Title: Entanglement structure of current driven diffusive fermion systems

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Abstract: <p>Applying a chemical potential bias to a conductor drives the system out of equilibrium into a current carrying non-equilibrium state. This current flow is associated with entropy production in the leads, but it remains poorly understood under what conditions the system is driven to local equilibrium by this process. We investigate this problem using two toy models for coherent quantum transport of diffusive fermions: Anderson models in the conducting phase and a class of random quantum circuits acting on a chain of qubits, which exactly maps to an interacting fermion problem. Under certain conditions, we find that the long-time states in both models exhibit volume-law mutual information and entanglement, in striking violation of local equilibrium. Extending this analysis to Anderson metal-insulator transitions, we find that the volume-law entanglement scaling persists at the critical point up to mobility edge effects. This work points towards a broad class of examples of physical systems where volume-law entanglement can be sustained, and potentially harnessed, despite strong coupling of the system to its surrounding environment. </p>

Entanglement Structure of Current Driven Diffusive Fermion Systems

Michael Gullans

Department of Physics, Princeton University, Princeton, NJ, USA

Joint work with David Huse

Statistical Mechanics of Entanglement

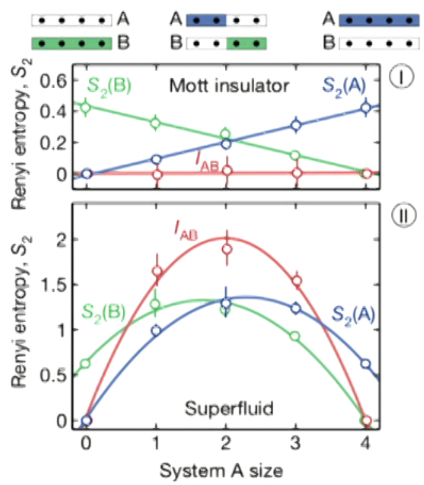
Entanglement is central to our understanding of few-body systems

Quantum information science has taught us that entanglement in many-body systems likely plays an even deeper role - "more is different"

- Entanglement is the central thermodynamic resource for a quantum computer
- Microscopic origin of entropy - eigenstate thermalization hypothesis

Entanglement in large-scale systems is becoming experimentally accessible!

Direct measurements



Islam, Ma, Preiss, Tai, Lukin, Rispoli, Greiner, Nature (2015)

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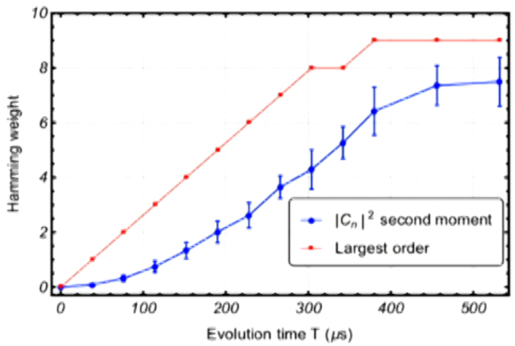
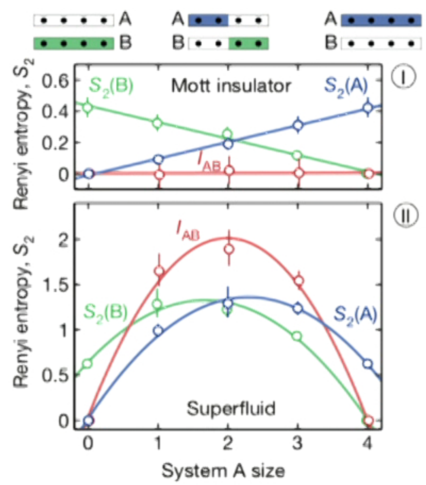
Indirect probes - Out-of-time-ordered correlators



Niknam, Santos, Cory, arxiv (2018)

Direct measurements



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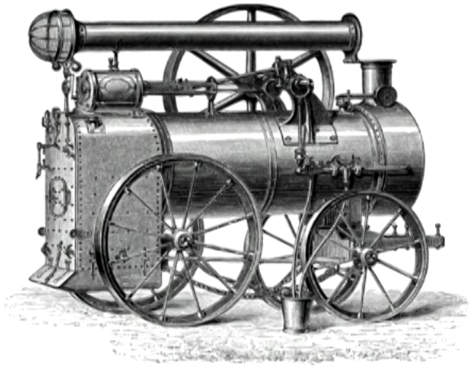
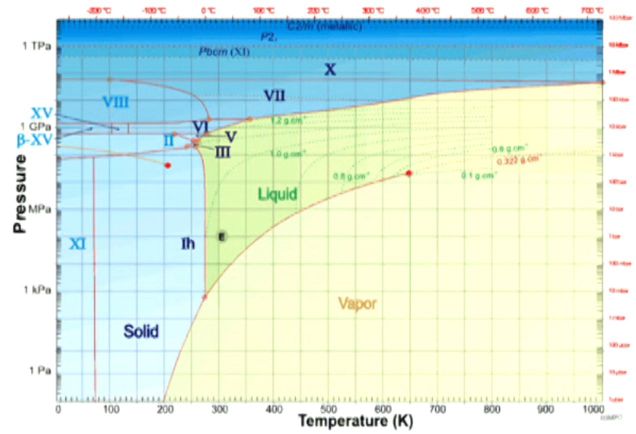
Statistical Mechanics of Entanglement

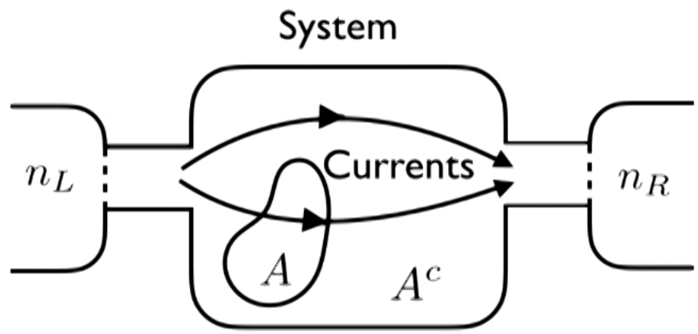
Steam engine



Phase diagram of water



Statistical Physics of Current-Driven Systems



Arrow of Time in Quantum Thermalization

How do we reconcile the reversible, unitary time evolution of quantum mechanics with the irreversible process of thermalization?

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One answer to this problem: eigenstate thermalization hypothesis (ETH)

Srednicki PRE (1994).
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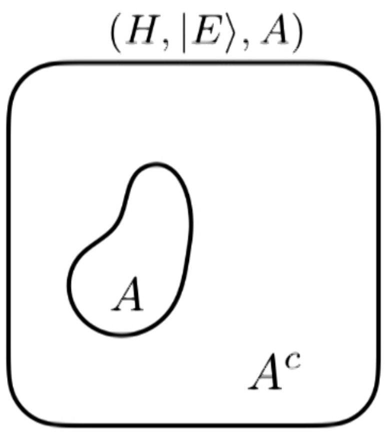


1) Single eigenstates are in thermal equilibrium

$$\mathrm{Tr}_{A^c}[|E\rangle\langle E|] \sim \mathrm{Tr}_{A^c}[e^{-\beta H}]$$

A^c acts as a bath for region A

Srednicki PRE (1994).
D'Alessio, Kafri, Polkovnikov, Rigol Adv Phys (2009).

Diffusion and the Arrow of Time

Diffusion describes how conserved quantities (energy, density, magnetization) spread through interacting or disordered systems - precursor to thermalization

Understanding the emergence (and failure) of diffusion in quantum many-body systems provides insight into the arrow of time

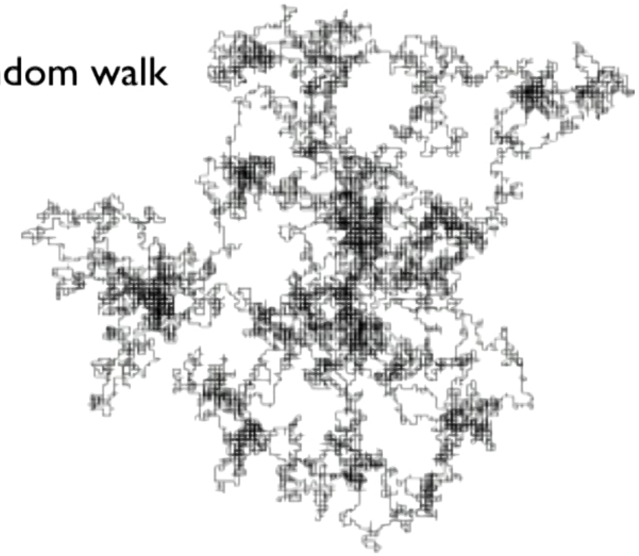
Why?

The diffusion equation is fundamentally a dissipative equation

$$\partial_t n = D\nabla^2 n$$

Emergence of Diffusion: Examples

2D random walk



Diffusion emerges only after coarse graining or averaging

Electron transport

Fermi liquid theory

$$\sigma(\omega) \propto D(\omega)\rho(\omega)$$

DOS

Castellani, Kotliar, Lee, PRL (1987)

Emergence of Diffusion: Dissipation

Scrambling is sufficient for emergence of dissipation needed to realize diffusion

Khemani, Vishwanath, Huse, arxiv (2017)
Rakovszky, Pollmann, von Keyserlingk, arxiv (2017)

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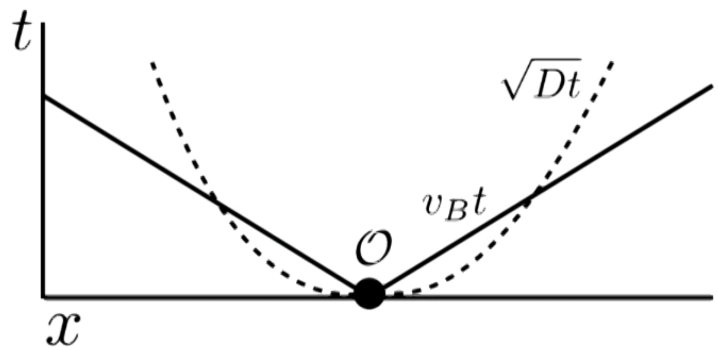

$$\mathcal{O}(t) = \mathcal{O}_c(t) + \mathcal{O}_{nc}(t)$$

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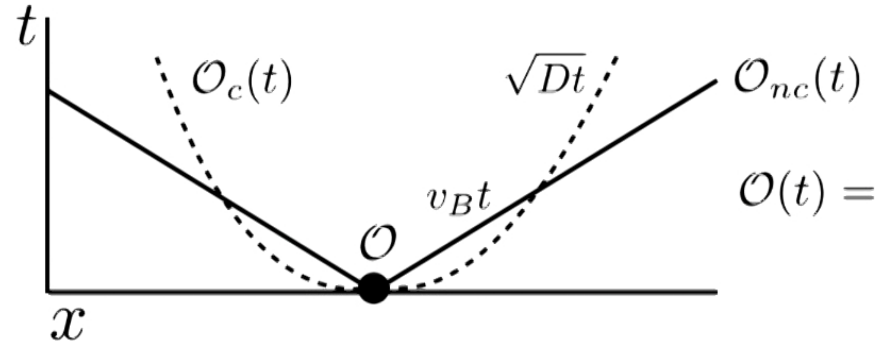

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Single operator dynamics - What happens to the full many-body state?

Progress in using similar ideas to develop approximate numerical methods for quantum chaotic 1D systems:

White, Zalatel, Mong, Refael, arxiv (2017)
Leviatan, Pollman, Bardensen, Huse, Altman, arxiv (2017)
Brandao, Haegeman, Scholz, Verstraete, arxiv (2017)

Boundary-Driven Random Circuit

Distribute nearest neighbor gates by a 2-parameter family:

$$d\mu = (1 - p_1 - p_2)d\mu_0 + p_1 d\mu_1 + p_2 d\mu_2$$

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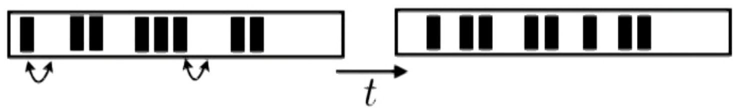
Diffusion constant: $D \sim 1$

Butterfly velocity: $v_B^2 \sim \min(\sqrt{p_1 p_2}, p_2)$

I $p_1 = 0$: Discrete Hopping



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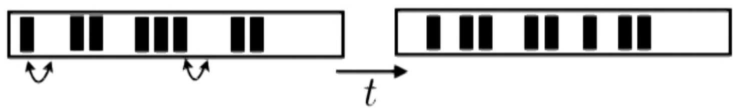
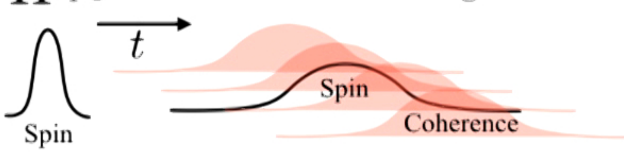
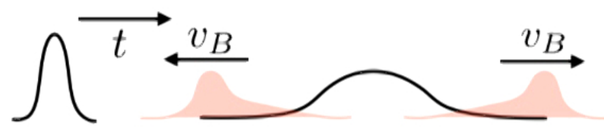


II $p_2 = 0:$ Non-Interacting Fermions



Represent qubits by fermions

III $p_1, p_2 \neq 0:$ Quantum Chaotic



Interacting fermions

Gate Set in Random Circuit

Gates are:

p_1

$$U = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & U_{ud} & U_{rl} & 0 \\ 0 & U_{lr} & U_{du} & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$

Middle matrix is Haar random on SU(2)

Generated by Hamiltonians that are bilinear in fermions

Terhal, DiVincenzo, PRA (2002)

Jordan-Wigner transformation

$$c_i = \prod_k^{i-1} \sigma_z^{(k)} \sigma_-^{(i)}$$

Induce interactions between fermions - no partial swaps

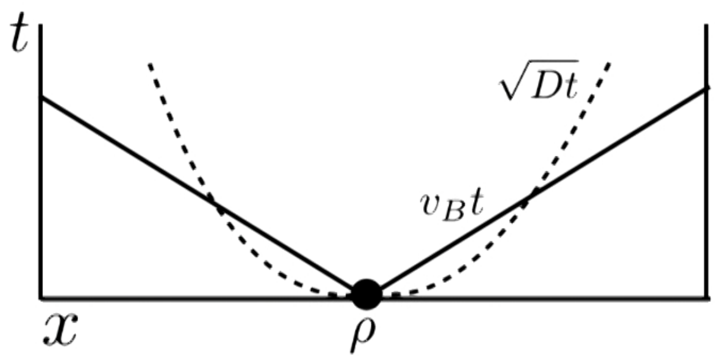
p_2

$$U_1 = e^{i\phi_1} u_1 u_2 + e^{i\phi_2} u_1 d_2 + e^{i\phi_3} d_1 u_2 + d_1 d_2,$$

$$U_2 = \mathrm{SWAP} \cdot U_1, \quad u_i = \frac{1+\sigma_z^{(i)}}{2} \quad d_i = \frac{1-\sigma_z^{(i)}}{2}$$

Qualitative Picture: Random Circuit Model

Generic behavior



- Boundary trace acts like a local density measurement and converts entanglement into classical correlations
- Classical violations of local hydrodynamics remain

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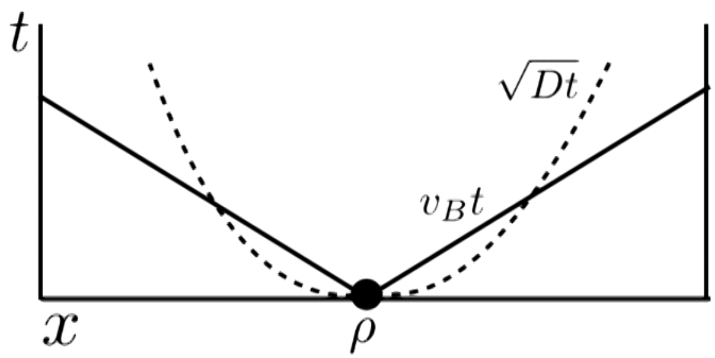

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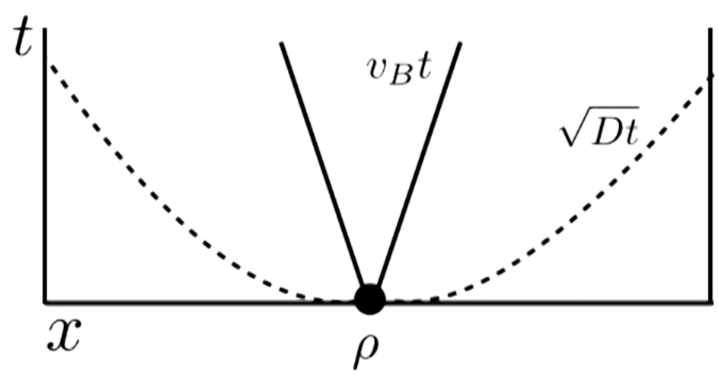
Fine tuned behavior: $v_B L^2/D \to 0$

Thouless time: L^2/D

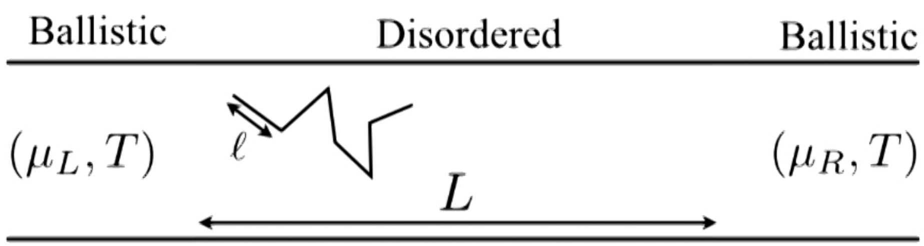


- Non-conserved operators transported to boundary by diffusion
- Violations of local hydrodynamics encoded in entanglement

Models. II. Driven Anderson Model

Anderson model — Coherent quantum transport - diffusion emerges only after disorder averaging

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j + \sum_i V_i c_i^\dagger c_i \qquad V_i \in [-W/2, W/2]$$



Anderson, *Absence of diffusion in certain random lattice models,* Phys Rev (1958)
Interacting case: Imbrie, PRL (2014); Review: Nandkishore, Huse, Ann Rev CMP (2015)

Implications for Mesoscopic Transport

Metals at low temperature governed by 4 length scales

ℓ	ℓ_φ	ℓ_{ee}	$\ell_{\rm eph}$
Mean free path	Phase coherence length	Electron scattering length	Electron phonon scattering length
50 nm	1 μm	10 μm	10 mm

Theory:

Altshuler, Aronov, Khmelnitsky (1982)

Outline

Overview

- Diffusion and the arrow of time
- Models and results

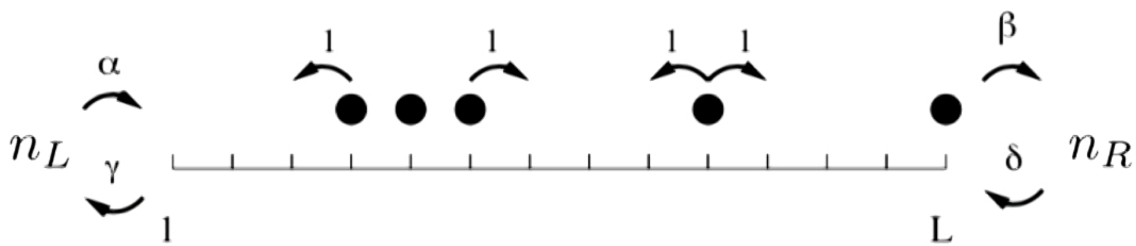
Analysis

- Non-equilibrium steady states in classical stochastic lattice gases
- Entanglement structure of current-driven diffusive fermion systems
  arXiv:1804.00010
- Entanglement phase transition in the driven Anderson model

Outlook

Non-equilibrium Steady States of Classical Systems

Stochastic lattice gas with hard core interactions: simple exclusion processes



Spohn (1983). Kipnis, Landim, *Scaling limits of interacting particle systems* (1999).

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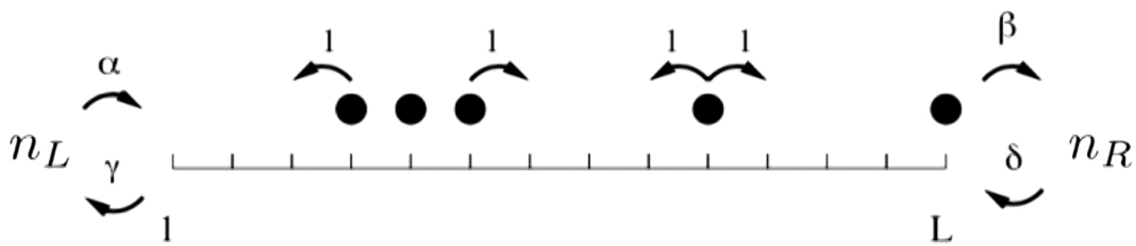


Configuration probability evolves according to master equation:

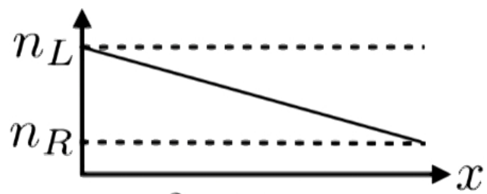
$$\frac{dP(C)}{dt} = \sum_{C'} W(C,C')P(C')$$

Classical analog of the random circuit

Spohn (1983). Kipnis, Landim, *Scaling limits of interacting particle systems* (1999).

Non-equilibrium Steady States of Classical Systems

Density:



$$\partial_x^2 n(x) = 0$$

Current: $J = \partial_x n(x) = \dfrac{\delta}{L}$

$$\delta = n_L - n_R$$

In 1D a matrix product state has been found for P(C) with bond dimension L

Exact solution: Derrida, *et al* (1991-93). Review: Derrida, J Stat Mech (2007)

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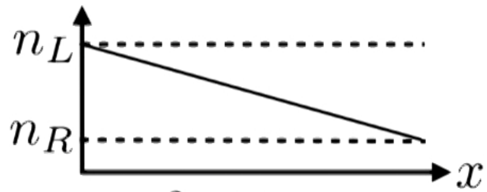

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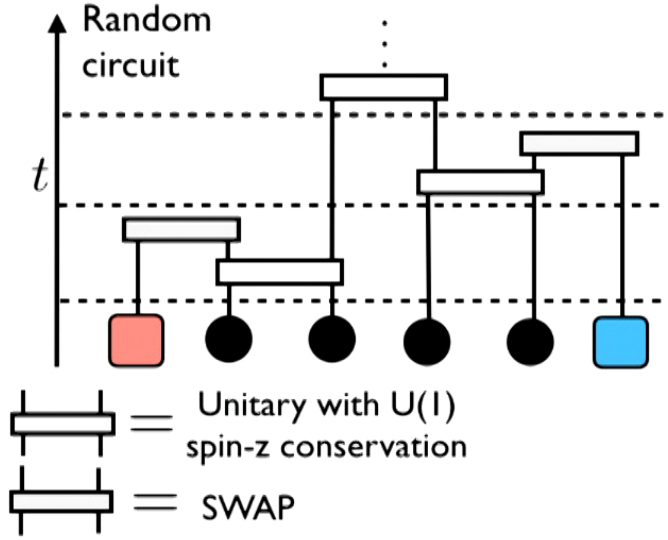
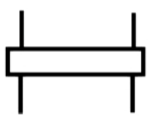
Density-density correlation: $\langle \tau_x \tau_y \rangle_c = -\dfrac{\delta^2}{L} x(1-y) \neq \mathcal{F}(n(x), J)$

Strong violation of local hydrodynamics!

Boundary-Driven Random Circuit



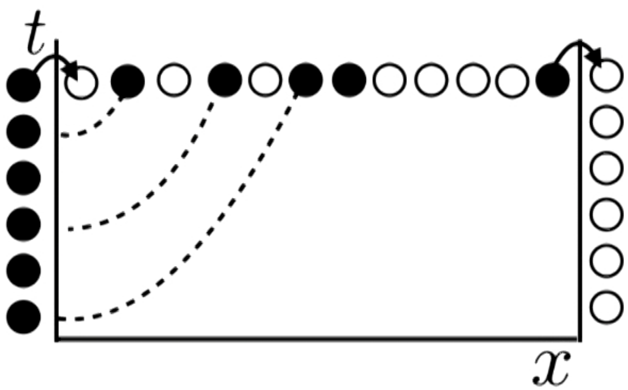
Local conservation of magnetization



$$= \begin{pmatrix} U_+ & 0 & 0 & 0 \\ 0 & U_{ud} & U_{rl} & 0 \\ 0 & U_{lr} & U_{du} & 0 \\ 0 & 0 & 0 & U_- \end{pmatrix}$$

Non-equilibrium Attracting States

Long-time state of system is independent of initial conditions $D \sim 1$



Random realizations of circuits induce a distribution over density matrices

$$\mathbb{P}(\rho)$$

How do we characterize this distribution?

Our approach: Look at moments using replica methods

Average behavior:

$$\partial_t \overline{\rho} = \mathcal{L}(\overline{\rho})$$

$$\overline{\rho} = \int d\rho \rho \mathbb{P}(\rho)$$

$\mathcal{L}(\overline{\rho})$ - Master equation for SSEP

Exact solution for steady-state average
- independent of (p_1,p_2)

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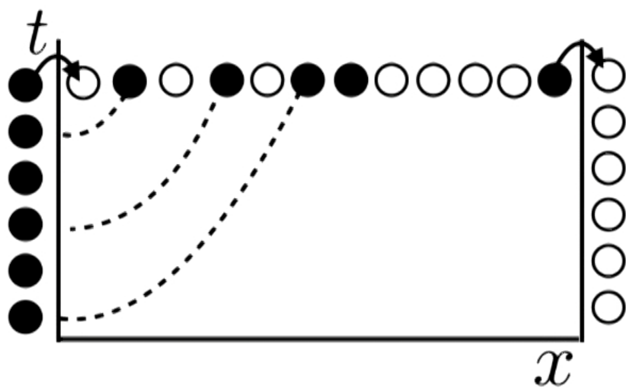

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Summary: Boundary-driven random circuit

Characterize deviation from local equilibrium through Renyi entropy

$$\Delta S_n(\rho) = S_n(\rho_{\mathrm{LE}}) - S_n(\rho) \qquad S_n(\rho) = \frac{1}{1-n}\log \mathrm{Tr}[\rho^n]$$

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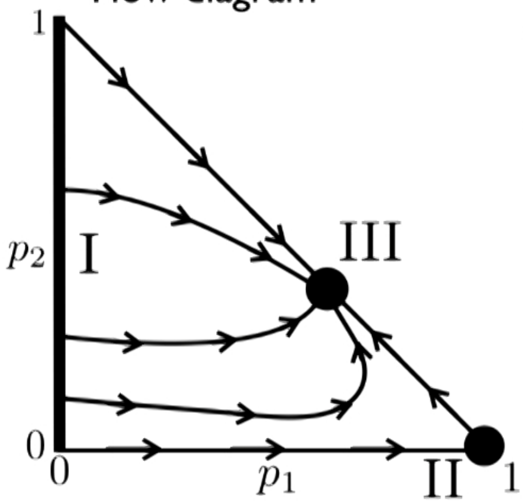
$$\Delta S_n(\rho) = S_n(\rho_{\mathrm{LE}}) - S_n(\rho) \qquad S_n(\rho) = \frac{1}{1-n}\log \mathrm{Tr}[\rho^n]$$

Replicas give direct access to: $\overline{\mathrm{Tr}[\rho^n]} = \overline{e^{(1-n)S_n(\rho)}}$

Computing $\overline{\Delta S_1(\rho)}$ reveals 3 distinct phases

Flow diagram

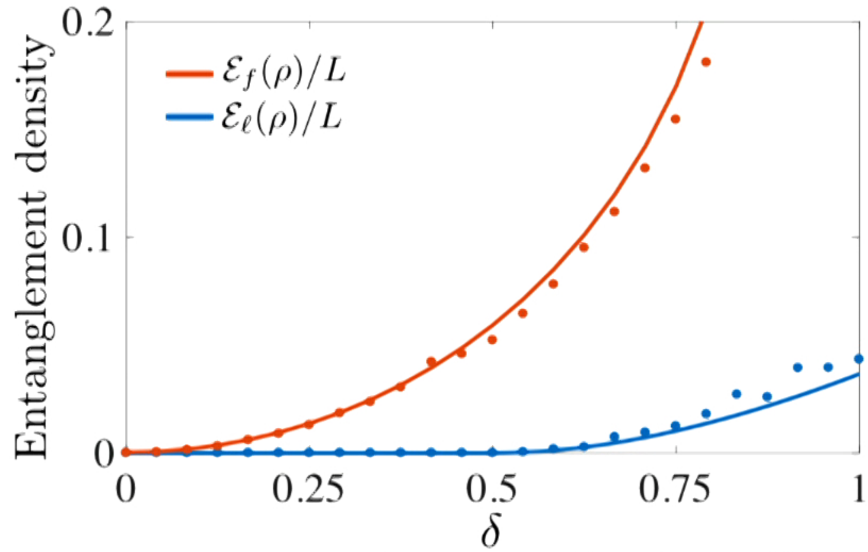


	Transport	Operator Spreading	ΔS	$I(L:R)$
I	Diffusive/ Ohm's law	Diffusive	Volume	0
II	"	Diffusive	Volume	Volume
III	"	Ballistic	Area	Area

II-III: Perturbative result in reservoir magnetization difference $\delta = m_L - m_R$

Entanglement in Phase II

Logarithmic negativity Vidal, Werner, PRA (2002)



Upper and lower bound on logarithmic negativity (both efficiently computable in number of fermions for Gaussian states)

Eisert, Eisler, Zimboras, arXiv (2016)

Shapourian, Shiozaki, Ryu, PRB (2017)

Phase II: (p_2=0) Non-interacting Fermions

Attracting density matrix is a Gaussian fermionic state - determined entirely by two-point function

$$G_{ij} = \mathrm{Tr}[\rho c_i^\dagger c_j]$$

$$\overline{|\langle c_{Lx}^\dagger c_{Ly}\rangle|^2} = \frac{x(1-y)}{L}\delta^2$$

Violations of local hydrodynamics encoded in long-range, off-diagonal coherences

Phase III: Quantum chaotic phase

Action of gates on fermion operators:

Non-interacting fermion gates:

$$p_1 : n_i \to c_i^\dagger c_{i+1}, \;\; c_i \to c_{i+1}$$

Interaction gates:

$$p_2 : n_i \to n_{i+1}, \;\; c_i \to c_i n_{i+1}$$

Need both gates to act for an operator to "grow" in length

Butterfly velocity

$$v_B^2 \sim \min(\sqrt{p_1 p_2}, p_2)$$

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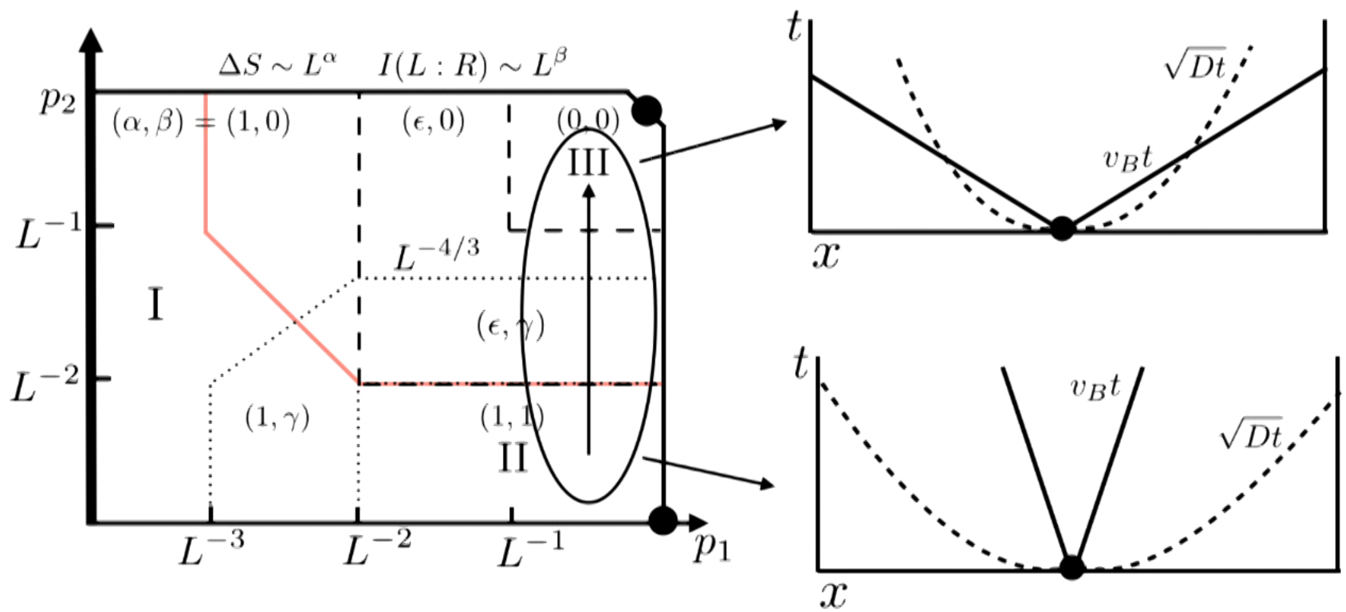
Analysis: Derived exact equations for $\overline{\rho \otimes \rho}$ in terms of 6-species stochastic lattice gas

Solved this model perturbatively in δ and $1/L$

$$\frac{\Delta S}{L} = \left(\frac{\alpha_1}{p_1} + \frac{\alpha_2}{p_2}\right) J^2 \qquad I(L:R) = \frac{\alpha_3}{p_2^{3/2}} J^2 \qquad J = \frac{\delta}{L}$$

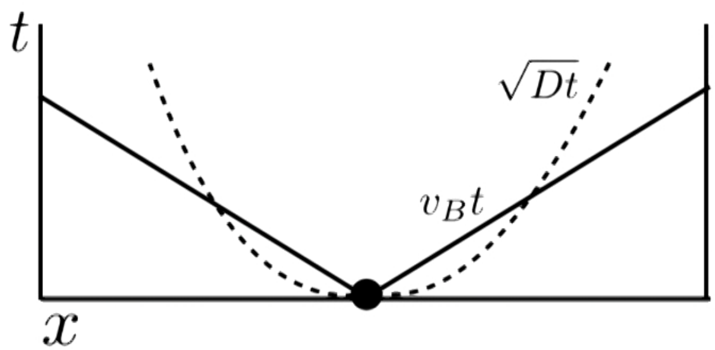
Crossover Scales

Derived hydrodynamic equations that describe entire phase diagram



Overview: Random Circuit

Phase III



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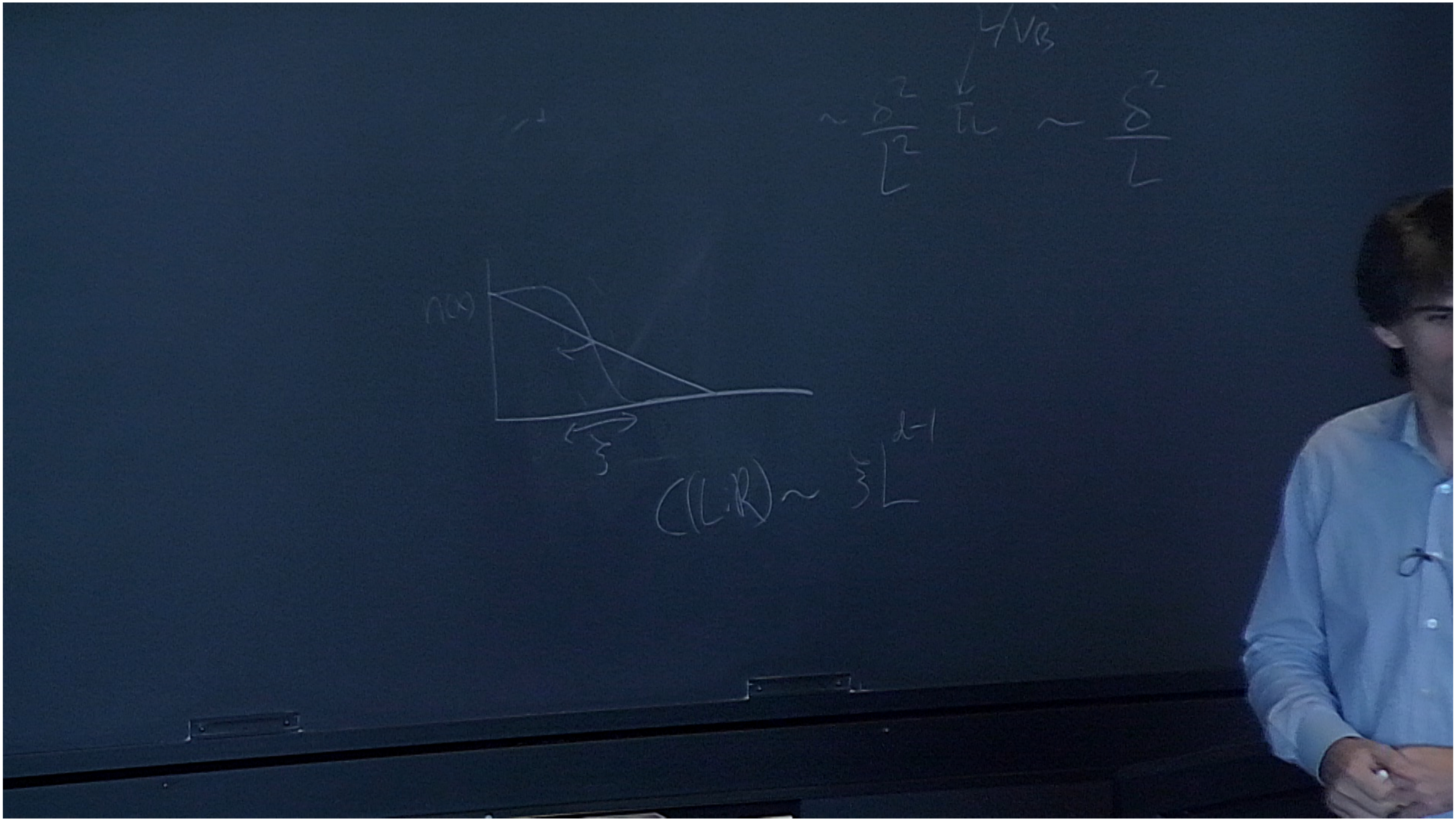
n(x)
ξ
C(L:R) ~ ξ L^{d-1}