Title: The role of entropy in topological quantum error correction

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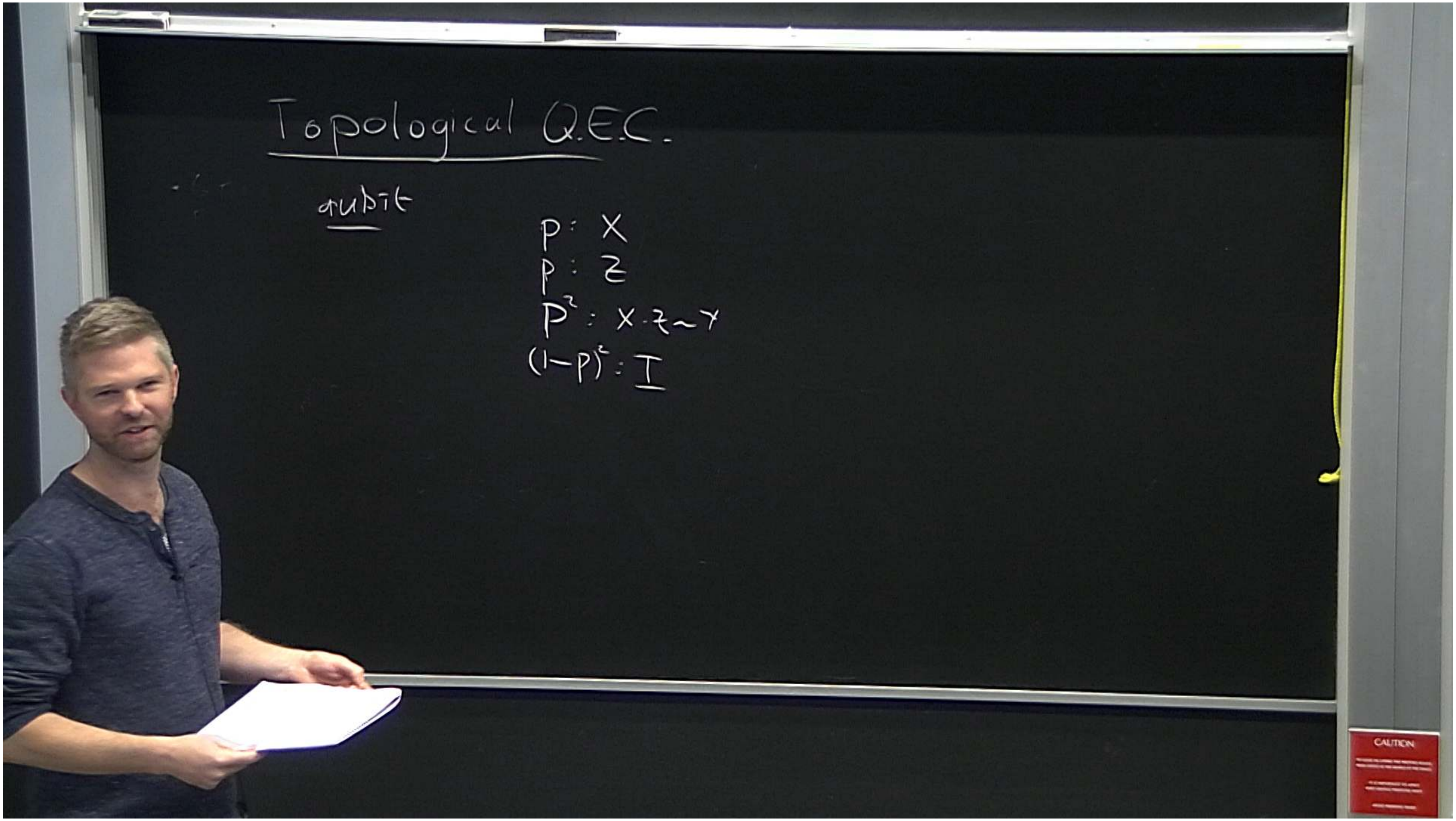
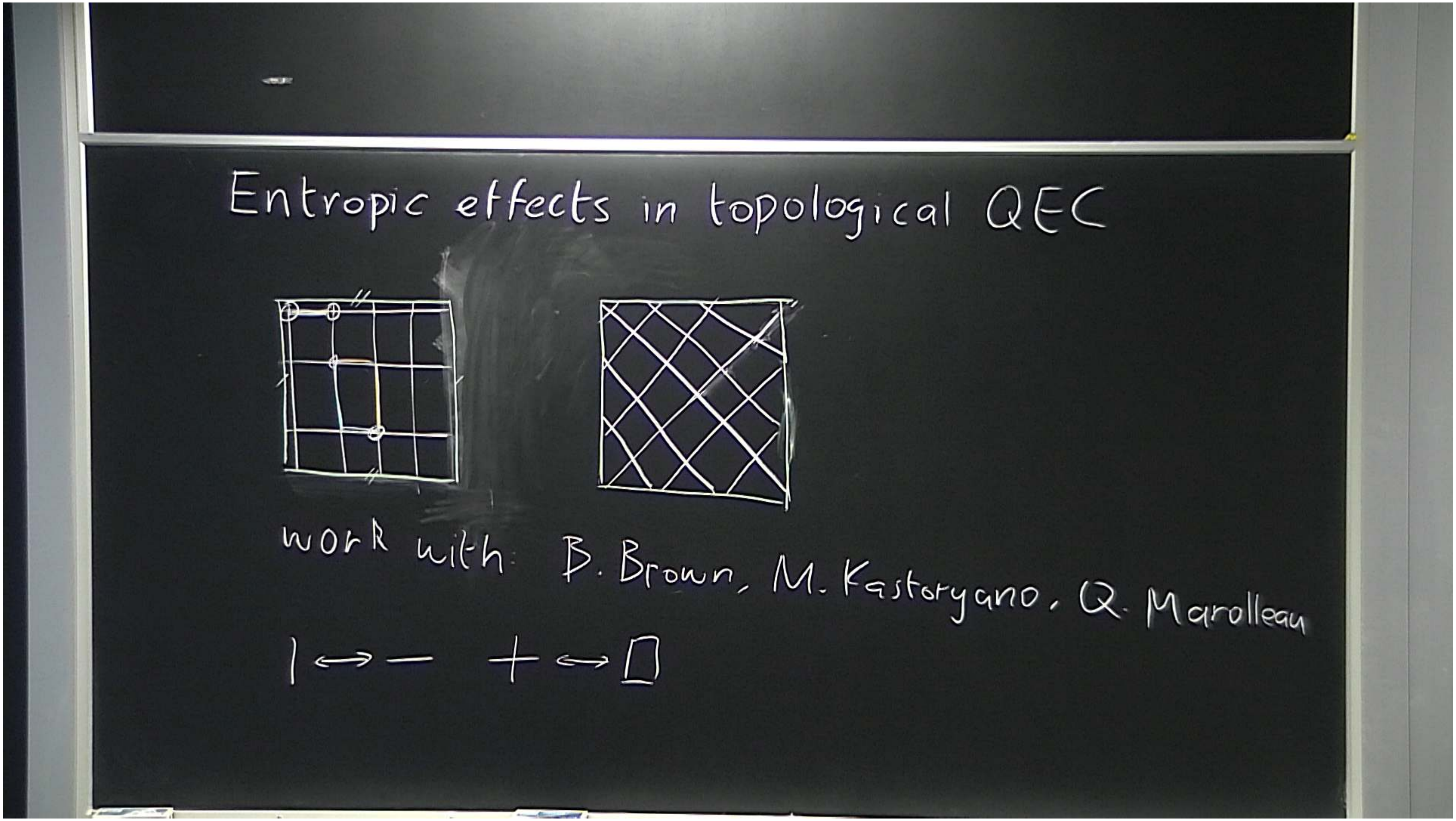
Abstract: <p>Ubiquitous in the behavior of physical systems is the competition between an energy term E and an entropy term S of their free energy F = E - beta S. These concepts are also relevant for error correction, where the `energy` E is the number of qubits afflicted by an error, the `entropy' S(E) is the logarithm of the number of energy-E failing errors, and beta relates to the probability of each qubit's error. Error-correction schemes with larger minimum free energy have better performance. Often distance d (which correct all errors with energy less than d/2) is used as a proxy for a code's performance since it increases the minimal E and therefore tends to increases the minimal F. However, a sufficiently large entropy S can counteract a large d and reduce the free energy (negatively impacting a code's performance). A great example of these principles is the surface code, which is at present the leading architecture for fault tolerant quantum computing. Rotating a square lattice geometry over the surface of the torus can increase the distance of the code by a factor of root two, but at the cost of increased entropy. We obtain exact expressions for this entropic effect in the low error regime, and introduce an analytical model that qualitatively describes the behavior for error rates all the way up to threshold. Our predictions are corroborated by numerical estimates of the low error failure rate, using the splitting method algorithm introduced by Bravyi et al. We find that although the rotated lattice outperforms the non-rotated lattice with the same number of qubits for low error rates, the two codes have very similar performance for error rates which are an appreciable fraction of the threshold error rate. Surprisingly, we also find some system sizes and error rates for which the non-rotated lattice has marginally better performance. </p>


Topological Q.E.C.
qubit
p : X
p : Z
p² : X·Z~Y
(1−p)² : I
CAUTION


Entropic effects in topological QEC
work with: B. Brown, M. Kastoryano, Q. Marolleau

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$| \longleftrightarrow -$ $\quad + \longleftrightarrow \square$

$(1-p)^2 : I$

X-errors stabs.

E set of edges

$s(E)$ syndrome $\partial E = s$

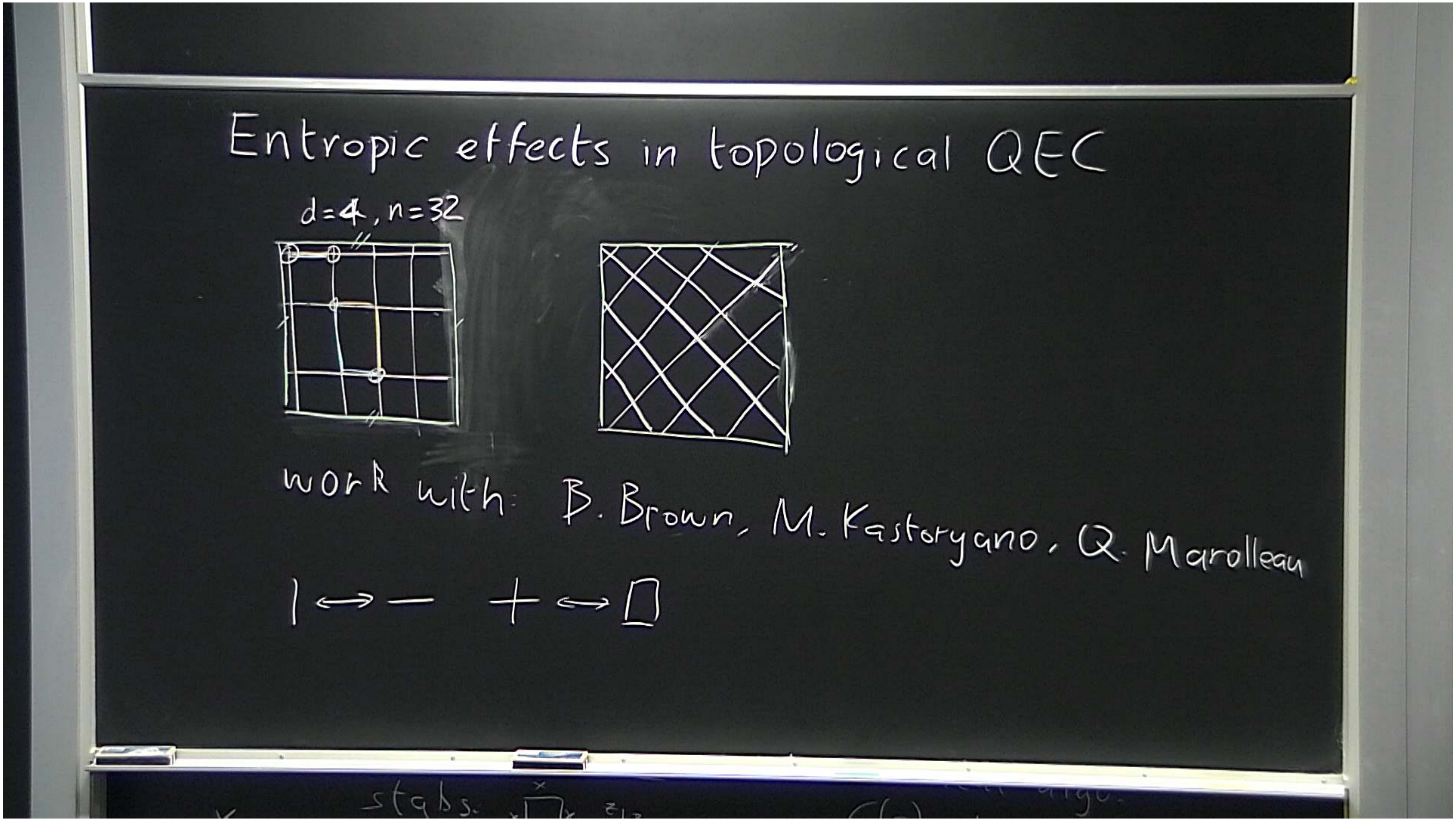
$C(s(E))$ set of edges $\partial C = s$

Classical algo.

$C(s)$ decoder

Assume:

- $C(s)$ min weight
- deterministic

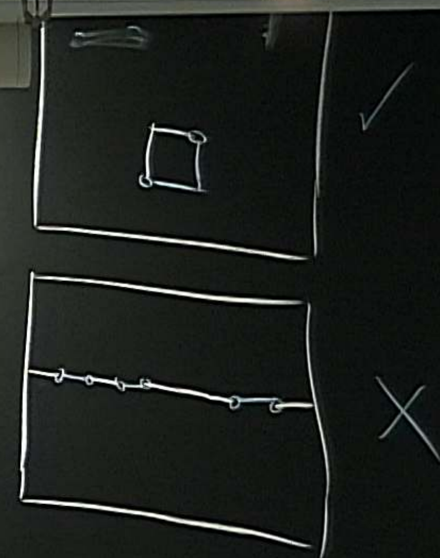

Entropic effects in topological QEC
d=4, n=32
work with: B. Brown, M. Kastoryano, Q. Marolleau

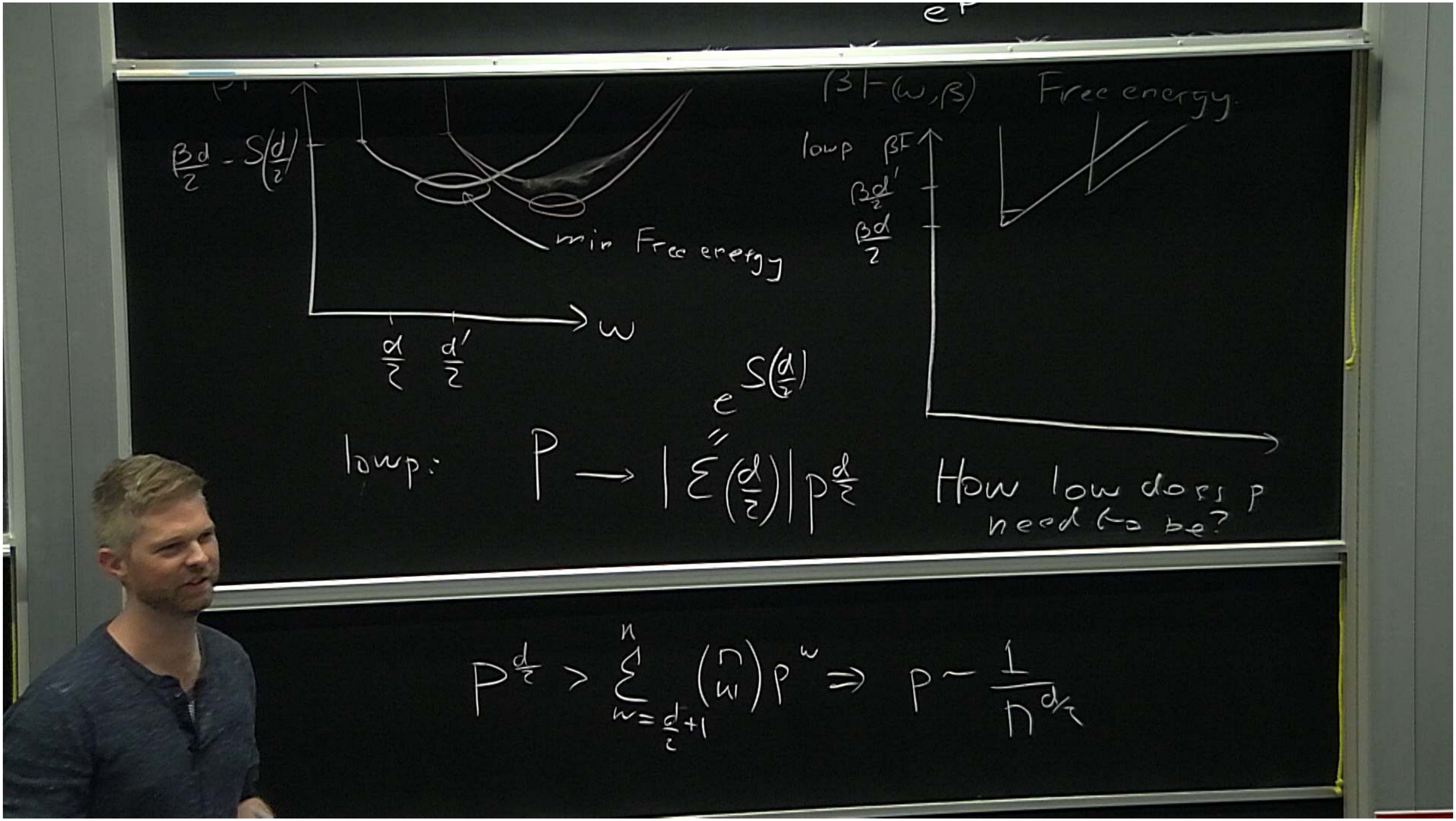
$$\mathcal{E} = \text{all failing errors}$$

$$\mathcal{E} = \bigsqcup_{w=\frac{d}{2}}^{n} \mathcal{E}(w)$$

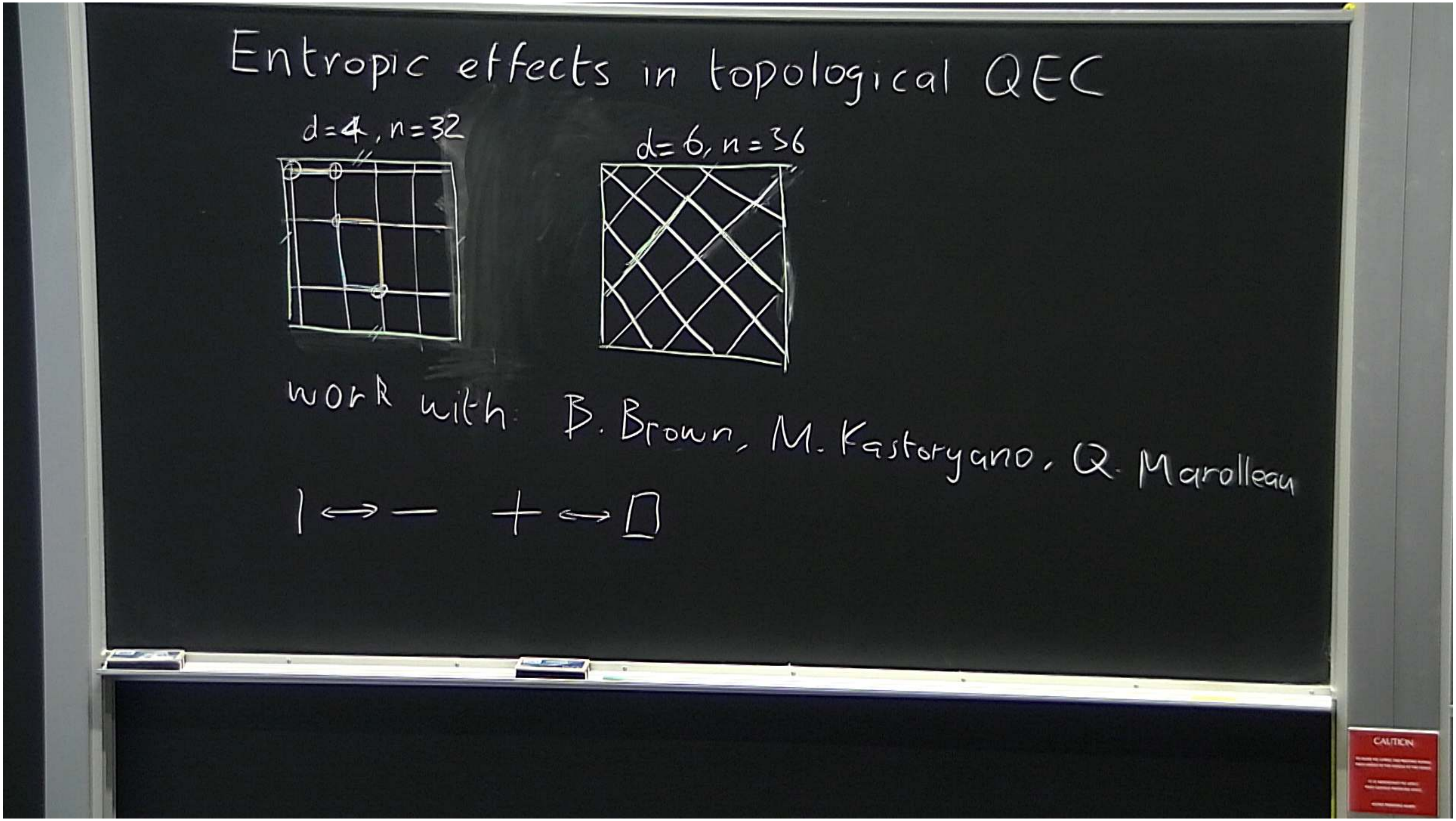
$$P(p) = \sum_{w=\frac{d}{2}}^{n} |\mathcal{E}(w)|\, p^{w} (1-p)^{n-w}$$

$$= (1-p)^{n} \sum_{w=\frac{d}{2}}^{n} \exp\big(\underbrace{\log|\mathcal{E}(w)|}_{S(w)}\big) \Big(\underbrace{\frac{p}{1-p}}_{e^{-\beta}}\Big)^{w}$$





$\beta F(\omega,\beta)$ Free energy
$\frac{\beta d}{2} - S(\frac{d}{2})$
min Free energy
ω
$\frac{d}{2}$ $\frac{d'}{2}$
low p βF
$\frac{\beta d'}{2}$
$\frac{\beta d}{2}$
$e^{S(\frac{d}{2})}$
low p: $P \longrightarrow |\mathcal{E}(\frac{d}{2})| p^{\frac{d}{2}}$
How low does p need to be?
$p^{\frac{d}{2}} > \sum_{w=\frac{d}{2}+1}^{n} \binom{n}{w} p^{w} \Rightarrow p \sim \frac{1}{n^{d/2}}$


Entropic effects in topological QEC
d=4, n=32
d=6, n=36
work with: B. Brown, M. Kastoryano, Q. Marolleau

Question: what are $P^{\square}(p,n)$, $P^{\diamond}(p,n)$

If have n qubits, p, which should use

$\square$ seems better for gates

$d^{\square} = \sqrt{\frac{n}{2}}$, $d^{\diamond} = \sqrt{n}$

Low p limit for $P^{\square}$

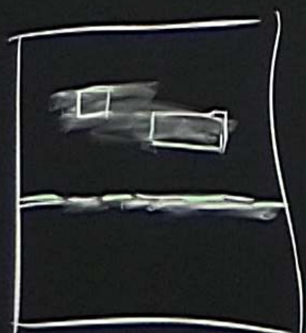
$$P^{\square} \to W^{\square} p^{\frac{d}{2}}$$

$$W^{\square} = \sum_{L} \#(L)$$

number of $d/2$ errors

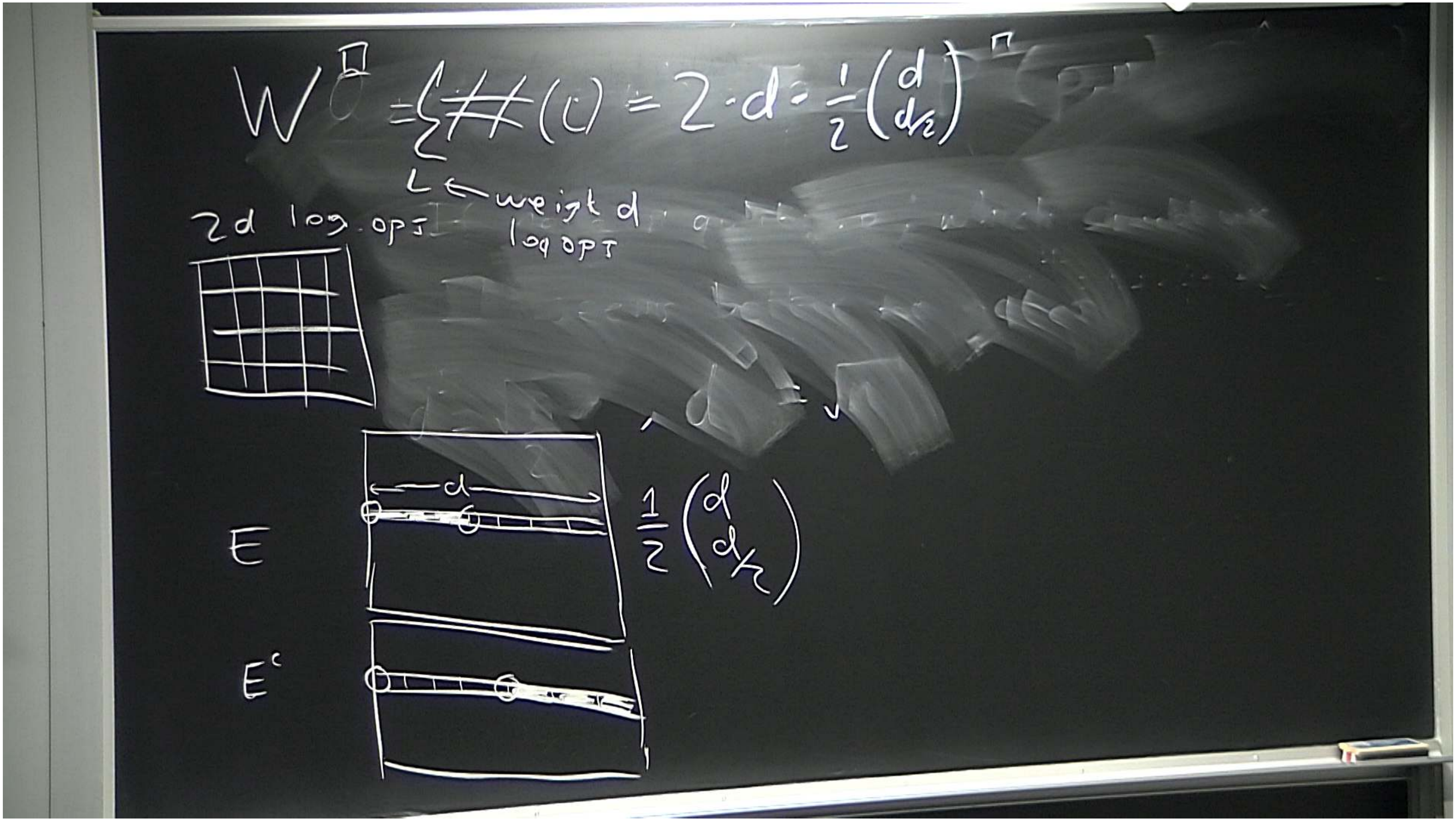
logical ops.

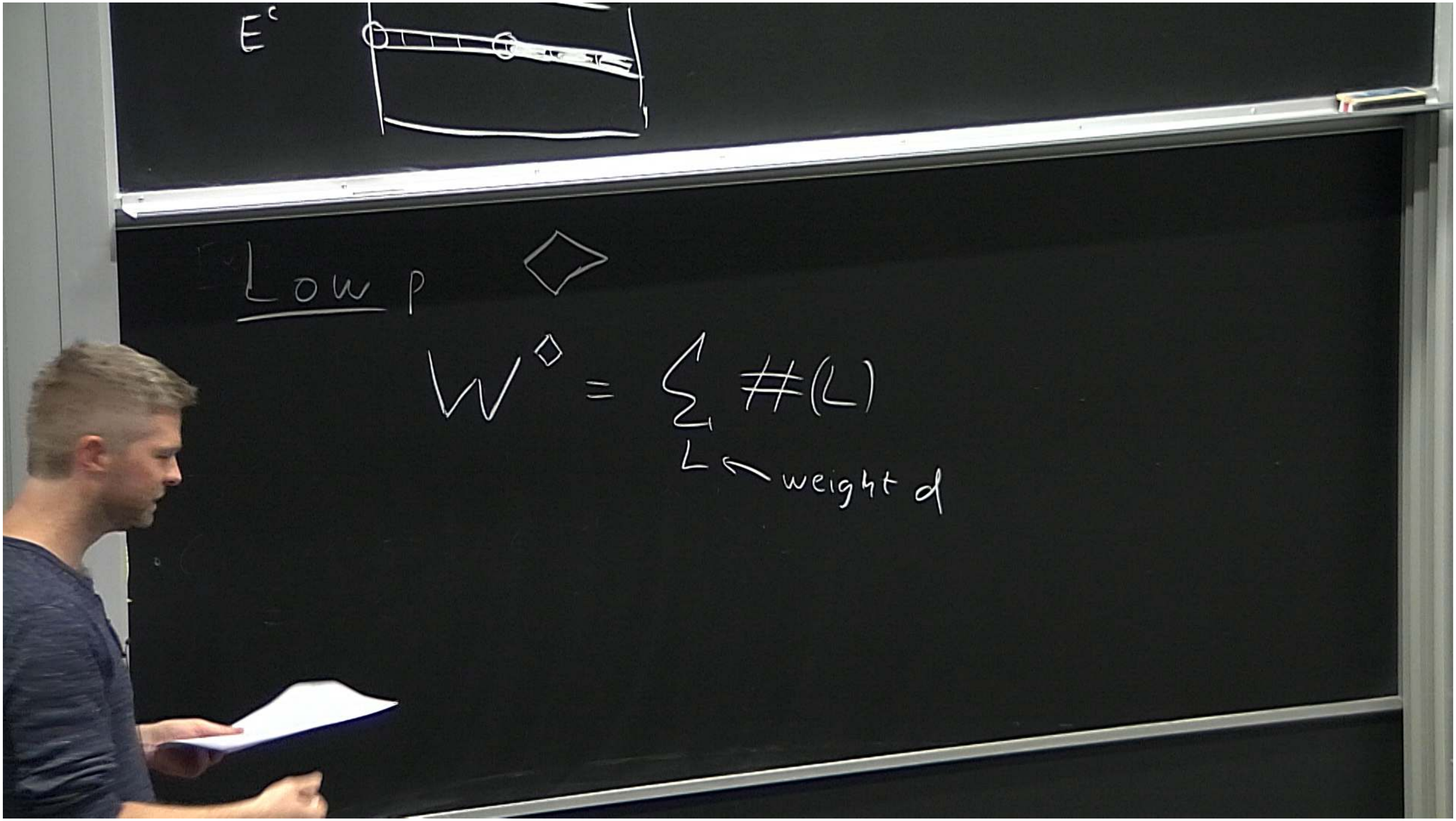
$$\#(L) = 0 \text{ if } wt(L) > d$$

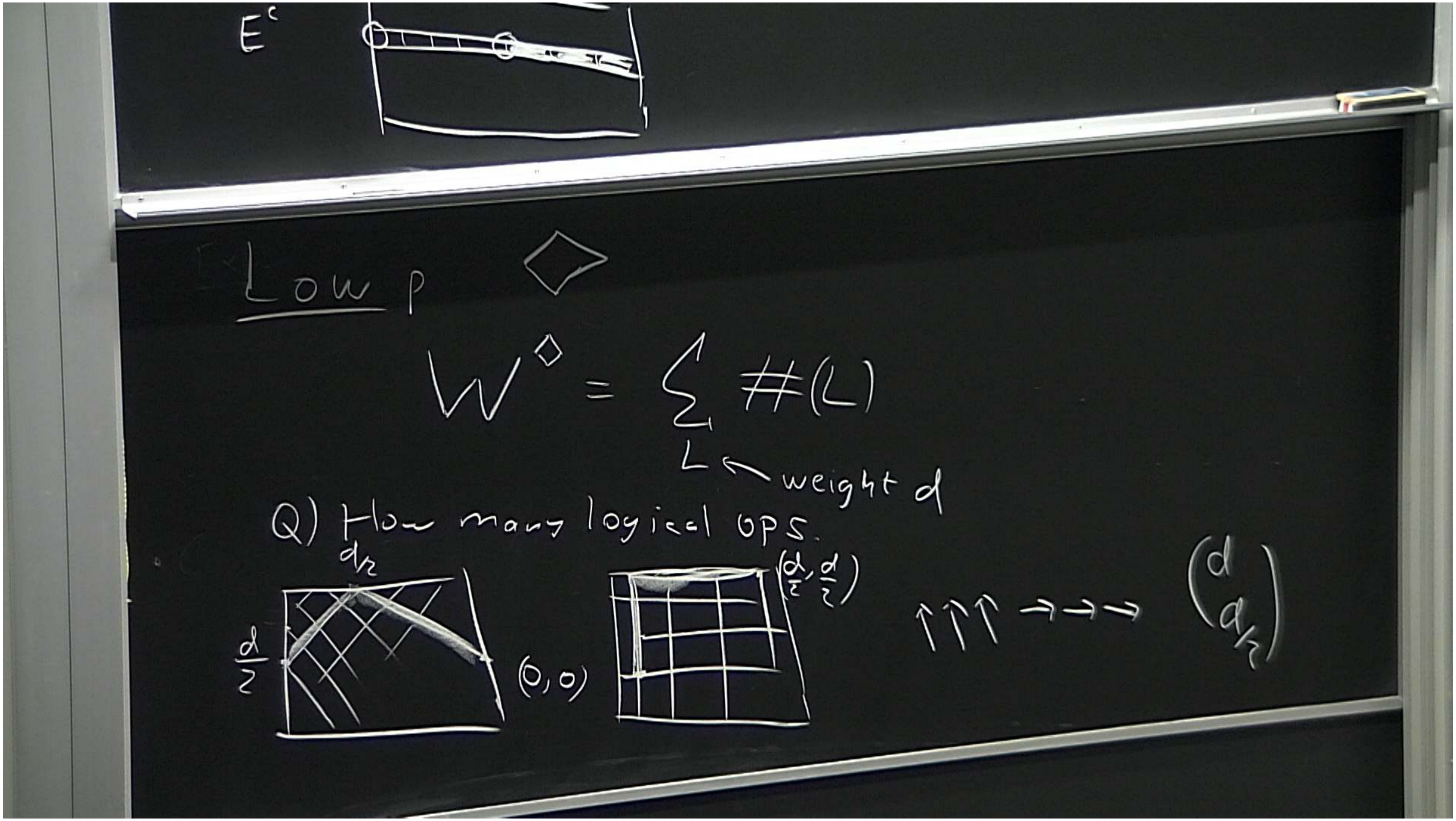


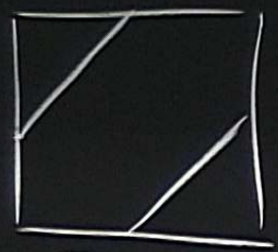
$$\underbrace{wt(E)}_{d/2} + wt(C) \geq wt(E \cdot C) > d$$

$$wt(C) > \frac{d}{2}$$

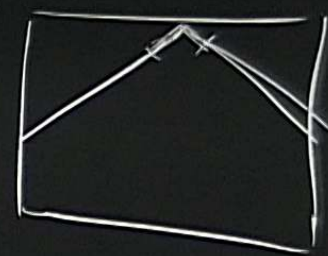

W^□ = Σ #(L) = 2·d = 1/2 (d d/2)
L ← weight d log ops
2d log. ops
E
d
1/2 (d d/2)
E^c


E^c
Low p
W^◇ = Σ_L #(L)
weight d


E^c
Low p
$W^{\diamond} = \sum_L \#(L)$
weight d
Q) How many logical ops.
$d/2$
$\frac{d}{2}$
$(0,0)$
$(\frac{d}{2}, \frac{d}{2})$
$\uparrow\uparrow\uparrow \rightarrow\rightarrow\rightarrow$
$\binom{d}{d/2}$

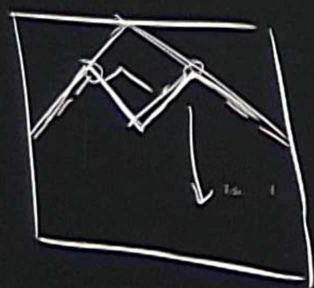


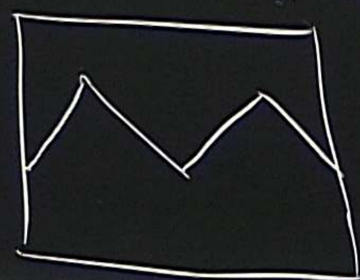
$$d \cdot \binom{d}{d/2} + d$$



$W =$

decoder not well defined





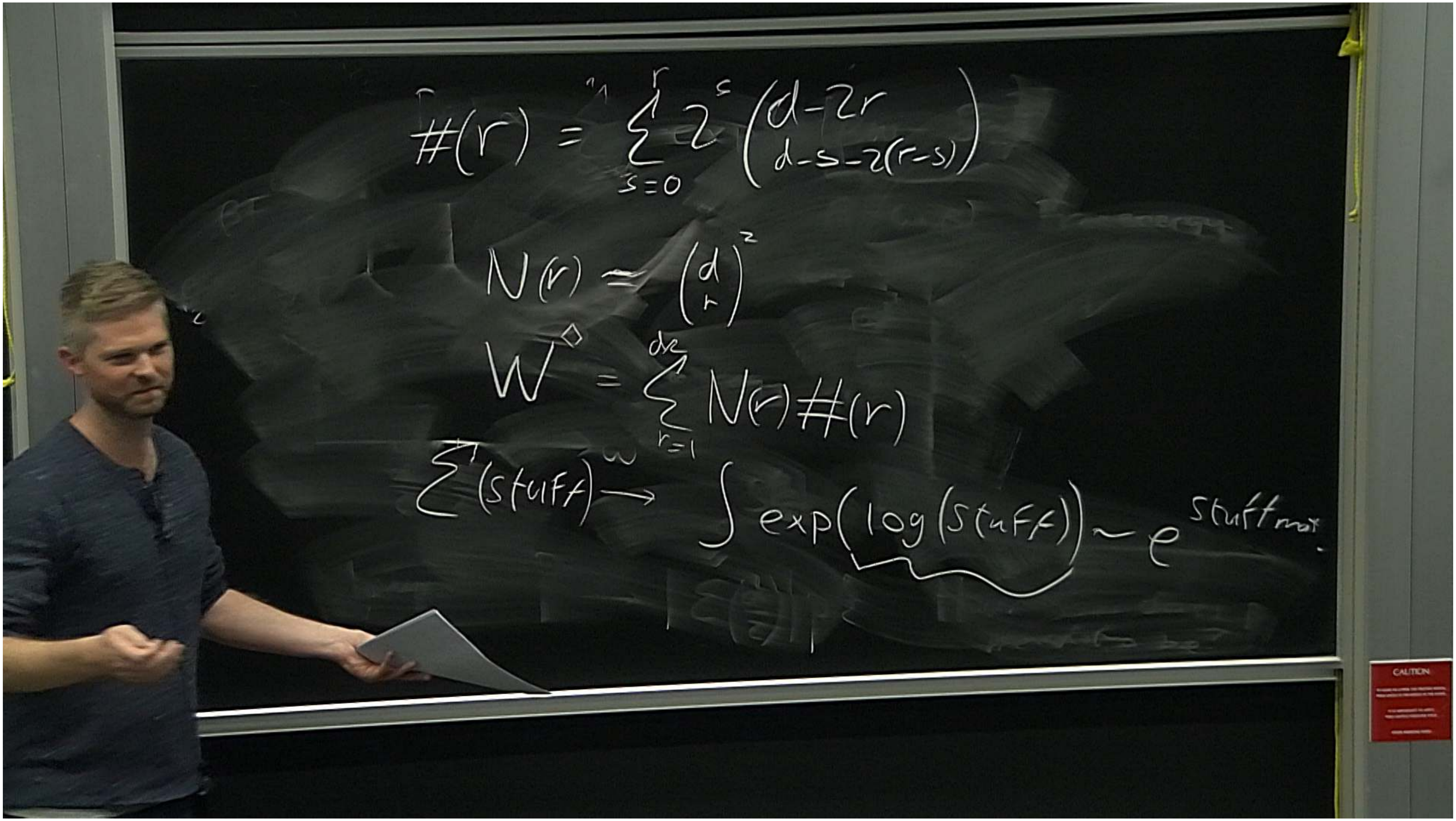
$$2 \cdot \binom{d-2}{\frac{d}{2}-1} + \binom{d-2}{\frac{d}{2}-2}$$

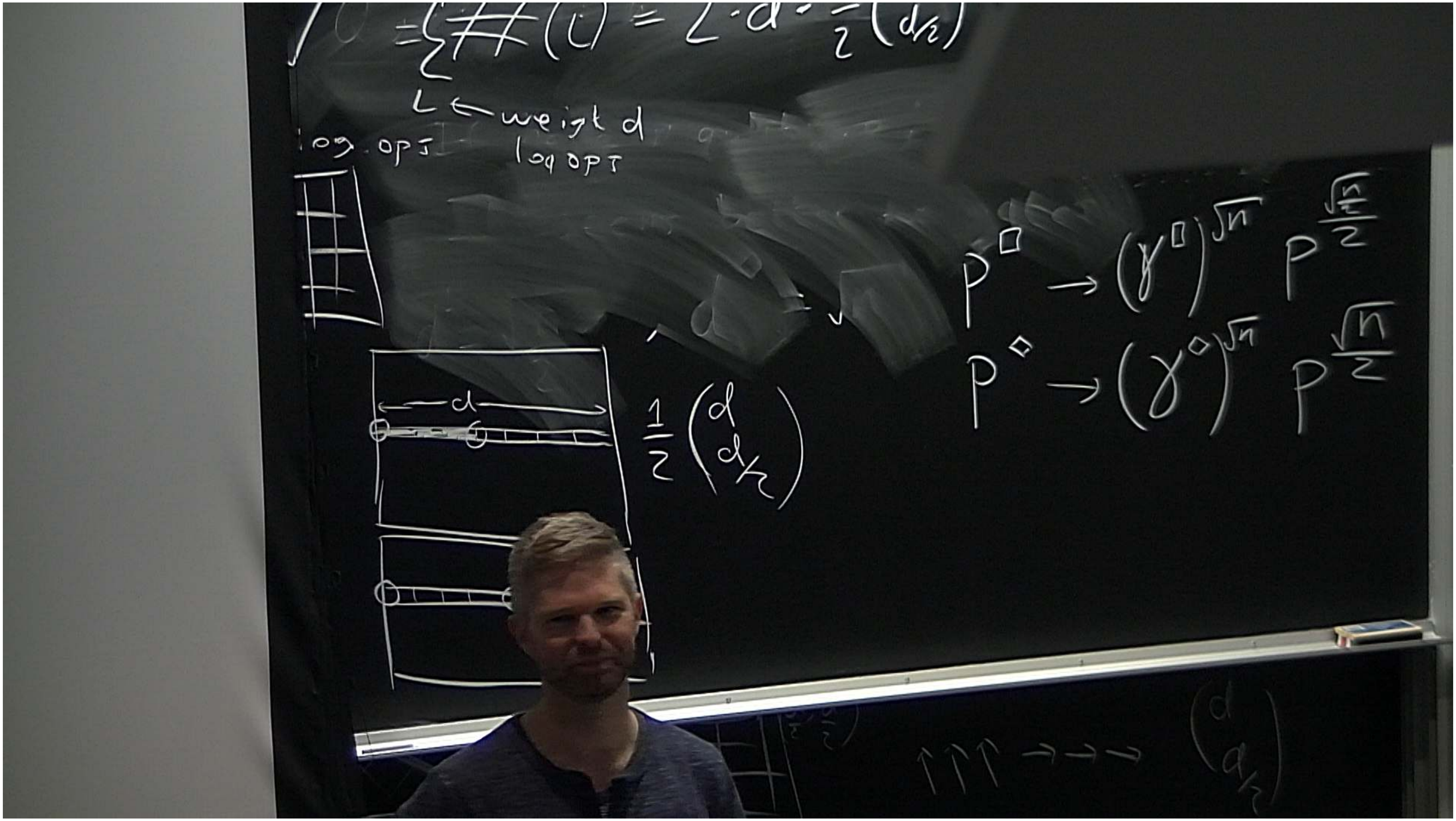
$C(s(E))$

syndrome $\partial E = s$

set of edges $\partial C = s$

· deterministic


$\#(r) = \sum_{s=0}^{r} 2^s \binom{d-2r}{d-s-2(r-s)}$
$N(r) = \binom{d}{r}^2$
$W^\diamond = \sum_{r=1}^{d/2} N(r)\#(r)$
$\sum (\mathrm{stuff}) \longrightarrow \int \exp(\log(\mathrm{stuff})) \sim e^{\mathrm{stuff}_{max}}$


L ← weight d
log. ops
log ops