

Title: Categories of line operators in 3d $N=4$ gauge theories

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Abstract:

A 3d $N=4$ gauge theory admits two topological twists, which we'll simply call A and B. The two twists are exchanged by 3d mirror symmetry. It is known that local operators in the A (resp. B) twist include the Coulomb-branch (resp. Higgs-branch) chiral rings. In this talk I will discuss the *line* operators preserved by the two twists, which in each case should have the structure of a braided tensor category.

Roughly speaking, one finds vortex lines in the A-twist and Wilson lines in the B-twist. I will propose a geometric identification of the corresponding categories, and explain how geometric calculations can be used to find the vector spaces of local operators at junctions of lines.

Combined with 3d mirror symmetry, this will lead to new dualities of braided tensor categories. [Joint work with N. Garner, M. Geracie, and J. Hilburn.]

w/

N. Garner, M. Geracie,
J. Hilburn

I. Categorical stuff

II. B-type lines

III. A-type lines

IV. Mirror sym

Given 3d SUSY^c th_y on $\mathbb{R}^3$
w/ a topological Q

$$Q^2 = 0 \quad P_u = \{Q, *\}$$

- $Ops_Q = H_Q^*(Ops)$
graded comm algebra
(Poisson)

- Line ops preserved by Q
form a category $\mathcal{C}$

$$Ob(\mathcal{C}) = \text{line ops}$$

$$L \overset{\cup}{,} M$$

$$Hom(L, M)$$

$$\begin{array}{c} | \\ M \\ | \\ L \end{array}$$

SUSY theory on $\mathbb{R}^3$

gical Q

$$\mathcal{M} = \{Q, *\}$$

$Q(Ops)$

ded comm algebra
(Poisson)

ned by Q
egory $\mathcal{C}$

$$Ob(\mathcal{C}) = \text{line ops}$$

$$L \overset{\cup}{,} M$$

$$Hom(L, M) = H_Q(Ops(L, M))$$



$Ob(e) = \text{line ops}$

L, M

$$\text{Hom}(L, M) = H_Q(\mathcal{O}_P(L, M))$$

- $\mathcal{C}$ is a braided $\otimes$ category

$$L \otimes M \sim L \otimes M$$

Braiding $\leadsto$ monodromy of
a local system $L \otimes M$
over S'

$$\beta \in \text{End}(L \otimes M)$$

$$L \otimes M$$

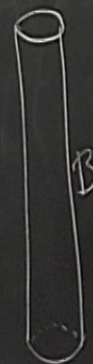
- Can recover bulk local ops from $\mathcal{C}$

$\mathbb{1}$ = trivial line

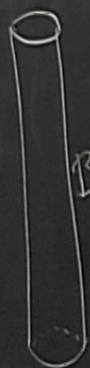
$$\text{Hom}(\mathbb{1}, \mathbb{1}) = \text{End}(\mathbb{1}) = \text{Ops}_{\mathcal{C}}$$

$\mathcal{Q}$ -closed

- Bdy conds wrapped on S^1
define objects of $\mathcal{C}$

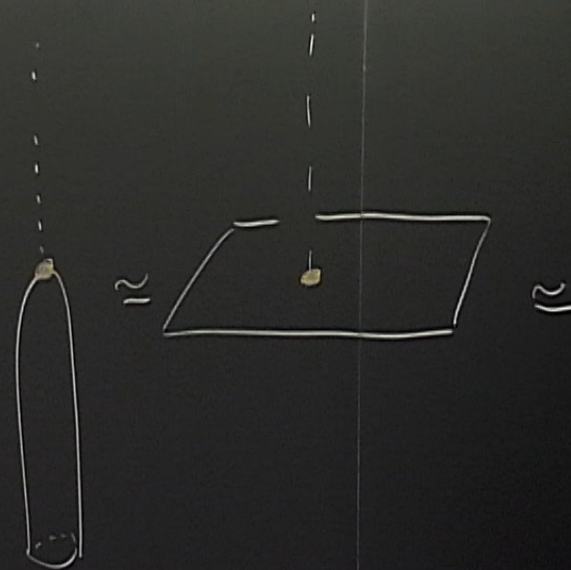


- Q-closed
- Bdy conds wrapped on S^1
define objects of $\mathcal{C}$



$B \in \text{Ob}(\mathcal{C})$

$\text{Hom}(B, \underline{1})$

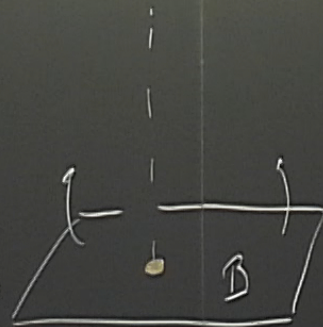


on S^1
 C

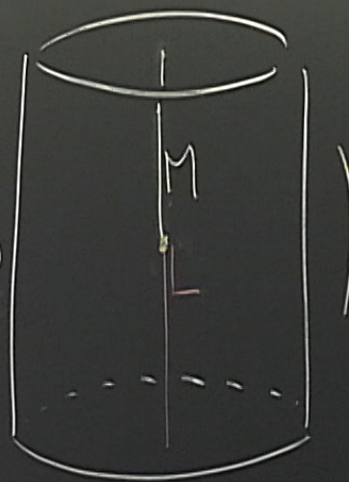
$$\text{Hom}(B, \mathbb{1})$$



$\simeq$



$$\simeq \mathcal{K}(B)$$



$$\text{Hom}(B, L)$$

$$\text{Hom}(L, M)$$

$$\text{Hom}(B, M)$$

3d $N=4$ gauge thys

two types of topological Q 's

Q_A

Q_B

R-sym: $SU(2)_H \times SU(2)_C$

3d mirror
sym

Q_A scalar under diag $[SU(2)_E \times SU(2)_H]$
 Q_B $SU(2)_C$

Given

w/ a

$Q^2 = 0$

Opsq

Line o

Data: $\text{cpt } G$, $R \oplus R^*$
 $\uparrow$
 f.d. cx rep

$\curvearrowright$
 $SU(2)_C$
 rotates a triplet
 of q -scalars

$\curvearrowright$
 $SU(2)_H$

Q'_S

$[SU(2)_H \times SU(2)_C]$
 $SU(2)_C$

Q_B

Rozansky-Witten,
 Kapustin-Poz-Saulina
 Blau-Thompson

Q_A

4d Donaldson twist

$\text{Ob}(e) =$

L, M

$\text{Hom}(L, M) =$

C is a braided

$L \rightarrow M$

Summary

$$C_B \simeq MF_G(G_0 \times R \times R^*, W)$$

$$\bigcup C_B^\circ \simeq MF_G(\overset{\alpha}{G_0} \times R \times R^*, W = \langle \alpha, \mu \rangle)$$

$\mu: () \rightarrow \overset{v}{G_0}$

Braiding $\leadsto$ monodromy
a local sys

$$\beta \in \text{End}(L \otimes M)$$



Summary

$$C_B \simeq MF_G(G_C \times R \times R^*, W)$$

$$\bigcup_{\alpha} C_B^{\alpha} \simeq MF_G(\underset{\text{is}}{g_C} \times R \times R^*, W = \langle \alpha, \mu \rangle)$$

$$\text{Coh}_G(\underset{\text{derived fiber}}{\mu^{-1}(0)})$$

$$\underline{1} \simeq \mathcal{O}_{\mu^{-1}(0)} \quad \text{End}(\underline{1}) \simeq \mathbb{C}[\mu^{-1}(0)]^G = \mathbb{C}[M_0]$$

Braiding $\leadsto$ monodromy
a local sys

$$\beta \in \text{End}(L \otimes M)$$



$$B \quad \simeq \quad MF_G(g \in \mathcal{R} \times \mathcal{R}^*, W = \langle \alpha, \mu \rangle)$$

is

$$\text{Coh}_G(\tilde{\mu}^{-1}(0))$$

derived fiber

$$\underline{1} \simeq \mathcal{O}_{\tilde{\mu}^{-1}(0)} \quad \text{End}(\underline{1}) \simeq \mathbb{C}[\tilde{\mu}^{-1}(0)]^G$$

$$= \mathbb{C}[M_H]$$

B-type lines $G, \quad \textcircled{X} \quad \textcircled{Y}$
 $R \oplus R^*$

Q_B -BPS eqs

triplet of scalars
 $\downarrow$
 ϕ

$$A_\mu = A_\mu + i\phi_\mu$$

$\mathbb{R}$ flat

X, Y covt^y constant

$\curvearrowright$
 $SU(2)_C$
 rotates a triplet
 of ϕ -scalars

Q_B Rozo
 Kar

Bl

Q_A

4d

B-type lines

$G, R \oplus R^+$

$\mathcal{Q}_B$ -BPS eqs

triplet of
of scalars

$$A_\mu = A_\mu + i\Phi_\mu$$

$\mathbb{R}$ flat

X, Y covt^y constant

$$\mu = 0$$

Wilson



Wilson lines W_p

$\rho: \text{fid}_{\text{rep}}$
of G



$$\text{Hol}_\rho \rho(A) = W_p$$

$$\text{Hom}(W_p, W_{p'})$$

$$\cong \left(\mathbb{C}[X, Y] / (Y=0) \otimes \rho^* \otimes \rho' \right)^G$$

Summary

$$C_B \cong MF_G$$

$$\bigcup C_B^0 \cong MF_G$$

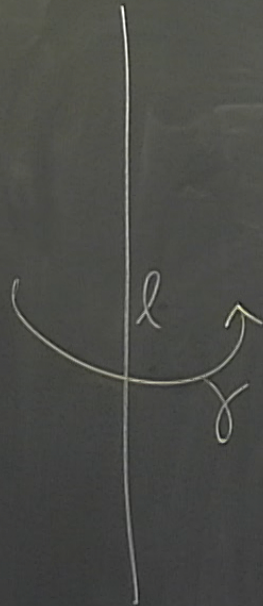
Coh

$$\underline{1} \cong \mathcal{O}_{\mu^{-1}(0)}$$

$f \downarrow \text{rep}$
of G

$= W_p$

$(\mathcal{O}/(\mu=0) \otimes p^* \otimes p')^G$



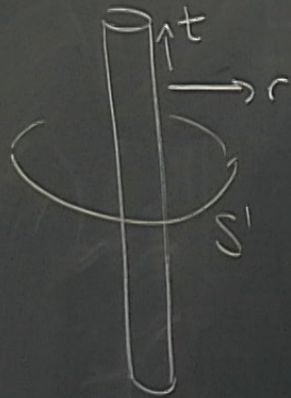
$$\mathcal{F} = \alpha \sum_l^{(2)}$$

$$\text{Hol}_g(A) = e^\alpha$$

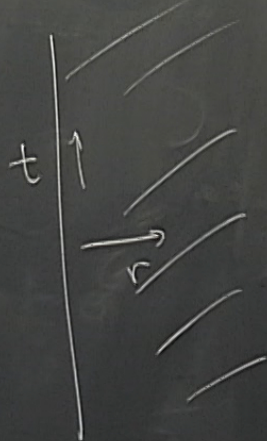
$$g \in G_{\mathbb{C}} / \text{Ad}(G_{\mathbb{C}})$$

((Conj))

To derive C_B :
excise $N(l) \in \mathbb{R}^3$



$\approx$



$\approx$ Bdy

Get 2d (2,2)

gauge gp LG

target $L(\mathbb{R} \oplus \mathbb{R}) \times L_{gc} \mathbb{R}$

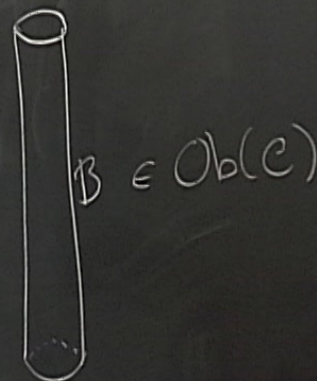
Get 2d (2,2)

gauge gp LG

target $L(R \oplus R) \times \underbrace{L_{\text{gauge}} \theta}_{\text{Conn on } S'}$

$$W = \int_{S'} \theta \wedge X \wedge D\theta Y$$

- Bdy conds wrapped on
define objects of C



$$C_B = MF_G(\hat{L}(R \oplus R^*) \times Lg_e, W)$$

$$\text{Hom}(B, \mathbb{1})$$

$$\text{If } G = \mathbb{1} \simeq \text{Coh}(R \oplus R^*)$$

$$\text{If } \text{Hol}_S A = e^\alpha \text{ } \alpha \text{ small}$$

$$\simeq MF_G((R \oplus R^*) \times g_c, W = \langle \alpha, \mu \rangle_\alpha)$$

$$\begin{array}{c} \text{Hom}(B, L) \\ \text{Hom}(B, M) \end{array} \xleftarrow{\text{Hom } L}$$

2)

G

$$(R \oplus R^*) \times \underbrace{Lg_e}_{\text{Conn on } S'}$$

$$X \circ D \circ Y$$

$$X \circ D \circ Y$$

A-type lines

Get $2d(2,2)$

A-twisted theory on $\mathbb{R}_r \times \mathbb{R}_t$

$\leadsto \text{Fut}_{\text{LG}}(L(\cdot))$

Wilson line



$\mathcal{Q}_A$ -BPS eqs

2)

$\downarrow$ th γ on $\mathbb{R}_r \times \mathbb{R}_t$

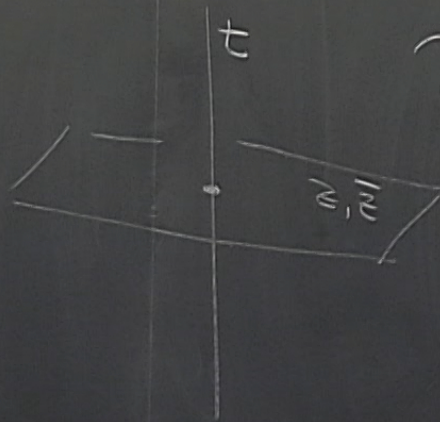
$L(\cdot, \cdot)$

E.g. free hypermultiplet

$$X, Y \in \mathbb{C} \oplus \mathbb{C}$$

$$\partial_{\bar{z}} X = -\partial_t \bar{Y}$$

$$\partial_{\bar{z}} Y = \partial_t \bar{X}$$



t.plet

$$\in \mathbb{C} \oplus \mathbb{C}$$

Singular solutions:

X, Y holomorphic on $\mathbb{C}$
(mero at $z=0$)

$$X \sim \frac{a}{z^p} \quad Y \sim \frac{b}{z^k}$$

In fact: $\begin{cases} X \sim \frac{a}{z^k} + \dots \\ Y \sim z^h (\text{regular}) \end{cases}$ or vice versa

Sing s.d must
 correspond to a Lagrangian
 in $L(\mathbb{C} \oplus \mathbb{C})$ wrt $Re(\Omega)$
 param'd by modes

$$X = \sum_n x_n z^n \quad Y = \sum_n y_n z^n$$

$$\Omega = \sum_n dx_n \wedge dy_{-n-1}$$

or vice
 versa

Get 2d (2,2)
 gauge gp LG
 target $L(\mathbb{R} \oplus \mathbb{R})$

$$W = \oint_{S'} d\theta X$$

$\swarrow$
 B-branes $(L(-))$
 $\searrow$
 A-branes $(L(-))$

$\searrow$
 B-branes (loc const loops)

$$LG(L(R \oplus R^*) \times Lg_c, W)$$

$$G=1 \simeq \text{Coh}(R \oplus R^*)$$

$$\text{Hol}_S A = e^\alpha \quad \alpha \text{ small}$$

$$\simeq \text{MF}_G((R \oplus R^*) \times g_c, W = \langle \alpha, \mu \rangle)$$

$$\begin{array}{c|c}
 L_k & k \\
 \hline
 H^*(L_0, L_k) & \\
 \hline
 L_0 & 0
 \end{array}$$

free hyper
 X, Y

3d MS
 $\longleftrightarrow$

$U(1) + \text{hyper}$
 $\tilde{X}, \tilde{Y}$
 $+1, -1$

isomorphic vxs

or vice
 versa

X
 Y

Pole order k
 zero "

$k \in \mathbb{Z}$

V_k

$$\mathbb{C} = H^*(L_0 \cap L_k)$$

or vice versa

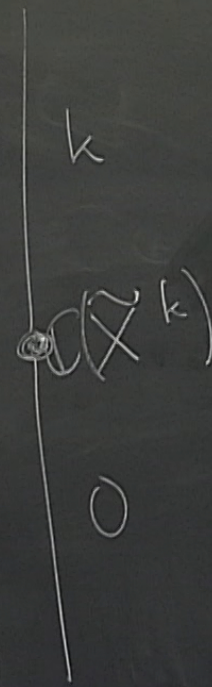
3d MS



$U(1) + \text{hyper}$
 $\tilde{X}, \tilde{Y}$
 $+1 \quad -1$

Wilson lines W_k
 in rep of $\frac{1}{T}$ charge k

polys in $\tilde{X}, \tilde{Y}$
 of degree k
 $(w | \tilde{X} \tilde{Y} = 0)$



Sing

X

Ih fa

k
 $\otimes (X^k)$
 0

G
 Φ_μ
 $\hookrightarrow$
 $SU(2)_C$
 Q_B Need $SU(2)_C$ (always!)

$\mathbb{R} \oplus \mathbb{R}^*$
 $\hookrightarrow$
 $SU(2)_H$
 (always!)

Q_A Need $SU(2)_H$
 If restrict $M_3 \approx \mathbb{R} \times \mathbb{C}$

Sing sd must
 correspond to
 in $L(\mathbb{C} \oplus \mathbb{C})$
 param^d by w
 $X = \sum_n x_n$
 $\Omega =$

$$\begin{array}{ccc}
 & R \oplus R^* & \\
 U(1)_H & 1 & 1 \\
 \downarrow & & \\
 \text{twist by a Ham isom} & & \\
 U(1)_H & \begin{array}{cc} 2 & 0 \\ \text{or} & \\ 0 & 2 \end{array} & \begin{array}{c} T^*(R^*) \\ T^*(R) \end{array}
 \end{array}$$

$$T^*Y$$

$$\begin{array}{l}
 \swarrow \text{B-branes } (L(-1)) \\
 \searrow \\
 \mathcal{A}_3(L(-1))
 \end{array}$$

$$R \oplus R$$

$$U(1)_H \quad 1 \quad 1$$

↓

twist by a Ham isom

$$U(1)_H \quad 2 \quad 0$$

or

$$0 \quad 2$$

$$T^+(R^+)$$

$$T^+(R)$$

$$\Omega_{LX} = \operatorname{Res}_{z=0} \Omega_X$$

$$G \subset X$$

$$\begin{aligned} T^+(R^+) \\ T^+(R) \end{aligned}$$

$$= \operatorname{Res}_{z=0} \Omega_X$$

$$MF_{LG}(\operatorname{Conn}(S') \times L\mathcal{X}, W) = \int_{S'} \phi^*(\Lambda)$$

$$\phi: S' \rightarrow \mathcal{X}$$

$$LG(L(R \oplus R^+) \times Lg_c, W)$$

$$G=1 \simeq \operatorname{Coh}(R \oplus$$

$$\operatorname{Hol}_{S'} A = e^\alpha \quad \alpha \text{ small}$$

$$\simeq MF_G((R \oplus R^+) \times g_c, W = \langle \alpha, \mu \rangle)$$