) = \frac{d}{dT} U_{el}(T) = 2N\,g(E_F)\,k_B^2\,T$
$k_B T$
E_F
E

(3)



T_D (Al) ~ 400 K

@ 4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



Electronic Energy (T)

electrons that are excited above E_F = $g(E_F)(k_B T)$

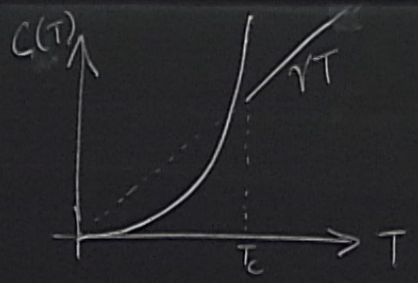
Excitation Energy = $(k_B T)$

$$U_{el}(T) = U_{el}(T=0) + N\left(g(E_F)(k_B T)\right)^2$$

$$C_V(T) = \frac{d}{dT} U_{el}(T) = 2N\, g(E_F)\, k_B^2\, T$$

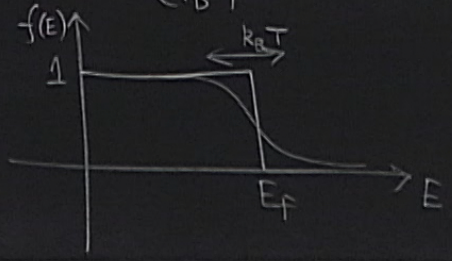
$$N = \frac{\pi^2}{6}$$

③



T_D (Al) ~ 400 K

@ 4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



Electronic Energy (T)

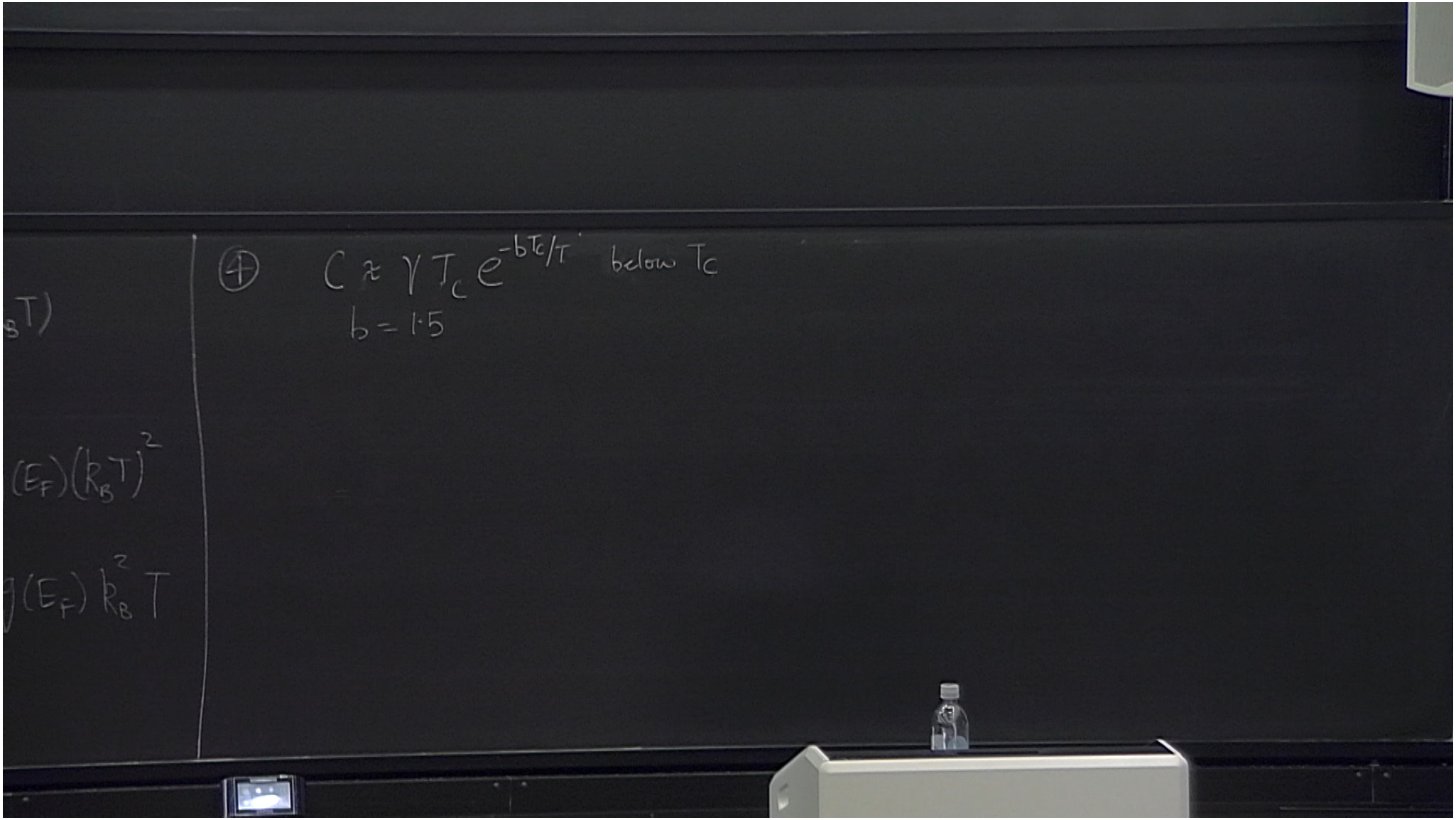
electrons that are excited above E_F = $g(E_F)(k_B T)$

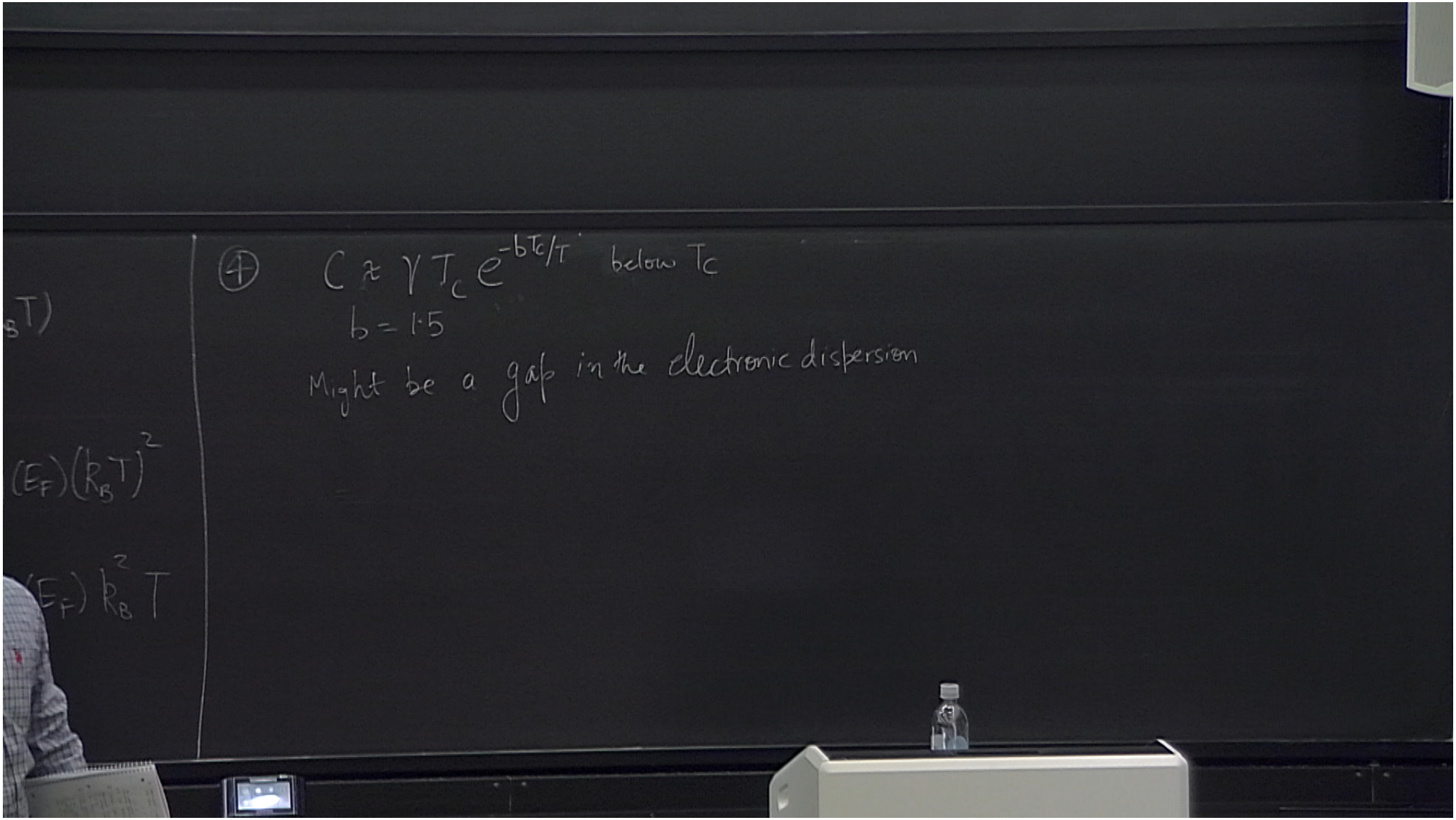
Excitation Energy = $(k_B T)$

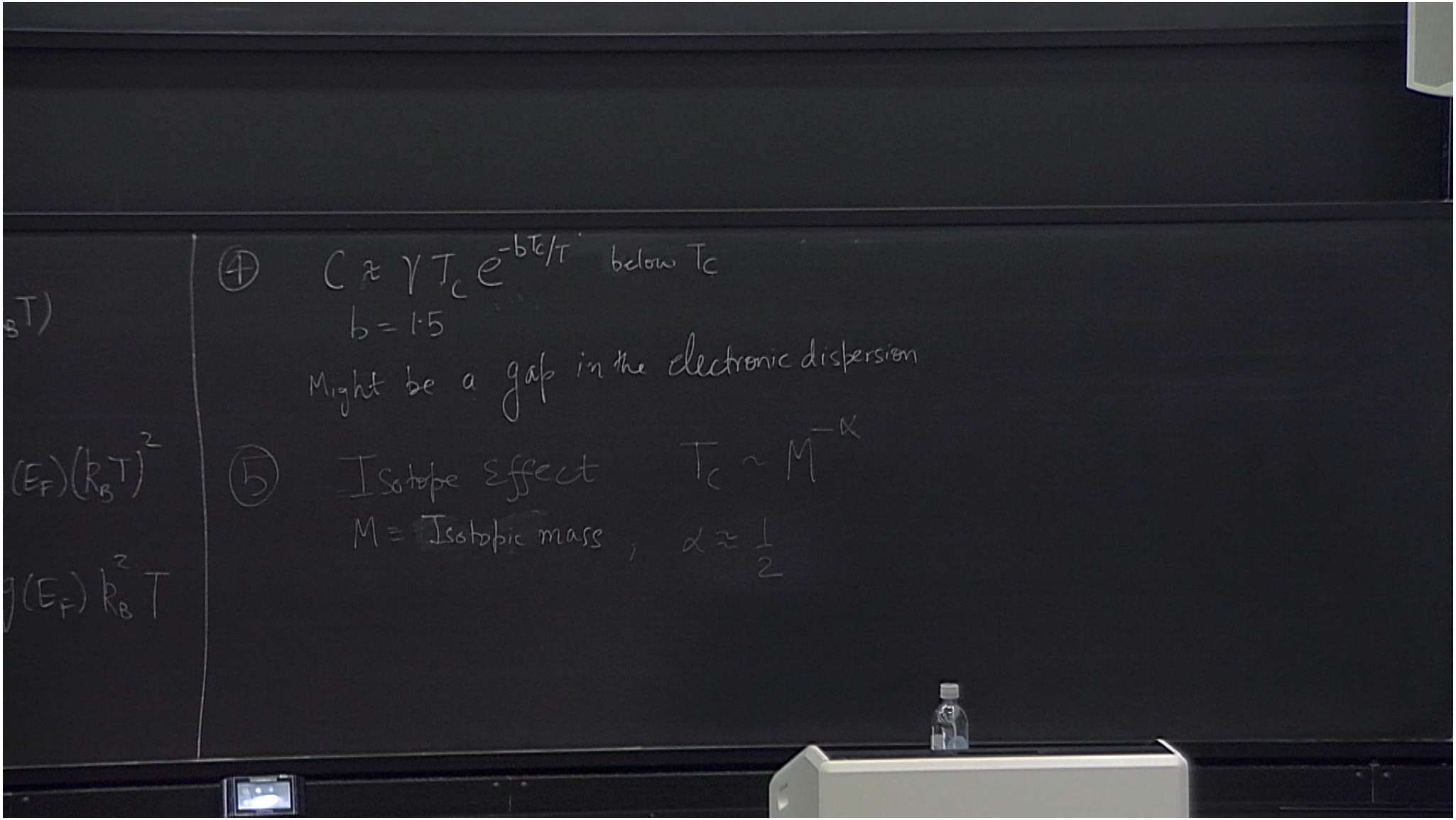
$$U_{el}(T) = U_{el}(T=0) + N\, g(E_F)(k_B T$$

$$C_V(T) = \frac{d}{dT} U_{el}(T) - 2N\, g(E$$

$$N = \frac{\pi^2}{6}$$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
$(E_F)(k_B T)^2$
$(E_F)\, k_B^2\, T$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
Might be a gap in the electronic dispersion
$(E_F)(k_B T)^2$
$(E_F)\, k_B^2\, T$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
Might be a gap in the electronic dispersion
⑤ Isotope effect $T_c \sim M^{-\alpha}$
M = Isotopic mass, $\alpha \approx \frac{1}{2}$

$T)$

$(E_F)(k_B T)^2$

$(E_F)\, k_B^2 T$

④ $C \approx \gamma T_c e^{-bT_c/T}$ below T_c

$b = 1.5$

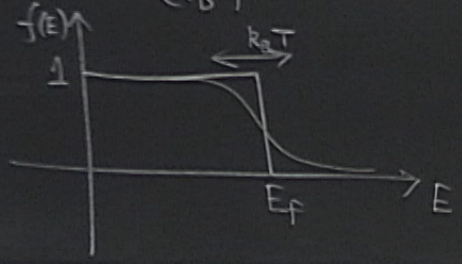
Might be a gap in the electronic dispersion

⑤ Isotope Effect $T_c \sim M^{-\alpha}$

M = Isotopic mass, $\alpha \approx \frac{1}{2}$

⑥ TWO electrons close to Fermi sea ⇒ Attractive interactions

@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



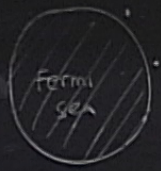
$C_V(T) = \frac{d}{dT} U_{el}(T) = 2 N g(E_F) k_B^2 T$

$N = \frac{\pi^2}{6}$

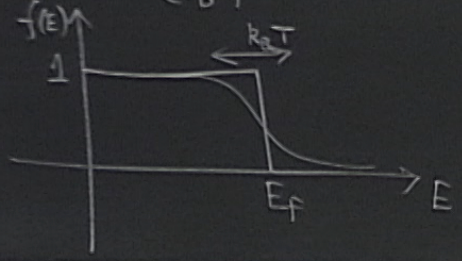
M =

(6) TW

Cooper Problem



@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



$C_V(T) = \frac{d}{dT} U_{el}(T) = 2 N g(E_F) k_B^2 T$

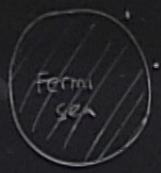
$N = \frac{\Pi^2}{6}$

M =

(6) TW

Cooper Problem

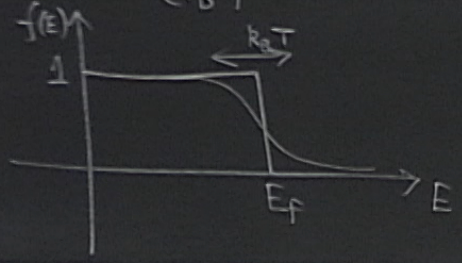
$\bar{r} = \bar{r}_1 - \bar{r}_2$



$\psi(\bar{r}_1 - \bar{r}_2) = \psi(\bar{r})$

$\psi(-\bar{r}) = -\psi(\bar{r}) \Rightarrow \psi(\bar{r}) = 0$

@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



$$C_V(T) = \frac{d}{dT} U_{el}(T) - 2 N g(E_F) k_B^2 T$$

$$N = \frac{\pi^2}{6}$$

M =

(6) TW

Cooper Problem

$\bar{r} = \bar{r}_1 - \bar{r}_2$



$$\psi(\bar{r}_1 - \bar{r}_2) = \psi(\bar{r})$$

$$\psi(-\bar{r}) = -\psi(\bar{r}) \Rightarrow \psi(\bar{r}) = 0$$

$$\psi(\bar{r}_1 - \bar{r}_2) \otimes \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$2 N g(E_F) k_B^2 T$

M = Isotopic mass, $\alpha \approx \frac{1}{2}$

(6) TWO electrons close to Fermi sea $\Rightarrow$ Attractive interactions

$H = H_{CM} + H_{rel}$

$H_{CM} = \frac{P_{CM}^2}{2M}$ $\quad P_{CM} = 0$

$H_{rel} = \frac{p_{rel}^2}{m} + V(\vec{r})$

Schrödinger Eq^n

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = (2\epsilon_F + E) \psi(\vec{r})$$

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Schrödinger Eq^n

$$-\frac{\hbar^2}{m}\nabla_r^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = (2\epsilon_F + E)\psi(\vec{r})$$

↓ Fourier

$$\frac{\hbar^2 k^2}{2m}\psi_k + \sum_{kk'} V_{kk'}\psi_{k'} = (2\epsilon_F + E)\psi_k$$

$\bar{r}) = (2\epsilon_F + E)\psi(\bar{r})$

$(2\epsilon_F + E)\psi_k$

$V_{kk'} = \frac{1}{\Omega}\int V(\bar{r})\, e^{i(\bar{k}'-\bar{k})\cdot\bar{r}}\, d^3r$

$\psi(\bar{r}) = 0$

$$H = H_{CM} + H_{rel}$$

$$H_{CM} = \frac{P_{CM}^2}{2M} \qquad P_{CM} = 0$$

$$H_{rel} = \frac{p_{rel}^2}{m} + V(\bar{r})$$

Schrödinger Eq^n

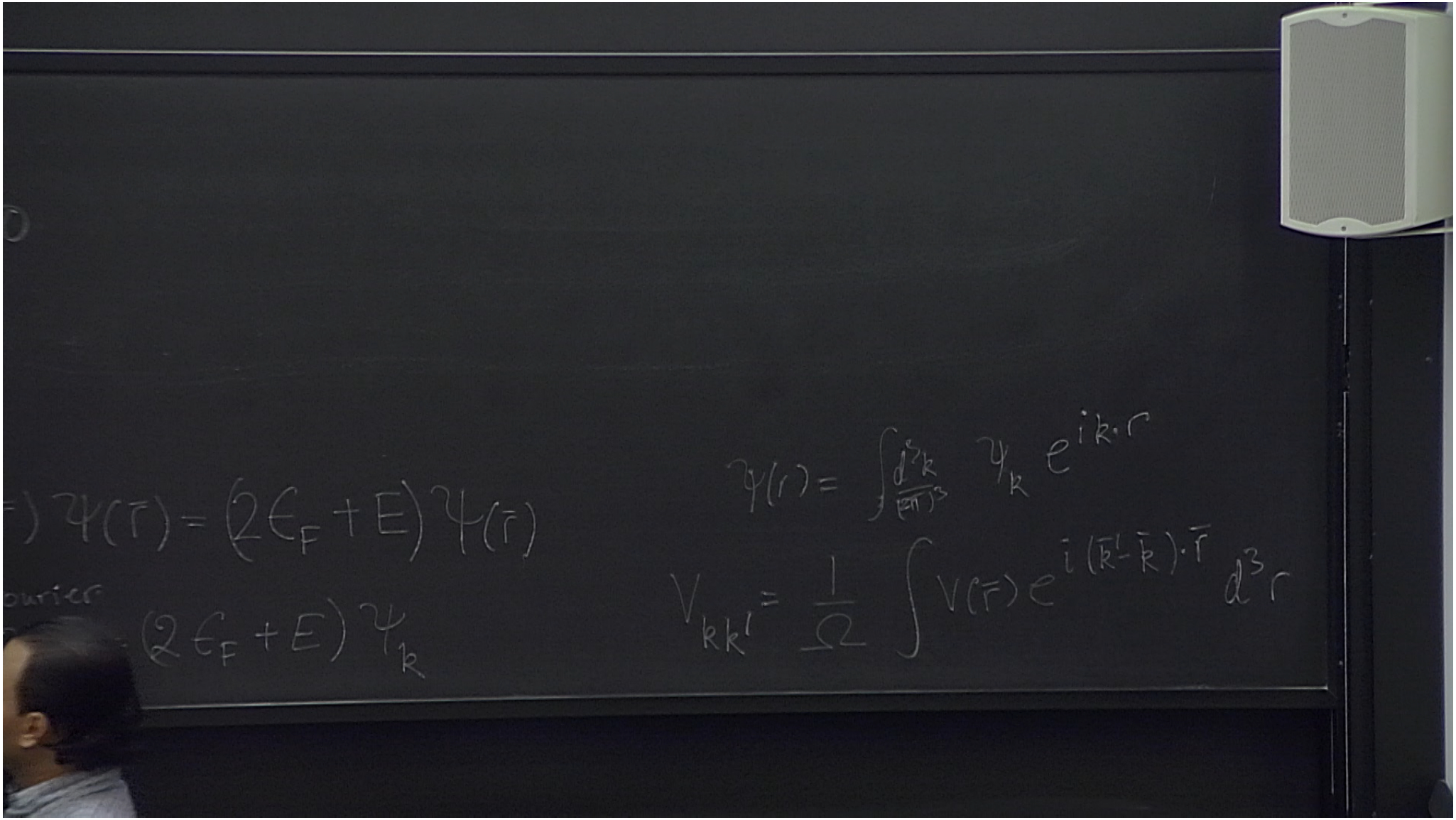
$$-\frac{\hbar^2}{m}\nabla_r^2 \psi(\bar{r}) + V(\bar{r})\psi(\bar{r}) = (2\epsilon_F + E)\psi(\bar{r})$$

$\psi(r$

↓ Fourier

$$\frac{\hbar^2 k^2}{m}\psi_k + \sum_{kk'} V_{kk'}\psi_{k'} = (2\epsilon_F + E)\psi_k$$

$V_{kk'} =$


$\psi(\vec{r}) = (2\epsilon_F + E)\psi(\vec{r})$
$(2\epsilon_F + E)\psi_k$
$\psi(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} \psi_k e^{i\vec{k}\cdot\vec{r}}$
$V_{kk'} = \frac{1}{\Omega}\int V(\vec{r}) e^{i(\vec{k}'-\vec{k})\cdot\vec{r}} d^3r$

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'}$$

$$\left(E - 2\xi_k\right)\psi_k = \sum_{k'}{}' V_{kk'}\psi_{k'}$$

$$\xi_k = \epsilon_k - \epsilon_F$$

$$= \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

$$\frac{\hbar^2 k^2}{m}\psi_k + \xi$$

$$(E - 2\xi_k)\psi_k = \sum_{k'}{}' V_{kk'}\psi_{k'}$$

$$\xi_k = \epsilon_k - \epsilon_F$$

$$= \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

On-shell Approximation

$$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c \\ & |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$$

