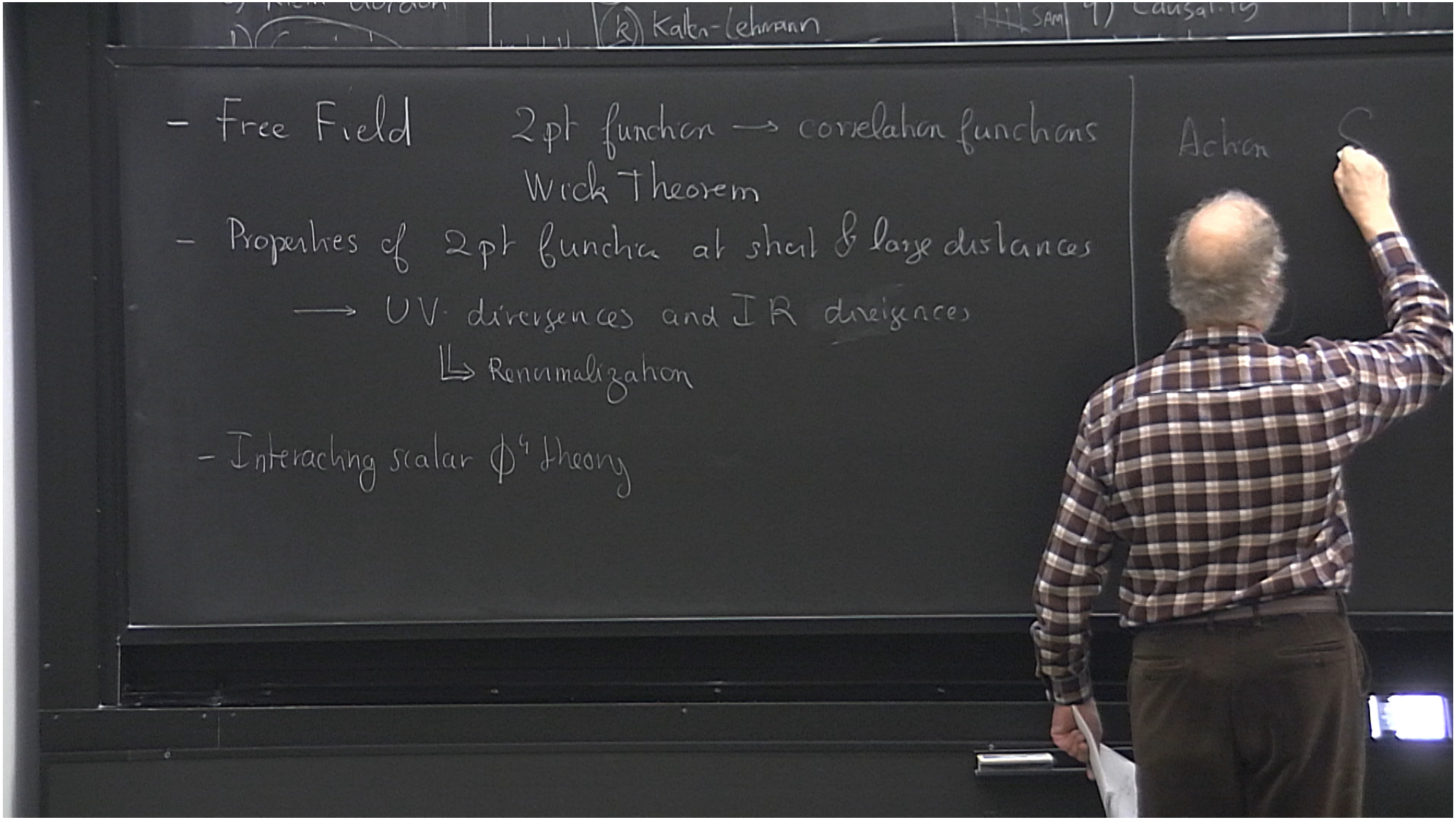
Title: PSI 2018/2019 - Quantum Field Theory II - Lecture 3

Date: Nov 15, 2018 09:00 AM

URL: http://pirsa.org/18110007

Abstract:


- Free Field 2pt function → correlation functions
Wick Theorem
- Properties of 2pt functions at short & large distances
→ UV divergences and IR divergences
↳ Renormalization
- Interacting scalar $\phi^4$ theory
Action S

Action (Euclidean) $S_E[\phi] = \int d^d x \left[ \frac{1}{2} (\partial_x \phi)^2 + \frac{m^2}{2} \phi^2 \right]$

Propagator $\langle \phi(x_1)\, \phi(x_2) \rangle = \frac{\hbar}{} G(x_1 - x_2)$

$$G(x) = \int \frac{d^d k}{(2\pi)^D} \frac{e^{ikx}}{k^2 + m^2} \xrightarrow{\text{Wick Rot}} G_F(x)$$

correlation functions

$\langle \phi(x_1) \cdots \phi(x_N) \rangle \Longrightarrow$ matrix elements of $a^\dagger \cdots a^\dagger |0\rangle \Rightarrow$ Hilbert Space and $\langle \cdot | \cdot \rangle$

$\Downarrow$

$\langle 0 | a \cdots a \, | a^\dagger \cdots a^\dagger | 0 \rangle$

$\dot{\phi})^2 + \frac{m^2}{2}\phi^2\Big]$

$G(x_1 - x_2)$

Wick Rd $\longrightarrow$ $G_F(x)$

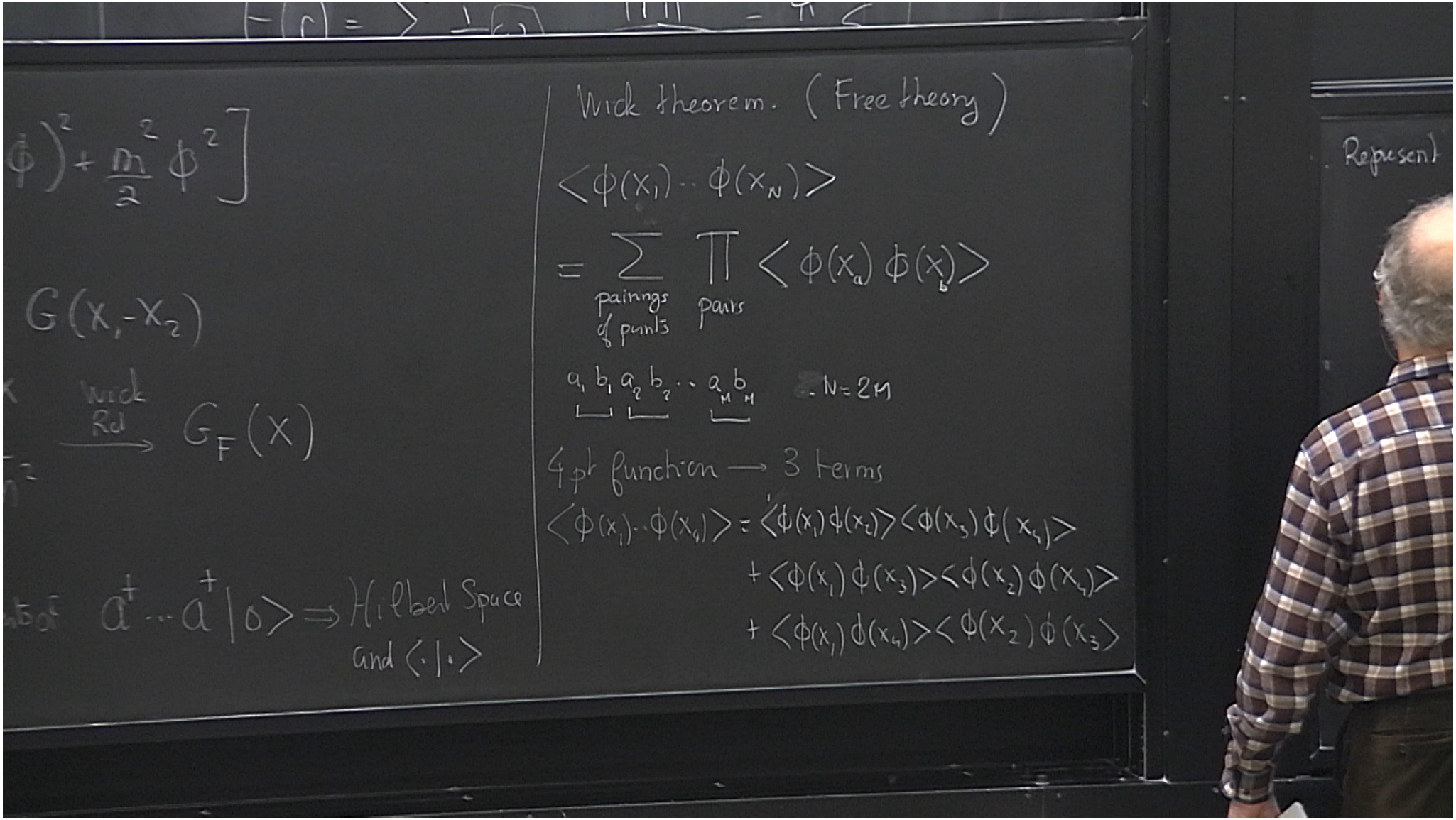
$a^\dagger \cdots a^\dagger |0\rangle \Rightarrow$ Hilbert Space and $\langle \cdot | \cdot \rangle$

Wick theorem.

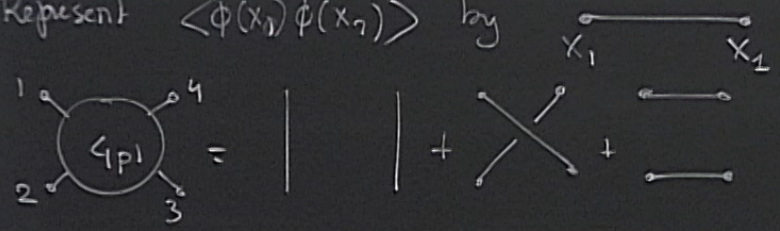
$$\langle \phi(x_1) \cdots \phi(x_N) \rangle$$

$$= \sum_{\substack{\text{pairings} \\ \text{of pairs}}} \prod_{\text{pairs}} \langle \phi(x_a)\, \phi(x_b) \rangle$$

$a_1 b_1 \; a_2 b_2 \cdots a_M b_M$ $\quad N = 2M$

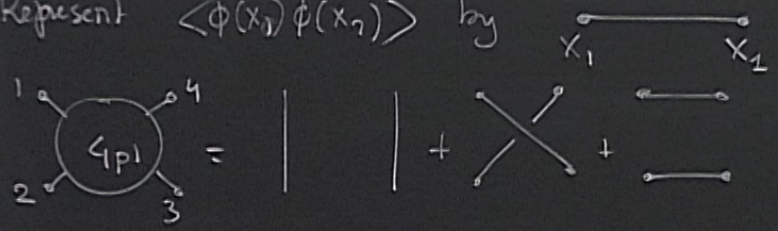

$\phi)^2 + \frac{m^2}{2}\phi^2]$
$G(x_1, x_2)$
Wick 2d
$G_F(x)$
$a^\dagger \cdots a^\dagger |0\rangle \Rightarrow$ Hilbert Space and $\langle \cdot | \cdot \rangle$
Wick theorem. (Free theory)
$\langle \phi(x_1) \cdots \phi(x_N) \rangle$
$= \sum_{\text{pairings of pairs}} \prod_{\text{pairs}} \langle \phi(x_a)\phi(x_b) \rangle$
$a_1 b_1 a_2 b_2 \cdots a_M b_M$ $N = 2M$
4pt function $\longrightarrow$ 3 terms
$\langle \phi(x_1) \cdots \phi(x_4) \rangle = \langle \phi(x_1)\phi(x_2) \rangle \langle \phi(x_3)\phi(x_4) \rangle$
$+ \langle \phi(x_1)\phi(x_3) \rangle \langle \phi(x_2)\phi(x_4) \rangle$
$+ \langle \phi(x_1)\phi(x_4) \rangle \langle \phi(x_2)\phi(x_3) \rangle$
Represent

Represent $\langle \phi(x_1)\phi(x_2) \rangle$ by




- Feynman diagram representation

Represent $\langle \phi(x_1)\phi(x_2)\rangle$ by




- Feynman diagram representation

Comes from the fact that

$$\langle \phi(x_1)\cdots\phi(x_N)\rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\,\phi(x_1)\cdots\phi(x_N)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$$

expectation

$\phi$ is a Gaussian random variable $= \mathbb{E}\left[\phi(x_1)\cdots\phi(x_N)\right]$

$F(r) = \sum \frac{1}{r} \frac{n\pi}{} - \pi$

$\left.\right]^2$

$\zeta_F(x)$

$+ \; |0\rangle \Rightarrow$ Hilbert Space and $\langle \cdot | \cdot \rangle$

**Wick theorem. (Free theory)**

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle$$

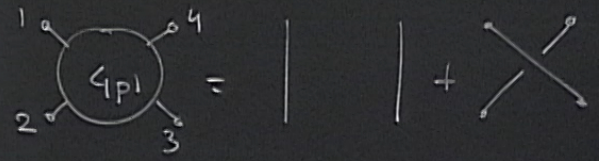
$$= \sum_{\substack{\text{pairings} \\ \text{of points}}} \prod_{\text{pairs}} \langle \phi(x_a)\, \phi(x_b) \rangle$$

$a_1 b_1 \; a_2 b_2 \cdots a_M b_M$ $\quad N = 2M$ $\quad N$ odd $\Rightarrow$ zero

4pt function $\longrightarrow$ 3 terms

$$\langle \phi(x_1) \cdots \phi(x_4) \rangle = \langle \phi(x_1)\phi(x_2) \rangle \langle \phi(x_3)\phi(x_4) \rangle + \langle \phi(x_1)\phi(x_3) \rangle \langle \phi(x_2)\phi(x_4) \rangle + \langle \phi(x_1)\phi(x_4) \rangle \langle \phi(x_2)\phi(x_3) \rangle$$

Represent $\langle \phi(x_1) \phi(x_2) \rangle$ by




- Feynman diagram representation

Comes from the fact that

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle = \frac{\int \mathcal{D}[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi}{\int \mathcal{D}[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}}$$

expectation

$\phi$ is a Gaussian random variable $= \mathbb{E}\left[ \phi(x_1) \cdots \phi(x_N) \right]$

$F(r) = \sum \frac{1}{(\quad)} \quad n\pi \quad - \pi$

$^2]$

)Hilbert Space
and $\langle \cdot | \cdot \rangle$

**Wick theorem. (Free theory)**

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle$$

$$= \sum_{\substack{\text{pairings} \\ \text{of points}}} \prod_{\text{pairs}} \langle \phi(x_a)\, \phi(x_b) \rangle$$

$a_1 b_1 \; a_2 b_2 \cdots a_M b_M \quad N = 2M$

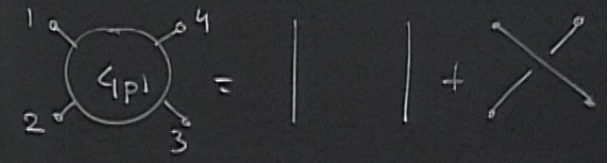
N odd $\Rightarrow$ zero

$\phi \to -\phi$ Symmetry

4pt function $\longrightarrow$ 3 terms

$$\langle \phi(x_1) \cdots \phi(x_4) \rangle = \langle \phi(x_1)\phi(x_2) \rangle \langle \phi(x_3)\phi(x_4) \rangle + \langle \phi(x_1)\phi(x_3) \rangle \langle \phi(x_2)\phi(x_4) \rangle + \langle \phi(x_1)\phi(x_4) \rangle \langle \phi(x_2)\phi(x_3) \rangle$$

Represent $\langle \phi(x_1) \phi(x_2) \rangle$ by




- Feynman diagram representation

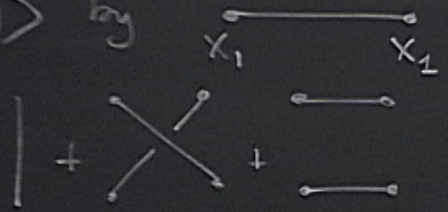
Comes from the fact that

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi}{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}}$$

expectation

$\phi$ is a Gaussian random variable $= \mathbb{E}\left[ \phi(x_1) \cdots \phi(x_N) \right]$

$x_2$)> by




representation

that

$$D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi(x_1) \cdots \phi(x_N)$$

$$D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}$$

$$\left[\phi(x_1) \cdots \phi(x_N)\right]$$

Short distance properties (Euclidean)

$$G(x_1, x_2) = G(|x_1 - x_2|)$$

## Short distance properties (Euclidean)

$$G(X_1, X_2) = G(|X_1 - X_2|) = |X_1 - X_2|^{2-d}\, \mathbb{C}(d)$$

unit vector

coefficient

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2 + m^2} = |x|^{2-d} \cdot \underbrace{\int \frac{d^d k\, e^{i\vec{\varphi}\cdot\vec{e}}}{(2\pi)^d\, \varphi^2}}_{\text{a number}}$$

$x$ small, $|x|\, m \ll 1$

then $k \cdot x \sim O(1)$

rescale $k = \varphi / |x|$

## Short distance properties (Euclidean)

$$G(X_1, X_2) = G(|X_1 - X_2|) = |X_1 - X_2|^{2-d}\, \mathbb{C}(d)$$

$\mathbb{C}(d)$: coefficient

$\to \infty$ at short distance if $d > 2$

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2 + m^2} = |x|^{2-d} \cdot \underbrace{\int \frac{d^d \varphi}{(2\pi)^d} \frac{e^{i\vec{\varphi}\cdot\vec{e}}}{\varphi^2}}_{\text{a number}}$$

unit vector

$X$ small, $|X| m \ll 1$

then $k \cdot x \sim O(1)$

rescale $k = \varphi / |x|$

works if $d > 2$

$$\int \frac{d^d \varphi}{\varphi^2} \to \int_0 d|\varphi| |\varphi|^{d-3} \times \text{angular}$$

near $\varphi = 0$

## Short distance properties (Euclidean)

$$G(X_1, X_2) = G(|X_1 - X_2|) = |X_1 - X_2|^{2-d} \, \mathbb{C}(d)$$

($\mathbb{C}(d)$: coefficient; $|X_1 - X_2|$: unit vector)

$\to \infty$ at short distance if $d > 2$

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2 + m^2} = |x|^{2-d} \cdot \underbrace{\int \frac{d^d \varphi}{(2\pi)^d} \frac{e^{i\vec{\varphi}\cdot\vec{e}}}{\varphi^2}}_{\text{a number}}$$

$x$ small, $|x| m \ll 1$

then $k \cdot x \sim O(1)$

rescale $k = \varphi / |x|$

works if $d > 2$

$$\int \frac{d^d \varphi}{\varphi^2} \to \int_0 d|\varphi| |\varphi|^{d-3} \times \text{angular}$$

near $\varphi = 0$

If $d = 2$ log divergence

$$G(x) = -\frac{1}{2\pi} \log\left(\frac{m \cdot |x|}{2}\right) + \frac{\gamma}{2\pi} + \cdots$$

($\gamma$: Euler constant)

New to QFT! in Q.M $d = 1$

$$G_{d=1}(x) = \frac{1}{2m} e^{-m\cdot|x|}$$ finite at $|x| = 0$

Euler constant

$$\frac{\gamma}{2\pi} + \cdots$$

New to QFT! in Q.M $d = 1$

$$G_{d=1}(x) = \frac{1}{2m} e^{-m\cdot|x|}$$ finite at $|x| = 0$

$X = (t)$, $\phi(x) = q(t)$ position of the particle

particle, mass $= 1$, in a harmonic potential $\frac{\omega^2}{2} q^2$

with $\omega$ = mass of the field

New to QFT! in Q.M $d = 1$

$$G_{d=1}(x) = \frac{1}{2m} e^{-m\cdot|x|}$$ finite at $|x| = 0$

$X = (t)$, $\phi(x) = q(t)$ position of the particle

particle, mass = 1, in a harmonic potential $\frac{\omega^2}{2} q^2$

with $\omega$ = mass of the field

harmonic oscillator

distance

Euler constant

$$\frac{\gamma}{2\pi} + \cdots$$

$\phi(x_N)]$

Rescale $k = \varphi/|x|$

$\int \frac{d\varphi}{\varphi^2} \to \int_0 d|\phi| \cdot |\phi| \times$ angular

near $\phi = 0$

...es

$= \infty$

...tion & time

...roblematic.

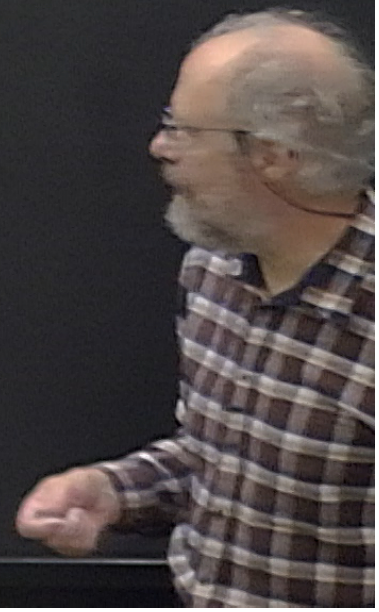
...rmalization

Euclidean space time

functional integral = probabilistic average

over a space of field configurations

$d \geq 2$. $\phi(x)$ is a random distribution



$\phi(x_N)]$

rescale $k = \varphi/|x|$

$\int \frac{d\varphi}{\varphi^2} \to \int_0 d|\phi| \cdot |\phi| \times$ angular

near $\phi = 0$

...es

... $= \infty$

...tion & time

...roblematic.

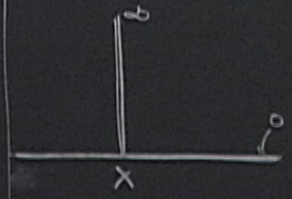
...rmalization

Euclidean space time

functional integral = probabilistic average

over a space of field configurations

$d \geq 2$. $\phi(x)$ is a random distribution




Dirac $\delta$-function

$\phi(x_N)]$

rescale $k = \varphi/|x|$

$\int \frac{d\varphi}{\varphi^2} \to \int_0 d|\varphi| \cdot |\varphi| \times$ angular

near $\varphi = 0$

$= \infty$

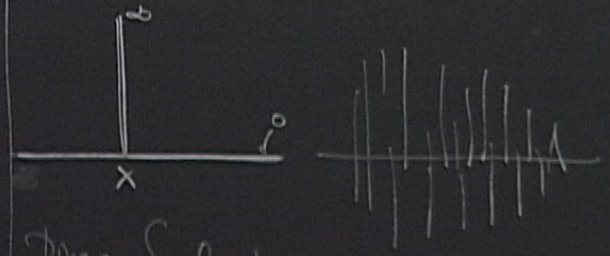
roblematic.

ormalization

Euclidean space time

functional integral = probabilistic average over a space of field configurations

$d \geq 2$. $\phi(x)$ is a random distribution




Dirac $\delta$-function

$\phi(x_N)]$

rescale $k = \varphi / |x|$

$$\int \frac{d\varphi}{\varphi^2} \to \int_0 d|\phi| \cdot |\phi| \times \text{angular}$$

near $\phi = 0$

...e s

$= \infty$

...tion & time

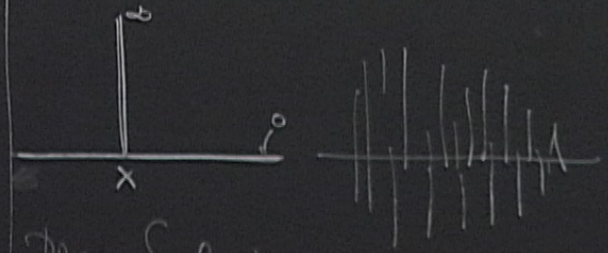
...roblematic.

...rmalization

Euclidean space time

functional integral = probabilistic average over a space of field configurations

$d \geq 2$. $\phi(x)$ is a random distribution




Dirac $\delta$-function



Gaussian free field

GFF

$\phi(x_N)]$

rescale $k = \varphi / |x|$

$$\int \frac{d\varphi}{\varphi^2} \to \int_0 d|\varphi| \cdot |\varphi| \times \text{angular}$$

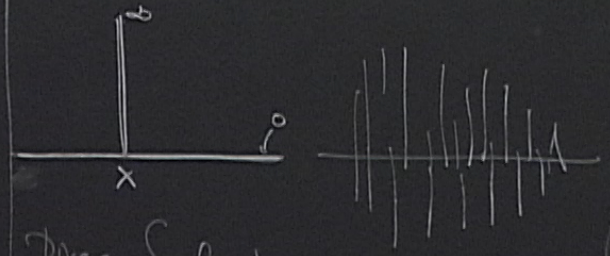
near $\varphi = 0$

$= \infty$

Euclidean space time

functional integral = probabilistic average over a space of field configurations

$d \geq 2$. $\phi(x)$ is a random distribution




Dirac $\delta$ function

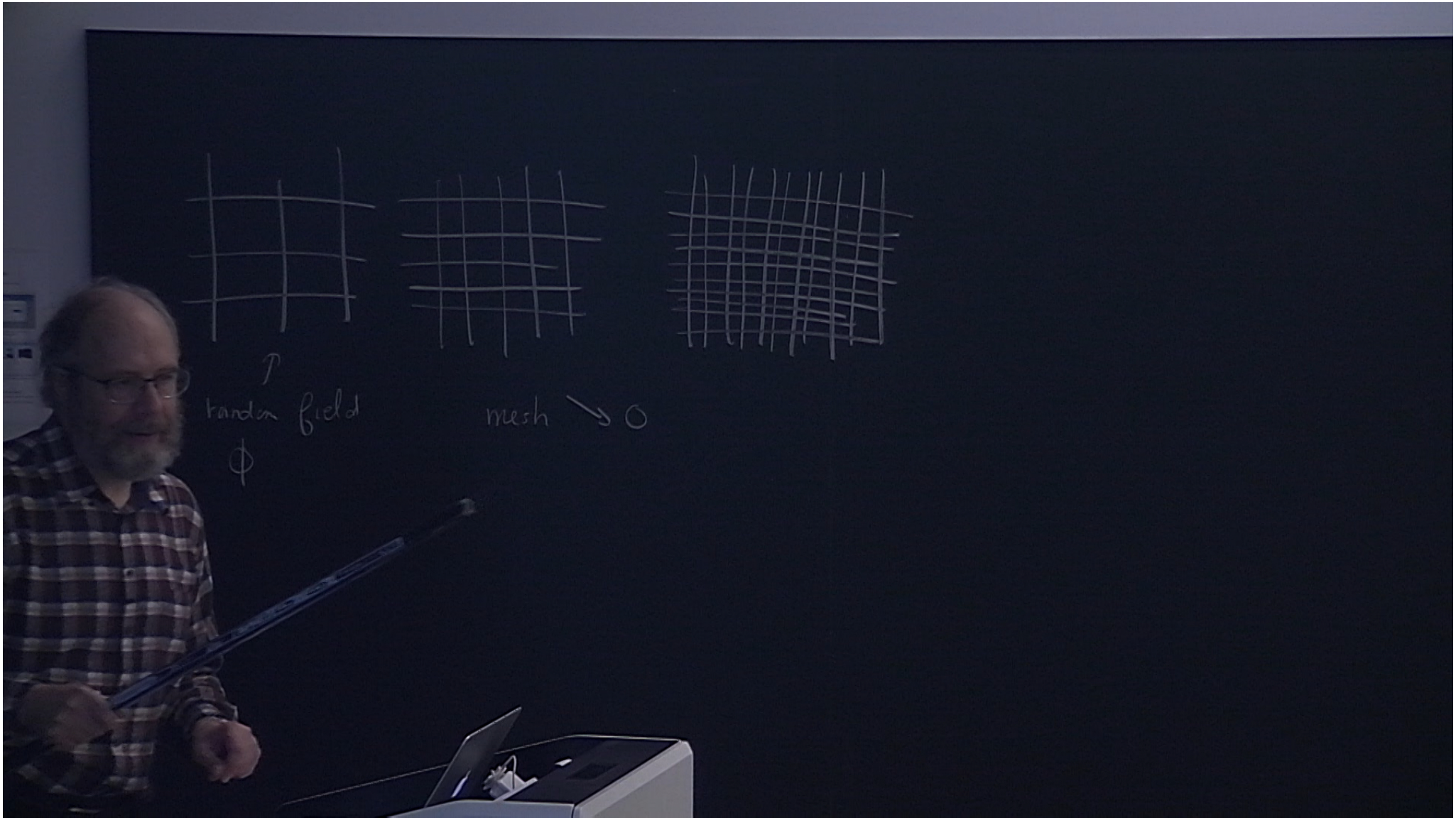


Gaussian free field

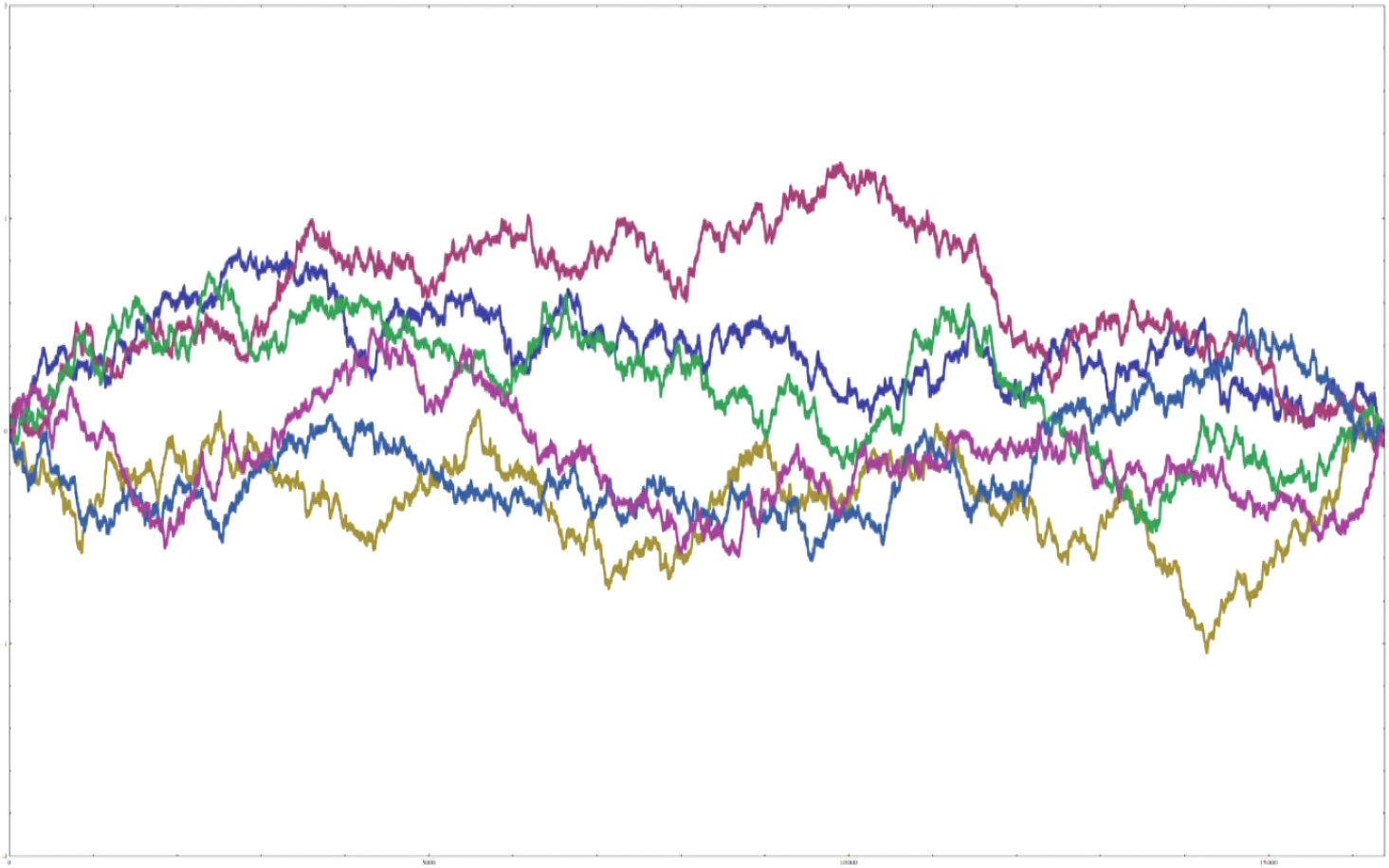
GFF

(active field in probability theory)

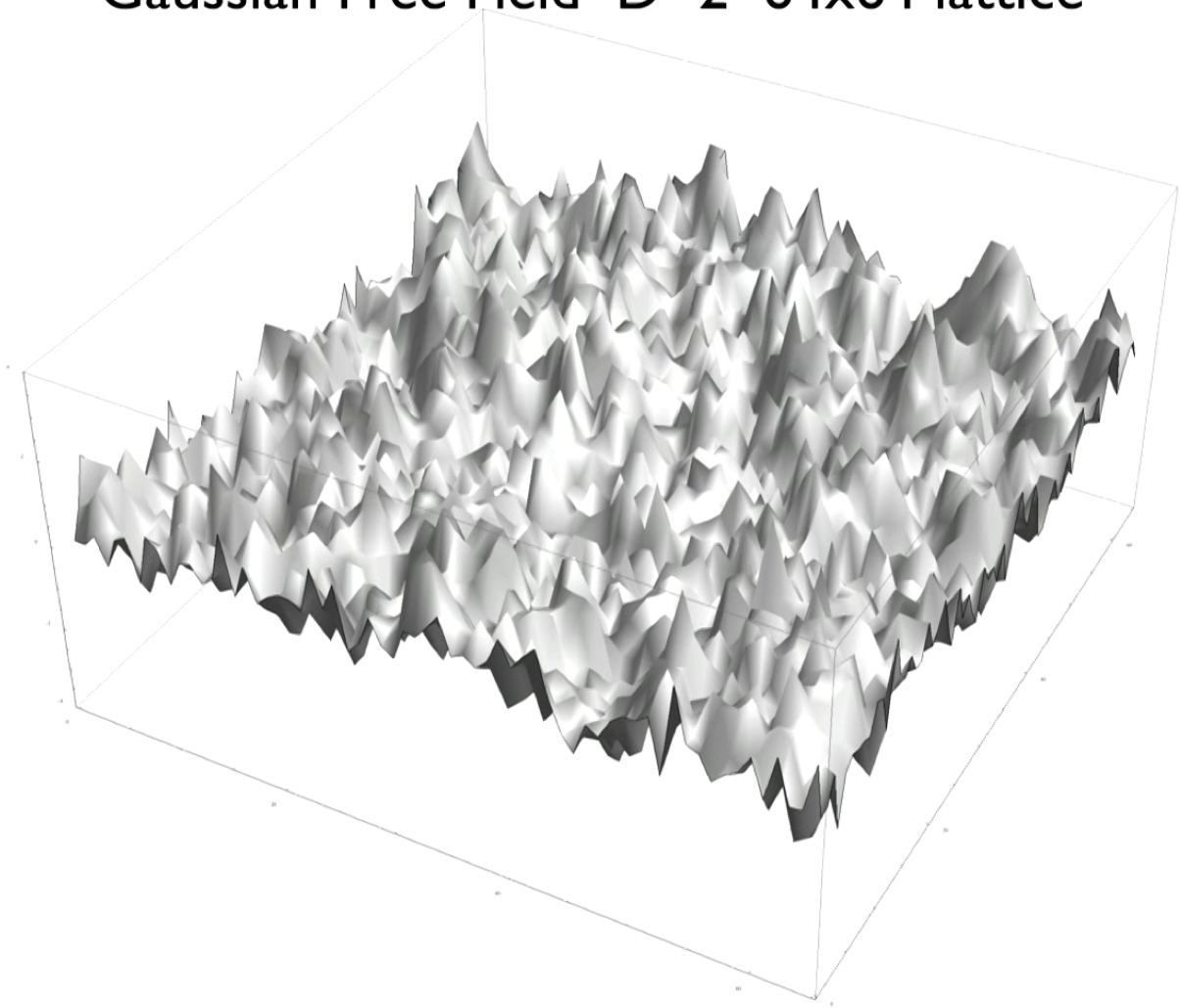
# Samples of Gaussian free field configurations (Euclidean, periodic b.c.)


Γ
random field
Φ
mesh → 0

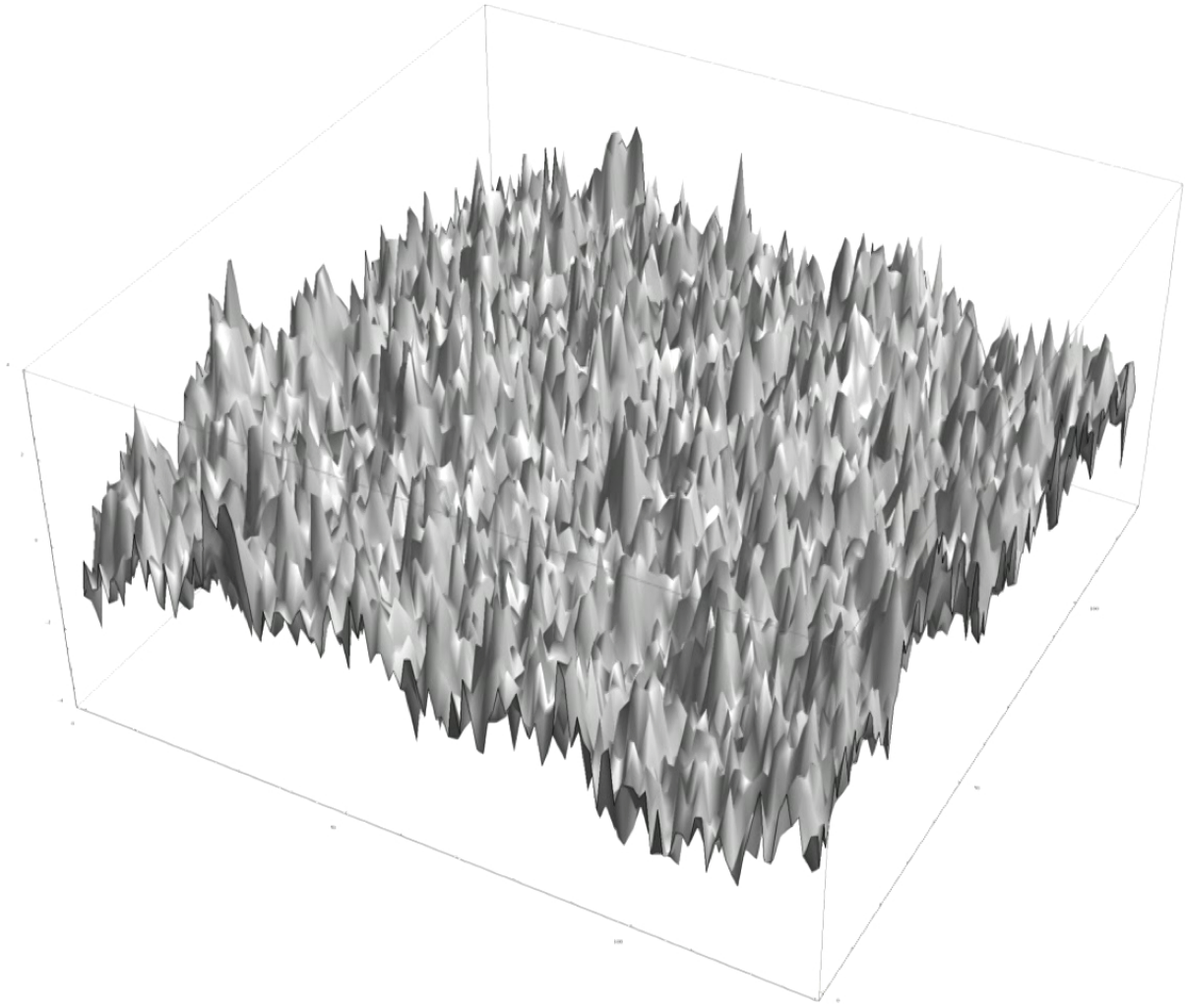
# D=1 : Gaussian Free Fields = Random Walk (i.e. Brownian or Wiener Process)



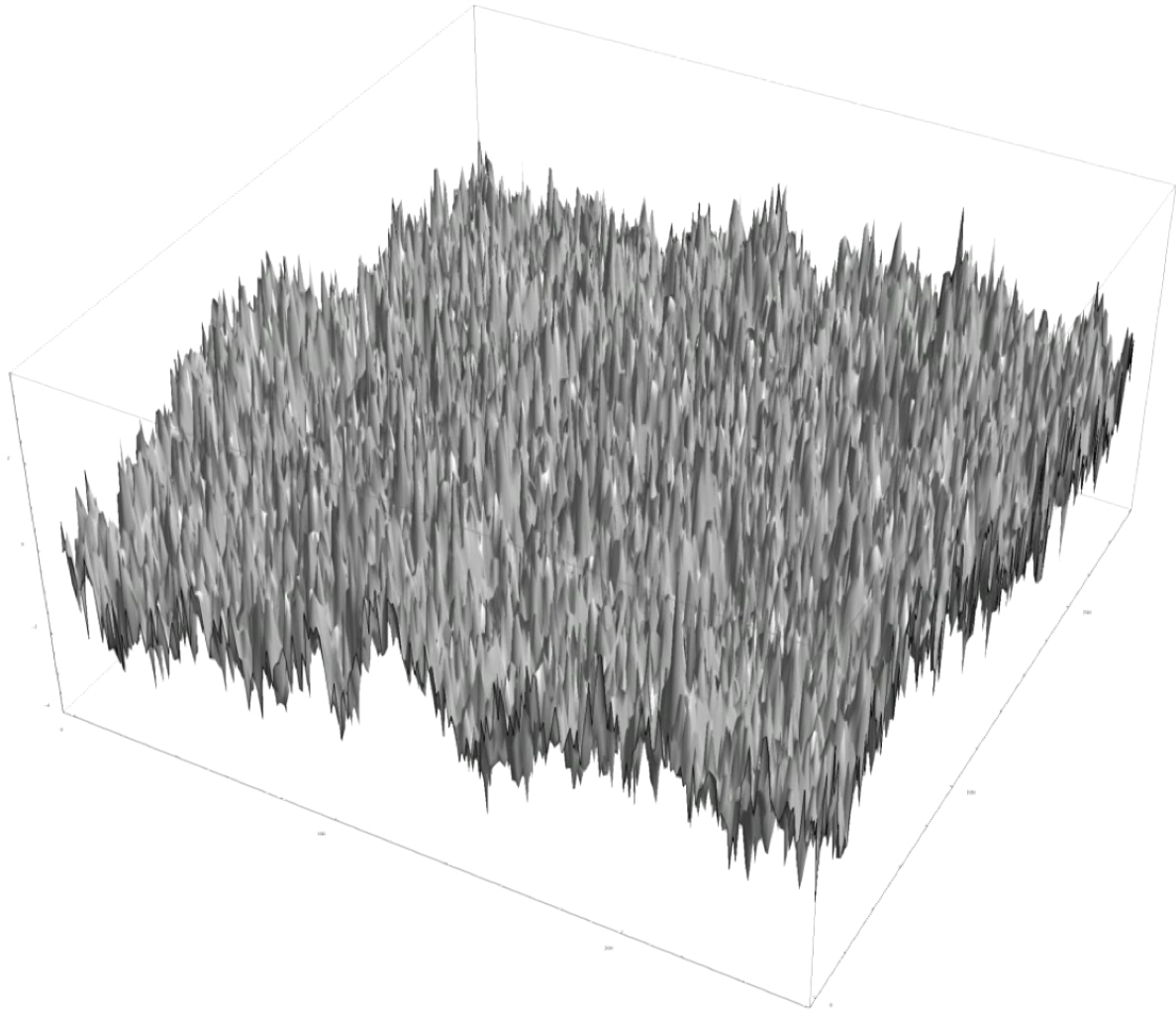
# Gaussian Free Field D=2 64x64 lattice



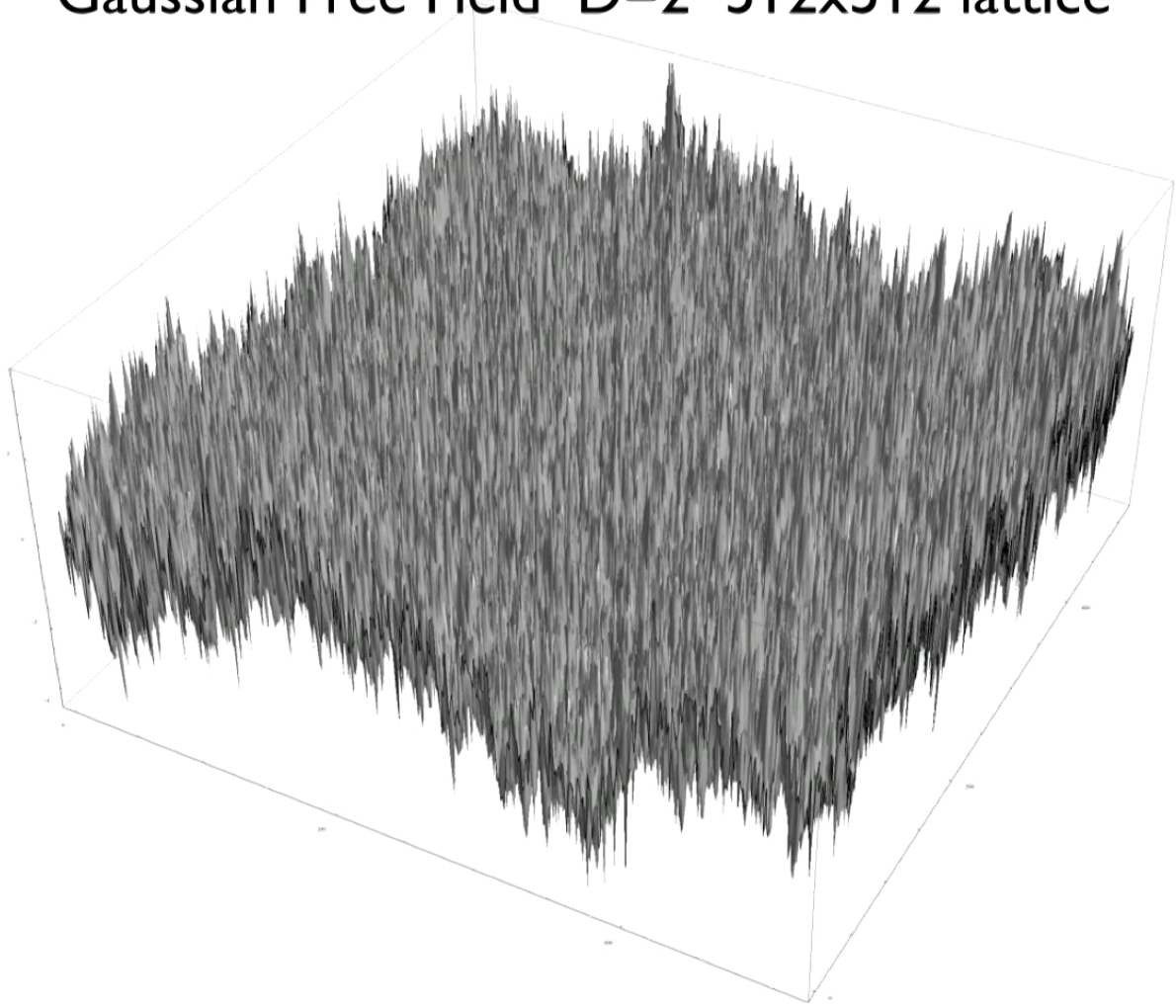
# Gaussian Free Field D=2 128x128 lattice

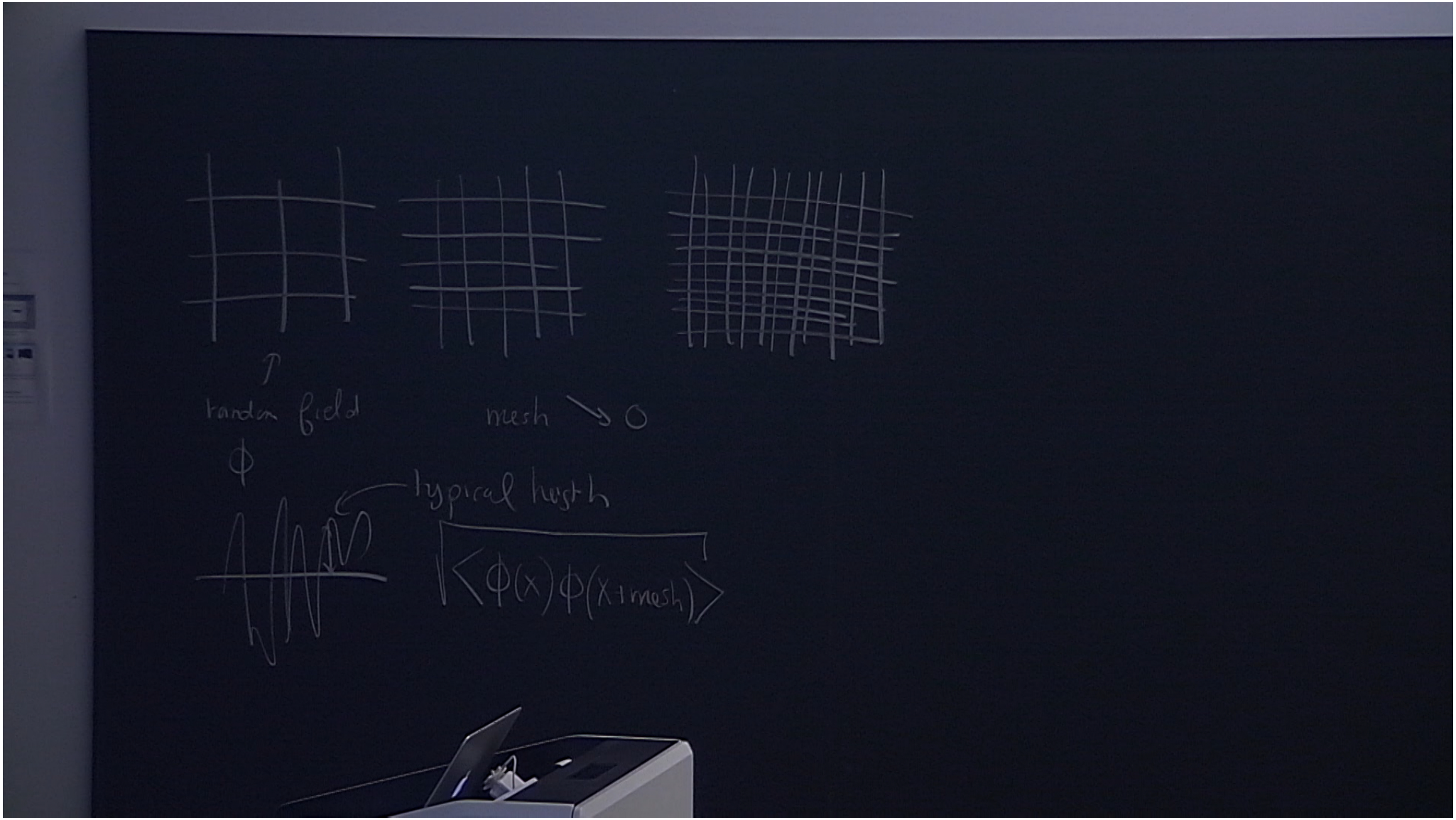


# Gaussian Free Field D=2 256x256 lattice

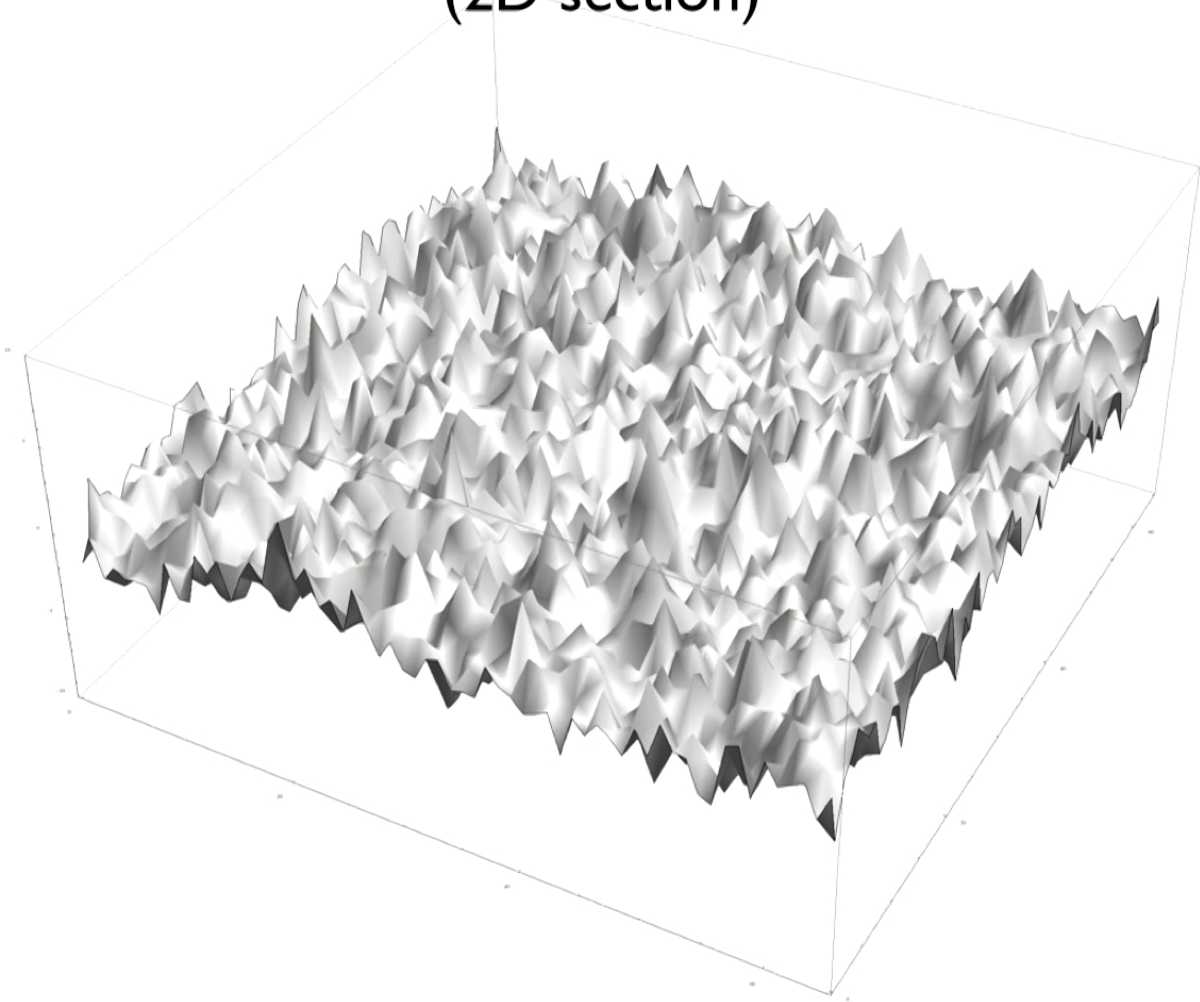


# Gaussian Free Field D=2 512x512 lattice

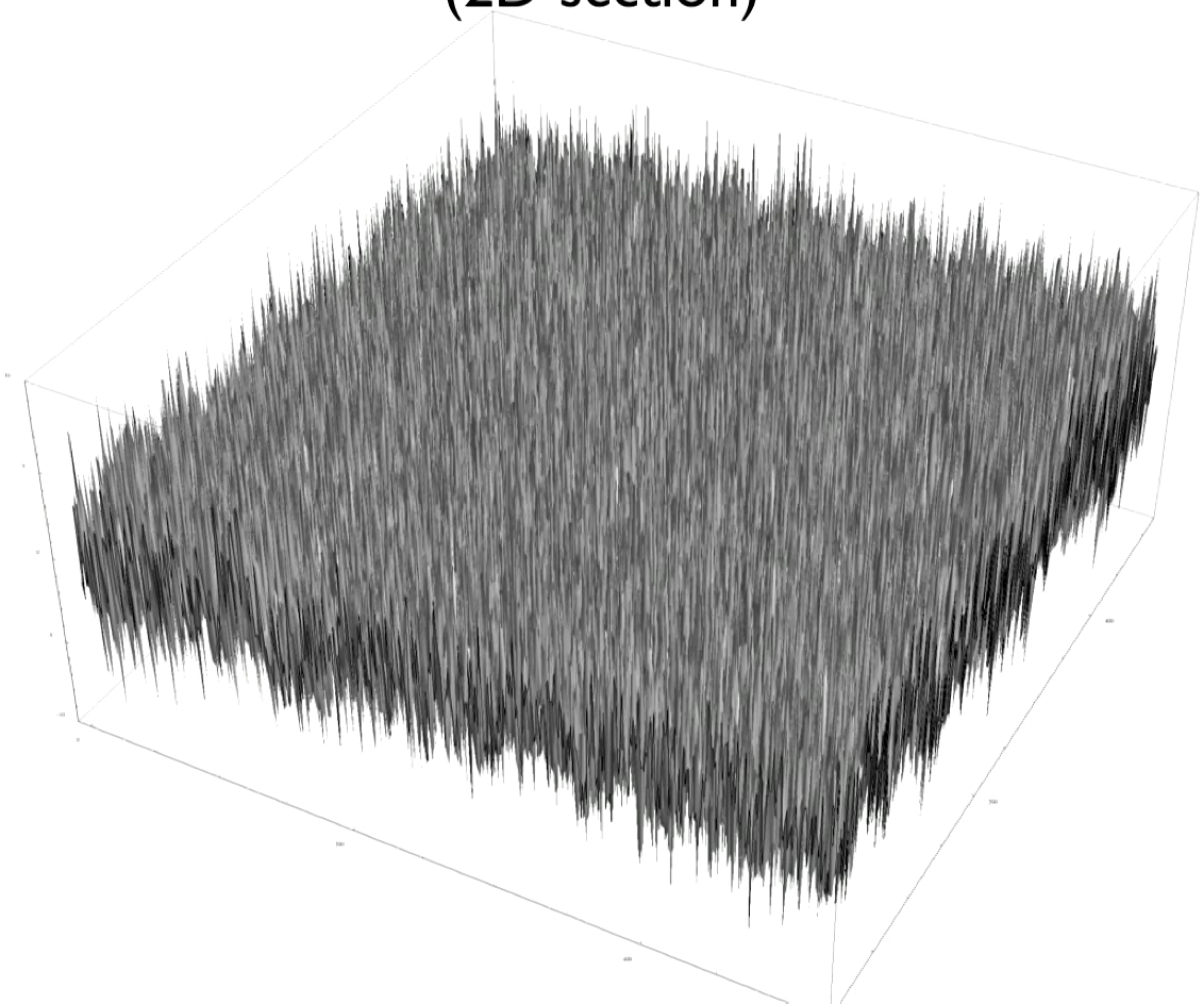


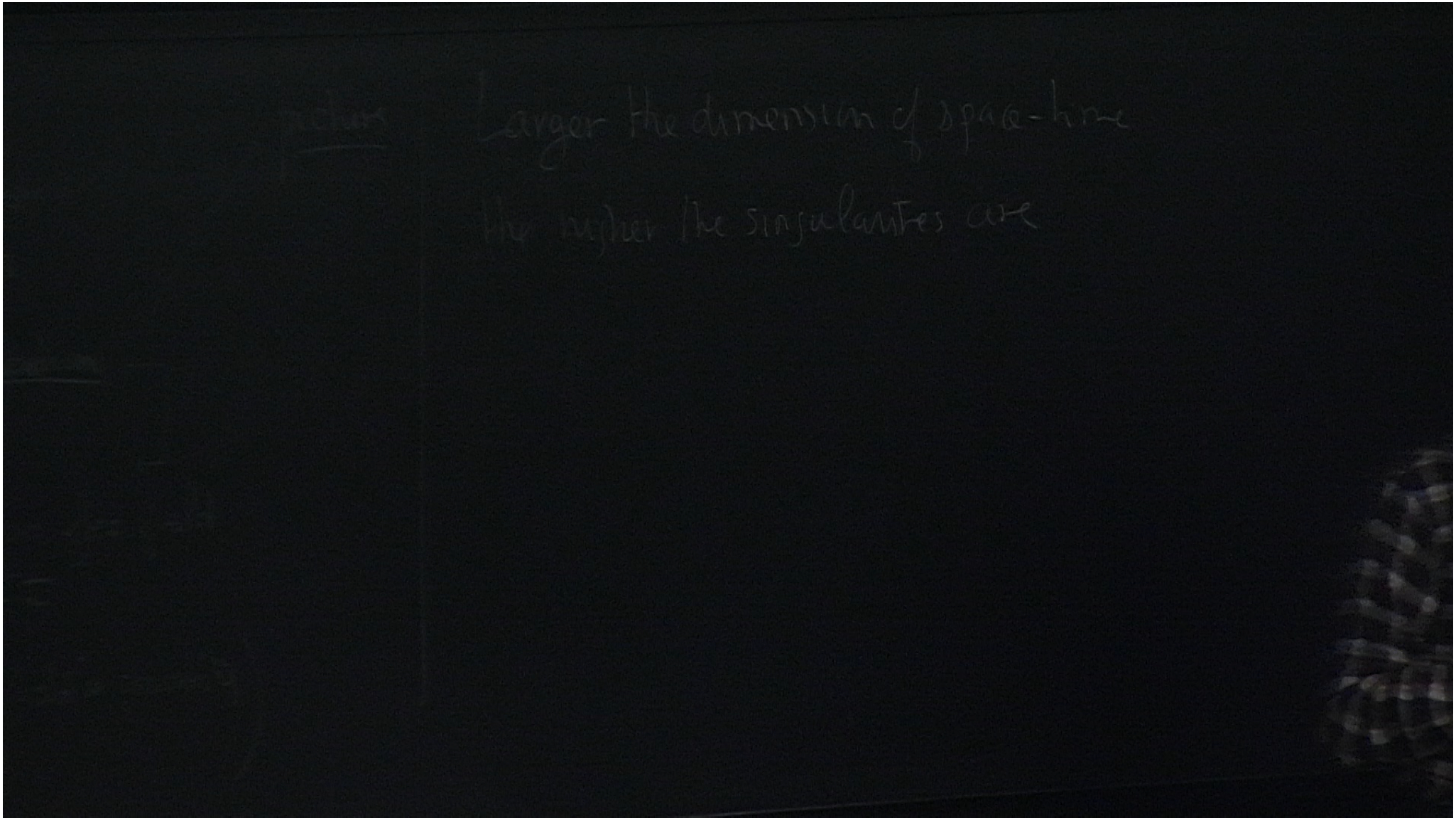

random field
mesh → 0
typical height
⟨φ(x)φ(x+mesh)⟩

# Gaussian Free Field D=3 64x64x64 lattice (2D section)



# Gaussian Free Field D=3 512x512x512 lattice (2D section)




larger the dimension of space-time
the richer the singularities are

$Q^2$ is well defined.

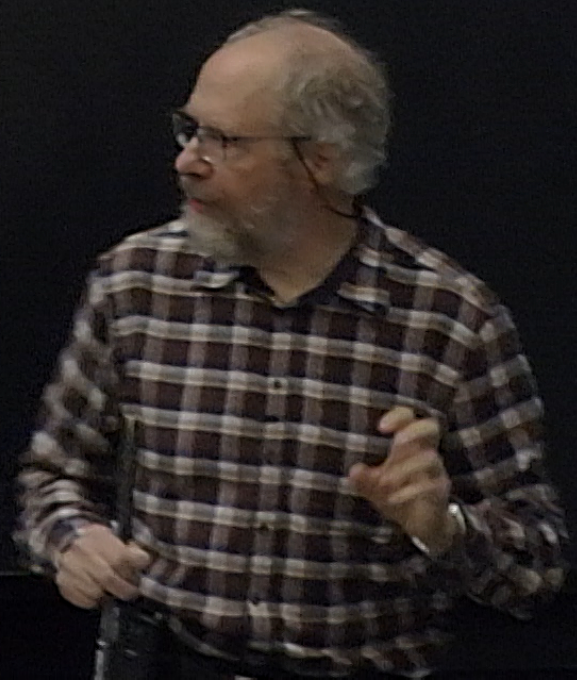
pichurs

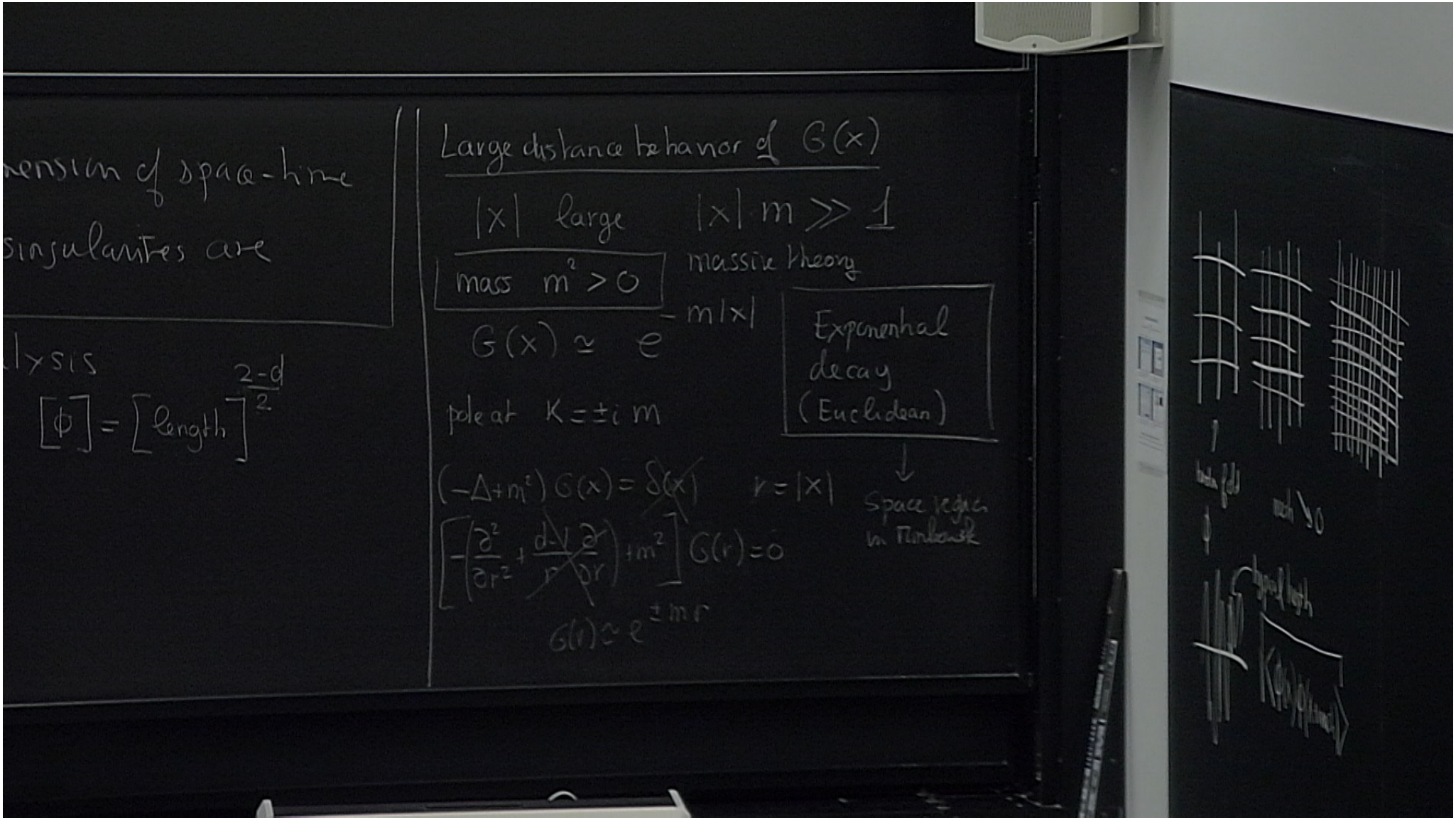
Larger the dimension of space-time
the higher the singularities are

dimensional analysis

dimension of Field $[\phi] = [\text{length}]^{\frac{2-d}{2}}$

$$\Delta_\phi = \frac{2-d}{2}$$




nension of space-time
singularities are
lysis
$[\phi] = [\text{length}]^{\frac{2-d}{2}}$
Large distance behavior of $G(x)$
$|x|$ large $|x|\, m \gg 1$
mass $m^2 > 0$
massive theory
$G(x) \simeq e^{-m|x|}$
Exponential decay (Euclidean)
pole at $K = \pm i\, m$
$(-\Delta + m^2)\, G(x) = \delta(x)$ $r = |x|$
Space region in Minkowski
$\left[-\left(\frac{\partial^2}{\partial r^2} + \frac{d-1}{r}\frac{\partial}{\partial r}\right) + m^2\right] G(r) = 0$
$G(r) \simeq e^{\pm m r}$

Dirac $\delta$-function

(active field in probability theory)

massless theory $\quad G(x) = |x|^{2-d} \quad \mathbb{C}(d) = \begin{cases} \frac{1}{4\pi} & d = 3 \\ \frac{1}{4\pi^2} & d = 4 \end{cases}$ Coulomb potential!

$\mathbb{C}(d)$ is finite $\quad d > 2$

Exact since the

Free massless field is scale invariant

Dirac $\delta$ function

(active field in probability theory

massless theory $G(x) = |x|^{2-d}$ $\mathbb{C}(d) = \begin{cases} \frac{1}{4\pi} & d=3 \\ \frac{1}{4\pi^2} & d=4 \end{cases}$ Coulomb potential!

$\mathbb{C}(d)$ is finite $d>2$

Exact since the

Free massless field is scale invariant

What about $d=2$

[Poincaré + Dilatations + inversions] = Conformal Theory

$X \to \lambda X$ $X \to \frac{X}{|X|^2}$



Dirac $\delta$ function

(active field in probability theory)

massless theory $\quad G(x) = |x|^{2-d} \quad \mathbb{C}(d) = \begin{cases} \frac{1}{4\pi} & d=3 \\ \frac{1}{4\pi^2} & d=4 \end{cases}$ coulomb potential!

$\mathbb{C}(d)$ is finite $\quad d>2$

Exact since the free massless field is scale invariant

[Poincaré + Dilatations + inversions] = Conformal Theory

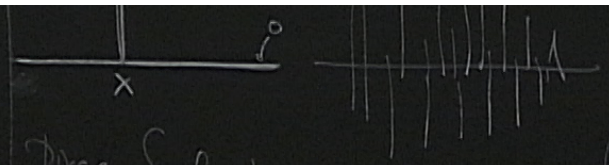
$X \to \lambda X \qquad X \to \frac{X}{|X|^2}$

What about $d=2$

$G(x) \sim -\frac{1}{2\pi} \log(m|x|) + \dots$

$m \to 0$ problematic!

Ultra-violet singularities
divergences




Gaussian free field
GFF
(active field in probability theory)

massless theory $G(x) = |x|^{2-d}$ $\mathbb{C}(d) = \begin{cases} \frac{1}{4\pi} & d=3 \\ \frac{1}{4\pi^2} & d=4 \end{cases}$ Coulomb potential!

$\mathbb{C}(d)$ is finite $d>2$

Exact since the
Free massless field is scale invariant

[Poincaré + Dilatations + inversions] = Conformal Theory

$X \to \lambda X$ $X \to \frac{X}{|X|^2}$

What about $d=2$?

$G(x) \sim -\frac{1}{2\pi} \log(m|x|) + \dots$

$m \to 0$ problematic!

new divergence: IR divergence
Infra-red

Phys

Gaussian free field

GFF

(active field in probability theory)

$$\Delta_\phi = \frac{2-d}{2}$$

$$(d) = \begin{cases} \frac{1}{4\pi} & d=3 \quad \text{Coulomb potential;} \\ \frac{1}{4\pi}\,2 & d=4 \end{cases}$$

What about $d=2$?

$$G(x) \sim -\frac{1}{2\pi}\log(m|x|) + \cdots$$

$m \to 0$ problematic!

new divergence: IR divergence

infra-red

Physical!

Non-interacting massless scalar field in $d=2$ is problematic

↕

Goldstone modes ⇔ spontaneous breaking of continuous sym.

GFF

(scalar field in probability theory)

$\Delta_\phi = \frac{d-2}{2}$

$\left[-\left(\frac{\partial^2}{\partial r^2} + \frac{d-1}{r}\frac{\partial}{\partial r}\right)\right]$

$G(r) \simeq$

$d = 3$ Coulomb potential;

$d = 4$

What about $d = 2$ ?

$$G(x) \sim -\frac{1}{2\pi} \log(m|x|) + \cdots$$

$m \to 0$

problematic!

new divergence: IR divergence

Infra-red

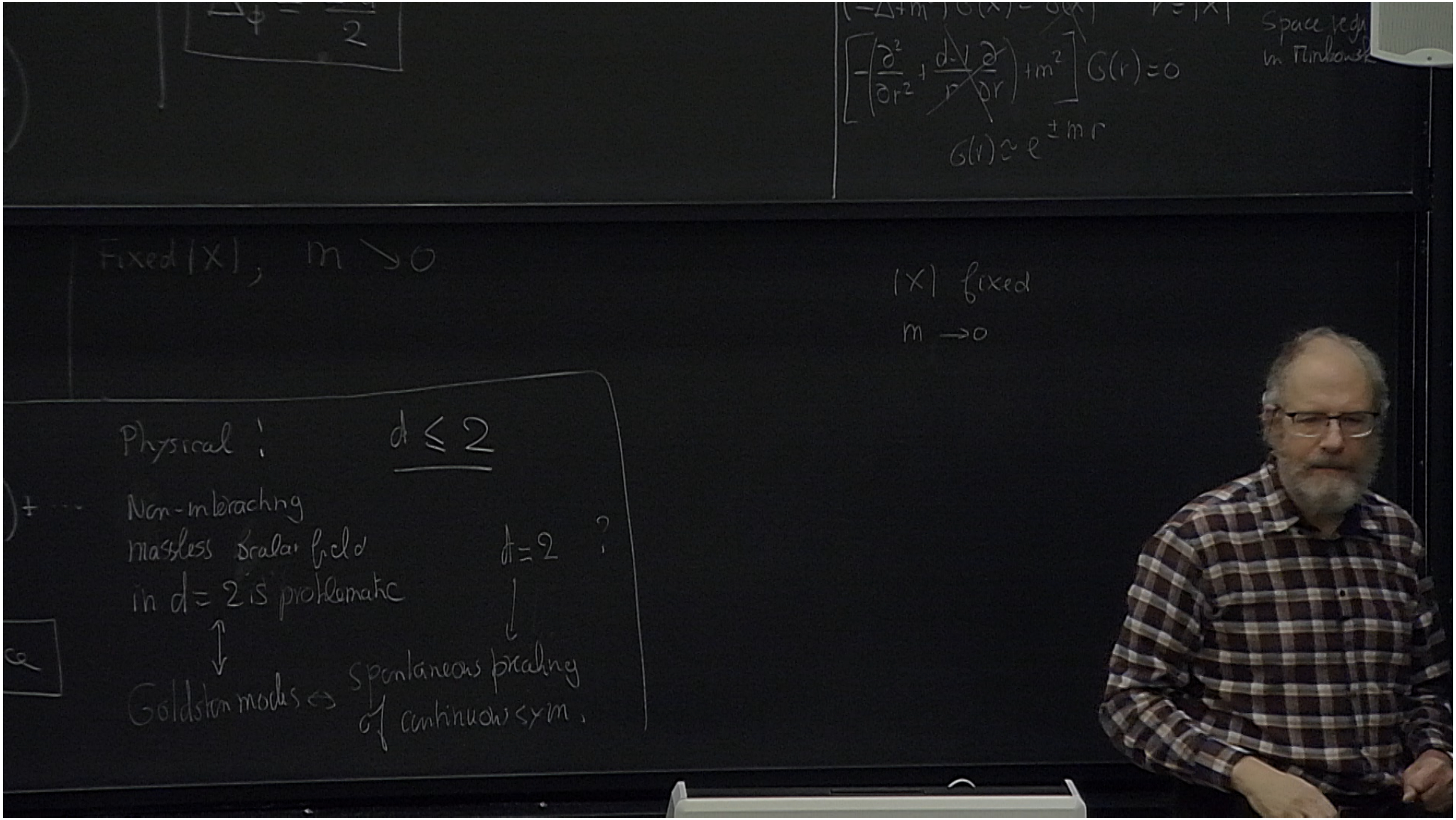
Physical; $d \leq 2$

Non-interacting massless scalar field in $d = 2$ is problematic

$d = 2$ ?

↕

Goldstone modes ⇔ spontaneous breaking of continuous sym.


$\left[-\left(\frac{\partial^2}{\partial r^2}+\frac{d-1}{r}\frac{\partial}{\partial r}\right)+m^2\right]G(r)=0$
$G(r)\simeq e^{\pm mr}$
Space region in Minkowski
Fixed $|x|$, $m>0$
$|x|$ fixed
$m\to 0$
Physical: $d\leq 2$
Non-interacting
massless scalar field
in $d=2$ is problematic
$d=2$ ?
Goldstone modes $\Leftrightarrow$ spontaneous breaking of continuous sym.

GFF

(active field in probability theory)

$\Delta_\phi = \frac{d-2}{2}$

$\frac{1}{4\pi}$ $d=3$ Coulomb potential;

$\frac{1}{4\pi}^2$ $d=4$

Fixed $|x|$, $m \to 0$

also in canonical quantization.

What about $d=2$?

$$G(x) \sim -\frac{1}{2\pi} \log(m|x|) + \cdots$$

$m \to 0$ problematic!

new divergence: IR divergence

Infra-red comes from $|k|$ small in the FT Transform

Physical: $d \leq 2$

Non-interacting massless scalar field in $d=2$ is problematic

$d=2$?

Goldstone modes $\leftrightarrow$ spontaneous breaking of continuous sym.

8) Klein-Gordon | (k) Kallen-Lehmann | SAM | 9) Causality

- Free Field  2pt function → correlation functions
  Wick Theorem
- Properties of 2pt functions at short & large distances
  IR divergences $d \leq 2$
  UV singularities at short distances when $d \geq 2$

Action (Euclidean)

Propagator

correlation fu

$\langle \phi(x_1) \rangle$

$\langle 0 | a \cdots a |$

divergences

massless theory $G(x) = |x|^{2-d}$ $\mathbb{C}(d) = \begin{cases} \frac{1}{4\pi} & d=3 \\ \frac{1}{4\pi}\,2 & d=4 \end{cases}$ Coulomb pole

$\mathbb{C}(d)$ is finite $d>2$

Exact since the

Free massless field is scale invariant

[Poincaré + Dilatations + inversions] = Conformal Theory

$X \to \lambda X$ $\quad X \to \frac{X}{|X|^2}$

mass = 0 strictly!

What about $d=2$

$G(x) \sim -\frac{1}{2\pi} \log(r$

$m \to 0$ problem

new divergence: IR d

Infra-red comes from

in the F. Th

Qtz Dirac
Anti particles
Causality

6-5) QED log

↳ Is vacuum energy real?

$E(r) = \sum \frac{1}{r^n}$ $n\pi$ $-\pi$ $\zeta$

## Products of local operators in a QFT $\Rightarrow$ Wilson's Operator Product Expansion OPE

$\phi(x)$ is well defined $\longleftrightarrow$ $\hat{\phi}(x)$ Field operator

$\phi^2(x)$ is infinite

how to construct $\phi(x)$ times $\phi(x)$ ?

$\langle \phi(x)\phi(y)\phi(z_1)\cdots\phi(z_N)\rangle$

QFT Dirac
Anti particles
Causality

6-5) QED bag

↳ Is vacuum energy real?

$E(r) = \sum \frac{1}{r}$ $n\pi$ $-\pi$ $c$

## Products of local operators in a QFT $\Rightarrow$ Wilson's Operator Product Expansion OPE

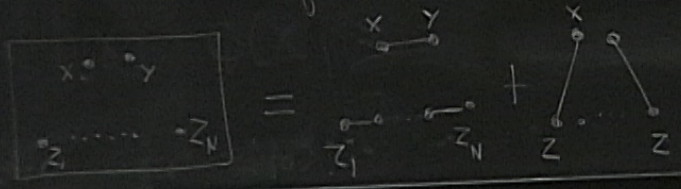
$\Phi(x)$ is well defined $\longleftrightarrow$ $\hat{\Phi}(x)$ Field operator

$\Phi^2(x)$ is infinite

how to construct $\phi(x)$ times $\phi(x)$ ?

$\langle \Phi(x)\,\Phi(y)\,\phi(z_1)\cdots\phi(z_N)\rangle$ N+2 points correlation function = Sum of Feynman diagrams Wick theorem

$y \to x$ becomes infinite



Qfz Dirac
Anti particles
Causality

III

6-5) QED log

↳ Is vacuum energy real?

$E(r) = \sum \frac{1}{r^n}$ $n\pi$ $\pi$ c

## Products of local operators in a QFT $\Rightarrow$ Wilson's Operator Product Expansion OPE

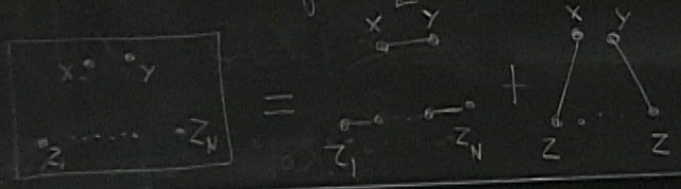
$\Phi(x)$ is well defined $\longleftrightarrow$ $\hat{\Phi}(x)$ Field operator

$\Phi^2(x)$ is infinite

how to construct $\phi(x)$ times $\phi(x)$ ?

$\langle \Phi(x)\Phi(y)\Phi(z_1)\cdots\Phi(z_N)\rangle$ N+2 points correlation function = Sum of Feynman diagrams Wick theorem

$y \to x$ becomes infinite singular regular



Qtz Dirac
Anti particles
Causality

6-5) QED log

↳ Is vacuum energy real?

$E(r) = \sum \frac{1}{n}$ $n\pi - \pi c$

## Products of local operators in a QFT ⇒ Wilson's Operator Product Expansion OPE

$\Phi(x)$ is well defined ⟷ $\hat{\Phi}(x)$ Field operator

$\Phi^2(x)$ is infinite

how to construct $\Phi(x)$ times $\Phi(x)$ ?

$\langle \Phi(x)\Phi(y)\Phi(z_1)\cdots\Phi(z_N)\rangle$ N+2 points correlation function = Sum of Feynman diagrams Wick theorem

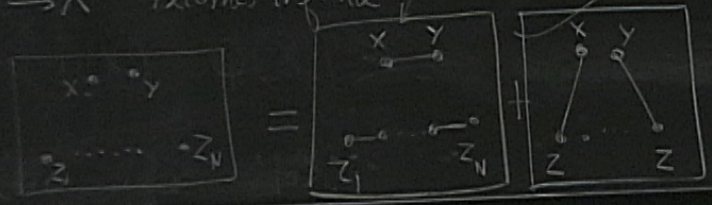
$y \to x$ becomes infinite

singular regular

$\langle\Phi(x)\Phi(y)\rangle\langle\Phi(z_1)\cdots\Phi(z_N)\rangle + \langle :\Phi(x)\Phi(y): \Phi(z_1)\cdots\Phi(z_N)\rangle$

normal product

take it as a formal definition



Extended into the definition of a new operator

Identity operator

$$:\phi(x_1)\phi(x_2): \;=\; \phi(x_1)\phi(x_N) \;-\; \langle\phi(x)\phi(y)\rangle \times \mathbb{I}\left(\frac{x+y}{2}\right)$$

operator — operator — number . function of $(x, y)$

Extended into the definition of a new operator

Identity operator

middle point

$$:\phi(x)\phi(y): \;=\; \phi(x)\phi(y) - \langle\phi(x)\phi(y)\rangle \times \mathbb{I}\left(\frac{x+y}{2}\right)$$

operator — operator — number . function of $(x, y)$

Extended into the definition of a new operator

Identity operator

middle point

$$:\phi(x)\phi(y): \;=\; \phi(x)\phi(y) - \langle\phi(x)\phi(y)\rangle \times \mathbb{I}\left(\frac{x+y}{2}\right)$$

operator — operator — number . function of $(x, y)$

$$\langle :\phi(x)\phi(y): \; \phi(z_1)\ldots\phi(z_N)\rangle = \sum_{\substack{\text{all pairings} \\ \text{but } x,y \text{ excluded}}} \prod \langle\phi\phi\rangle$$

$$\lim_{Y \to X} : \phi(x)\phi(y) : = : \phi(x)^2 : \quad \text{exists}$$

Extended into the definition of a new operator

Identity operator

middle point

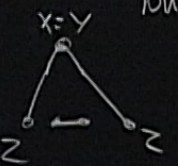
$$:\phi(x)\phi(y): = \phi(x)\phi(y) - \langle\phi(x)\phi(y)\rangle \times \mathbb{I}\left(\frac{x+y}{2}\right)$$

operator — operator — number, function of $(x, y)$

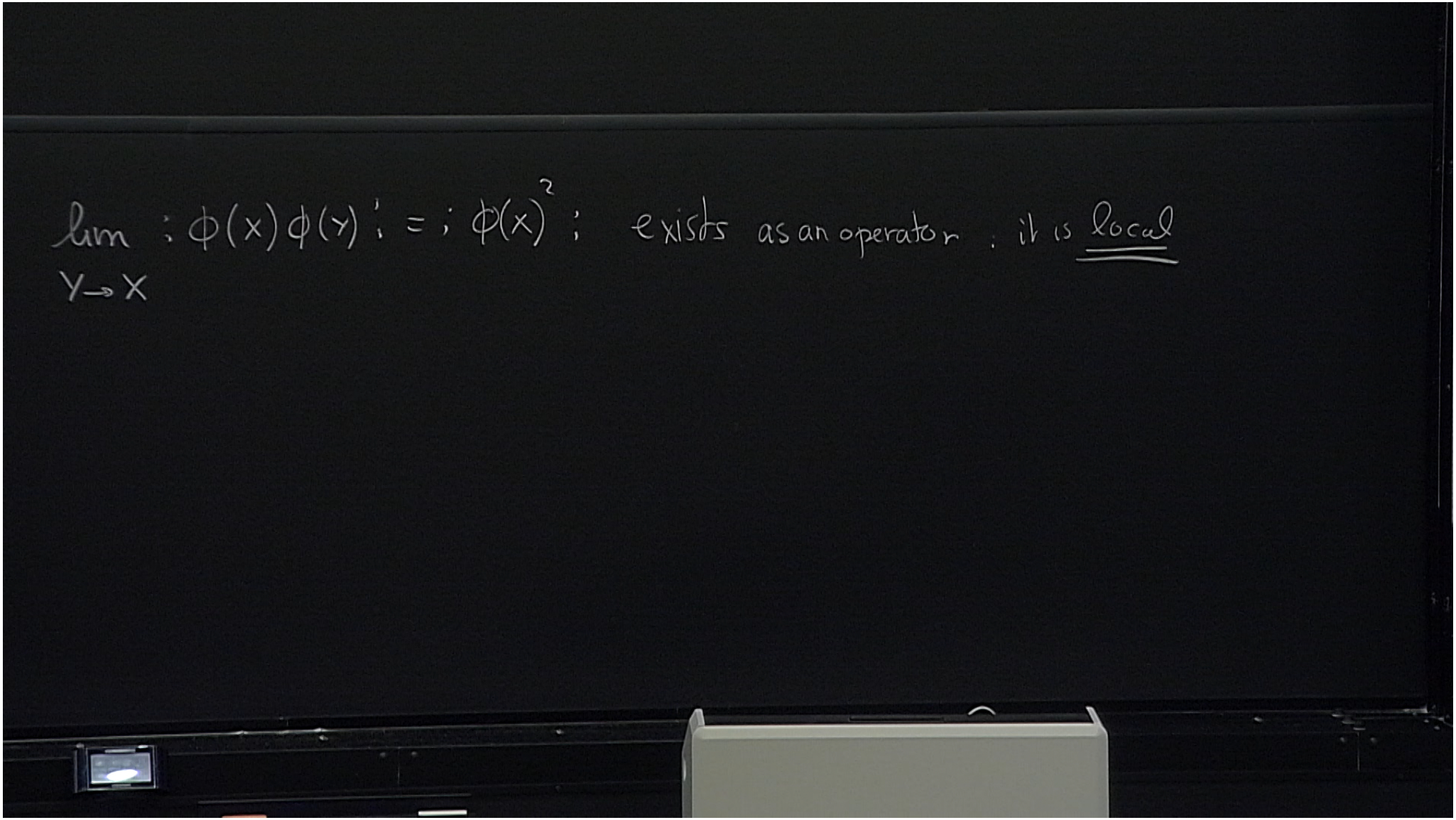
$$\lim_{y\to x} :\phi(x)$$

$$\langle :\phi(x)\phi(y): \phi(z_1)\cdots\phi(z_N)\rangle = \sum_{\substack{\text{all pairings} \\ \text{but } xy \text{ excluded}}} \prod \langle\phi\phi\rangle$$

$$\langle :\phi(x)^2: \phi(z_1)\cdots\phi(z_N)\rangle = \sum_{\text{all pairings}}$$



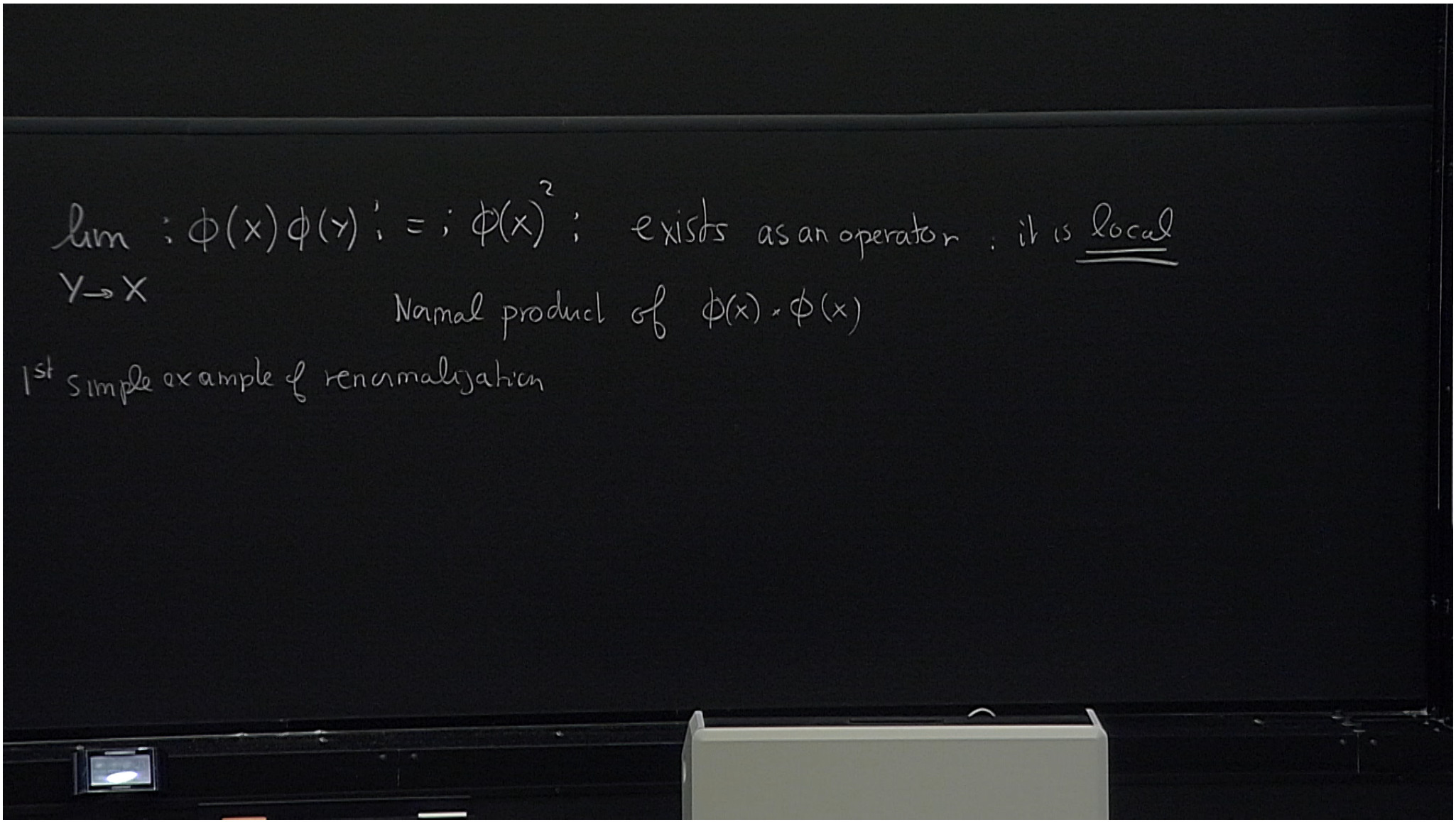

but no $x=y$ pairing allowed


$\lim_{Y \to X} :\phi(x)\phi(y): = :\phi(x)^2:$ exists as an operator : it is local

$\lim_{Y \to X} : \phi(x)\phi(y) : = : \phi(x)^2 :$ exists as an operator : it is <u>local</u>

Normal product of $\phi(x) \cdot \phi(x)$

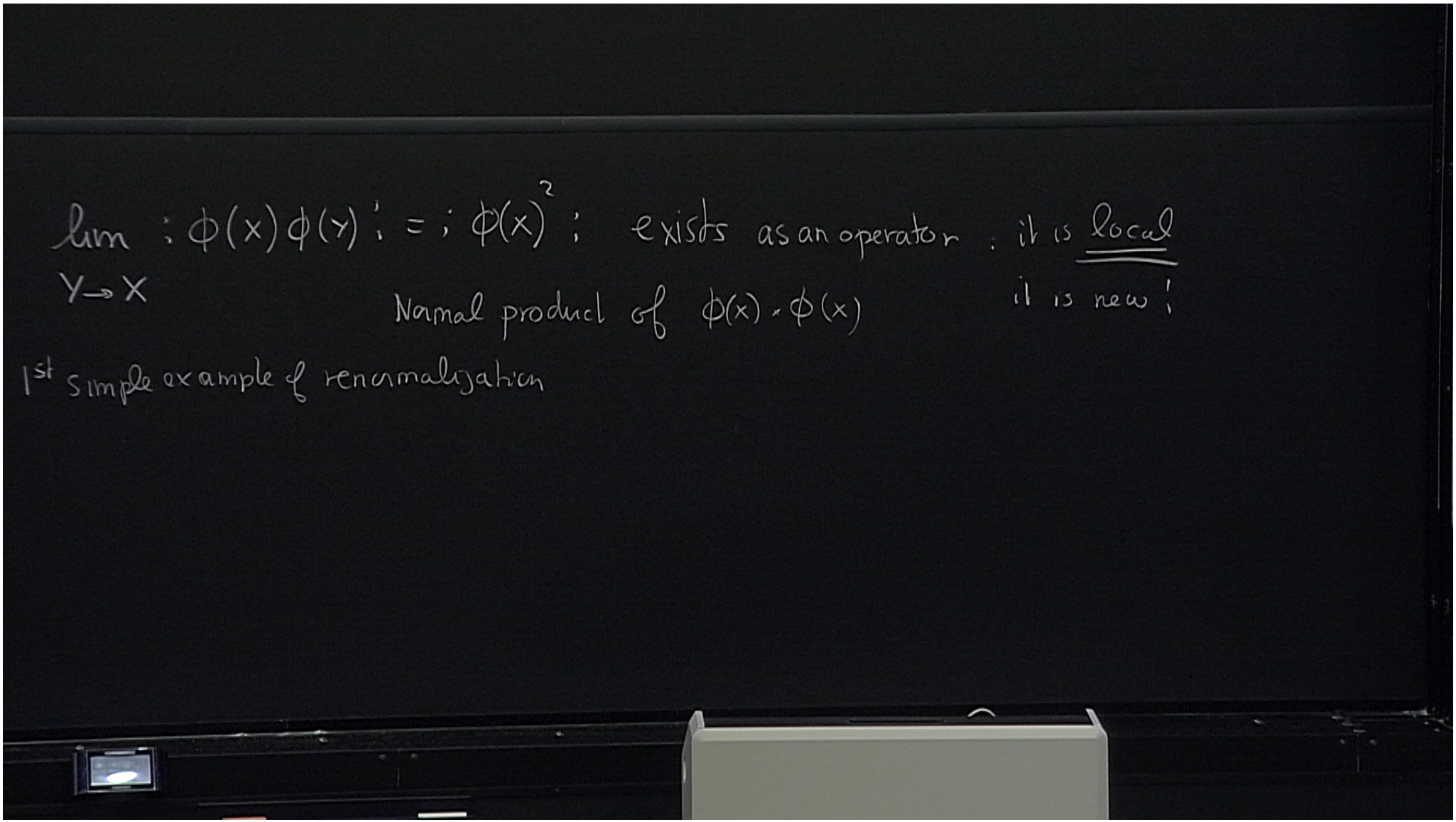
1st simple example of renormalization

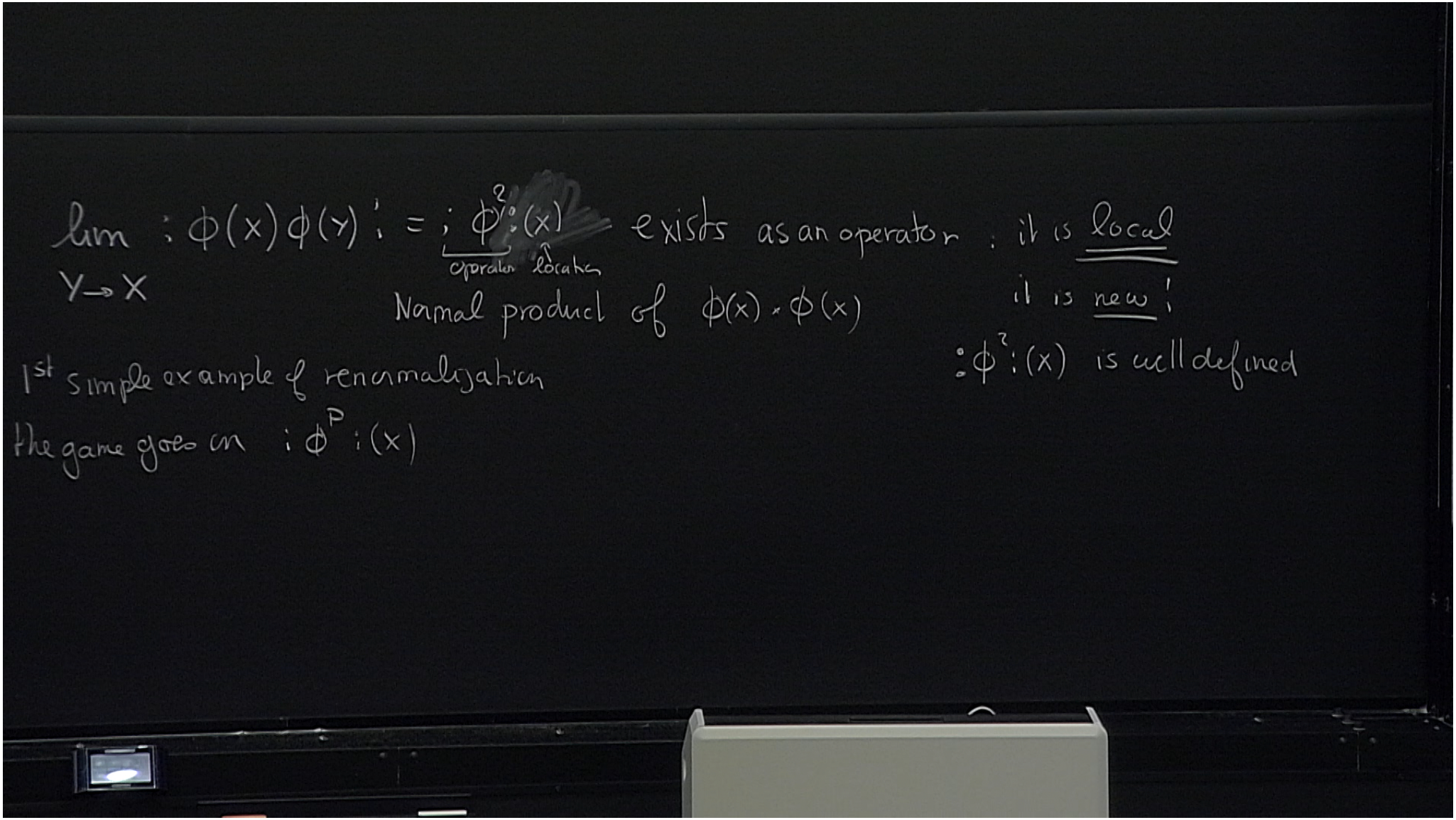


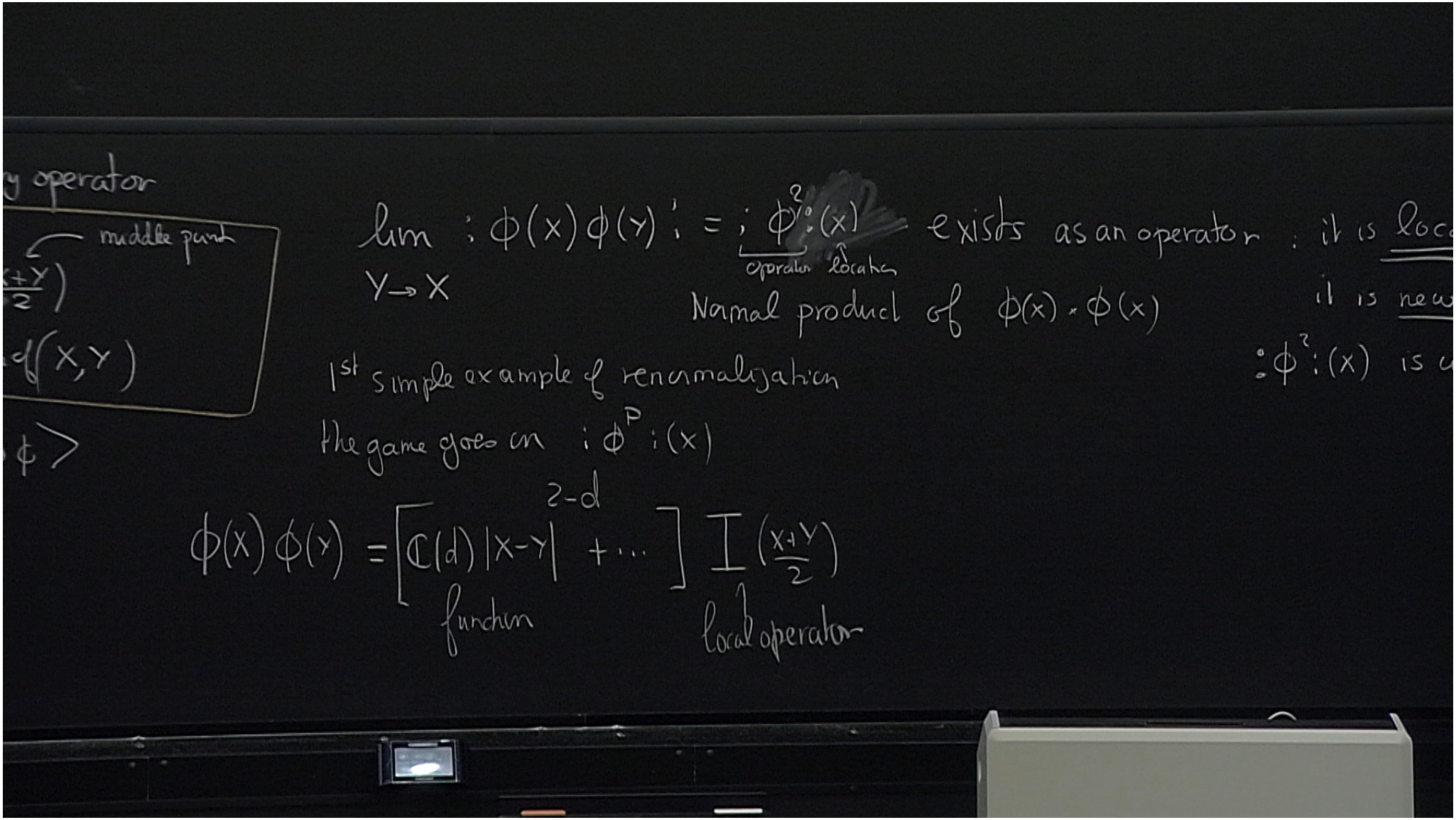
$\lim_{Y \to X} : \phi(x)\phi(y) : = : \phi(x)^2 :$ exists as an operator : it is <u>local</u>

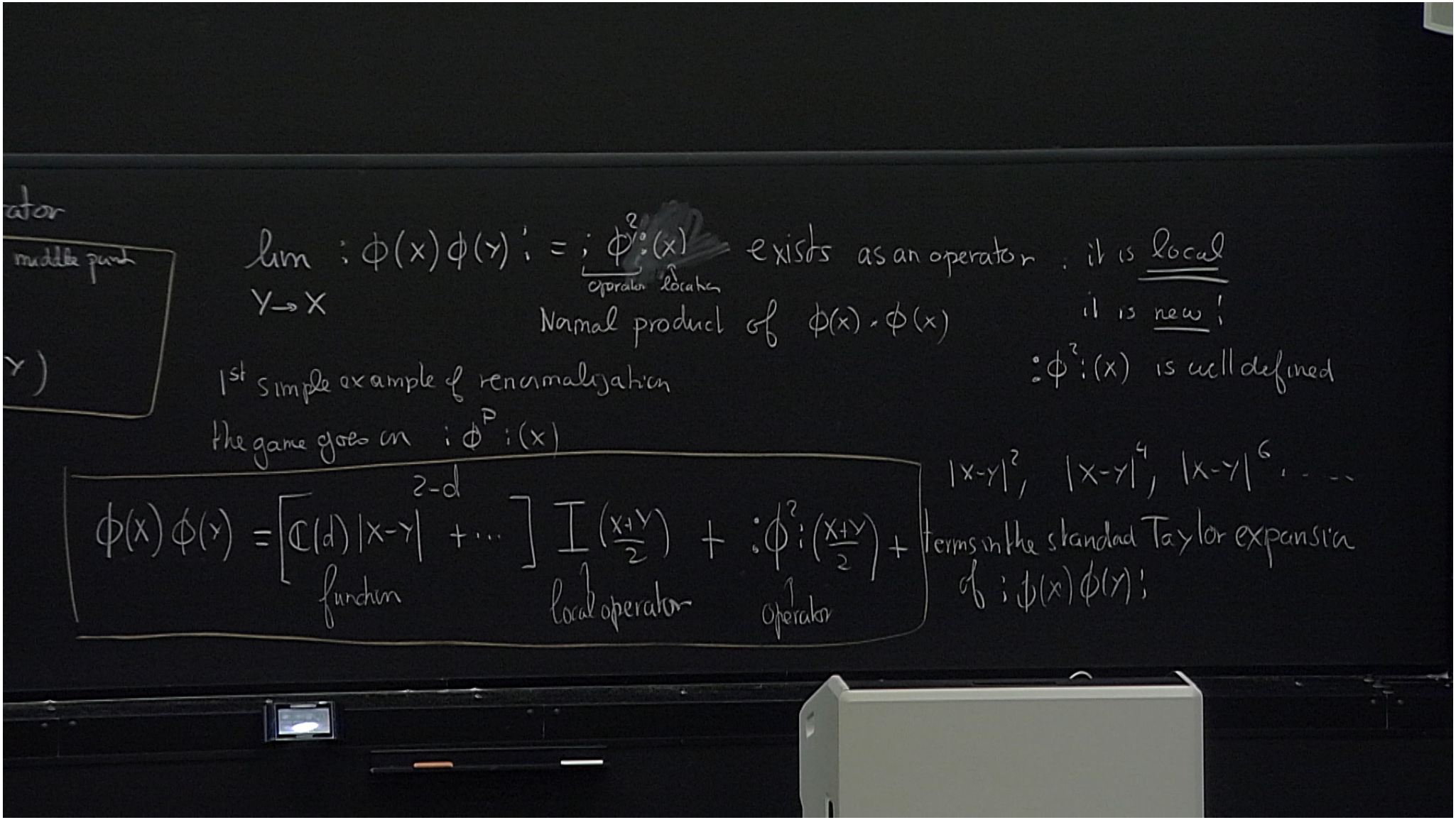
Normal product of $\phi(x) \cdot \phi(x)$ it is new!

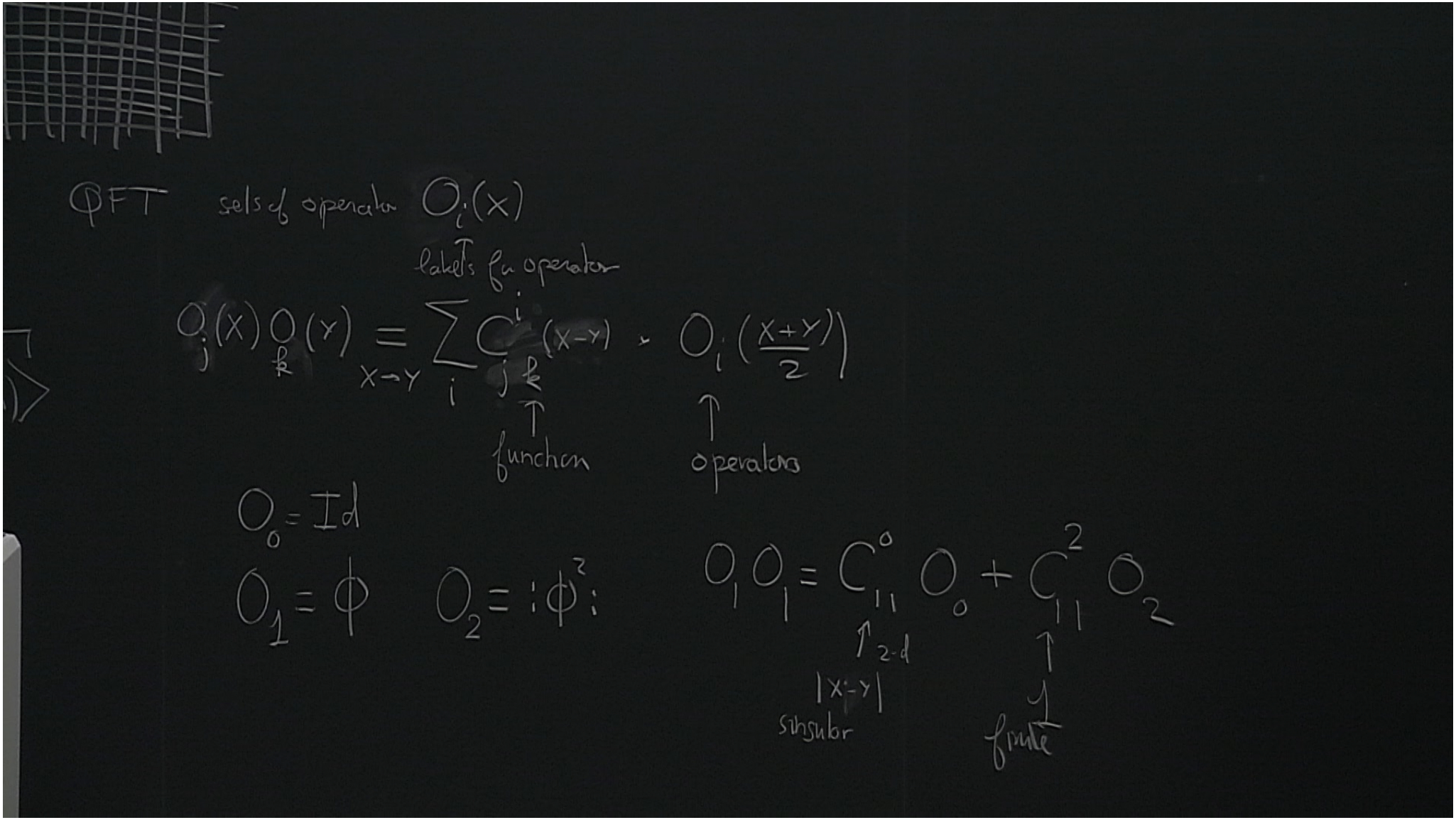
1st simple example of renormalization




lim : φ(x)φ(y) : = : φ²(x) exists as an operator : it is local
y→x
operator location
Normal product of φ(x) · φ(x)
it is new !
1st simple example of renormalization
: φ² : (x) is well defined
the game goes on : φ^p : (x)


y operator
middle point
d(x,y)
$\lim_{y \to x} :\phi(x)\phi(y): = :\phi^2:(x)$ exists as an operator : it is loc
operator
local
Normal product of $\phi(x) \cdot \phi(x)$
it is new
$:\phi^2:(x)$ is
1st simple example of renormalization
the game goes on $:\phi^p:(x)$
$\phi(x)\phi(y) = \left[ C(d)\,|x-y|^{2-d} + \cdots \right] I\left(\frac{x+y}{2}\right)$
function
local operator


middle part
$\lim_{Y \to X} :\phi(x)\phi(y): = :\phi^2:(x)$ exists as an operator : it is local
operator location
Normal product of $\phi(x) \cdot \phi(x)$
it is new!
$:\phi^2:(x)$ is well defined
1st simple example of renormalization
the game goes on $:\phi^p:(x)$
$\phi(x)\phi(y) = \left[C(d)\,|x-y|^{2-d} + \dots\right] I\left(\frac{x+y}{2}\right) + :\phi^2:\left(\frac{x+y}{2}\right) +$ terms in the standard Taylor expansion of $:\phi(x)\phi(y):$
function
local operator
operator
$|x-y|^2$, $|x-y|^4$, $|x-y|^6$, ...


QFT sets of operators $O_i(x)$
labels for operators
$O_j(x)\,O_k(y) \underset{x\to y}{=} \sum_i C_{jk}^{i}(x-y)\cdot O_i\left(\frac{x+y}{2}\right)$
function
operators
$O_0 = Id$
$O_1 = \phi \quad O_2 = :\phi^2:$
$O_1 O_1 = C_{11}^{0}\, O_0 + C_{11}^{2}\, O_2$
$\uparrow_{2-d}$
$|x-y|$
singular
finite