Title: PSI 2018/2019 - Condensed Matter - Lecture 9

Date: Nov 22, 2018 10:45 AM

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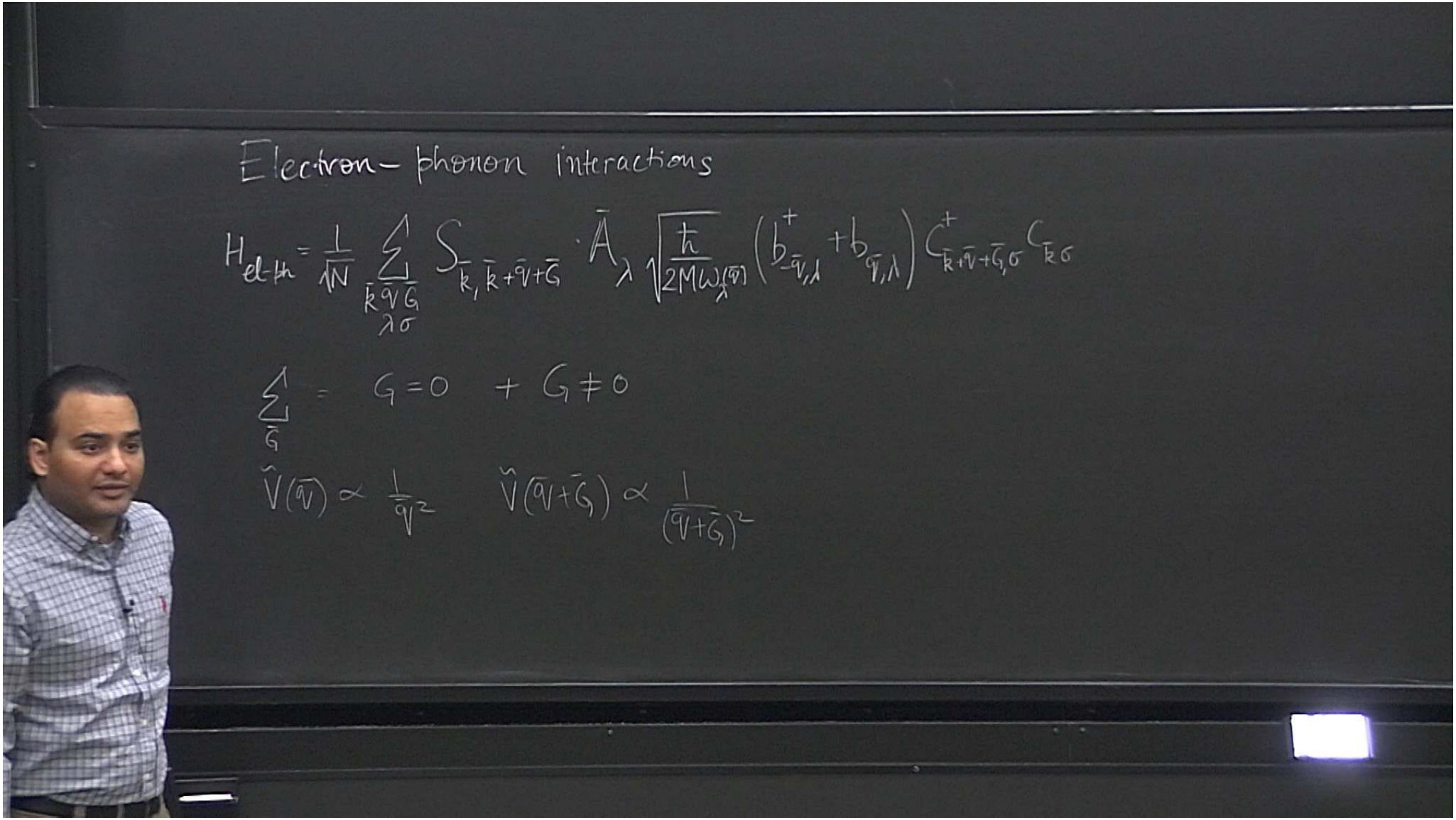
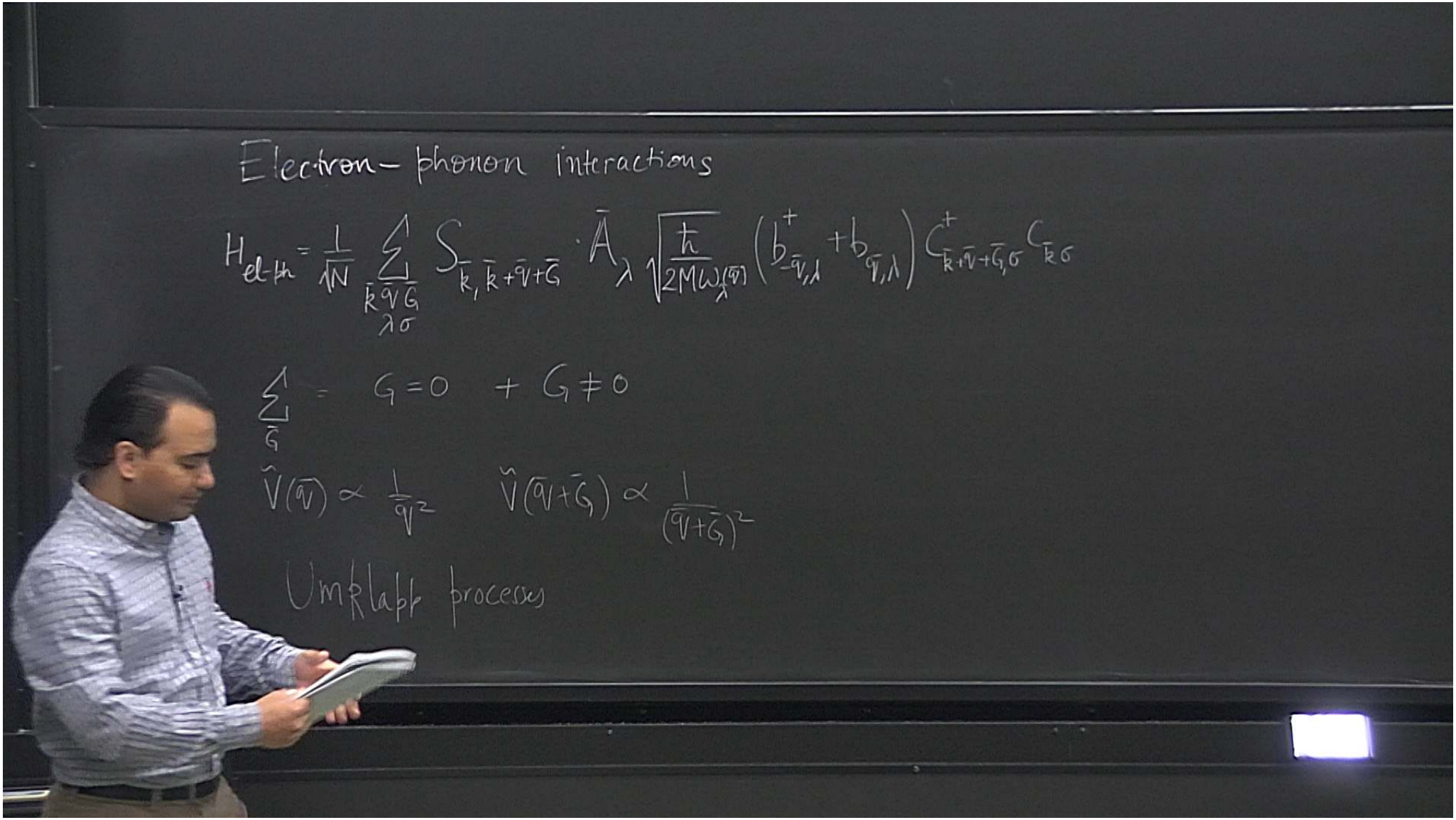
Abstract:

Electron–phonon interactions

$$H_{el\text{-}ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}}{}' S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_\lambda \sqrt{\frac{\hbar}{2M\omega_\lambda(\bar{q})}} \left(b^+_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^+_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$$


Electron – phonon interactions
$H_{el-ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}} S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_\lambda \sqrt{\frac{\hbar}{2M\omega_\lambda(\bar{q})}} \left(b^+_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^+_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$
$\sum_{\bar{G}} = G=0 + G\neq 0$
$\hat{V}(\bar{q}) \propto \frac{1}{\bar{q}^2} \quad \hat{V}(\bar{q}+\bar{G}) \propto \frac{1}{(\bar{q}+\bar{G})^2}$


Electron – phonon interactions
$H_{el-ph} = \frac{1}{\sqrt{N}} \sum_{\substack{\bar{k}\bar{q}\bar{G} \\ \lambda\sigma}} S_{\bar{k},\bar{k}+\bar{q}+\bar{G}} \cdot \bar{A}_\lambda \sqrt{\frac{\hbar}{2M\omega_\lambda(\bar{q})}} \left(b^+_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) C^+_{\bar{k}+\bar{q}+\bar{G},\sigma} C_{\bar{k}\sigma}$
$\sum_{\bar{G}} = G=0 + G\neq 0$
$\hat{V}(\bar{q}) \propto \frac{1}{\bar{q}^2} \quad \hat{V}(\bar{q}+\bar{G}) \propto \frac{1}{(\bar{q}+\bar{G})^2}$
Umklapp processes

$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma} c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el-ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}} \, c^{+}_{\bar{k}} c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \frac{1}{2}\right)$$

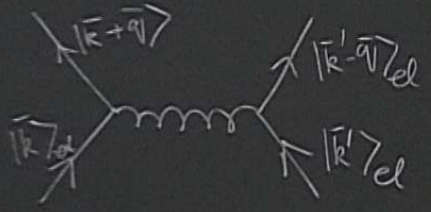
$$H_{el-ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q}) \, c^{+}_{\bar{k}+\bar{q}} \, c_{\bar{k}} \left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right)$$

$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma}\, c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el-ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}}\, c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \frac{1}{2}\right)$$

$$H_{el-ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{+}_{-\bar{q}} + b_{\bar{q}}\right)$$

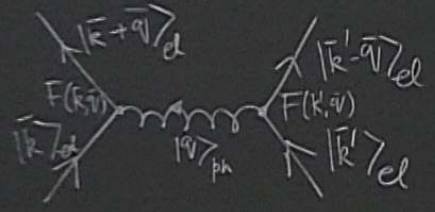


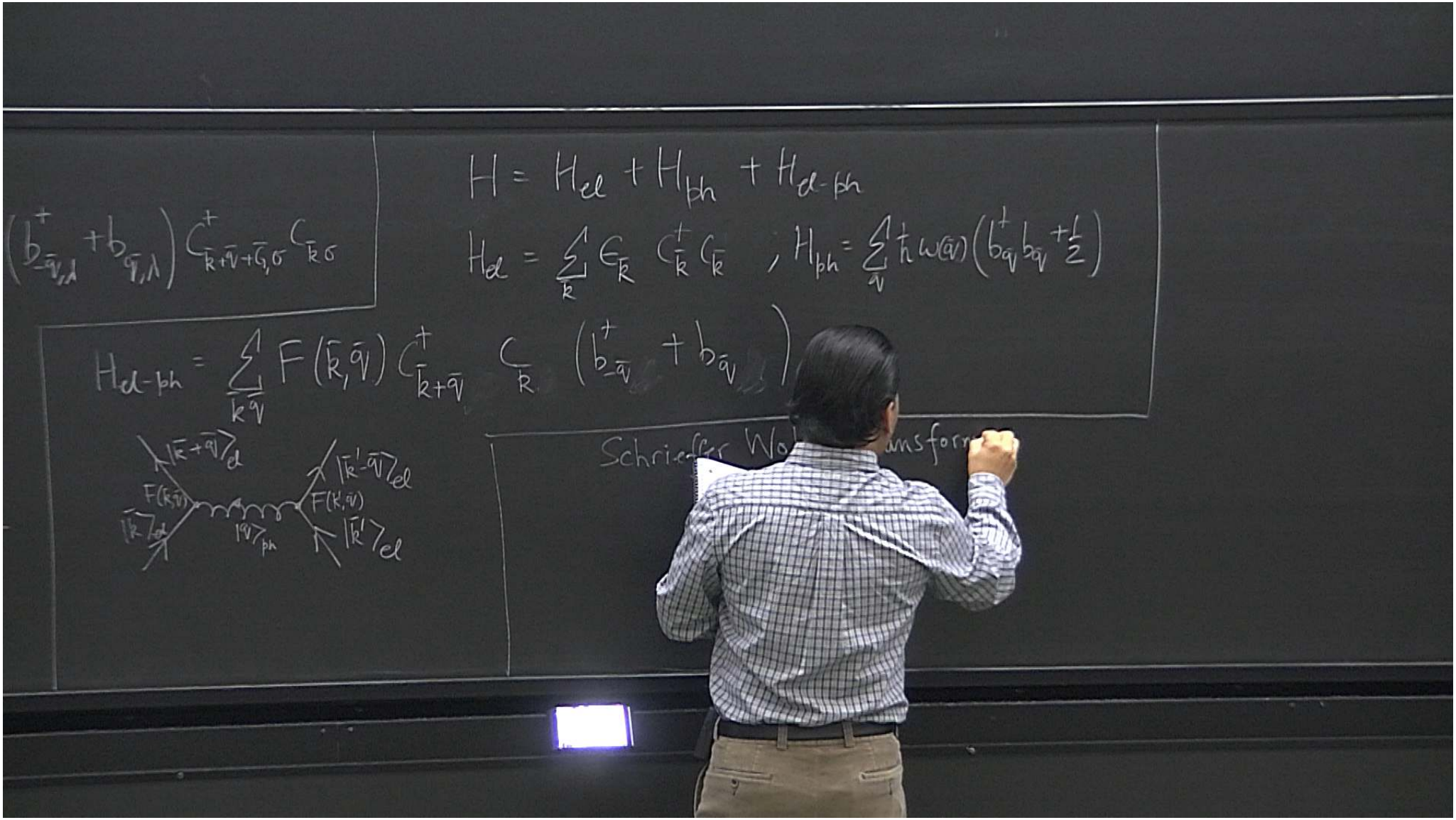
$$\left(b^{+}_{-\bar{q},\lambda} + b_{\bar{q},\lambda}\right) c^{+}_{\bar{k}+\bar{q}+\bar{G},\sigma}\, c_{\bar{k}\sigma}$$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{+}_{\bar{k}}\, c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{+}_{\bar{q}} b_{\bar{q}} + \tfrac{1}{2}\right)$$

$$H_{el\text{-}ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{+}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{+}_{-\bar{q}} + b_{\bar{q}}\right)$$




$H = H_{el} + H_{ph} + H_{el-ph}$
$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{\dagger}_{\bar{k}} c_{\bar{k}}$, $H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{\dagger}_{\bar{q}} b_{\bar{q}} + \frac{1}{2} \right)$
$H_{el-ph} = \sum_{\bar{k}\bar{q}} F(\bar{k},\bar{q})\, c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{\dagger}_{-\bar{q}} + b_{\bar{q}} \right)$
$\left(b^{\dagger}_{-\bar{q},\lambda} + b_{\bar{q},\lambda} \right) c^{\dagger}_{\bar{k}+\bar{q}+\bar{G},\sigma}\, c_{\bar{k}\sigma}$
$|\bar{k}+\bar{q}\rangle_{el}$
$F(\bar{k},\bar{q})$
$|\bar{k}\rangle_{el}$
$|\bar{q}\rangle_{ph}$
$|\bar{k}'-\bar{q}\rangle_{el}$
$F(\bar{k}',\bar{q})$
$|\bar{k}'\rangle_{el}$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{\dagger}_{\bar{k}} c_{\bar{k}} \quad ; \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{\dagger}_{\bar{q}} b_{\bar{q}} + \frac{1}{2} \right)$$

$$c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{\dagger}_{-\bar{q}} + b_{\bar{q}} \right)$$

Schrieffer Wolff transformation

$$U = e^{S}, \quad S^{\dagger} = -S$$

$$\tilde{H} = e^{-S} H e^{S} = H - [S, H] + \frac{(-1)^2}{2} [S, [S, H]] + (\dots)$$

$$H = H_{el} + H_{ph} + H_{el\text{-}ph}$$

$$H_{el} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, c^{\dagger}_{\bar{k}} c_{\bar{k}} \quad , \quad H_{ph} = \sum_{\bar{q}} \hbar\omega(\bar{q}) \left(b^{\dagger}_{\bar{q}} b_{\bar{q}} + \frac{1}{2} \right)$$

$$c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(b^{\dagger}_{-\bar{q}} + b_{\bar{q}} \right)$$

Schrieffer Wol[illegible]formation

$$U = e^{S} \quad , \quad S^{\dagger} =$$

$$\tilde{H} = e^{-S} H [\text{illegible}] \frac{(-1)^2}{2} [S,[S,H]] + (\cdots)$$

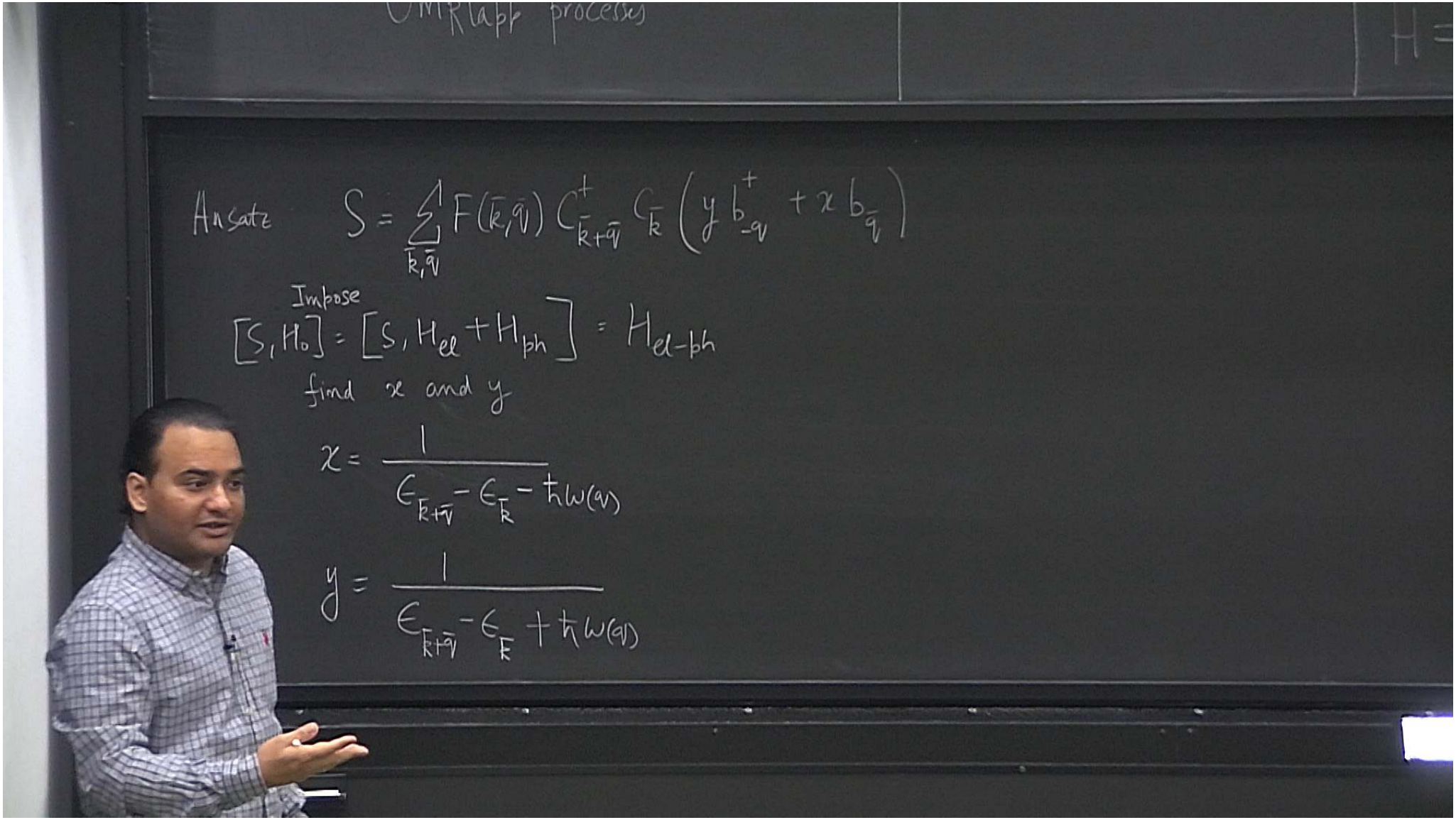
$$H = H_0 + H_{el\text{-}ph} \; ; \quad H_0 = H_{el} + H_{ph}$$

Choose, $[S, H_0] = H_{el\text{-}ph} \propto F(\bar{k}, \bar{q})$

$$\tilde{H} = H_0 + H_{el\text{-}ph} - [S, H_0] - [S, H_{el\text{-}ph}] + \frac{1}{2}[S,[S,H_0]]$$

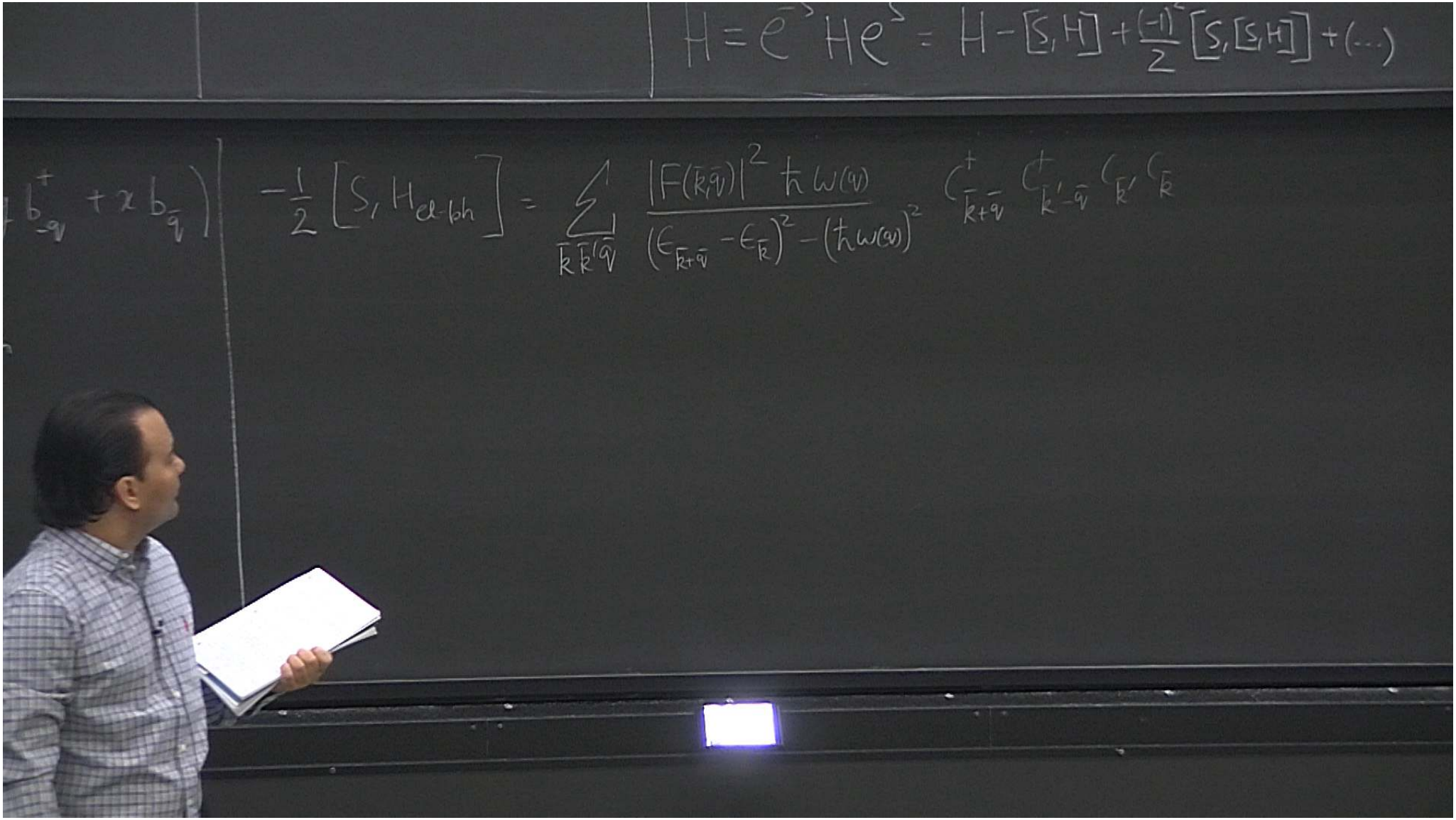
$$+ \frac{1}{2}[S,[S,H_{el\text{-}ph}]] + (\cdots)$$

$$\tilde{H} = H_0 - \frac{1}{2}[S, H_{el\text{-}ph}] + O\left(F(\bar{k},\bar{q})^3\right)$$


Umklapp processes
H=
Ansatz $S = \sum_{\bar{k},\bar{q}} F(\bar{k},\bar{q})\, C^{+}_{\bar{k}+\bar{q}}\, C_{\bar{k}} \left(y\, b^{+}_{-q} + x\, b_{\bar{q}} \right)$
Impose
$[S, H_0] = [S, H_{el} + H_{ph}] = H_{el-ph}$
find x and y
$x = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} - \hbar\omega(q)}$
$y = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} + \hbar\omega(q)}$

$$H = e^{S} H e^{S} = H - [S,H] + \frac{(-1)^2}{2}[S,[S,H]] + (\dots)$$

$$+ b_{-\bar{q}}^{\dagger} + x\, b_{\bar{q}}) \qquad -\frac{1}{2}[S, H_{el-ph}] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2 \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2}\, c^{\dagger}_{\bar{k}+\bar{q}}\, c^{\dagger}_{\bar{k}'-\bar{q}}\, c_{\bar{k}'}\, c_{\bar{k}}$$



$$H = e^{S} H e^{S} = H - [S,H] + \frac{(-1)^2}{2}[S,[S,H]] + (\dots)$$

$$\left(b_{-q}^{\dagger} + \alpha\, b_{\bar{q}} \right)$$

$$-\frac{1}{2}\left[S, H_{el-ph}\right] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2\, \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2}\, c^{\dagger}_{\bar{k}+\bar{q}}\, c^{\dagger}_{\bar{k}'-\bar{q}}\, c_{\bar{k}'}\, c_{\bar{k}}$$

If $(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 > (\hbar\omega(q))^2$: Repulsive

$(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 < (\hbar\omega(q))^2$: Attractive

$$H = e^{S} H e^{S} = H - [S, H] + \frac{(-1)^2}{2}[S,[S,H]] + (\cdots)$$

$$b_{-\bar{q}}^{\dagger} + \alpha\, b_{\bar{q}}$$

$$-\frac{1}{2}[S, H_{el-ph}] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k},\bar{q})|^2 \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2}\, c^{\dagger}_{\bar{k}+\bar{q}}\, c^{\dagger}_{\bar{k}'-\bar{q}}\, c_{\bar{k}'}\, c_{\bar{k}}$$

If $(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 > (\hbar\omega(q))^2$: Repulsive

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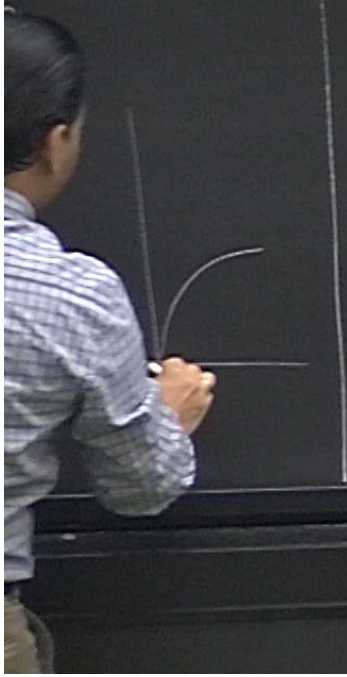
Optical phonons $\hbar\omega(q) \xrightarrow[q\to 0]{}$ Constant



$$S = \sum_{\bar{k},\bar{q}} F(k,q)\, c^{\dagger}_{\bar{k}+\bar{q}}\, c_{\bar{k}} \left(g^{b}_{-q} \right)$$

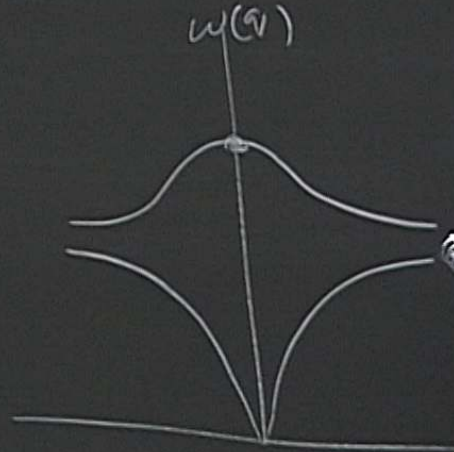
Impose

$$[\,\cdot\,, H_{el} + H_{ph}] = H_{el-ph}$$

find x and y

$$x = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} - \hbar\omega(q)}$$

$$y = \frac{1}{\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} + \hbar\omega(q)}$$



If

$$H = e^{S} H e^{-S} = H - [S, H] + \frac{(-1)}{2}[S,[S,H]] + (\dots)$$

$$-\frac{1}{2}[S, H_{el\text{-}bh}] = \sum_{\bar{k}\bar{k}'\bar{q}} \frac{|F(\bar{k}\bar{q})|^2 \hbar\omega(q)}{(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 - (\hbar\omega(q))^2} c^{\dagger}_{\bar{k}+\bar{q}} c^{\dagger}_{\bar{k}'-\bar{q}} c_{\bar{k}'} c_{\bar{k}}$$

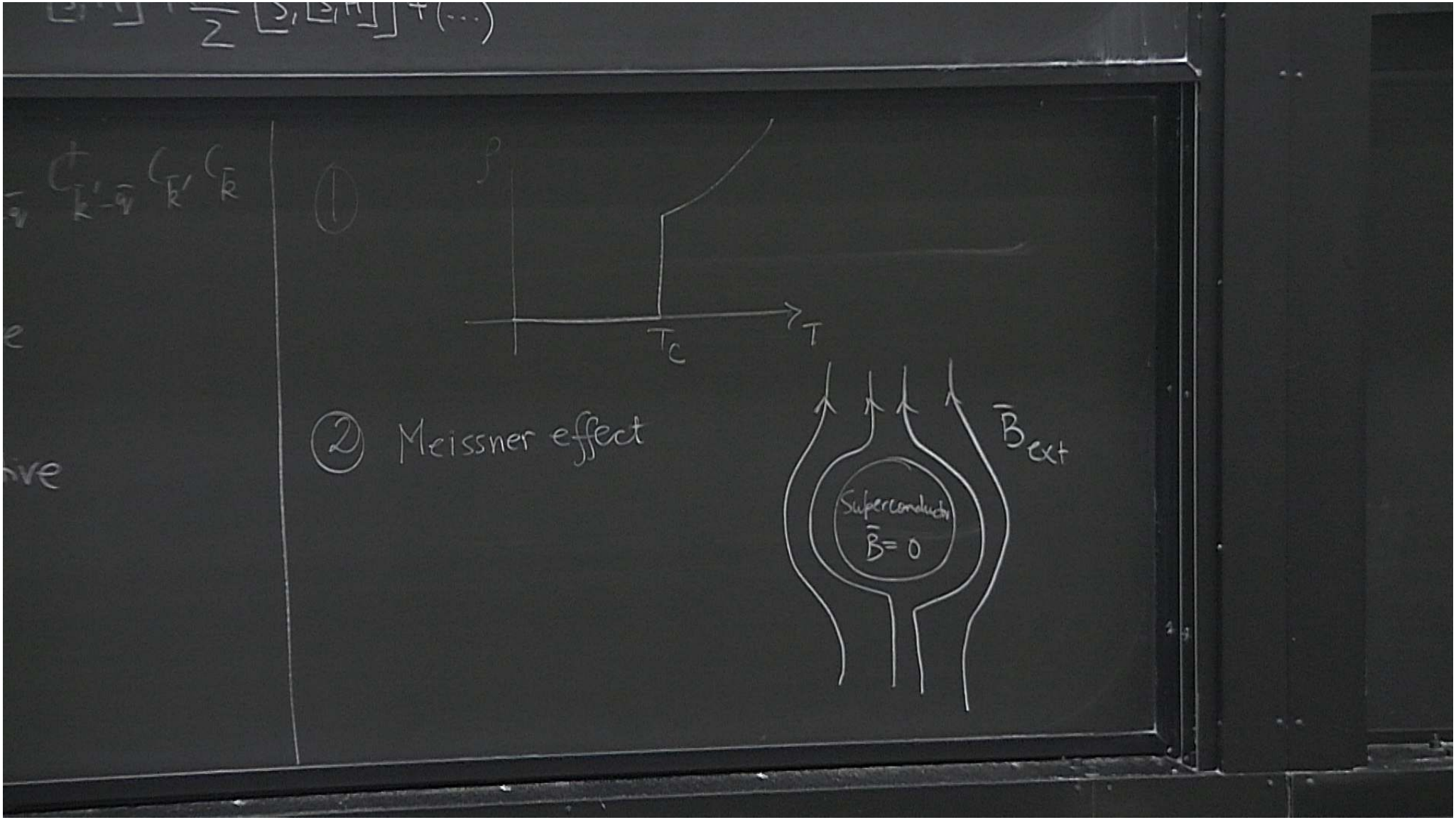
If $(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 > (\hbar\omega(q))^2$: Repulsive

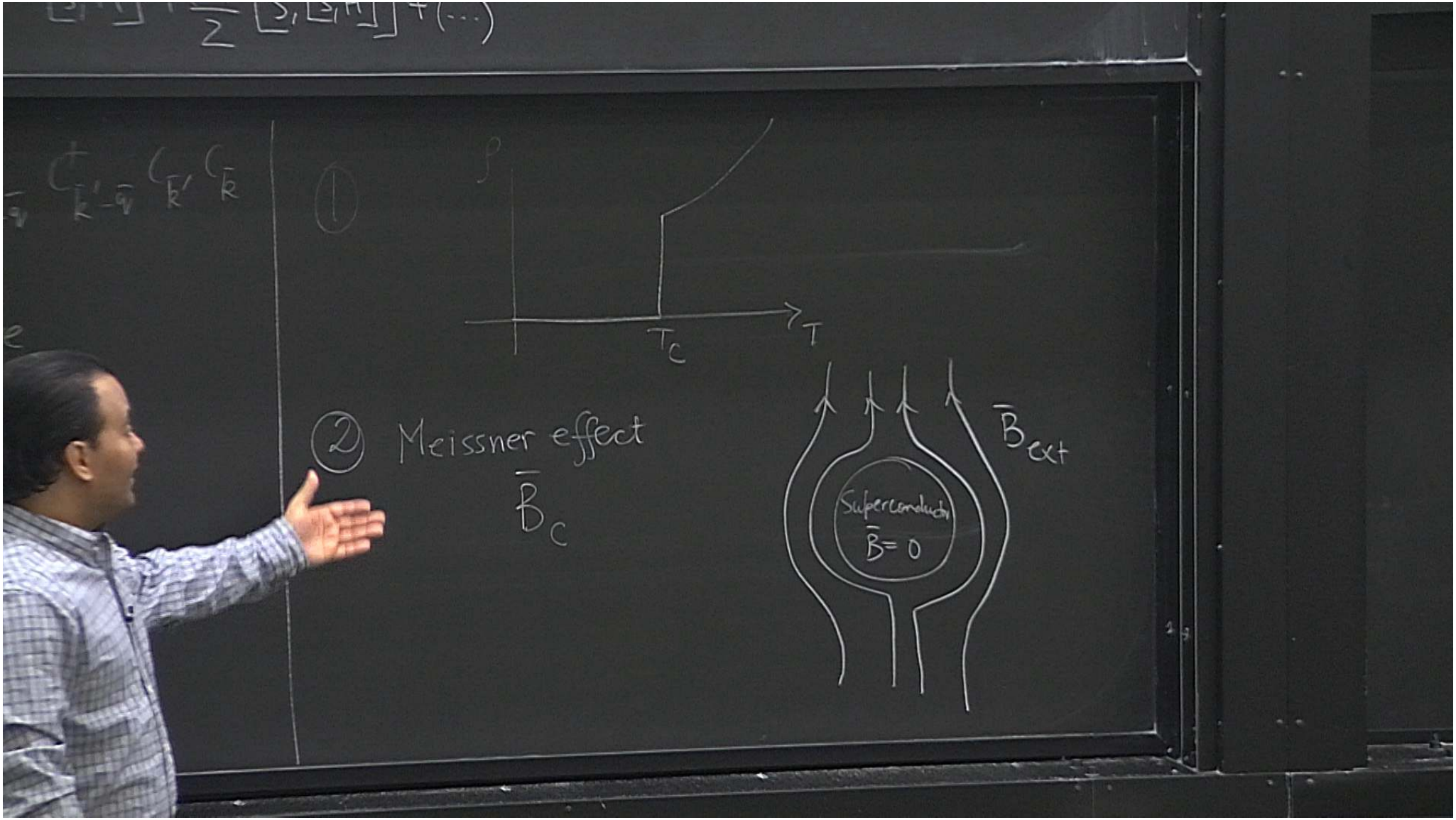
$(\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}})^2 < (\hbar\omega(q))^2$: Attractive

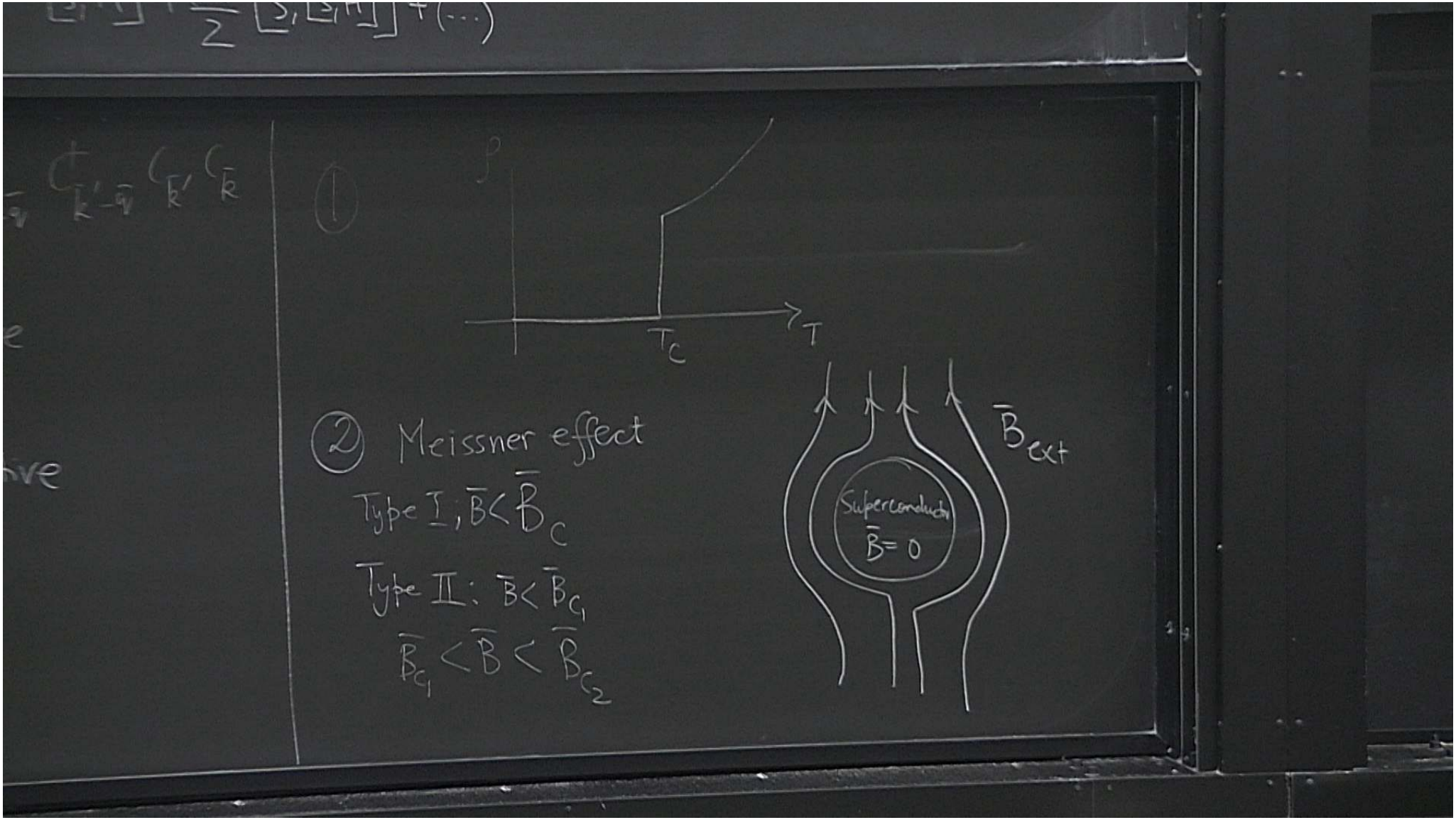
Optical phonons $\hbar\omega(q) \xrightarrow[q\to 0]{}$ Constant

Two electrons close to Fermi sea

$$\epsilon_{\bar{k}+\bar{q}} - \epsilon_{\bar{k}} = \frac{\hbar^2(k_F+q)^2}{2m} - \frac{\hbar^2 k_F^2}{2m} \xrightarrow{q\to 0} 0$$


①
ρ
T_c
T
② Meissner effect
$\bar{B}_{ext}$
Superconductor
$\bar{B}=0$


①
ρ
T_c
T
② Meissner effect
$\bar{B}_c$
$\bar{B}_{ext}$
Superconductor
$\bar{B}=0$



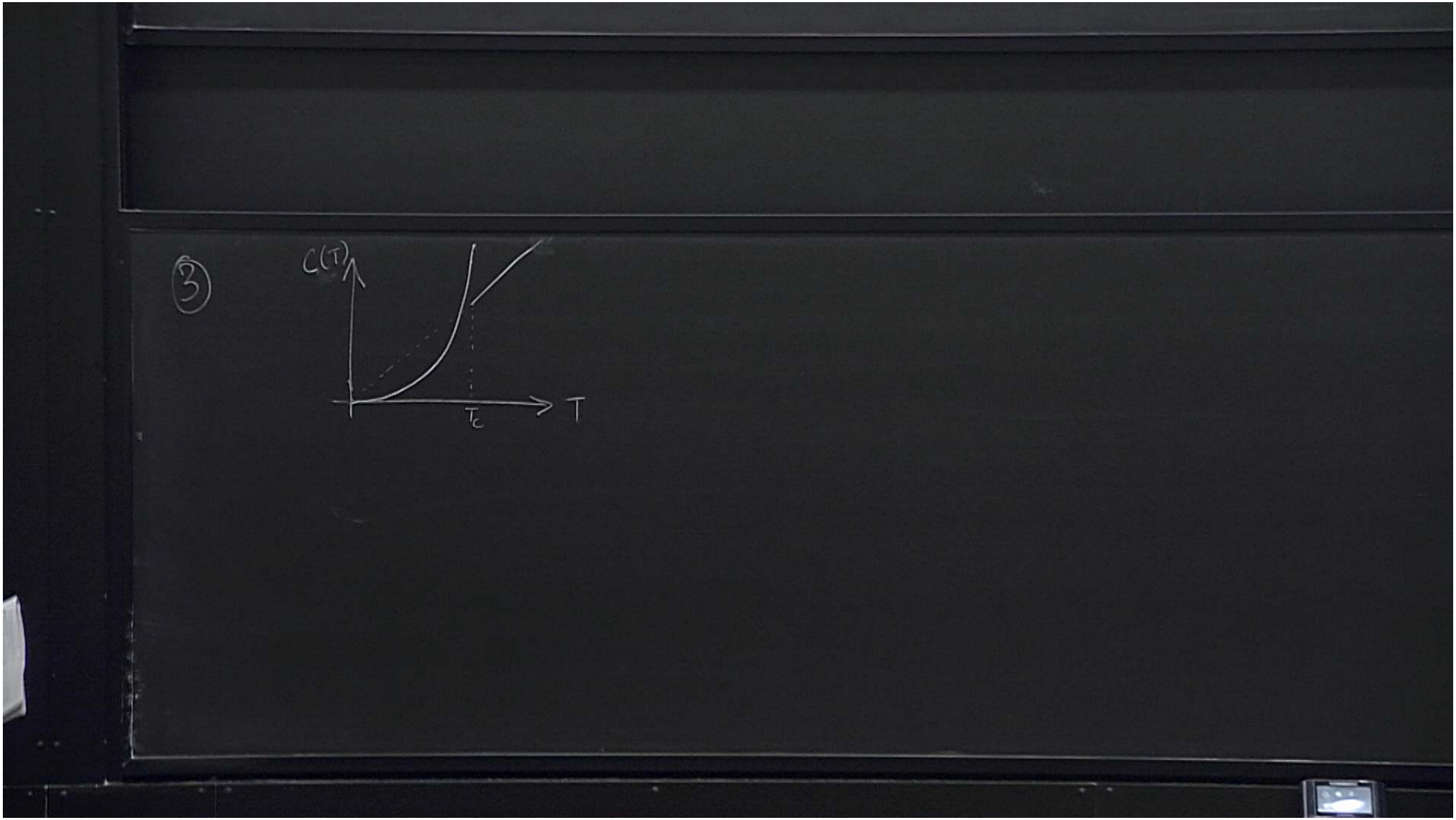
① ρ vs T, T_c

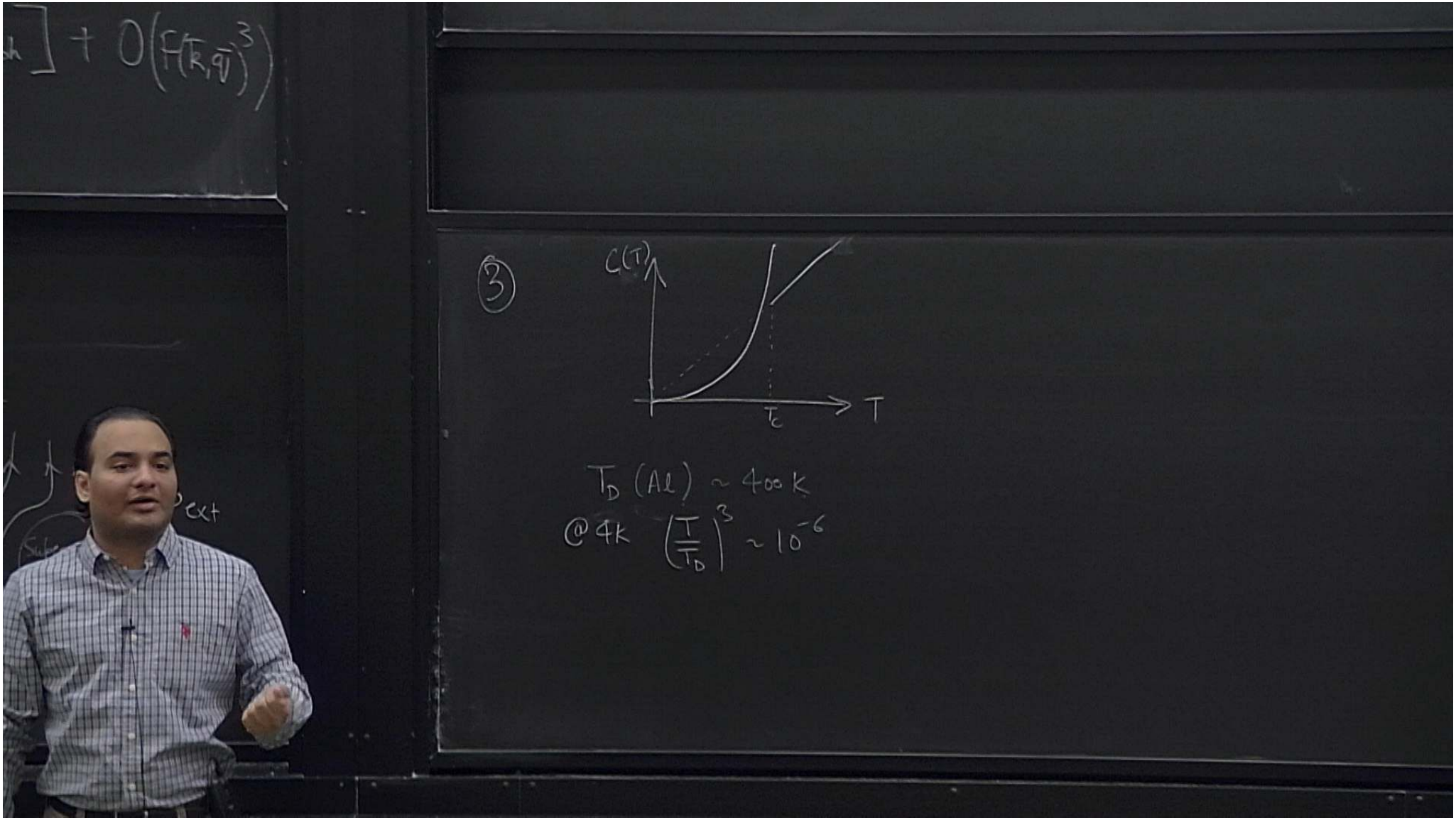
② Meissner effect

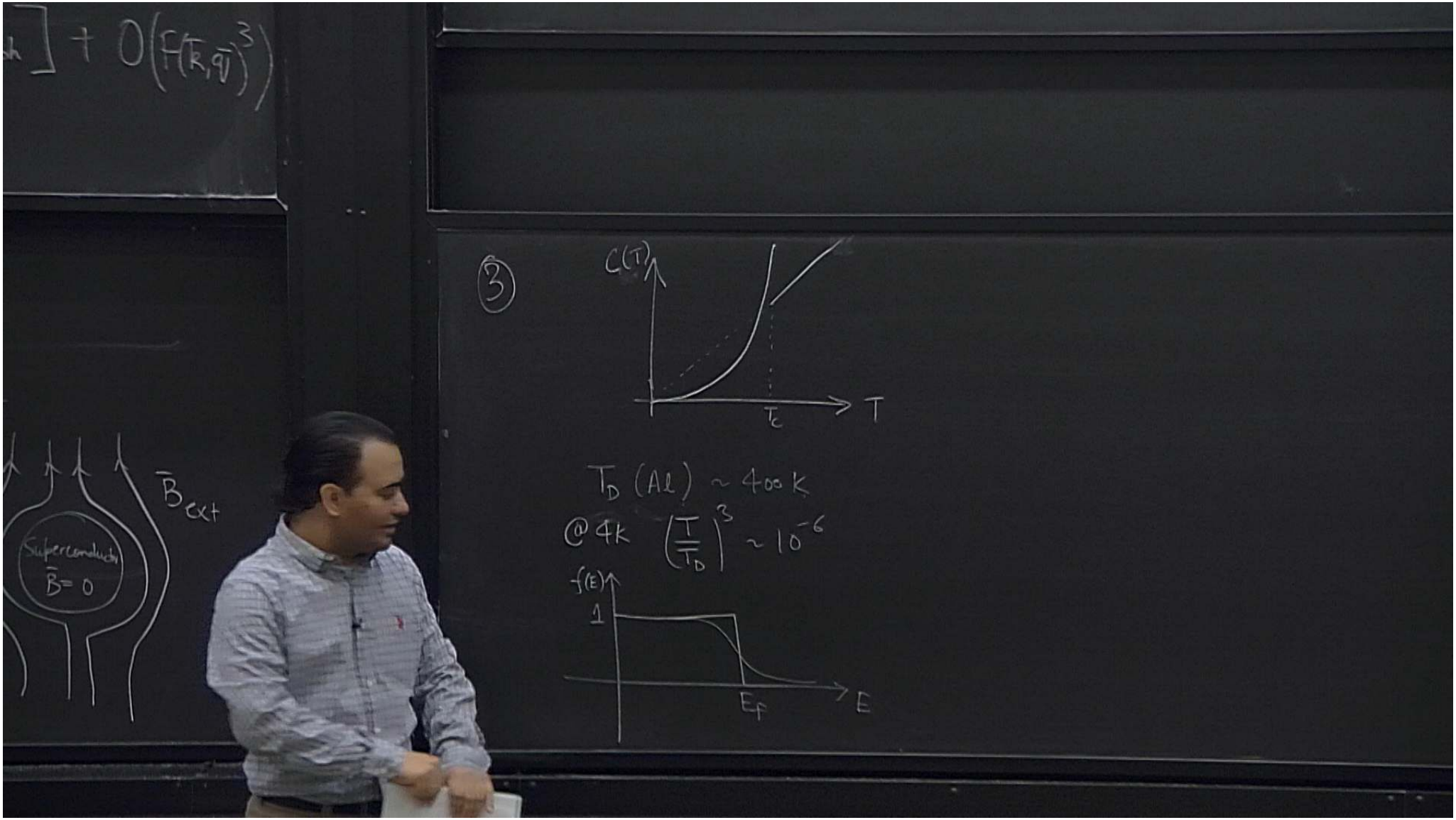
Type I, $\bar{B} < \bar{B}_c$

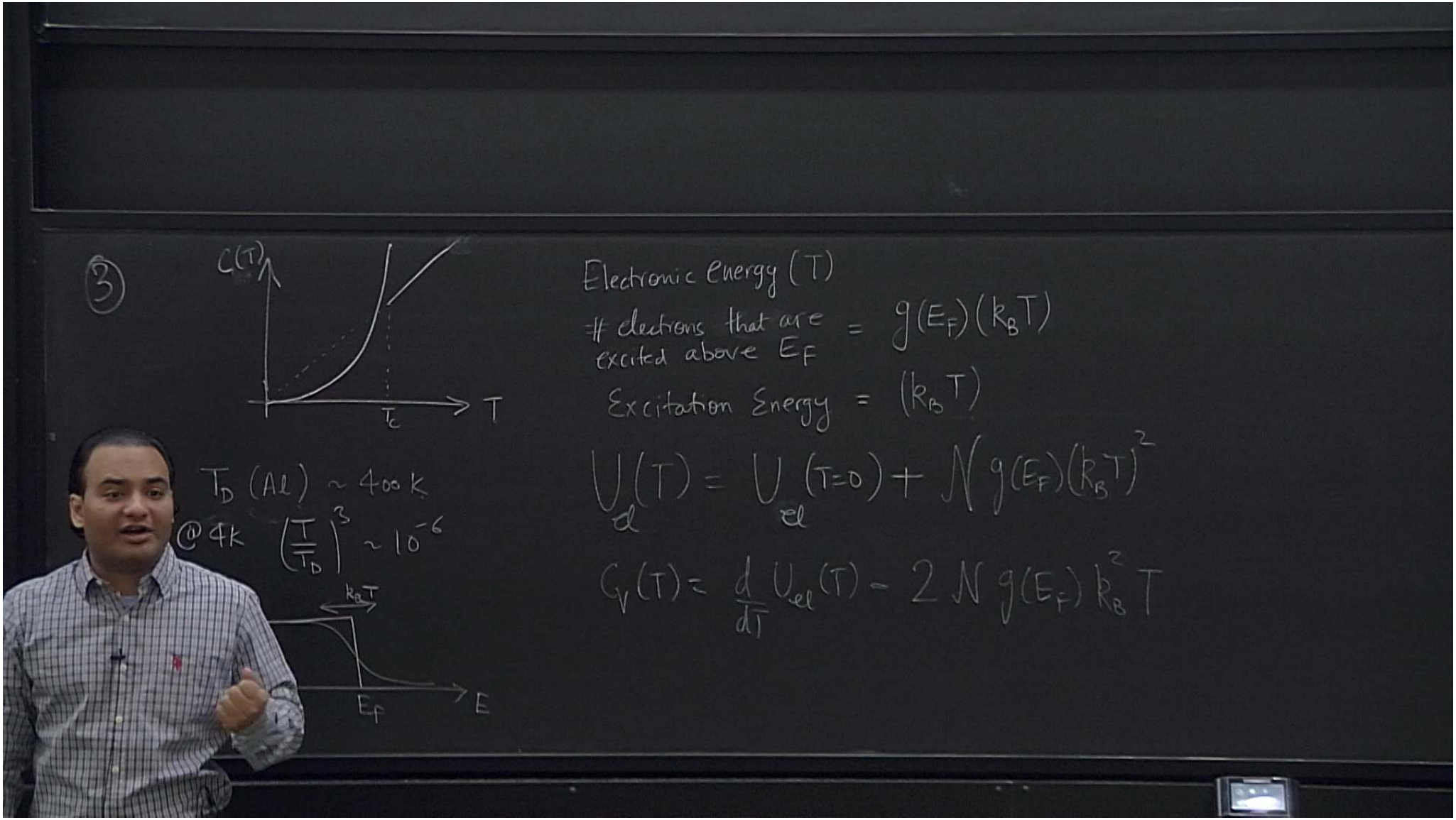
Type II: $\bar{B} < \bar{B}_{c_1}$

$\bar{B}_{c_1} < \bar{B} < \bar{B}_{c_2}$

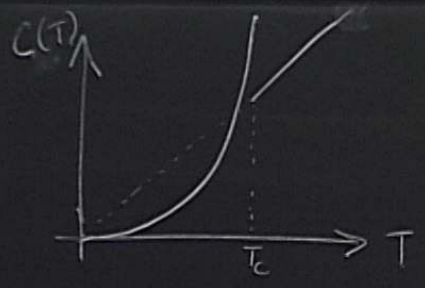

3
C(T)
T
T_c


$+ O(F(R,q)^3)$
③
C(T)
T
T_c
T_D (Al) ~ 400 K
@ 4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$


$+ O(F(\vec{k},\vec{q})^3)$
③
C(T)
T
T_c
T_D (Al) ~ 400 K
@ 4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$
f(E)
1
E_f
E
$\vec{B}_{ext}$
Superconductor
$\vec{B}=0$

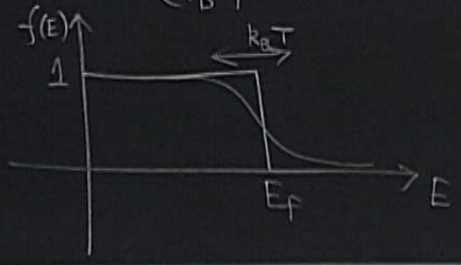

3
C(T)
T
T_c
T_D (Al) ~ 400 K
@ 4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$
$k_B T$
E_F
E
Electronic Energy (T)
electrons that are excited above E_F = $g(E_F)(k_B T)$
Excitation Energy = $(k_B T)$
$U_{el}(T) = U_{el}(T=0) + N(g(E_F)(k_B T)^2$
$C_V(T) = \frac{d}{dT} U_{el}(T) - 2 N g(E_F) k_B^2 T$

3)



T_D (Al) ~ 400 K

@ 4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



Electronic Energy (T)

electrons that are excited above E_F = $g(E_F)(k_B T)$

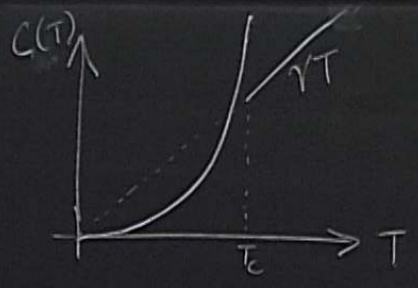
Excitation Energy = $(k_B T)$

$$U_{el}(T) = U_{el}(T=0) + N\left(g(E_F)(k_B T)\right)^2$$

$$C_V(T) = \frac{d}{dT} U_{el}(T) = 2 N g(E_F) k_B^2 T$$

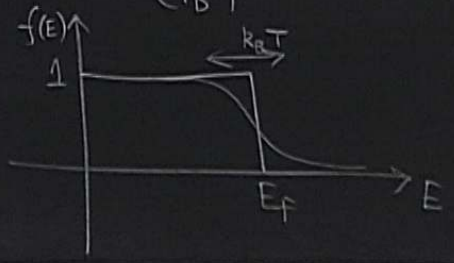
$$N = \frac{\pi^2}{6}$$

③



T_D (Al) ~ 400 K

@ 4K $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



Electronic Energy (T)

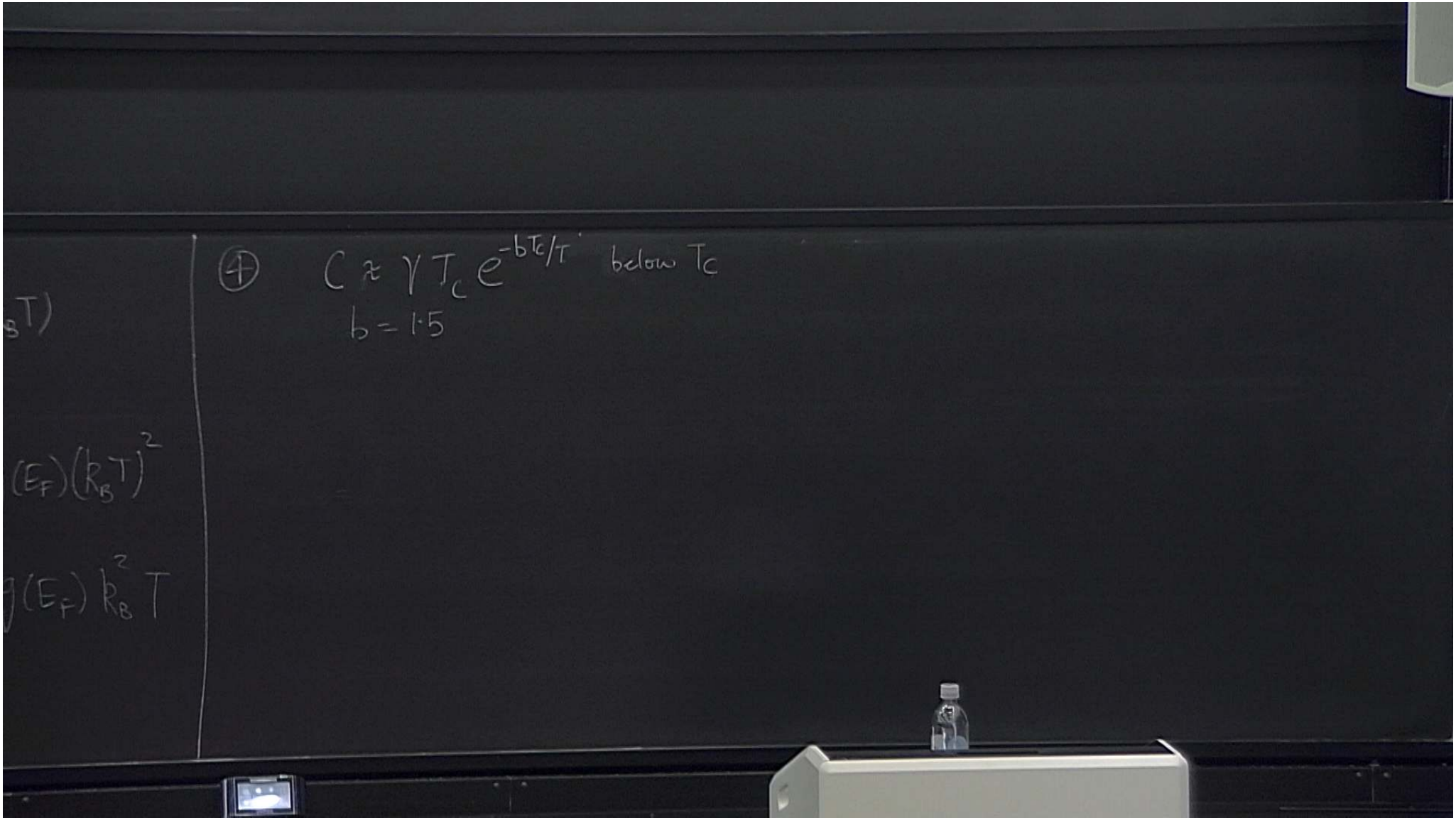
electrons that are excited above E_F = $g(E_F)(k_B T)$

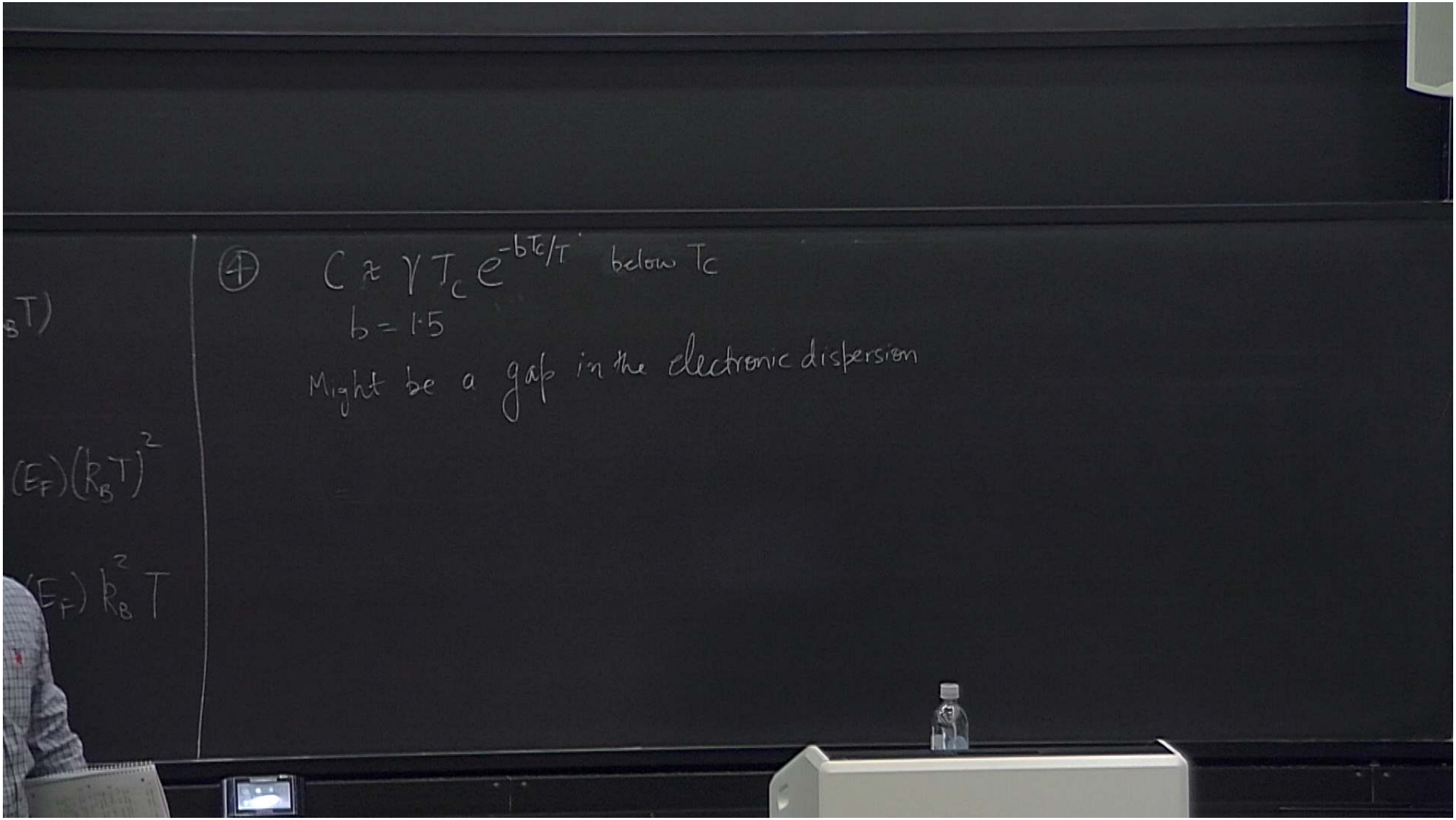
Excitation Energy = $(k_B T)$

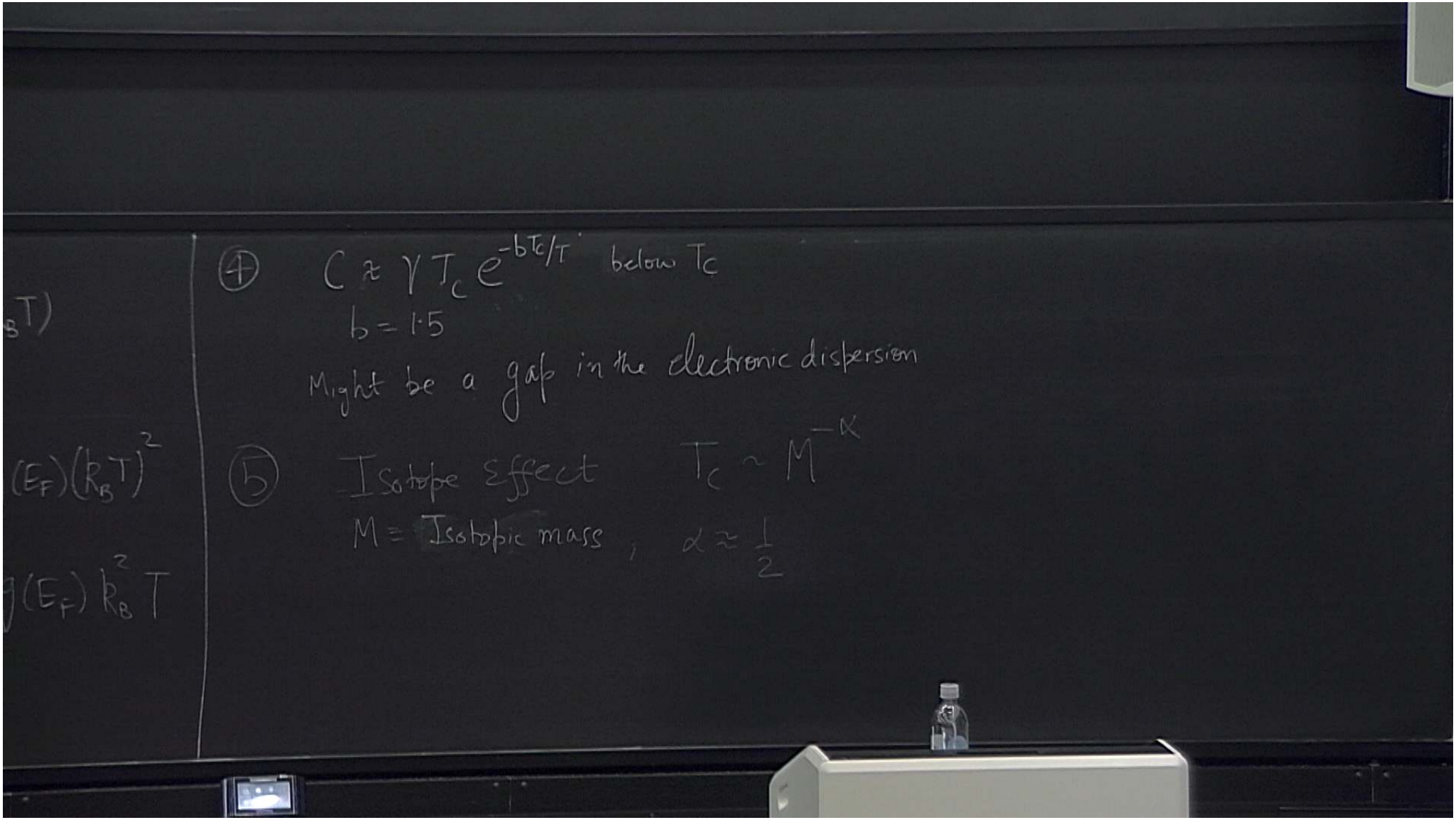
$$U_{el}(T) = U_{el}(T=0) + N\, g(E_F)(k_B T$$

$$C_V(T) = \frac{d}{dT} U_{el}(T) - 2N\, g(E$$

$$N = \frac{\pi^2}{6}$$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
$(E_F)(k_B T)^2$
$(E_F) k_B^2 T$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
Might be a gap in the electronic dispersion
$(E_F)(k_B T)^2$
$(E_F)\, k_B^2\, T$


④ $C \propto \gamma T_c e^{-bT_c/T}$ below T_c
$b = 1.5$
Might be a gap in the electronic dispersion
⑤ Isotope effect $T_c \sim M^{-\alpha}$
M = Isotopic mass, $\alpha \approx \frac{1}{2}$

$T)$

$(E_F)(k_B T)^2$

$(E_F)\, k_B^2 T$

④ $C \approx \gamma T_c e^{-bT_c/T}$ below T_c

$b = 1.5$

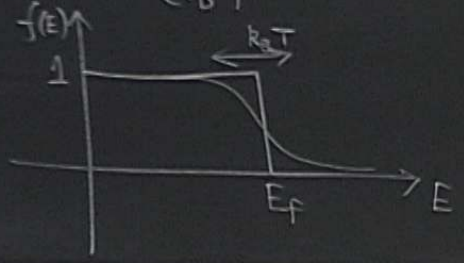
Might be a gap in the electronic dispersion

⑤ Isotope effect $T_c \sim M^{-\alpha}$

M = Isotopic mass, $\alpha \approx \frac{1}{2}$

⑥ TWO electrons close to Fermi sea ⇒ Attractive interactions

@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



$$C_V(T) = \frac{d}{dT} U_{el}(T) = 2 N g(E_F) k_B^2 T$$

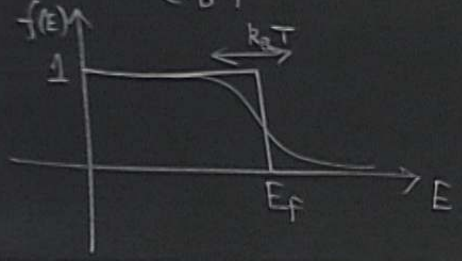
$$N = \frac{\pi^2}{6}$$

M =

(6) TW

Cooper Problem

@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



$$C_V(T) = \frac{d}{dT} U_{el}(T) = 2 N g(E_F) k_B^2 T$$

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M =

6 TW

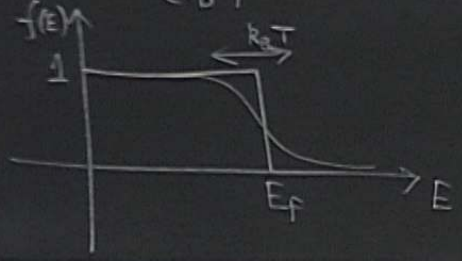
Cooper Problem

$\bar{r} = \bar{r}_1 - \bar{r}_2$

$$\psi(\bar{r}_1 - \bar{r}_2) = \psi(\bar{r})$$

$$\psi(-\bar{r}) = -\psi(\bar{r}) \Rightarrow \psi(\bar{r}) = 0$$

@4k $\left(\frac{T}{T_D}\right)^3 \sim 10^{-6}$



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Cooper Problem

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$\psi(\bar{r}_1 - \bar{r}_2) = \psi(\bar{r})$

$\psi(-\bar{r}) = -\psi(\bar{r}) \Rightarrow \psi(\bar{r}) = 0$

$\psi(\bar{r}_1 - \bar{r}_2) \otimes \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

$2 N g(E_F) k_B^2 T$

M = Isotopic mass, $\alpha \approx \frac{1}{2}$

(6) TWO electrons close to Fermi sea $\Rightarrow$ Attractive interactions

$H = H_{CM} + H_{rel}$

$H_{CM} = \frac{P_{CM}^2}{2M}$ $\quad P_{CM} = 0$

$H_{rel} = \frac{p_{rel}^2}{m} + V(\vec{r})$

Schrödinger Eq^n

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = (2\epsilon_F + E) \psi(\vec{r})$$

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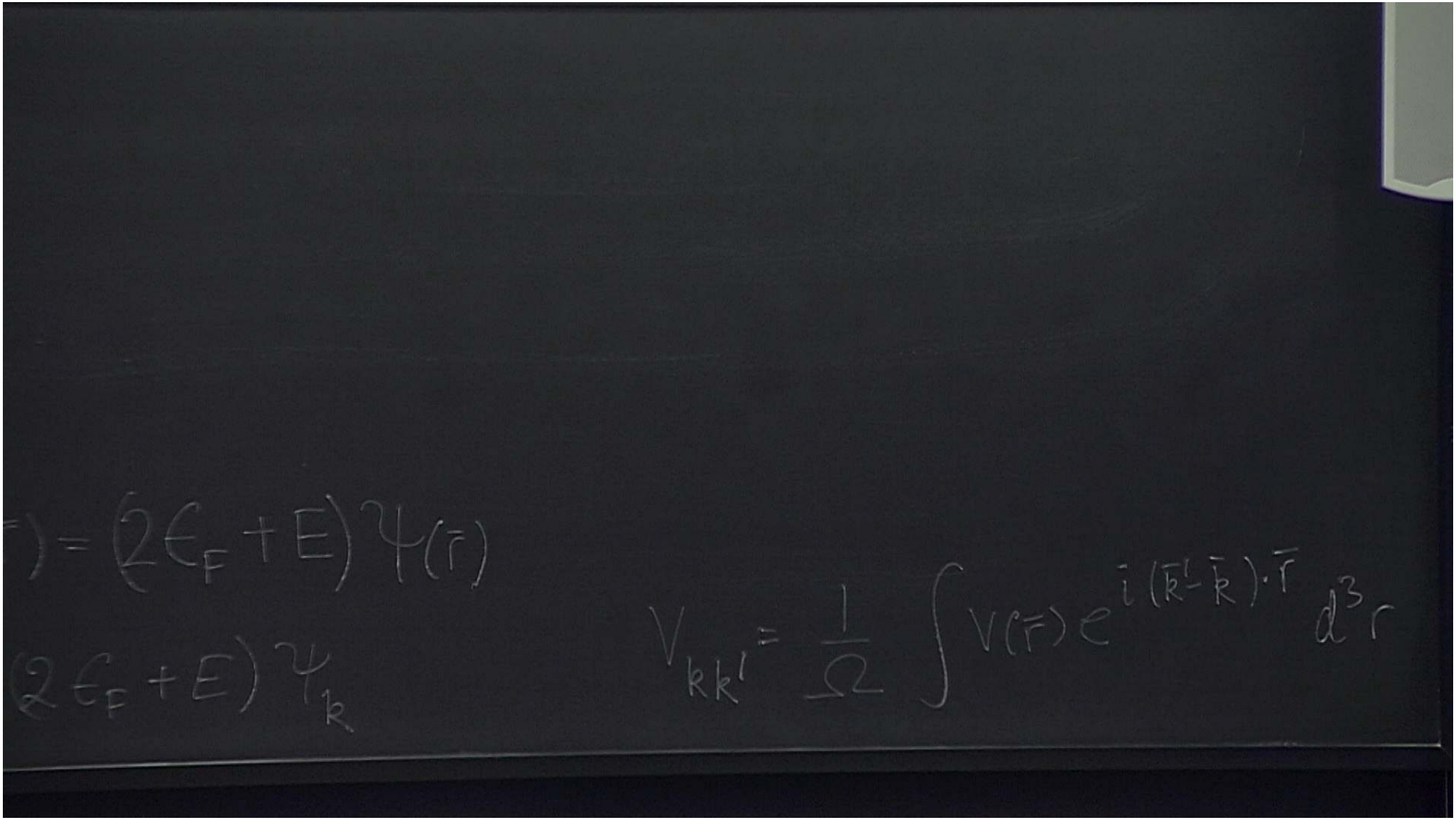
Schrödinger Eq^n

$$-\frac{\hbar^2}{m}\nabla_r^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = (2\epsilon_F + E)\psi(\vec{r})$$

↓ Fourier

$$\frac{\hbar^2 k^2}{2m}\psi_k + \sum_{kk'}' V_{kk'}\psi_{k'} = (2\epsilon_F + E)\psi_k$$


$= (2\epsilon_F + E)\psi(\vec{r})$
$(2\epsilon_F + E)\psi_k$
$V_{kk'} = \frac{1}{\Omega}\int V(\vec{r})\, e^{i(\vec{k}'-\vec{k})\cdot\vec{r}}\, d^3r$

$\psi(\bar{r}) = 0$

$$H = H_{CM} + H_{rel}$$

$$H_{CM} = \frac{P_{CM}^2}{2M} \qquad P_{CM} = 0$$

$$H_{rel} = \frac{p_{rel}^2}{m} + V(\bar{r})$$

Schrödinger Eq^n

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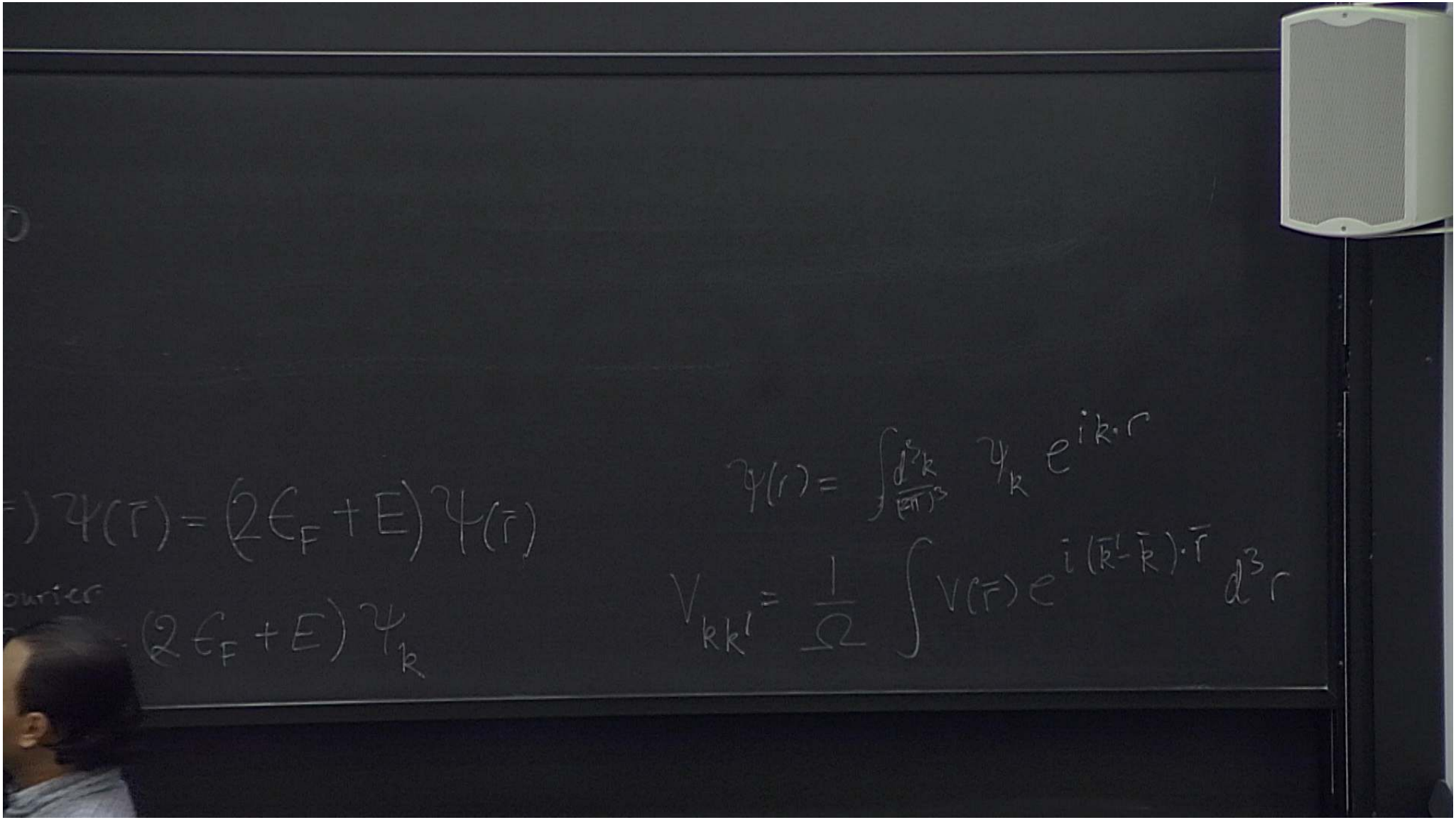
$\psi(r$

↓ Fourier

$$\frac{\hbar^2 k^2}{m}\psi_k + \sum_{kk'} V_{kk'}\psi_{k'} = (2\epsilon_F + E)\psi_k$$

$V_{kk'} =$


$\psi(r) = \int \frac{d^3k}{(2\pi)^3} \psi_k e^{ik\cdot r}$
$V_{kk'} = \frac{1}{\Omega} \int V(r) e^{i(k'-k)\cdot r} d^3r$
$(2\epsilon_F + E)\psi_k$

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'}$$

$$\left(E - 2\xi_k\right)\psi_k = \sum_{k'}{}' V_{kk'}\psi_{k'}$$

$$\xi_k = \epsilon_k - \epsilon_F$$

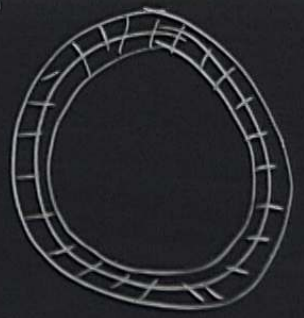
$$= \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'} \ldots$$

$$(E - 2\xi_k)\psi_k = \sum_{k'}' V_{kk'} \psi_{k'}$$

$$\xi_k = \epsilon_k - \epsilon_F$$

$$= \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

On-shell Approximation

$$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c \\ & |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$$