

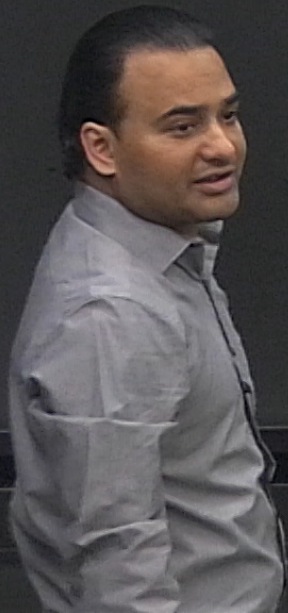
Title: PSI 2018/2019 - Condensed Matter - Lecture 5

Date: Nov 16, 2018 10:45 AM

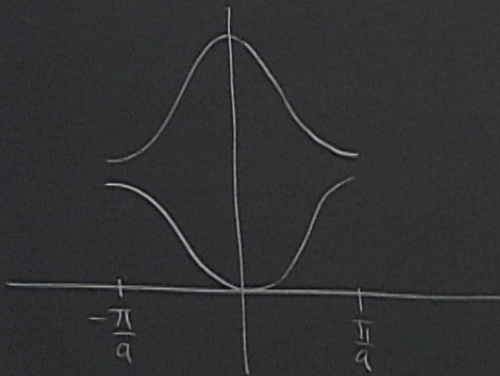
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Abstract:

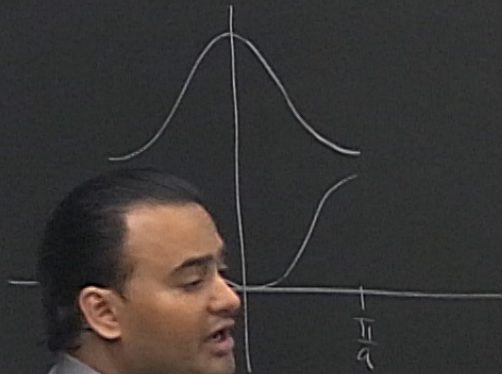
$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$



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Peierl's theorem

FBZ $k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right)$

$$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$$

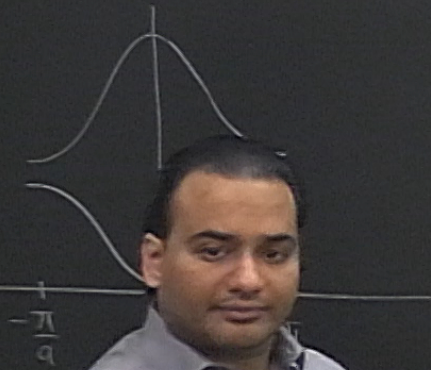
$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$

Peierl's theorem

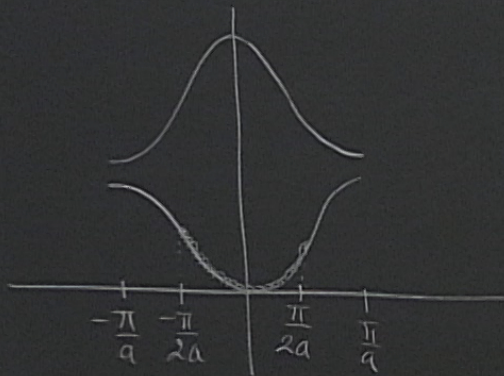
$$\text{FBZ} \quad k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \Rightarrow n \in \left(-\frac{N}{2}, \frac{N}{2}\right)$$

$$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$$

In FBZ N states



$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$

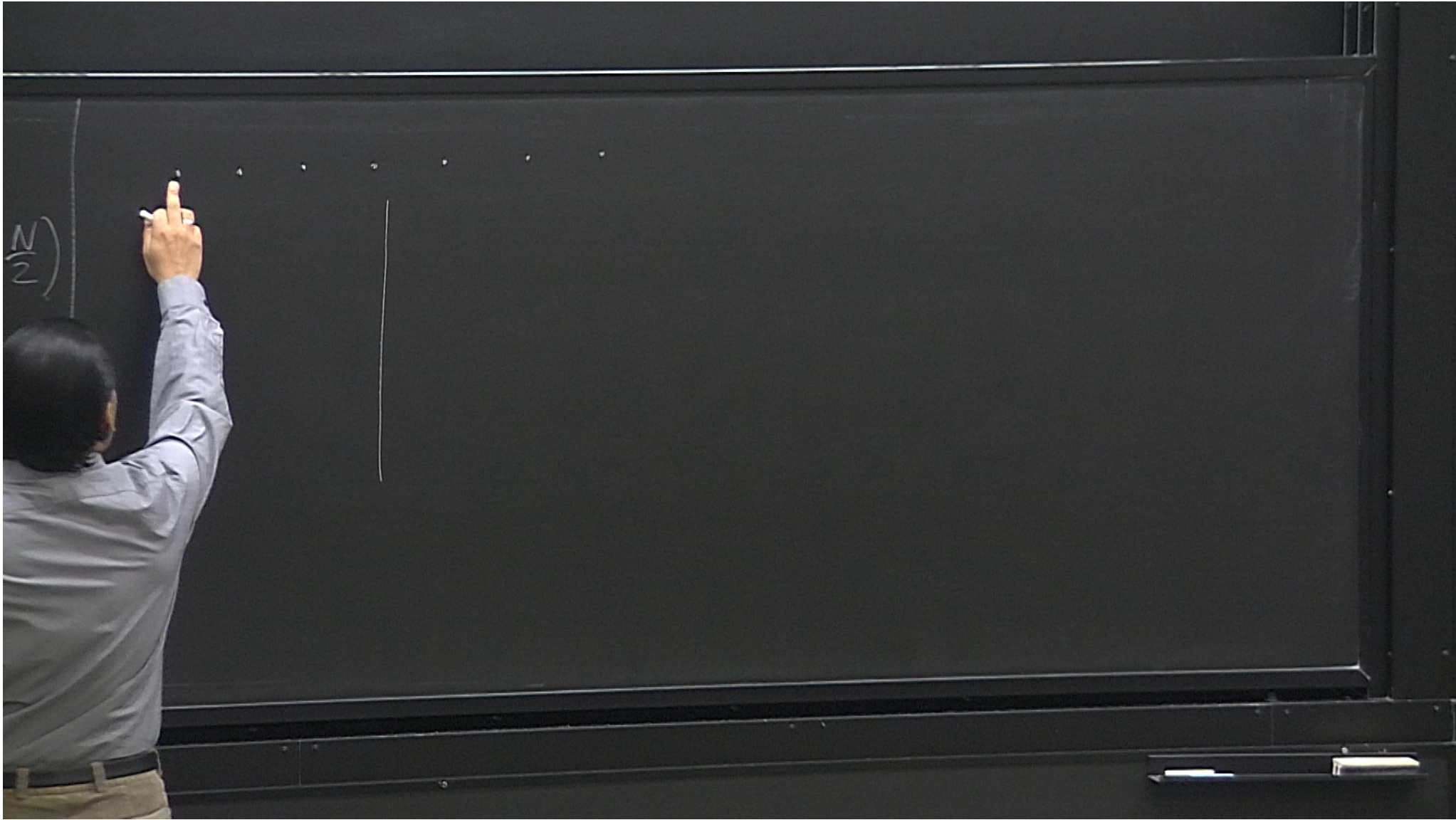


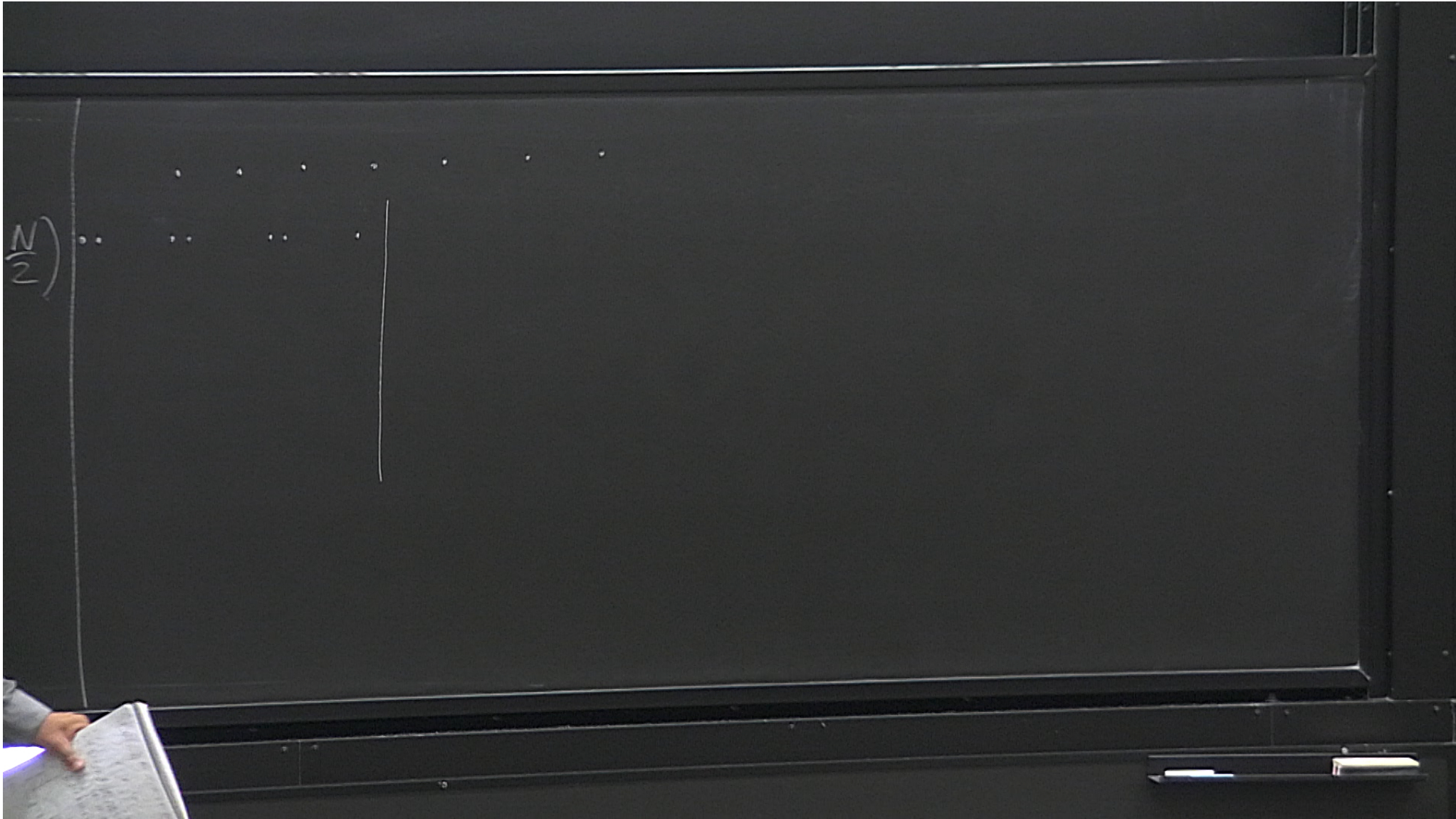
Peierl's theorem

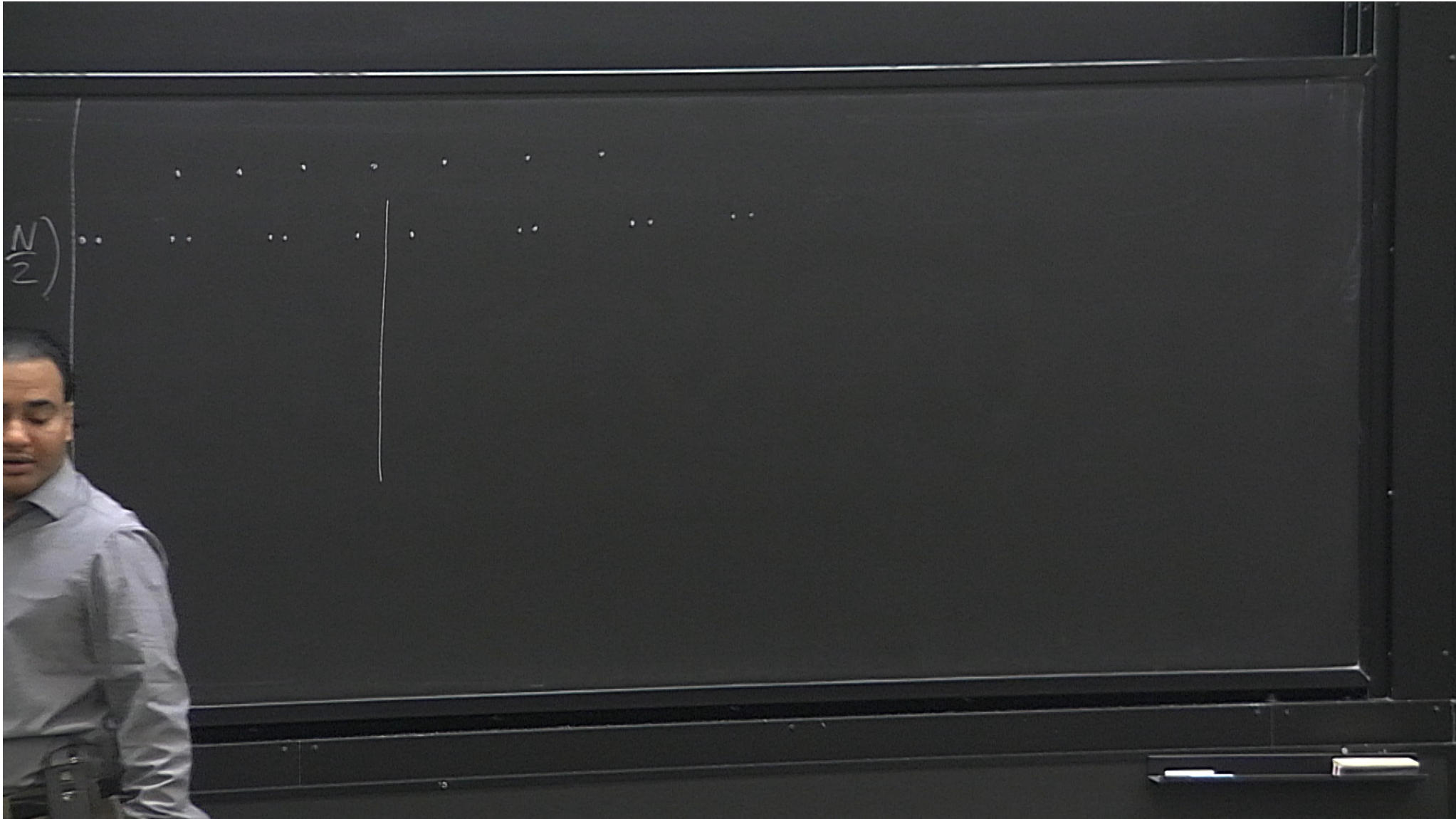
FBZ $k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \Rightarrow n \in \left(-\frac{N}{2}, \frac{N}{2}\right)$

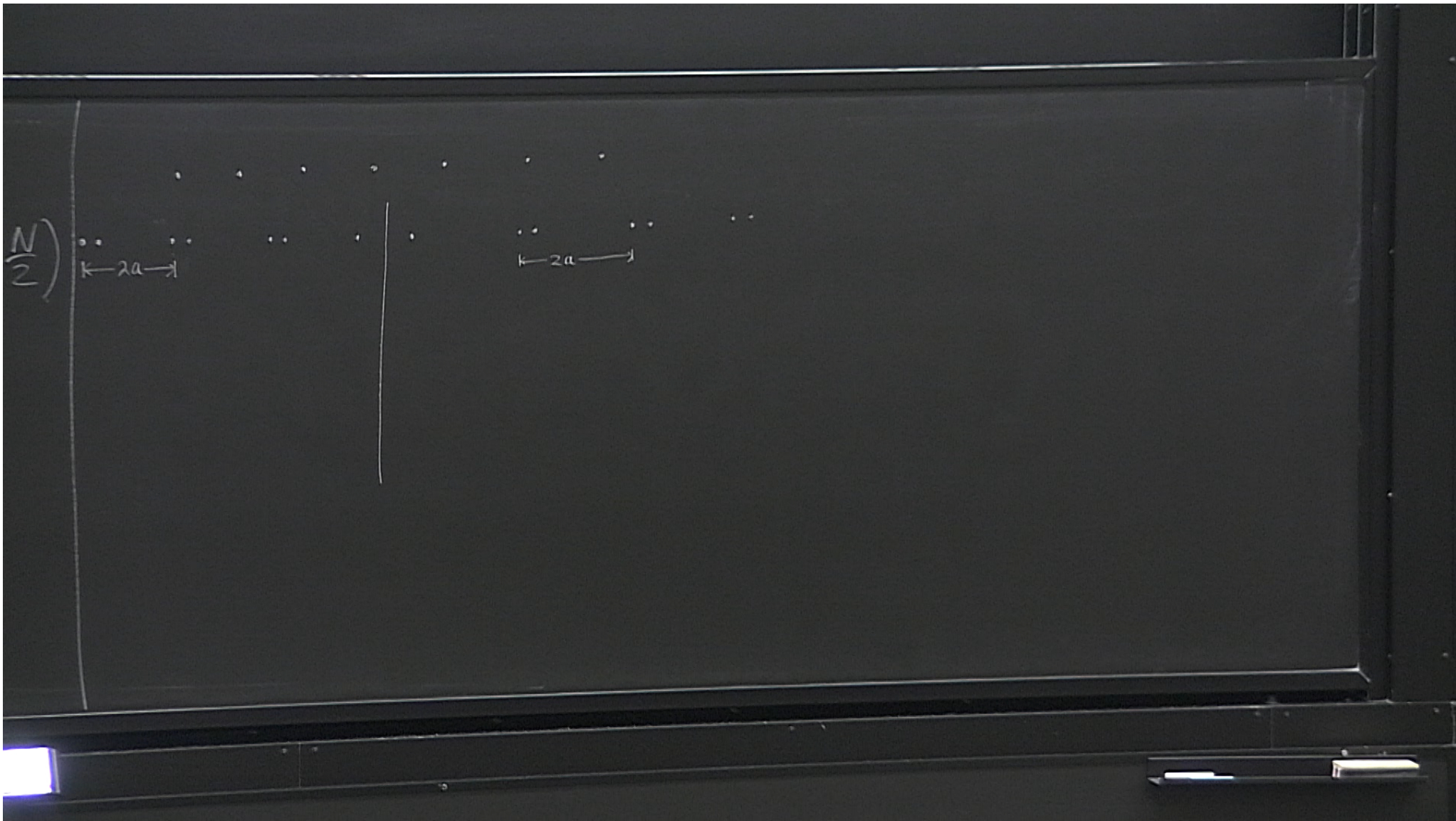
$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$

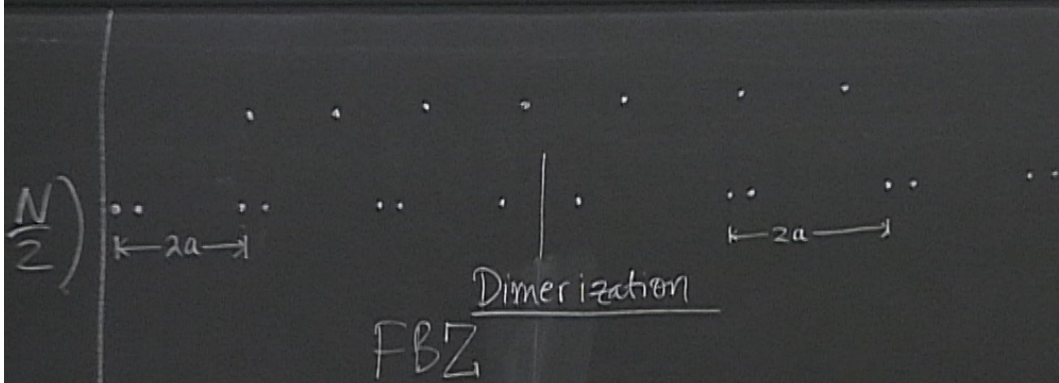
In FBZ N states











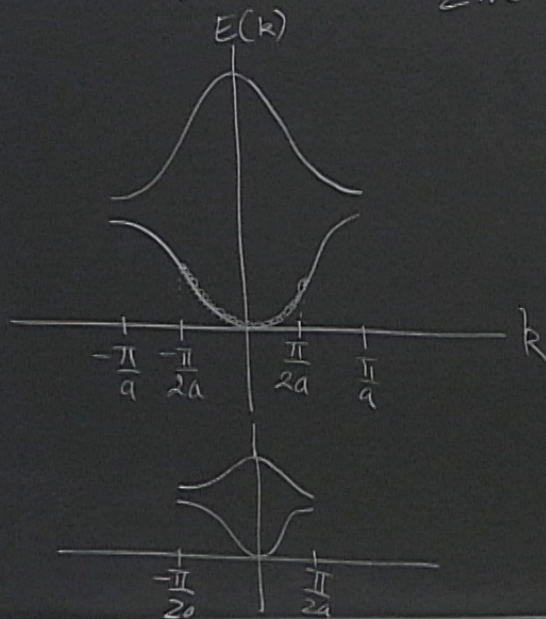
$\frac{N}{2}$



Dimerization

New FBZ, $k \in (-\frac{\pi}{2a}, \frac{\pi}{2a})$

$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$



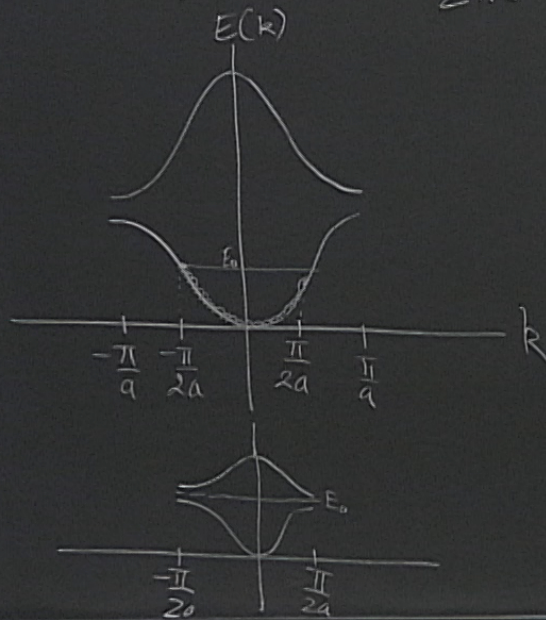
Peierl's theorem

$$\text{FBZ} \quad k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \Rightarrow n \in \left(-\frac{N}{2}, \frac{N}{2}\right) \quad \text{with } k \leftarrow k + 2\pi/a$$

$$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$$

In FBZ N states

$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$



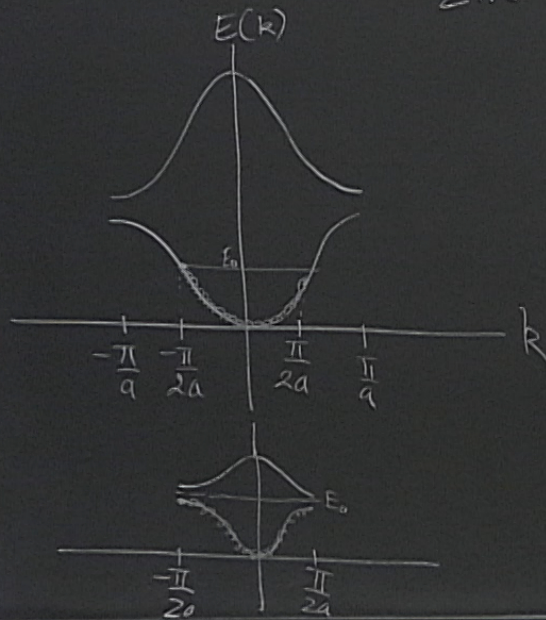
Peierl's theorem

$$\text{FBZ} \quad k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \Rightarrow n \in \left(-\frac{N}{2}, \frac{N}{2}\right) \quad \leftarrow \frac{\pi}{a}$$

$$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$$

In FBZ N states

$$E_p^{(0)}(k) = \frac{\hbar^2 (k+q)^2}{2m}$$

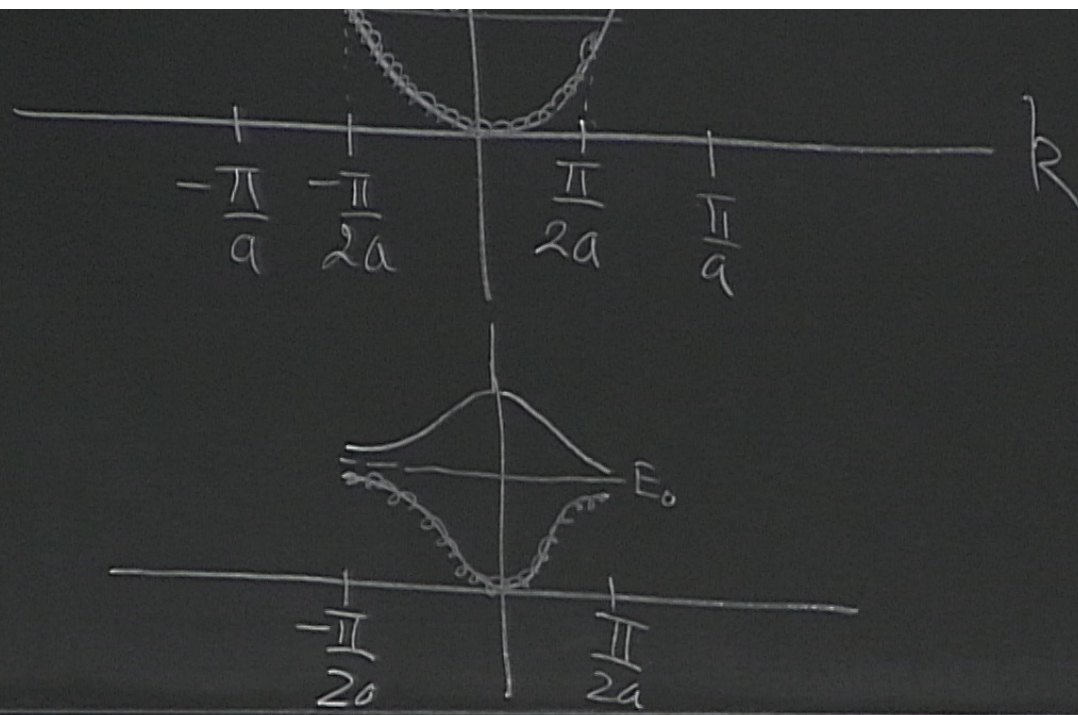


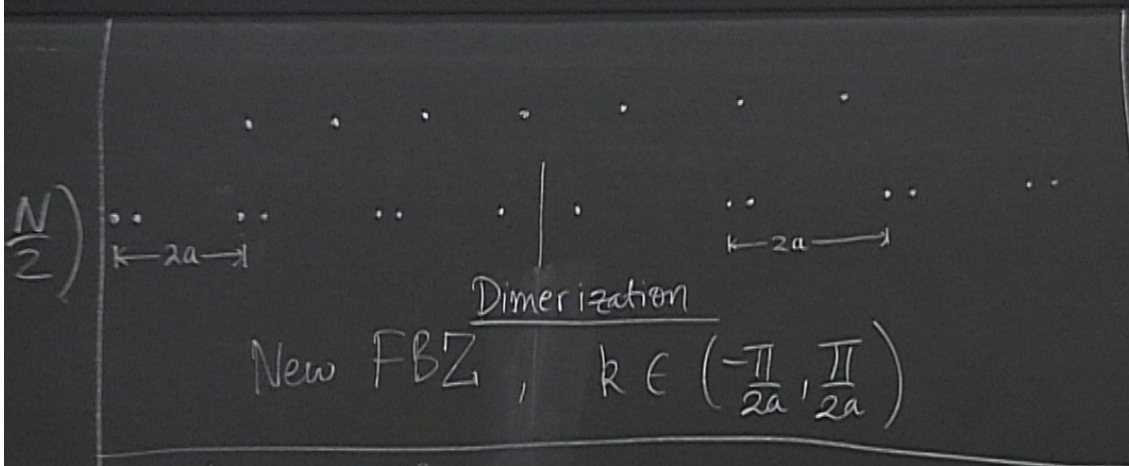
Peierl's theorem

$$\text{FBZ} \quad k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \Rightarrow n \in \left(-\frac{N}{2}, \frac{N}{2}\right) \quad \dots \quad k \leftarrow 2a$$

$$k = \frac{2\pi}{a} \frac{n}{N} \quad n \in \mathbb{Z}$$

In FBZ N states





$$\hat{T}_{el} = \sum_{\sigma} \int d^3r \psi_{\sigma}^{\dagger}(\vec{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi_{\sigma}(\vec{r}) = \sum_{\vec{k}\sigma} \epsilon(\vec{k}) c_{\vec{k}\sigma}^{\dagger} c_{\vec{k}\sigma}$$

$$c_{\vec{k}\sigma} = \frac{1}{\sqrt{N}} \int d^3r e^{i\vec{k}\cdot\vec{r}} \psi_{\sigma}(\vec{r})$$



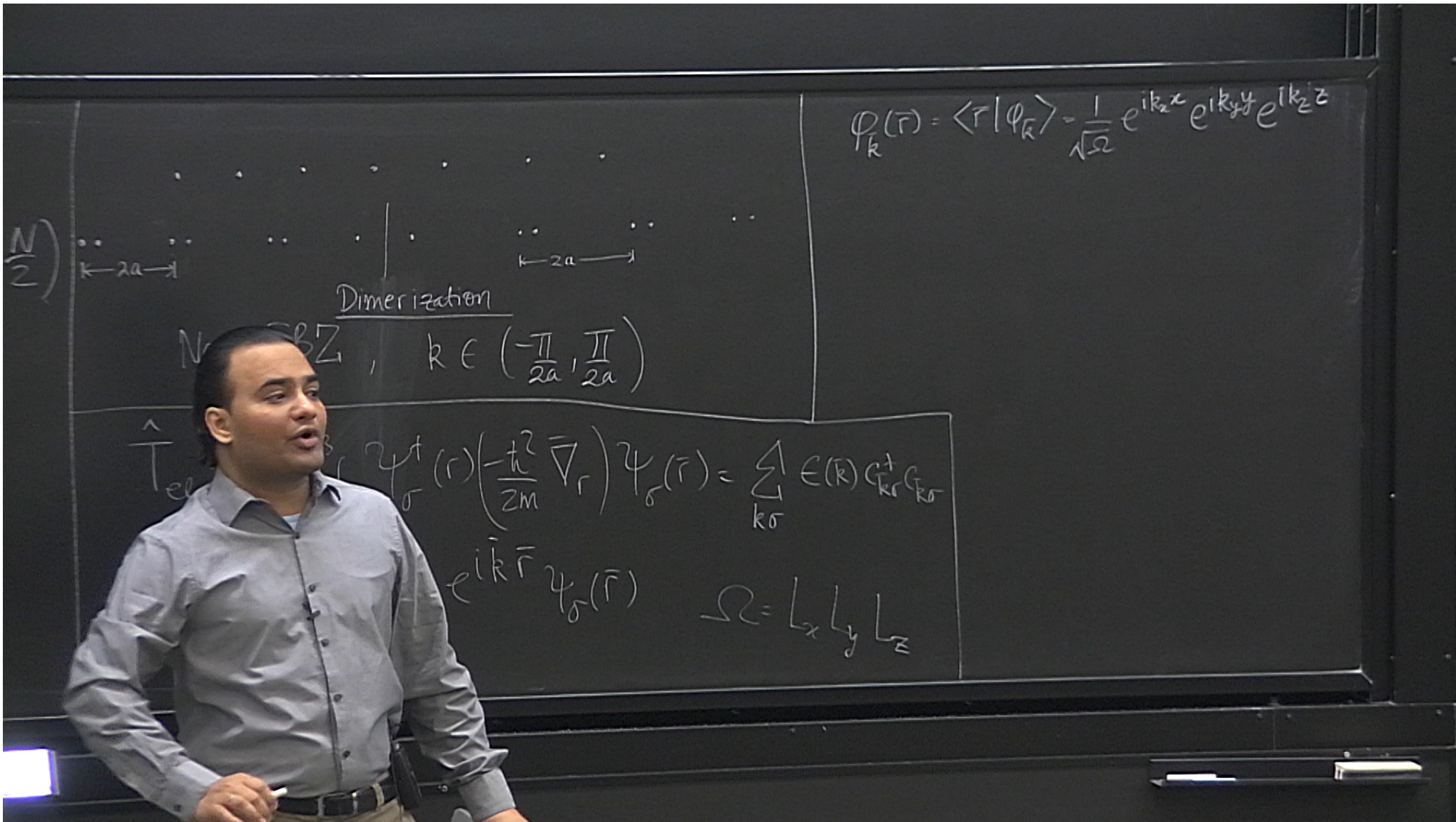
$\frac{N}{2}$ $\cdots \leftarrow 2a \rightarrow \cdots$ $\cdots \leftarrow 2a \rightarrow \cdots$

Dimerization

New FBZ, $k \in \left(-\frac{\pi}{2a}, \frac{\pi}{2a}\right)$

$$\hat{T}_{el} = \sum_{\sigma} \int d^3r \psi_{\sigma}^{\dagger}(\vec{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi_{\sigma}(\vec{r}) = \sum_{\vec{k}\sigma} \epsilon(\vec{k}) c_{\vec{k}\sigma}^{\dagger} c_{\vec{k}\sigma}$$

$$c_{\vec{k}\sigma} = \frac{1}{\sqrt{\Omega}} \int d^3r e^{i\vec{k}\cdot\vec{r}} \psi_{\sigma}(\vec{r}) \quad \Omega = L_x L_y L_z$$



$\left(\frac{N}{2}\right)$
 $\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$
 $\leftarrow 2a \rightarrow$
 $\leftarrow 2a \rightarrow$

Dimerization

New FB7

$$k \in \left(-\frac{\pi}{2a}, \frac{\pi}{2a}\right)$$

$$\hat{T}_{el} = \sum_{\sigma} \int d\vec{r} \psi_{\sigma}^{\dagger}(\vec{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi_{\sigma}(\vec{r}) = \sum_{\vec{k}, \sigma} \epsilon(\vec{k}) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

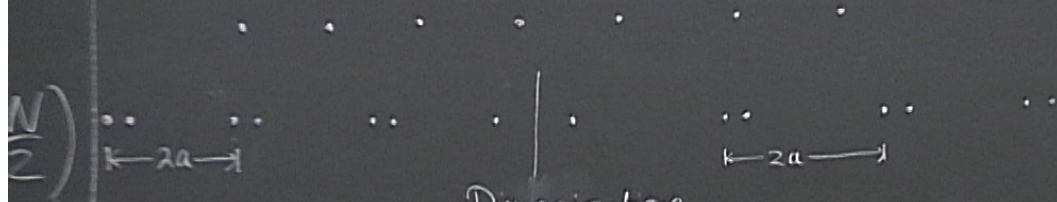
$$i\vec{k} \cdot \vec{r} \psi_{\sigma}(\vec{r})$$

$$\Omega = L_x L_y L_z$$

$$\phi_{\vec{k}}(\vec{r}) = \langle \vec{r} | \phi_{\vec{k}} \rangle = \frac{1}{\sqrt{\Omega}} e^{ik_x x} e^{ik_y y} e^{ik_z z}$$

$$\phi_{\vec{k}}(\vec{r} + L_x \hat{x}) = \phi_{\vec{k}}(\vec{r}) = \phi_{\vec{k}}(\vec{r} + L_y \hat{y}) = \phi_{\vec{k}}(\vec{r} + L_z \hat{z})$$

$$\Rightarrow \begin{aligned} k_x &= \frac{2\pi}{L_x} n_x \\ k_y &= \frac{2\pi}{L_y} n_y \end{aligned} \quad \& \quad \begin{aligned} k_z &= \frac{2\pi}{L_z} n_z \\ (n_x, n_y, n_z) &\in \mathbb{Z} \end{aligned}$$



Dimerization

New FBZ, $k \in (-\frac{\pi}{2a}, \frac{\pi}{2a})$

$$\hat{T}_{el} = \sum_{\sigma} \int d^3r \psi_{\sigma}^{\dagger}(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi_{\sigma}(\mathbf{r}) = \sum_{\mathbf{k}\sigma} \epsilon(\mathbf{k}) c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

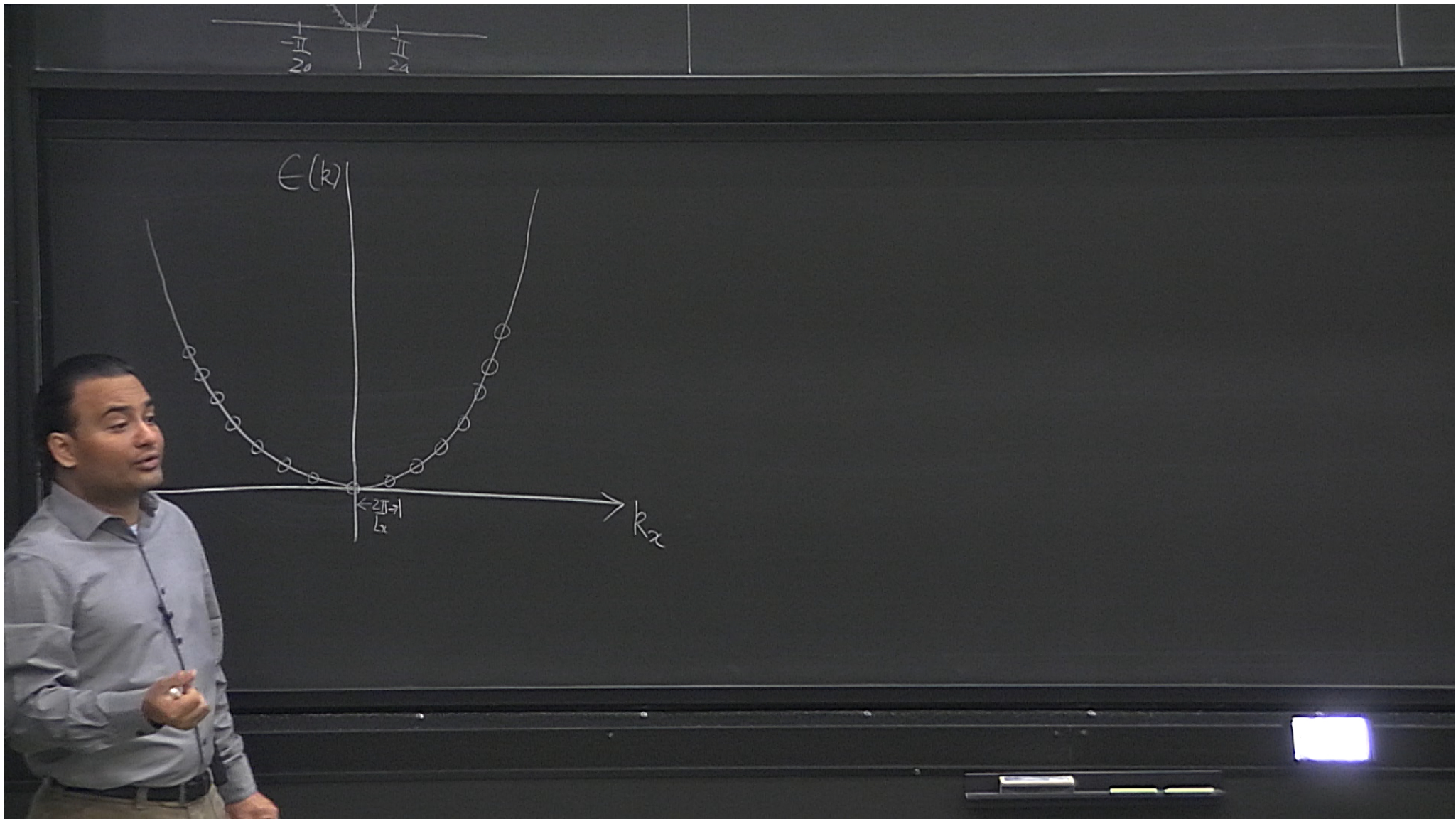
$$c_{\mathbf{k}\sigma} = \frac{1}{\sqrt{\Omega}} \int d^3r e^{i\mathbf{k}\cdot\mathbf{r}} \psi_{\sigma}(\mathbf{r}) \quad \Omega = L_x L_y L_z$$

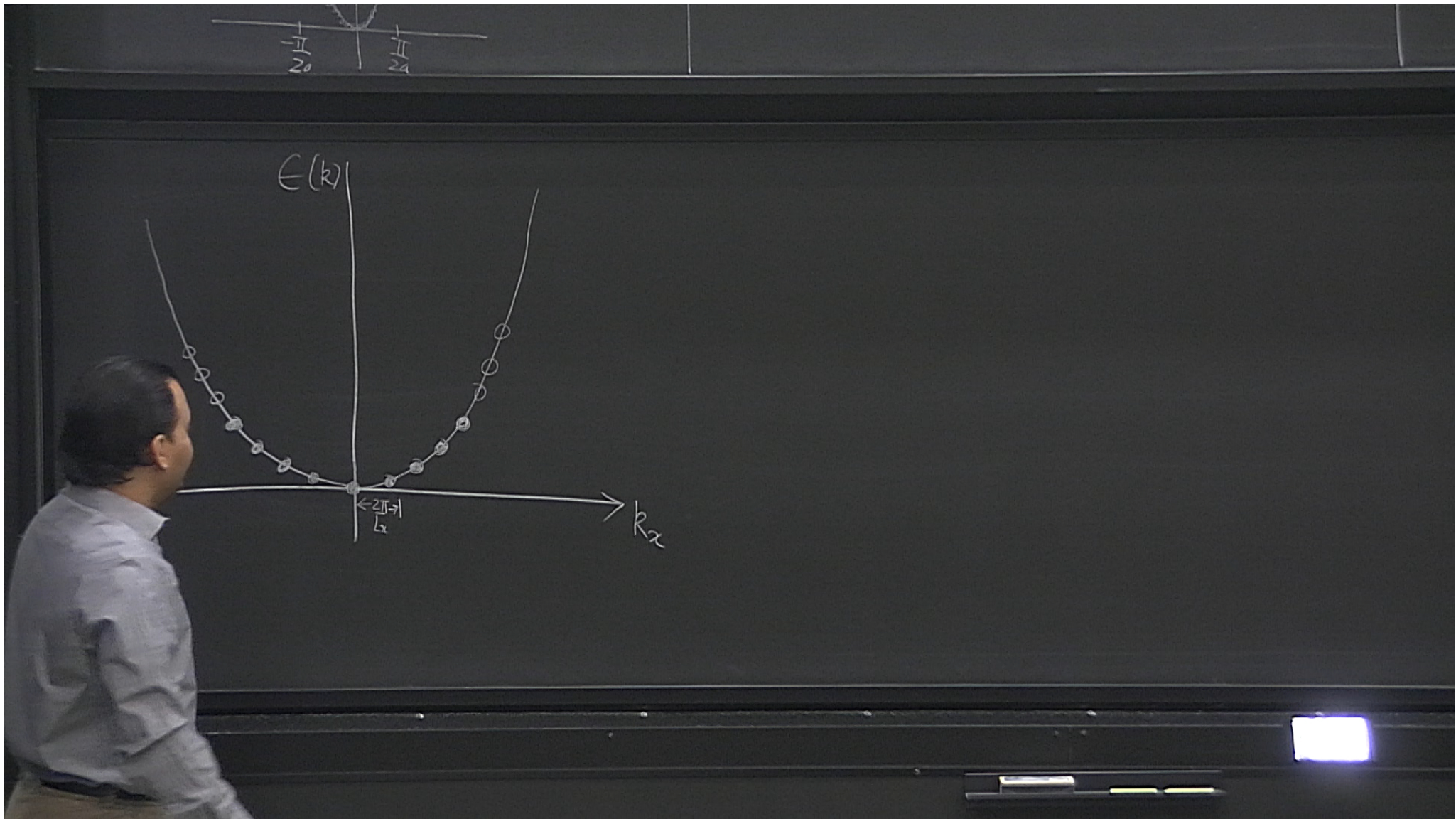
$$\phi_{\mathbf{k}}(\mathbf{r}) = \langle \mathbf{r} | \phi_{\mathbf{k}} \rangle = \frac{1}{\sqrt{\Omega}} e^{ik_x x} e^{ik_y y} e^{ik_z z}$$

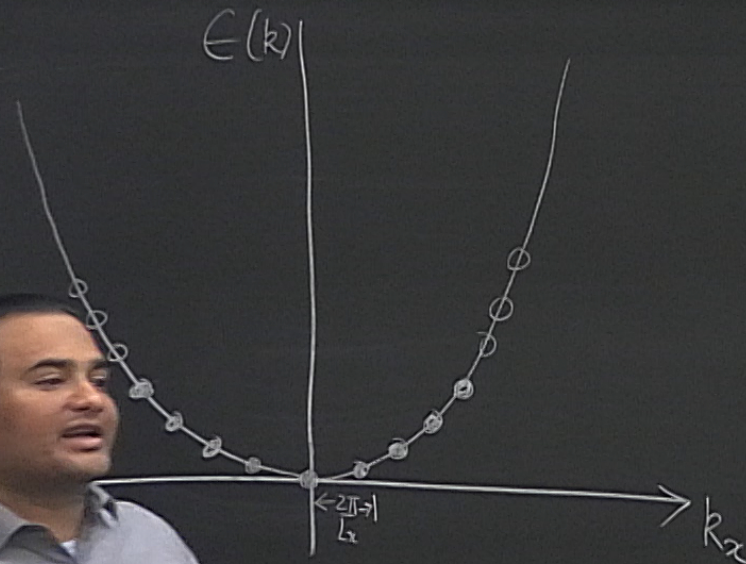
$$\phi_{\mathbf{k}}(\mathbf{r} + L_x \hat{x}) = \phi_{\mathbf{k}}(\mathbf{r}) = \phi_{\mathbf{k}}(\mathbf{r} + L_y \hat{y}) = \phi_{\mathbf{k}}(\mathbf{r} + L_z \hat{z})$$

$$\Rightarrow \begin{aligned} k_x &= \frac{2\pi}{L_x} n_x & k_y &= \frac{2\pi}{L_y} n_y & k_z &= \frac{2\pi}{L_z} n_z \\ (n_x, n_y, n_z) &\in \mathbb{Z} \end{aligned}$$

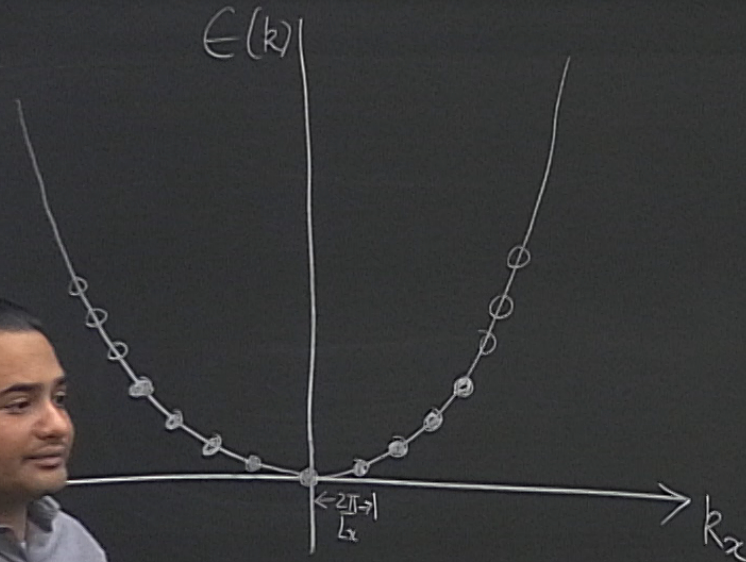
$$\epsilon(\mathbf{k}) = \frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m}$$





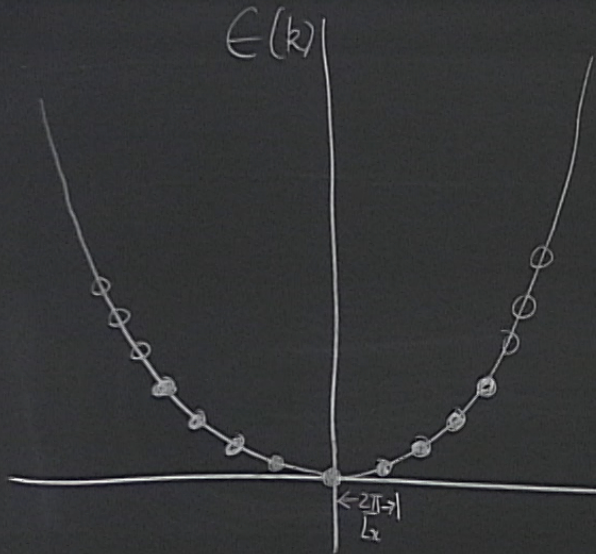


N_{el} will result in states being occupied till E_F



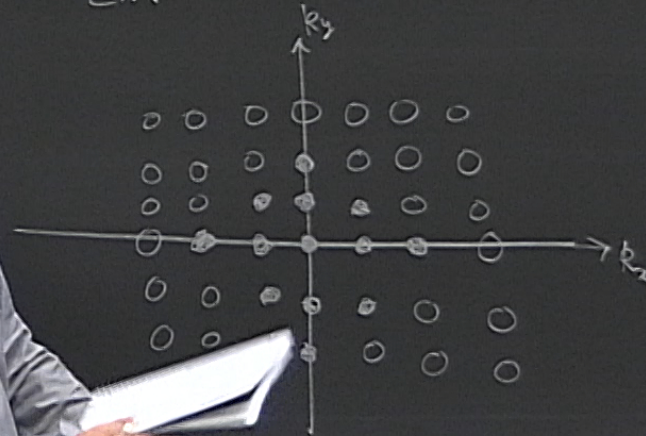
N_{el} will result in states being occupied till E_F

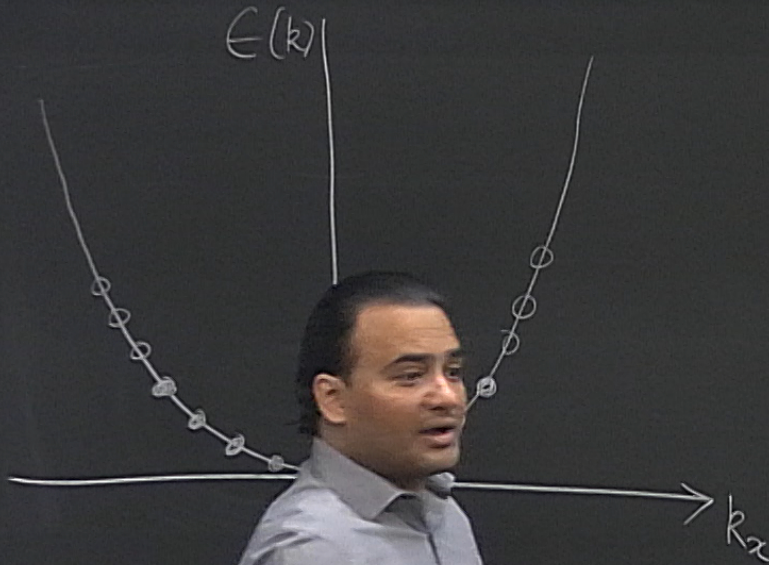
$$E_F = \frac{\hbar^2 k_F^2}{2m} \Rightarrow k_F = \frac{\sqrt{2mE_F}}{\hbar}$$



N_{el} will result in states being occupied till E_F

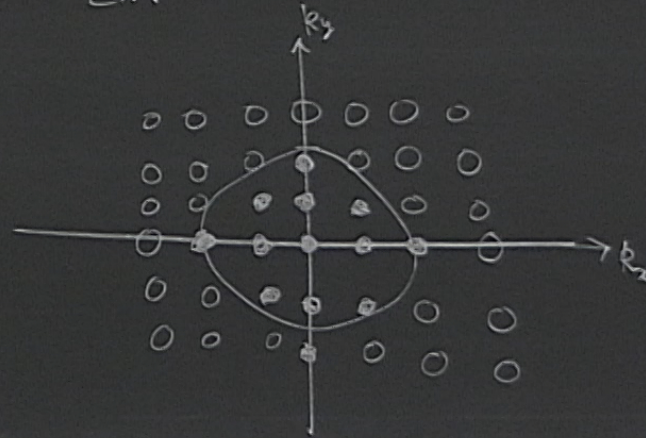
$$E_F = \frac{\hbar^2 k_F^2}{2m} \Rightarrow k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$$

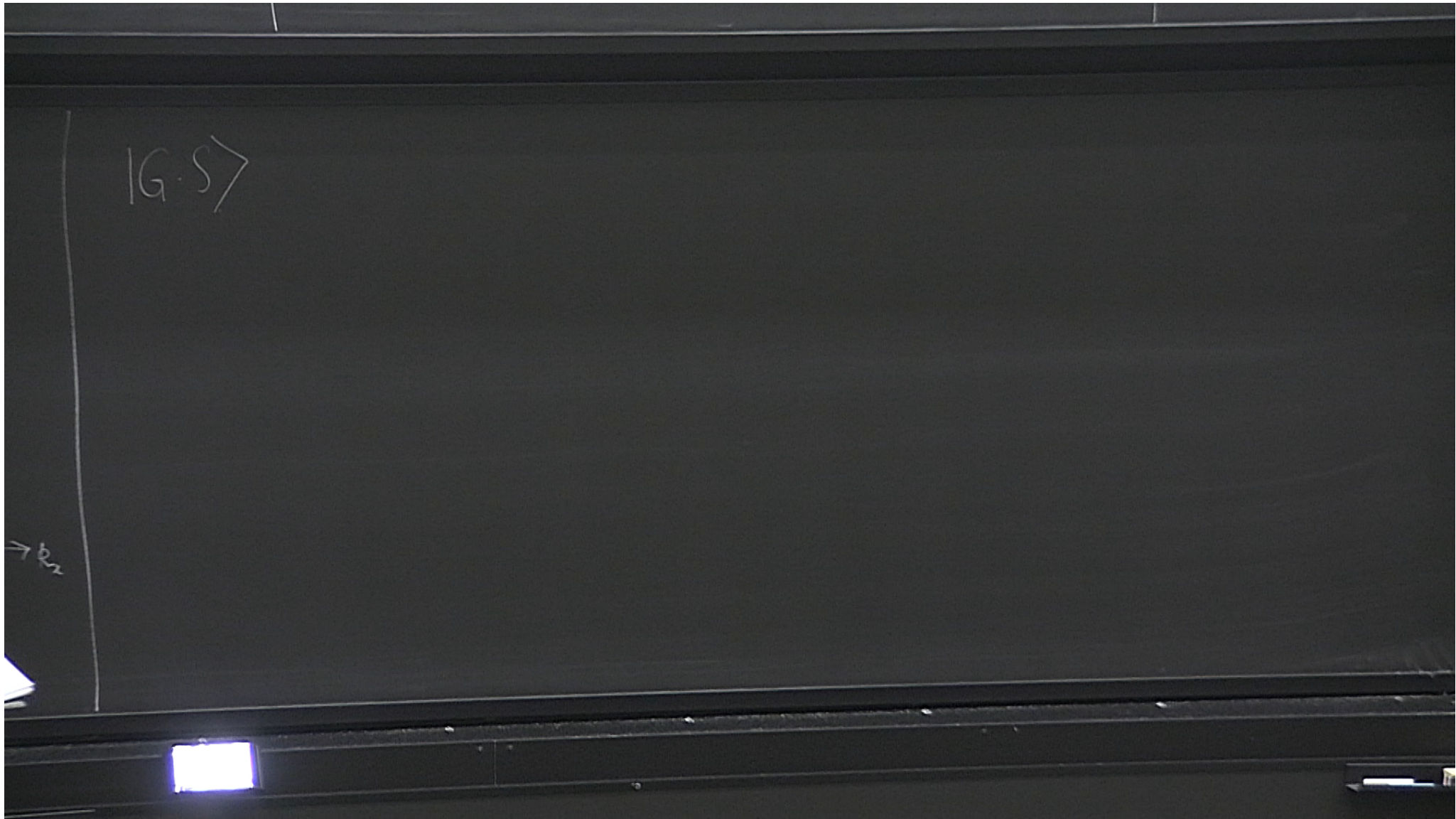




N_{el} will result in states being occupied till E_F

$$E_F = \frac{\hbar^2 k_F^2}{2m} \Rightarrow k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$$





$|G \cdot S\rangle$

$|0\rangle$

$\rightarrow k_z$

$$|G.S\rangle = \prod_{|\vec{k}| < k_F} c_{\vec{k}\uparrow}^{\dagger} c_{\vec{k}\downarrow} |0\rangle$$

$\rightarrow k_z$

$$|G.S\rangle = \prod_{|\vec{k}| < k_F} c_{\vec{k}\uparrow}^{\dagger} c_{\vec{k}\downarrow}^{\dagger} |0\rangle$$

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$$\lambda_F = \frac{2\pi}{k_F}$$

$$v_F = \frac{1}{\hbar} \left. \frac{\partial \epsilon(k)}{\partial k} \right|_{k_F} = \frac{\hbar k_F}{m}$$

$\rightarrow k_z$

$$|G-S\rangle = \prod_{|\mathbf{k}| < k_F} c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\downarrow}^\dagger |0\rangle$$

$$N_{el} = \sum_{\mathbf{k}\sigma} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle$$

$$=$$

$$\lambda_F = \frac{2\pi}{k_F}$$

$$v_F = \frac{1}{\hbar} \left. \frac{\partial E(k)}{\partial k} \right|_{k_F} = \frac{\hbar k_F}{m}$$

$$|G.S\rangle = \prod_{|\mathbf{k}| < k_F} c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\downarrow}^\dagger |0\rangle$$

$$N_{el} = \sum_{\mathbf{k}\sigma} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle$$

$$= 2_{\text{spin}} \sum_{\mathbf{k}} (\text{over all occupied states})$$

$$\lambda_F = \frac{2\pi}{k_F}$$

$$v_F = \frac{1}{\hbar} \left. \frac{\partial E(\mathbf{k})}{\partial k} \right|_{k_F} = \frac{\hbar k_F}{m}$$

$$= 2 \Omega \int_{|\mathbf{k}| < k_F} \frac{d^3 k}{(2\pi)^3} =$$



$$|G.S\rangle = \prod_{|\mathbf{k}| < k_F} c_{\bar{\mathbf{k}}\uparrow}^\dagger c_{\bar{\mathbf{k}}\downarrow}^\dagger |0\rangle$$

$$N_{el} = \sum_{\mathbf{k}\sigma} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle$$

$$= 2_{\text{spin}} \sum_{\mathbf{k}} (\text{over all occupied states})$$

$$= 2 \Omega \int_{|\mathbf{k}| < k_F} \frac{d^3 \mathbf{k}}{(2\pi)^3} = \frac{2\Omega}{(2\pi)^3} \frac{4}{3} \pi k_F^3 = \frac{\Omega k_F^3}{3\pi^2}$$

$$\lambda_F = 2\pi$$

$$v_F = \left. \frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}} \right|_{k_F} = \frac{\hbar k_F}{m}$$

$$|G.S\rangle = \prod_{|\mathbf{k}| < k_F} c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\downarrow}^\dagger |0\rangle$$

$$\lambda_F = \frac{2\pi}{k_F}$$

$$v_F = \frac{1}{\hbar} \left. \frac{\partial E(\mathbf{k})}{\partial k} \right|_{k_F} = \frac{\hbar k_F}{m}$$

$$N_{el} = \sum_{\mathbf{k}\sigma} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle$$

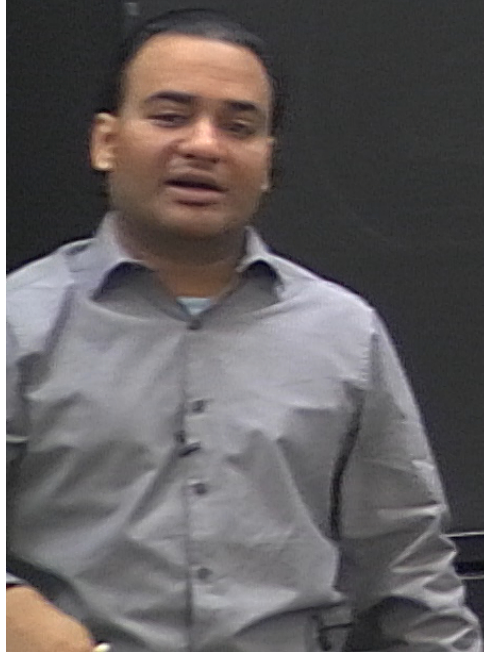
$$= 2_{\text{spin}} \sum_{\mathbf{k}} (\text{over all occupied states})$$

$$= 2 \Omega \int_{|\mathbf{k}| < k_F} \frac{d^3 k}{(2\pi)^3} = \frac{2\Omega}{(2\pi)^3} \frac{4}{3} \pi k_F^3 = \frac{\Omega k_F^3}{3\pi^2}$$

$$\rho = \frac{N_{el}}{\Omega} = \frac{k_F^3}{3\pi^2} \Rightarrow k_F = (3\pi^2 \rho)^{1/3}$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m} (3\pi^2 \rho)^{2/3}$$

$$E^{(0)} = \sum_{\mathbf{k}\sigma} \frac{\hbar^2 \mathbf{k}^2}{2m} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle$$



$$E^{(0)} = \sum_{k\sigma} \frac{\hbar^2 k^2}{2m} \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle = \frac{3}{5} \epsilon_F N_{el}$$

Average kinetic energy per particle $\bar{\epsilon}_{kin} = \frac{E^{(0)}}{N_{el}} = \frac{3}{5} \epsilon_F \propto$

$$E^{(0)} = \sum_{\mathbf{k}\sigma} \frac{\hbar^2 \mathbf{k}^2}{2m} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle = \frac{3}{5} \epsilon_F N_{el}$$

Average kinetic energy per particle $\bar{\epsilon}_{kin} = \frac{E^{(0)}}{N_{el}} = \frac{3}{5} \epsilon_F \propto \rho^{2/3}$

$$E^{(0)} = \sum_{\mathbf{k}\sigma} \frac{\hbar^2 \mathbf{k}^2}{2m} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle = \frac{3}{5} \epsilon_F N_{el}$$

Average kinetic energy per particle $\bar{\epsilon}_{kin} = \frac{E^{(0)}}{N_{el}} = \frac{3}{5} \epsilon_F \propto \rho^{2/3}$

Average distance between electrons $\bar{d}$

$$\Rightarrow N_{el} = \frac{\Omega}{\bar{d}^3}$$

$$E^{(0)} = \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle = \frac{3}{5} \epsilon_F N_{el}$$

Average kinetic energy per particle $\bar{\epsilon}_{kin} = \frac{E^{(0)}}{N_{el}} = \frac{3}{5} \epsilon_F \propto \rho_{el}^{2/3}$

Average distance between electrons $\bar{d}$

$$\Rightarrow N_{el} = \frac{\Omega}{\bar{d}^3} \Rightarrow \rho_{el} \propto \frac{1}{\bar{d}^3} \Rightarrow \bar{d} \propto \rho_{el}^{-1/3}$$

Average potential energy, $\epsilon_{pot} = \frac{e^2}{4\pi\epsilon_0 \bar{d}} \propto \rho_{el}^{1/3}$

$\bar{\epsilon}_1$

$$\frac{\bar{E}_{\text{pot}}}{\bar{E}_{\text{kin}}} \propto \frac{\rho_{\text{el}}^{1/3}}{\rho_{\text{el}}^{2/3}} \propto \rho_{\text{el}}^{-1/3}$$

$$\frac{\bar{E}_{\text{pot}}}{\bar{E}_{\text{kin}}} \propto \frac{\rho_{\text{el}}^{1/3}}{\rho_{\text{el}}^{2/3}} \propto \rho_{\text{el}}^{-1/3}$$

Density of states $\rho(\epsilon)$

$$\rho(\epsilon) = \underline{2}_{\text{spin}}$$

$$\frac{\bar{E}_{\text{pot}}}{\bar{E}_{\text{kin}}} \propto \frac{\rho_{\text{el}}^{1/3}}{\rho_{\text{el}}^{2/3}} \propto \rho_{\text{el}}^{-1/3}$$

Density of states $\rho(\epsilon)$

$$\rho(\epsilon) = \frac{2_{\text{spin}}}{\Omega} \sum_{\mathbf{k}} \delta(\epsilon - \epsilon(\mathbf{k}))$$

$$\frac{\bar{E}_{\text{pot}}}{\bar{E}_{\text{kin}}} \propto \frac{\rho_{\text{el}}^{1/3}}{\rho_{\text{el}}^{2/3}} \propto \rho_{\text{el}}^{-1/3}$$

Density of states $\rho(\epsilon)$

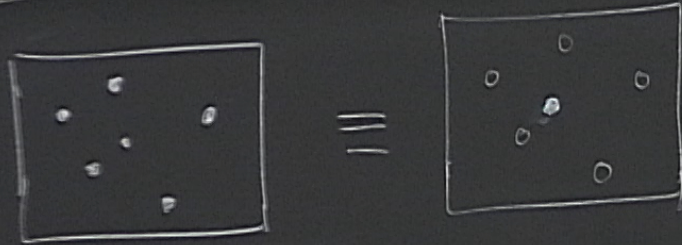
$$\begin{aligned} \rho(\epsilon) &= \frac{2_{\text{spin}}}{\Omega} \sum_{\mathbf{k}} \delta(\epsilon - \epsilon(\mathbf{k})) \\ &= 2 \int_{\text{BZ}} \frac{d^3 \mathbf{k}}{(2\pi)^3} \delta(\epsilon - \epsilon(\mathbf{k})) \end{aligned}$$

$$\frac{\bar{E}_{\text{pot}}}{\bar{E}_{\text{kin}}} \propto \frac{\rho_{\text{el}}^{1/3}}{\rho_{\text{el}}^{2/3}} \propto \rho_{\text{el}}^{-1/3}$$

Density of states $\rho(E)$

$$\begin{aligned} \rho(E) &= \frac{2_{\text{spin}}}{\Omega} \sum_{\mathbf{k}} \delta(E - E(\mathbf{k})) \\ &= 2 \int_{\text{BZ}} \frac{d^3 \mathbf{k}}{(2\pi)^3} \delta(E - E(\mathbf{k})) \\ &= 2 \int_{S(E)} \frac{dS}{(2\pi)^3} \frac{1}{|\nabla_{\mathbf{k}} E(\mathbf{k})|} \end{aligned}$$

Mean field theory



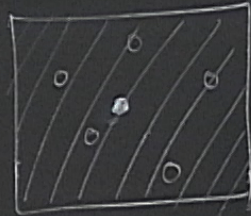
Real System



Mean field theory



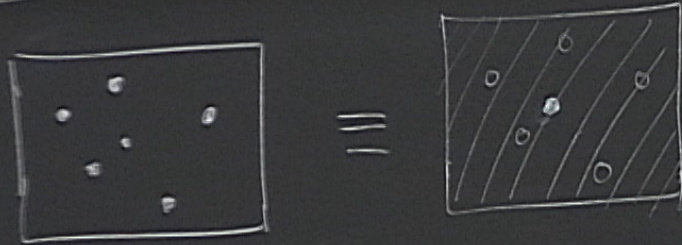
$\equiv$



Real System



Mean field theory

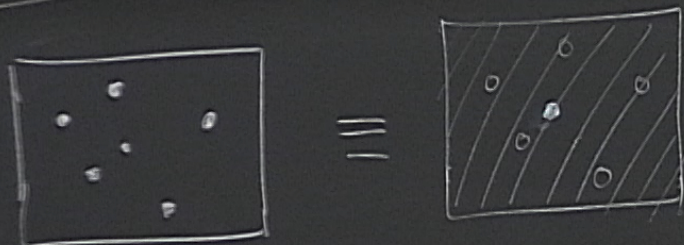


Real System

$$H = H_0 + V_{\text{int}}$$

$$H_0 = \sum_j \epsilon_j^a a_j^\dagger a_j + \sum_\mu \epsilon_\mu^b b_\mu^\dagger b_\mu$$

Mean field theory



Real System

$$H = H_0 + V_{int}$$

$$H_0 = \sum_{\nu} \epsilon_{\nu}^a a_{\nu}^{\dagger} a_{\nu} + \sum_{\mu} \epsilon_{\mu}^b b_{\mu}^{\dagger} b_{\mu}$$

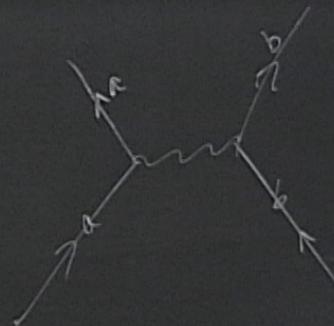
$$V_{int} = \sum_{\substack{\nu \nu' \\ \mu \mu'}} V_{\nu \mu}^{\nu' \mu'} a_{\nu}^{\dagger} b_{\mu}^{\dagger} b_{\mu'} a_{\nu'}$$



theory

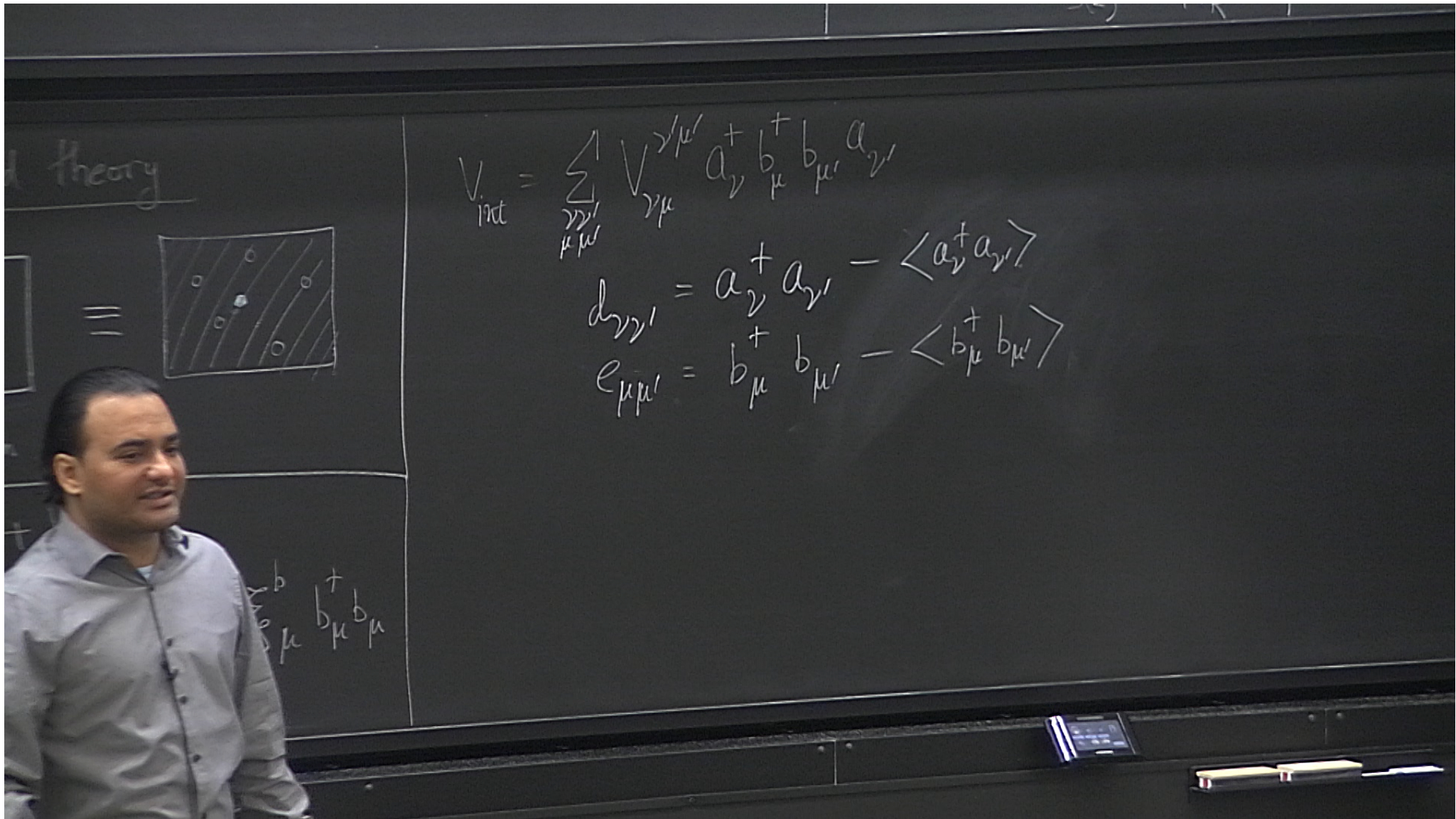


$$V_{int} = \sum_{\substack{\nu \nu' \\ \mu \mu'}} V_{\nu \mu}^{\nu' \mu'} a_{\nu}^{\dagger} b_{\mu}^{\dagger} b_{\mu'} a_{\nu'}$$



$$+ V_{int}$$

$$a_{\nu}^{\dagger} a_{\nu} + \sum_{\mu} \sum_{\nu} b_{\mu}^{\dagger} b_{\mu}$$



$$V_{int} = \sum_{\substack{\nu\nu' \\ \mu\mu'}} V_{\nu\mu'} a_{\nu}^{\dagger} b_{\mu}^{\dagger} b_{\mu'} a_{\nu'}$$

$$d_{\nu\nu'} = a_{\nu}^{\dagger} a_{\nu'} - \langle a_{\nu}^{\dagger} a_{\nu'} \rangle$$

$$e_{\mu\mu'} = b_{\mu}^{\dagger} b_{\mu'} - \langle b_{\mu}^{\dagger} b_{\mu'} \rangle$$

$$\sum_{\mu} b_{\mu}^{\dagger} b_{\mu}$$

$$V_{\text{int}} = \sum_{\nu\nu', \mu\mu'} V_{\nu\mu}^{\nu'\mu'} a_{\nu}^{\dagger} b_{\mu}^{\dagger} b_{\mu'} a_{\nu'}$$

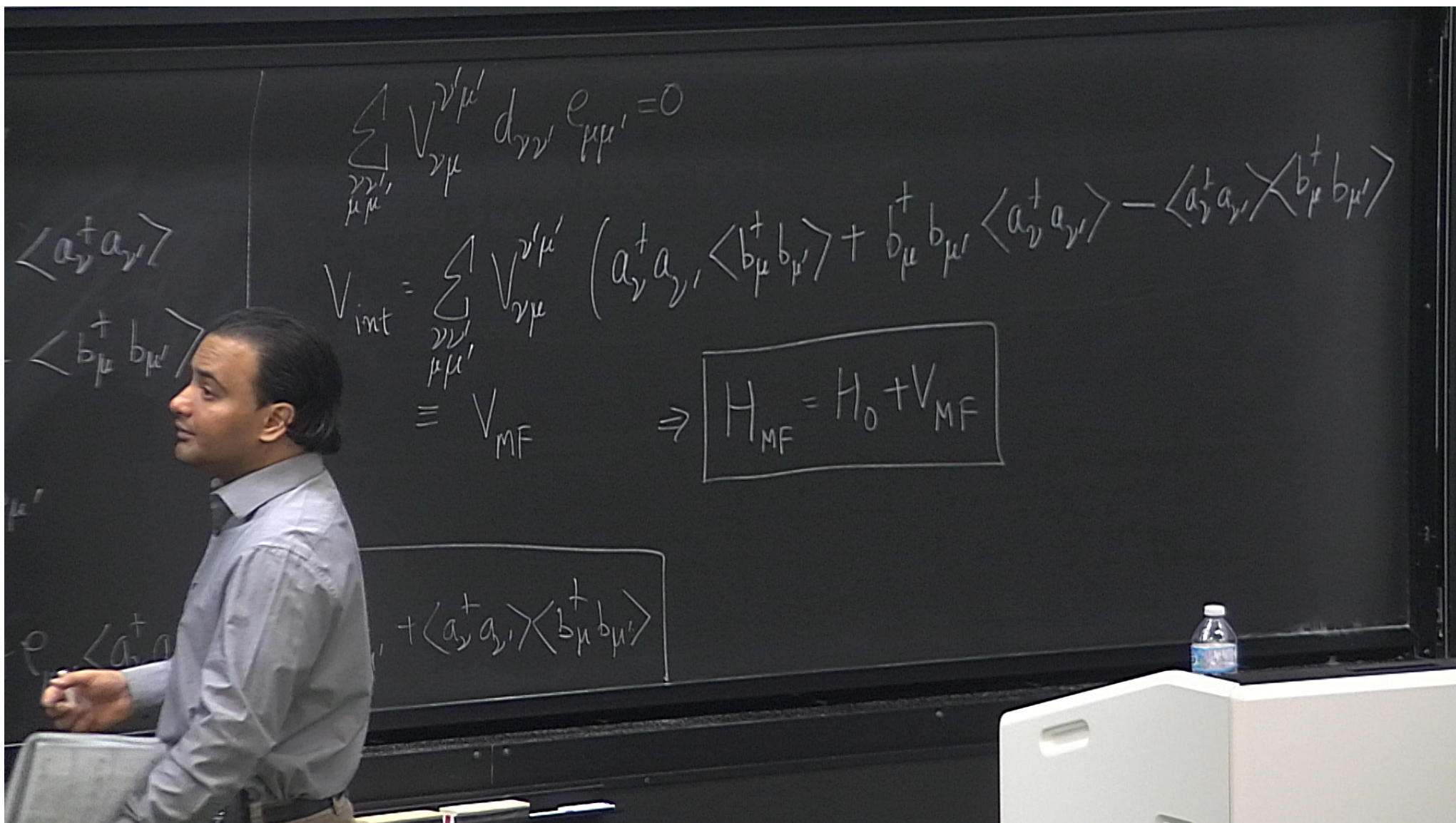
$$d_{\nu\nu'} = a_{\nu}^{\dagger} a_{\nu'} - \langle a_{\nu}^{\dagger} a_{\nu'} \rangle$$

$$e_{\mu\mu'} = b_{\mu}^{\dagger} b_{\mu'} - \langle b_{\mu}^{\dagger} b_{\mu'} \rangle$$

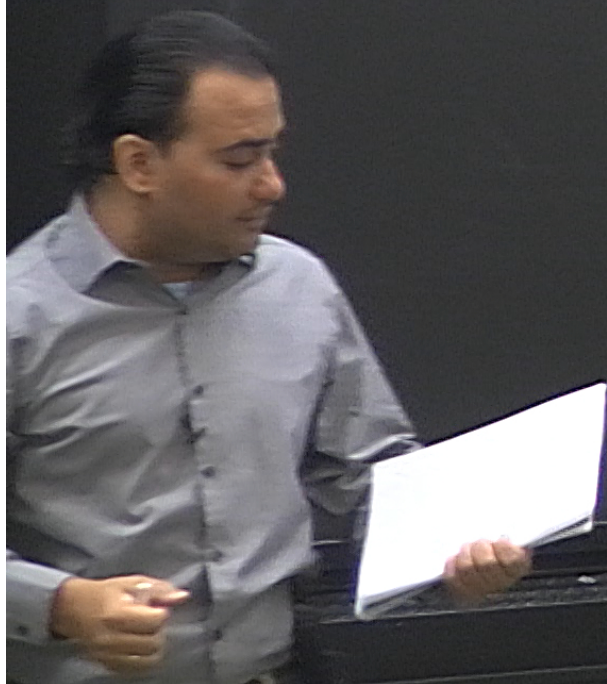
$$V_{\text{int}} = \sum_{\nu\nu', \mu\mu'} V_{\nu\mu}^{\nu'\mu'} \underbrace{a_{\nu}^{\dagger} a_{\nu'}}_{d_{\nu\nu'} + \langle a_{\nu}^{\dagger} a_{\nu'} \rangle} b_{\mu}^{\dagger} b_{\mu'}$$

$$= \sum_{\nu\nu', \mu\mu'} V_{\nu\mu}^{\nu'\mu'} \left(d_{\nu\nu'} \langle b_{\mu}^{\dagger} b_{\mu'} \rangle + e_{\mu\mu'} \langle a_{\nu}^{\dagger} a_{\nu'} \rangle + d_{\nu\nu'} e_{\mu\mu'} + \langle a_{\nu}^{\dagger} a_{\nu'} \rangle \langle b_{\mu}^{\dagger} b_{\mu'} \rangle \right)$$

$$\sum_{\nu\nu', \mu\mu'} V_{\nu\mu}^{\nu'\mu'} d_{\nu\nu'} e_{\mu\mu'} = 0$$



$$\langle a_\nu^\dagger a_{\nu'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_\nu^\dagger a_{\nu'})$$



$$\langle a_\nu^\dagger a_{\nu'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_\nu^\dagger a_{\nu'})$$

guess $\langle a_\nu^\dagger a_\nu \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

Calculate H
 // $\langle a_\nu^\dagger a_\nu \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

$$\langle a_\nu^\dagger a_{\nu'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_\nu^\dagger a_{\nu'})$$

guess $\langle a_\nu^\dagger a_\nu \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

Calculate H

// $\langle a_\nu^\dagger a_\nu \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

Calculate new H

I $\langle a_\nu^\dagger a_\nu \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

$$\langle a_r^\dagger a_{r'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_r^\dagger a_{r'})$$

guess $\langle a_r^\dagger a_r \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

Calculate H

// $\langle a_r^\dagger a_r \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

Calculate new H

I $\langle a_r^\dagger a_r \rangle$ & $\langle b_\mu^\dagger b_{\mu'} \rangle$

$$H = H_{MF}$$

$$\langle a_r^\dagger \rangle$$

$$\langle a_{\nu}^{\dagger} a_{\nu'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_{\nu}^{\dagger} a_{\nu'})$$

guess $\langle a_{\nu}^{\dagger} a_{\nu'} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

Calculate H

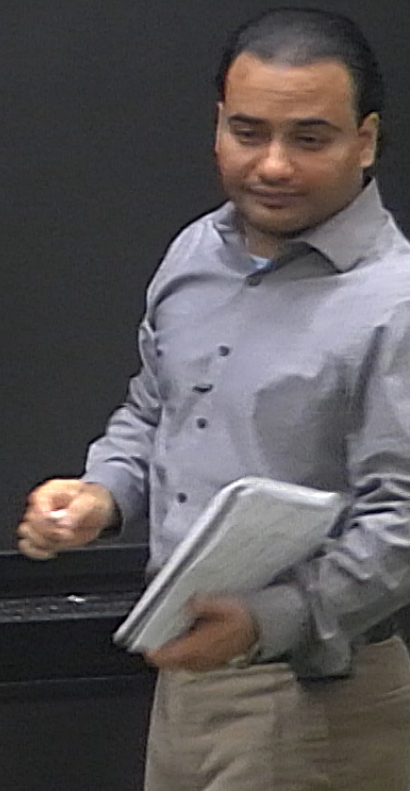
" $\langle a_{\nu}^{\dagger} a_{\nu'} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

Calculate new H

" $\langle a_{\nu}^{\dagger} a_{\nu'} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

$$H = H_{MF}$$

$$\langle a_{\nu}^{\dagger} a_{\nu'} \rangle_{MF} = \frac{1}{Z_{MF}} \text{Tr} (e^{-\beta H_{MF}} a_{\nu}^{\dagger} a_{\nu'})$$



$$\langle a_{\nu}^{\dagger} a_{\nu'} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} a_{\nu}^{\dagger} a_{\nu'})$$

guess $\langle a_{\nu}^{\dagger} a_{\nu} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

Calculate H

// $\langle a_{\nu}^{\dagger} a_{\nu} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

Calculate new H

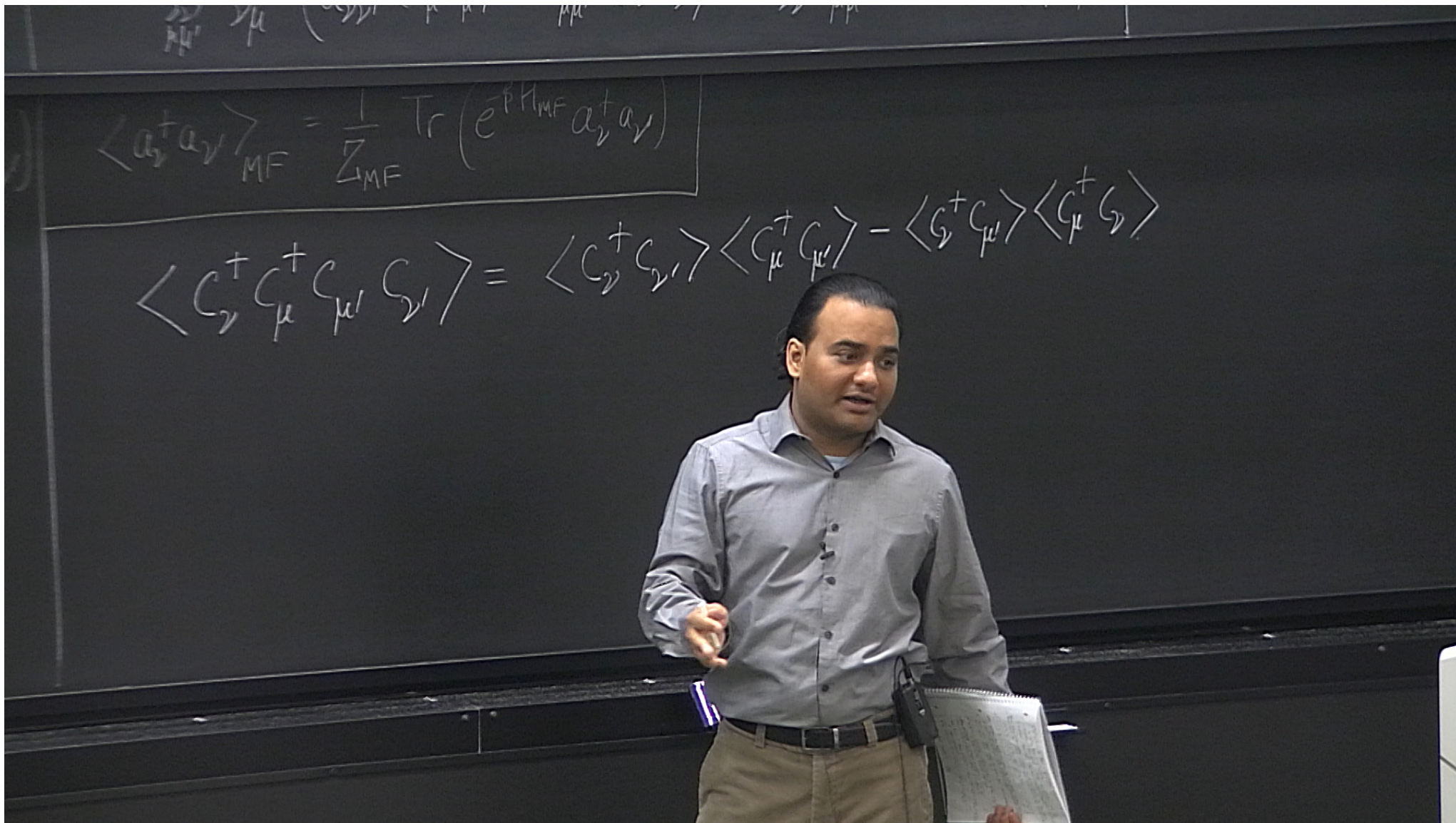
// $\langle a_{\nu}^{\dagger} a_{\nu} \rangle$ & $\langle b_{\mu}^{\dagger} b_{\mu'} \rangle$

$$H = H_{MF}$$

$$\langle a_{\nu}^{\dagger} a_{\nu'} \rangle_{MF} = \frac{1}{Z_{MF}} \text{Tr} (e^{-\beta H_{MF}} a_{\nu}^{\dagger} a_{\nu'})$$

$$\langle c_{\nu}^{\dagger} c_{\mu}^{\dagger} c_{\mu'} c_{\nu'} \rangle =$$



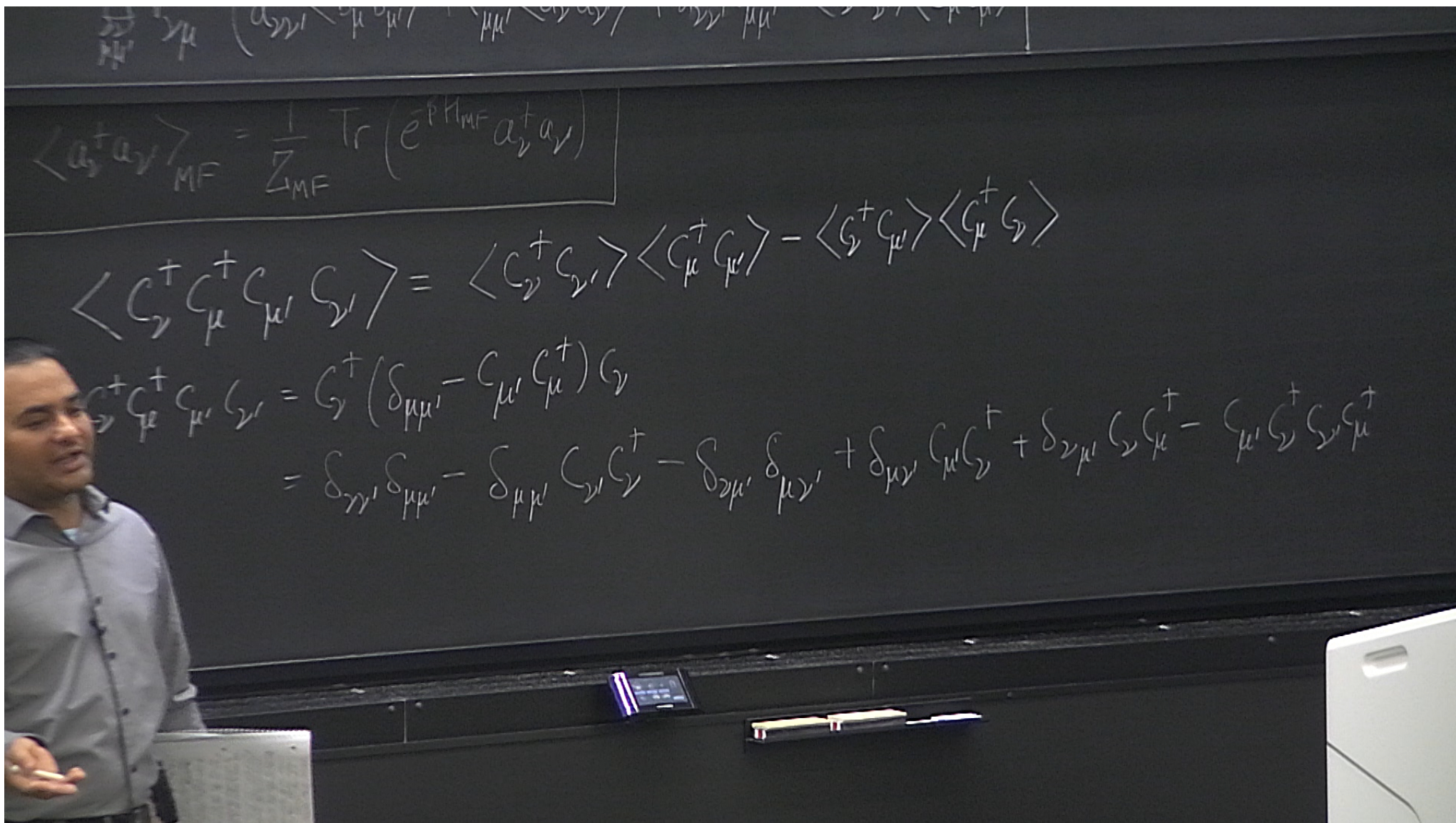


$$\langle a_\nu^\dagger a_\nu \rangle_{MF} = \frac{1}{Z_{MF}} \text{Tr} (e^{-\beta H_{MF}} a_\nu^\dagger a_\nu)$$

$$\langle c_\nu^\dagger c_\mu^\dagger c_{\mu'} c_{\nu'} \rangle = \langle c_\nu^\dagger c_{\nu'} \rangle \langle c_\mu^\dagger c_{\mu'} \rangle - \langle c_\nu^\dagger c_{\mu'} \rangle \langle c_\mu^\dagger c_{\nu'} \rangle$$

$$c_\nu^\dagger c_\mu^\dagger c_{\mu'} c_{\nu'} = c_\nu^\dagger (\delta_{\mu\mu'} - c_{\mu'} c_\mu^\dagger) c_{\nu'}$$

$$= \delta_{\nu\nu'} \delta_{\mu\mu'} - \delta_{\mu\mu'} c_{\nu'} c_\nu^\dagger - \delta_{\nu\mu'} \delta_{\mu\nu'} + \delta_{\mu\nu'} c_{\mu'} c_\nu^\dagger + \delta_{\nu\mu'} c_{\mu'} c_\nu^\dagger$$



$$\langle a_{\nu}^{\dagger} a_{\nu} \rangle_{MF} = \frac{1}{Z_{MF}} \text{Tr} (e^{-\beta H_{MF}} a_{\nu}^{\dagger} a_{\nu})$$

$$\langle c_{\nu}^{\dagger} c_{\mu}^{\dagger} c_{\mu'} c_{\nu'} \rangle = \langle c_{\nu}^{\dagger} c_{\nu'} \rangle \langle c_{\mu}^{\dagger} c_{\mu'} \rangle - \langle c_{\nu}^{\dagger} c_{\mu'} \rangle \langle c_{\mu}^{\dagger} c_{\nu} \rangle$$

$$c_{\nu}^{\dagger} c_{\mu}^{\dagger} c_{\mu'} c_{\nu'} = c_{\nu}^{\dagger} (\delta_{\mu\mu'} - c_{\mu'} c_{\mu}^{\dagger}) c_{\nu'}$$

$$= \delta_{\nu\nu'} \delta_{\mu\mu'} - \delta_{\mu\mu'} c_{\nu'} c_{\nu}^{\dagger} - \delta_{\nu\mu'} \delta_{\mu\nu'} + \delta_{\mu\nu'} c_{\mu'} c_{\nu}^{\dagger} + \delta_{\nu\mu'} c_{\nu} c_{\mu}^{\dagger} - c_{\mu'} c_{\nu}^{\dagger} c_{\nu'} c_{\mu}^{\dagger}$$

$$\langle c_{\nu}^{\dagger} c_{\nu'} \rangle \langle c_{\mu}^{\dagger} c_{\mu'} \rangle$$