Title: The Large Charge Expansion and the Universal Correlation Functions in Rank-1 SCFT

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Abstract: <p>The two-point functions <O^n(x) Obar^n(y)> for generator O of Coulomb branch chiral rings in D=4 N=2 SCFT will be determined universally to all orders in 1/n by the theory's a-anomaly.<br />
The calculation will be done using the method of large-charge expansion presented in [1706.05743]; the absence of F-terms in the (R-charge)^(-1) expansion of the effective action ensures this universality.<br />
I will also comment on the non-universal and non-perturbative corrections to the two-point functions whose leading piece was numerically shown to go as O(exp(-sqrt(n))).  </p>

# The Large Charge Expansion and Universal Correlation Functions in Rank-1 SCFT

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Based on
[1505.01537] with
Simeon Hellerman, Domenico Orlando, Susanne Reffert
[1804.01535] with
Simeon Hellerman, Shunsuke Maeda, Domenico Orlando, Susanne Reffert

The Large Charge Expansion

$D = 3$, $O(2)$ Wilson-Fisher fixed point at large charge

$D = 4$, $\mathcal{N} = 2$ rank one SCFTs

## Strongly-coupled theories are difficult.

- Strongly-coupled quantum field theories are interesting.
- You cannot really avoid them if you even remotely set out to study real physics because of RG flows.
- Studying strongly-coupled theories is, however, intrinsically difficult.
- You might even not have a Lagrangian description, or even when you have one, you do not have a small parameter to have control over physical quantities.

## The large charge expansion

- For strongly-coupled theories, semi-classical analysis can sometimes be of use, e.g., large-$N$ expansion.
- Likewise, when a system we consider has a global symmetry, going to a sector of large global charge, $J$, gives us some simplifications.
- We can then sometimes analyse strongly-coupled theory in the semi-classical regime, where the Lagrangian is weakly coupled in units of $1/J$.
- We call this the large charge (large-$J$) expansion.
- (One example of the physical realisation will be the superfluid, where you take the number of He atoms large. Perfectly a reasonable limit.)

## Why is the large charge expansion is semi-classical?

- Let us consider a strongly-coupled QFT on the cylinder, with radius $R$.
- We give a state a large charge density $\rho$, where $J \propto \rho R^2 \gg 1$.
- By the standard Wilsonian argument, $\Lambda_{\mathrm{UV}} = \sqrt{\rho}$ and $\Lambda_{\mathrm{IR}} = 1/R$.

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# RG flow of the $O(2)$ model

- Let me start with the simplest model of all to be analysed at large charge, $J$. We put the system on $S^2 \times \mathbb{R}$ with radius $R$.
- We start with the following UV action using a complex field $\phi \equiv a \times e^{i\chi}$,

$$\mathcal{L}_{\mathrm{UV}} = -\partial\phi\partial\bar{\phi} - m^2|\phi|^2 - g^2|\phi|^4$$

  This system has the $O(2)$ symmetry.
- By fine-tuning the value of $m^2$, we get the conformal Wilson-Fisher fixed point in the IR.

## RG flow of the $O(2)$ model at large charge

- We can add the chemical potential term $\omega \times \rho$ to the action to fix the charge density.
- This is the same thing as giving $\langle a \rangle = \langle | \phi | \rangle \propto \sqrt{\rho}$ a large dimensionful VEV proportional to $\sqrt{\rho}$, spontaneously breaking the $O(2)$ symmetry.
- (Why not explicit breaking? Just set $\phi_{\mathrm{new}} = e^{i\omega t}\phi$, and you recover the Lagrangian with the $O(2)$ symmetry, which is a combination of the original $O(2)$ and translation in time.)
- Now $\chi$, the flat direction, becomes the Goldstone boson of the theory from which we construct the effective Lagrangian.

## RG flow of the $O(2)$ model at large charge

- Scale everything far below $\Lambda_0 \equiv \sqrt{\rho}$ now.
- The IR Lagrangian has to be classically conformally invariant. Quantum fluctuations come in positive powers of $\Lambda/\Lambda_0$ (no diagrams to generate singularities).
- (We will not care about dressings in $\Lambda/\Lambda_0$, because this can be done systematically and will not affect what I want to consider.)
- All our tasks, therefore, has reduced to listing all the conformal invariant operators according to $J$-scaling.

## Classically conformally invariant Lagrangian at large charge

- I will now give you the leading order $O(2)$ invariant IR Lagrangian, which is also classically conformally invariant,

$$\mathcal{L}_{\rm IR} = -\frac{1}{2}(\partial a)^2 - \frac{1}{2}\kappa a^2(\partial\chi)^2 - \frac{h^2}{12}a^6 + \circ\circ\circ,$$

where $\circ\circ\circ$ are higher derivative terms.

- Or, much better, you can integrate out the $a$ field, whose mass is of order $\sqrt{\rho} \gg \Lambda_{\rm IR}$, resulting in

$$\mathcal{L}_{\rm IR} = b_\chi |\partial\chi|^3 + \circ\circ\circ$$

where $b_\chi = \frac{\sqrt{2}\kappa^{3/2}}{3h}$.

## Equation of motion at large charge

- The solution to the equation of motion of this leading order Lagrangian is $\chi_0 = \omega t$.
- By virtue of the Noether theorem, we have $\rho = 3b_\chi |\partial\chi|^2$ and $J = 4\pi R^2 \rho$.
- Therefore, we have $\omega = O(\sqrt{\rho})$.
- When we separate $\chi = \chi_0 + \chi_{\mathrm{fluc}} = \chi_0 + |\partial\chi_0|^{-1/2}\hat{\chi}$ into VEV and fluctuations, we have

$$\mathcal{L}_{\mathrm{leading}}/b_\chi = |\partial\chi_0^3| + \frac{3}{2}\hat{\chi}\left(\partial_t^2 + \frac{1}{2}\triangle_{S^2}\right)\hat{\chi}$$

where $\chi_{\mathrm{fluc}}$ are $O(1)$.

## Rules to sort operators at large-$J$

- Now we want to write down sub-leading terms in $1/J$.
- After integrating out the $a$ field, you are free to put its mass to the denominator of each effective operator, which is, in this case, $|\partial\chi|$.
- The scaling of $|\partial\chi|$ is as follows,

$$|\partial\chi| = |\partial\chi_0| \propto \sqrt{J}$$

- Also you have

$$\partial^{k>1}\chi = \partial^k \chi_{\text{fluc}} \propto J^{-1/4}$$

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- One more rule to remember is that you can use the EOM for the leading action. That is, $\partial_\mu \left(|\partial\chi|\partial^\mu\chi\right) = 0$ can be used to eliminate operators (they can be traded for something of the lower $J$ scaling)

## Sorting operators larger than $O(J^0)$

- Order $J^{3/2}$

$$|\partial\chi|^3$$

- Order $J^{1/2}$

$$\mathtt{Ric}_3|\partial\chi| + \frac{2(\partial|\partial\chi|)^2}{|\partial\chi|} = O(J^{1/2}) + O(J^{-1})$$

- Note that the Weyl completion to $\mathtt{Ric}_3|\partial\chi|$ is lower in scaling than the first term. The latter term is still there even on the flat background.
- These are actually the only operators that appear at or above $O(J^0)$!

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## Universal IR Lagrangian at large charge

- Now we know everything about the IR Lagrangian at large charge.
- This Lagrangian is universally determined when the theory is on the WF fixed point,

$$\mathcal{L}_{\mathrm{IR}} = c_{3/2}|\partial\chi|^3 + c_{1/2}\mathtt{Ric}_3|\partial\chi| + O(J^{-1/4}),$$

  where $c_{3/2,1/2}$ are (UV) theory-dependent coefficients
- We are now in a position to calculate the operator dimensions at large charge.

## Operator dimension at large charge

- Let us calculate the operator dimension at large charge.
- The classical piece is given by just

$$c_{3/2}J^{3/2} + c_{1/2}J^{1/2}$$

- This WF universality class includes SUSY models, e.g., $D = 3$, $\mathcal{N} = 2$ with $W = g\Phi^3$ at large-$R$-charge. This means that the lowest operator dimensions at large-$J$ are far above the BPS bounds, instead of saturating them. More comments are in order later.

## Operator dimension at large charge

- The Casimir energy from $|\partial\chi|^{3/2}$ is the leading quantum contribution, which goes as $O(J^0)$.
- The Lagrangian for the normalised fluctuation goes as

$$\hat{\chi}\left(\partial_t^2 + \frac{1}{2}\triangle_{S^2}\right)\hat{\chi}$$

- So the contribution to the Casimir energy is as follows

$$E_0 = \frac{1}{2\sqrt{2}}\sum_{\ell=0}^{\ell=\infty}(2\ell+1)\sqrt{\ell(\ell+1)}.$$

## Operator dimension at large charge

- It is actually a bit tricky because you can sometimes naively use zeta-function regularization to get a wrong result.
- But there is certainly a way to correctly compute this using zeta-functions, which gives

$$E_0 = -0.094$$

- There were no operators available at $O(J^0)$, so this is the only contribution to the dimension at this order. We therefore get

$$c_{3/2}J^{3/2} + c_{1/2}J^{1/2} - 0.094$$

## Conformal Regge Trajectories in the EFT

- All the operators $O(1)$ above the lowest one can be written down – at spin $\ell$, the energy of the excited state increases by $\Delta(\ell) = \sqrt{\ell(\ell+1)/2}$. This is the conformal Regge trajectory for this EFT.
- The spin $\ell = 1$ state is the descendent of the ground state, as required by the conformal symmetry. Others are just new primaries.
- The fact that the speed of the GB is $1/\sqrt{2}$ times the speed of light, therefore, follows directly from conformal symmetry. More specifically, the existence of the descendent.

## SUSY theories without moduli

- $\mathcal{N}=2$, $D=3$ SUSY theory with superpotential $W=\Phi^3$ at large $R$-charge is in the same universality class too.
- In this case, there is no moduli space of vacua, which makes the theory lie in the same universality class.
- In this case, superconformal symmetry fixes the fermion's mass at $O(\sqrt{J})$ (called massive Goldstini).
- Then you can integrate the fermions out, and you get the same EFT as in the non-SUSY case.
- When there is a moduli space of vacua, the story is totally different, and I will talk about this in the latter half of my talk.

## Numerical simulations

- To make sure we didn't make any mistakes, let me show you a few numerical results that back up our formula, from two other groups than ours.
- In [1707.00711] the authors calculated the lowest operator dimension at fixed charges up to $J \sim 10$ and see if our formula fits well.
- The result actually suggests a remarkable fit even down to $J \sim 1$!
- In [1805.00501] the author calculated the lowest operator dimension up to $J = 100$, using large-$N$ technique. This even recovers our universal prediction, $-0.094$.

## Bootstrap at large charge?

- It is clear from the construction that the straightforward numerical bootstrap program slows down at larger charges.

## Large-$N$ [Anton de la Fuente]

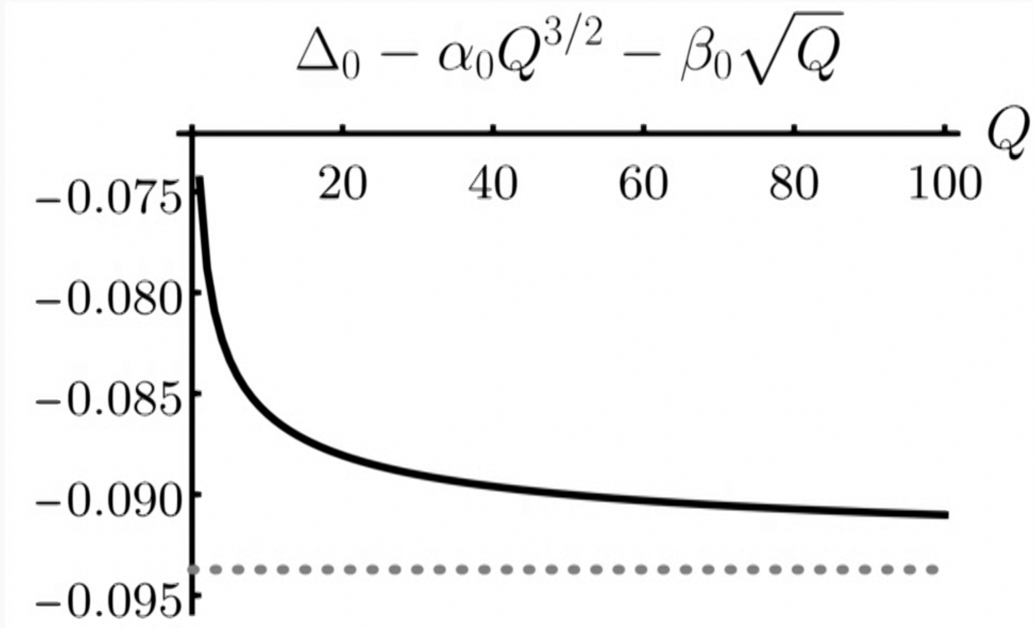


**Figure 2:** $O(N)$ contribution to the $O(Q^0)$ piece of the lowest dimension can be shown to be analytically vanishing. The plot shows the $O(N^0)$ contribution to the $O(Q^0)$ piece, and it quickly reaches $-0.094$ at large-$Q$.

## Bootstrap at large charge?

- It is clear from the construction that the straightforward numerical bootstrap program slows down at larger charges.
- Analytic bootstrap should be interesting, but not exactly parallel with analytic bootstrap at large spin. The theory is not going to be (generalised) free at leading order (the dimension scales as $J^{3/2}$ instead of $J$).
- Recently Jafferis, Mukhametzhanov and Zhiboedov have put out a nice paper studying large-$J$ bootstrap. (Question: can we relate this with the recent result about large-$\Delta$?)
- It shows that the EFT we derived is the only possible EFT when there is only one Regge trajectory, and that there are far more possibilities on the spectrum if you allow more than one Regge trajectories.

## SUSY theories with moduli space of vacua

- SUSY theories can have moduli space of vacua, and in those cases the large-$J$ analysis is totally different.
- Most importantly, the leading order action is just the free Lagrangian, as opposed to the case without the moduli.
- This is because the flat direction of the theory is the entire multiplet, as opposed to just the angular field, $\chi$, as in the previous cases without moduli.
- The effective action, therefore, preserves supersymmetry. This restricts the effective action all the more, compared to the case without supersymmetry.

## The low-energy action of the $XYZ$ model on the $X$ branch

- One example of this type of theories include the $D = 3$, $\mathcal{N} = 2$, $W = gXYZ$ model.
- On the $X$ branch, the field content of the effective action is just $\Phi \equiv X^{3/4}$ (normalised for a unit kinetic term).
- You can write down the effective action in terms of the $1/J$ scaling, where $|\phi| \propto \sqrt{J}$.
- The anomalous dimension for the lowest operator that receives quantum corrections is

$$\gamma(J) = -\frac{c}{J^3} < 0 \quad \text{(causality)},$$

# $D = 4$, $\mathcal{N} = 2$, rank-1 SCFT

- We now consider $D = 4$, $\mathcal{N} = 2$, rank-1 SCFTs.
- Instead of the operator dimensions, we here would like to consider two-point functions of operators in the chiral ring (which is one-dimensional).
- First of all, they are independent of any $D$-terms (because the two-point functions preserve some SUSY).
- Now then, we can only care about $F$-terms. They are highly constrained because there are so many supercharges. Actually, there will be no $F$-terms (aside from WZ term.).

## Vanishing of the subleading $F$-terms in rank one SCFTs (1)

- The symmetries we use are $\mathcal{N} = 2$ SUSY, $U(1)_R$ and the conformal symmetry.
- They transform the field living on the one-dimensional Coulomb branch as

$$\phi(x) \mapsto e^{\sigma}\phi(x), \quad \sigma = \rho + i\gamma$$

- Let's promote $\sigma$ into a chiral multiplet, $\Sigma(x, \theta)$ (whose scalar component I call $\sigma(x)$). Then the symmetry of the theory becomes spurionic – you transform both $\phi$ and the background field appropriately to get the real symmetry.

## Vanishing of the subleading $F$-terms in rank one SCFTs (2)

- Background fields coupled to $\phi$ are background metric $g_{\mu\nu}$ for the Weyl transformation, and the background gauge field $a_\mu$ for the $U(1)_R$ transformation respectively.
- Now set $\sigma(x) = \log(\phi(x)/\mu)$, and SUSY partners of $\sigma(x)$ appropriately to set $\phi_{\rm new}(x) \equiv \mu$, with all other components vanishing everywhere.
- Note that we have turned on background fields $g_{\mu\nu}$ and the $R$-gauge field in trade.

# Vanishing of the subleading $F$-terms in rank one SCFTs (3)

- Let us only consider the background $\mathbb{R}^4$ or what are conformally equivalent to that, e.g., $S^4$ and $S^3 \times \mathbb{R}$.
- Then the Weyl tensor is obviously vanishing. The $R$-gauge fluxes are also vanishing, because we only consider maximally supersymmetric background (which can be seen by varying gravitini and setting it to zero, and all other auxiliary fields vanish too).
- We therefore can only consider terms involving the Ricci curvature, its derivatives, and the flat $R$-gauge field.

# Vanishing of the subleading $F$-terms in rank one SCFTs (4)

- We only have one field, $\phi$, on the Coulomb branch; we thus have four building blocks in the effective action. $\phi$, $\bar{\phi}$, $g_{\mu\nu}$, and $a_\mu$.
- The $F$-term must be a scalar, Weyl-weight $2$, $R$-gauge invariant, and all the more, chiral. It is of the form

$$\int d\theta^4 \, \phi^2 \times \mathcal{I}$$

where $\mathcal{I}$ is a scalar, Weyl-weight $0$, $R$-gauge invariant, and chiral.

## Vanishing of the subleading $F$-terms in rank one SCFTs (5)

- Let us try to construct $\mathcal{I}$.
- Out of $a_\mu$ only, you can only construct gauge invariant combinations out of its flux, $d\alpha$. This is vanishing on $\mathbb{R}^4$ as I have already explained. (CS term is not invariant when dressed with $\phi^2$)
- Out of $\hat{g}_{\mu\nu} \equiv |\phi|^2 g_{\mu\nu}$, which is not chiral, you can only construct things made out of the hatted Weyl tensor. But since Weyl tensors are Weyl covariant, things out of the hatted Weyl tensor are just the same as ones created out of the original Weyl tensor. This vanishes on $\mathbb{R}^4$
- The combination of $a_\mu$ and $\hat{g}_{\mu\nu}$ is possible, but only reduces to the multiplication of the above two and hence vanishes.
- We therefore conclude that there are no subleading $F$-terms in the effective action.

## Weyl anomaly matching and the Wess-Zumino term.

- So, is the effective Lagrangian just comprised of the free one? No.
- In four dimensions instead of three, you have to add the Wess-Zumino term to match the theory's Weyl and $U(1)$ anomaly.
- This is essentially of order $\log(\phi/\mu)$ which goes as $\log J$. This is bigger than any possible subleading terms present in the effective theory (even if there were $F$-terms).

# The Wess-Zumino term

- The only candidate for the complex axiodilation which comes into Wess-Zumino term compensating the anomaly mismatch is $\tau + i\beta \equiv \log(\phi/\mu)$, where $\mu$ is some scale.
- To compensate for the anomaly mismatch, the term goes as

$$L_{\rm WZ} \equiv \alpha \times L[\tau + i\beta]$$

  where careful matching of coefficients tell you that $\alpha \equiv \frac{5}{12} \frac{a}{a_{U(1)\ \text{vector multiplet}}}$.
- I don't even want to tell you what the form of $L[\tau + i\beta]$ is.

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# Normalisation of $\phi$

- I have been agnostic about the definition of $\phi$ up until now.
- We would like to consider rank one SCFTs, and will call the Coulomb branch generator $\mathcal{O}$, whose dimension is $\Delta_{\mathcal{O}}$.
- We will define $\phi$ by $\mathcal{O} \equiv (N_{\mathcal{O}}\phi)^{\Delta_{\mathcal{O}}}$ and set its kinetic term to be unit normalised.
- In Lagrangian theories, $N_{\mathcal{O}} = \sqrt{\text{Im}\,\tau}$.

## Correlation functions at large charge

- We would like to compute the correlation function $\langle \bar{\mathcal{O}}(x)^n \mathcal{O}(y)^n \rangle$. This is equivalent to computing $\langle \bar{\phi}(x)^J \phi(y)^J \rangle$, where $J \equiv n\Delta_{\mathcal{O}}$.
- This can be computed by inserting operators in the path integral which we call $Z_n$. The Lagrangian is, as I have already explained,

$$L_{\text{eff}} = L_{\text{free}}^{O(J)} + J\hat{\alpha} L_{\text{WZ}}^{O(\log J)}.$$

where $\hat{\alpha} \equiv \alpha/J$ and $J$ becomes the loop counting parameter.

## The structure of the correlation function (1)

- The structure of $q_n = \log Z_n$ is as follows.
- We separate the field into VEV ($O(\sqrt{J})$) and fluctuations.
- We only compute the connected vacuum diagram because source terms are already in the action itself.
- The number of loops $\ell$ are counted by $1/J^{\ell-1}$, while number vertices $m$ are counted by $\hat{\alpha}^m$.
- We also should not forget the most important $\alpha \log J$ term from the classical WZ action itself.

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# Example of diagrams at $O(1/J)$

| description | term | diagrams |
|---|---|---|
| Two-loop with no $\alpha$-vertices | $\hat{K}_{1,0}$ |  |
| One-loop with one $\alpha$-vertex | $\hat{K}_{1,1}\alpha$ |  |
| Tree-level with two $\alpha$-vertices | $\hat{K}_{1,2}\alpha^2$ |  |

**Table 1** – Diagrams appearing at order $1/\mathcal{J}$.

## The structure of the correlation function (2)

- Putting $\hat{\alpha} \equiv \alpha/J$ back in, we see that $q_n$ is represented by an infinite sum

$$q_n = (\text{free}) + \alpha \log J + \sum_{m=1}^{\infty} \frac{P_{m+1}(\alpha)}{J^m}$$

where $P_{m+1}(\alpha)$ is a polynomial of order $m+1$ with $P_{m+1}(0) = 0$

- The free correlator, $(\text{free})$, is given by

$$AJ + B + \log(\Gamma(J+1)) = \circ\circ\circ + \frac{1}{2}\log J + \sum_{m=1}^{\infty} \frac{(-)^m B_{m+1}}{m(m+1)}$$

from the Wick contraction.

- Note that $A$ is convention dependent (normalisation for the partition function) and $B$ is theory dependent.

## Universality of correlators

- Now you know how to compute $q_n$, and the result should be only dependent on $\alpha$. It is universal because of the vanishing of the subleading $F$-terms.
- This should in theory be the end of the story. But it is difficult to conduct the calculation in practice.

# A trick

- Because of the structure of $q_n(\alpha)$, it is possible to determine everything using an countably infinite number of $\alpha$ as reference points.
- You can prepare them in Lagrangian theories with a marginal coupling, $\tau$. In those theories, it is known that $q_n$ satisfies the Toda equation

$$\partial\bar{\partial}q_n(\tau,\bar{\tau}) = \exp[q_{n+1}(\tau,\bar{\tau}) - q_n(\tau,\bar{\tau})] - \exp[q_n(\tau,\bar{\tau}) - q_{n-1}(\tau,\bar{\tau})].$$

- Examples of such theories include the free theory ($\alpha = 0$), $\mathcal{N} = 4$ SYM with $g = SU(2)$ ($\alpha = 1$), and $\mathcal{N} = 2$ SQCD with $N_f = 4$ ($\alpha = 3/2$).
- It is also possible to prepare infinitely many such theories by adding infinite numbers of matter hypermultiplets and ghost hypermultiplets.

# Toda equation solved at large-$n$

- Because of the structure of $q_n$, however, it should not depend on $\tau$, $\bar{\tau}$ or on $\Delta_{\mathcal{O}}$. This is a prediction from the EFT.
- You can exploit this fact to solve the Toda equation exactly,

$$q_n = An + B + \log\left(\Gamma(n - n_+(\alpha))\Gamma(n - n_-(\alpha))\right),$$

where again I remind you that $J = n\Delta_{\mathcal{O}} = 2n$ and $1 + n_+ + n_- = -(\alpha + \frac{1}{2})$ (matching of $O(\log(J))$).

- At order $1/J$ the term goes like

$$\frac{1 + 3n_+(n_+ + 1) + 3n_-(n_- + 1)}{6n}$$

and we match this with EFT to determine $n_\pm$.

## One real job to be done

- Now, by straightforward computation (which is tedious), combined with the exactly known result for the correlator in $\mathcal{N}=4$ SYM, we get the $1/n$ contribution as

$$\frac{1}{2}\left(\alpha^2+\alpha+\frac{1}{6}\right).$$

- We finally get

$$n_+=-\frac{\alpha+1}{2}, \quad n_-=-\frac{\alpha+2}{2}.$$

- The final result, therefore becomes,

$$q_n = AJ + B + \log\left(\Gamma(J+\alpha+1)\right)$$

This applies to any rank-1 theories, including non-Lagrangian theories!

# Worldline instantons

- BPS dyons are integrated out already in our effective field description, whose mass is of order $\sqrt{n}$

## Worldline instantons

- BPS dyons are integrated out already in our effective field description, whose mass is of order $\sqrt{n}$
- This should contribute to $q_n$ as $\exp(-\sqrt{n})$, a non-perturbative and non-universal correction.
- We test this by comparing the result with exact localization computations.

# Numerics (Localisation)

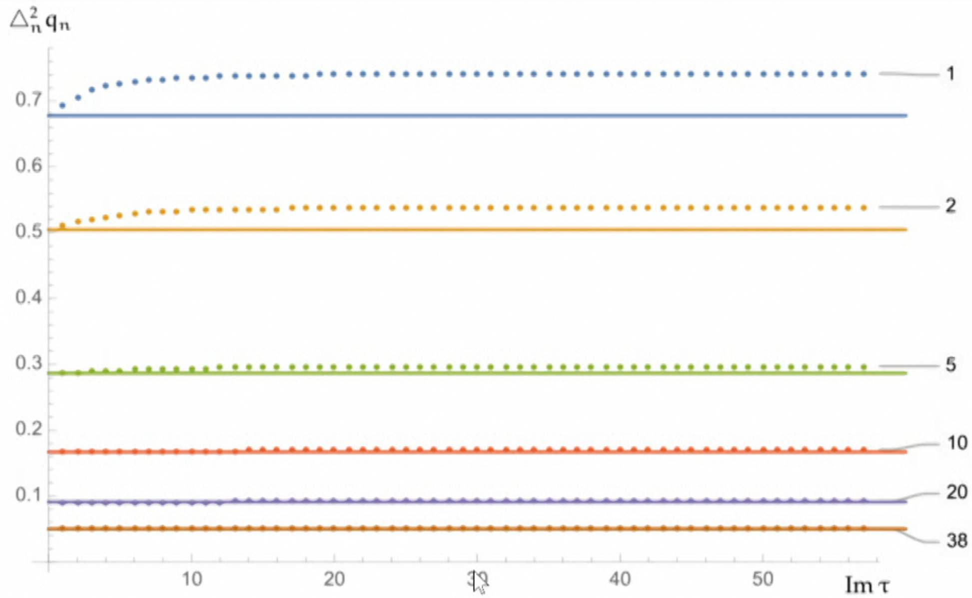


**Figure 4.1** – Second difference in $n$ for $\triangle_n^2 q_n^{(\mathrm{loc})}$ (dots) and for $\triangle_n^2 q_n^{\mathrm{EFT}}$ (continuous lines) as function of $\operatorname{Im}\tau$ at fixed values of $n$. The numerical results quickly reach a $\tau$-independent value that is well approximated by the asymptotic formula when $n$ is larger than $n \gtrsim 5$.

# Numerics (Localisation)

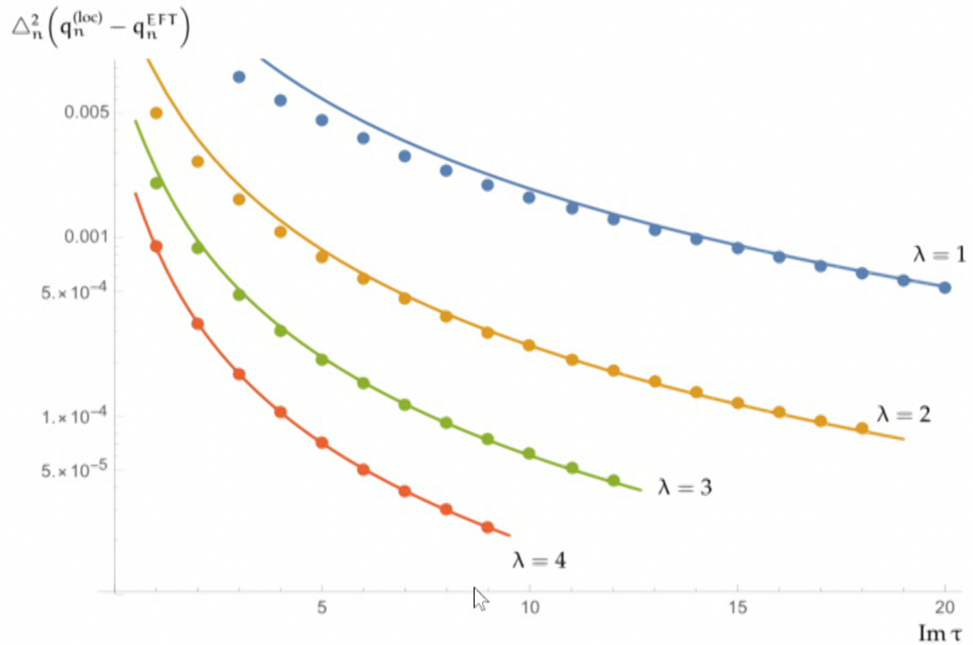


**Figure 6.1** – Second difference in $n$ for the discrepancy between localization and EFT results $\triangle_n^2\left(q_n^{(\mathrm{loc})} - q_n^{\mathrm{EFT}}\right)$ (dots) compared to $\triangle_n^2(1.6\,e^{-\sqrt{\pi\lambda}/2})$ (continuous lines) as functions of $\mathrm{Im}\,\tau$ at fixed values of $n/\,\mathrm{Im}\,\tau = \lambda/(4\pi)$. The agreement is quite good already for $\lambda = 3$.

# Comments

- The operator dimension of the Coulomb branch operator is conjectured to be a rational number by Argyres. Therefore, it is always possible to make $n$ and $J$ integers, where $\mathcal{O}^n \sim \phi^J$.
- Don't worry about non-freely generated chiral rings. It is actually proven that in rank-1 theories, the chiral ring is always freely generated.

## Take-home messages

- Large charge expansion is interesting, and makes it possible to analyse a strongly-coupled theory like a weakly-coupled one.
- Theories without moduli has an interesting behaviour at large charge, starting from an interacting theory at leading order.

## Take-home messages

- Theories with moduli space are more restricted at large charge. There could even be cases where the subleading corrections vanish entirely.
- Because of this, we derived the correlation functions between BPS operators universally, only depending on the theories $a$-anomaly, in rank one SCFTs.
- The result is also checked by standard methods like localisation.
- Future directions would include clearing up the logic a bit, or actually determining the size of the non-perturbative corrections. Higher ranks and Higgs branches are interesting too.