Title: PSI 2018/2019 - Quantum Field Theory II - Lecture 11

Date: Nov 29, 2018 09:00 AM

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Abstract:

Today

1) Wilson RG $\longleftrightarrow$ QFT

2) Fermionic Path Integrals

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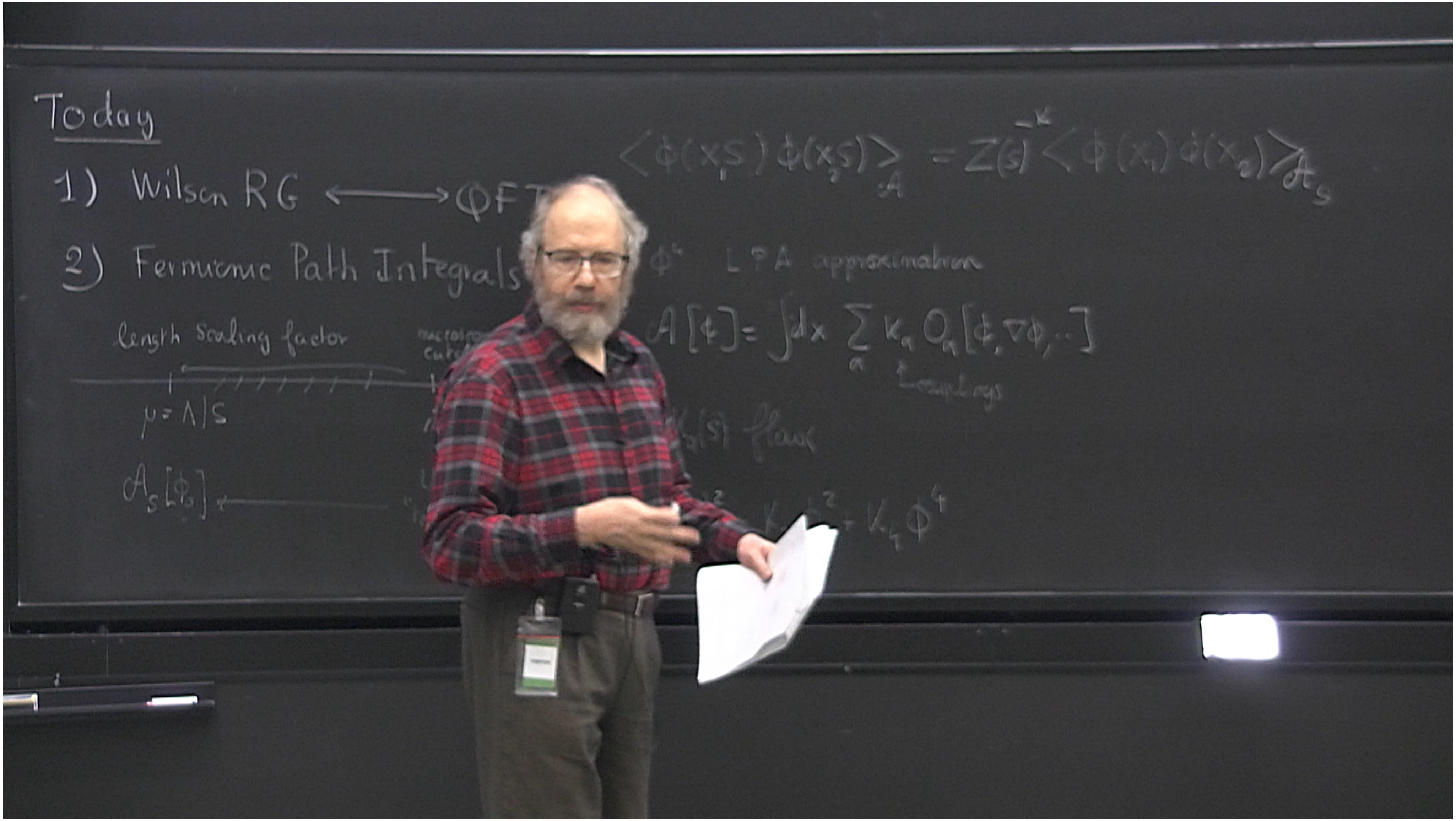
1) Wilson RG $\longleftrightarrow$ QFT

2) Fermionic Path Integrals


Today
1) Wilson RG ⟷ QFT
2) Fermionic Path Integrals
length scaling factor
$\mu = \Lambda / s$
$A_s[\phi_s]$
$\langle \phi(x_1 s)\phi(x_2 s)\rangle_A = Z(s)^{-K} \langle \phi(x_1)\phi(x_2)\rangle_{A_s}$
ϕ^4 LPA approximation
$A[\phi] = \int dx \sum_a K_a O_a[\phi, \nabla\phi, \ldots]$
couplings
$K_a(s)$ flow
$K_2\phi^2 + K_4\phi^4$

Today

1) Wilson RG $\longleftrightarrow$ ΦFT

2) Fermionic Path Integrals

length scaling factor

microscopic cutoff

$\mu = \Lambda / S$ ... $\Lambda = \frac{\pi}{a}$ Energy

$\mathcal{A}_S[\phi_S] \longleftarrow \mathcal{A}[\phi]$

"microscopic Action"

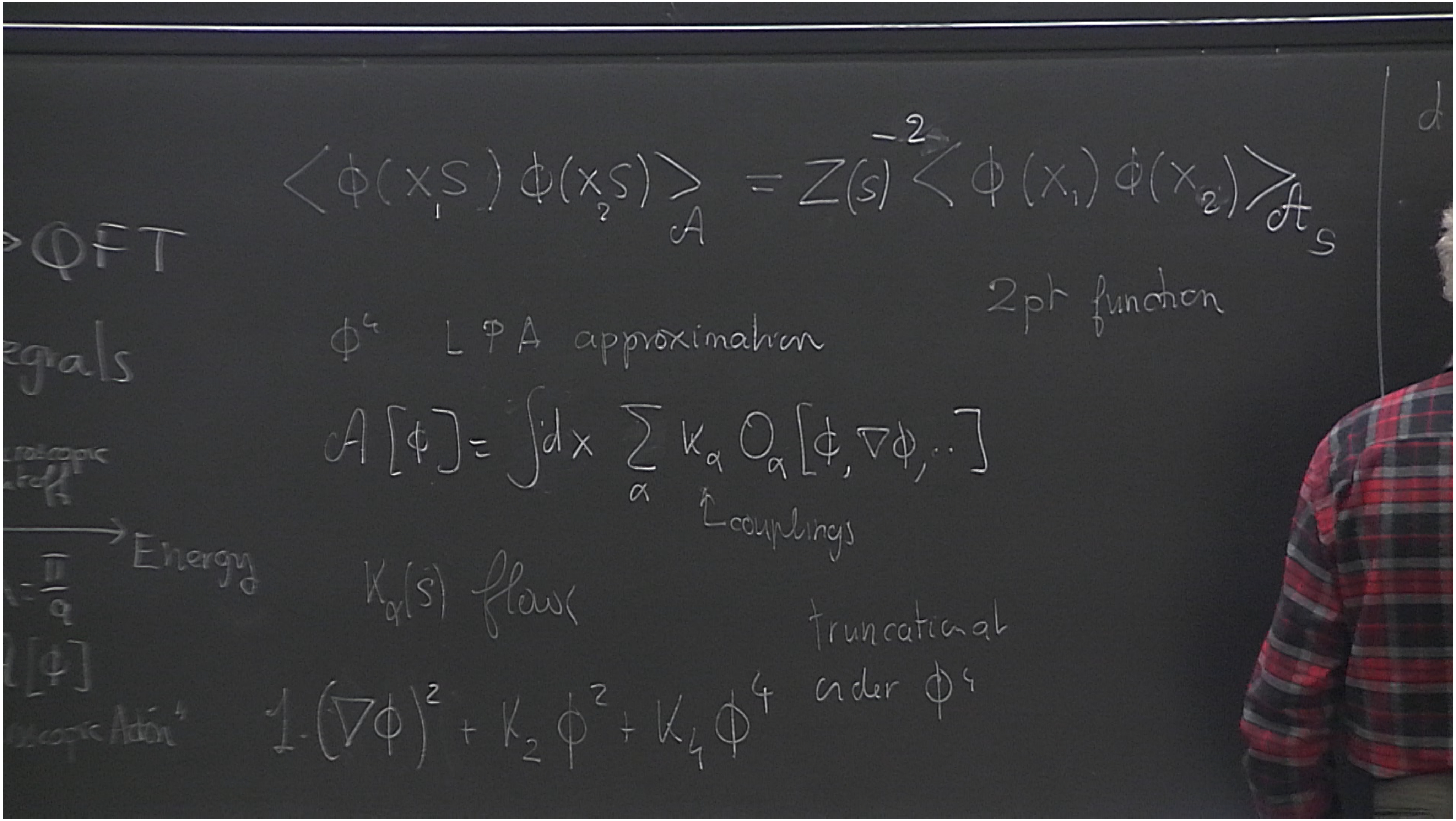
$\langle \phi(x_1 S)$

ϕ^4 L ϕ

$\mathcal{A}[\phi] =$

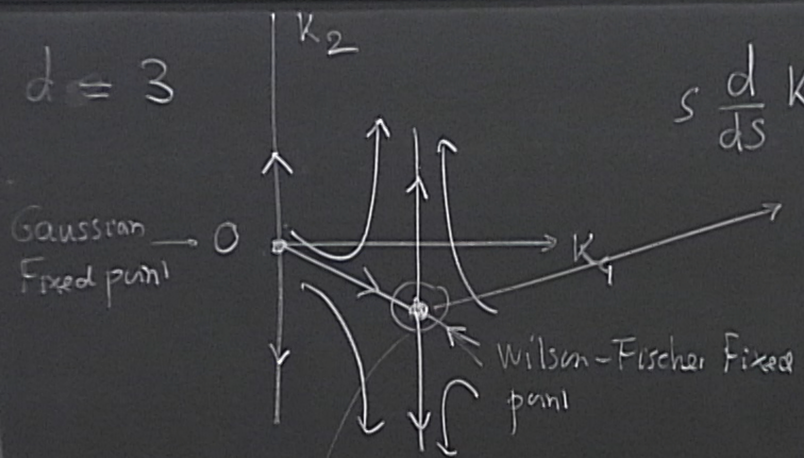
$K_\alpha(S)$ flo

$1 \cdot (\nabla\phi)^2 + K$


$\langle \phi(x_1 s)\, \phi(x_2 s) \rangle_{\mathcal{A}} = Z(s)^{-2} \langle \phi(x_1)\, \phi(x_2) \rangle_{\mathcal{A}_s}$
2pt function
ϕ^4 LPA approximation
$\mathcal{A}[\phi] = \int dx \sum_\alpha K_\alpha O_\alpha[\phi, \nabla\phi, ..]$
couplings
$K_\alpha(s)$ flows
truncation at order ϕ^4
$1 \cdot (\nabla\phi)^2 + K_2 \phi^2 + K_4 \phi^4$
QFT
Energy

$\geq \mathcal{A}_s$

$d = 3$



$$s \frac{d}{ds} K_\alpha(s) = W_\alpha(\{K(s)\})$$ Wilson Flow Equation

near a fixed point — linearize the flow

λ_β are the eigenvalues of $\left[\frac{\partial W_\alpha}{\partial K_\beta}\right]\Big|_{\text{at } K = K^*}$

eigenvectors

valid when g_β are small

$$\mathcal{A}[\phi] = \mathcal{A}_*[\phi] + \int dx \sum_\beta g_\beta \tilde{O}_\beta$$

action at the fixed point

choose a coordinate system $K_\alpha = K^*_\alpha + L_{\alpha\beta}\, g_\beta$

Fixed point

scaling couplings (fields)

Scaling operators — lin. comb. of the O

such that

$$\boxed{s \frac{d}{ds} g_\beta(s) = \lambda_\beta \cdot g_\beta(s)}$$

eigenvalue

dimensions of the coupling

$$g_\beta(s) = s^{\lambda_\beta} g_\beta$$

$\uparrow$ scaling dimension

ϕ is a scaling operator

j is the scaling field

external source

At the fixed point $g_\beta = 0$

$\tilde{O}_\beta$ has dim $\quad g_\beta$ has dim λ_β

$$\dim \tilde{O}_\beta = \Delta_{\tilde{O}_\beta} = \Delta_\beta = d - \lambda_\beta$$

dimension of an operator

$$g_\beta(s) = s^{\lambda_\beta}\, g_\beta$$

scaling dimension

ϕ is a scaling operator

g_ϕ is the scaling coupling

{ external source

At the fixed point $g_\beta = 0$

$\tilde{O}_\beta$ has dim $\quad g_\beta$ has dim λ_β

$$\dim \tilde{O}_\beta = \Delta_{\tilde{O}_\beta} = \Delta_\beta = d - \lambda_\beta$$

dimension of an operator

g = gé ji

j = ji / jay

$$\int \cdots + g_\phi \phi = \int \cdots j \cdot \phi = \int \cdots h \cdot \phi$$

source

QFT

external field

magnetisation

Stat. Mech

$\phi \longrightarrow \Delta_\phi$

$g_\phi \longrightarrow \lambda_\phi =$

$= d - \lambda_g$

ation

$\phi \longrightarrow \Delta_\phi$ scaling dimension

$g_\phi \longrightarrow \lambda_\phi = d - \Delta_\phi$

$A = A_*$

At a fixed point :

$$\boxed{\langle \phi(s x_1) \phi(s x_2) \rangle_* = s^{-2\Delta_\phi} \langle \phi(x_1) \phi(x_2) \rangle_*}$$

scale invariance

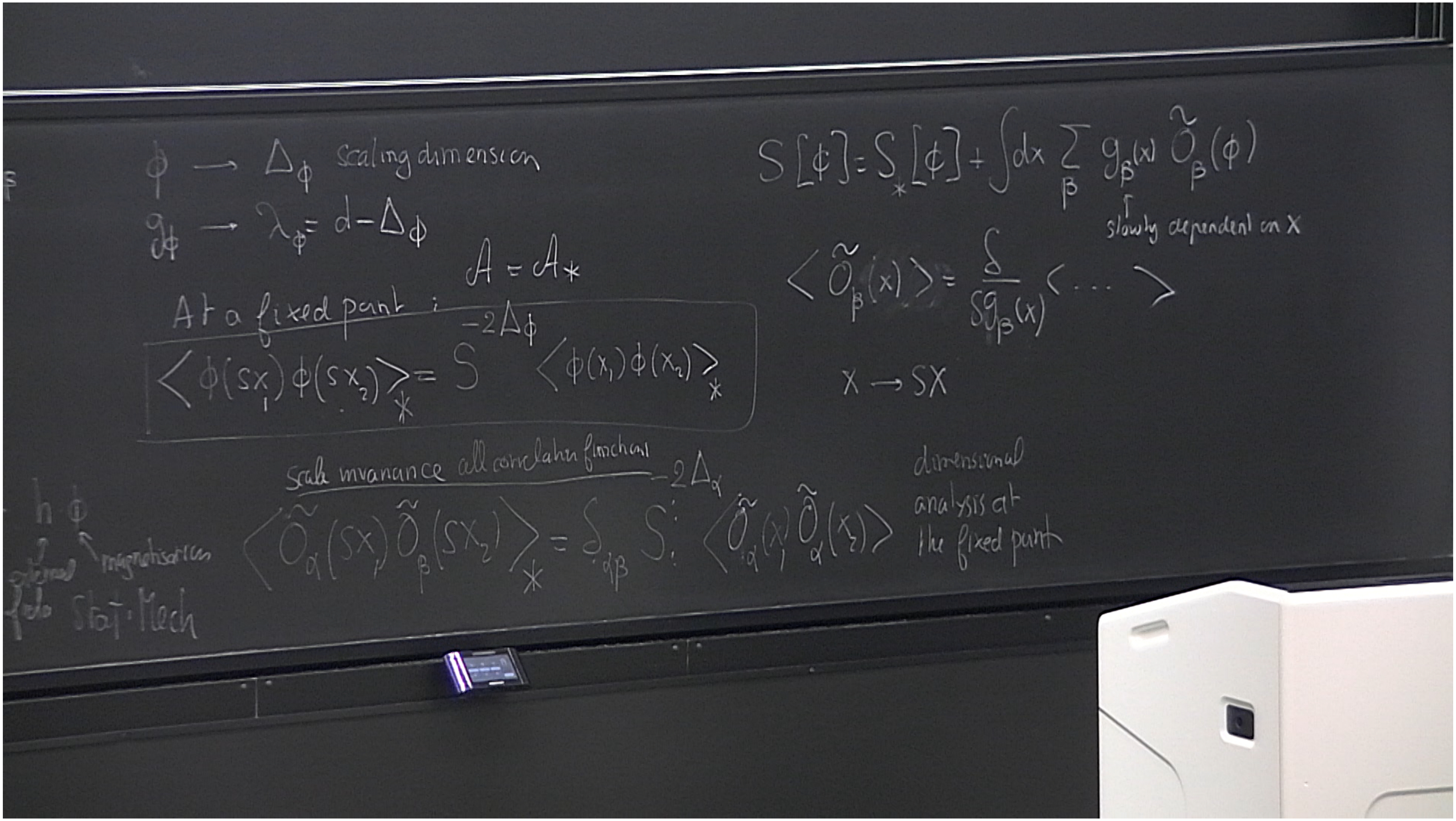
$$\langle \tilde{O}_\alpha(s x_1) \tilde{O}_\beta(s x_2) \rangle_* = \delta_{\alpha\beta}\, s^{-2\Delta_\alpha} \langle \tilde{O}_\alpha(x_1) \tilde{O}_\alpha(x_2) \rangle$$

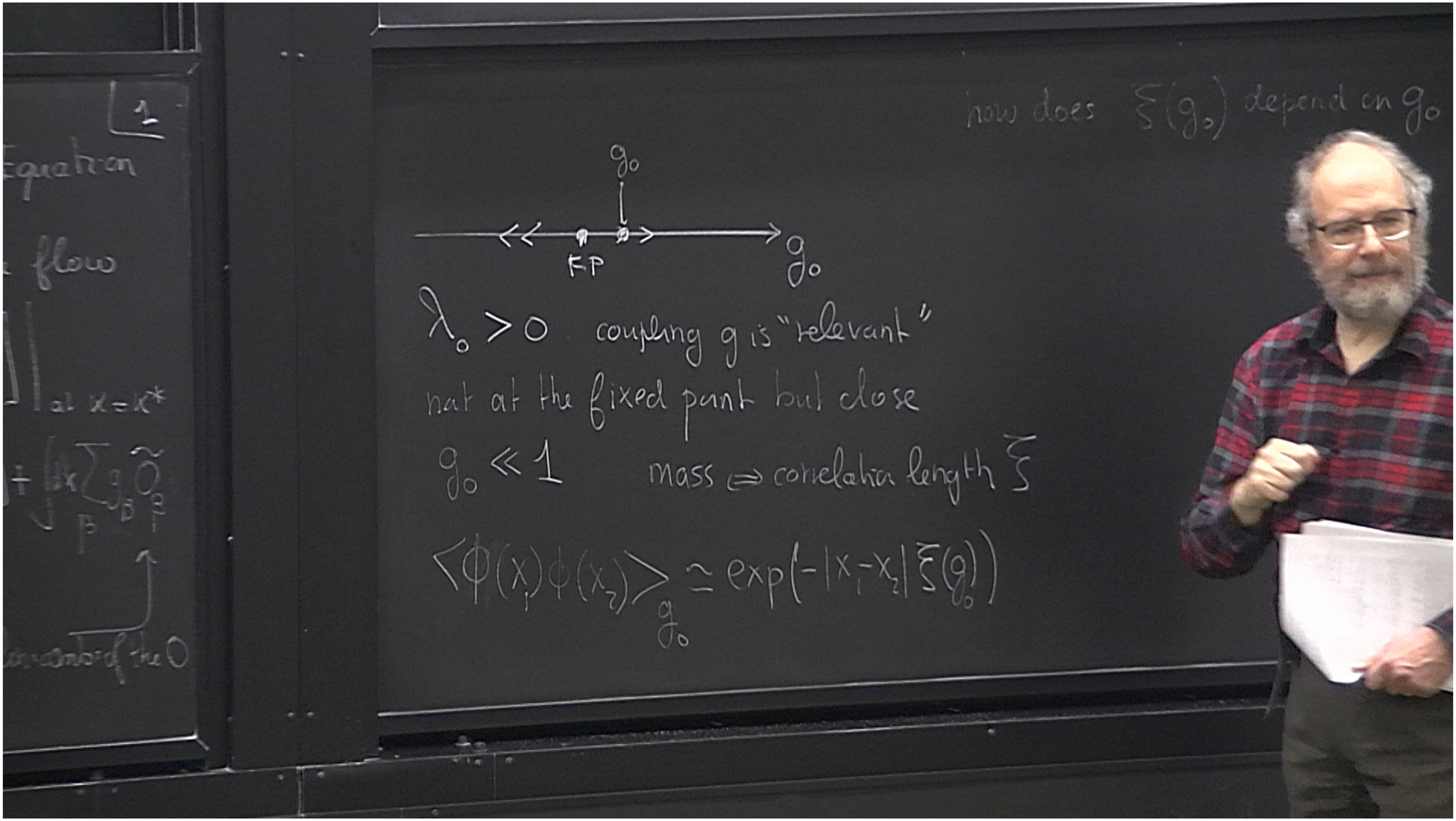
$\phi = \int \cdots h \cdot \phi$

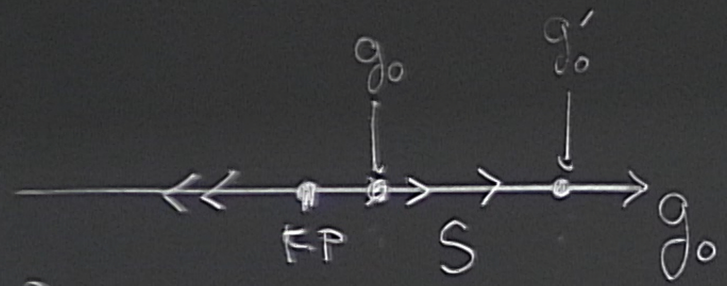
external field magnetisation

ue

Stat. Mech


$\phi \to \Delta_\phi$ scaling dimension
$g_\phi \to \lambda_\phi = d - \Delta_\phi$
$A = A_*$
At a fixed point:
$\langle \phi(sx_1)\phi(sx_2)\rangle_* = s^{-2\Delta_\phi}\langle\phi(x_1)\phi(x_2)\rangle_*$
Scale invariance all correlation functions
$\langle \tilde{O}_\alpha(sx_1)\tilde{O}_\beta(sx_2)\rangle_* = \delta_{\alpha\beta}\, s^{-2\Delta_\alpha}\langle \tilde{O}_\alpha(x_1)\tilde{O}_\alpha(x_2)\rangle$
dimensional analysis at the fixed point
h ϕ
external field
magnetisation
Stat. Mech
$S[\phi] = S_*[\phi] + \int dx \sum_\beta g_\beta(x)\,\tilde{O}_\beta(\phi)$
slowly dependent on x
$\langle \tilde{O}_\beta(x)\rangle = \frac{\delta}{\delta g_\beta(x)}\langle \dots \rangle$
$x \to sx$


how does ξ(g0) depend on g0
g0
FP
g0
λ0 > 0 coupling g is "relevant"
not at the fixed point but close
g0 ≪ 1 mass ⇔ correlation length ξ
⟨φ(x1)φ(x2)⟩g0 ≃ exp(−|x1−x2| ξ(g0))
Equation
flow
at x = x*



$\lambda_0 > 0$ coupling g is "relevant"

not at the fixed point but close

$g_0 \ll 1$ mass $\Leftrightarrow$ correlation length ξ

$$\langle \phi(x_1)\phi(x_2)\rangle_{g_0} \simeq \exp(-|x_1 - x_2|\, \xi(g_0))$$

how does $\xi(g_0)$ depend on g_0 ?

find $\underline{\underline{s}}$ such that $g_0(s) = g_0'$

then. $\xi(g_0') = \xi(g_0)/s$

$$\xi = \frac{1}{\text{mass}}$$

How does $\xi(g_0)$ depend on g_0 ?

find $\underline{\underline{S}}$ such that $g_0(s) = g_0'$

$$\frac{g_0'}{g_0} = S^{\lambda_0}$$

then $\xi(g_0') = \xi(g_0)/S$

$$\xi = \frac{1}{\text{mass}}$$

$$\boxed{\xi(g_0) = g_0^{-\frac{1}{\lambda_0}}} \longleftrightarrow \xi \sim |T_c - T|^{-\nu}$$

Ising Model

critical temperature

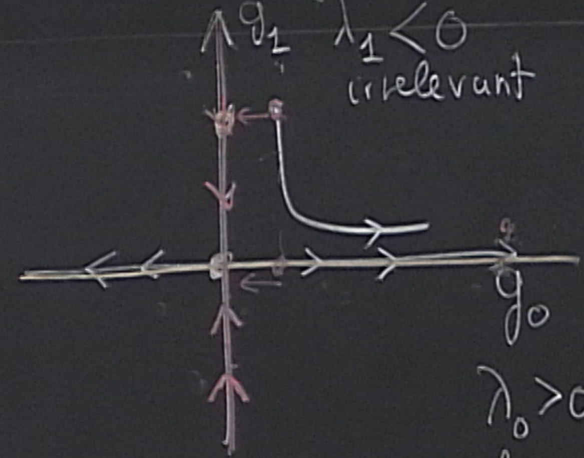
$$\boxed{\nu = \frac{1}{\lambda_0}}$$

the λ_α are character

length ξ

$\xi(g_0)$

the λ_α are characteristics of the fixed point



(g_0, g_1)

the large distances are the same

one relevant coupling & some irrelevant couplings

$$g_0(s) = s^{\lambda_0} g_0 \nearrow$$

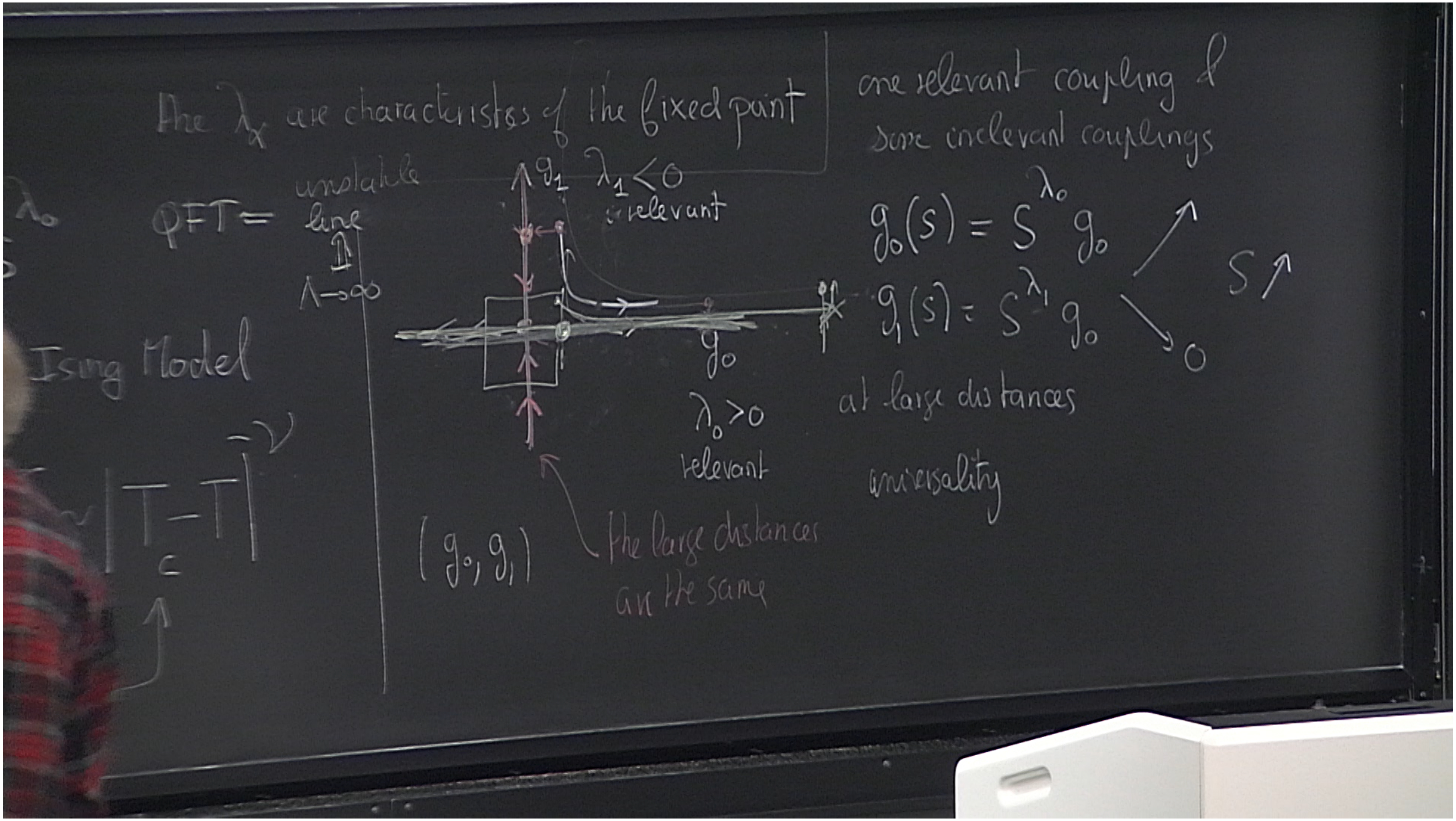
$$g_1(s) = s^{\lambda_1} g_0 \searrow 0$$

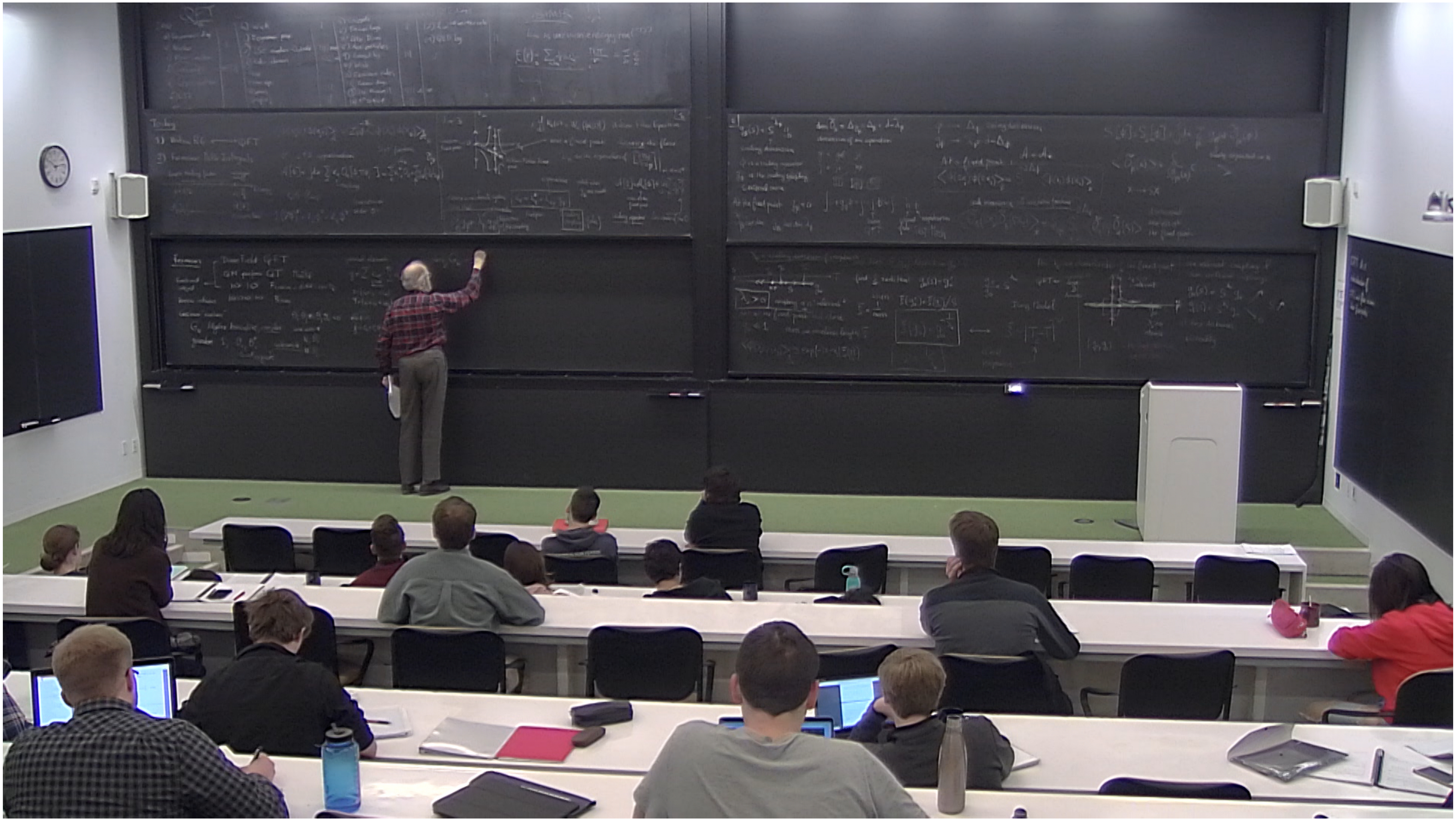
$s \nearrow$

at large distances

sing Model

$\sim |T_c - T|^{-\nu}$


The λ_α are characteristics of the fixed point
one relevant coupling &
some irrelevant couplings
QFT = unstable line
Λ→∞
g_1 λ_1<0
irrelevant
g_0
λ_0>0
relevant
g_0(s) = s^λ_0 g_0
g_1(s) = s^λ_1 g_0
s ↗
0
at large distances
universality
Ising Model
|T_c − T|^−ν
(g_0, g_1)
the large distances are the same



Fermions

Dirac Field QFT

Q.M pure fermion QT Maïté

Functional integral

$|0\rangle \quad |1\rangle$ Fermion = QuBit = spin 1/2

Berezin calculus

$|0\rangle |1\rangle |2\rangle \cdots |n\rangle$ Boson

Grassmann "numbers"

$\theta_i \theta_j + \theta_j \theta_i = 0$

G_N Algebra Associative, complex

Same with *

generators $1, \quad \theta_i, \quad \theta_i^*$ (conjugate)

anticommute N, θ_i

$N \quad \theta_j$

general elem

$g = \sum c_{I\bar{J}}$

complex numb

$I = \{ i_1 < i_2 \cdots <$

$\bar{J} = \{ j_1 < j_2 <$

$N = 1$

$g = c_0 + c_1 \theta$

of the coupling

Maïté

= QuBit = spin $1/2$

$$\Theta_i \Theta_j + \Theta_j \Theta_i = 0$$

Same with *

N, Θ_i

$N \; \Theta_j$

general element

$$g = \sum \underbrace{c_{I\bar{J}}}_{\text{complex number}} \prod_{a=1}^{|I|} \theta_{i_a} \prod_{b=1}^{|\bar{J}|} \bar{\theta}_{j_b}$$

$I = \{ i_1 < i_2 \cdots < i_{|I|} \}$

$\bar{J} = \{ j_1 < j_2 < j_{|\bar{J}|} \}$

$N = 1$

$$g = c_0 + c_1 \theta + c_2 \bar{\theta} + c_3 \theta \bar{\theta}$$

$$\dim_{\mathbb{C}} G_N = 2^{2N}$$

general element

$$g = \sum C_{I\bar{J}} \prod_{a=1}^{|I|} \theta_{i_a} \prod_{b=1}^{|\bar{J}|} \bar{\theta}_{j_b}$$

complex number

spin $1/2$

$I = \{ i_1 < i_2 \cdots < i_{|I|} \}$

$\bar{J} = \{ j_1 < j_2 < j_{|\bar{J}|} \}$

$\theta_j \theta_i = 0$

$N = 1$

$$g = c_0 + c_1 \theta + c_2 \bar{\theta} + c_3 \theta \bar{\theta}$$

$g =$ "function of the θ's and $\bar{\theta}$'s"

$$\dim_{\mathbb{C}} G_N = 2^{2N}$$

define a product $g_1 \cdot g_2$

define integration and derivation

$$\int d\theta_1 = \frac{\partial}{\partial \theta_1} \; ; \quad \frac{\partial}{\partial \theta_1} \theta_1 = 1 \, , \quad dx = 0$$

= spin 1/2

$\theta_j \theta_i = 0$

*

general element

$$g = \sum \underset{\text{complex number}}{C_{I\bar{J}}} \prod_{a=1}^{|I|} \theta_{i_a} \prod_{b=1}^{|\bar{J}|} \bar{\theta}_{j_b}$$

$I = \{ i_1 < i_2 \cdots < i_{|I|} \}$

$\bar{J} = \{ j_1 < j_2 < j_{|\bar{J}|} \}$

$N = 1$

$$g = c_0 + c_1 \theta + c_2 \bar{\theta} + c_3 \theta\bar{\theta}$$

$g =$ " function of the θ's and $\bar{\theta}$'s "

$$\dim_{\mathbb{C}} G_N = 2^{2N}$$

define a product $g_1 \cdot g_2$

define integration and derivation

$$\int d\theta_i = \frac{\partial}{\partial \theta_i} \quad , \quad \frac{\partial}{\partial \theta_i} \theta_i = 1 \quad , \quad dx = 0$$

$$\frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} + \frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} = 0$$

anticommutation

roduct $g_1 \cdot g_2$

egration and derivation

$\frac{\partial}{\partial \theta_i} \theta_i = 1$, else $= 0$

$\frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} = 0$ anticommutation

Wick theorem and Gaussian Integral

$[A_{ij}] = A$ a $N \times N$ self-adjoint matrix

$$\exp(-\bar{\theta} \cdot A \cdot \theta) = \exp\left(-\sum_{i,j=1}^{N} \bar{\theta}_i A_{ij} \theta_j\right)$$

$$= 1 - \bar{\theta} \cdot A \theta + \frac{1}{2} \bar{\theta} A \theta \cdot \bar{\theta} A \theta + \cdots$$ stops at order N

it is a polynormial !

$$\boxed{\int \prod_{j=1}^{N} d\bar{\theta}_j \prod_{i=1}^{N} d\theta_i \exp(-\bar{\theta} A \theta) = \det A}$$

⚠ order is important

G. conjucate $N\,\theta_j$

$z_i, \bar{z}_i$ are complex numbers

$$\int \prod_{i=1}^{N} \frac{dz_i\, d\bar{z}_i}{(2\pi)} \exp\left(-\sum_{ij} \bar{z}_j A_{ji} z_i\right) = \left(\det A\right)^{-1}$$

$$dz\, d\bar{z} = d(\mathrm{Re}\, z) \cdot d(\mathrm{Im}\, z)$$

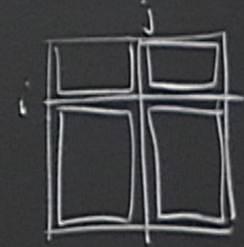
Charged Bosons (U(1) charge $\leftarrow$ complex)

Fermions

Correlation functions

$$\langle \theta_i \bar{\theta}_j \rangle = \frac{\int \prod d\bar{\theta} \prod d\theta \exp(-\bar{\theta} A \theta)\ \theta_i \bar{\theta}_j}{\int \prod d\bar{\theta} \prod d\theta \cdot \exp(-\bar{\theta} \cdot A \cdot \theta)}$$

$$= \frac{[\text{Minor}]_{ij}}{\det A}$$



$$\boxed{\langle \theta_i \bar{\theta}_j \rangle = (A^{-1})_{ij}}$$

But $\langle \bar{\theta}_j \theta_i \rangle = -(A^{-1})_{ij}$

G. generators 1, θ_i, $\bar\theta_i$ anticommute N θ_j g = function of the

$\theta_i, \bar\theta_i$ conjugate

$z_i, \bar z_i$ are complex numbers

$$\int \prod_{i=1}^{N} \frac{dz_i\, d\bar z_i}{(2\pi)} \exp\left(-\sum_{ij} \bar z_j A_{ji} z_i\right) = (\det A)^{-1}$$

$$dz\, d\bar z = d(\mathrm{Re}\, z)\, d(\mathrm{Im}\, z)$$

Charged Bosons (U(1) charge ← complex)

$$\langle z_i \bar z_j \rangle = (A^{-1})_{ij} \qquad \langle z_i \bar z_j z_k \bar z_l \rangle = A^{-1}_{ij} A^{-1}_{kl} + A^{-1}_{il} A^{-1}_{kj}$$

Wick theorem

of the θ's and $\bar{\theta}$'s

$$\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j}+\frac{\partial}{\partial\theta_j}\frac{\partial}{\partial\theta_i}=0$$

anti commutation

⚠ order is important

Fermions

4 pts

$$\boxed{\langle\theta_i\bar{\theta}_j\theta_k\bar{\theta}_l\rangle = A^{-1}_{ij}\cdot A^{-1}_{kl} - A^{-1}_{il}\cdot A^{-1}_{kj}}$$

$$\overset{i}{\circ}\!\longrightarrow\!\overset{\bar{j}}{\circ}\quad\overset{k}{\circ}\!\longrightarrow\!\overset{\bar{l}}{\circ}\quad - \quad \overset{i}{\circ}\quad \overset{j}{\circ}\!\longleftarrow\!\overset{k}{\circ}\quad\overset{l}{\circ}$$

− sign of Wick Theorem for Dirac Fields

QFT $\hookrightarrow a, a^\dagger \quad \{a,a^\dagger\}=1$

Correlation functions

$$\langle\theta_i\bar{\theta}_j\rangle = \frac{\int\Pi d\bar{\theta}\,\Pi d\theta\,\exp(-\bar{\theta}A\theta)\;\theta_i\bar{\theta}_j}{\int\Pi d\bar{\theta}\,\Pi d\theta\,\exp(-\bar{\theta}\cdot A\cdot\theta)}$$

$$\overset{i}{\circ}\!\longrightarrow\!\overset{j}{\circ} \qquad = \frac{[\text{Minor}]_{ij}}{\det A}$$

$$\boxed{\langle\theta_i\bar{\theta}_j\rangle = (A^{-1})_{ij}}$$

But $\langle\bar{\theta}_j\theta_i\rangle = -(A^{-1})_{ij}$