Title: Macroscopic Effects of the Quantum Conformal Anomaly: Scalar Gravitational Waves, Black Holes, and Dark Energy

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Abstract: <p>In the second lecture, I will extend the previous discussion to gravity, and show that the conformal trace anomaly must play a special role in the effective field theory of low energy gravity. Classical General Relativity receives an infrared relevant modification from the conformal trace anomaly of the energy-momentum tensor of massless, or nearly massless, quantum fields, which implies the existence of a new long range massless scalar degree of freedom, not present in the classical Einstein theory, that contributes to gravitational scattering processes and has long range gravitational effects. Similar to the axial anomaly, the local form of the effective action associated with the conformal anomaly is expressible in terms of a dynamical scalar field that couples to the conformal factor of the spacetime metric, allowing it to propagate over macroscopic distances. Among the significant implications of this effective field theory of gravity are the prediction of scalar gravitational wave solutionsâ€”a spin-0 breather modeâ€” in addition to the transversely polarized tensor waves of the classical Einstein theory. Astrophysical sources for scalar gravitational waves are considered, with the excited gluonic condensates in the interiors of neutron stars in merger events with other compact objects likely to provide the strongest burst signals. The conformal anomaly also implies generically large quantum back reaction effects and conformal correlators in the vicinity of black hole horizons which are relevant to the formation of a non-singular interior, as well as an additional scalar degree of freedom in cosmology, providing a theoretical foundation for dynamical dark energy.</p>

Effective Field Theory & Quantum Anomalies

- EFT = Expansion of Effective Action in *Local* Invariants
- Assumes Decoupling of Short (*UV*) from Long Distance (*IR*)
- But *Massless* Modes do not decouple
- Massless Chiral, Conformal Symmetries are *Anomalous*
- Macroscopic Effects of Short Distance physics
- Special Non-Local Terms Must be Added to Low Energy EFT
- *IR* Sensitivity to *UV* degrees of freedom
- Important on horizons because of large blueshift/redshift

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a{}_a \rangle = + \frac{c_m}{24\pi} R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g}\,\langle T^a{}_a \rangle$$

determines the Wess-Zumino Action by *inspection*

2D Anomaly Effective Action

- **Integrating the anomaly linear in σ gives**

$$\Gamma_{WZ}[\bar{g},\sigma] = \frac{c_m}{24\pi}\int d^2x\sqrt{\bar{g}}\,\left(-\sigma\,\bar{\Box}\sigma + \bar{R}\,\sigma\right)$$

- This is local but **non-covariant**. Note **kinetic** term for σ
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ}[\bar{g},\sigma] = S_{anom}[g = e^{2\sigma}\bar{g}] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi}\int d^2x\sqrt{g}\int d^2x'\sqrt{g'}\;R_x\left(\Box^{-1}\right)_{x,x'}R_{x'}$$

- A covariant local form implies a **dynamical scalar** field

$$S_{anom}[g;\varphi] = \frac{c}{96\pi}\int d^2x\sqrt{g}\left[g^{ab}(\nabla_a\varphi)(\nabla_b\varphi) + 2R\varphi\right]$$

$$-\Box\varphi = R \qquad\qquad \varphi \leftrightarrow 2\sigma$$

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Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

Conservation of T_{ab} Ward Identity in 2D implies

$$\boxed{\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\,\Pi(k^2)}$$

Anomalous Trace Ward Identity in 2D implies

$\boxed{k^2\,\Pi(k^2) \neq 0}$ at $k^2 = 0$ **massless boson pole**

Correlated Pair State → Boson

$$a_n^\dagger \sim \sum_{q=\frac{1}{2}}^{n-\frac{1}{2}} b_{n-q}^\dagger d_q^\dagger$$

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2D Anomaly Stress Tensor

- The stress-energy tensor of the 2D anomaly action is

$$\boxed{T_{ab}^{(anom)}[g;\varphi] \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{anom}[g;\varphi]}{\delta g^{ab}} = \frac{c}{24\pi}\left[\nabla_a\nabla_b\varphi - g_{ab}\Box\varphi + \frac{1}{2}\nabla_a\varphi\nabla_b\varphi - \frac{g_{ab}}{4}\nabla_c\varphi\nabla^c\varphi\right]}$$

- Static 'Schwarzschild' or 2D de Sitter : $ds^2 = f(r^*)(-dt^2 + dr^{*2})$
- General soln. to $\Box\varphi = -R = f''$ with $\varphi = \varphi(r^*)$ + linear in t

$$\varphi = (qr^* + pt)/2M + \ln f$$

$$T_t^{\ t} = \frac{N}{24\pi}\left\{-\frac{1}{4f}\left(\frac{p^2+q^2}{4M^2} - \frac{4M^2}{r^4}\right) + \frac{4M}{r^3}\right\}$$

$$T_t^{\ r^*} = \frac{N}{192\pi M^2}\frac{pq}{f}$$

$$T_{r^*}^{\ r^*} = \frac{N}{96\pi f}\left(\frac{p^2+q^2}{4M^2} - \frac{4M^2}{r^4}\right)$$

- Full Quantum stress tensor determined by anomaly
- Generally divergent at $f = 1\text{-}2M/r = 0$
- Finite iff $pq = 0$ & $p^2 + q^2 = 1$

2D Anomaly Effective Action

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Quantum Effects of 2D Anomaly Action

- **Modification** of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Potentially **Large Effects on Horizons**
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes **dynamical** & itself fluctuates freely ($c - 26 \rightarrow c - 25$)
- Gravitational 'Dressing' of critical exponents at 2nd order phase transitions -- long distance macroscopic physics
- Non-perturbative conformal fixed point of 2D gravity
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energies

Quantum Trace Anomaly in 4D Flat Space

Eg. QED in an External EM Field A_μ

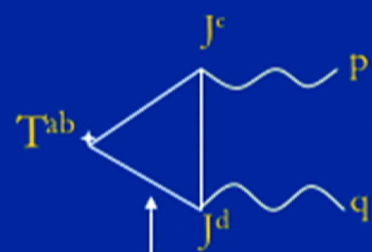
$$\boxed{\left\langle T^{m}_{\ m} \right\rangle = \frac{e^2}{24\pi^2} F^{mn} F_{mn}}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2) + \text{(traceless terms)}$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:

$k = p + q$



$$\boxed{F_1(k^2) = \frac{e^2}{18\pi^2 k^2}}$$

$$\boxed{\rho_T(s) \to \frac{e^2}{6\pi^2}\,\delta(s)}$$

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in
the two-particle correlated spin-0 state

<TJJ> Triangle Amplitude in QED

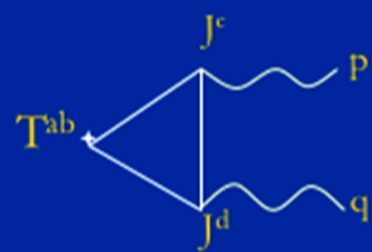
Determining the Amplitude by Symmetries and Its Finite Parts

M. Giannotti & E. M. ***Phys. Rev. D*** **79, 045014 (2009)**

$$\Gamma^{abcd}(p,q) = \int d^4x \int d^4y\, e^{ip\cdot x + iq\cdot y}\, \frac{\delta^2 \langle T^{ab}(0)\rangle_A}{\delta A_c(x)\delta A_d(y)}\Big|_{A=0}$$

Γ^{abcd}: Mass Dimension 2   Use low energy symmetries:

$k = p + q$



1. By Lorentz invariance, can be expanded in a complete set of 13 tensors $t_i^{abcd}(p,q)$, $i = 1, \ldots 13$:

$$\Gamma^{abcd}(p,q) = \Sigma_i\, F_i\, t_i^{abcd}(p,q)$$

2. By current conservation: $p_c t_i^{abcd}(p,q) = 0 = q_d t_i^{abcd}(p,q)$

All (but one) of these 13 tensors are dimension ≥ 4, so $\dim(F_i) \leq -2$ so these scalar $F_i(k^2; p^2, q^2)$ are completely UV Convergent

Spectral Representation & Finite Sum Rule

$$F_1(k^2;p^2,q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2+s-i\epsilon}\left[(k^2+s)\rho_{\scriptscriptstyle T} - m^2\rho_m\right]$$

Im $F_1(k^2 = -s)$: Non-anomalous,vanishes when m=0, s≠0

$$\rho_{\scriptscriptstyle T}(s;p^2,q^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\int_0^{1-x} dy\;(1-4xy)\;\delta\left(s - \frac{(p^2x+q^2y)(1-x-y)+m^2}{xy}\right)$$

$$\boxed{\int_0^\infty ds\,\rho_{\scriptscriptstyle T}(s;p^2,q^2) = \frac{e^2}{6\pi^2}}$$

Obeys a UV <u>Finite</u> Sum Rule

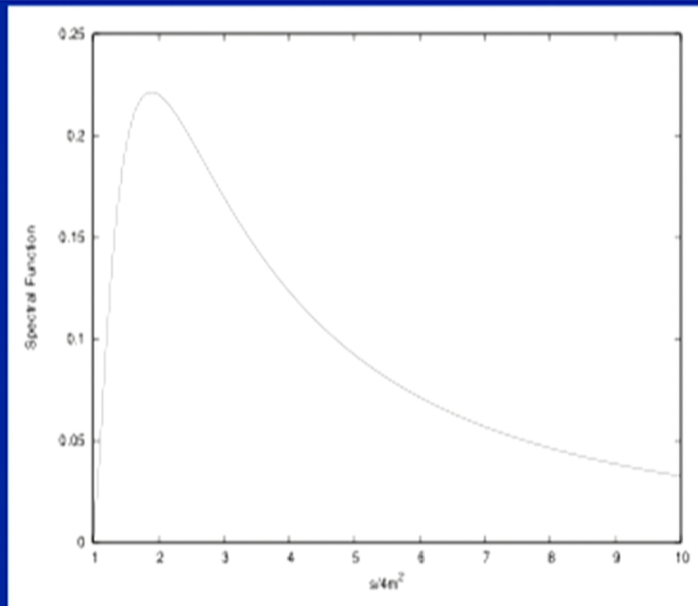
<u>independent</u> of p^2, q^2, m^2

As m^2/p^2, $m^2/q^2 \to 0^+$

$$\boxed{F_1(k^2) \to \frac{e^2}{18\pi^2 k^2}}$$

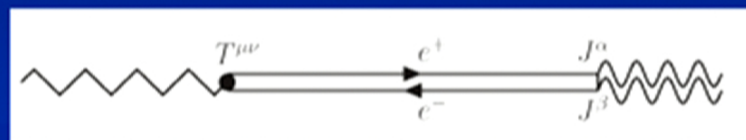
$$\boxed{\rho_{\scriptscriptstyle T}(s) \to \frac{e^2}{6\pi^2}\,\delta(s)}$$

<u>Massless</u> <u>scalar</u>
Intermediate
Two-particle pair
state



Massless Anomaly Pole

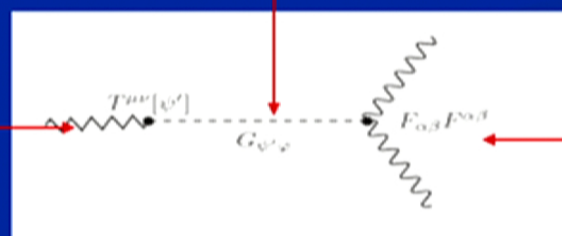
For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a massless $e^+ e^-$ pair moving at v=c collinearly, with opposite helicities in a total spin-0 state (relativistic *Cooper pair* in QFT *vacuum*)



$\Rightarrow$ a massless scalar 0^+ state which couples to gravity

Effective vertex

$h_{\mu\nu}\,(g^{\mu\nu}\Box - \partial^\mu\partial^\nu)\psi'$



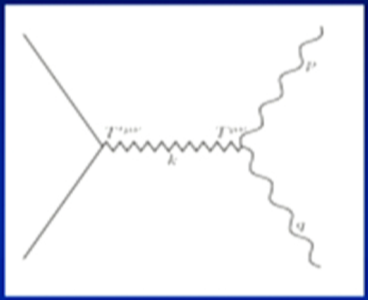
$\varphi\, F^{\alpha\beta}F_{\alpha\beta}$

Effective Action

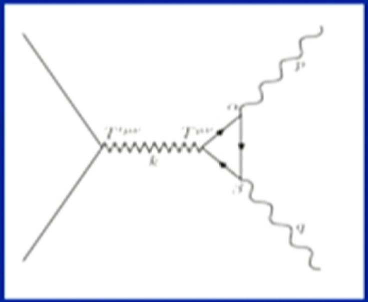
$$\int d^4x\sqrt{-g}\left\{-\psi'\Box\varphi - \frac{R}{3}\psi' - \frac{e^2}{48\pi^2}\varphi F^{\alpha\beta}F_{\alpha\beta}\right\}$$

special case of general form

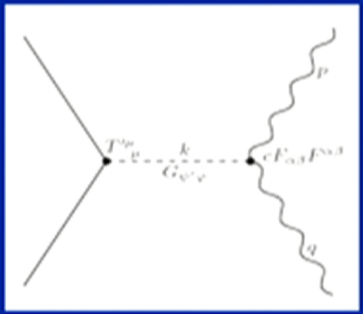
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (<u>spin-2</u>) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)



- But for $m_e = 0$ there is a scalar pole in the $\langle TJJ \rangle$ triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$



- Couples to trace $T'^{\mu}{}_{\mu}$
- $\langle TTT \rangle$ triangle of massless photons has similar pole
- Induced scalar degrees of freedom in EFT

Constructing the EFT of Gravity

- Assume ***Equivalence Principle*** (Symmetry)
- Metric Order Parameter Field g_{ab}
- Only two strictly ***relevant*** operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity **+** Quantum Corrections
- Semi-classical Einstein Eqs. $(m << k << M_{pl})$:

$$G_{ab} + \Lambda\, g_{ab} = 8\pi\, G \langle T_{ab} \rangle$$

- But there is also a quantum (trace) anomaly:

$$\langle T_a^{\ a} \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R$$

$$F = C_{abcd} C^{abcd} = C^2 \qquad E = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2$$

- ***Massless Poles ⇒New*** (marginally) relevant operator in gravitational sector

4D Anomalous Effective Action

Conformal Parametization

$$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$$

Since $\sqrt{g}\,F = \sqrt{\bar{g}}\,\bar{F}$ $\quad F = C_{abcd}C^{abcd} = C^2$

is independent of σ, and

$$\sqrt{g}\,\left(E - \tfrac{2}{3}\Box R\right) = \sqrt{\bar{g}}\,\left(\bar{E} - \tfrac{2}{3}\bar{\Box}\bar{R}\right) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is only linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F + b'\sqrt{g}\,\left(E - \tfrac{2}{3}\Box R\right)$$

determines the Wess-Zumino Action by *inspection*

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma + \int d^4x\sqrt{\bar{g}}\left[b\bar{F} + b'\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\,,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

Effective Action for the Trace Anomaly

- **Non-Local Covariant Form (logarithmic propagator)**

$$S_{anom}[g] = \frac{1}{8}\int d^4x\sqrt{g_x}\left(E - \tfrac{2}{3}\Box R\right)_x \int d^4x'\sqrt{g_{x'}}\,(\Delta_4)^{-1}_{x,x'}\,\mathcal{A}_{x'}$$

$$\boxed{\mathcal{A} = b'\left(E - \tfrac{2}{3}\Box R\right) + b\,C^2 + c\,F^2 + c_s\,\mathrm{tr}\,G^2}$$

- **Local Covariant Form in Terms of New Scalar Field**

$$\boxed{S_{anom}[g;\varphi] = -\frac{b'}{2}\int d^4x\sqrt{g}\,\varphi\,\Delta_4\,\varphi + \frac{1}{2}\int d^4x\sqrt{g}\,\mathcal{A}\,\varphi}$$

- **Dynamical Scalar in Conformal Sector: 'Conformalon'**

$$\boxed{\Delta_4\varphi = \frac{1}{2b'}\,\mathcal{A}}$$

$$\boxed{\begin{aligned} b &= \frac{\hbar}{120(4\pi)^2}\,(N_s + 6N_f + 12N_v) \\ b' &= -\frac{\hbar}{360(4\pi)^2}\,(N_s + 11N_f + 62N_v) \end{aligned}}$$

Exact Effective Action & Wilson Effective Action

- Integrating out Matter + … Fields in Fixed Gravitational Background gives the Exact 1PI Quantum Effective Action
- The possible terms in $S_{exact}[g]$ can be classified according to their repsonse to local Weyl rescalings $g \to e^{2\sigma} g$

$$S_{exact} = S_{local} + S_{anom} + S_{Weyl}$$

- $S_{local} = (1/16\pi G) \int d^4x \sqrt{g}\, (R - 2\Lambda) + \Sigma_{n\geq 4} M_{Pl}^{4-n} S^{(n)}_{local}[g]$

  Ascending series of higher derivative local terms, **n>4 irrelevant**
- Non-local but Weyl-invariant (neutral under rescalings)

  $S_{Weyl}[g] = S_{Weyl}[e^{2\sigma} g]$ **Marginally Irrelevant**
- S_{anom} scales linearly with σ, logarithmic w. distance, non-trivial cohomology of Weyl conformal group <u>Marginally Relevant</u>
- Wess-Zumino Consistency requires conformal property

$$\boxed{S_{anom}[e^{-2\sigma} g; \varphi] = S_{anom}[g; \varphi + 2\sigma] - S_{anom}[g; 2\sigma]}$$

QCD Source of Scalar Gravitational Waves

- The QCD Trace Anomaly is also a Source for φ

$$\boxed{\Box^2 \varphi = \frac{1}{2b'}\,\mathcal{A}_{QCD}}$$

- Gluonic Condensate much larger (10 Orders of Magnitude)

$$\boxed{\mathcal{A}_{QCD} = -(11N_c - 2N_f)\,\frac{\alpha_s}{24\pi}\,\langle G^a_{\mu\nu} G^{a\mu\nu}\rangle \simeq -5.6 \times 10^{36}\,\mathrm{erg/cm^3}}$$

- Neutron Star Cores contain Density Dependent Gluon Condensate
- In a Neutron Star Merger with another Compact Object this Gluonic Condensate ('Bag Constant') is almost certainly disturbed
- Scalar GW Mode most likely excited in <u>Neutron Star Mergers</u>
- Requires quantitative control of nuclear physics in NS mergers
- Condensate excited also in Gravastar Alternative to BH's

Sources of Scalar Gravitational Waves

- Sources of **φ** are all the trace anomaly terms

$$\Delta_4\,\varphi = \frac{E}{2} - \frac{\Box R}{3} + \frac{1}{2b'}\Big(b\,C^2 + cF_{\mu\nu}F^{\mu\nu} + \dots\Big)$$

- Curvature Invariants are extremely small
- QED and QCD Gauge Field Anomalies are <u>much larger</u>
- For a Magnetar **B ~ 10^{15} Gauss**

$$\mathcal{A}_{mag} = -\frac{e^2}{24\pi^2}F_{\mu\nu}F^{\mu\nu} = -\frac{\alpha B^2}{3\pi} \simeq -8\times 10^{26}\left(\frac{B}{10^{15}\,\mathrm{Gauss}}\right)^2 \mathrm{erg/cm^3}$$

$$\frac{\delta L}{L} \simeq -\frac{G}{3r}\int d^3x\,\mathcal{A}_{mag} \simeq 5\times 10^{-26}\left(\frac{\mathrm{kpc}}{r}\right)$$

- Still not large enough to be observable by aLIGO

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Anomaly Stress Tensor: General Form

$$T_{\mu\nu}[\varphi] = b'\, E_{\mu\nu} + b\, C_{\mu\nu} + \sum_i \beta_i\, T^{(i)}_{\mu\nu}[\varphi]$$

$$\begin{aligned}
E_{\mu\nu} \equiv & -2\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\Box\varphi) + 2\nabla^\alpha\big[(\nabla_\alpha\varphi)(\nabla_\mu\nabla_\nu\varphi)\big] - \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big] \\
& + \tfrac{2}{3}\,R_{\mu\nu}\,(\nabla_\alpha\varphi)(\nabla^\alpha\varphi) - 4\,R^\alpha_{\ (\mu}\,\big[(\nabla_{\nu)}\varphi)(\nabla_\alpha\varphi)\big] + \tfrac{2}{3}\,R\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\varphi) \\
& + \tfrac{1}{6}\,g_{\mu\nu}\,\big\{-3\,(\Box\varphi)^2 + \Box\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big] + 2\,\big(3R^{\alpha\beta} - Rg^{\alpha\beta}\big)(\nabla_\alpha\varphi)(\nabla_\beta\varphi)\big\} \\
& - \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\Box\varphi - 4\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R^\alpha_{(\mu}\nabla_{\nu)}\nabla_\alpha\varphi + \tfrac{8}{3}\,R_{\mu\nu}\,\Box\varphi + \tfrac{4}{3}\,R\,\nabla_\mu\nabla_\nu\varphi \\
& - \tfrac{2}{3}\,\big(\nabla_{(\mu}R\big)\,\nabla_{\nu)}\varphi + \tfrac{1}{3}\,g_{\mu\nu}\,\big[2\,\Box^2\varphi + 6\,R^{\alpha\beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R\,\Box\varphi + (\nabla^\alpha R)\nabla_\alpha\varphi\big]
\end{aligned}$$

$$C_{\mu\nu} = -4\,\nabla_\alpha\nabla_\beta\big(C_{(\mu\ \nu)}^{\ \ \alpha\ \ \beta}\,\varphi\big) - 2\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,R_{\alpha\beta}\,\varphi$$

Anomaly Stress Tensor Near Horizons

- An horizon is a null surface, conformal to flat space light cone & conformally invariant
- Fields become effectively massless there
- The near horizon region is conformal to $EAdS_3 \otimes$ time
- Conformal Anomaly becomes the dominant term in the effective action in the near horizon region
- Stress Tensor from S_{anom} determines $\langle T_{ab} \rangle$
- Stress Tensor is generally singular there
- Singular behavior has invariant meaning in terms of conformalon scalar degree of freedom on horizon

Anomaly Stress Tensor in de Sitter Space

- General soln. for φ as fn. of static r and linear in t is

$$\left.\varphi(r,t)\right|_{dS} = c_0 + 2Hpt + \ln\left(1 - H^2r^2\right) + \frac{q}{2}\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{2c_H - 2 - q}{2Hr}\ln\left(\frac{1-Hr}{1+Hr}\right)$$

- Bunch-Davies state has $p = 1$, $q = 0$, $c_H = 1$ again

$$\left.T_{ab}\right|_{BD,dS} = 6b' H^4 g_{ab} = -\frac{H^4}{960\pi^2}\, g_{ab}\left(N_s + 11N_f + 62N_v\right)$$

- This is the soln. for conformal map to flat spacetime

$$ds^2 = e^{\varphi_{BD}}\,(ds^2)_{flat}$$

- Otherwise T_{ab} is generally divergent at the static horizon $r = H^{-1}$ behaving like $(1-H^2r^2)^{-2}$