Title: Superrotations and Flat Space Holography

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Abstract: <p>We consider implications of superrotations as an asymptotic symmetry of asymptotically flat spacetimes. Beginning with a review of the rich structure of interconnections between soft theorems, asymptotic symmetries, and   memory effects, we describe the superrotation iteration. The subleading soft graviton theorem can be cast as a Ward identity for this asymptotic symmetry in 4D, and also as one for the stress tensor of a putative CFT2. We detail the change of scattering basis motivated by this asymptotic symmetry and discuss recent progress.</p>

Superrotations and Flat Space Holography

SABRINA GONZALEZ PASTERSKI

Motivation

- What can IR physics teach us about gravitational scattering?

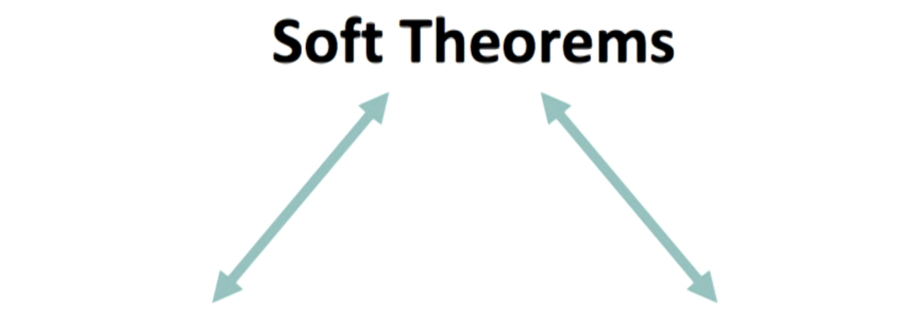
More Symmetries $\Rightarrow$ More Constraints

The **asymptotic symmetry group** of *asymptotically flat* spacetimes is much larger than Poincare

A Triangle of Relations

- What can IR physics teach us about gravitational scattering?

There exists a generic pattern of connections between asymptotic symmetries, soft theorems, and memory effects…

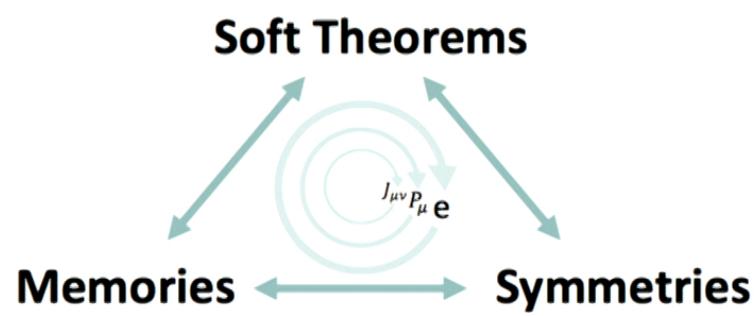


A Triangle of Relations

- What can IR physics teach us about gravitational scattering?

There exists a generic pattern of connections between asymptotic symmetries, soft theorems, and memory effects...

Such that by understanding simpler examples we can identify missing components of new iterations...

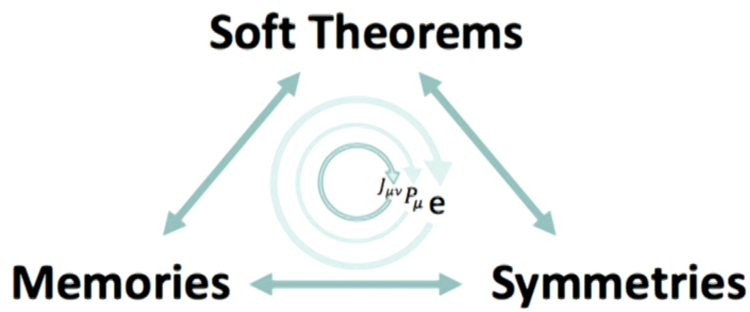


A Triangle of Relations

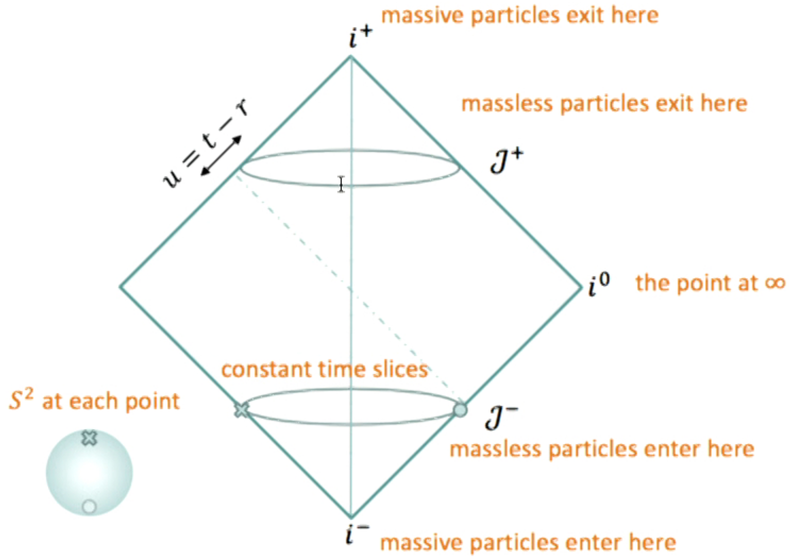
- What can IR physics teach us about gravitational scattering?

There exists a generic pattern of connections between asymptotic symmetries, soft theorems, and memory effects…

In this manner a brand new iteration was completed corresponding to ***superrotations***. This iteration is related to a generalization of Lorentz transformations and has motivated looking at ***S***-matrix elements in a new basis with definite SL(2,C) weights

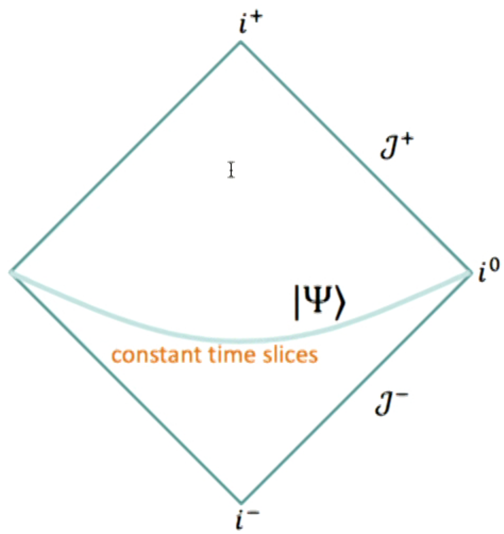


A Simple Example

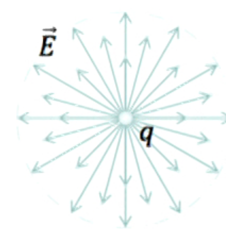


- Consider the conformal compactification of Minkowski spacetime

A Simple Example

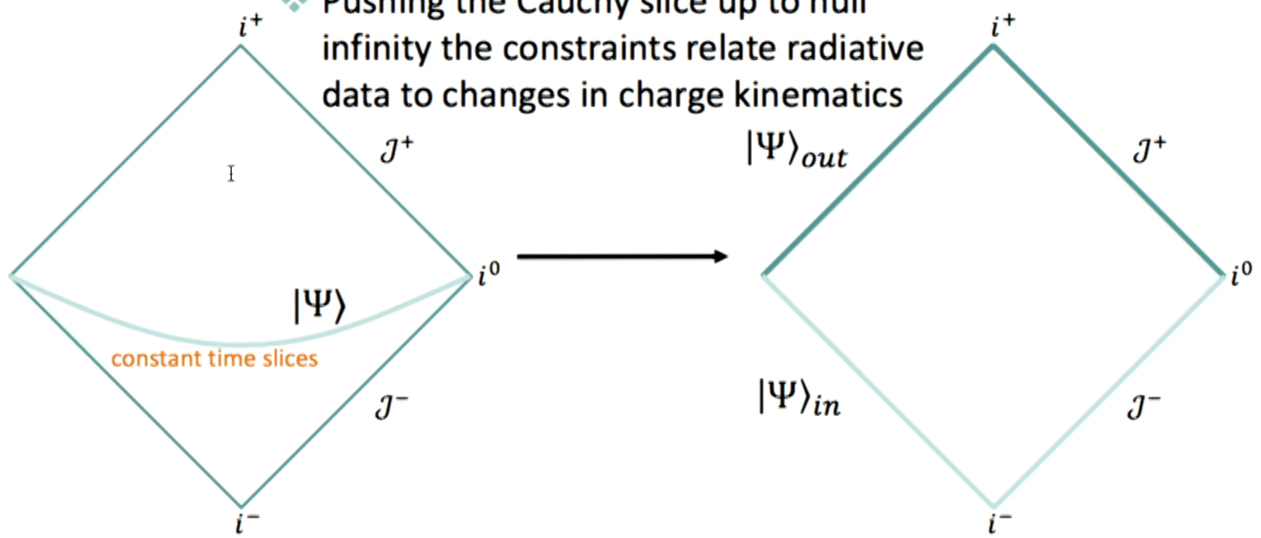


- In gauge theories there are constraints that need to be satisfied for the initial data on a Cauchy slice



A Simple Example

- Pushing the Cauchy slice up to null infinity the constraints relate radiative data to changes in charge kinematics



A Simple Example

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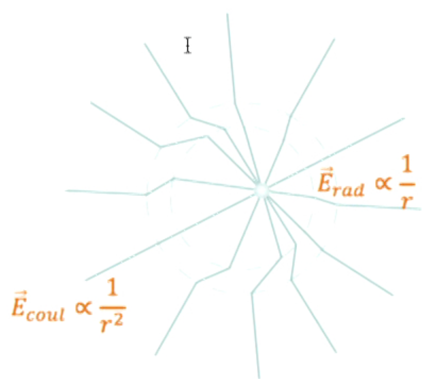
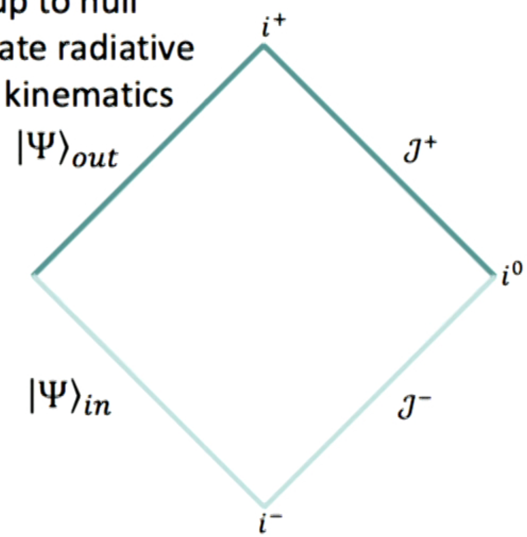




A Simple Example

Some more details:

- Radial Expansion:

$$\mathcal{A}_z(r,u,z,\bar{z}) = A_z(u,z,\bar{z}) + \sum_{n=1}^{\infty} \frac{A_z^{(n)}(u,z,\bar{z})}{r^n}$$

$$\mathcal{A}_u(r,u,z,\bar{z}) = \frac{1}{r}A_u(u,z,\bar{z}) + \sum_{n=1}^{\infty} \frac{A_u^{(n)}(u,z,\bar{z})}{r^{n+1}}$$

$$F_{ur} = A_u$$
$$F_{z\bar{z}} = \partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z$$
$$F_{uz} = \partial_u A_z$$

- ASG that preserves this expansion:

$$\delta_\epsilon A_z(u,z,\bar{z}) = \partial_z \epsilon(z,\bar{z})$$

- Mode Expansion:

$$\mathcal{A}_\mu(x) = e\sum_{\alpha=\pm}\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}\left[\epsilon_\mu^{\alpha^*}(\vec{q})a_\alpha(\vec{q})e^{iq\cdot x} + \epsilon_\mu^{\alpha}(\vec{q})a_\alpha(\vec{q})^\dagger e^{-iq\cdot x}\right]$$

- Constraint Equation:

[arXiv:1407.3789]

$$\partial_u A_u = \partial_u(D^z A_z + D^{\bar{z}} A_{\bar{z}}) + e^2 j_u$$

Coordinate Conventions:

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

$$z = e^{i\phi}\tan\frac{\theta}{2} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

A Simple Example

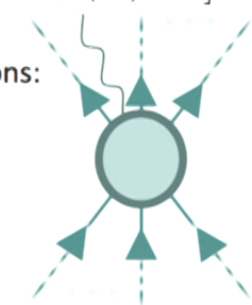
Two key points:

- Saddle point at large r picks out a gauge boson momentum pointing in the same direction as where an observer near null infinity would detect it. As a result, one ends up with a mode expansion where the angular integral localizes, and (u, ω) remain as Fourier conjugates.

$$e^{iq\cdot x} = e^{-i\omega u - i\omega r(1-\hat{q}\cdot\hat{x})} \quad \Rightarrow \quad A_z(u,z,\bar{z}) = -\frac{i}{8\pi^2}\frac{\sqrt{2}e}{1+z\bar{z}}\int_0^\infty d\omega \left[a_+(\omega\hat{x})e^{-i\omega u} - a_-(\omega\hat{x})^\dagger e^{i\omega u}\right]$$

- $\int du$ picks out $\omega \to 0$. As such we can relate the soft factors to the constraint equations: Fourier transform of a pole $\frac{1}{\omega}$ is a step function

$$S^{(0)-} = \sum_k eQ_k \frac{p_k \cdot \epsilon^-}{p_k \cdot q}$$



$$\langle z_{n+1}, z_{n+2}, \ldots | a_-(q)\mathcal{S}|z_1, z_2, \ldots\rangle = S^{(0)-}\langle z_{n+1}, z_{n+2}, \ldots|\mathcal{S}|z_1, z_2, \ldots\rangle + \mathcal{O}(1)$$

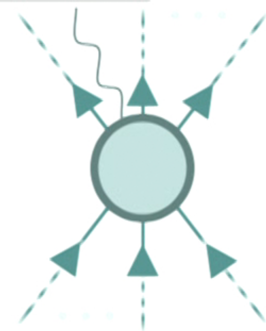
[arXiv:1407.3789, arXiv:1505.00716]

A Simple Example

Integrate the constraint equation along u

$$E_r = \frac{Q}{4\pi r^2}\frac{1}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^2}$$

$$\Delta A_u = 2D^z \Delta A_z + e^2 \int du j_u$$



$$-\frac{e}{4\pi}\lim_{\omega\to 0}\omega[D^z\hat{\epsilon}_z^{*+}S_p^{(0)+} + D^{\bar{z}}\hat{\epsilon}_{\bar{z}}^{*-}S_p^{(0)-}] \quad = -e^2\frac{Q}{4\pi}\frac{1}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^2}$$

The soft factor indicates that typical scattering processes will produce a nonzero u integrated electric field.

Some Conventions:

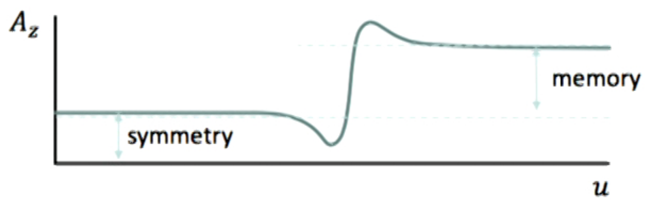
$$p^\mu = m\gamma(1,\vec{\beta}) \qquad S_p^{(0)\pm} = eQ\frac{p\cdot\epsilon^\pm}{p\cdot q}$$

$$\Delta A_z = -\frac{e}{4\pi}\hat{\epsilon}_z^{*+}\omega S^{(0)+}$$

[arXiv:1505.00716]

A Simple Example

- Upshot: The residue of the Weinberg pole indicates a nonzero value for certain low-energy radiation observables aka **"memory effects"**
- Since setting these modes to zero would trivialize the allowed scattering events, we get with this class of boundary conditions a larger class of gauge transformations that preserve the radial order of the falloffs while shifting the boundary values aka **"large gauge transformations"**



A Triangle of Relations

relate $\mathcal{S}$-matrix elements for states with and without extra soft gauge particle

Soft Theorems

Fourier transform:
long time ↔ low energy

Ward identities
[arXiv:1308.0589 arXiv:1312.2229]

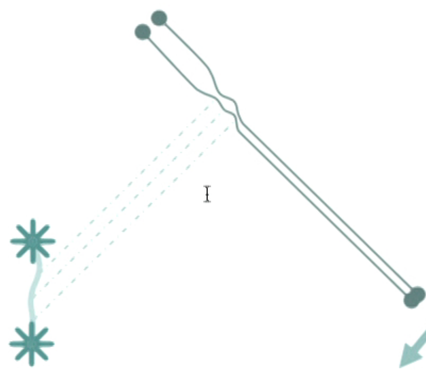
Memories ↔ **Symmetries**

non-zero net effects in a typical scattering process forces us to have asymptotic behavior that allows them, these extra symmetries then act non-trivially

step (net change)
vs baseline (starting point)

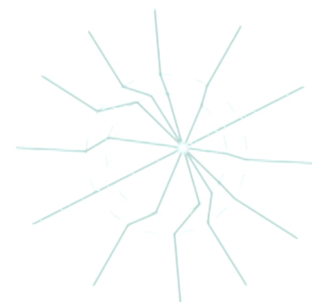
what we're after:
More Symmetries ⇒ More Constraints on $\mathcal{S}$-matrix

A Triangle of Relations



i) Weinberg – photon $O(\frac{1}{\omega})$
ii) Weinberg – graviton $O(\frac{1}{\omega})$
iii) Cachazo & Strominger – graviton O(1)

Soft Theorems



Memories ⟷ **Symmetries**

i) Liénard-Wiechert / Bieri & Garfinkle
ii) Zeldovich & Polnarev / Christodoulou
iii) Pasterski, Strominger, & Zhiboedov

(global)	(asymptotic)
i) e-charge	large U(1)
ii) P_μ	supertranslations
iii) $J_{\mu\nu}$	superrotations

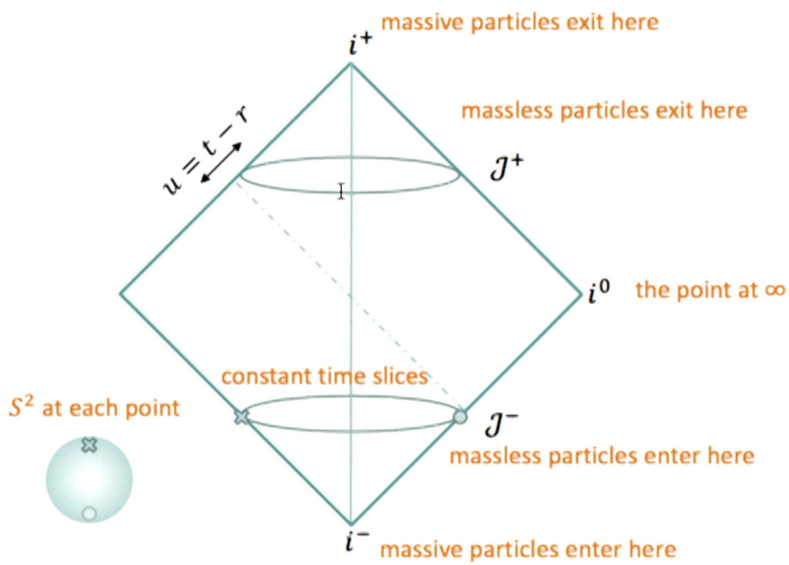
Asymptotically Flat Spacetimes

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

BMS 1960's

- Want to consider non-trivial gravitational backgrounds that are ``close” to being flat
  - Approach flat spacetime far away from sources

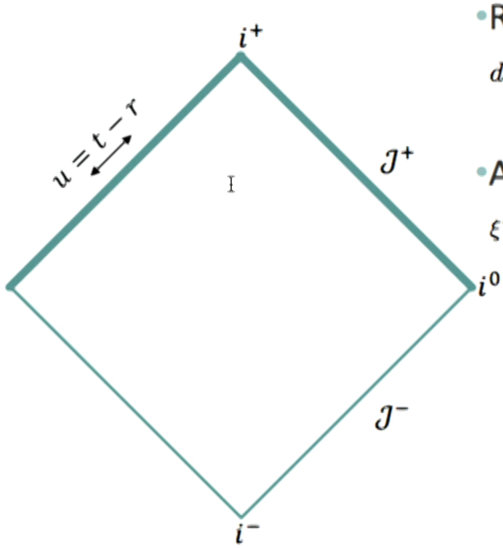
Asymptotically Flat Spacetimes



- Interested in set of diffeomorphisms that preserve class of asymptotically flat metrics, characterized by radial fall-off near null infinity

- $\text{ASG} = \dfrac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$

Asymptotically Flat Spacetimes



- **Radial Expansion:**

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\tfrac{m_B}{r}du^2 \\ &+\big(rC_{zz}dz^2 + D^zC_{zz}dudz + \tfrac{1}{r}(\tfrac{4}{3}N_z - \tfrac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + \text{c.c.}\big) + \ldots \end{aligned}$$

- **ASG that preserves this expansion:**

$$\begin{aligned} \xi^+ =&(1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c \\ &+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r \end{aligned}$$

Coordinate Conventions:

$$z = e^{i\phi}\tan\frac{\theta}{2} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$f^+ = f^+(z,\bar{z}) \qquad \partial_{\bar{z}}Y^{+z} = 0$$

Superrotation Charge

- We can demonstrate a semiclassical Ward identity for superrotations using the subleading soft graviton theorem [arXiv:1406.3312].

$$< out|Q^+[Y]\mathcal{S} - \mathcal{S}Q^-[Y]|in >= 0$$

$$8\pi G Q^+[Y] = \int du \int d^2z \sqrt{\gamma}\partial_u[-uY^A D_A m_B + Y^A N_A + ...]$$

$$\partial_u m_B = \frac{1}{4}\partial_u\left[D_z^2 C^{zz} + D_{\bar{z}}^2 C^{\bar{z}\bar{z}}\right] - T_{uu}$$

$$\partial_u N_z = \frac{1}{4}\partial_z\left[D_z^2 C^{zz} - D_{\bar{z}}^2 C^{\bar{z}\bar{z}}\right] + \partial_z m_B - T_{uz}$$

Superrotation Charge

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$$Q^+[Y] = Q^+_S[Y] + Q^+_H[Y]$$

$$Q^+_S[Y] = \frac{1}{2}\int_{\mathcal{I}^+} du d^2 z D^3_z Y^z u \partial_u C^z_{\bar{z}} \qquad Q^+_H[Y] = \lim_{\Sigma \to \mathcal{I}^+} \int_\Sigma d\Sigma \, \xi^\mu n^\nu_\Sigma T^M_{\mu\nu}$$

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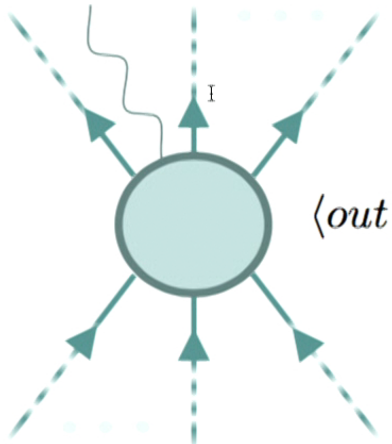

as Fourier mode of field operator

$$e^{iq\cdot x} = e^{-i\omega u - i\omega r(1-\hat{q}\cdot\hat{x})}$$

$\hat{q}_{pos} \Leftrightarrow \hat{q}_{mom}$ & $\lim_{\omega\to 0} \Leftrightarrow \int du$

$$\langle out|a_-(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

$$S^{(0)-} = \sum_k \frac{(p_k\cdot\epsilon^-)^2}{p_k\cdot q} \qquad S^{(1)-} = -i\sum_k \frac{p_{k\mu}\epsilon^{-\mu\nu}q^\lambda J_{k\lambda\nu}}{p_k\cdot q}$$

Superrotation Charge

- Looking again at the superrotation vector field near null infinity, we notice we have two copies of the Witt algebra:

$$\xi^+|_{\mathcal{J}^+} = Y^{+z}\partial_z + \frac{u}{2}D_z Y^{+z}\partial_u + c.c.$$

- Moreover, for a particular choice of $Y^z \sim \frac{1}{z-w}$ we find that the soft part of the charge takes the form of a putative 2D stress tensor [arXiv:1609.00282] .

$$T_{zz} \equiv \frac{i}{8\pi G}\int d^2w \frac{1}{z-w} D_w^2 D^{\bar{w}} \int du u \partial_u C_{\bar{w}\bar{w}}$$

$$\langle T_{zz}\mathcal{O}_1 \cdots \mathcal{O}_n\rangle = \sum_{k=1}^{n}\left[\frac{h_k}{(z-z_k)^2} + \frac{\Gamma^{z_k}_{z_k z_k}}{z-z_k}h_k + \frac{1}{z-z_k}\left(\partial_{z_k} - |s_k|\Omega_{z_k}\right)\right]\langle\mathcal{O}_1\cdots\mathcal{O}_n\rangle$$

Weight Conventions:

$$h = \frac{1}{2}(s+1+iE_R) \quad \bar{h} = \frac{1}{2}(-s+1+iE_R)$$

$$\Delta = h + \bar{h} \quad s = h - \bar{h}$$

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We need Rindler energy eigenstates!

Constructing A Conformal-Primary Basis

- Using that the Lorentz group SO(1,d+1) in $\boldsymbol{R}^{1,d+1}$ acts as the conformal group on $\boldsymbol{R}^{d}$ define the *massive scalar conformal primary wavefunction* to:

  - satisfy the (d+2)-dimensional massive Klein-Gordon equation of mass *m*:

$$\left(\frac{\partial}{\partial X^{\nu}}\frac{\partial}{\partial X_{\nu}} - m^2\right)\phi_{\Delta}(X^{\mu};\vec{w}) = 0$$

  - transform covariantly as a scalar conformal primary operator in d dimensions under an SO(1,d+1) transformation:

$$\phi_{\Delta}\left(\Lambda^{\mu}{}_{\nu}X^{\nu};\vec{w}\,'(\vec{w})\right) = \left|\frac{\partial \vec{w}\,'}{\partial \vec{w}}\right|^{-\Delta/d}\phi_{\Delta}(X^{\mu};\vec{w})$$

[arXiv:1705.01027]

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[arXiv:1705.01027]

Constructing A Conformal-Primary Basis

$$ds^2_{H_{d+1}} = \frac{dy^2 + d\vec{z}\cdot d\vec{z}}{y^2}$$

$\hat{p}$

$m \neq 0$

$p = m\hat{p}$

$$\hat{p}(y,\vec{z}) = \left(\frac{1+y^2+|\vec{z}|^2}{2y}, \frac{\vec{z}}{y}, \frac{1-y^2-|\vec{z}|^2}{2y}\right)$$

$$\hat{p}^\mu(y',\vec{z}\,') = \Lambda^\mu{}_\nu \hat{p}^\nu$$

q

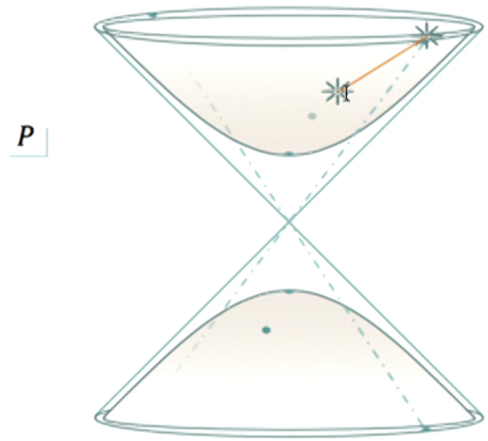
$\vec{w} \in \boldsymbol{R}^d$

$$q^\mu(\vec{w}) = \left(1+|\vec{w}|^2, 2\vec{w}, 1-|\vec{w}|^2\right)$$

$$q^\mu(\vec{w}\,') = \left|\frac{\partial \vec{w}\,'}{\partial \vec{w}}\right|^{1/d} \Lambda^\mu{}_\nu q^\nu(\vec{w})$$

Constructing A Conformal-Primary Basis

- The desired properties are met by the convolution:



$$\phi^{\pm}_{\Delta}(X^{\mu};\vec{w}) = \int_{H_{d+1}} [d\hat{p}]\, G_{\Delta}(\hat{p};\vec{w})\ \exp\left[\,\pm im\hat{p}\cdot X\,\right]$$

- Interpretation as bulk-to-boundary propagation in momentum space
- Have **plane wave** $\Rightarrow$ **highest-weight**, what about reverse?

Constructing A Conformal-Primary Basis

- The orthogonality conditions $$\int_{-\infty}^{\infty} d\nu\, \mu(\nu) \int d^d\vec{w}\, G_{\frac{d}{2}+i\nu}(\hat{p}_1;\vec{w}) G_{\frac{d}{2}-i\nu}(\hat{p}_2;\vec{w}) = \delta^{(d+1)}(\hat{p}_1,\hat{p}_2)$$

$$\int_{H_{d+1}} [d\hat{p}]\, G_{\frac{d}{2}+i\nu}(\hat{p};\vec{w}_1) G_{\frac{d}{2}+i\bar{\nu}}(\hat{p};\vec{w}_2) =$$

$$2\pi^{d+1}\frac{\Gamma(i\nu)\Gamma(-i\nu)}{\Gamma(\frac{d}{2}+i\nu)\Gamma(\frac{d}{2}-i\nu)}\delta(\nu+\bar{\nu})\delta^{(d)}(\vec{w}_1-\vec{w}_2) + 2\pi^{\frac{d}{2}+1}\frac{\Gamma(i\nu)}{\Gamma(\frac{d}{2}+i\nu)}\delta(\nu-\bar{\nu})\frac{1}{|\vec{w}_1-\vec{w}_2|^{2(\frac{d}{2}+i\nu)}}$$

$$\mu(\nu) = \frac{\Gamma(\frac{d}{2}+i\nu)\Gamma(\frac{d}{2}-i\nu)}{4\pi^{d+1}\Gamma(i\nu)\Gamma(-i\nu)}$$

[arXiv:1404.5625]

- Imply we can go in the opposite direction **highest-weight ⇒ plane wave**

$$e^{\pm im\hat{p}\cdot X} = 2\int_0^{\infty} d\nu\, \mu(\nu) \int d^d\vec{w}\; G_{\frac{d}{2}-i\nu}(\hat{p};\vec{w})\; \phi^{\pm}_{\frac{d}{2}+i\nu}(X^\mu;\vec{w})$$

$\Delta \in \frac{d}{2} + i\boldsymbol{R}_{\geq 0}$ ×

$\vec{w} \in \boldsymbol{R}^d$

Constructing A Conformal-Primary Basis

- By forming the combination $\omega = \frac{m}{2y}$ we can further use the boundary behavior of G_Δ to explore the massless analog:

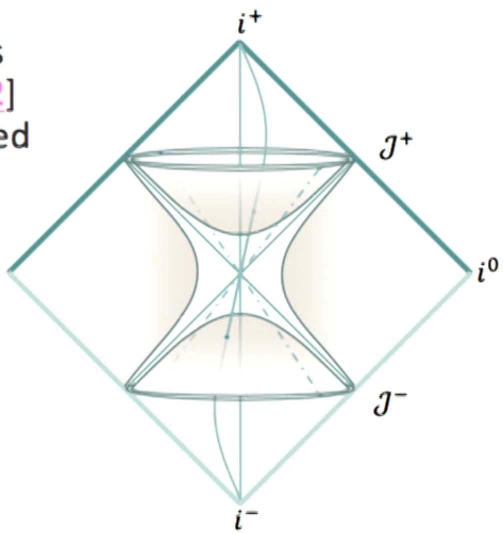
$$G_\Delta(y,\vec{z};\vec{w}) \underset{m\to 0}{\longrightarrow} \pi^{\frac{d}{2}} \frac{\Gamma(\Delta-\frac{d}{2})}{\Gamma(\Delta)} y^{d-\Delta}\delta^{(d)}(\vec{z}-\vec{w}) + \frac{y^\Delta}{|\vec{z}-\vec{w}|^{2\Delta}} + \cdots$$

- The first term results in a Mellin transform of the energy, in which the reference direction is the same as the momentum, and satisfies the desired properties of a massless conformal primary.

$$\boxed{\varphi^{\pm}_{\Delta}(X^\mu;\vec{w}) \equiv \int_0^\infty d\omega\, \omega^{\Delta-1}\, e^{\pm i\omega q\cdot X - \epsilon\omega} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q(\vec{w})\cdot X \mp i\epsilon)^\Delta}}$$

Amplitude Transforms

- Note that transforming momentum space amplitudes directly, is an alternative to previous approaches [hep-th/0303006,arXiv:1609.00732] towards flat space holography, which have looked at a foliation of Minkowski space to reproduce AdS/CFT, dS/CFT on each slice.



Constructing A Conformal-Primary Basis

- Photon

$$\left(\frac{\partial}{\partial X^\sigma}\frac{\partial}{\partial X_\sigma}\delta^\mu_\nu - \frac{\partial}{\partial X^\nu}\frac{\partial}{\partial X_\mu}\right) A^{\Delta\pm}_{\mu a}(X^\rho;\vec{w}) = 0$$

$$A^{\Delta\pm}_{\mu a}\left(\Lambda^\rho{}_\nu X^\nu; \vec{w}'(\vec{w})\right) = \frac{\partial w^b}{\partial w'^a}\left|\frac{\partial \vec{w}'}{\partial \vec{w}}\right|^{-(\Delta-1)/d}\Lambda_\mu{}^\sigma A^{\Delta\pm}_{\sigma b}(X^\rho;\vec{w})$$

- Graviton

$$\partial_\sigma\partial_\nu h^\sigma{}_{\mu;a_1a_2} + \partial_\sigma\partial_\mu h^\sigma{}_{\nu;a_1a_2} - \partial_\mu\partial_\nu h^\sigma{}_{\sigma;a_1a_2} - \partial^\rho\partial_\rho h_{\mu\nu;a_1a_2} = 0$$

$$h^{\Delta,\pm}_{\mu_1\mu_2;a_1a_2} = h^{\Delta,\pm}_{\mu_2\mu_1;a_1a_2}\,,\quad h^{\Delta,\pm}_{\mu_1\mu_2;a_1a_2} = h^{\Delta,\pm}_{\mu_1\mu_2;a_2a_1}\,,\quad \delta^{a_1a_2}h^{\Delta,\pm}_{\mu_1\mu_2;a_1a_2} = 0$$

$$h^{\Delta,\pm}_{\mu_1\mu_2;a_1a_2}\left(\Lambda^\rho{}_\nu X^\nu;\vec{w}'(\vec{w})\right) = \frac{\partial w^{b_1}}{\partial w'^{a_1}}\frac{\partial w^{b_2}}{\partial w'^{a_2}}\left|\frac{\partial \vec{w}'}{\partial \vec{w}}\right|^{-(\Delta-2)/d}\Lambda_{\mu_1}{}^{\sigma_1}\Lambda_{\mu_2}{}^{\sigma_2}h^{\Delta,\pm}_{\sigma_1\sigma_2;b_1b_2}(X^\rho;\vec{w})$$

$\Delta \in \frac{d}{2} + iR$

$\vec{w} \in R^d$

Constructing A Conformal-Primary Basis

- Photon

$$\left(\frac{\partial}{\partial X^\sigma}\frac{\partial}{\partial X_\sigma}\delta^\mu_\nu - \frac{\partial}{\partial X^\nu}\frac{\partial}{\partial X_\mu}\right) A^{\Delta\pm}_{\mu a}(X^\rho;\vec{w}) = 0 \qquad A^{\Delta\pm}_{\mu a}\left(\Lambda^\rho{}_\nu X^\nu;\vec{w}'(\vec{w})\right) = \frac{\partial w^b}{\partial w'^a}\left|\frac{\partial \vec{w}'}{\partial \vec{w}}\right|^{-(\Delta-1)/d}\Lambda_\mu{}^\sigma A^{\Delta\pm}_{\sigma b}(X^\rho;\vec{w})$$

- Graviton

$$\partial_\sigma\partial_\nu h^\sigma{}_{\mu;a_1a_2} + \partial_\sigma\partial_\mu h^\sigma{}_{\nu;a_1a_2} - \partial_\mu\partial_\nu h^\sigma{}_{\sigma;a_1a_2} - \partial^\rho\partial_\rho h_{\mu\nu;a_1a_2} = 0$$

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$$h^{\Delta,\pm}_{\mu_1\mu_2;a_1a_2}\left(\Lambda^\rho{}_\nu X^\nu;\vec{w}'(\vec{w})\right) = \frac{\partial w^{b_1}}{\partial w'^{a_1}}\frac{\partial w^{b_2}}{\partial w'^{a_2}}\left|\frac{\partial \vec{w}'}{\partial \vec{w}}\right|^{-(\Delta-2)/d}\Lambda_{\mu_1}{}^{\sigma_1}\Lambda_{\mu_2}{}^{\sigma_2}h^{\Delta,\pm}_{\sigma_1\sigma_2;b_1b_2}(X^\rho;\vec{w})$$

$\Delta \in \frac{d}{2} + i\mathbb{R}$ $\times$ $\vec{w} \in \mathbb{R}^d$

Constructing A Conformal-Primary Basis

- The shadow is linearly independent.
- Demanding conformal profile fixes residual gauge transformations but within gauge equivalence class can return to Mellin representative.

$$A^{\Delta,\pm}_{\mu a}(X^\mu;\vec{w}) = \frac{\partial_a q_\mu}{(-q\cdot X \mp i\epsilon)^\Delta} + \frac{\partial_a q\cdot X}{(-q\cdot X \mp i\epsilon)^{\Delta+1}} q_\mu$$

$$-\,const.\ \frac{\partial}{\partial X^\mu}\left(\frac{\partial_a q\cdot X}{(-q\cdot X \mp i\epsilon)^\Delta}\right)$$

Constructing A Conformal-Primary Basis

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Constructing A Conformal-Primary Basis

- The orthogonality conditions $$\int_{-\infty}^{\infty} d\nu\, \mu(\nu) \int d^d\vec{w}\, G_{\frac{d}{2}+i\nu}(\hat{p}_1;\vec{w}) G_{\frac{d}{2}-i\nu}(\hat{p}_2;\vec{w}) = \delta^{(d+1)}(\hat{p}_1,\hat{p}_2)$$

$$\int_{H_{d+1}} [d\hat{p}]\, G_{\frac{d}{2}+i\nu}(\hat{p};\vec{w}_1) G_{\frac{d}{2}+i\bar{\nu}}(\hat{p};\vec{w}_2) =$$

$$\mu(\nu) = \frac{\Gamma(\frac{d}{2}+i\nu)\Gamma(\frac{d}{2}-i\nu)}{4\pi^{d+1}\Gamma(i\nu)\Gamma(-i\nu)}$$

$$2\pi^{d+1}\frac{\Gamma(i\nu)\Gamma(-i\nu)}{\Gamma(\frac{d}{2}+i\nu)\Gamma(\frac{d}{2}-i\nu)}\delta(\nu+\bar{\nu})\delta^{(d)}(\vec{w}_1-\vec{w}_2) + 2\pi^{\frac{d}{2}+1}\frac{\Gamma(i\nu)}{\Gamma(\frac{d}{2}+i\nu)}\delta(\nu-\bar{\nu})\frac{1}{|\vec{w}_1-\vec{w}_2|^{2(\frac{d}{2}+i\nu)}}$$

[arXiv:1404.5625]

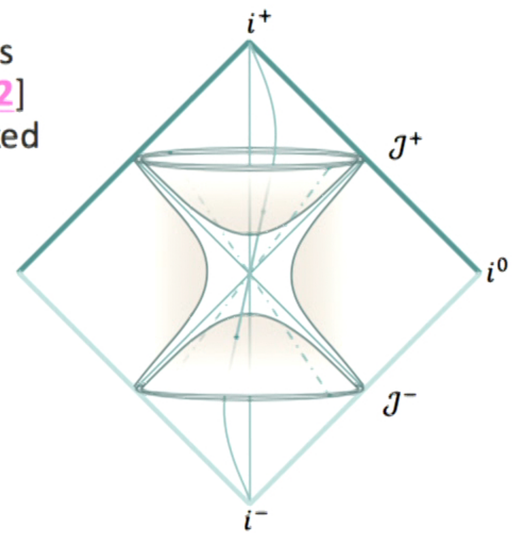
- Imply we can go in the opposite direction **highest-weight ⇒ plane wave**

$$e^{\pm i m \hat{p}\cdot X} = 2\int_0^{\infty} d\nu\, \mu(\nu) \int d^d\vec{w}\; G_{\frac{d}{2}-i\nu}(\hat{p};\vec{w})\; \phi^{\pm}_{\frac{d}{2}+i\nu}(X^\mu;\vec{w})$$

$\Delta \in \frac{d}{2} + i\mathbf{R}_{\geq 0}$ × $\vec{w} \in \mathbf{R}^d$

Amplitude Transforms

- Note that transforming momentum space amplitudes directly, is an alternative to previous approaches [hep-th/0303006,arXiv:1609.00732] towards flat space holography, which have looked at a foliation of Minkowski space to reproduce AdS/CFT, dS/CFT on each slice.



What Needs To Be Done

- The current map interpreting S-matrix elements as 2D CFT correlators seems to imply either an exotic CFT 2 or that the map needs to be finessed… **Options?**

  - *Is there a better shadow-related basis?*

$$\mathcal{O}^{+}_{i\lambda}(w,\bar{w}) = \phi^{+}_{i\lambda}(w,\bar{w}) + C_{+,\lambda}\int d^2z \frac{1}{(z-w)^{2+i\lambda}(\bar{z}-\bar{w})^{i\lambda}}\phi^{-}_{-i\lambda}(z,\bar{z})$$
$$\mathcal{O}^{-}_{i\lambda}(w,\bar{w}) = \phi^{-}_{i\lambda}(w,\bar{w}) + C_{-,\lambda}\int d^2z \frac{1}{(z-w)^{i\lambda}(\bar{z}-\bar{w})^{2+i\lambda}}\phi^{+}_{-i\lambda}(z,\bar{z})$$

  - The mode combination that decouples in the soft limit is precisely a linear combination of Mellin and Mellin+shadow in the limit where $\mathrm{Im}\,\Delta = 0$:

$$\mathbf{a}_{-} \equiv a_{-}(\omega\hat{x}) - \frac{1}{2\pi}\int d^2w \frac{1}{\bar{z}-\bar{w}}\partial_{\bar{w}} a_{+}(\omega\hat{y})$$

Understand the conformally soft limit!

What Has Been Done

- Beautiful expressions for full mellin transform (which inherently probe UV structure) of string amplitudes [arXiv:1806.05688]
- Systematic n-pt N^kMHV [arXiv:1711.08435]
- 3D example of CB decomposition [arXiv:1711.06138]
- Interesting statments about symplectic pairing of conformally soft modes [arXiv:1810.05219]