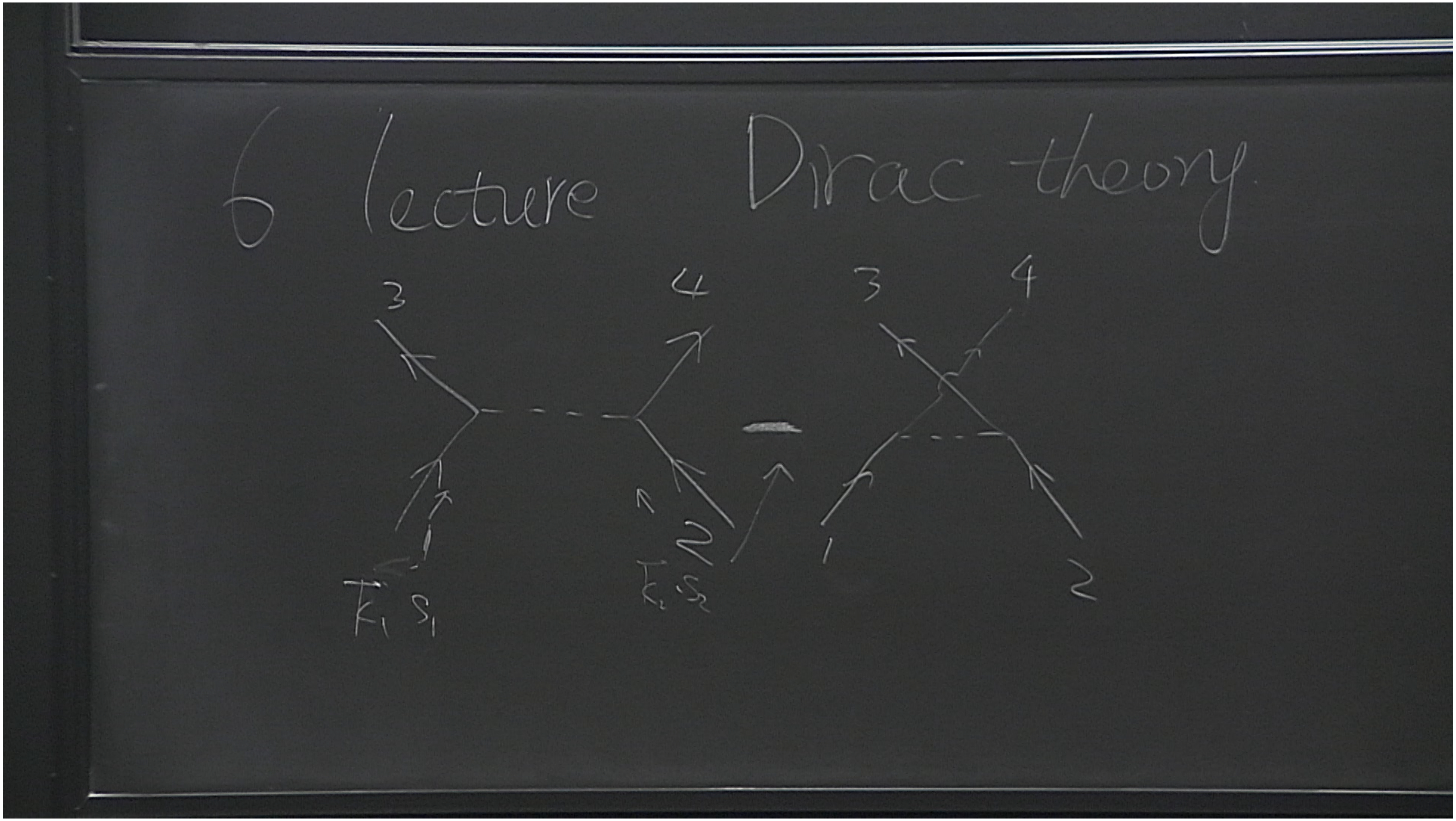
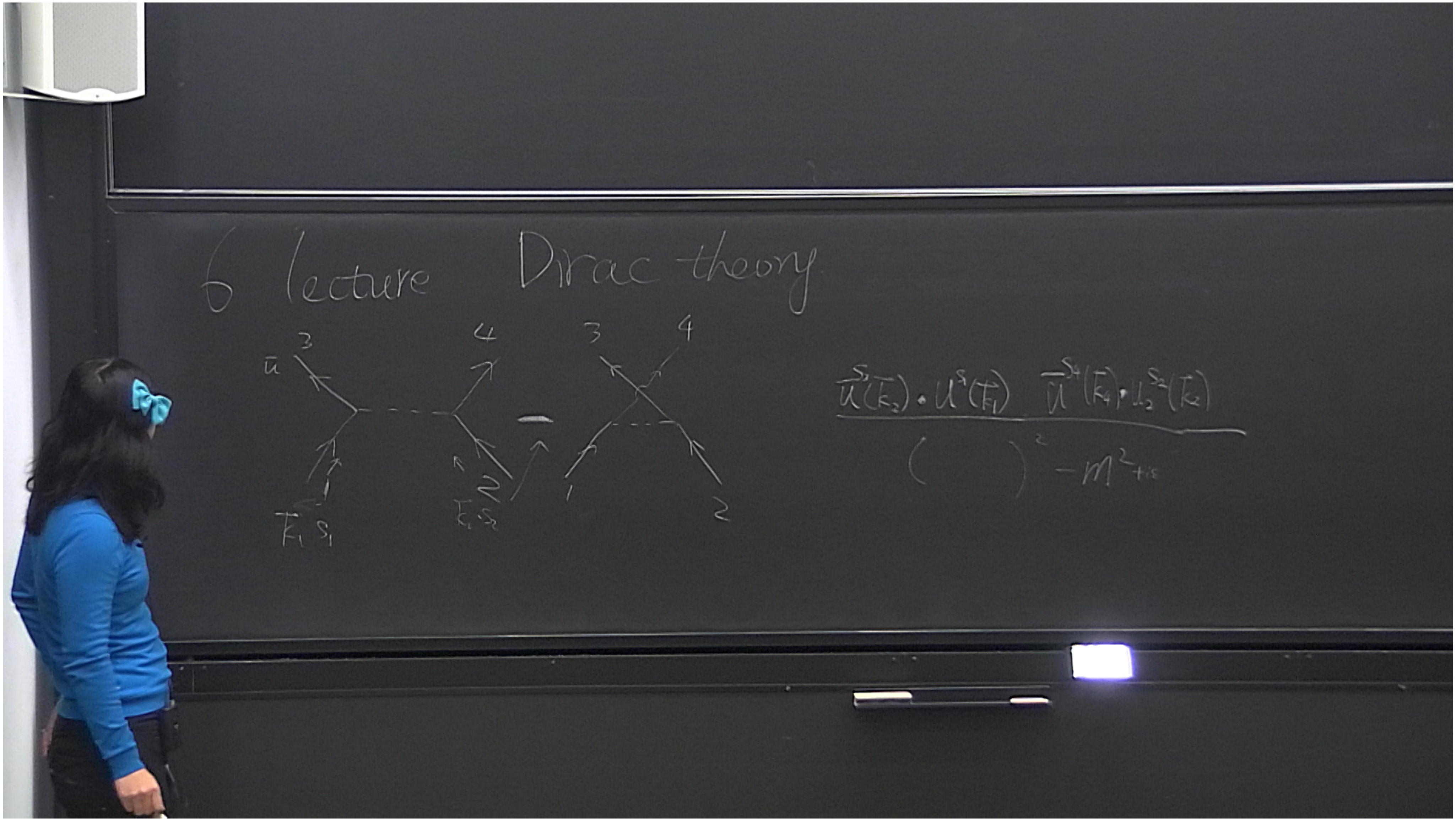
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 14

Date: Nov 01, 2018  09:00 AM

URL: http://pirsa.org/18110002

Abstract:

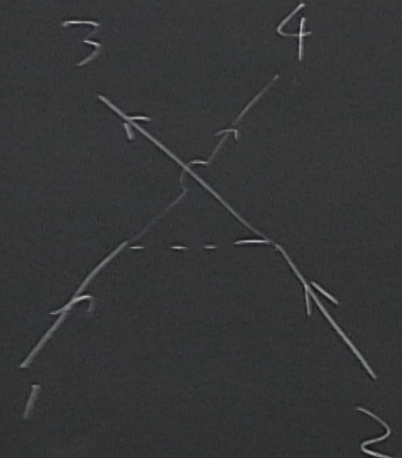

6 lecture Dirac theory
3
4
3
4
$k_1\ s_1$
$k_2\ s_2$
2
1
2
−


6 lecture Dirac theory

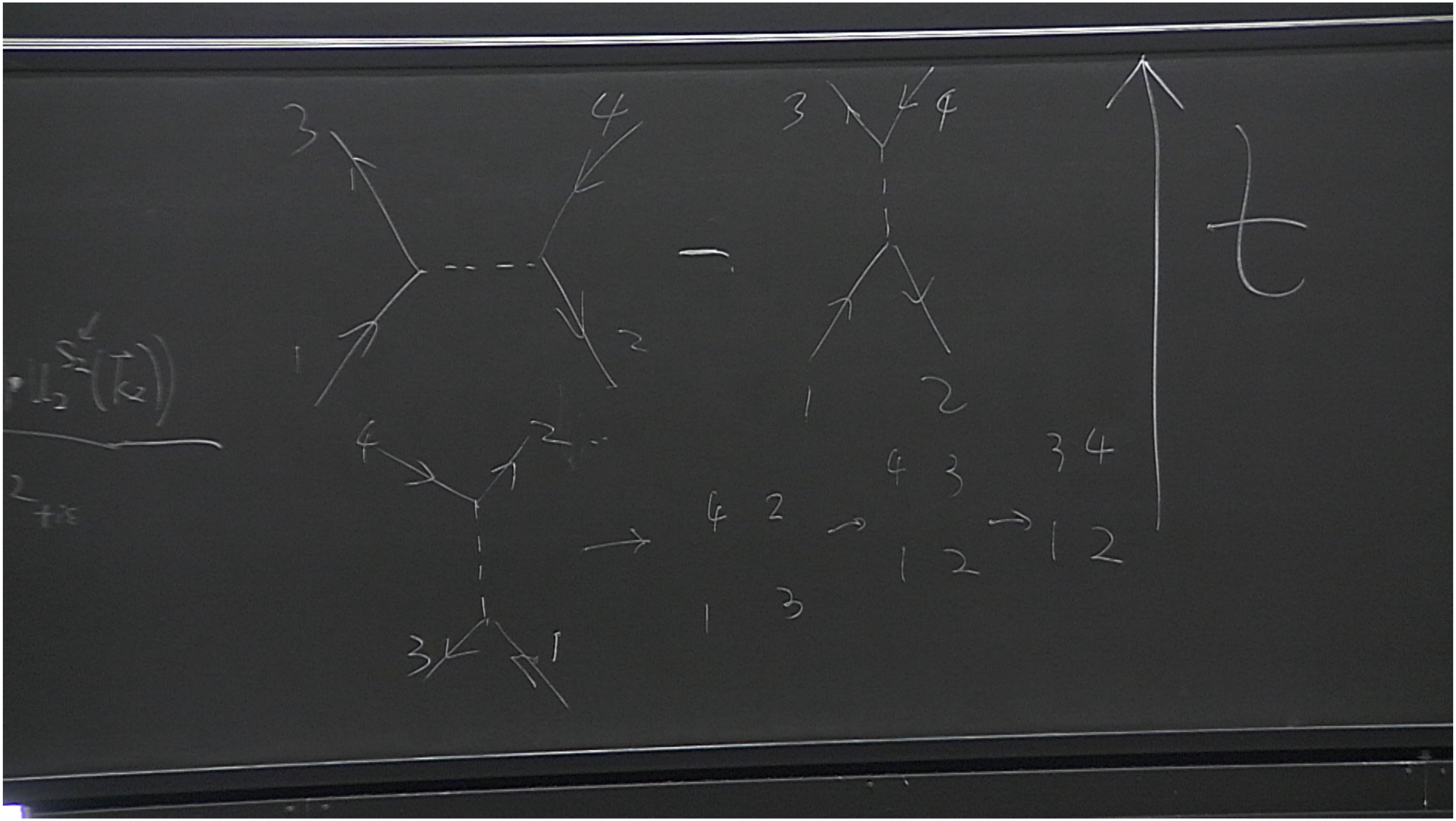
# Dirac theory.

tomorrow Average initial

sum final




$$(-i\lambda)^2 \frac{\bar{u}^{s_3}(\vec{k}_2)\cdot u^{s_1}(\vec{k}_1)\;\bar{u}^{s_4}(\vec{k}_4)\cdot u_2^{s_2}(\vec{k}_2)}{(k_1-k_3)^2-m^2+i\varepsilon}$$

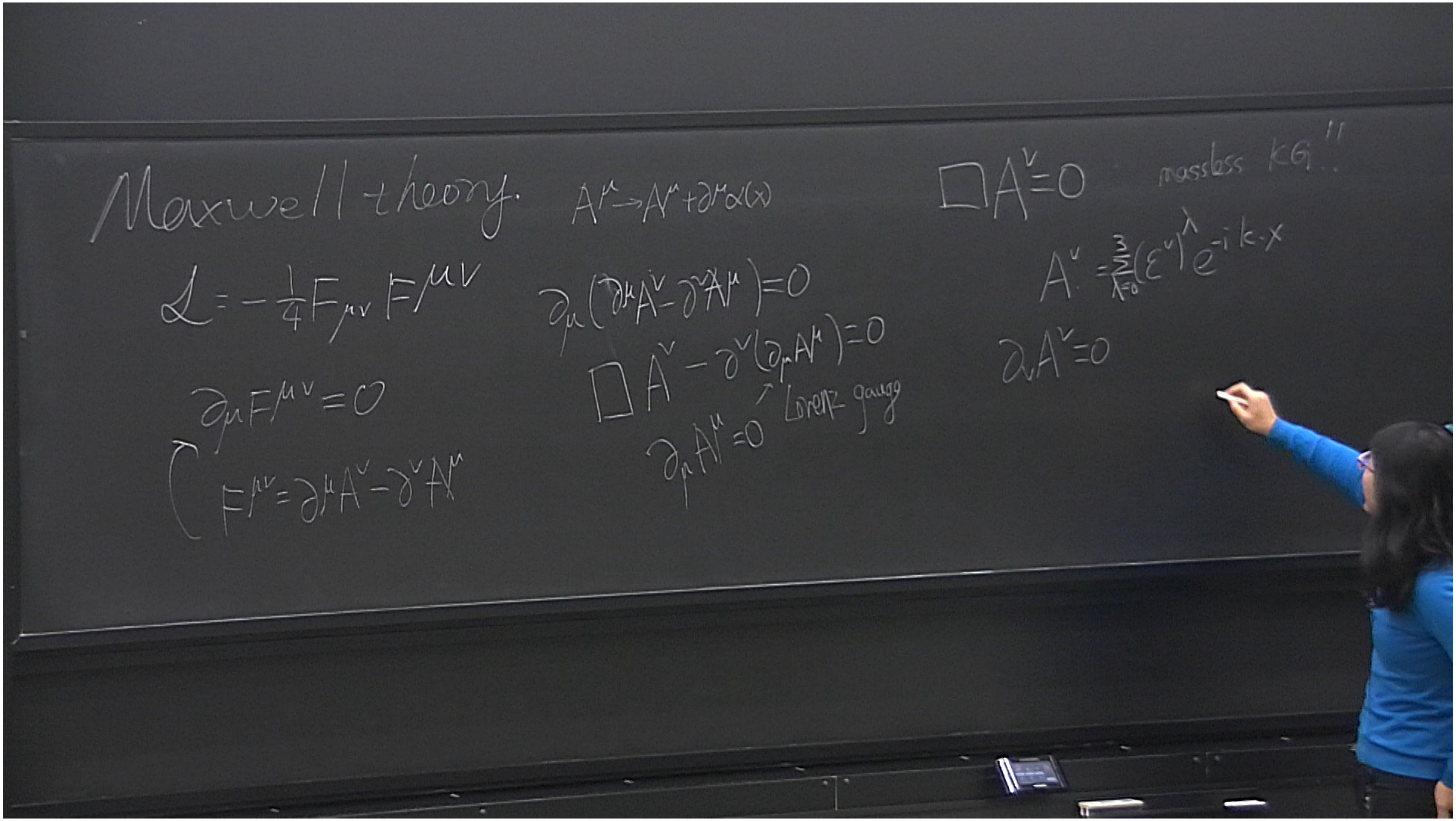


Maxwell theory. $A^\mu \to A^\mu + \partial^\mu \alpha(x)$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\partial_\mu F^{\mu\nu} = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$


Maxwell theory.
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\partial_\mu F^{\mu\nu} = 0$
$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$
$A^\mu \to A^\mu + \partial^\mu \alpha(x)$
$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = 0$
$\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = 0$
$\partial_\mu A^\mu = 0$ Lorenz gauge
$\Box A^\nu = 0$ massless KG!!
$A^\nu = \sum_{\lambda=0}^{3} (\epsilon^\nu)^\lambda e^{-ik\cdot x}$
$\partial_\nu A^\nu = 0$

$$\Box A^\nu = 0 \quad \text{massless KG!!}$$

$$A^\nu = \sum_{\lambda=0}^{3} (\varepsilon^\nu)^\lambda e^{-ik\cdot x}$$

$$\partial_\nu A^\nu = 0 \quad \sum_{\lambda=0}^{3} (k_\nu \varepsilon^\nu)^\lambda = 0 \rightarrow k_\nu(\varepsilon^\nu)^0 + k_\nu(\varepsilon^\nu)^3 = 0$$

reference momentum $(k, 0, 0, k)$

$$\varepsilon^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \varepsilon^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \varepsilon^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \varepsilon^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

In general. $k^\mu$.

$$\varepsilon^1 \cdot k = \varepsilon^2 \cdot k = 0$$

$$\varepsilon^0 \cdot k + \varepsilon^3 \cdot k = 0$$

$$(\varepsilon^\lambda)_\mu (\varepsilon^{\lambda'})_\nu \eta^{\mu\nu} = \eta^{\lambda\lambda'}$$

$$(\varepsilon^\lambda)_\mu (\varepsilon^{\lambda'})_\nu \eta_{\lambda\lambda'} = \eta_{\mu\nu}$$

In general. $k^\mu$.

$$\varepsilon^1 \cdot k = \varepsilon^2 \cdot k = 0$$

$$\varepsilon^0 \cdot k + \varepsilon^3 \cdot k = 0$$

$$(\varepsilon^\lambda)_\mu (\varepsilon^{\lambda'})_\nu \, \eta^{\mu\nu} = \eta^{\lambda\lambda'}$$

$$(\varepsilon^\lambda)_\mu (\varepsilon^{\lambda'})_\nu \, \eta_{\lambda\lambda'} = \eta_{\mu\nu}$$

# Recipes

1. pick a Lagrangian
2. $\pi$, $H$
3. impose commutor.
4. normal ordering.

step 1 $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$\mathcal{L} \longrightarrow$ EOM $\xrightarrow{\text{Gauge condition}}$ $\Box A^\nu = 0$ ✓

postpone

ngian step 2 $\Pi^0 = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0$

oops step1 $\mathcal{L}' \longrightarrow$ EOM $\Box A^\nu = 0$

$[A^0, \Pi^0] = i\delta \neq 0$

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2$

utor

ning.

$$\mathcal{L} \longrightarrow \text{EOM} \xrightarrow{\text{gauge condition}} \Box A^\nu = 0 \ \checkmark$$

postpone

$=0$

oops step1 $\mathcal{L}' \longrightarrow$ EOM $\Box A^\nu = 0$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2$$

$$\mathcal{L}_{\varphi} = -\frac{1}{2} \partial_\mu \overset{\varphi^i_{,i}}{A_\nu} \partial^\mu \overset{\varphi_i}{A^\nu}$$

$$\mathcal{L}_g = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu$$

$$\Pi^\mu = \frac{\partial \mathcal{L}_g}{\partial(\partial_0 A_\mu)} = -\partial^0 A^\mu = -\dot{A}^\mu$$

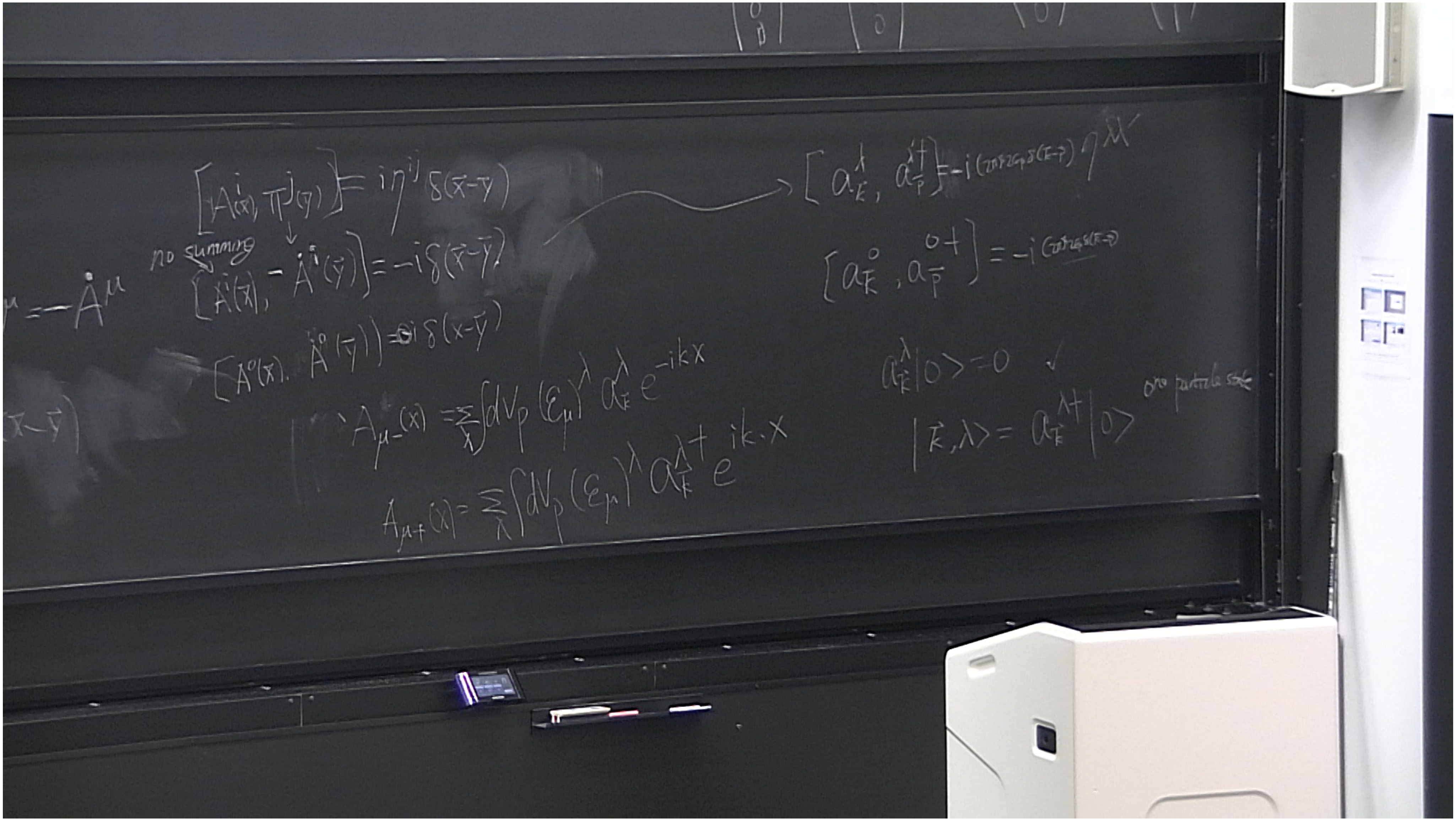
$$[A^\mu(\vec{x}), \Pi^\nu(\vec{y})] = i\eta^{\mu\nu}\delta(\vec{x}-\vec{y})$$

$$[A^i(\vec{x}), \Pi^j(\vec{y})] = i\eta^{ij}\delta(\vec{x}-\vec{y})$$

no summing

$$[A^i(\vec{x}), -\dot{A}^i(\vec{y})] = -i\delta(\vec{x}-\vec{y})$$

$$[A^0(\vec{x}), \dot{A}^0(\vec{y})] = -i\delta(\vec{x}-\vec{y})$$


$[A^i(\vec{x}), \pi^j(\vec{y})] = i\eta^{ij}\delta(\vec{x}-\vec{y})$
no summing
$[\dot{A}^i(\vec{x}), -\dot{A}^i(\vec{y})] = -i\delta(\vec{x}-\vec{y})$
$[A^0(\vec{x}), \dot{A}^0(\vec{y})] = i\delta(\vec{x}-\vec{y})$
$A_{\mu-}(x) = \sum_\lambda \int dV_p (\epsilon_\mu)^\lambda a_k^\lambda e^{-ikx}$
$A_{\mu+}(x) = \sum_\lambda \int dV_p (\epsilon_\mu)^\lambda a_k^{\lambda\dagger} e^{ik\cdot x}$
$[a_k^\lambda, a_p^{\lambda'\dagger}] = -i(2\pi)^3 2\omega_p \delta(\vec{k}-\vec{p})\eta^{\lambda\lambda'}$
$[a_k^0, a_p^{0\dagger}] = -i(2\pi)^3 2\omega_p \delta(\vec{k}-\vec{p})$
$a_k^\lambda|0\rangle = 0$
$|\vec{k},\lambda\rangle = a_k^{\lambda\dagger}|0\rangle$
one particle state

$$\left|\,|\vec{k}, \underset{\lambda=0}{0}\rangle\right|^2 = \langle 0| a^0_{\vec{k}}\, a^{0\dagger}_{\vec{k}} |0\rangle$$

$$= -(2\pi)^3 2E_{\vec{k}}\, \delta(0)$$

How to cure?

you are bad states

remove you!

from physical states

Systematic way

is to use $\partial_\mu A^\mu(x) = 0$

$$\partial^x_\mu \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle = \int d^4p \, \frac{-i\eta_{\mu\nu}}{p^2 - m^2} e^{-ip\cdot(x-y)} \neq 0$$

2N

$$\langle \text{phys} | \partial_\mu A^\mu | \text{phys} \rangle = 0 .$$

$$\partial_\mu A^{\mu -} \sim \sum_\lambda k_\mu (\varepsilon^\mu)^\lambda a^\lambda_{\vec{k}} \, | \text{phys} \rangle = 0$$

$$\left( k \cdot \varepsilon^0 a^0_{\vec{k}} + k \cdot \varepsilon^3 a^3_{\vec{k}} \right) | \text{phys} \rangle = 0$$

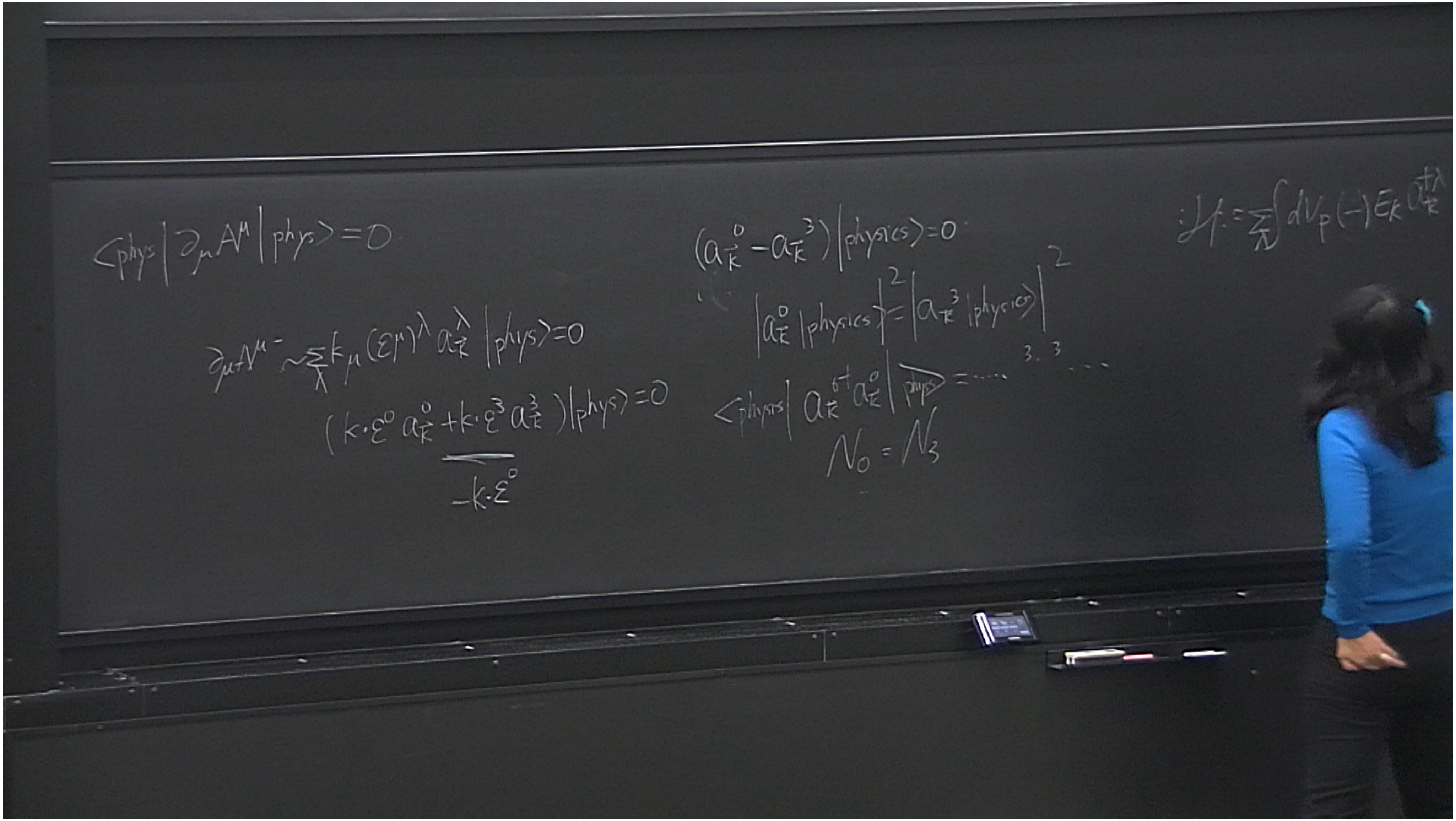
$$A_{\mu+}(x) = \sum_\lambda \int dV_p \, (\epsilon_\mu)^\lambda a_{\vec{k}} \, e \ldots$$

$$\langle phys | \partial_\mu A^\mu | phys \rangle = 0$$

$$(a_{\vec{k}}^0 - a_{\vec{k}}^3)|physics\rangle = 0$$

$$\partial_\mu A^{\mu -} \sim \sum_\lambda k_\mu (\epsilon^\mu)^\lambda a_{\vec{k}}^\lambda |phys\rangle = 0$$

$$(k\cdot\epsilon^0 a_{\vec{k}}^0 + \underbrace{k\cdot\epsilon^3}_{-k\cdot\epsilon^0} a_{\vec{k}}^3)|phys\rangle = 0$$


$\langle phys | \partial_\mu A^\mu | phys \rangle = 0$
$\partial_\mu A^\mu \sim \sum_\lambda k_\mu (\varepsilon^\mu)^\lambda a^\lambda_{\vec{k}} | phys \rangle = 0$
$(k \cdot \varepsilon^0 a^0_{\vec{k}} + k \cdot \varepsilon^3 a^3_{\vec{k}}) | phys \rangle = 0$
$-k \cdot \varepsilon^0$
$(a^0_{\vec{k}} - a^3_{\vec{k}}) | physics \rangle = 0$
$| a^0_{\vec{k}} | physics \rangle |^2 = | a^3_{\vec{k}} | physics \rangle |^2$
$\langle physics | a^{0\dagger}_{\vec{k}} a^0_{\vec{k}} | phys \rangle = \dots$
$N_0 = N_3$
$:H: = \sum_\lambda \int dV_p (-) E_k a^{\dagger\lambda}_{\vec{k}}$

$$:\mathcal{H}: = \sum_{\lambda} \int dV_p \, (-) E_k \, a_{\vec{k}}^{\dagger\lambda} a_{\vec{k}}^{\lambda} \, \eta^{\lambda\lambda}$$

$$\lambda = 0 \quad -a_{\vec{k}}^{\dagger 0} a_{\vec{k}}^{0}$$

$$= 0$$

$$\lambda = 3 \quad a_{\vec{k}}^{\dagger 3} a_{\vec{k}}^{3}$$