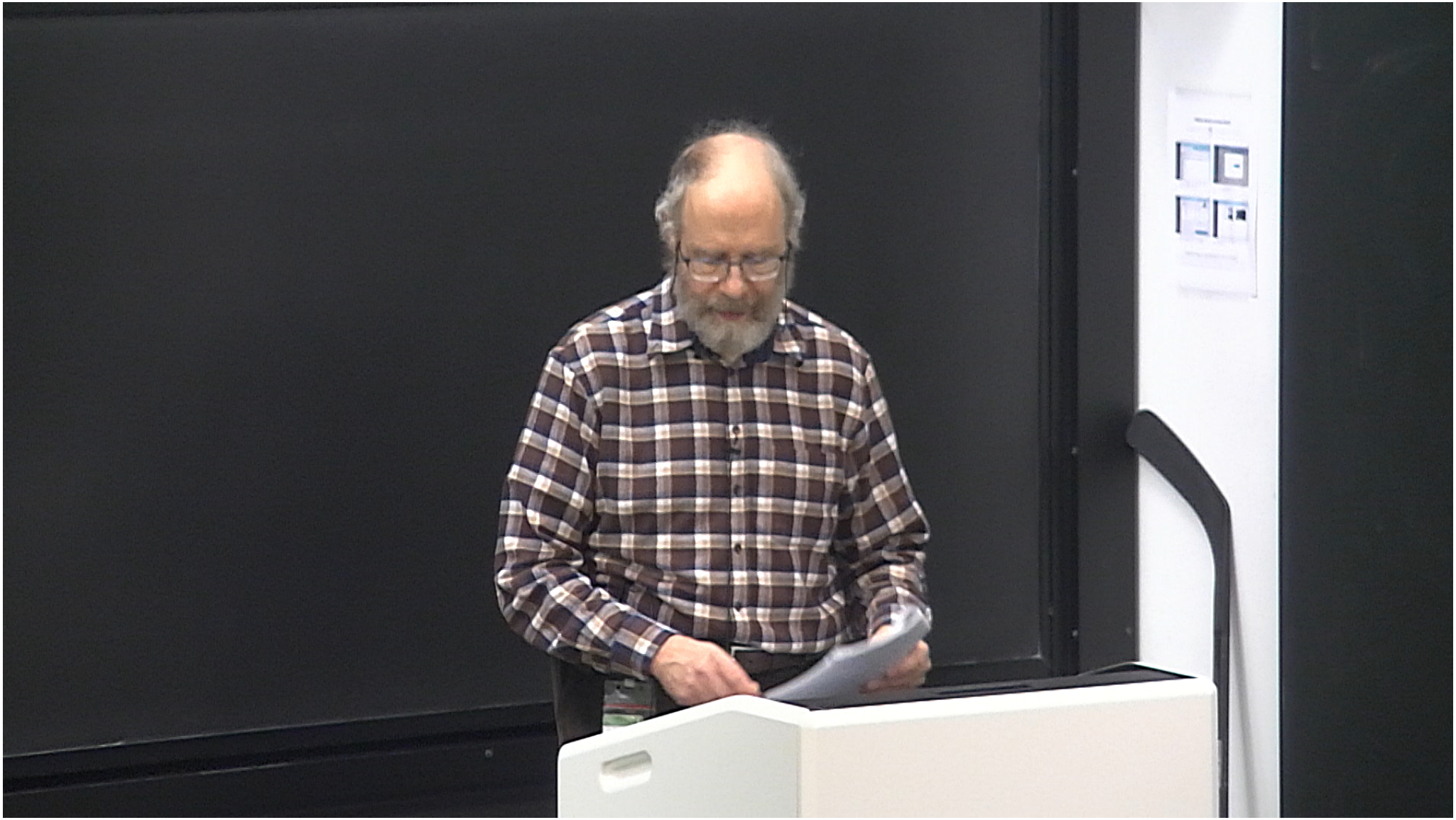
Title: PSI 2018/2019 - Quantum Field Theory II - Lecture 8

Date: Nov 23, 2018 09:00 AM

URL: http://pirsa.org/18110013

Abstract:



Renormalization procedure

μ choosen, g_R is fixed, $M_{phys} = 0$, $\Lambda \to \infty$

in this limit. 1 loop theory is finite

$$\lim_{\Lambda \to \infty} \Gamma_R(\varphi; \Lambda) \text{ exists} = \Gamma_R[\varphi; g_R, \mu]$$

$$\lim_{\Lambda \to \infty} \langle \phi(z_1) \cdots \phi(z_N) \rangle_R = G_R^{(N)}(z_1 \cdots z_N; g_R, \mu)$$

renormalized correlation functions

Renormalization ϕ^4 1 loop massless $d=4$

$$\int D[\phi]\, e^{-\frac{1}{\hbar} S_R[\phi]}$$

quantize with this

$$S_R[\phi] = \int d^4x \left[\frac{A}{2} (\partial_\nu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right]$$

Renormalized action ; + cut off Λ, $|k| < \Lambda$

$\Gamma_R[\varphi]$ renormalized effective action

renormalization scale μ

massless : $\Gamma_{(2)}(p) = p^2 +$ 2 loops terms

continuum limit

renormalized coupling $\Gamma_{(4)}(\{p^3\}) = g_R$ $\{p\} = \{p^{REF}\} \sim \mu$, fixed $\mu \ll \Lambda \nearrow \infty$

$$A = 1$$

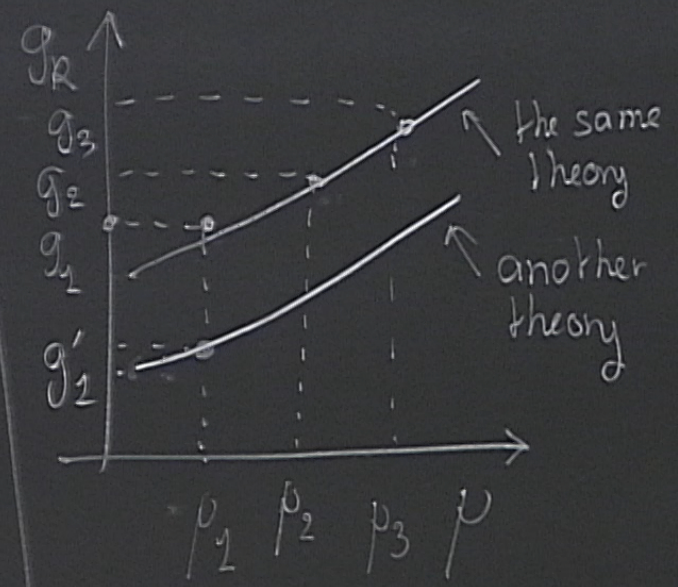
$$B = -\, g_R \, \frac{1}{2} \frac{1}{(4\pi)^2} \cdot \Lambda^2$$

$$C = g_R + g_R^2 \, \frac{3}{2} \, \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

explicit calculation

Th: This works at all orders for "renormalizable theories", also for the massive theory renormalized mass m_R

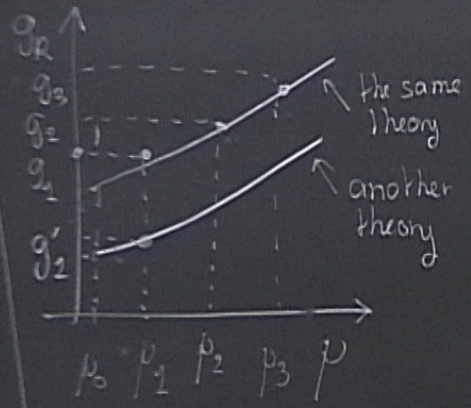
Meaning of μ ? $\longrightarrow$ energy scale where g_R is defined



change of parametrization of the space of ϕ^4 theories

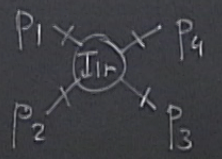
for the
theory
ized mass M_R

Meaning of μ ? → energy scale where g_R is defined



change of parametrization of the space of ϕ^4 theories

4 pt function



$$\Gamma_{(4)}[\{p\}, g_R, \mu] = g_R - g_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + p_2 \to p_3 + p_2 \to p_4\right] + \dots$$

start from μ_0, $g_R = g_0$

change μ, $g(\mu)$

$$\Gamma_{(4)}[\{p\}, g_0, \mu_0] = \Gamma_{(4)}[\{p\}, g(\mu), \mu]$$

$$g_0 = g(\mu) - g(\mu)^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\mu^2}{\mu_0^2}\right) + \dots$$

explicit expression

$g(\mu)$ effective coupling at energy scale μ

for ϕ^4 $g(\mu)$ ↗ with μ strong at small distances, weak at large distances

massless: $\Gamma_{(2)}(p) = p^2 + 2\text{ loops terms}$ continuum limit

renormalized coupling $\Gamma_{(4)}(\{p_i^2\}) = g_R$ $\{p\} = \{p^{REF}\} \sim \mu$, fixed $\mu \ll \Lambda \nearrow \infty$

QED has similar feature (Stückelberg, Peterman, Gell-Mann & Low, Callan-Symazik)

beta functions, β-functions

For the coupling g: $\beta(g) = \mu \frac{d}{d\mu} g(\mu)\Big|_{\mu_0, g_0 \text{ fixed}}$

the theory is fixed

massless: $\Gamma_{(2)}(p) = p^2 + 2\text{ loops terms}$ — continuum limit

renormalized coupling $\Gamma_{(4)}(\{p_i^2\}) = g_R \quad \{p_i\} = \{p_i^{REF}\} \sim \mu$, fixed $\mu \ll \Lambda \nearrow \infty$

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beta functions, β-functions

For the coupling g: the theory is fixed

$$\beta(g(\mu)) = \mu \frac{d}{d\mu} g(\mu) \Big|_{\mu_0, g_0 \text{ fixed}}$$

general definition

For ϕ^4: 1-loop β-function

$$\beta(g) = \frac{3}{(4\pi)^2} g^2 + \dots$$

2 loop terms $\alpha(g^3)$

RGT ... renormalization scale

massless: $\Gamma_{(2)}(p) = p^2 +$ 2 loops terms — continuum limit μ — $\lim_{\Lambda\to\infty} \langle \phi(z_1) \cdot \phi$

renormalized coupling $\Gamma_{(4)}(\{p_i^2\}) = g_R \quad \{p_i\} = \{p_i^{REF}\} \sim \mu$, fixed $\mu \ll \Lambda \nearrow \infty$

QED has similar feature (Stückelberg, Peterman, Gell-Mann & Low, Callan-Symanzik)

beta functions, β-functions

$\mu \frac{d}{d\mu} = \frac{d}{d(\log \mu)}$

For the coupling g: the theory is fixed

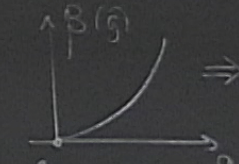
$$\beta(g(\mu)) = \mu \frac{d}{d\mu} g(\mu) \Big|_{\mu_0, g_0 \text{ fixed}}$$

General definition

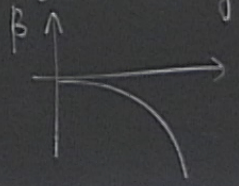
For ϕ^4: 1-loop β-function

$$\beta(g) = \frac{3}{(4\pi)^2} g^2 + \dots$$

2 loop terms $\leftarrow \mathcal{O}(g^3)$



Asymptotic IR freedom
$\Rightarrow$ Strongly coupled in the UV.
ϕ^4, QED



YM, Gauge theories (non-abelian)
$\beta(g) < 0$ Asymptotic Freedom in the UV

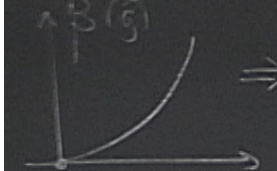
uum limit $\lambda \to \infty$ correlation functions

$\nu \ll \Lambda \nearrow \infty$

, Callan-Symazik)

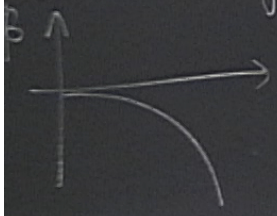
$\nu \frac{d}{d\nu} = \frac{d}{d(\log \nu)}$

$\beta(g)$



$\Rightarrow$ Strongly coupled in the UV.

ϕ^4, QED



YM, Gauge theories (non-abelian)

$\beta(g) < 0$ Asymptotic Freedom in the UV

ϕ^4 theory is not consistent at high energy (Landau)

$\Uparrow$

QED

↑

EW theory

↑

GUT

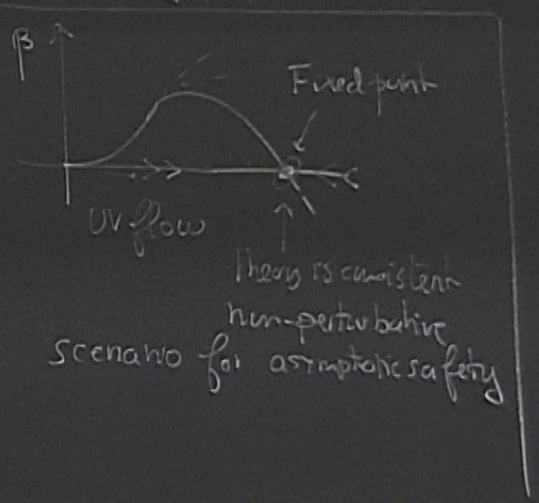
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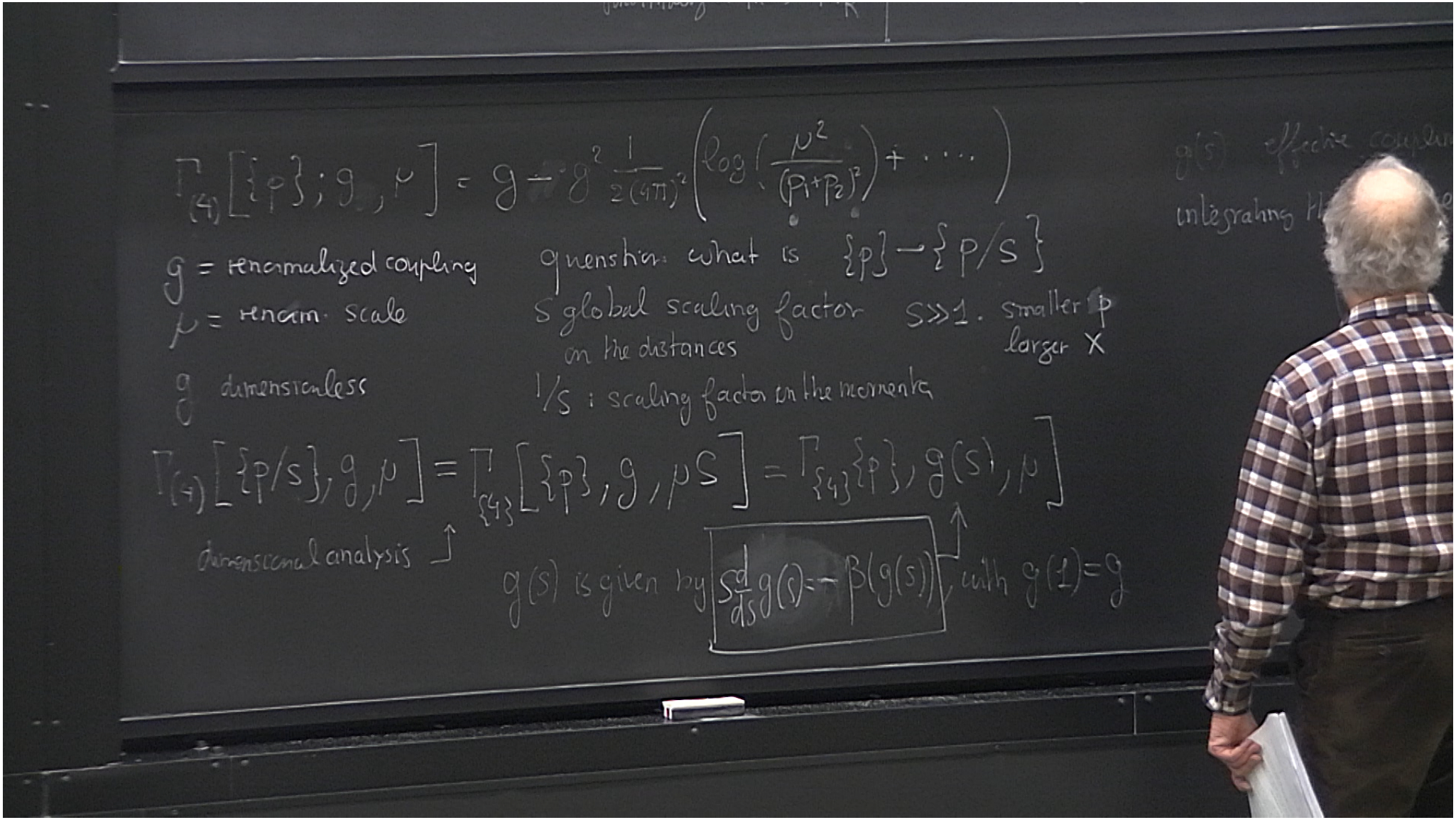
String T, or M theory

or

Asymptotic IR freedom

How $\Gamma^{(n)}_{\{i\}}(\{p_i\})$ varies with the momenta $\{p\}$




$\Gamma_{(4)}[\{p\}; g, \mu] = g - g^2 \frac{1}{2(4\pi)^2}\left(\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \dots\right)$
g = renormalized coupling
μ = renorm. scale
g dimensionless
question: what is $\{p\} \to \{p/s\}$
s global scaling factor on the distances $s \gg 1$. smaller p larger x
$1/s$: scaling factor on the momenta
$\Gamma_{(4)}[\{p/s\}, g, \mu] = \Gamma_{\{4\}}[\{p\}, g, \mu s] = \Gamma_{\{4\}}[\{p\}, g(s), \mu]$
dimensional analysis
$g(s)$ is given by $\boxed{s\frac{d}{ds}g(s) = -\beta(g(s))}$ with $g(1) = g$

$\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \dots \Big)$

it is $\{p\} \to \{p/s\}$

g factor $s \gg 1$. smaller p
larger x

on the momenta

$= \Gamma_{\{q\}}\{p\}, g(s), \mu \Big]$

$\frac{d}{ds} g(s) = -\beta(g(s))$ with $g(1) = g$

$g(s)$ effective coupling constant when we rescale the distances

integrating the Flow equation

$[g, \mu_0] \equiv [g(\frac{\mu}{\mu_0}), \mu]$

$[g, \mu s] \equiv [g(s), \mu]$

choose $\mu_0 = \mu s$ $\quad s = \mu_0/\mu$

$\beta(g(\frac{\mu}{\mu_0})) = (\frac{\mu}{\mu_0}) \frac{d\, g(\mu/\mu_0)}{d(\mu/\mu_0)}$ $\quad g(1) = g$, μ_0 fixed

$\mu \frac{d}{d\mu} g(\mu/\mu_0) = \beta(g(\mu/\mu_0))$

$g(1) = g$

$\mu \frac{d}{d\mu} \to -s \frac{d}{ds}$

$$-\sum_i p_i \frac{\partial}{\partial p_i} \Gamma_4[p_i, g, \mu] = \underbrace{s \frac{dg(s)}{ds}}_{-\beta(g)} \cdot \frac{\partial}{\partial g} \Gamma_4(p, g, \mu]$$

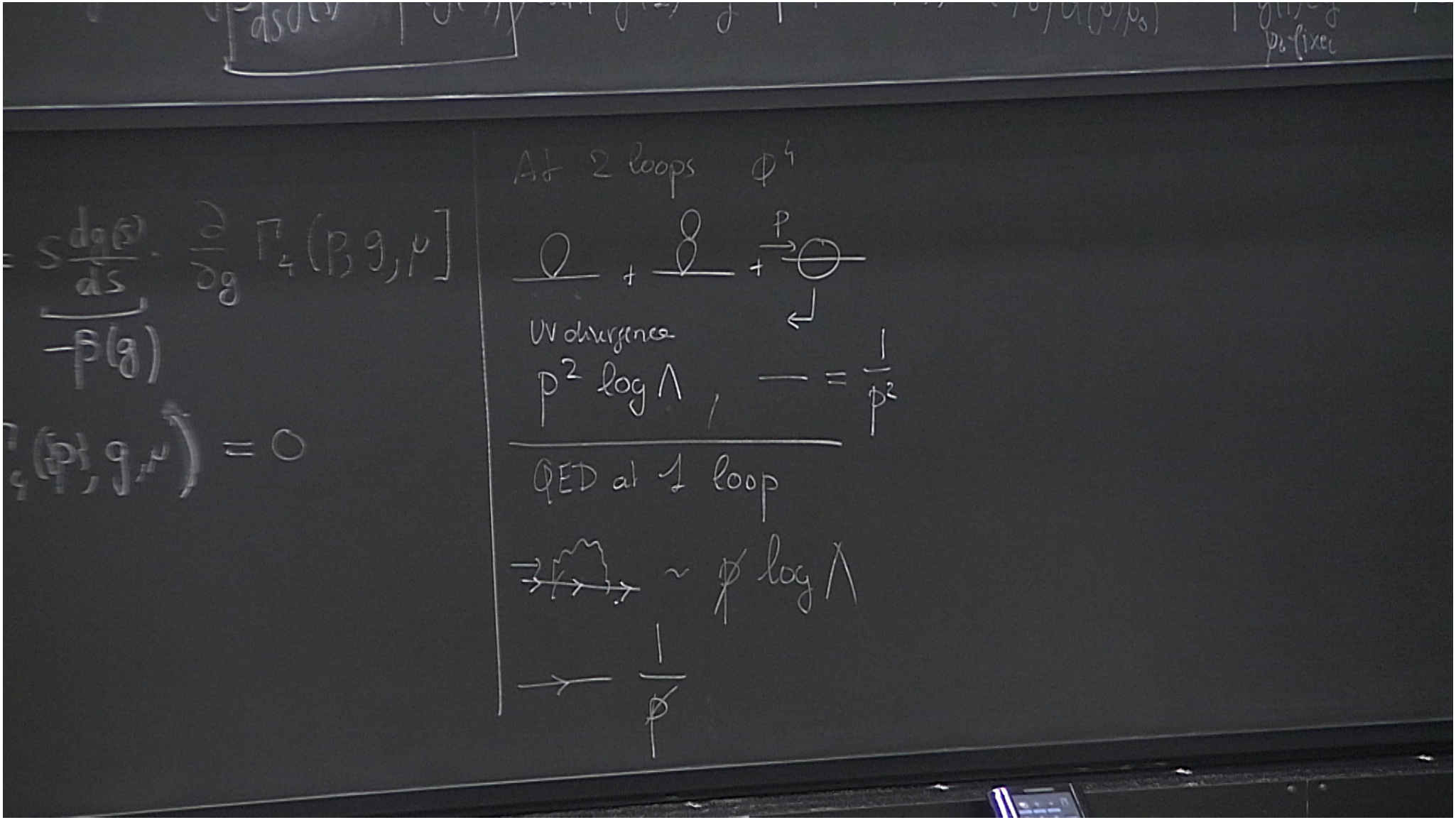
$$\left[\sum_i p_i \frac{\partial}{\partial p_i} - \beta(g) \frac{\partial}{\partial g}\right] \Gamma_4(\{p\}, g, \mu) = 0$$

Callan-Symanzik equation

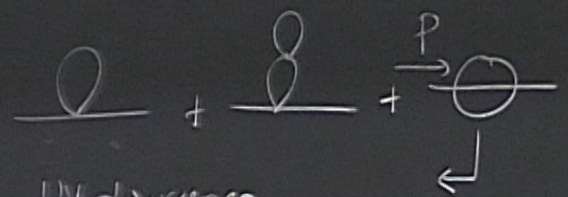
valid at 1 loop

At

0


At 2 loops ϕ^4
UV divergence
$p^2 \log \Lambda$, $\frac{1}{} = \frac{1}{p^2}$
QED at 1 loop
$\sim \not{p} \log \Lambda$
$\frac{1}{\not{p}}$

At 2 loops ϕ^4



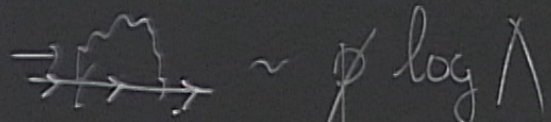
UV divergence

$p^2 \log \Lambda$, $\frac{}{} = \frac{1}{p^2}$

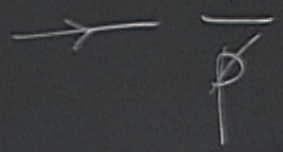
wave function or field renormalization

$A = 1 + \cdot \log \Lambda$

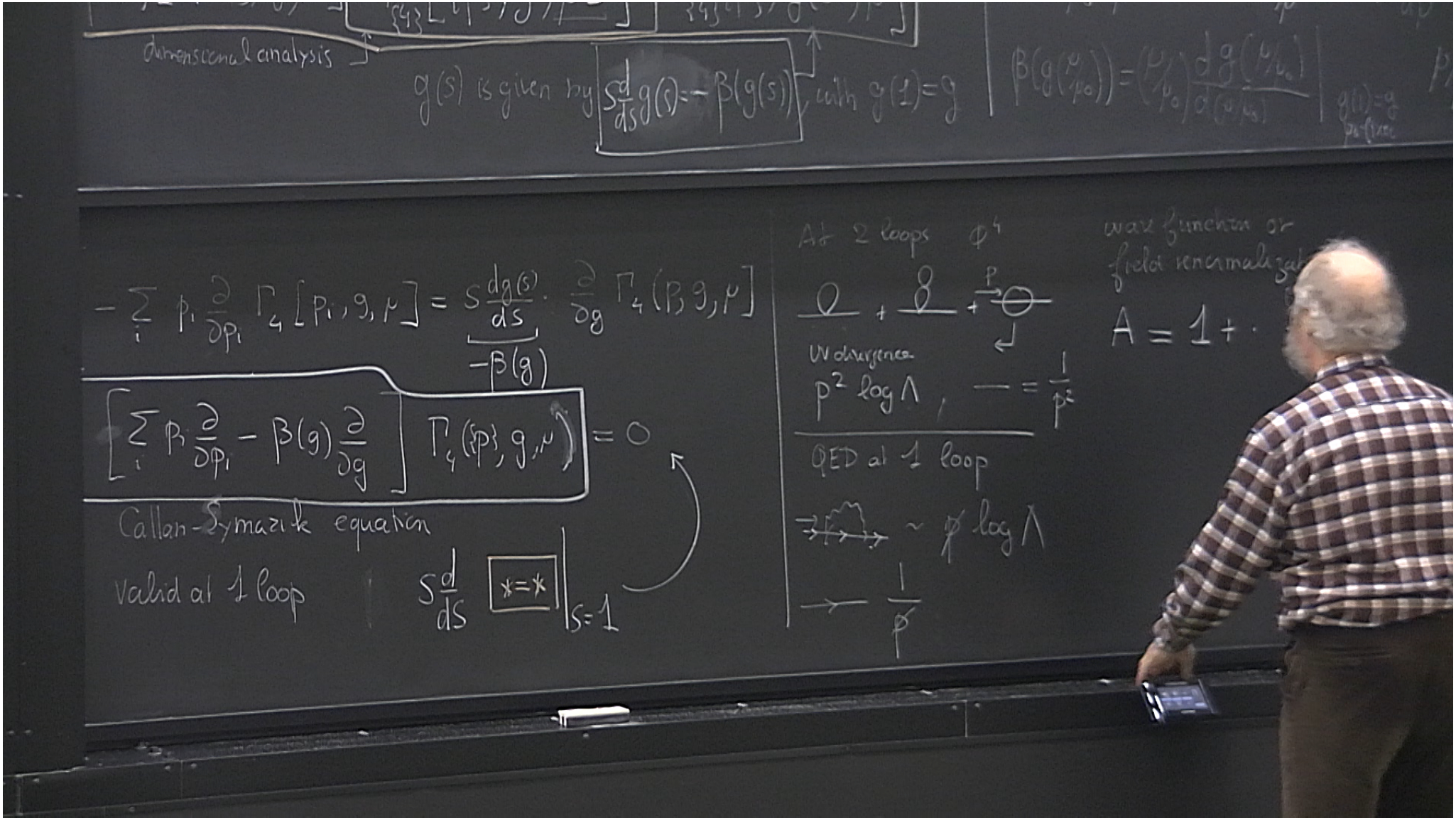
QED at 1 loop

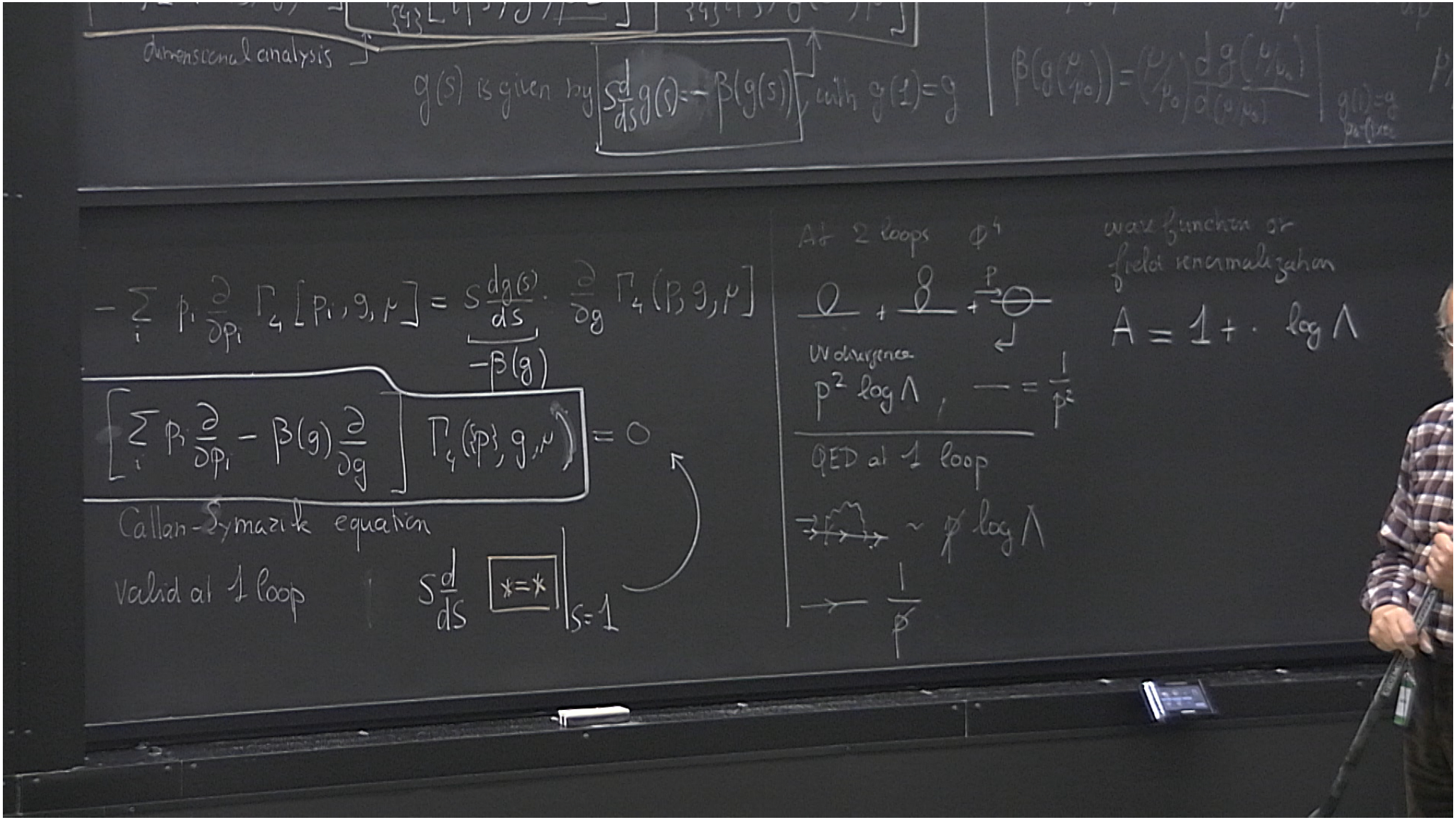


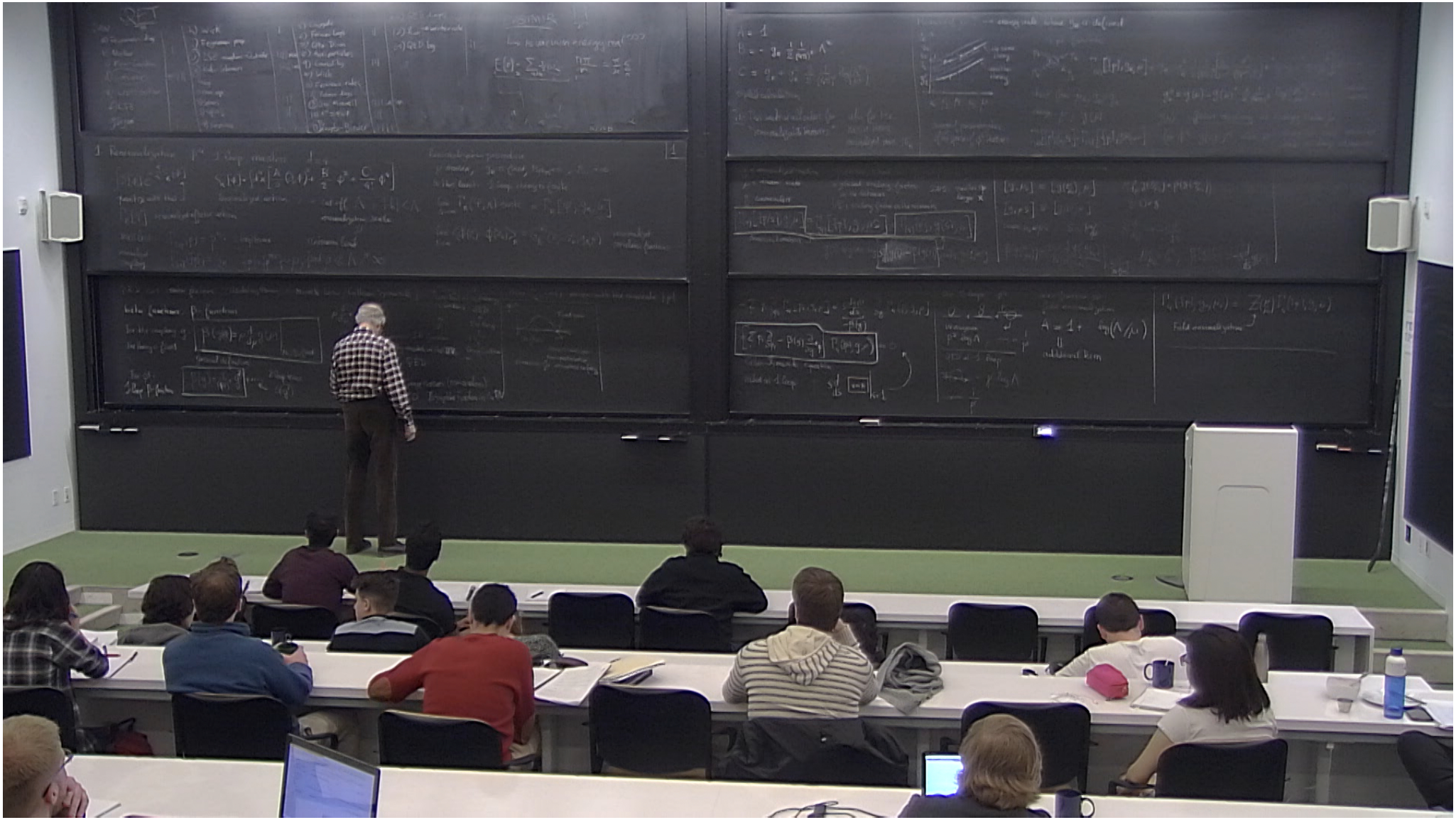
$\sim \not{p} \log \Lambda$



$\frac{1}{\not{p}}$


dimensional analysis
g(s) is given by $s\frac{d}{ds}g(s) = -\beta(g(s))$ with $g(1) = g$
$\beta(g(\frac{\mu}{\mu_0})) = (\frac{\mu}{\mu_0})\frac{dg(\frac{\mu}{\mu_0})}{d(\frac{\mu}{\mu_0})}$
$-\sum_i p_i \frac{\partial}{\partial p_i}\Gamma_4[p_i, g, \mu] = s\frac{dg(s)}{ds}\cdot\frac{\partial}{\partial g}\Gamma_4(p, g, \mu)$
$-\beta(g)$
$\left[\sum_i p_i \frac{\partial}{\partial p_i} - \beta(g)\frac{\partial}{\partial g}\right]\Gamma_4(\{p\}, g, \mu) = 0$
Callan-Symanzik equation
valid at 1 loop
$s\frac{d}{ds}$ *=* $|_{s=1}$
At 2 loops ϕ^4
UV divergence
$p^2 \log\Lambda$, $\frac{1}{p^2}$
QED at 1 loop
$\sim \not{p}\log\Lambda$
$\frac{1}{\not{p}}$
wave function or field renormalization
$A = 1 +$


dimensional analysis
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$-\sum_i p_i \frac{\partial}{\partial p_i}\Gamma_4[p_i, g, \mu] = s\frac{dg(s)}{ds}\cdot\frac{\partial}{\partial g}\Gamma_4(p, g, \mu)$
$-\beta(g)$
$\left[\sum_i p_i \frac{\partial}{\partial p_i} - \beta(g)\frac{\partial}{\partial g}\right]\Gamma_4(\{p\}, g, \mu) = 0$
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QED at 1 loop
$\sim \not{p}\log\Lambda$
$\frac{1}{\not{p}}$
wave function or field renormalization
$A = 1 + \cdot \log\Lambda$



s : scaling factor in the increments

$$\Gamma^{(4)}[\{p/s\}, g, \mu] = \Gamma_{\{4\}}[\{p\}, g, \mu s] = \Gamma_{\{4\}}[\{p\}, g(s), \mu]$$

dimensional analysis

$g(s)$ is given by $s\frac{d}{ds}g(s) = -\beta(g(s))$ with $g(1) = g$

$$[g, \mu s] = [g(s), \mu]$$

choose $\mu_0 = \mu s$ $\quad s = \mu_0/\mu$ $\quad \mu\frac{d}{d\mu} \to -$

$$\beta\left(g\left(\frac{\mu}{\mu_0}\right)\right) = \left(\frac{\mu}{\mu_0}\right)\frac{d\, g\left(\frac{\mu}{\mu_0}, 1\right)}{d\left(\frac{\mu}{\mu_0}\right)}$$

$g(1) = g$ perturbative $\quad \mu/\mu_0 =$

$$-\sum_i p_i \frac{\partial}{\partial p_i}\Gamma_4[p_i, g, \mu] = s\frac{dg(s)}{ds}\cdot\frac{\partial}{\partial g}\Gamma_4(\{p\}, g, \mu)$$

$$\underbrace{s\frac{dg(s)}{ds}}_{-\beta(g)}$$

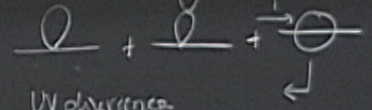
$$\left[\sum_i p_i\frac{\partial}{\partial p_i} - \beta(g)\frac{\partial}{\partial g} + \gamma\right]\Gamma_4(\{p\}, g, \mu) = 0$$

Callan-Symanzik equation

valid at 1 loop

$$s\frac{d}{ds}\boxed{*=*}\Big|_{s=1}$$

At 2 loops ϕ^4



UV divergence

$p^2 \log\Lambda$, $\quad - = \frac{1}{p^2}$

QED at 1 loop



$\sim \not{p}\log\Lambda$

$\to \frac{1}{\not{p}}$

wave function or field renormalization

$$A = 1 + \cdot\log(\Lambda/\mu)$$

$\Downarrow$

additional term.

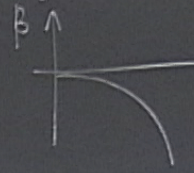
$$\Gamma_4(\{p\}, g_0, \mu_0) = Z\left(\frac{\mu}{\mu_0}\right) \Gamma_4(\{p\}, g, \mu)$$

Field renormalization

General definition

For ϕ^4:

1-loop β-function $\beta(g) = \frac{3}{(4\pi)^2} g^2$ + ... ← 2 loop terms $\mathcal{O}(g^3)$



ϕ^4, QED

YM, Gauge theories (non-abelian)

$\beta(g) < 0$ Asymptotic Freedom in the UV

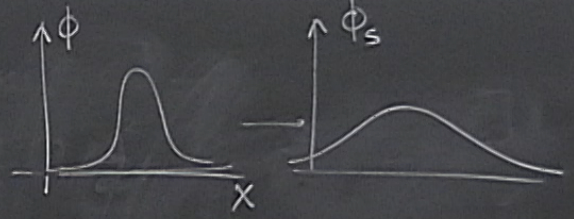
Scena

1. Renormalization and scale invariance

Quantum

$d = 4$ massless ϕ^4 is scale invariant

$$S[\phi] = \int d^4x \left(\frac{1}{2}\partial_\nu \phi\right)^2 + \frac{g}{4!}\phi^4$$



scale transformation $\phi \longrightarrow \phi_s$ $\quad \phi_s(x) = \frac{1}{s}\phi(x/s)$

$S[\phi_s] = S[\phi]$ for any ϕ, and same g

$\Rightarrow$ off shell global symmetry

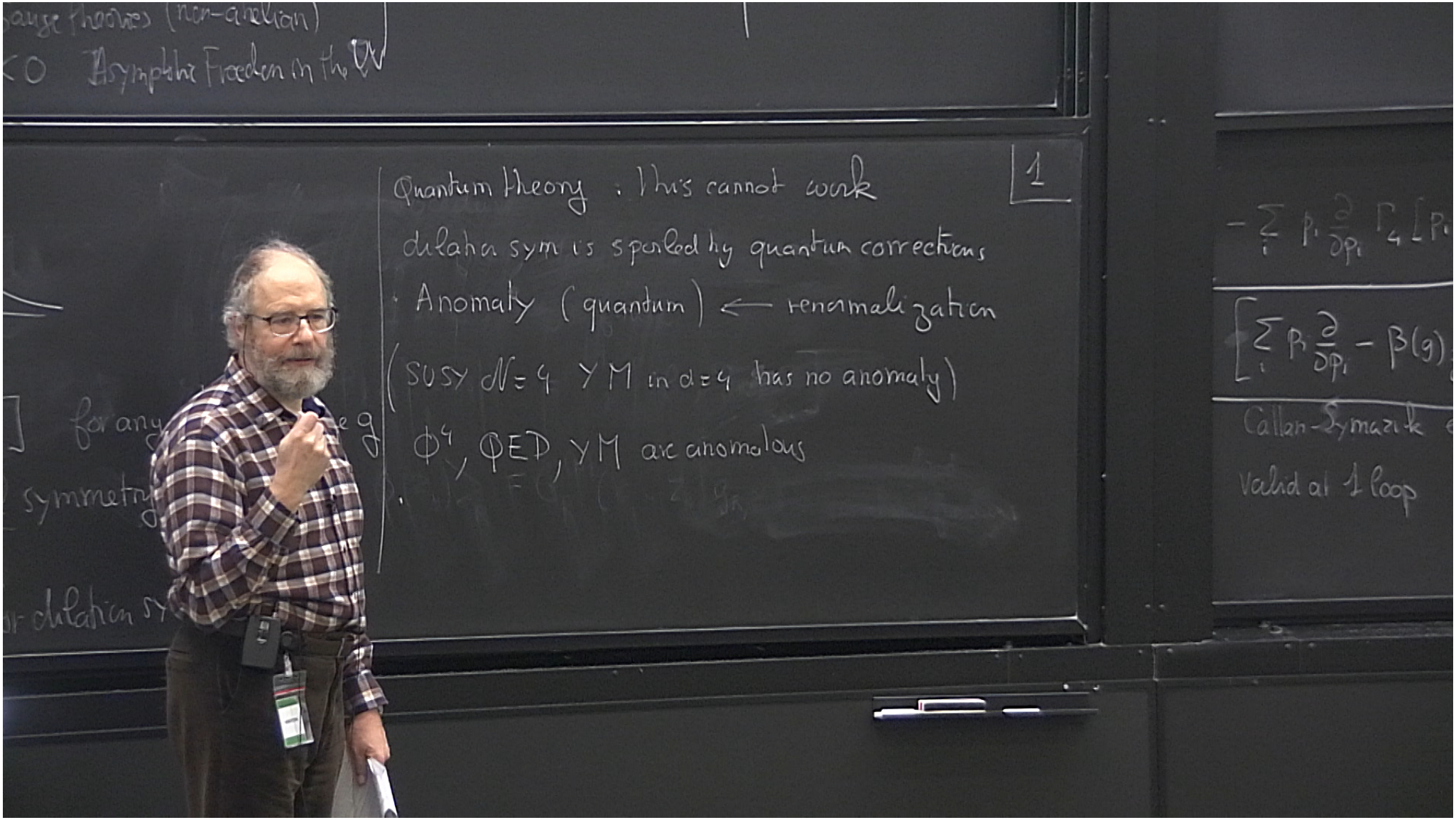
mixed ϕ and X

Scale symmetry or dilation sym.

Scaling dimension of ϕ is $(\text{length})^{-1}$

action is dimensionless and g also

classical


gauge theories (non-abelian)
< 0 Asymptotic Freedom in the UV
Quantum theory . This cannot work
dilatation sym is spoiled by quantum corrections
· Anomaly (quantum) ← renormalization
(SUSY $\mathcal{N}=4$ YM in $d=4$ has no anomaly)
ϕ^4, QED, YM are anomalous
1
Callan-Symanzik
Valid at 1 loop

λ in 6 dimensional less ang g also classical

Möether theorem $\Rightarrow$ conserved current

$d \neq 4$

$$J^\nu_{\text{scale}} = T^\nu_\lambda x^\lambda + \phi \partial^\nu \phi$$

T: E.M. tensor

ϕ: dim of $\phi = -1$

$$\partial_\nu J^\nu = (d-4)\frac{g}{4!}\phi^4$$

classically this is 0 in $d=4$

But qua

Classical

mixed ϕ and X

Scale symmetry or dilation sym.

But quantum theory $\partial_\mu J^\mu = \beta(g) \frac{1}{4!} \phi^4$

β functon = anomaly

Σ the beta function

ent

of $\phi = -1$

ically this is

n $d=4$