Title: The Conformal Anomaly Effective Theory of Gravity III: Scalar Gravitational Waves, Black Holes and Dark Energy

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Abstract: In this third of 3 talks I will discuss the effects of the conformal anomaly in the low energy infrared relevant correction to General Relativity. Among the significant implications of this effective field theory of gravity are the prediction of scalar gravitational wave solutionsâ€”a spin-0 breather modeâ€” in addition to the transversely polarized tensor waves of the classical Einstein theory. Astrophysical sources for scalar gravitational waves are considered, with the excited gluonic condensates in the interiors of neutron stars in merger events with other compact objects likely to provide the strongest burst signals. The conformal anomaly also implies generically large quantum back reaction effects and conformal correlators in the vicinity of black hole horizons which are relevant to the formation of a non-singular interior, as well as an additional scalar degree of freedom in cosmology, providing a theoretical foundation for dynamical vacuum dark energy.

Macroscopic Effects of the Quantum Conformal Anomaly:

Scalar Gravitational Waves, Black Holes & Dark Energy

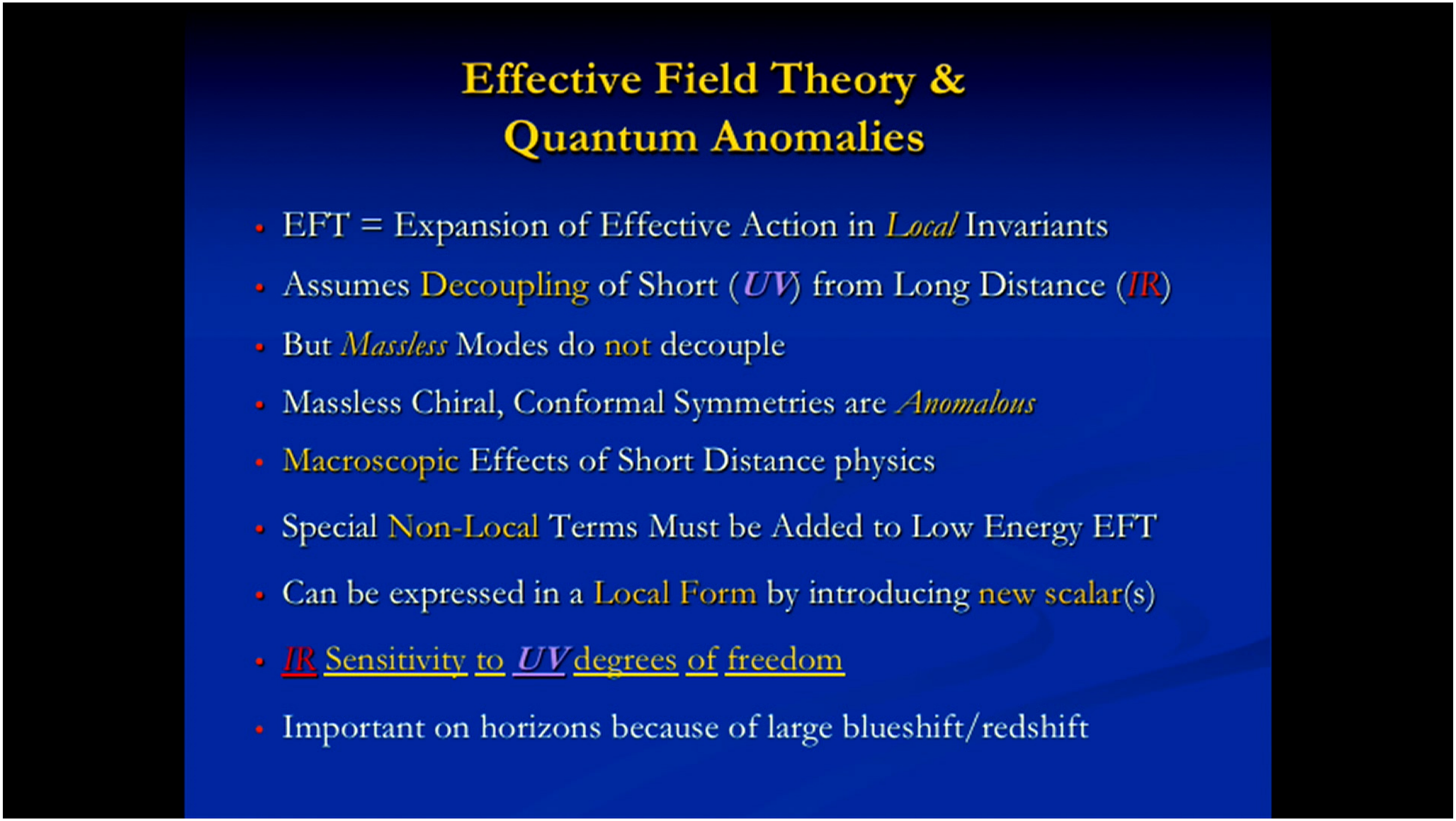
E. M.

Los Alamos & Perimeter

- w. R. Vaulin, Phys. Rev. D 74, 064004 (2006)
- w. M. Giannotti, Phys. Rev. D 79, 045014 (2009)
- Dynamical Dark Energy & Non-Gaussianity CMB:
  w. I. Antoniadis & P. O. Mazur, N. Jour. Phys. 9, 11 (2007)
  JCAP 09, 024 (2012)
- Conformal Scalar in Cosmology:
  w. P. R. Anderson & C. Molina-Paris, Phys. Rev. D 80, 084005 (2009)
- Review: Acta. Phys. Pol. B 41, 2031 (2010)
- w. D. Blaschke, R. Caballo-Rubio, JHEP 1412 (2014) 153
- Scalar Gravitational Waves: JHEP 07 (2017) 043
- Axion-Like Particle from Axial Anomaly: w. A. Sadofyev (to appear)

Outline

- Macroscopic Effects of the Chiral Anomaly
  - The Chiral/Axial Anomaly in QED in 2D and 4D
  - Massless Scalar Poles in Anomaly Amplitudes
  - Ideal Anomalous Hydrodynamics & Superfluidity
  - Axion-Like Collective Boson from the Axial Anomaly
- Macroscopic Effects of the Conformal Anomaly
  - The Conformal/Trace Anomaly in 2D and 4D
  - Massless Scalar from the Conformal Anomaly
  - Conformal Part of Metric becomes Dynamical
- Effective Field Theory of Low Energy Gravity
  - Black Holes & Horizons
  - Scalar Gravitational Waves
  - Dark Energy & Cosmology


Effective Field Theory &
Quantum Anomalies
EFT = Expansion of Effective Action in Local Invariants
Assumes Decoupling of Short (UV) from Long Distance (IR)
But Massless Modes do not decouple
Massless Chiral, Conformal Symmetries are Anomalous
Macroscopic Effects of Short Distance physics
Special Non-Local Terms Must be Added to Low Energy EFT
Can be expressed in a Local Form by introducing new scalar(s)
IR Sensitivity to UV degrees of freedom
Important on horizons because of large blueshift/redshift

Constructing the EFT of Gravity

- Assume ***Equivalence Principle*** (Symmetry)
- Metric Order Parameter Field g_{ab}
- Only two strictly ***relevant*** operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity **+** Quantum Corrections
- Semi-classical Einstein Eqs. $(m << k << M_{pl})$:

$$G_{ab} + \Lambda\, g_{ab} = 8\pi\, G\, \langle T_{ab} \rangle$$

- But there is also a quantum (trace) anomaly:

$$\langle T_a{}^a \rangle = b\, C^2 + b'\, (E - \tfrac{2}{3} \Box R) + b''\, \Box R$$

$$F = C_{abcd}C^{abcd} = C^2 \qquad E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

- ***Massless Poles*** $\Rightarrow$ ***New*** (marginally) relevant operator in gravitational sector

Effective Action for the Trace Anomaly

- **Non-Local Covariant Form (logarithmic propagator)**

$$S_{anom}[g] = \frac{1}{8}\int d^4x\sqrt{g_x}\left(E - \tfrac{2}{3}\Box R\right)_x \int d^4x'\sqrt{g_{x'}}\,(\Delta_4)^{-1}_{x,x'}\,\mathcal{A}_{x'}$$

$$\boxed{\mathcal{A} = b'\left(E - \tfrac{2}{3}\Box R\right) + b\,C^2 + c\,F^2 + c_s\,\mathrm{tr}\,G^2}$$

- **Local Covariant Form in Terms of New Scalar Field**

$$\boxed{S_{anom}[g;\varphi] = -\frac{b'}{2}\int d^4x\sqrt{g}\,\varphi\,\Delta_4\,\varphi + \frac{1}{2}\int d^4x\sqrt{g}\,\mathcal{A}\,\varphi}$$

- **Dynamical Scalar in Conformal Sector: 'Conformalon'**

$$\boxed{\Delta_4\varphi = \frac{1}{2b'}\,\mathcal{A}}$$

$$\boxed{\begin{aligned} b &= \frac{\hbar}{120(4\pi)^2}\,(N_s + 6N_f + 12N_v) \\ b' &= -\frac{\hbar}{360(4\pi)^2}\,(N_s + 11N_f + 62N_v) \end{aligned}}$$

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given in powers of Local Curvature

$$\boxed{S_{eff}[g] = S_{EH}[g] + S_{anom}[g,\varphi]}$$

This is a non-trivial modification of classical General Relativity required by quantum effects in the Std. Model

A scalar-tensor effective theory
but very different than e.g. Brans-Dicke

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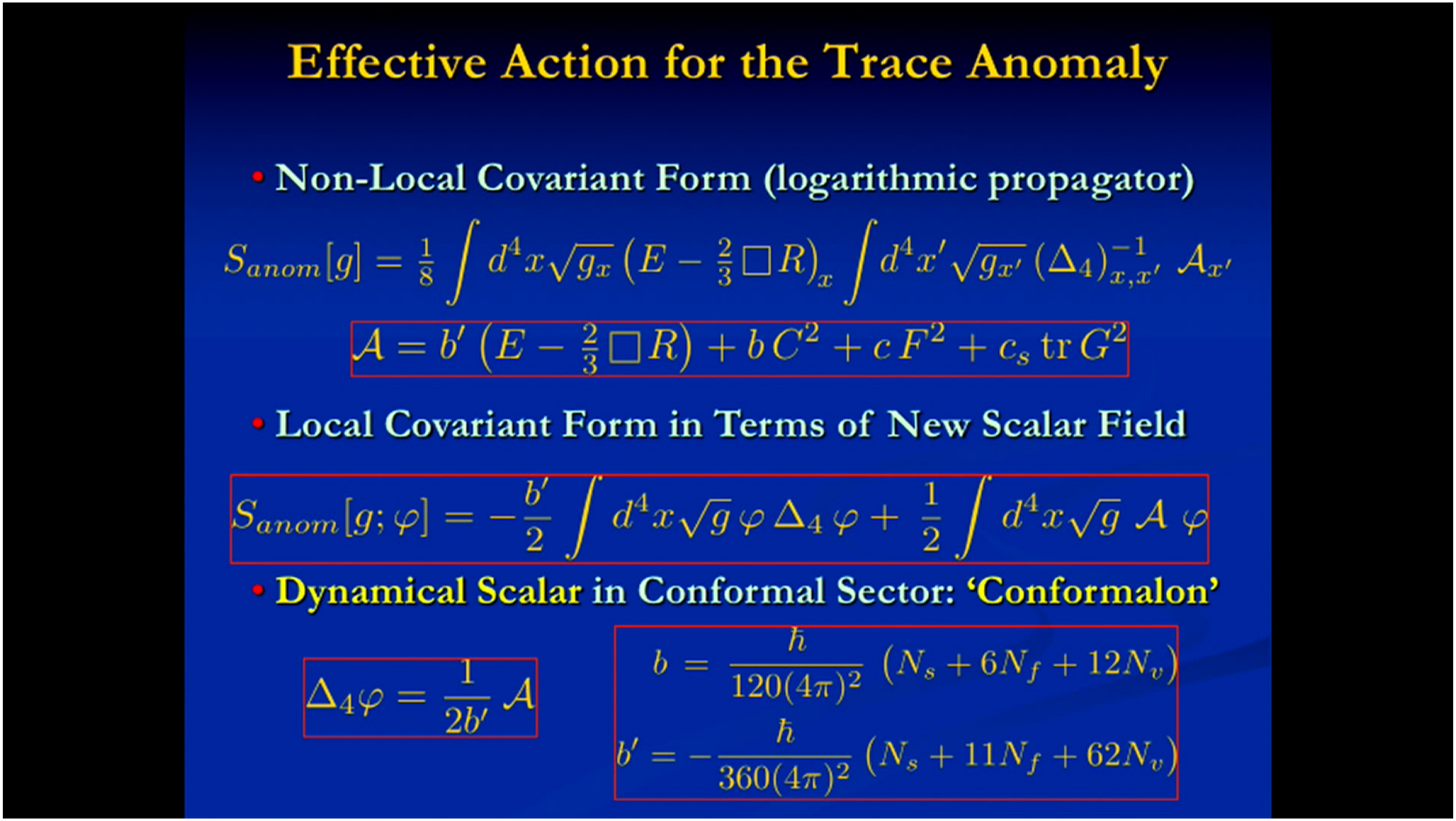
Effective Action for the Trace Anomaly
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Stress Tensor of the Anomaly Action

Variation of the Effective Action with respect to the metric gives conserved stress-energy tensor

$$T_{\mu\nu}[\varphi] \equiv -\frac{2}{\sqrt{g}}\,\frac{\delta}{\delta g^{\mu\nu}}\,S_{anom}[g;\varphi]$$

- Quantum Vacuum Polarization in Terms of (Semi-)Classical Scalar Conformalon
- φ depends upon the global Topology of spacetime and its boundaries, horizons

Anomaly Effective Field Theory
Stress Tensor

$$T_{\mu\nu}[\varphi] = b'\,E_{\mu\nu} + b\,C_{\mu\nu} + \sum_i \beta_i\,T^{(i)}_{\mu\nu}[\varphi]$$

Euler-Gauss-Bonnet—Quadratic & Linear in **φ**

$$\begin{aligned}
E_{\mu\nu} \equiv &-2\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\Box\varphi) + 2\nabla^\alpha\big[(\nabla_\alpha\varphi)(\nabla_\mu\nabla_\nu\varphi)\big] - \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big]\\
&+ \tfrac{2}{3}\,R_{\mu\nu}\,(\nabla_\alpha\varphi)(\nabla^\alpha\varphi) - 4\,R^\alpha_{\ (\mu}\,\big[(\nabla_{\nu)}\varphi)(\nabla_\alpha\varphi)\big] + \tfrac{2}{3}\,R\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\varphi)\\
&+ \tfrac{1}{6}\,g_{\mu\nu}\,\big\{-3\,(\Box\varphi)^2 + \Box\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big] + 2\,\big(3R^{\alpha\beta} - Rg^{\alpha\beta}\big)(\nabla_\alpha\varphi)(\nabla_\beta\varphi)\big\}\\
&- \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\Box\varphi - 4\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R^\alpha_{(\mu}\nabla_{\nu)}\nabla_\alpha\varphi + \tfrac{8}{3}\,R_{\mu\nu}\,\Box\varphi + \tfrac{4}{3}\,R\,\nabla_\mu\nabla_\nu\varphi\\
&- \tfrac{2}{3}\,(\nabla_{(\mu}R)\,\nabla_{\nu)}\varphi + \tfrac{1}{3}\,g_{\mu\nu}\,\big[2\,\Box^2\varphi + 6\,R^{\alpha\beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R\,\Box\varphi + (\nabla^\alpha R)\nabla_\alpha\varphi\big]
\end{aligned}$$

Weyl—Purely Linear in **φ**

$$C_{\mu\nu} = -4\,\nabla_\alpha\nabla_\beta\big(C_{(\mu\ \nu)}^{\ \ \alpha\ \beta}\,\varphi\big) - 2\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,R_{\alpha\beta}\,\varphi$$

Trace recovered using Eq. of Motion

$$\boxed{\Delta_4\,\varphi = \nabla_\mu\big(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \tfrac{2}{3}Rg^{\mu\nu}\big)\,\nabla_\nu\varphi = \frac{E}{2} - \frac{\Box R}{3} + \frac{1}{2b'}\Big(b\,C^2 + \sum_i \beta_i\mathcal{L}_i\Big)}$$

Casimir Effect from the Anomaly Stress Tensor

In flat space the stress tensor simplifies to

$$E_{ab}\Big|_{flat} = -2\,(\nabla_{(a}\varphi)(\nabla_{b)}\Box\varphi) + 2(\Box\varphi)(\nabla_a\nabla_b\varphi)$$
$$+\frac{2}{3}(\nabla_c\varphi)(\nabla^c\nabla_a\nabla_b\varphi) - \frac{4}{3}\,(\nabla_a\nabla_c\varphi)(\nabla_b\nabla^c\varphi)$$
$$+\frac{1}{6}\,g_{ab}\,\left\{-3\,(\Box\varphi)^2 + \Box(\nabla_c\varphi\nabla^c\varphi)\right\} - \frac{2}{3}\,\nabla_a\nabla_b\Box\varphi\,,$$

$\Box^2\varphi = 0$ **Particular Solution:** $\varphi = c_1\,\dfrac{z^2}{a^2}$

Casimir Stress tensor between parallel plates:

$$T_{ab}^{(anom)} = \frac{C}{a^4}\;\mathrm{diag}\,(-1,1,1,-3)$$

A Macroscopic Quantum Effect captured by this Stress Tensor

Scalar Gravitational Waves in EFT

- Linearize Vacuum Einstein Eqs. around Flat Space with $T_{\mu\nu}^{(\varphi)}$ source

$$R_{ab} - \tfrac{1}{2} g_{ab} R = 8\pi G\, T_{ab}^{(\varphi)}$$

- Linear Metric Decomposition (Covariant)

$$h_{ab} = h_{ab}^{\perp} + \nabla_a v_b^{\perp} + \nabla_b v_a^{\perp} + \left(\nabla_a \nabla_b - \tfrac{1}{4}\eta_{ab}\Box\right) w + \tfrac{1}{4}\eta_{ab} h$$

- Usual Einstein Constraints now Dynamical

$$\delta G_{tt}^{(S)} = -\frac{1}{4}\,\vec{\nabla}^2\,(h - \Box w) = -\frac{16\pi G b'}{3}\,\vec{\nabla}^2(\Box\varphi)$$

$$\delta G_{ti}^{(S)} = -\frac{1}{4}\,\partial_t \vec{\nabla}_i\,(h - \Box w) = -\frac{16\pi G b'}{3}\,\partial_t \vec{\nabla}_i(\Box\varphi)$$

- Solved by $\boxed{\frac{1}{4}\,(h - \Box w) = \frac{16\pi G b'}{3}\,\Box\varphi}$ gauge invariant

$$\Box^2 \varphi = 0 \qquad\qquad \Box\varphi \sim \exp(-i\omega_k t + i\vec{k}\cdot\vec{x})$$

- Scalar 'Breathing' Mode GW: Only half of Solns. Couple to metric

Gauge Invariant Space + Time Split

- Linear Metric Decomposition

$$h_{tt} = -2A \qquad h_{ti} = \mathcal{B}_i^{\perp} + \vec{\nabla}_i B$$

$$h_{ij} = \mathcal{H}_{ij}^{\perp} + \vec{\nabla}_i \mathcal{E}_j^{\perp} + \vec{\nabla}_j \mathcal{E}_i^{\perp} + 2\,\eta_{ij}\,C + 2\left(\vec{\nabla}_i \vec{\nabla}_j - \tfrac{1}{3}\,\eta_{ij} \vec{\nabla}^2\right) D$$

- Gauge Invariant Scalar Potentials $\Upsilon_A \equiv A + \partial_t B - \partial_t^2 D$
- Linearized Einstein Eqs. $\Upsilon_C \equiv C - \tfrac{1}{3}\vec{\nabla}^2 D$

$$\vec{\nabla}^2 \Upsilon_C = \frac{8\pi G b'}{3}\,\vec{\nabla}^2(\Box\varphi)$$

$$\partial_t \vec{\nabla}_i \Upsilon_C = \frac{8\pi G b'}{3}\,\partial_t \vec{\nabla}_i(\Box\varphi)$$

$$(\eta_{ij}\vec{\nabla}^2 - \vec{\nabla}_i\vec{\nabla}_j)(\Upsilon_A + \Upsilon_C) - 2\eta_{ij}\,\partial_t^2 \Upsilon_C = -\frac{16\pi G b'}{3}\,\vec{\nabla}_i\vec{\nabla}_j(\Box\varphi)$$

- Solved by

$$\boxed{\begin{gathered}\Upsilon_A = \Upsilon_C = \frac{8\pi G b'}{3}\,\Box\varphi \\ \Box\Upsilon_A = \Box\Upsilon_C = \frac{8\pi G b'}{3}\,\Box^2\varphi = 0 = \delta R\end{gathered}}$$

Potentials obey
<u>2nd order</u> wave eq.

Localized Sources of Scalar Gravitational Waves

- Retarded Green's Fn. $\Box^2 D_R(t-t';\vec{x}-\vec{x}') = \delta(t-t')\,\delta(\vec{x}-\vec{x}')$

$$D_R(t-t';\vec{x}-\vec{x}') = \frac{1}{8\pi}\,\theta(t-t'-|\vec{x}-\vec{x}'|)\,\theta(t-t')$$

- Anomaly Scalar **Conformalon Field**

$$\varphi(t,\vec{x}) = \frac{1}{16\pi b'}\int d^3\vec{x}'\int_{-\infty}^{t-|\vec{x}-\vec{x}'|} dt'\,\mathcal{A}(t',\vec{x}')$$

- Scalar Metric Peturbation in Far (Radiation) Zone

$$\boxed{\frac{\delta L}{L} = \frac{16\pi G b'}{3}\,\Box\,\varphi \to -\frac{G}{3r}\int d^3\vec{x}\,\mathcal{A}(t_{ret},\vec{x})}$$

- Power Radiated for Time Harmonic Source $\quad \mathcal{A}(t,\mathbf{x}) = e^{-i\omega t}\mathcal{A}_\omega(\mathbf{x})$

$$\left(\frac{dP}{d\Omega}\right)_{SGW}(\hat{\mathbf{r}}) = \frac{G\,\omega^2}{72\pi c^5}\left|\int d^3\mathbf{x}\,e^{-i\omega\hat{\mathbf{r}}\cdot\mathbf{x}/c}\mathcal{A}_\omega(\mathbf{x})\right|^2$$

$$P_{SGW}\big|_{monopole} = \frac{G\omega^2}{18c^5}\left|\int d^3\mathbf{x}\,\mathcal{A}_\omega(\mathbf{x})\right|^2$$

Sources of Scalar Gravitational Waves

- Sources of φ are all the trace anomaly terms

$$\Delta_4\,\varphi = \frac{E}{2} - \frac{\Box R}{3} + \frac{1}{2b'}\Big(b\,C^2 + cF_{\mu\nu}F^{\mu\nu} + \dots\Big)$$

- Curvature Invariants are extremely small
- QED and QCD Gauge Field Anomalies are much larger
- Magnetar Field **B ~ 10^{15} Gauss**

$$\mathcal{A}_{mag} = -\frac{e^2}{24\pi^2}F_{\mu\nu}F^{\mu\nu} = -\frac{\alpha B^2}{3\pi} \simeq -8\times 10^{26}\left(\frac{B}{10^{15}\,\text{Gauss}}\right)^2 \text{erg/cm}^3$$

$$\frac{\delta L}{L} \simeq -\frac{G}{3r}\int d^3x\,\mathcal{A}_{mag} \simeq 5\times 10^{-26}\left(\frac{\text{kpc}}{r}\right)$$

- Still not large enough to be observable by aLIGO
- No effects on solar system or direct terrestrial tests of GR

QCD Source of Scalar Gravitational Waves

- QCD Trace Anomaly is also a Source for φ

$$\boxed{\Box^2 \varphi = \frac{1}{2b'} \mathcal{A}_{QCD}}$$

- Gluonic Condensate much larger than $\mathcal{A}_{mag}$ (10 Orders of Magnitude)

$$\boxed{\mathcal{A}_{QCD} = -(11N_c - 2N_f)\,\frac{\alpha_s}{24\pi}\,\langle G^a_{\mu\nu} G^{a\mu\nu}\rangle \simeq -5.6 \times 10^{36}\ \mathrm{erg/cm^3}}$$

- Neutron Star Cores contain Density Dependent Gluon Condensate
- In a Neutron Star Merger with another Compact Object this Gluonic Condensate ('Bag Constant') is disturbed
- Scalar GW Mode most likely excited in Neutron Star Mergers
- Estimates require quantitative control of nuclear physics in NS mergers
- Condensate excited also in Gravastar Alternative to BH's

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Localized Sources of Scalar Gravitational Waves

- **Retarded Green's Fn.** $\Box^2 D_R(t-t';\vec{x}-\vec{x}') = \delta(t-t')\,\delta(\vec{x}-\vec{x}')$

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Anomaly Stress Tensor Near Horizons

- An apparent horizon is a null surface, where outgoing null rays are first marginally trapped
- Near horizon region is conformal to $EAdS_3 \otimes$ time
- Fields become effectively massless there, CFT
- Conformal Anomaly becomes the dominant term in effective action in the near horizon region
- Stress Tensor from S_{anom} determines $\langle T_{ab} \rangle$
- Stress Tensor is generally singular there
- Singular behavior has invariant meaning in terms of conformalon φ scalar degree of freedom on horizon

Schwarzschild Spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\, dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2\, d\Omega^2$$

$$\varphi = \sigma = \ln \sqrt{f} = \frac{1}{2}\, \ln \left(1 - \frac{2M}{r}\right) \to \infty$$

solves homogeneous $\Delta_4 \varphi = 0$

Timelike Killing field is Invariant of Geometry

$K^a = (1, 0, 0, 0) \quad e^{\sigma} = (-K_a K^a)^{\frac{1}{2}} = \sqrt{f}$

Energy density scales like $e^{-4\sigma} = f^{-2}$

Scalar Potential φ gives Geometric (Coordinate Invariant) Meaning to Non-Local Quantum correlations becoming Large on Horizon

Conformalon Scalar in Schwarzschild Space

- General solution for $\varphi(r)$ for Schwarzschild is

$$\left.\frac{d\varphi}{dr}\right|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right) +$$

$$\frac{q-2}{6M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right)\ln\left(1 - \frac{2M}{r}\right) - \frac{q}{6r}\left[\frac{4M}{r-2M}\ln\left(\frac{r}{2M}\right) + \frac{r}{2M} + 3\right]$$

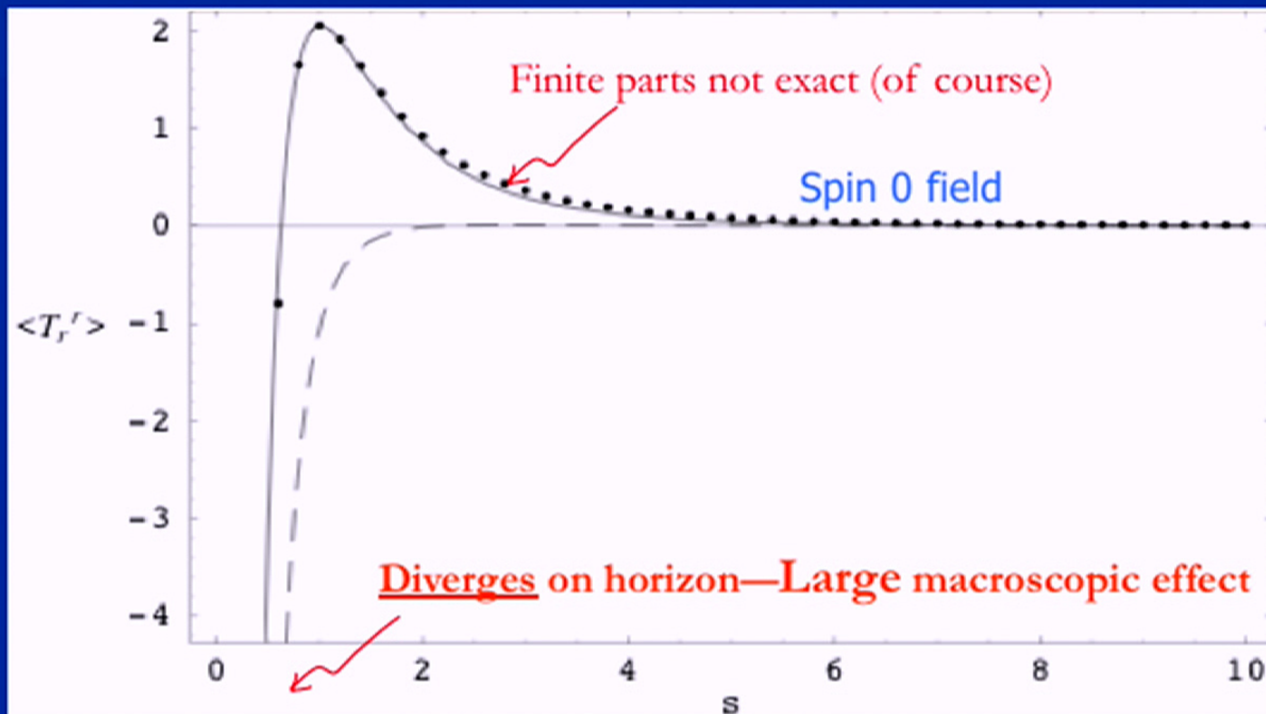
- q, c_H, c_∞ are state dependent integration constants
- Linear time dependence can be added
- Only way to have φ fall off as $r \to \infty$ is $\boxed{c_\infty = q = 0}$
- But only way to have finiteness on the horizon is

$$\boxed{c_H = 0,\ q = 2}$$

- Topological obstruction to finiteness vs. falloff as $r \to \infty$
- 2 conditions on 3 integration constants for horizon finiteness

Stress-Energy Tensor in Boulware State
Radial Component

Dots – Direct Numerical Evaluation of $<T_a^{\ b}>$ (Jensen et. al. 1992)
Solid – Stress Tensor from the Anomaly (E.M. & R. Vaulin 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)



Conformal Behavior on Horizon

Wave Eq. $\quad \left(-\Box + \mu^2\right)\Phi = 0$

Mode decomposition $\quad \Phi_{\omega\ell m} = e^{-i\omega t}\, Y_{\ell m}(\theta,\phi)\,\frac{\psi_{\omega\ell}(r)}{r}$

Radial Mode Eq. in Wheeler coordinate r* $\quad \left(-\frac{d^2}{dr^{*2}} + V_\ell\right)\psi_{\omega\ell} = \omega^2\,\psi_{\omega\ell}$

$$V_\ell = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right] \xrightarrow{r\to 2M} 0$$

Becomes Conformal at Horizon
All masses become irrelevant

$$ds^2 = f(r)\left(-dt^2 + ds^2_{opt}\right)$$

$$ds^2_{opt} \xrightarrow{r\to r_H} d\text{Ł}^2 = \frac{r_H^2}{z^2}\left(dx^2 + dy^2 + dz^2\right)$$

3D Lobachevsky Space (Euclidean AdS_3)

Symmetry Group SO(3,1) is Conformal Group of Horizon S^2

Powers of $z \propto \sqrt{f(r)} = e^{\sigma} \to 0$ measure Conformal Weights

Anomaly Stress Tensor in de Sitter Space

- Conformally Flat

$$ds^2 = -d\tau^2 + a^2(\tau)\, d\vec{x}^2 = a^2\left(-d\eta^2 + d\vec{x}^2\right) \qquad a(\tau) = e^{H\tau}$$

- Eq. of Motion Operator factorizes

$$\Delta_4 \varphi = \Box(\Box - 2H^2)\varphi = 12H^4$$

- Inhomogeneous Soln. $\varphi_{BD} = 2\ln a = 2H\tau$ gives

$$\boxed{T_{ab}\big|_{BD,dS} = 6b' H^4 g_{ab} = -\frac{H^4}{960\pi^2}\, g_{ab}\,(N_s + 11N_f + 62N_v)}$$

- This is the soln. for conformal map to flat spacetime

$$ds^2 = e^{\varphi_{BD}}(ds^2)_{\text{flat}}$$

Otherwise $T^a{}_b$ is generally divergent at the static horizon $r=H^{-1}$ behaving like $(1-H^2r^2)^{-2}$ Conformal Weight= 4

- Fluctuations? Since BD is a self-consistent de Sitter solution we may linearize the EFT Stress Tensor & Einstein Eqs. about it

Scalar Waves in de Sitter Space

- In de Sitter space remarkably there is another factorization

$$\Delta_4\Big|_{dS} = \left(\frac{\partial^2}{\partial\tau^2} + 5H\frac{\partial}{\partial\tau} + 6H^2 - \frac{\vec{\nabla}^2}{a^2}\right)\left(\frac{\partial^2}{\partial\tau^2} + H\frac{\partial}{\partial\tau} - \frac{\vec{\nabla}^2}{a^2}\right)$$

- And Linearized Einstein Eqs. of the Anomaly EFT depend only upon

$$u \equiv \left(\frac{\partial^2}{\partial\tau^2} + H\frac{\partial}{\partial\tau} - \frac{\vec{\nabla}^2}{a^2}\right)\delta\varphi \neq 0$$

- Again only half of the solns. obeying the 2nd order wave eq.

$$\Delta_4\,\delta\varphi = \left(\frac{\partial^2}{\partial\tau^2} + 5H\frac{\partial}{\partial\tau} + 6H^2 - \frac{\vec{\nabla}^2}{a^2}\right) u = 0$$

- Couple to the Gauge Invariant Scalar Metric Potentials

$$\boxed{\begin{aligned} \Upsilon_{\mathcal{A}} + \Upsilon_{\mathcal{C}} &= -\frac{8\pi G b'}{3}\,u \\ \frac{\vec{\nabla}^2}{a^2}\left(\Upsilon_{\mathcal{A}} - \Upsilon_{\mathcal{C}}\right) &= 8\pi G H b'\left(\frac{\partial}{\partial\tau} + 2H\right) u \end{aligned}}$$

Cosmological Horizon Modes

- In de Sitter static time coordinates

$$\Upsilon_A - \Upsilon_C \overset{Hr \to 1}{\longrightarrow} e^{-i\omega t}\, Y_{\ell m}(\hat{n})\,(1 - Hr)^{\pm \frac{i\omega}{2H}}$$

oscillate rapidly on the cosmological horizon

- General soln. for as fn. of r

$$\Upsilon_A - \Upsilon_C \;\propto\; \frac{c_1}{r}\,\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{c_2}{r}\,\ln\left(1 - H^2 r^2\right)$$

- Behaves logarithmically---Conformal Weight Zero Field
- Correct Conformal Weight to give Large Scale CMB Anisotropy

$$\Big\langle (\Upsilon_A - \Upsilon_C)(\hat{n})\;(\Upsilon_A - \Upsilon_C)(\hat{n}')\Big\rangle \;\propto\; \frac{1}{4\pi}\ln\left(1 - \hat{n}\cdot\hat{n}'\right) = \sum_{\ell \neq 0} \frac{Y_{\ell m}(\hat{n})Y_{\ell m}(\hat{n}')}{\ell(\ell+1)}$$

$$\boxed{c_\ell\, \ell(\ell+1) = const.}$$

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CFT Behavior also predicts Different Non-Gaussianity than Slow Roll

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Conformal Symmetry & de Sitter Horizon

- The de Sitter group SO(4,1) has 10 Killing vectors (isometries)

$$\nabla_a \xi_b^{(i)} + \nabla_b \xi_a^{(i)} = 0 \quad , \quad i = 1, \dots, 10$$

- The conformal group of S^2 is the Lorentz group SO(3,1) realized projectively, includes 3 rotations and 3 rigid special conformal transformations

$$\hat{n}^i = \frac{X^i}{X^0} \rightarrow \frac{L^i{}_0 + L^i{}_j \hat{n}^j}{L^0{}_0 + L^0{}_k \hat{n}^k} \,, \qquad \delta n^i = -v^i + \hat{n}^i (v \cdot \hat{n})$$

- The 10 isometries of SO(4,1) decompose on the de Sitter horizon into
  - 3 rotations
  - 3 conformal transformations (above) x 2 = 6
  - 1 time translation
- Any SO(4,1) de Sitter invariant Green's fn. becomes SO(3,1) conformally invariant (and t independent) on the de Sitter horizon (deS/CFT) eg.

$$\Delta_4^{-1}(x, x') \rightarrow -\frac{1}{16\pi^2} \ln(1 - \hat{n} \cdot \hat{n}') + c_0 \propto \Delta_2^{-1}(\hat{n}, \hat{n}')$$

- Dimension 0 field correlator = logarithm

Conformal Behavior on Horizon

Wave Eq. $\left(-\Box + \mu^2\right)\Phi = 0$

Mode decomposition $\Phi_{\omega\ell m} = e^{-i\omega t}\, Y_{\ell m}(\theta,\phi)\,\frac{\psi_{\omega\ell}(r)}{r}$

Radial Mode Eq. in Wheeler coordinate r* $\left(-\frac{d^2}{dr^{*2}} + V_\ell\right)\psi_{\omega\ell} = \omega^2\, \psi_{\omega\ell}$

$$V_\ell = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right] \xrightarrow{r\to 2M} 0$$

Becomes Conformal at Horizon
All masses become irrelevant

$$ds^2 = f(r)\left(-dt^2 + ds^2_{opt}\right)$$

$$ds^2_{opt} \xrightarrow{r\to r_H} d\text{Ł}^2 = \frac{r_H^2}{z^2}\left(dx^2 + dy^2 + dz^2\right)$$

3D Lobachevsky Space (Euclidean AdS_3)

Symmetry Group SO(3,1) is Conformal Group of Horizon S^2

Powers of $z \propto \sqrt{f(r)} = e^{\sigma} \to 0$ measure Conformal Weights

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New Cosmological Scalar Fluctuations

- Linear Variation of $\langle T_{\mu\nu} \rangle$ in de Sitter space contains contributions from S_{anom} of scalar conformalon field φ
- Relevant gauge inv. scalar modes satisfy second order wave eqs. (Scalar Field without an inflaton)
- New conformalon scalar degree of freedom in cosmology
- In static de Sitter coordinates the modes grow large $\ln(1 - H^2 r^2)$ on the horizon
- Corresponding stress tensor perturbation

$$\delta \langle T^a{}_b \rangle \sim H^4 (1 - H^2 r^2)^{-2} \,\mathrm{diag}\,(-3, 1, 1, 1)$$

diverges on the horizon—Suggests inhomogeneous Cosmology

Correct log scaling for scale invariant Harrison spectrum
Fluctuations of Anomaly Fields can generate CMB w/o inflaton

Additional Implication:
Vacuum Energy is Dynamical

- Conformal part of the metric, $g_{ab} = e^{2\sigma}\ \bar{g}_{ab}$ constrained --frozen--by trace of Einstein's eq. $R=4\Lambda$ becomes dynamical and can fluctuate due to φ
- Fluctuations of φ describe a conformally invariant phase of gravity in 4D $\Rightarrow$ non-Gaussian statistics of CMB

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770; Phys. Rev. Lett. 79 (1997) 14; N. Jour. Phys. 9, 11 (2007)

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- The Quantum Phase Transition to this phase characterized by the 'melting' of the scalar condensate Λ

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Fluctuations of conformalon scalar d. of f. allow Λ_{eff} to vary dynamically, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{eff} \rightarrow 0$

Summary

- Einstein's classical theory receives Quantum Corrections relevant at macroscopic Distances from Trace Anomaly
- This is a necessary quantum modification of classical GR
- Scalar 'Conformalon' φ degree of freedom in the EFT of Low Energy Gravity derived from Conformal Anomaly
- Scalar-Tensor Theory quite different from Brans-Dicke
- Does not couple to Classical Matter directly—only thru Conformal/Trace Anomaly
- IR Modification of GR passes all Observational Tests
- Form of Effective Action fixed with no free parameters (b, b')

The inner dark matter distribution of the Cosmic Horseshoe with gravitational lensing and dynamics

Thursday Talk at Perimeter Institute in Waterloo

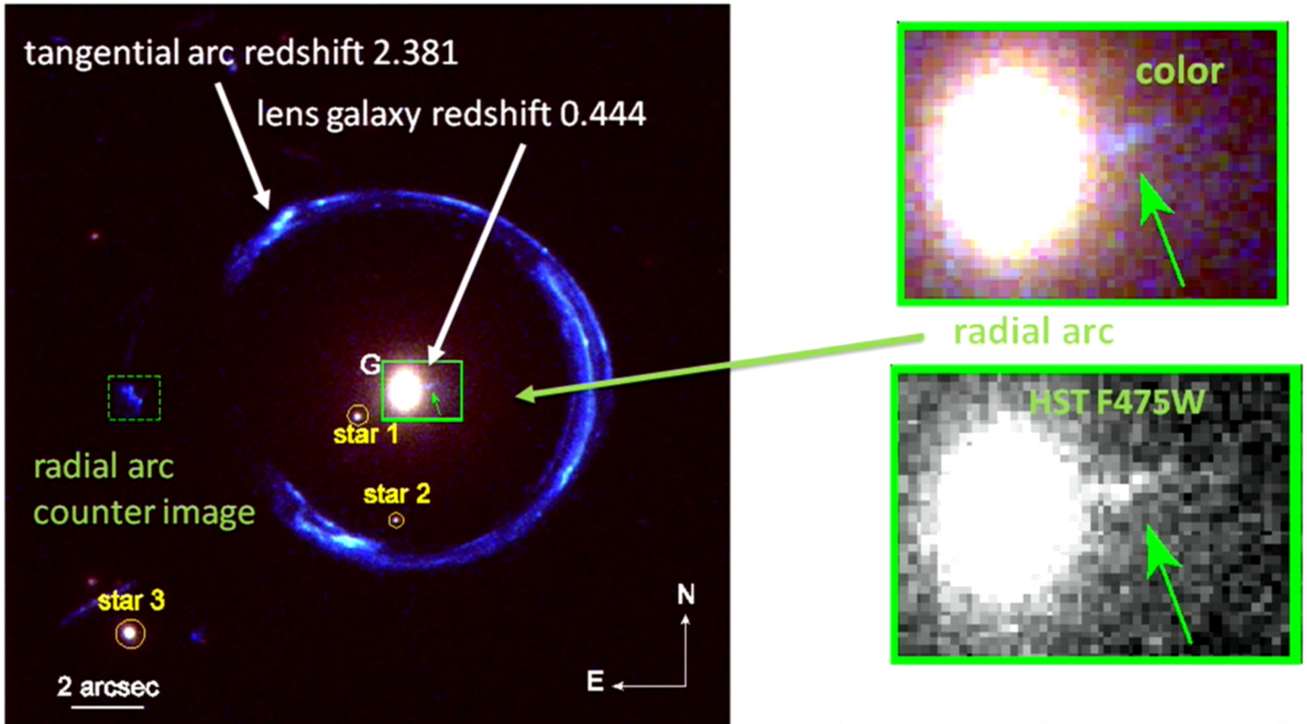
Stefan Schuldt

Max-Planck Institute for Astrophysics
Karl-Schwarzschild Str. 1
Garching Germany

Collaborators
Sherry Suyu, Giulia Chirivì, Akın Yıldırım, Alessandro Sonnenfeld

October 25, 2018

The Cosmic Horseshoe



Credit: Schuldt et al. (in prep)

discovery: Belokurov et al. (2007)