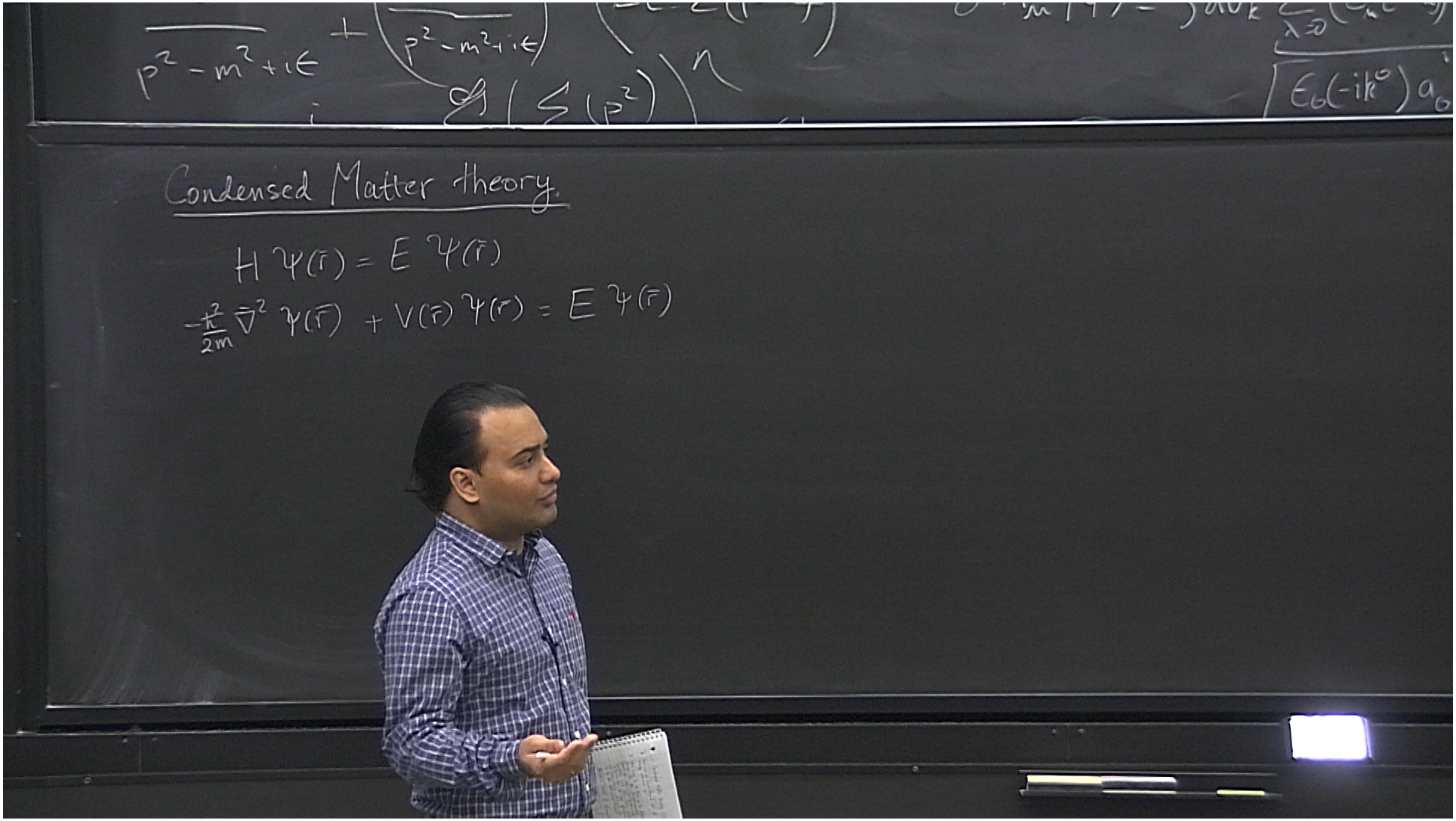
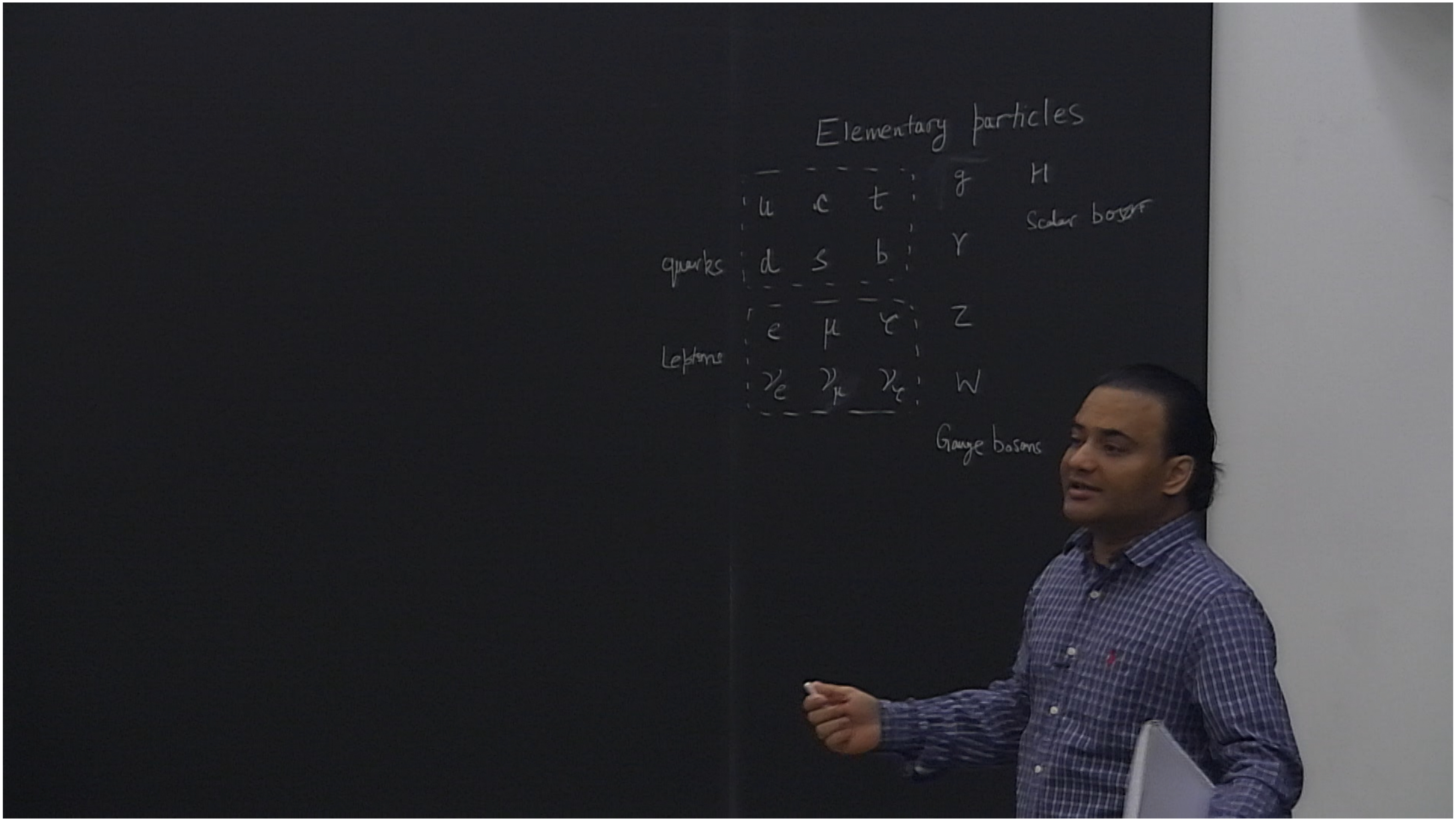
Title: PSI 2018/2019 - Condensed Matter - Lecture 1

Date: Nov 12, 2018 10:45 AM

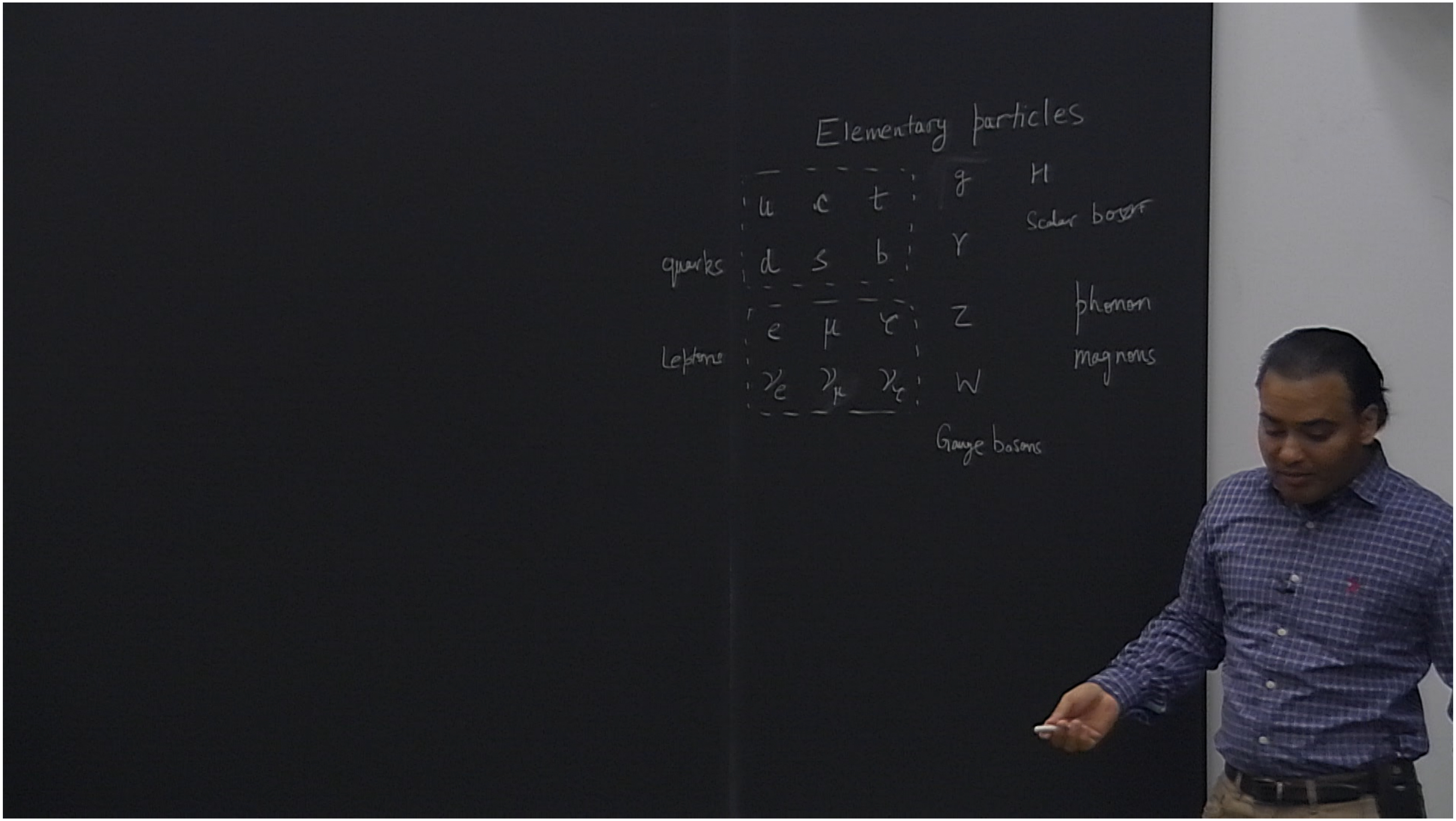
URL: http://pirsa.org/18110019

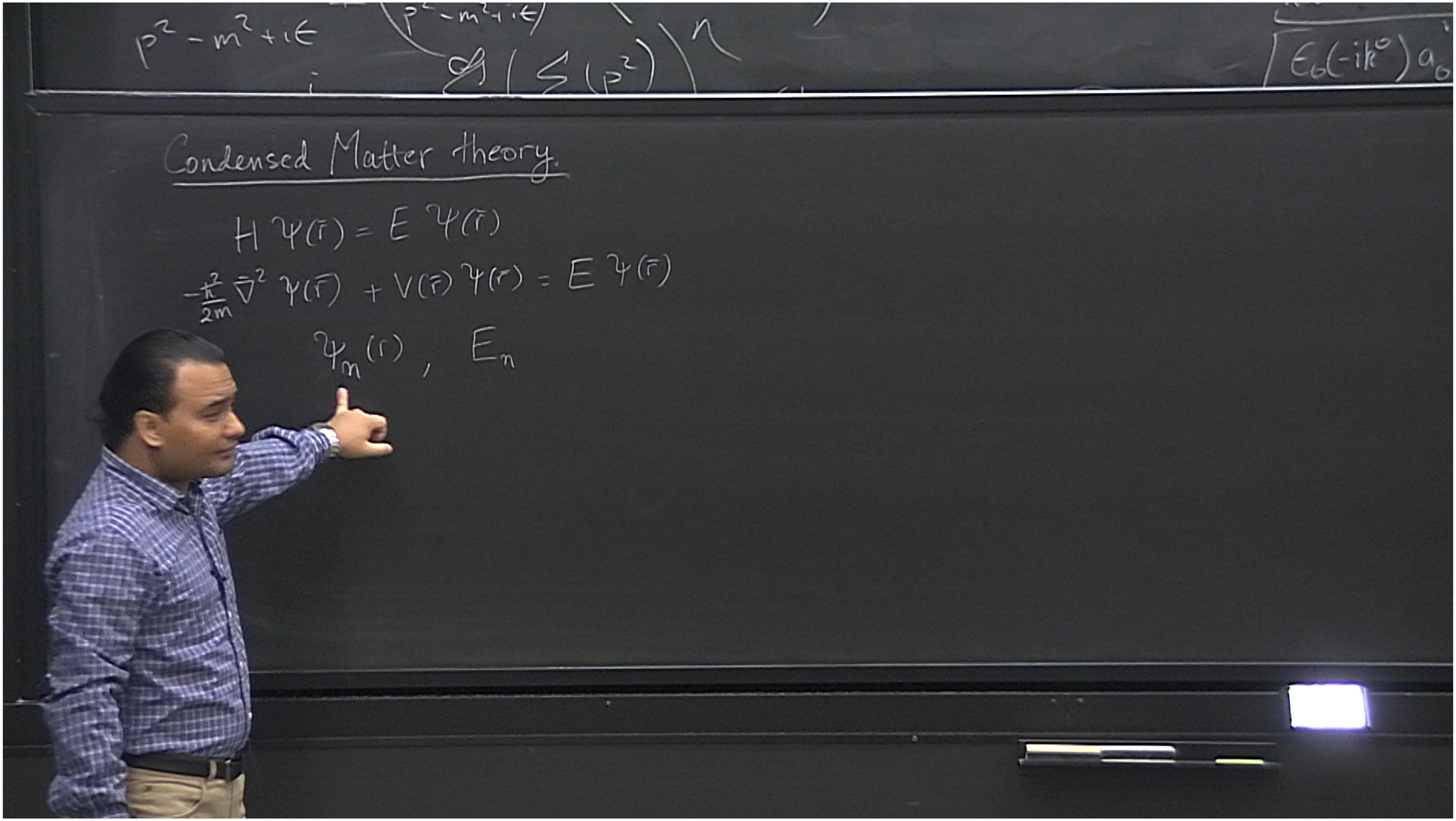
Abstract:

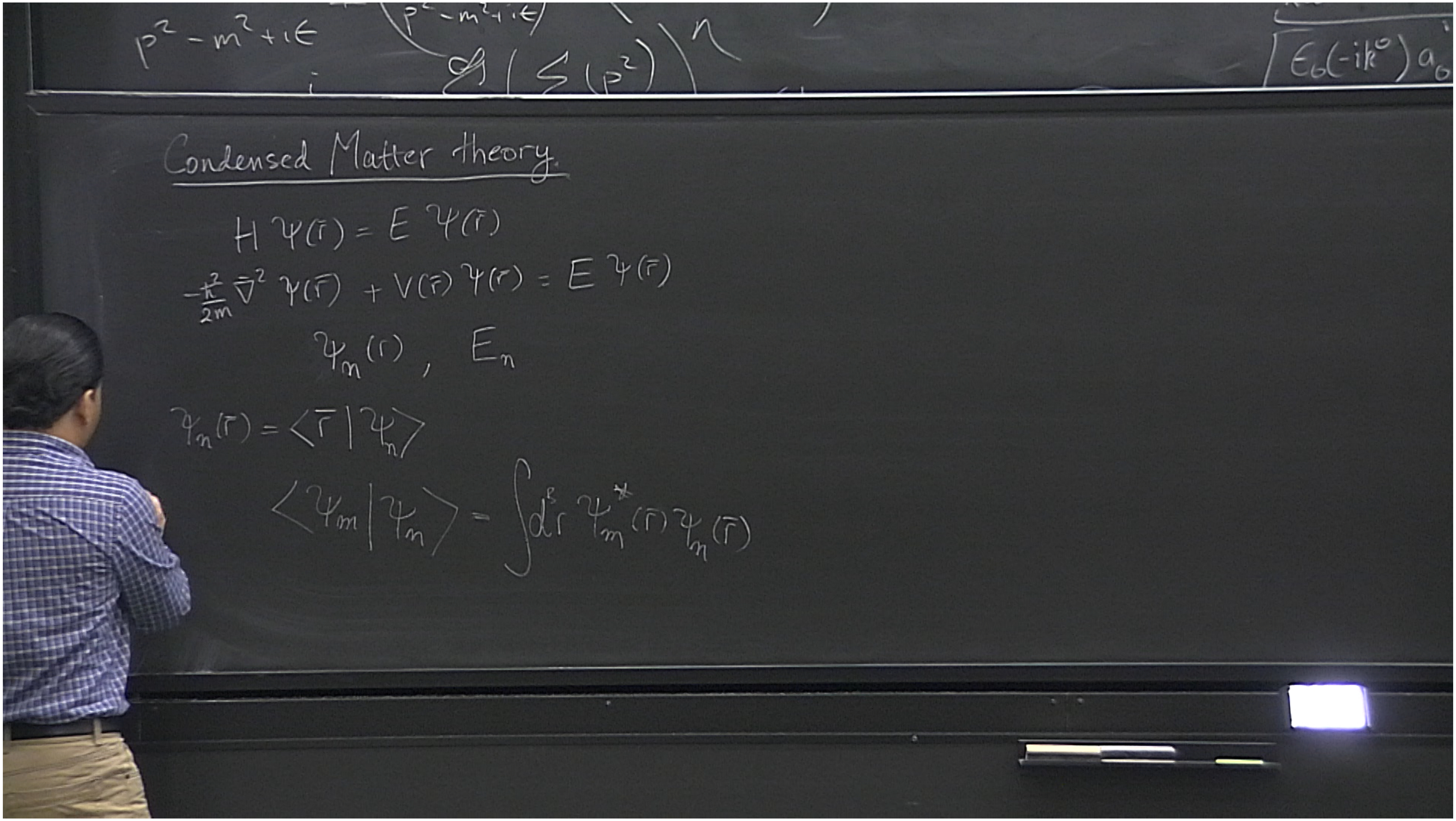

Condensed Matter theory.
H ψ(r̄) = E ψ(r̄)
-ħ²/2m ∇̄² ψ(r̄) + V(r̄) ψ(r̄) = E ψ(r̄)



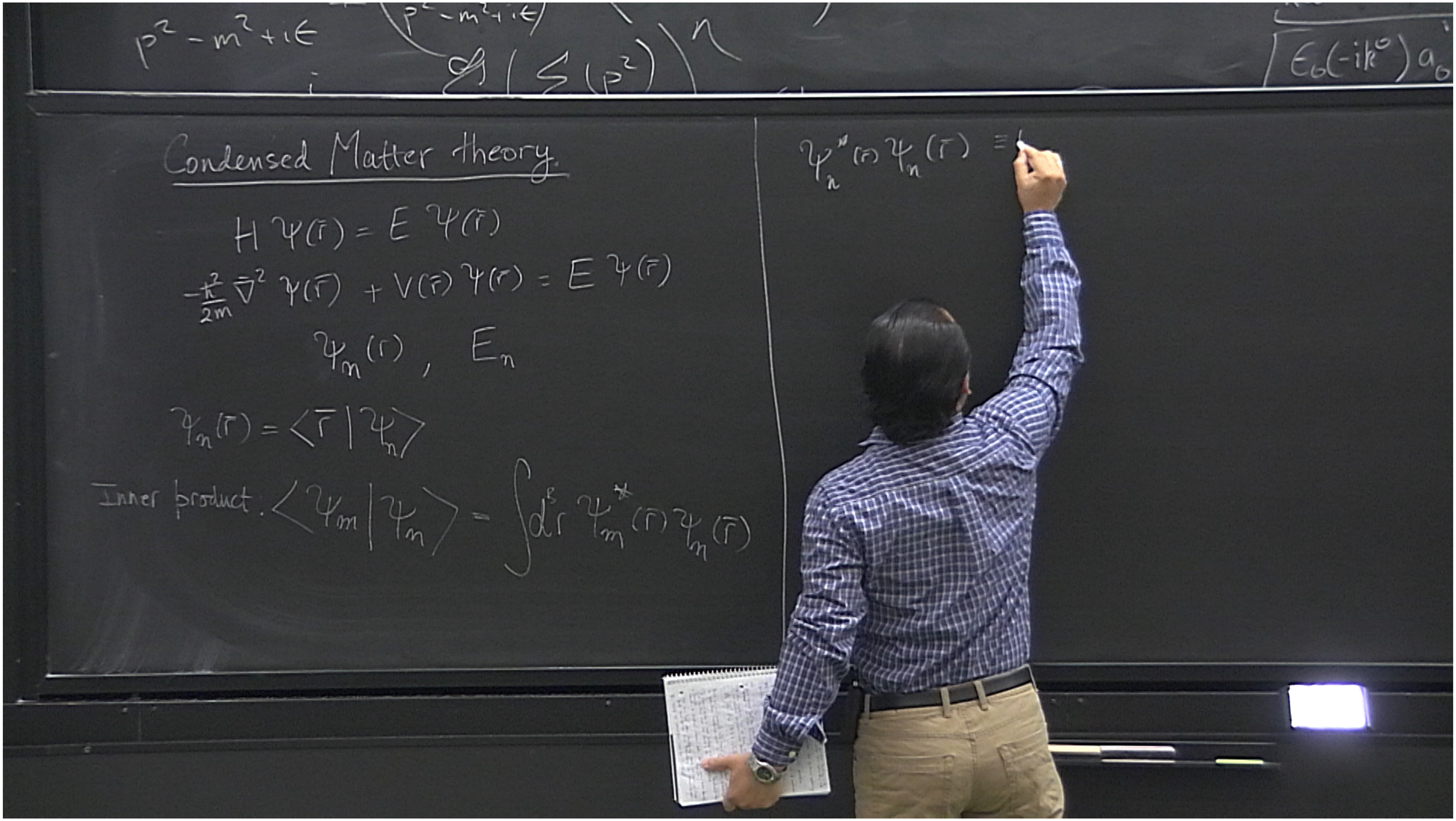
Elementary particles
u c t
d s b
quarks
e μ τ
ν_e ν_μ ν_τ
Leptons
g
γ
Z
W
Gauge bosons
H
Scalar boson

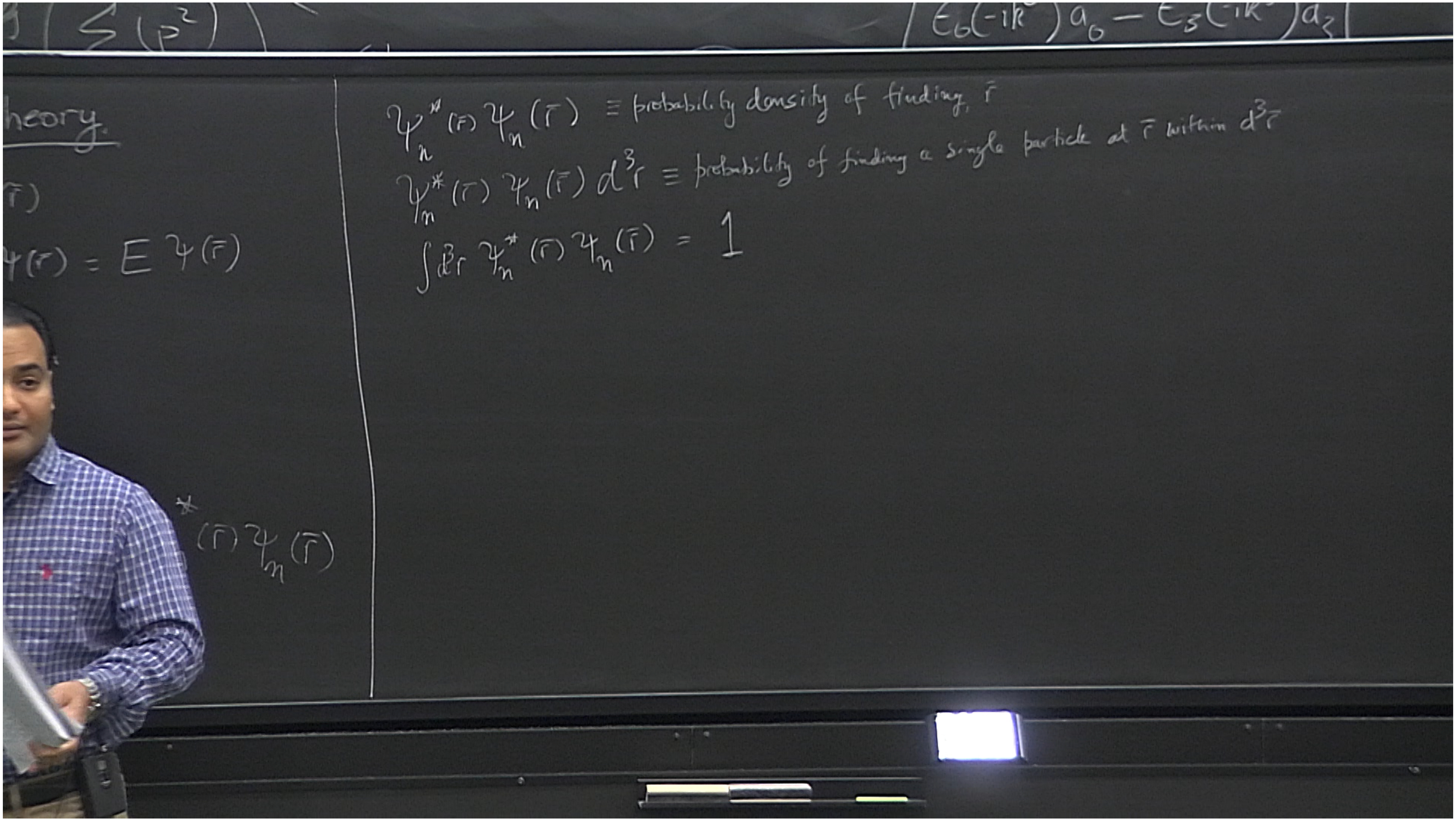

Elementary particles
u c t
d s b
quarks
e μ τ
ν_e ν_μ ν_τ
Leptons
g
γ
Z
W
Gauge bosons
H
Scalar boson
phonon
magnons

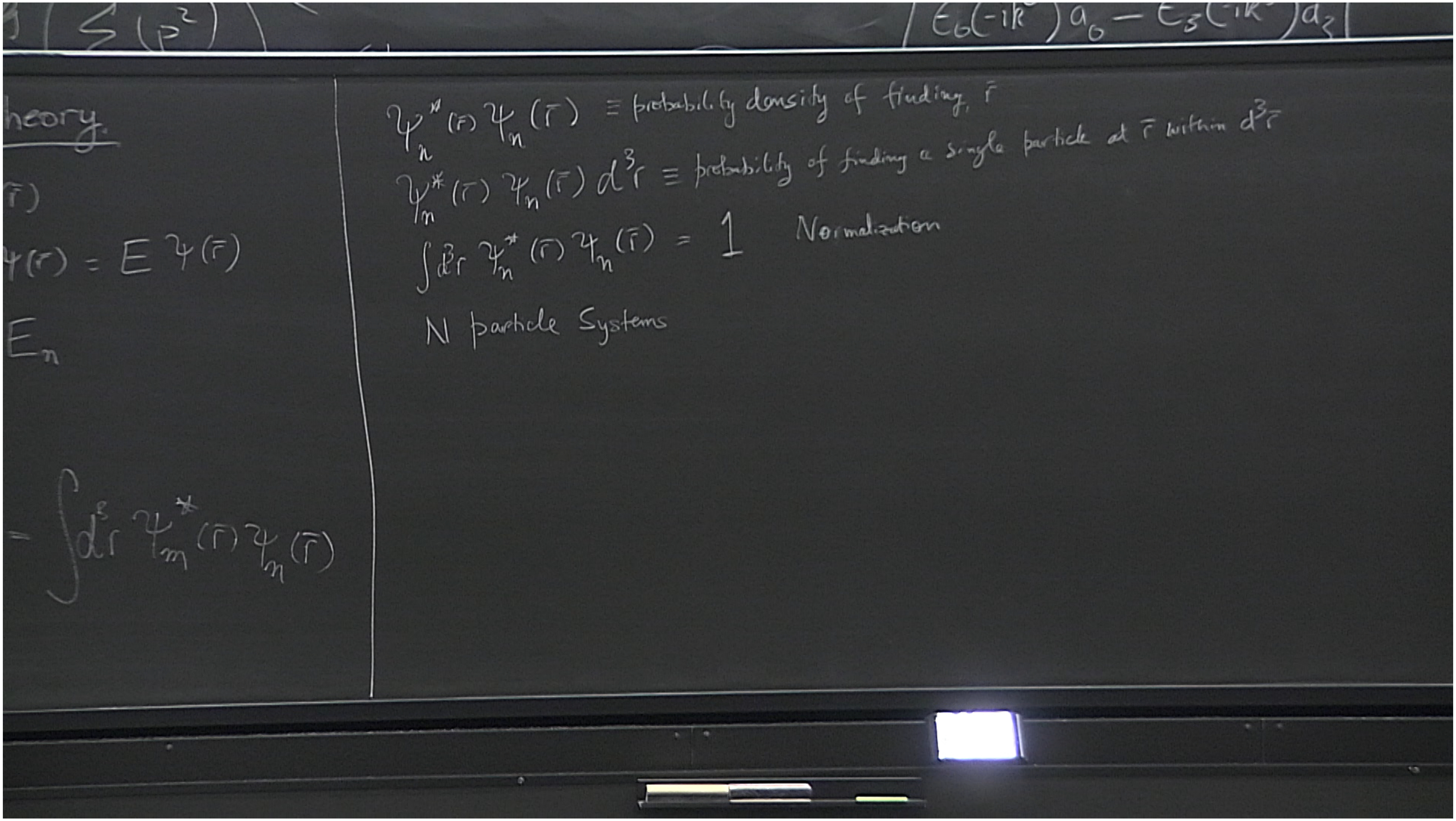

Condensed Matter theory
H Ψ(r) = E Ψ(r)
-ħ²/2m ∇² ψ(r) + V(r) ψ(r) = E ψ(r)
ψ_n(r) , E_n

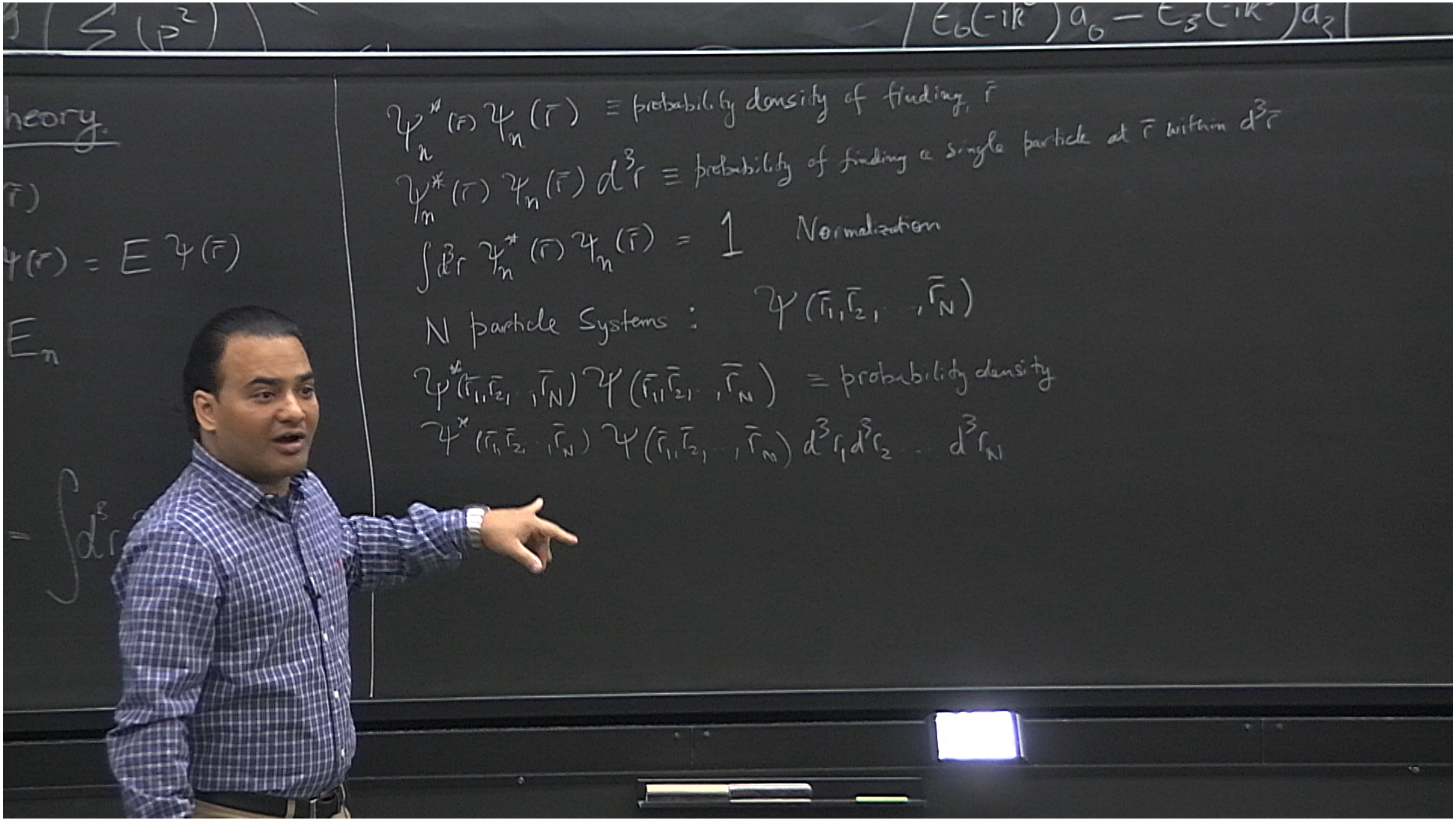


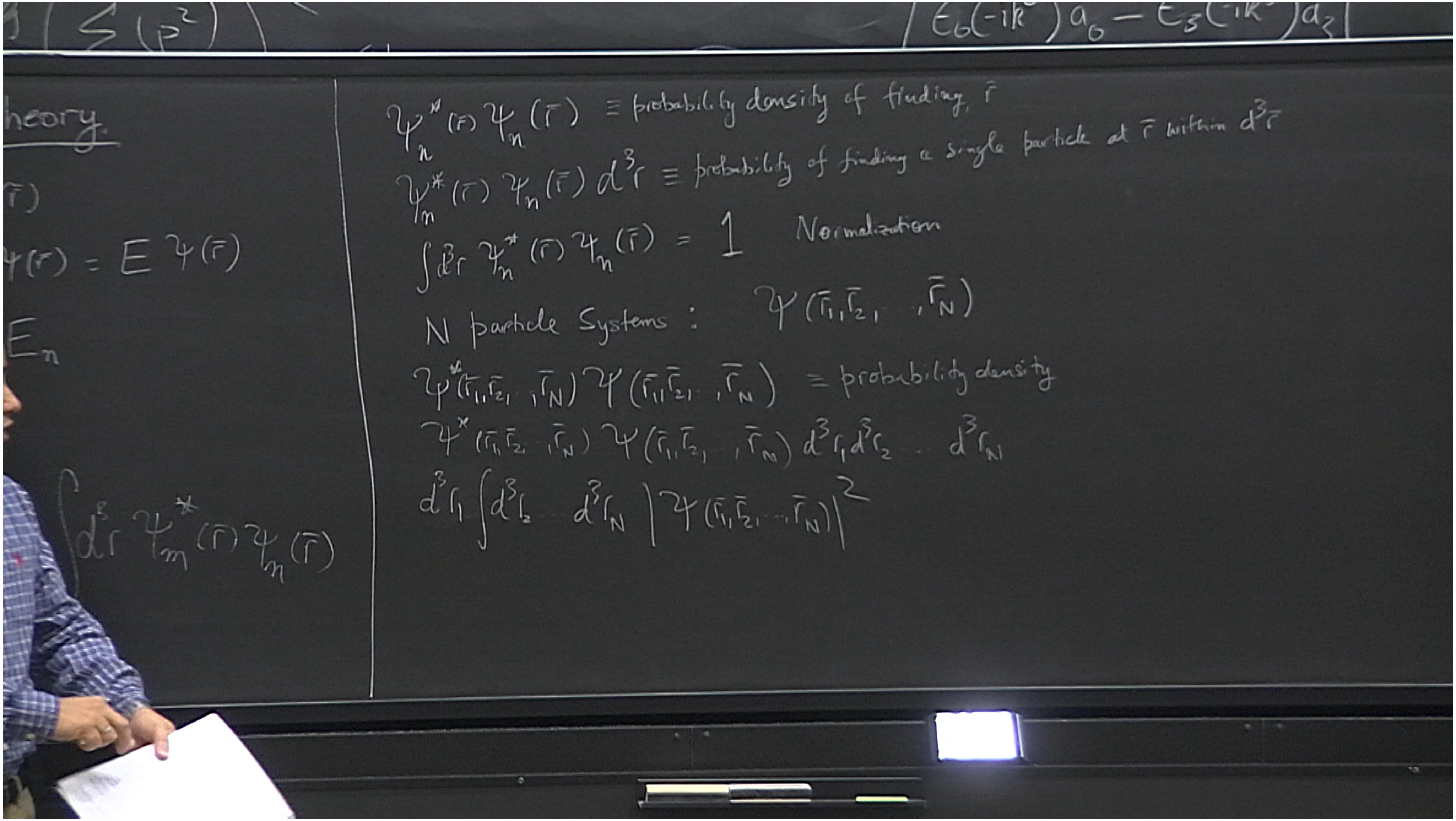
Condensed Matter theory
$H\,\psi(\bar r) = E\,\psi(\bar r)$
$-\frac{\hbar^2}{2m}\bar\nabla^2\psi(\bar r) + V(\bar r)\psi(\bar r) = E\,\psi(\bar r)$
$\psi_n(\bar r)\,,\quad E_n$
$\psi_n(\bar r) = \langle \bar r | \psi_n \rangle$
$\langle \psi_m | \psi_n \rangle = \int d^3r\, \psi_m^*(\bar r)\,\psi_n(\bar r)$


Condensed Matter theory.
H Ψ(r̄) = E Ψ(r̄)
-ħ²/2m ∇̄² Ψ(r̄) + V(r̄) Ψ(r̄) = E Ψ(r̄)
Ψn(r̄) , En
Ψn(r̄) = ⟨r̄ | Ψn⟩
Inner product: ⟨Ψm | Ψn⟩ = ∫d³r Ψm*(r̄) Ψn(r̄)
Ψn*(r̄) Ψn(r̄) ≡

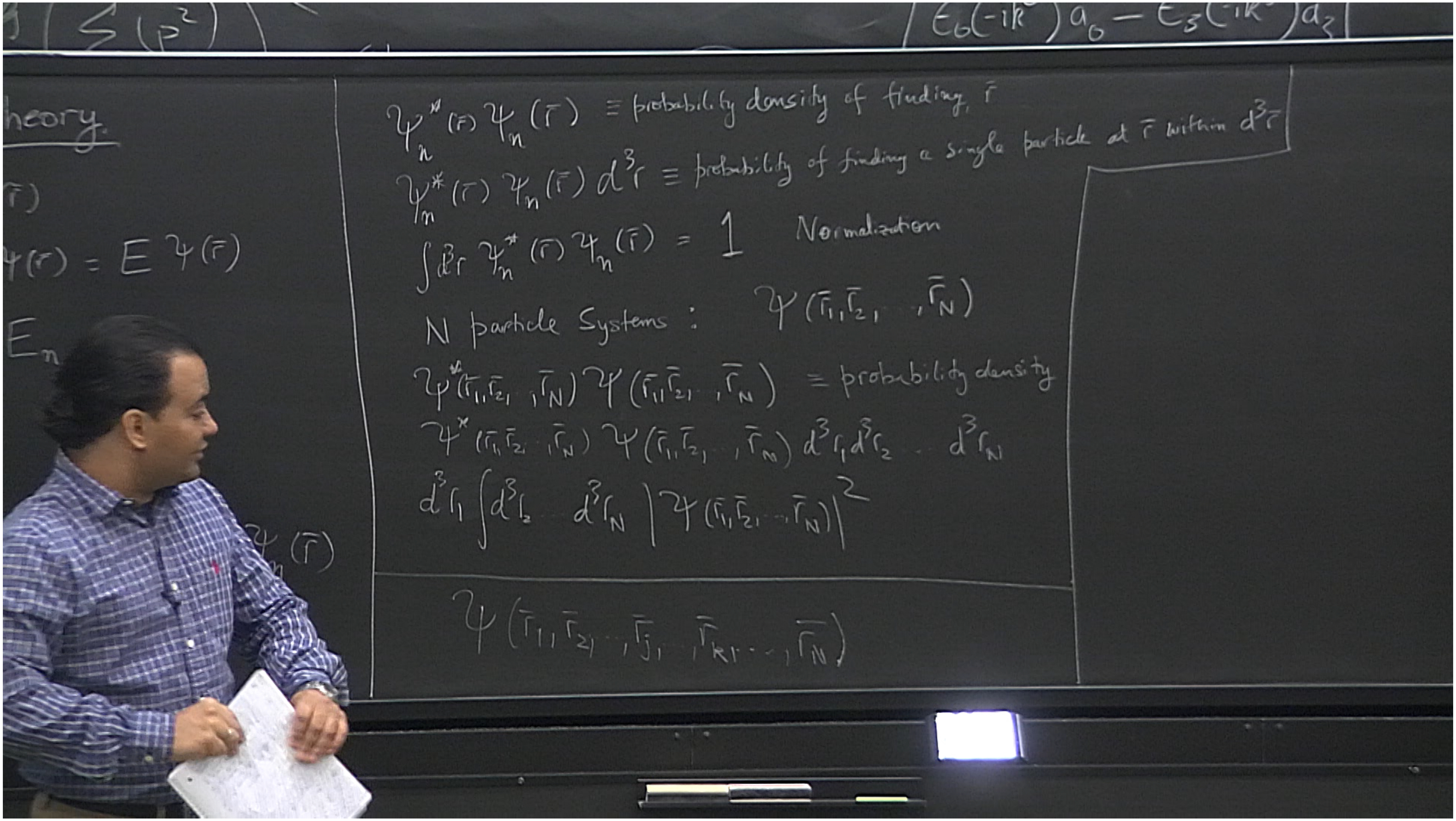

ψ_n^*(r̄) ψ_n(r̄) ≡ probability density of finding r̄
ψ_n^*(r̄) ψ_n(r̄) d^3r ≡ probability of finding a single particle at r̄ within d^3r̄
∫d^3r ψ_n^*(r̄) ψ_n(r̄) = 1

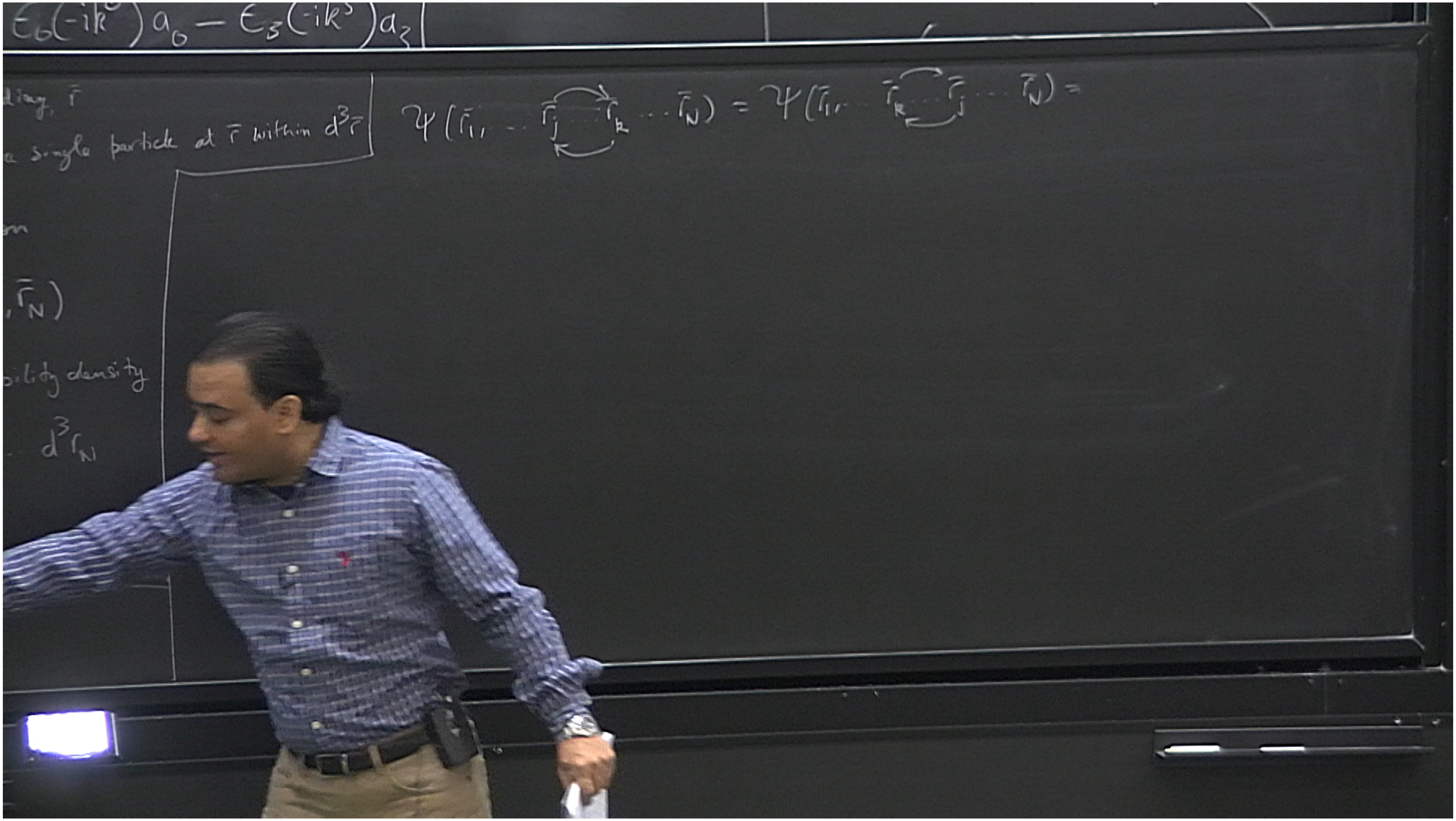

$\psi_n^*(\bar r)\,\psi_n(\bar r) \equiv$ probability density of finding $\bar r$
$\psi_n^*(\bar r)\,\psi_n(\bar r)\, d^3 r \equiv$ probability of finding a single particle at $\bar r$ within $d^3\bar r$
$\int d^3 r\, \psi_n^*(\bar r)\,\psi_n(\bar r) = 1$ Normalization
N particle Systems
heory
$= E\,\psi(\bar r)$
E_n
$= \int d^3 r\, \psi_m^*(\bar r)\,\psi_n(\bar r)$

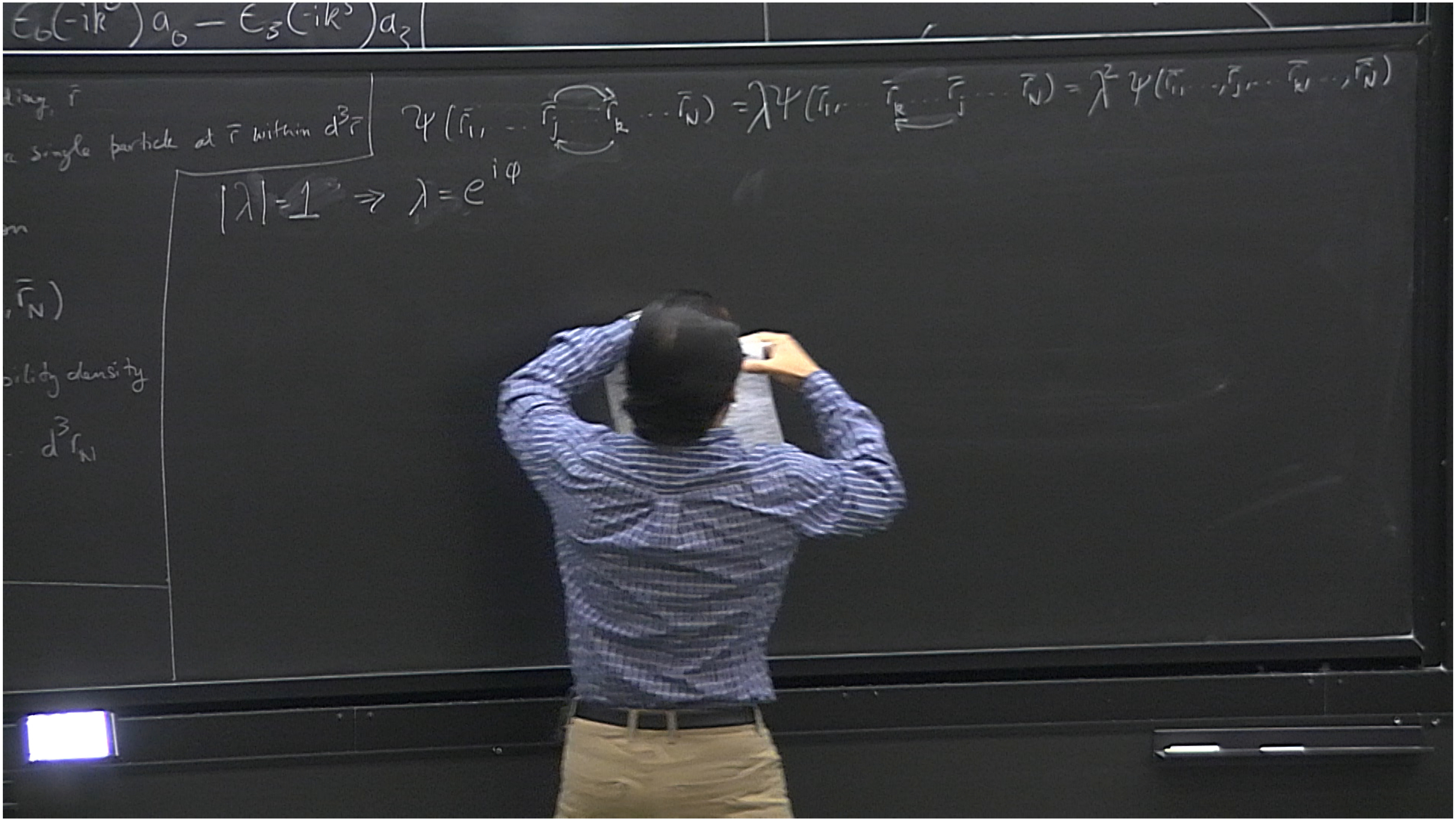

ψ*n(r̄) ψn(r̄) ≡ probability density of finding r̄
probability of finding a single particle at r̄ within d³r̄
ψ*n(r̄) ψn(r̄) d³r ≡ probability of finding a single particle at r̄ within d³r̄
∫d³r ψ*n(r̄) ψn(r̄) = 1 Normalization
N particle Systems : ψ(r̄1, r̄2, …, r̄N)
ψ*(r̄1, r̄2, …, r̄N) ψ(r̄1, r̄2, …, r̄N) ≡ probability density
ψ*(r̄1, r̄2, …, r̄N) ψ(r̄1, r̄2, …, r̄N) d³r1 d³r2 … d³rN
theory
ψ(r̄) = E ψ(r̄)
En

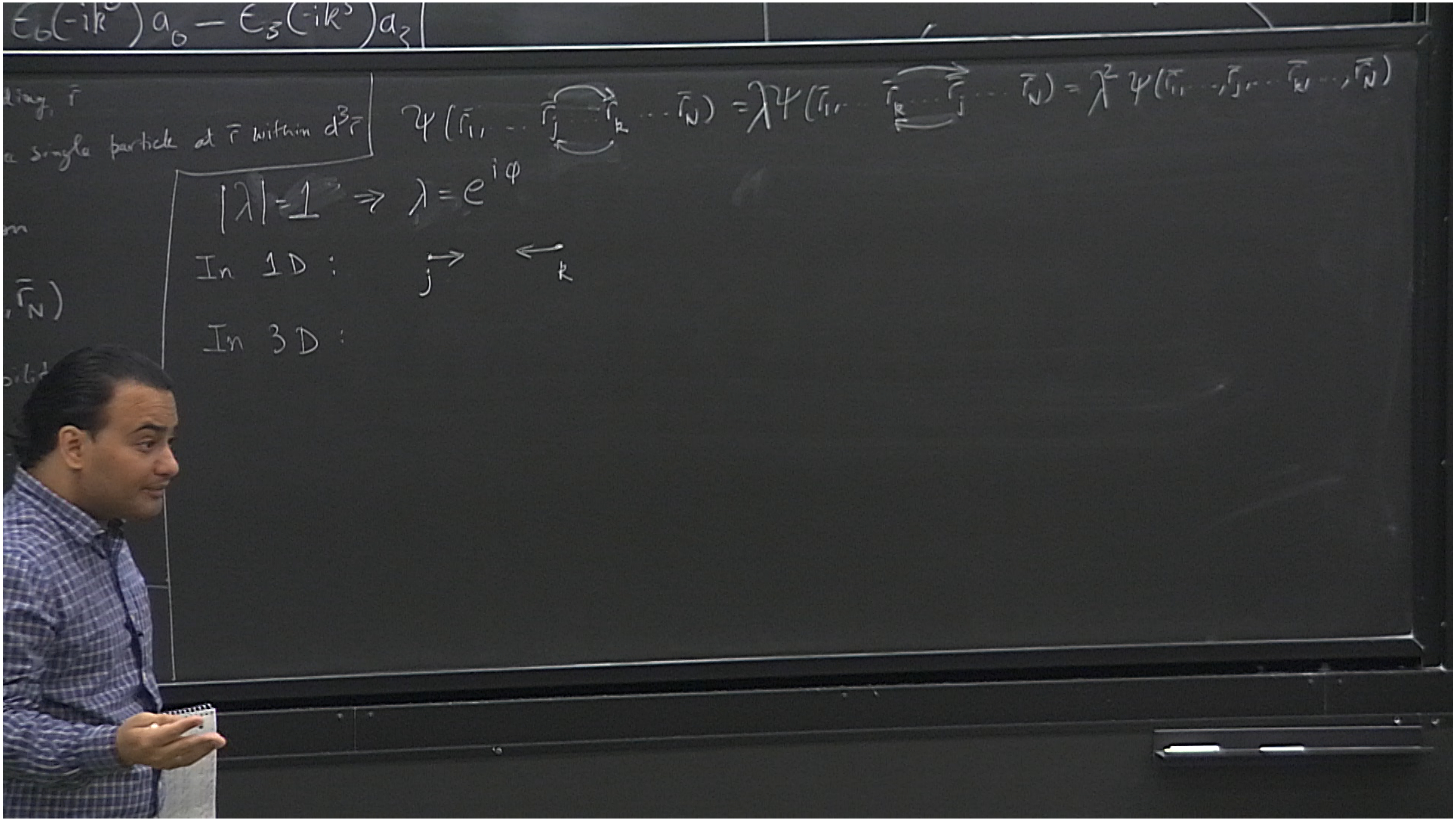


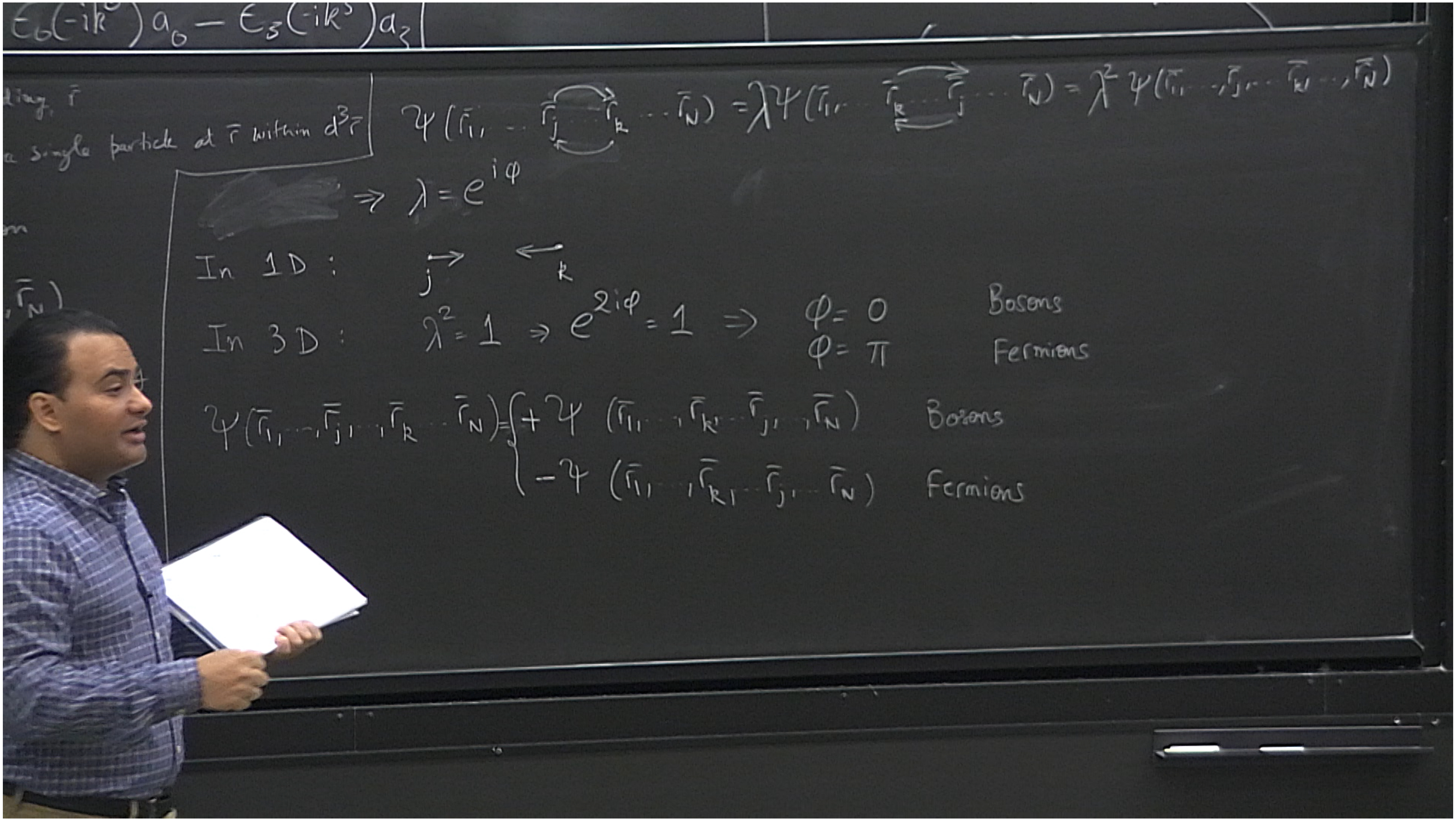
$\psi_n^*(\bar r)\psi_n(\bar r)$ ≡ probability density of finding $\bar r$
$\psi_n^*(\bar r)\psi_n(\bar r)\,d^3r$ ≡ probability of finding a single particle at $\bar r$ within $d^3\bar r$
$\int d^3r\, \psi_n^*(\bar r)\psi_n(\bar r) = 1$ Normalization
N particle Systems : $\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)$
$\psi^*(\bar r_1, \bar r_2, \ldots, \bar r_N)\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)$ = probability density
$\psi^*(\bar r_1, \bar r_2, \ldots, \bar r_N)\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)\, d^3r_1 d^3r_2 \ldots d^3r_N$
$d^3r_1 \int d^3r_2 \ldots d^3r_N\, |\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)|^2$
heory
$\psi(\bar r) = E\,\psi(\bar r)$
E_n
$\int d^3r\, \psi_m^*(\bar r)\psi_n(\bar r)$

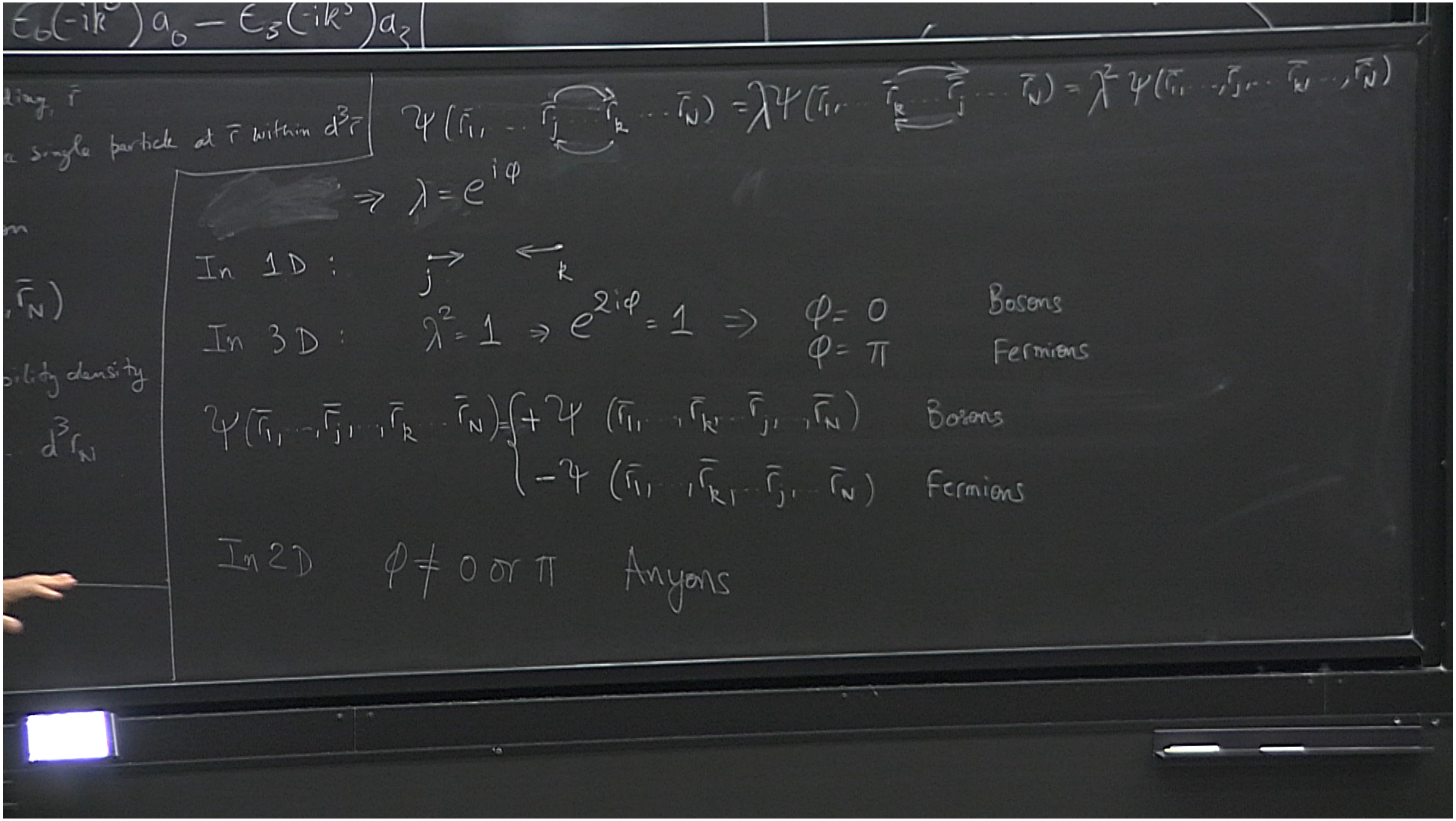

$\psi_n^*(\bar r)\psi_n(\bar r) \equiv$ probability density of finding $\bar r$
particle at $\bar r$ within $d^3\bar r$
$\psi_n^*(\bar r)\psi_n(\bar r)\, d^3 r \equiv$ probability of finding a single particle at $\bar r$ within $d^3\bar r$
$\int d^3r\, \psi_n^*(\bar r)\psi_n(\bar r) = 1$ Normalization
N particle Systems : $\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)$
$\psi^*(\bar r_1, \bar r_2, \ldots, \bar r_N)\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)$ = probability density
$\psi^*(\bar r_1, \bar r_2, \ldots, \bar r_N)\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)\, d^3r_1 d^3r_2 \ldots d^3r_N$
$d^3r_1 \int d^3r_2 \ldots d^3r_N\, |\psi(\bar r_1, \bar r_2, \ldots, \bar r_N)|^2$
$\psi(\bar r_1, \bar r_2, \ldots, \bar r_j, \ldots, \bar r_k, \ldots, \bar r_N)$

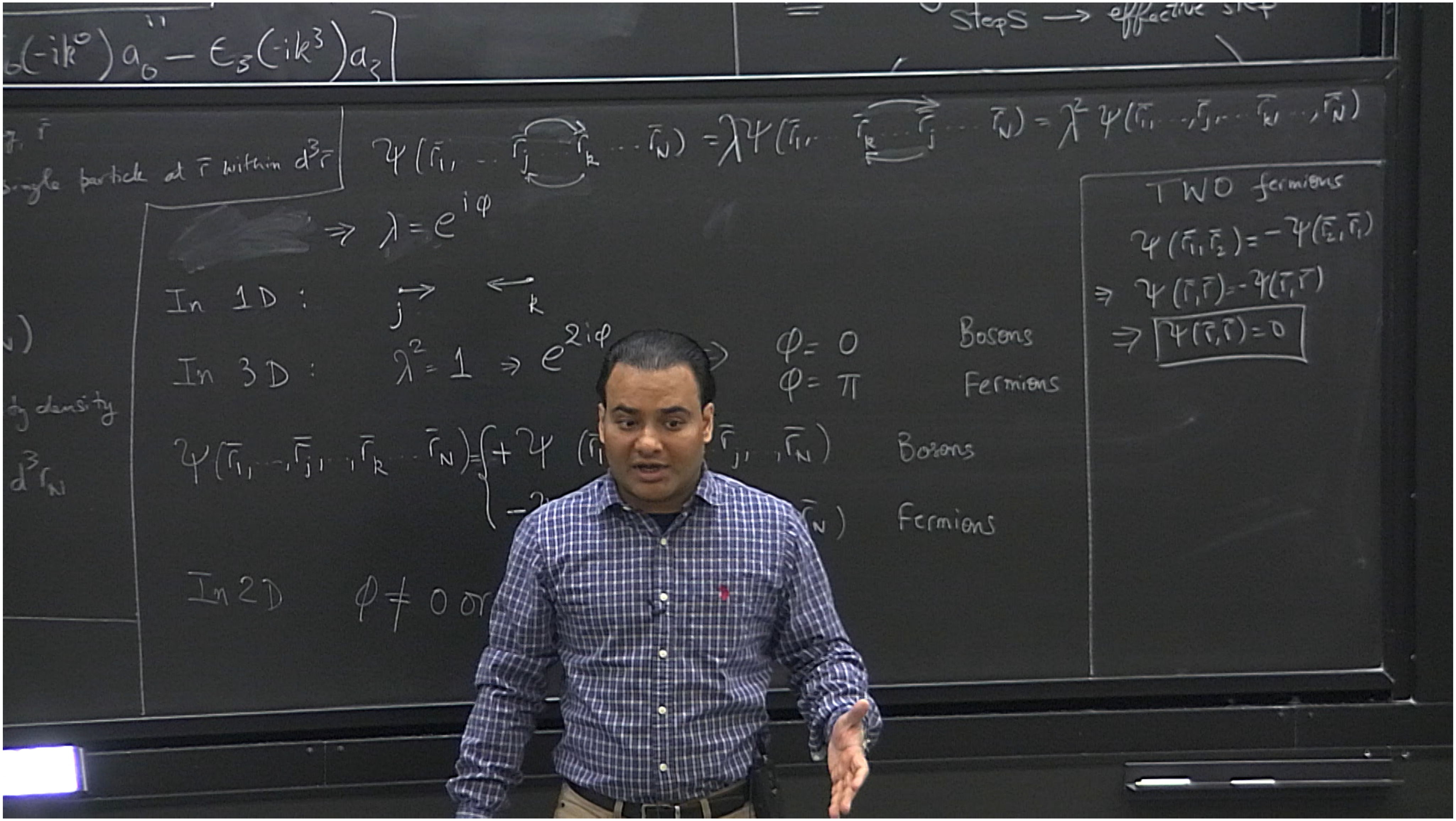

a single particle at r̄ within d³r̄
probability density

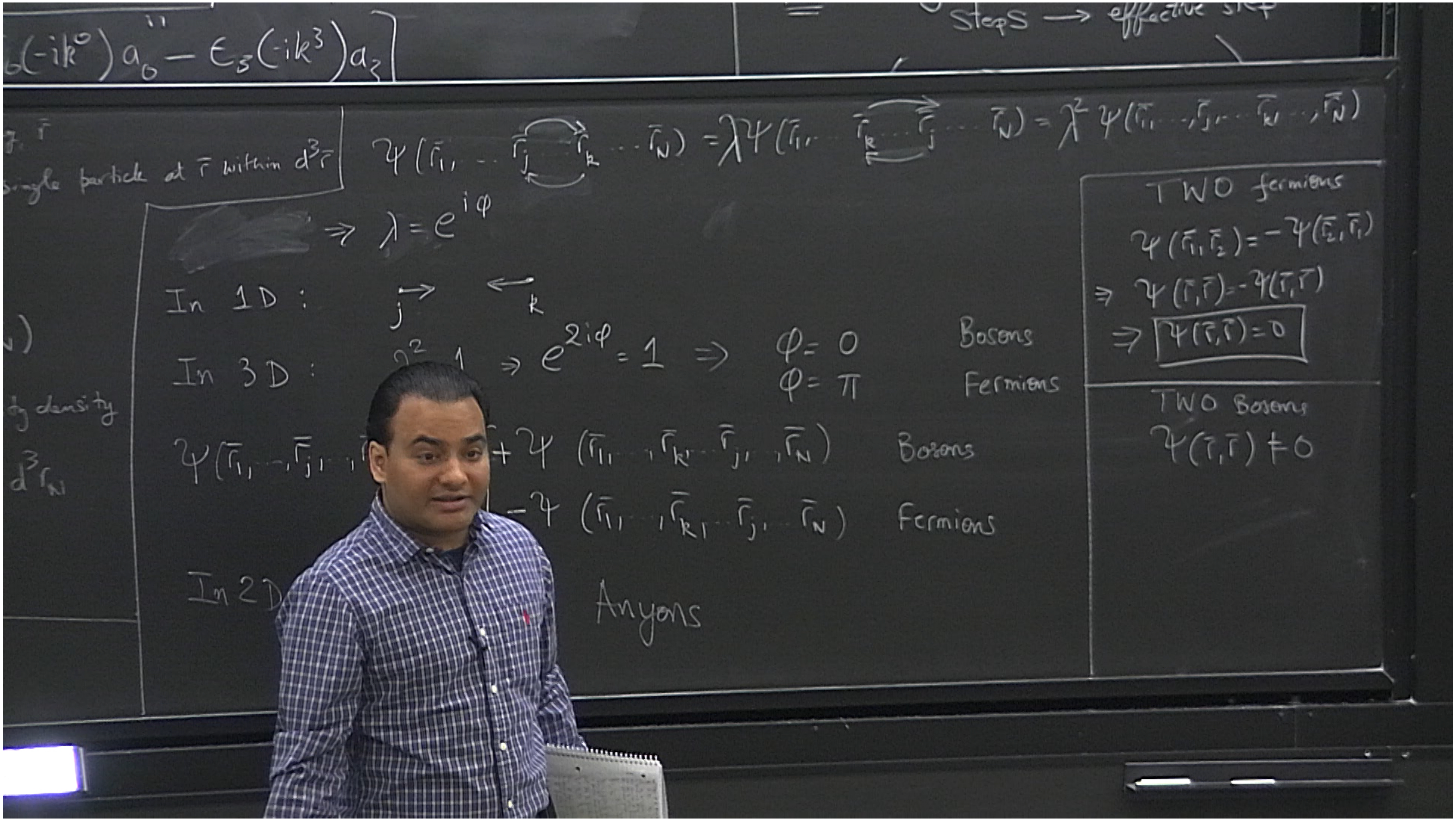

single particle at r within d³r
|λ|=1 ⇒ λ=e^{iφ}
density

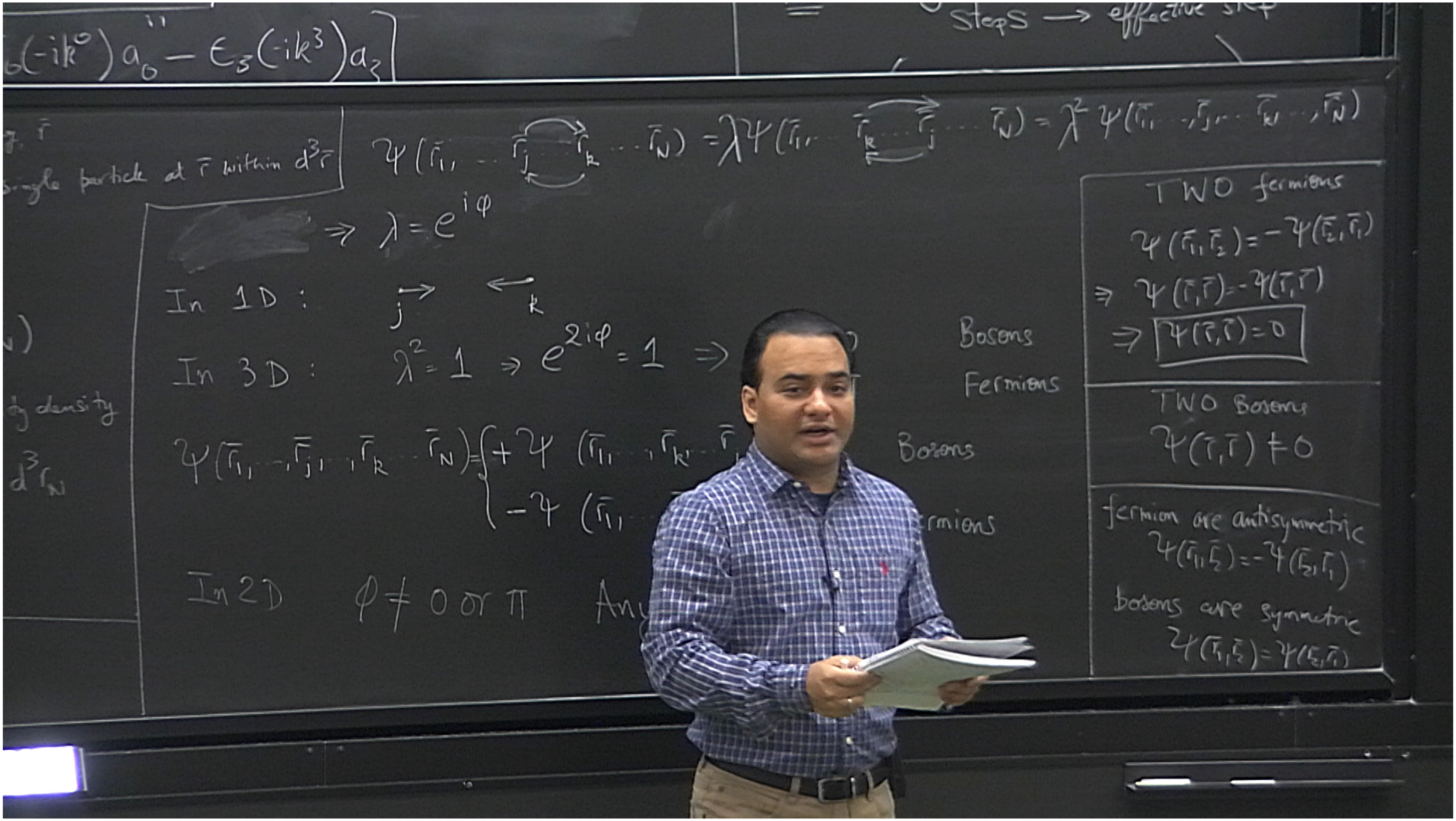

single particle at r within d3r
In 1D :
In 3D :

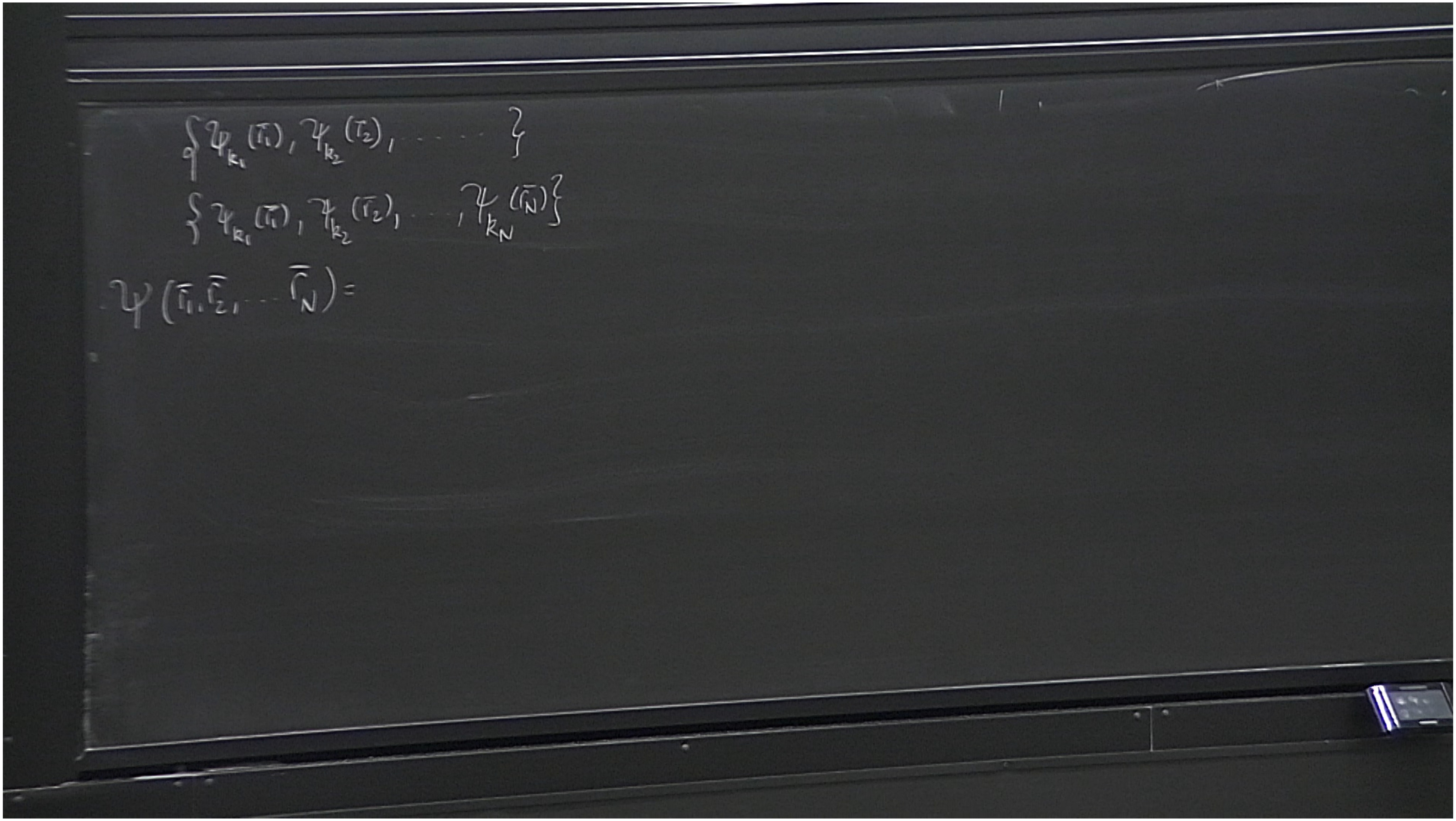

ε₀(-ik⁰)a₀ − ε₃(-ik³)a₃
a single particle at r̄ within d³r̄
Ψ(r̄₁, … r̄_j … r̄_k … r̄_N) = λΨ(r̄₁, … r̄_k … r̄_j … r̄_N) = λ²Ψ(r̄₁, …, r̄_j, … r̄_k, …, r̄_N)
⇒ λ = e^{iφ}
In 1D :
In 3D : λ² = 1 ⇒ e^{2iφ} = 1 ⇒ φ = 0 Bosons
φ = π Fermions
Ψ(r̄₁, …, r̄_j, …, r̄_k … r̄_N) = +Ψ(r̄₁, …, r̄_k, … r̄_j, …, r̄_N) Bosons
−Ψ(r̄₁, …, r̄_k, … r̄_j, … r̄_N) Fermions

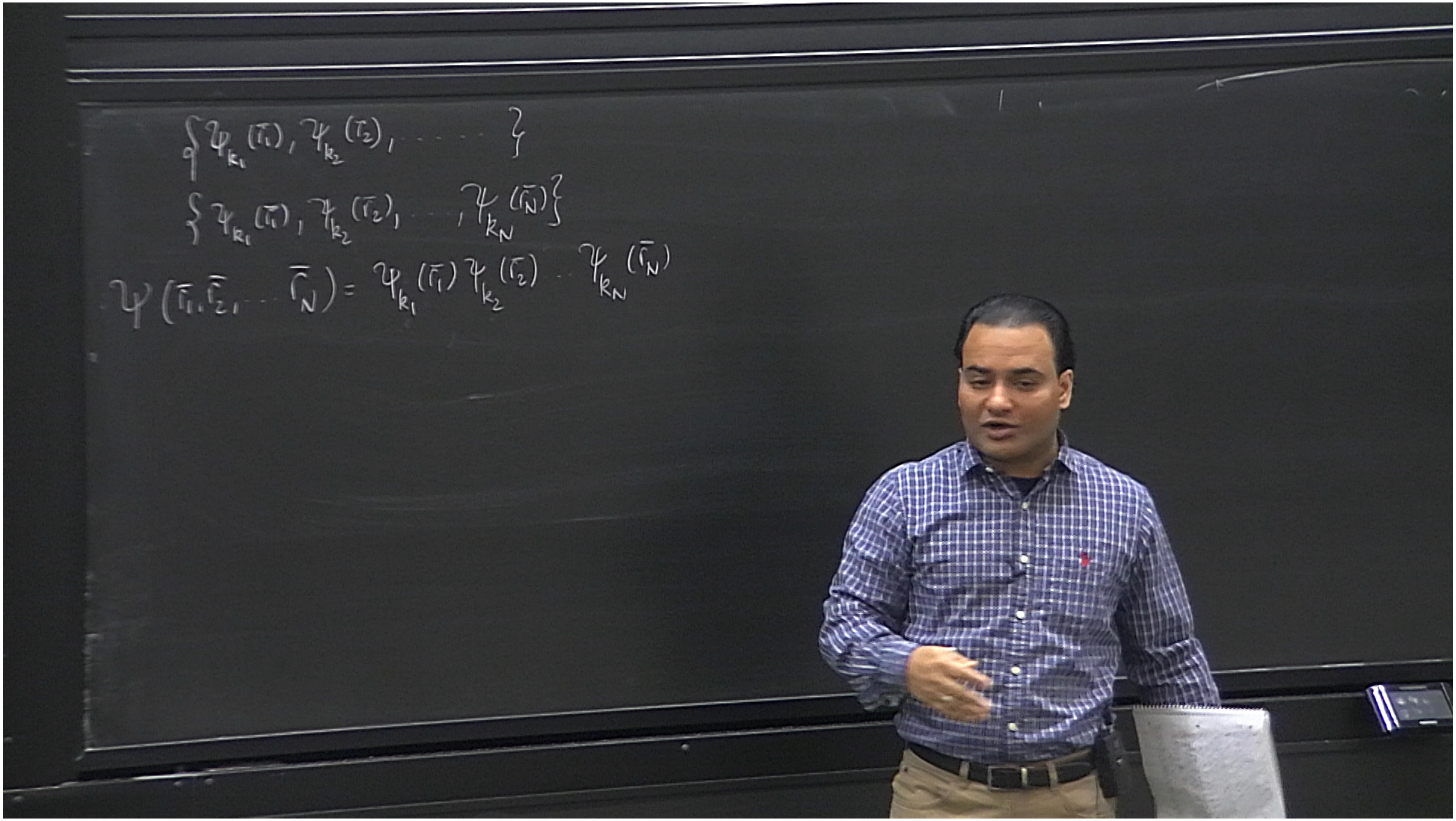

single particle at r̄ within d³r̄
ψ(r̄1, … r̄j … r̄k … r̄N) = λψ(r̄1, … r̄k … r̄j … r̄N) = λ² ψ(r̄1, … r̄j, … r̄k … , r̄N)
⇒ λ = e^{iφ}
In 1D :
In 3D : λ² = 1 ⇒ e^{2iφ} = 1 ⇒ φ = 0 Bosons
φ = π Fermions
ψ(r̄1, …, r̄j, …, r̄k … r̄N) = +ψ (r̄1, …, r̄k, … r̄j, …, r̄N) Bosons
−ψ (r̄1, …, r̄k, … r̄j, … r̄N) Fermions
In 2D φ ≠ 0 or π Anyons
probability density


steps → effective step
single particle at r̄ within d³r̄
ψ(r̄₁, ... r̄ⱼ ... r̄ₖ ... r̄_N) = λψ(r̄₁, ... r̄ₖ ... r̄ⱼ ... r̄_N) = λ² ψ(r̄₁, ..., r̄ⱼ, ... r̄ₖ ..., r̄_N)
⇒ λ = e^{iφ}
In 1D :
In 3D : λ² = 1 ⇒ e^{2iφ}
φ = 0 Bosons
φ = π Fermions
Bosons
Fermions
In 2D φ ≠ 0 or
TWO fermions
ψ(r̄₁, r̄₂) = −ψ(r̄₂, r̄₁)
⇒ ψ(r̄, r̄) = −ψ(r̄, r̄)
⇒ ψ(r̄, r̄) = 0


single particle at r within d³r
⇒ λ = e^{iφ}
In 1D :
In 3D :
e^{2iφ} = 1 ⇒ φ = 0 Bosons
φ = π Fermions
Bosons
Fermions
In 2D
Anyons
TWO fermions
ψ(r₁, r₂) = − ψ(r₂, r₁)
⇒ ψ(r, r) = − ψ(r, r)
⇒ ψ(r, r) = 0
TWO Bosons
ψ(r, r) ≠ 0
steps → effective step


steps → effective step
single particle at r within d³r
TWO fermions
In 1D :
Bosons
In 3D :
Fermions
TWO Bosons
Bosons
fermion are antisymmetric
In 2D
bosons are symmetric


{ψ_{k_1}(r̄_1), ψ_{k_2}(r̄_2), ... }
{ψ_{k_1}(r̄_1), ψ_{k_2}(r̄_2), ... , ψ_{k_N}(r̄_N)}
Ψ(r̄_1, r̄_2, ... r̄_N) =


{ψ_{k1}(r̄1), ψ_{k2}(r̄2), ... }
{ψ_{k1}(r̄1), ψ_{k2}(r̄2), . . . , ψ_{kN}(r̄N)}
ψ(r̄1, r̄2, . . . r̄N) = ψ_{k1}(r̄1) ψ_{k2}(r̄2) . . . ψ_{kN}(r̄N)

$$\left\{ \psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots \right\}$$

$$\left\{ \psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots, \psi_{k_N}(\bar{r}_N) \right\}$$

$$\Psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2) \ldots \psi_{k_N}(\bar{r}_N) = \prod_{k_j} \psi_{\bar{k}_j}(\bar{r}_j)$$

$$\hat{S}_{\pm} \prod_{j=1}^{N} \psi_{k_j}(\bar{r}_j) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \cdots & \psi_{k_1}(\bar{r}_N) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \cdots & \psi_{k_2}(\bar{r}_N) \\ \vdots & & & \\ \psi_{k_N}(\bar{r}_1) & \psi_{k_N}(\bar{r}_2) & \cdots & \psi_{k_N}(\bar{r}_N) \end{vmatrix}$$

$$\{\psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots \}$$

$$\{\psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots, \psi_{k_N}(\bar{r}_N)\}$$

$$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2) \ldots \psi_{k_N}(\bar{r}_N) = \prod_{k_j} \psi_{\bar{k}_j}(\bar{r}_j)$$

$$\hat{S}_{\pm} \prod_{j=1}^{N} \psi_{k_j}(\bar{r}_j) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \cdots & \psi_{k_1}(\bar{r}_N) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \cdots & \psi_{k_2}(\bar{r}_N) \\ \vdots & & & \\ \psi_{k_N}(\bar{r}_1) & \psi_{k_N}(\bar{r}_2) & \cdots & \psi_{k_N}(\bar{r}_N) \end{vmatrix}_{\pm}$$

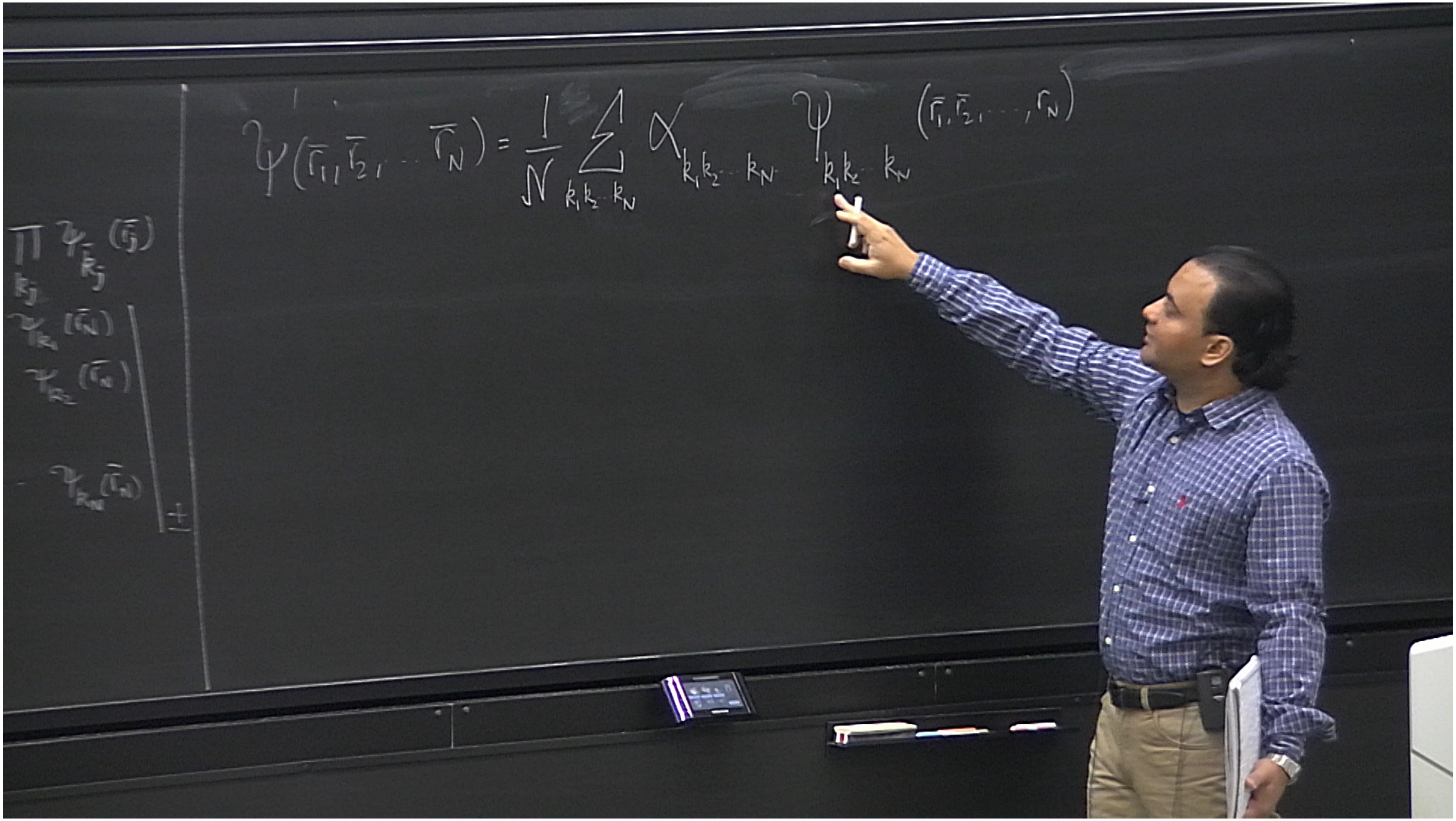
$$\{\psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots \}$$

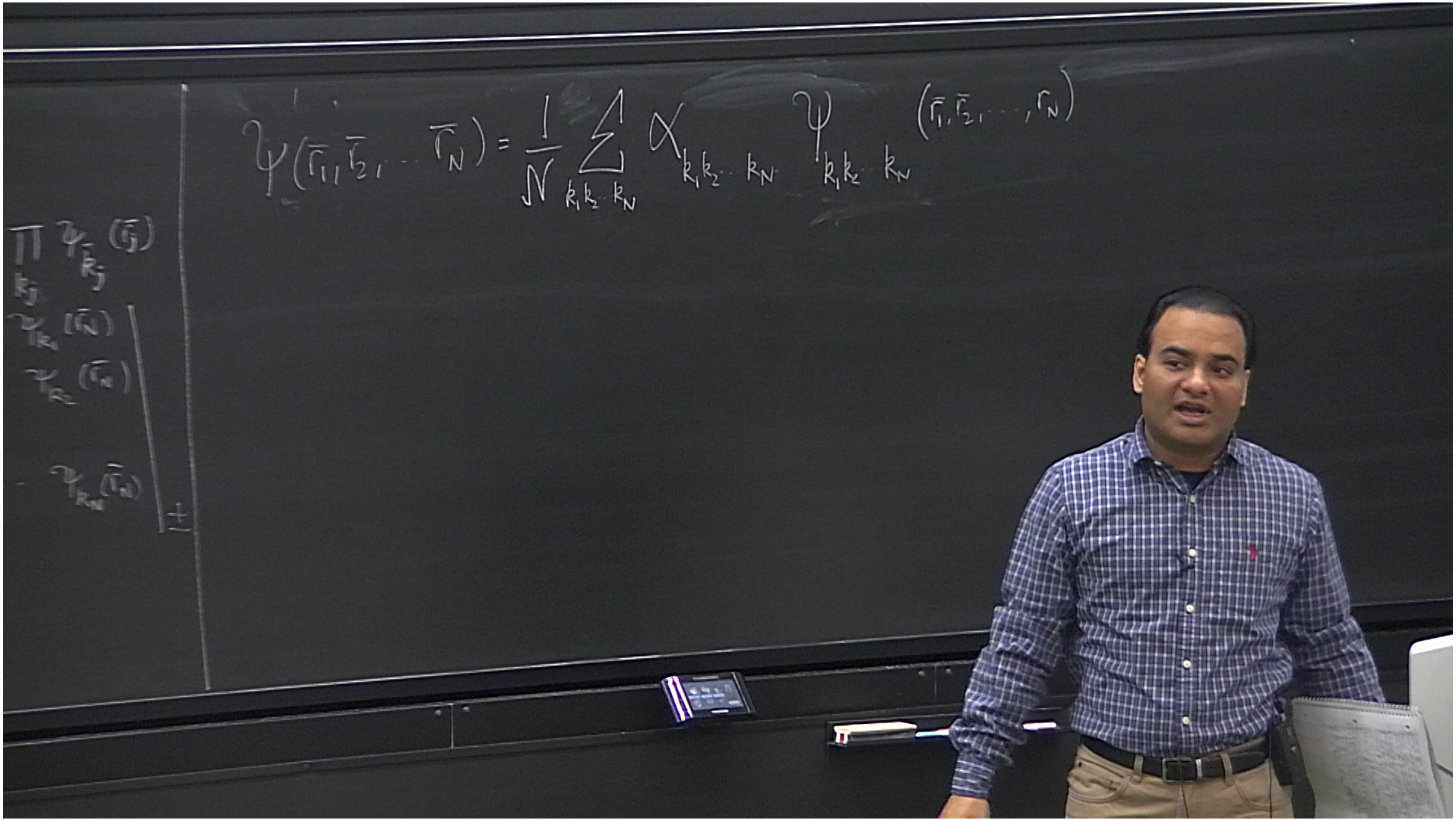
$$\{\psi_{k_1}(\bar{r}_1), \psi_{k_2}(\bar{r}_2), \ldots, \psi_{k_N}(\bar{r}_N)\}$$

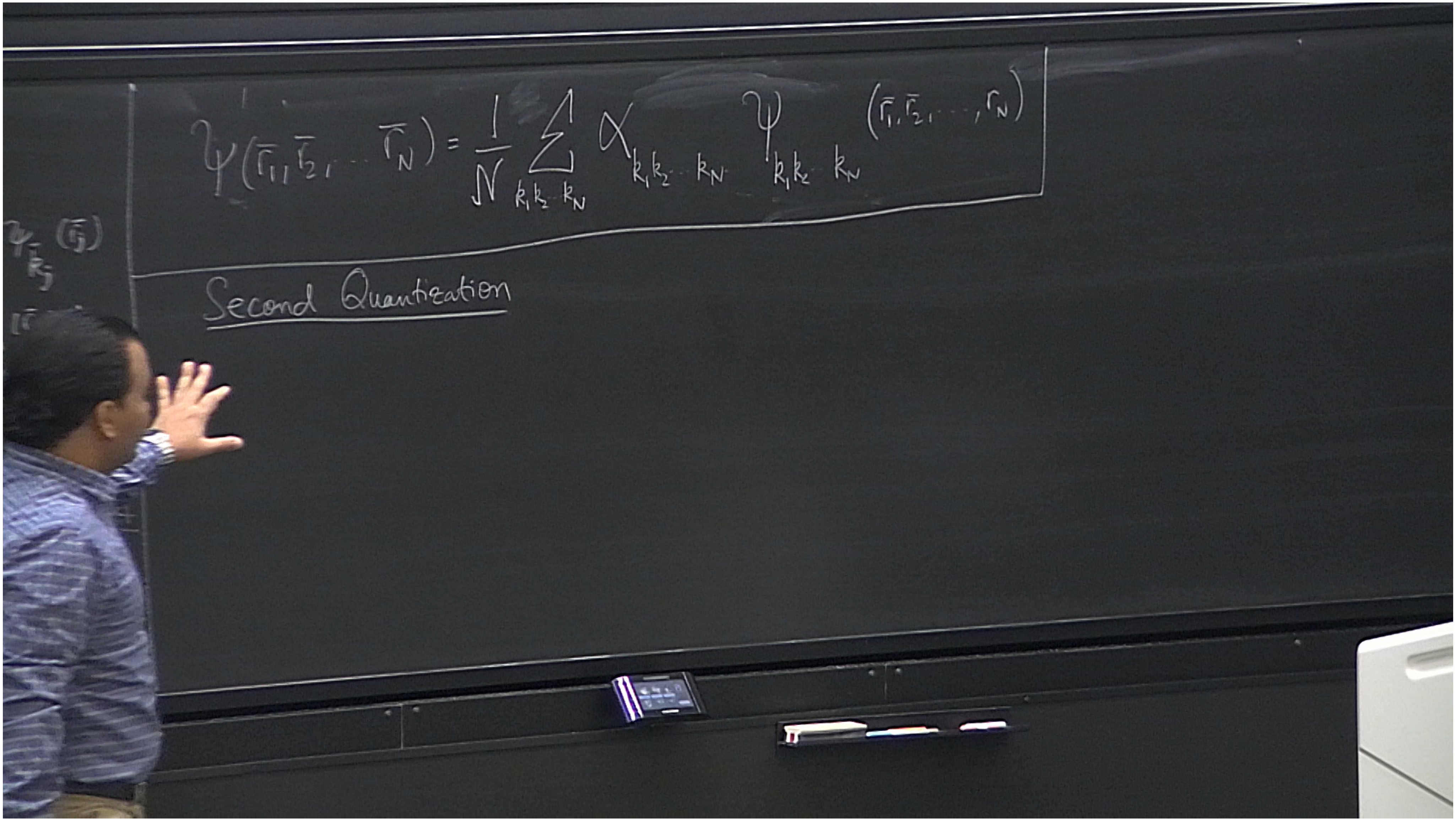
$$\Psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2) \ldots \psi_{k_N}(\bar{r}_N) = \prod_{k_j} \psi_{\bar{k}_j}(\bar{r}_j)$$

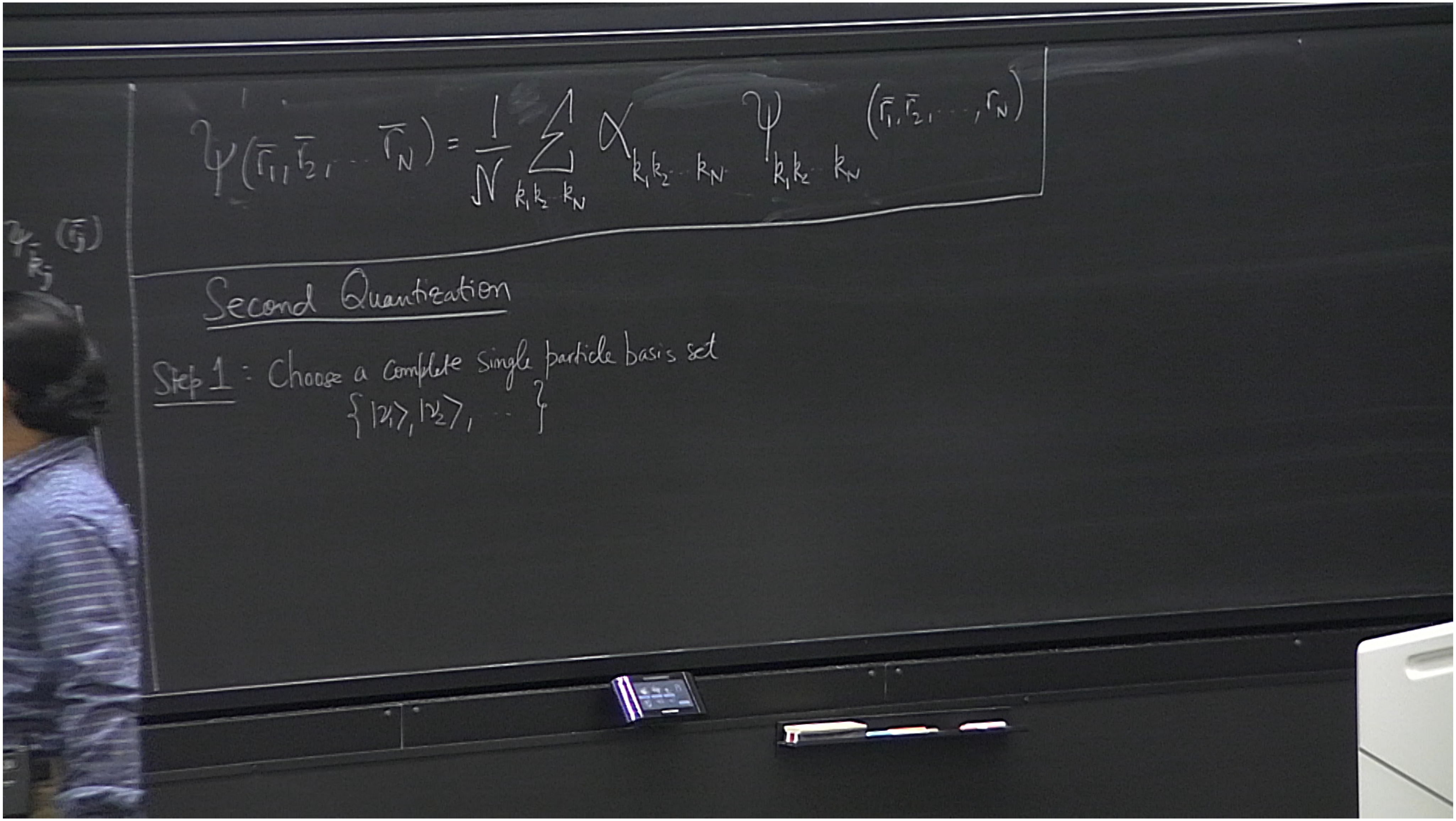
$$\hat{S}_{\pm} \prod_{j=1}^{N} \psi_{k_j}(\bar{r}_j) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \cdots & \psi_{k_1}(\bar{r}_N) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \cdots & \psi_{k_2}(\bar{r}_N) \\ \vdots & & & \\ \psi_{k_N}(\bar{r}_1) & \psi_{k_N}(\bar{r}_2) & \cdots & \psi_{k_N}(\bar{r}_N) \end{vmatrix}_{\pm}$$

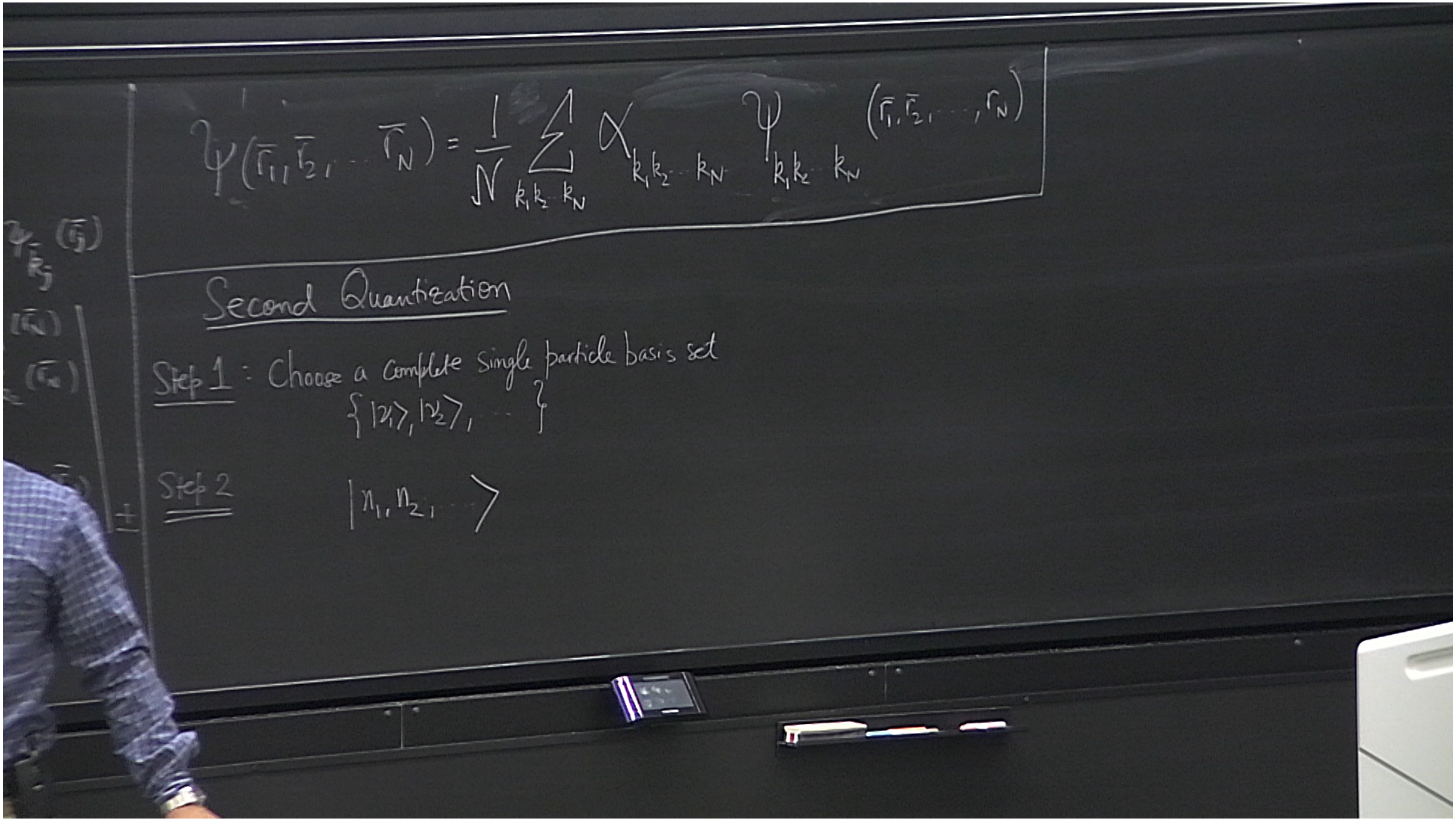
$$\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2) \ldots \psi_{k_N}(\bar{r}_N)$$

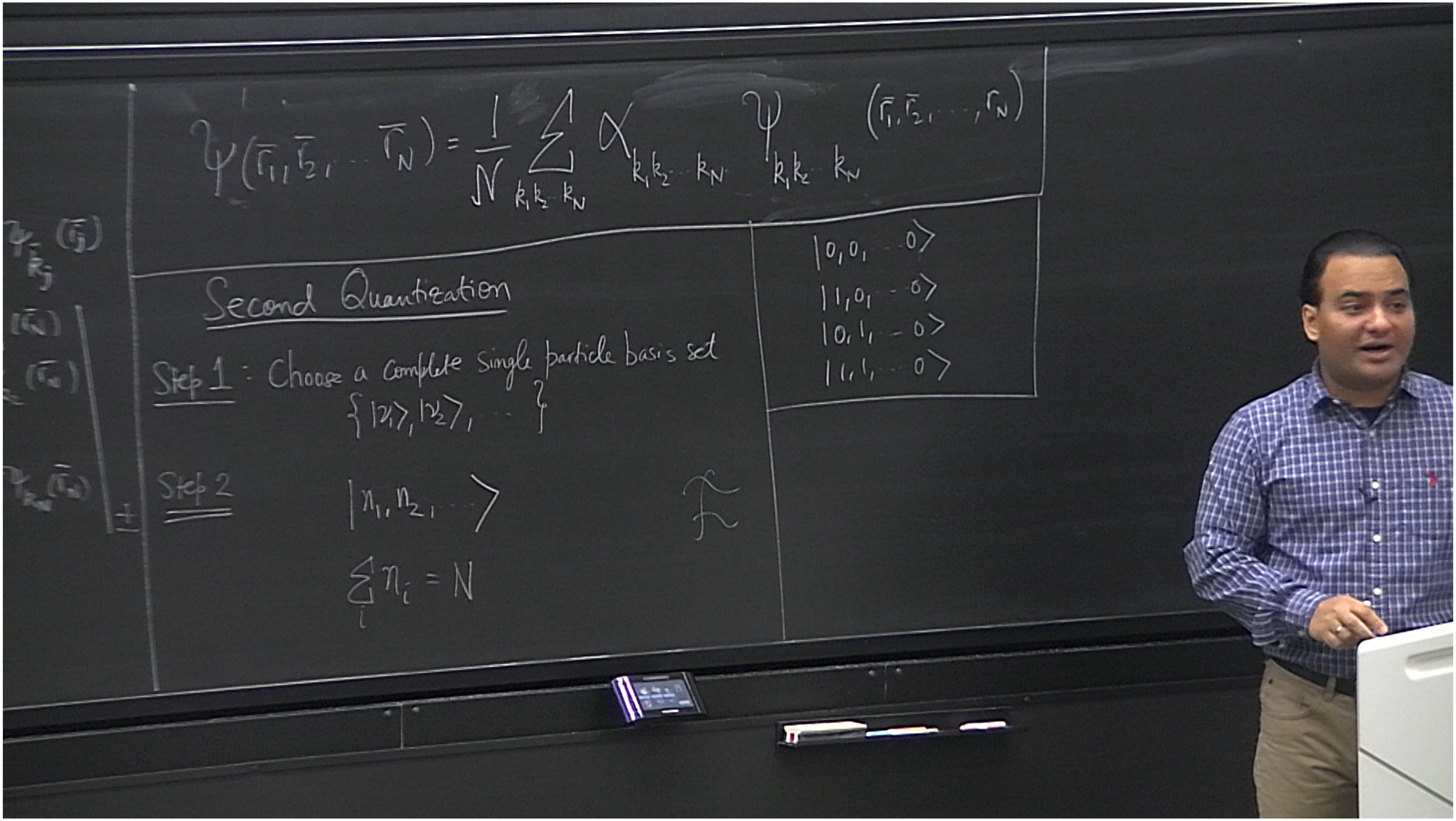

ψ(r̄1, r̄2, ... r̄N) = 1/√N Σ k1k2...kN α k1k2...kN ψ k1k2...kN (r̄1, r̄2, ..., rN)
Π kj ψ kj (r̄j)
ψ k1 (r̄N)
ψ k2 (r̄N)
ψ kN (r̄N)
±

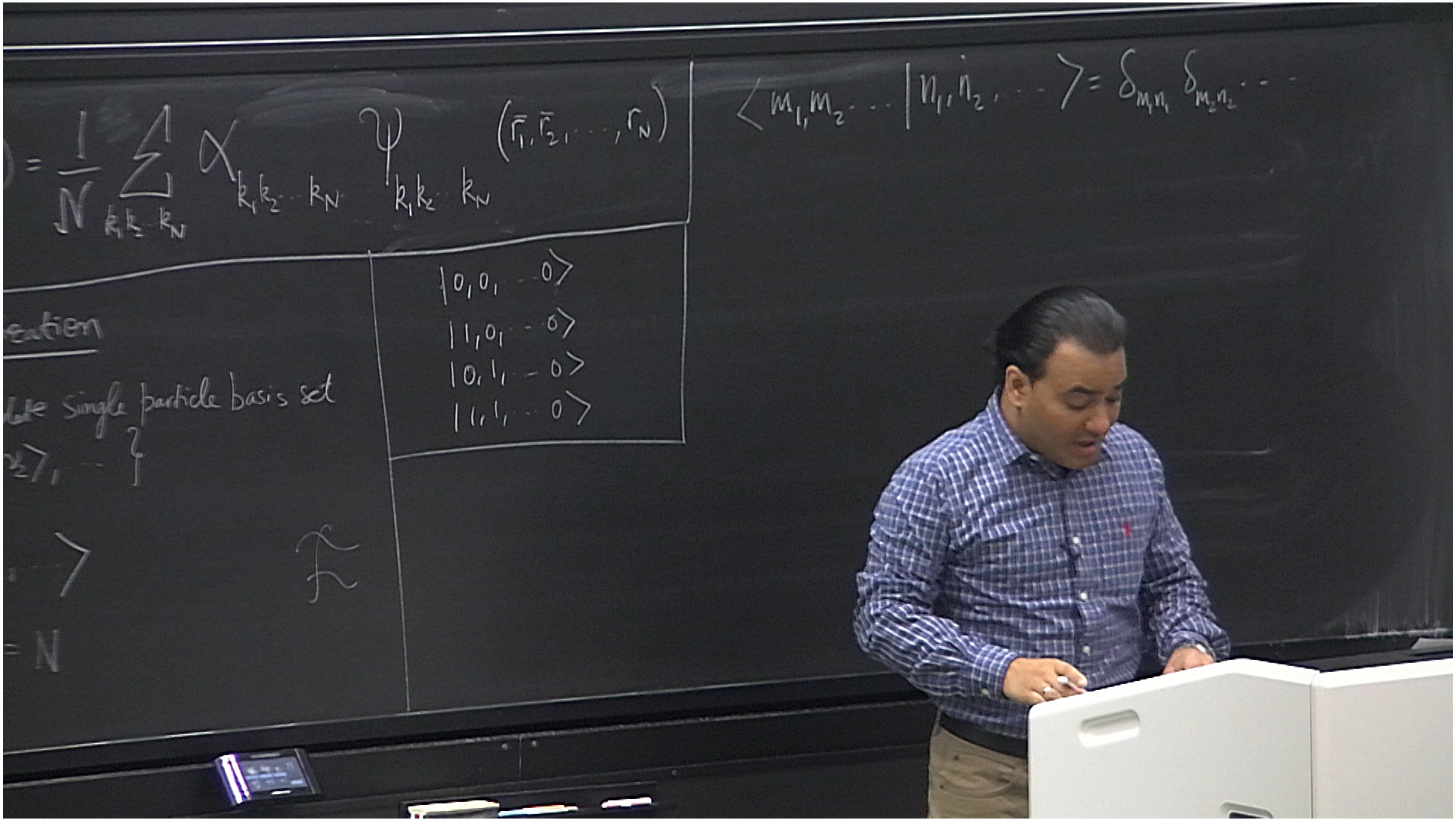

$\psi(\bar{r}_1, \bar{r}_2, \dots \bar{r}_N) = \frac{1}{\sqrt{N}} \sum_{k_1 k_2 \dots k_N} \alpha_{k_1 k_2 \dots k_N} \psi_{k_1 k_2 \dots k_N} (\bar{r}_1, \bar{r}_2, \dots, r_N)$

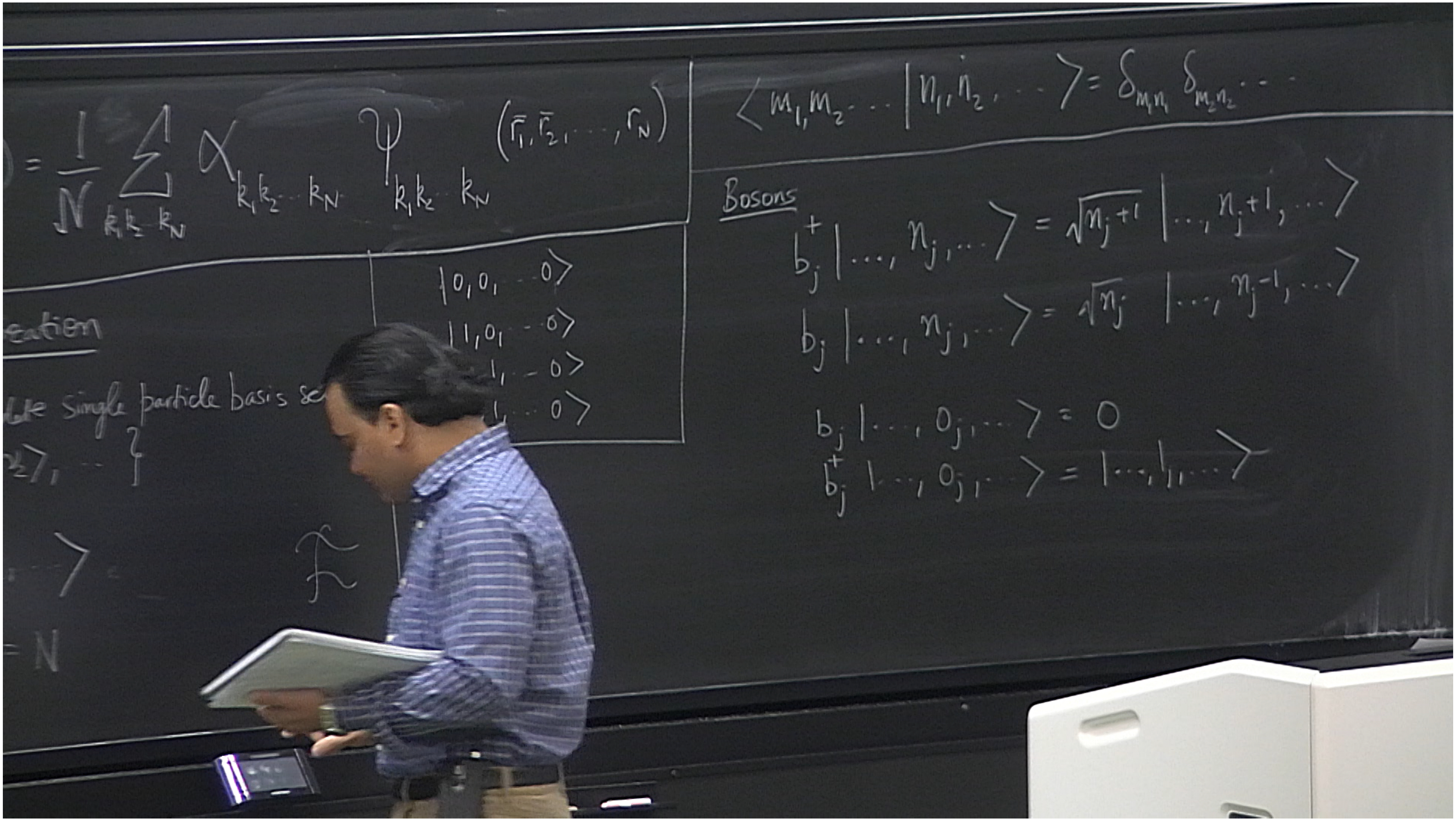

$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \frac{1}{\sqrt{N}} \sum_{k_1 k_2 \ldots k_N} \alpha_{k_1 k_2 \ldots k_N} \psi_{k_1 k_2 \ldots k_N}(\bar{r}_1, \bar{r}_2, \ldots, r_N)$
Second Quantization


$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \frac{1}{\sqrt{N}} \sum_{k_1 k_2 \ldots k_N} \alpha_{k_1 k_2 \ldots k_N} \psi_{k_1 k_2 \ldots k_N}(\bar{r}_1, \bar{r}_2, \ldots, r_N)$
$\psi_{\bar{k}_j}(\bar{r}_j)$
Second Quantization
Step 1 : Choose a complete single particle basis set
$\{ |\nu_1\rangle, |\nu_2\rangle, \ldots \}$


$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N) = \frac{1}{\sqrt{N}} \sum_{k_1 k_2 \ldots k_N} \alpha_{k_1 k_2 \ldots k_N} \psi_{k_1 k_2 \ldots k_N}(\bar{r}_1, \bar{r}_2, \ldots, r_N)$
Second Quantization
Step 1: Choose a complete single particle basis set
$\{ |\nu_1\rangle, |\nu_2\rangle, \ldots \}$
Step 2
$|n_1, n_2, \ldots \rangle$

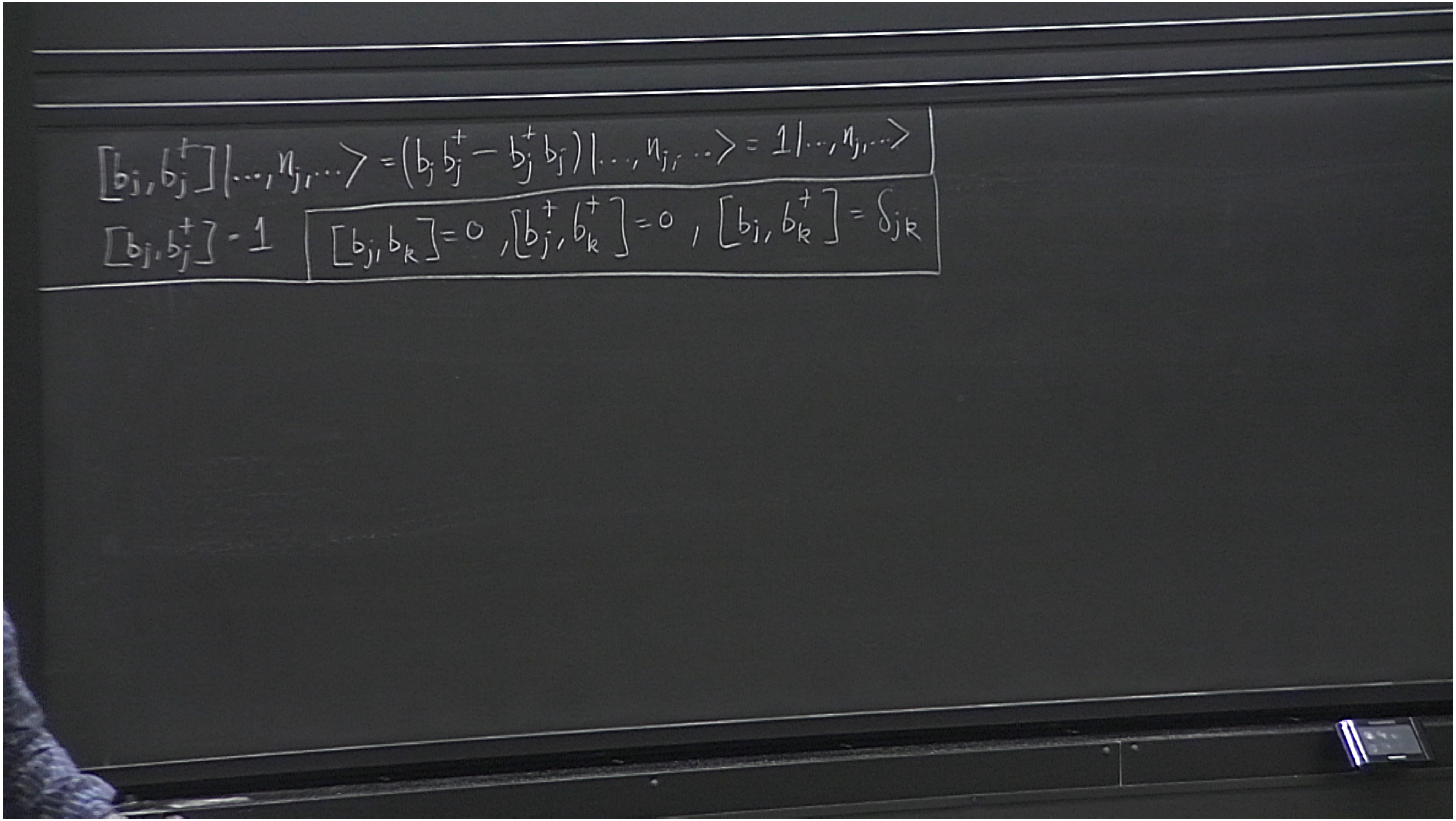

Ψ(r̄₁, r̄₂, … r̄_N) = 1/√N Σ_{k₁k₂…k_N} α_{k₁k₂…k_N} ψ_{k₁k₂…k_N}(r̄₁, r̄₂, …, r_N)
Second Quantization
Step 1: Choose a complete single particle basis set
{|ν₁⟩, |ν₂⟩, … }
Step 2
|n₁, n₂, … ⟩
Σ_i n_i = N
|0, 0, … 0⟩
|1, 0, … 0⟩
|0, 1, … 0⟩
|1, 1, … 0⟩


$\langle m_1, m_2 \ldots | n_1, n_2, \ldots \rangle = \delta_{m_1 n_1} \delta_{m_2 n_2} \ldots$
$= \frac{1}{N} \sum_{k_1 k_2 \ldots k_N} \alpha_{k_1 k_2 \ldots k_N} \psi_{k_1 k_2 \ldots k_N} (\bar{r}_1, \bar{r}_2, \ldots, r_N)$
$|0, 0, \ldots 0\rangle$
$|1, 0, \ldots 0\rangle$
$|0, 1, \ldots 0\rangle$
$|1, 1, \ldots 0\rangle$
single particle basis set
$= N$


⟨m1, m2 ... | n1, n2, ... ⟩ = δm1n1 δm2n2 ...
Bosons
b†j | ..., nj, ... ⟩ = √(nj+1) | ..., nj+1, ... ⟩
bj | ..., nj, ... ⟩ = √nj | ..., nj−1, ... ⟩
bj | ..., 0j, ... ⟩ = 0
b†j | ..., 0j, ... ⟩ = | ..., 1j, ... ⟩
|0, 0, ... 0⟩
|1, 0, ... 0⟩
single particle basis se
= N

$$[b_j, b_j^{+}] \, |\ldots, n_j, \ldots\rangle = (b_j b_j^{+} - b_j^{+} b_j) \, |\ldots, n_j, \ldots\rangle = 1 \, |\ldots, n_j, \ldots\rangle$$

$$[b_j, b_j^{+}] = 1$$


$[b_j, b_j^+] |\ldots, n_j, \ldots\rangle = (b_j b_j^+ - b_j^+ b_j) |\ldots, n_j, \ldots\rangle = 1 |\ldots, n_j, \ldots\rangle$
$[b_j, b_j^+] = 1$
$[b_j, b_k] = 0$, $[b_j^+, b_k^+] = 0$, $[b_j, b_k^+] = \delta_{jk}$

$$[b_j, b_j^+]\,|\ldots, n_j, \ldots\rangle = (b_j b_j^+ - b_j^+ b_j)\,|\ldots, n_j, \ldots\rangle = 1\,|\ldots, n_j, \ldots\rangle$$

$$[b_j, b_j^+] = 1 \qquad [b_j, b_k] = 0\,,\ [b_j^+, b_k^+] = 0\,,\ [b_j, b_k^+] = \delta_{jk}$$

Number operator: $\hat{n}_j = b_j^+ b_j$

$$\hat{n}_j\,|\ldots, n_j, \ldots\rangle = n_j\,|\ldots, n_j, \ldots\rangle$$

$$[b_j, b_j^+]|\ldots, n_j, \ldots\rangle = (b_j b_j^+ - b_j^+ b_j)|\ldots, n_j, \ldots\rangle = 1|\ldots, n_j, \ldots\rangle$$

$$[b_j, b_j^+] = 1 \qquad \boxed{[b_j, b_k] = 0 \,, [b_j^+, b_k^+] = 0 \,, [b_j, b_k^+] = \delta_{jk}}$$

Number operator: $\hat{n}_j = b_j^+ b_j$

$$\hat{n}_j|\ldots, n_j, \ldots\rangle = n_j|\ldots, n_j, \ldots\rangle$$

$$|\ldots, n_j, \ldots\rangle = \frac{(b_j^+)^{n_j}}{\sqrt{n_j!}}|\ldots, 0_j, \ldots\rangle$$

$$[b_j, b_j^+]\,|\ldots, n_j, \ldots\rangle = (b_j b_j^+ - b_j^+ b_j)\,|\ldots, n_j, \ldots\rangle = 1\,|\ldots, n_j, \ldots\rangle$$

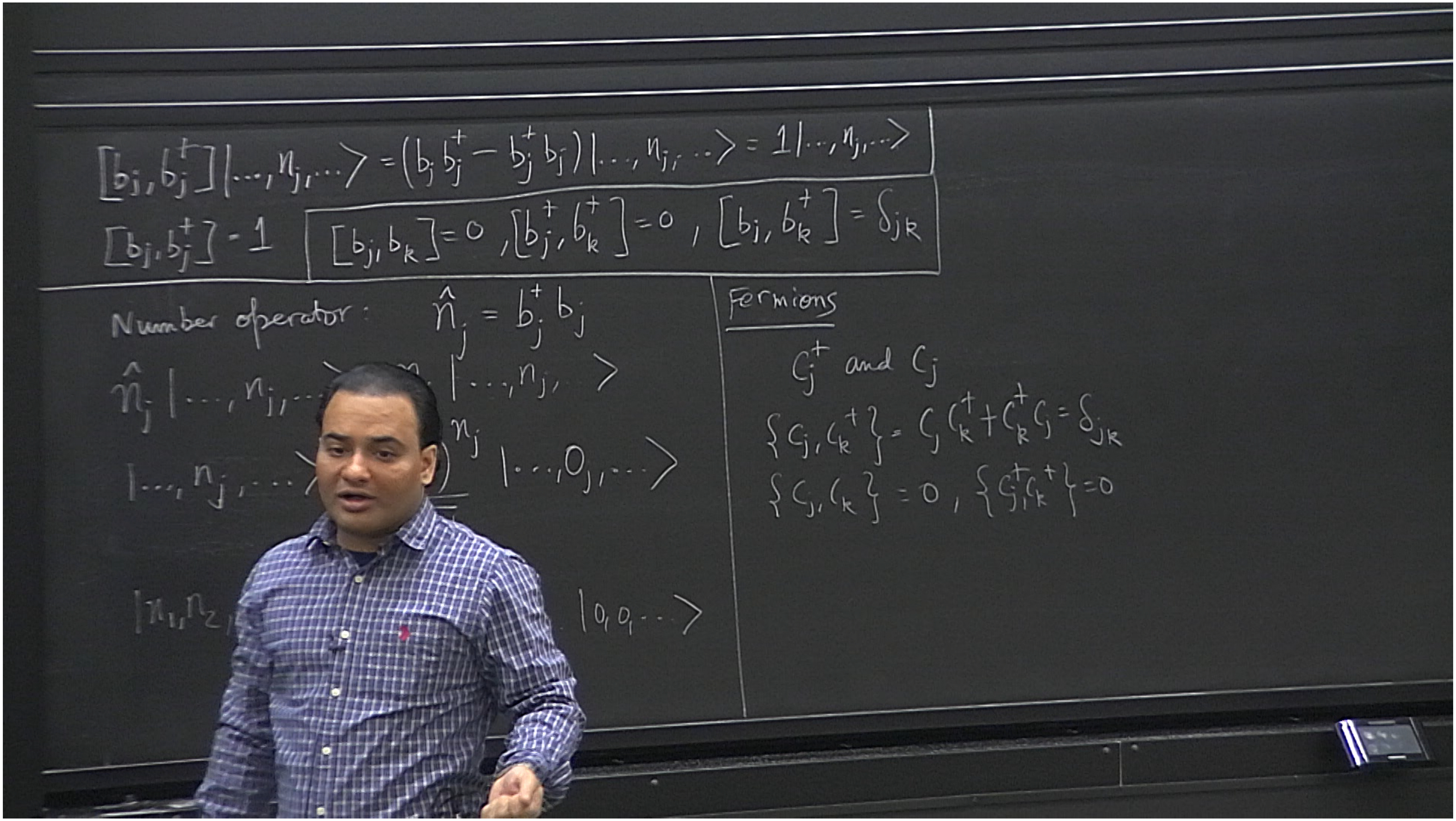
$$[b_j, b_j^+] = 1 \qquad [b_j, b_k] = 0, \; [b_j^+, b_k^+] = 0, \; [b_j, b_k^+] = \delta_{jk}$$

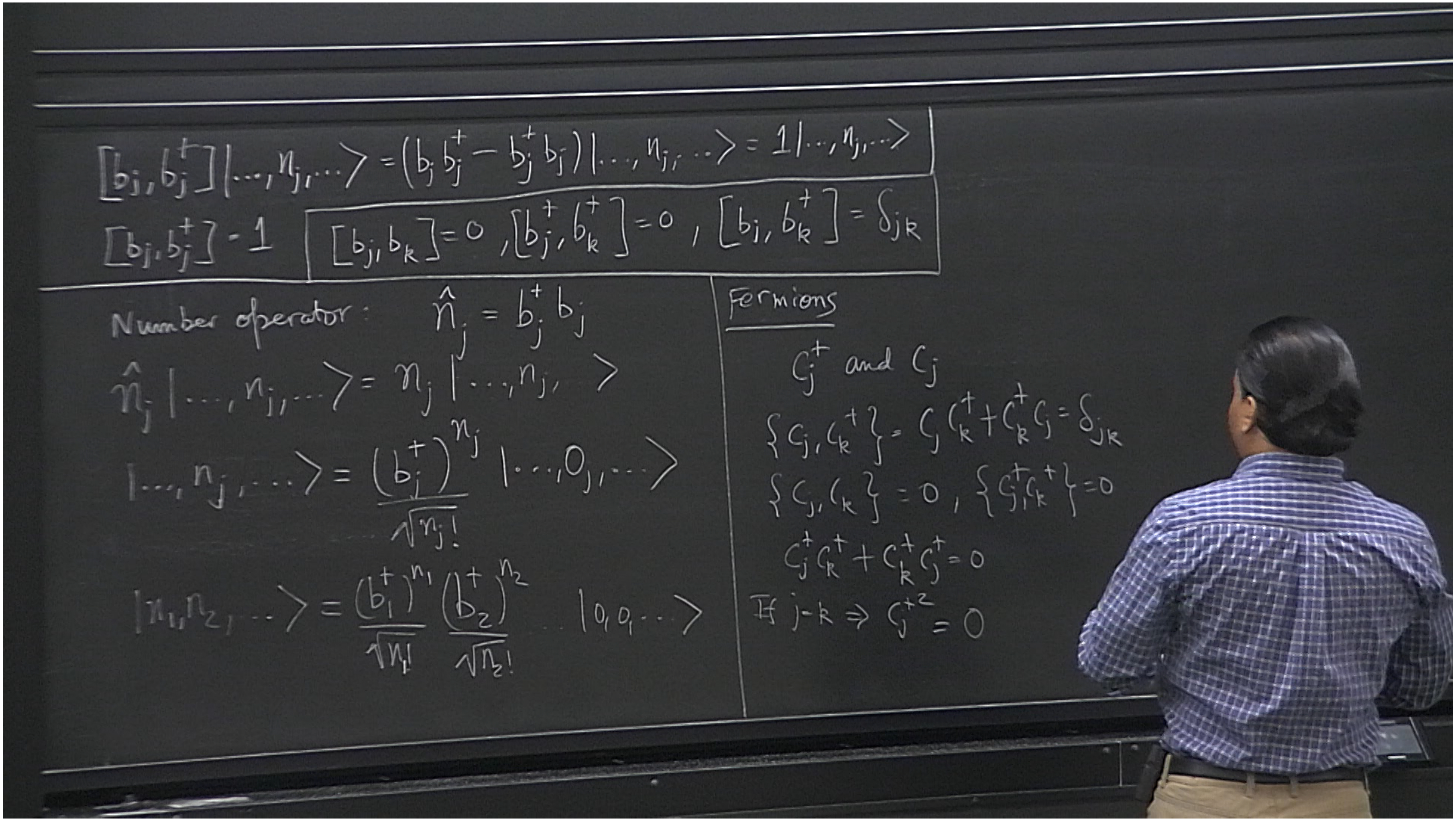
Number operator: $\hat{n}_j = b_j^+ b_j$

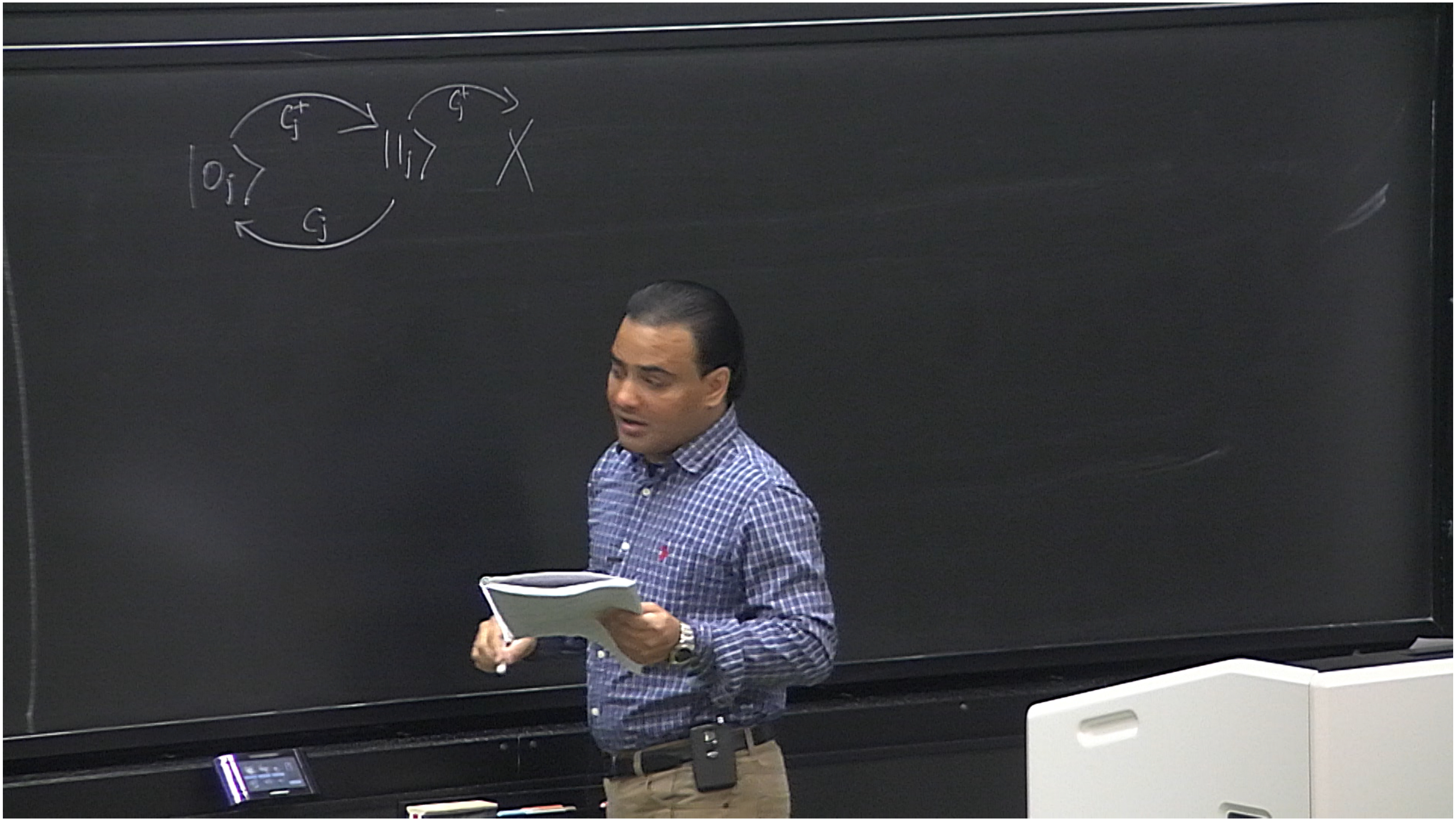
$$\hat{n}_j\,|\ldots, n_j, \ldots\rangle = n_j\,|\ldots, n_j, \ldots\rangle$$

$$|\ldots, n_j, \ldots\rangle = \frac{(b_j^+)^{n_j}}{\sqrt{n_j!}}\,|\ldots, 0_j, \ldots\rangle$$

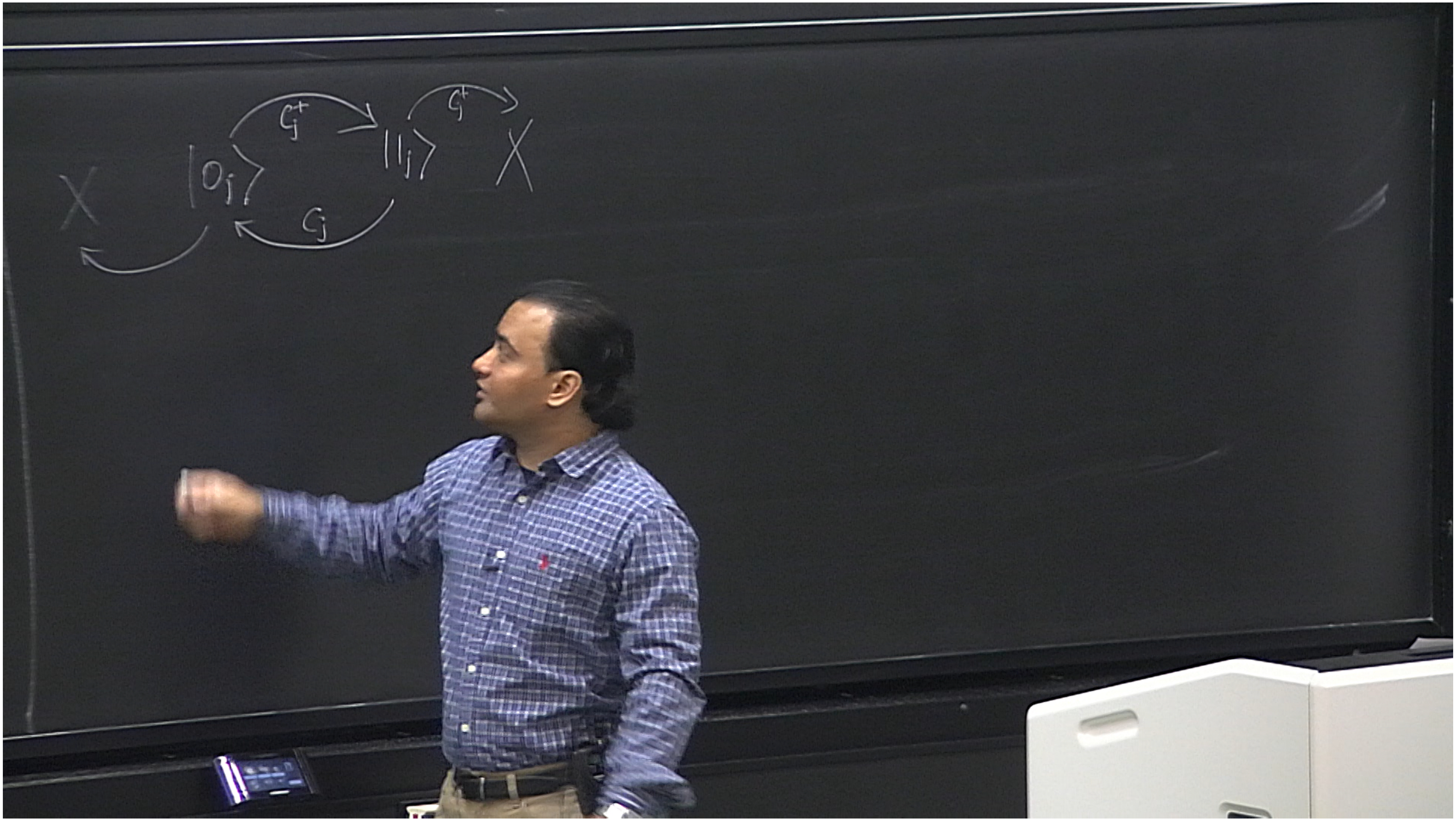
$$|n_1, n_2, \ldots\rangle = \frac{(b_1^+)^{n_1}}{\sqrt{n_1!}}\frac{(b_2^+)^{n_2}}{\sqrt{n_2!}}\cdots|0, 0, \ldots\rangle$$

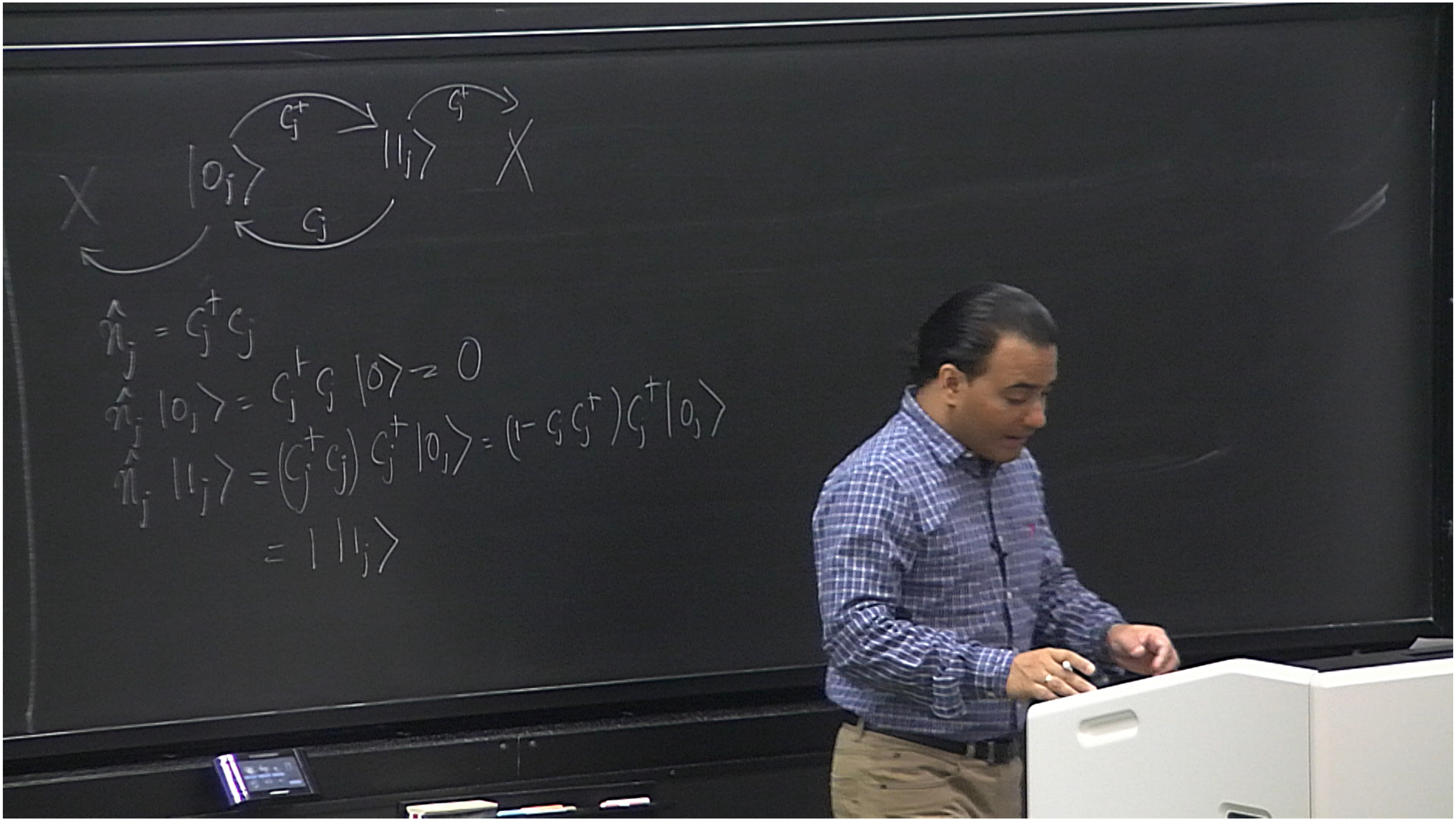

[bj, bj+] |..., nj, ...> = (bj bj+ − bj+ bj) |..., nj, ...> = 1 |..., nj, ...>
[bj, bj+] = 1
[bj, bk] = 0 , [bj+, bk+] = 0 , [bj, bk+] = δjk
Number operator: n̂j = bj+ bj
Fermions
cj+ and cj
{cj, ck+} = cj ck+ + ck+ cj = δjk
{cj, ck} = 0 , {cj+, ck+} = 0


$[b_j, b_j^+]|\ldots, n_j, \ldots\rangle = (b_j b_j^+ - b_j^+ b_j)|\ldots, n_j, \ldots\rangle = 1|\ldots, n_j, \ldots\rangle$
$[b_j, b_j^+] = 1$
$[b_j, b_k] = 0$, $[b_j^+, b_k^+] = 0$, $[b_j, b_k^+] = \delta_{jk}$
Number operator: $\hat{n}_j = b_j^+ b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = n_j |\ldots, n_j, \ldots\rangle$
$|\ldots, n_j, \ldots\rangle = \frac{(b_j^+)^{n_j}}{\sqrt{n_j!}} |\ldots, 0_j, \ldots\rangle$
$|n_1, n_2, \ldots\rangle = \frac{(b_1^+)^{n_1}}{\sqrt{n_1!}} \frac{(b_2^+)^{n_2}}{\sqrt{n_2!}} \ldots |0, 0, \ldots\rangle$
Fermions
c_j^+ and c_j
$\{c_j, c_k^+\} = c_j c_k^+ + c_k^+ c_j = \delta_{jk}$
$\{c_j, c_k\} = 0$, $\{c_j^+, c_k^+\} = 0$
$c_j^+ c_k^+ + c_k^+ c_j^+ = 0$
If $j = k \Rightarrow c_j^{+2} = 0$



$|0_j\rangle$
c_j^+
$|1_j\rangle$
c_j^+
c_j




$|0_j\rangle \xrightarrow{c_j^\dagger} |1_j\rangle$
$\hat{n}_j = c_j^\dagger c_j$
$\hat{n}_j |0_j\rangle = c_j^\dagger c_j |0\rangle = 0$
$\hat{n}_j |1_j\rangle = (c_j^\dagger c_j) c_j^\dagger |0_j\rangle = (1 - c_j c_j^\dagger) c_j^\dagger |0_j\rangle$
$= 1 |1_j\rangle$