Title: Solvable models of correlated metals with interactions and disorder, and their transport properties

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Abstract: <p>Despite much theoretical effort, there is no complete theory of the â€œstrangeâ€• metal phase of the high temperature </p>

<p>superconductors, and its linear-in-temperature resistivity. This phase is believed to be a strongly-interacting metallic </p>

<p>phase of matter without fermionic quasiparticles, and is virtually impossible to model accurately using traditional</p>

<p>perturbative field-theoretic techniques. Recently, progress has been made using large-N techniques based on the </p>

<p>solvable Sachdev-Ye-Kitaev (SYK) model, which do not involve expanding about any weakly-coupled limit. I will </p>

<p>describe constructions of solvable models of strange metals based on SYK-like large-N limits, which can reproduce </p>

<p>some of the experimentally observed features of strange metals and adjoining phases. These models, and further </p>

<p>extensions, could possibly pave the way to developing a controlled theoretical understanding of the essential building </p>

<p>blocks of the electronic state in correlated-electron superconductors near optimal doping.</p>

Solvable models of metals with interactions and disorder, and their transport properties

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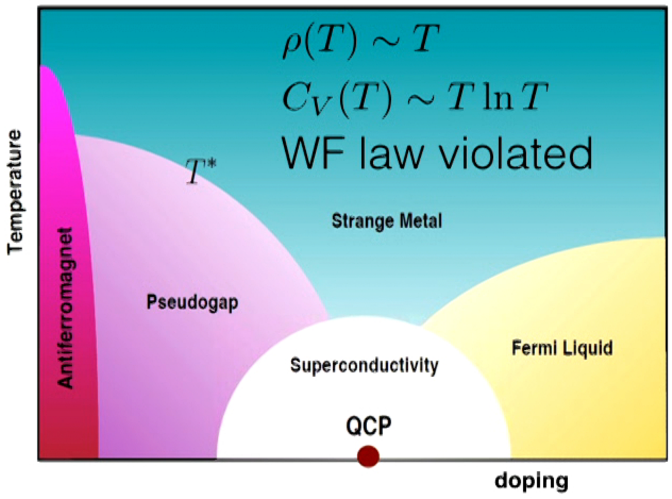
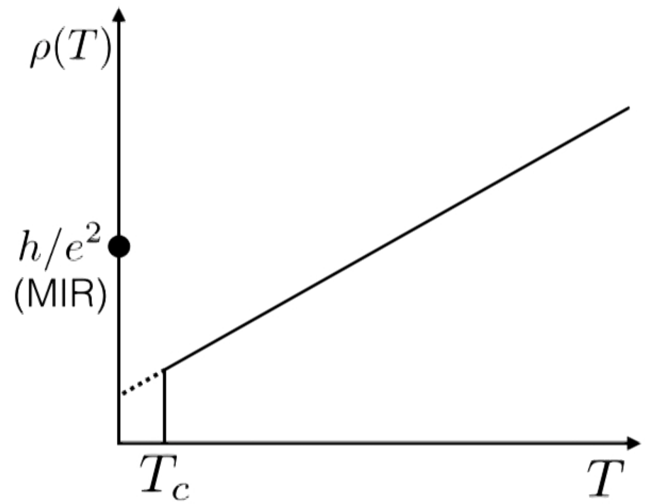
10/16/2018

A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev,
Phys. Rev. X 8, 021049 (2018)

A. A. Patel and S. Sachdev,
Phys. Rev. B 98, 125134 (2018)

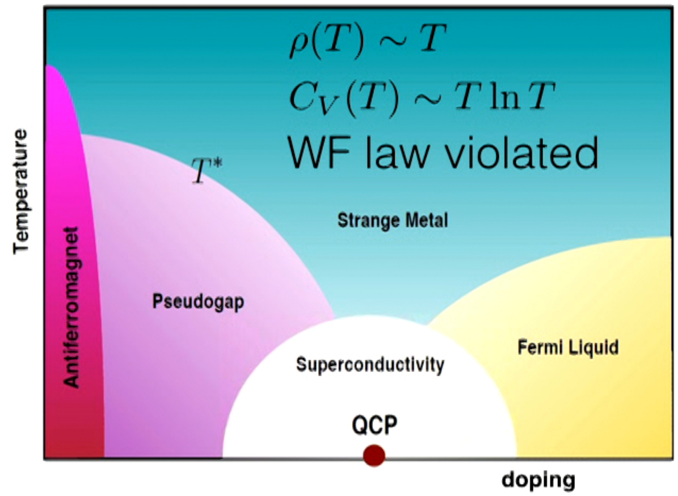
Strange metals





- Resistivity is linear over a large range of temperatures.
- Exceeds the Mott-Ioffe-Regel "limit" at high enough T, where the semiclassical mean-free-path becomes smaller than a lattice spacing, no signs of saturation even at highest possible T.

Strange metals: Theoretically challenging

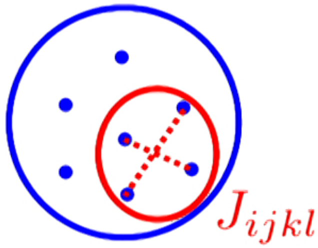


- The strange metal is not a weakly interacting Landau Fermi liquid.
- Metals with strong interactions in 2+1 dimensions and a finite density of electron states are usually not amenable to controlled field-theoretic calculations, due to the large number of gapless modes on a Fermi surface.
- Even if controlled, a continuum field-theoretic description is not good enough for transport, need to consider how momentum is relaxed as well.
- Field-theoretic situation has so far been nearly hopeless when disorder is added on top of the mess.

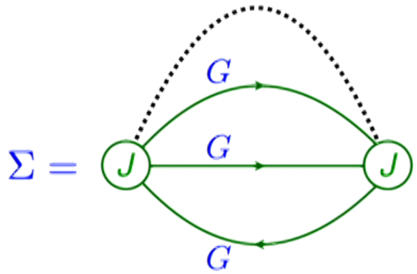
SYK Model: Solvable Non-Fermi liquid at a point

$$H = \sum_{i,j,k,l=1}^{N\to\infty} J_{ijkl} f_i^\dagger f_j^\dagger f_k f_l, \quad \{f_i^\dagger, f_j\} = \delta_{ij}$$

$$\ll J_{ijkl} \gg = 0, \quad \ll |J_{ijkl}|^2 \gg = J^2/(8N^3)$$



- Consists of large-*N* number of sites on a single "quantum dot", with random all-to-all interactions.
- The hamiltonian has no quadratic kinetic terms.
- The randomness self-averages in the large-*N* limit, leading to a gapless non-Fermi liquid ground state.

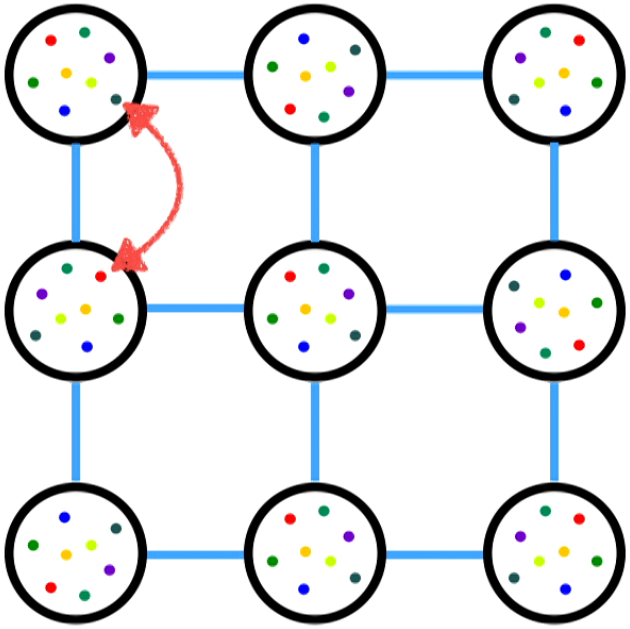


$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau),$$

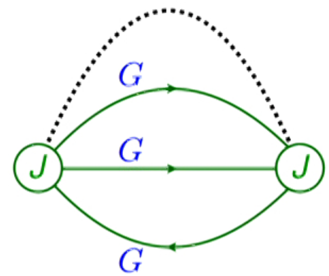
$$G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)}.$$

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
S. Sachdev, PRX 041025 (2015)
A. Kitaev, Unpublished

Lattice of SYK Models

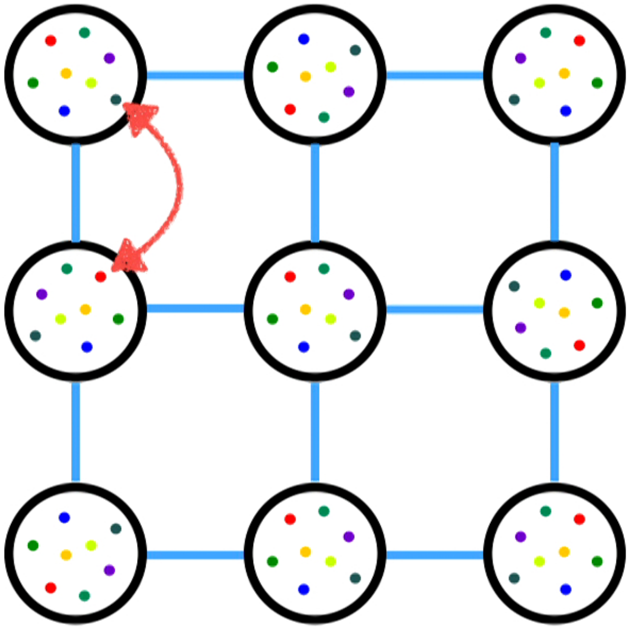


$$H = -t \sum_{\langle xy \rangle, i=1}^{N\to\infty} (c_{ix}^\dagger c_{iy} + \text{h.c.})$$
$$+ \sum_{x, ijkl=1}^{N\to\infty} J_{ijkl}^x c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx}$$

$\Sigma =$ 

X.-Y. Song, C.-M. Jian and L. Balents, PRL 119, 216601 (2017)
P. Zhang, PRB 96, 205138 (2017)
D. Chowdhury, Y. Werman, E. Berg, and T. Senthil, PRX 8, 031024 (2018)

Lattice of SYK Models



$$t \ll J, \quad T \ll t^2/J$$

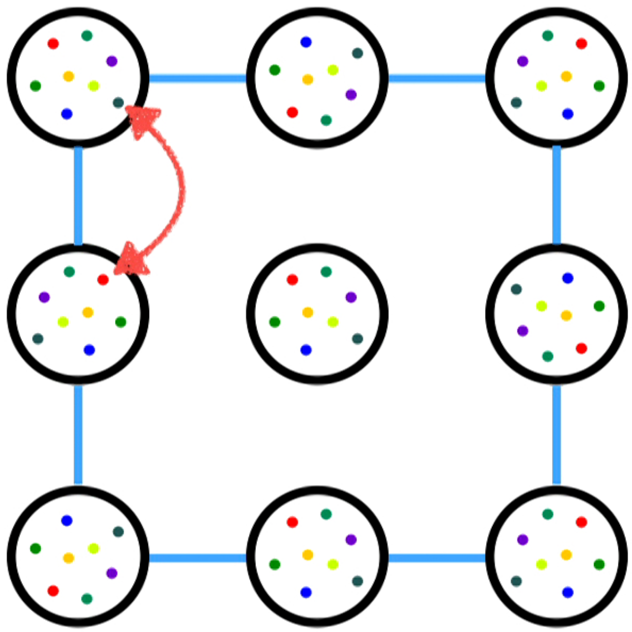
SYK interaction is irrelevant in the low-energy limit. Resistance comes from weak inelastic, momentum non-conserving scattering of plane-wave states.

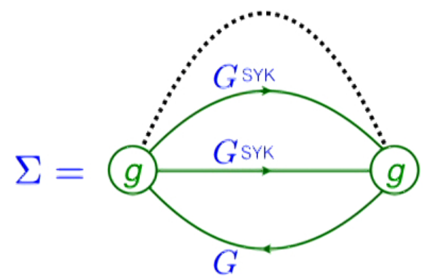
$$\rho(T) \sim T^2 \qquad L = L_0$$

Renormalized Fermi liquid at low temperatures, with $Z < 1$.

X.-Y. Song, C.-M. Jian and L. Balents, PRL 119, 216601 (2017)
P. Zhang, PRB 96, 205138 (2017)
D. Chowdhury, Y. Werman, E. Berg, and T. Senthil, PRX 8, 031024 (2018)

Lattice of SYK Models: Low temperature linear-in-T





$$G_{\text{SYK}} \sim \text{sgn}(\omega_n)|\omega_n|^{-1/2}, \quad \int_k G \sim \text{sgn}(\omega),$$

$$\Sigma(\omega_n) - \Sigma(0) \sim \omega_n \ln|\omega_n|$$

$$\text{Kubo formula} \to \rho(T) \sim (\text{const.} + (g^2 T/(t^2 J)))(h/e^2)$$

$$L < L_0$$

"Marginal Fermi liquid" (MFL) with momentum dissipation. $C_V \sim T \ln T$.

Resistivity is linear-in-T (plus constant residual resistivity), and smaller than the MIR value. Conduction occurs due to non-quasiparticle approximately plane wave states. A true non-Fermi liquid.

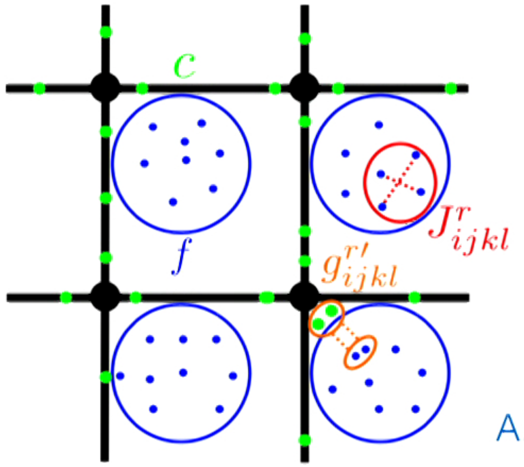
A. A. Patel et. al., Unpublished

Lattice of SYK Models: Low temperature linear-in-T

- Idealization where the logic on the previous slide works exactly, without any residual resistivity at $T = 0$. No weak localization at large-N.

$$H = -t \sum_{\langle xy \rangle, i=1}^{M\to\infty} (c^\dagger_{xi} c_{yi} + \text{h.c.}) + \frac{1}{NM^{1/2}} \sum_{x,ij=1}^{N\to\infty} \sum_{kl=1}^{M\to\infty} g^x_{ijkl} f^\dagger_{xi} f_{xj} c^\dagger_{xk} c_{xl}$$

$$+ \frac{1}{N^{3/2}} \sum_{x,ijkl=1}^{N\to\infty} J^x_{ijkl} f^\dagger_{xi} f^\dagger_{xj} f_{xk} f_{xl}.$$

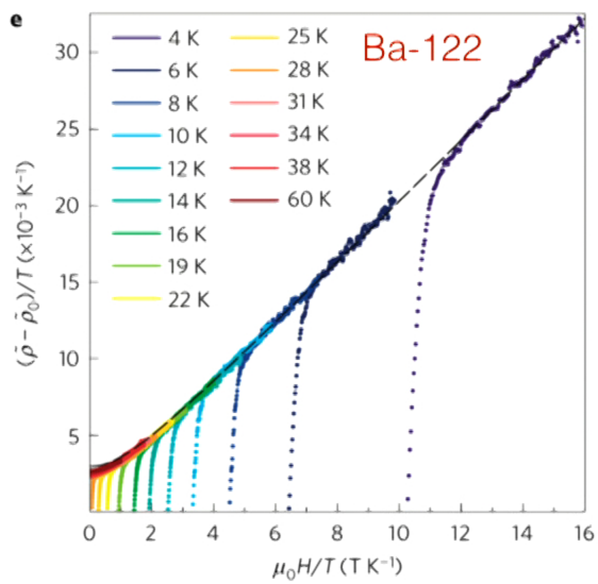


Locally and randomly couple a sea of itinerant electrons to a lattice of SYK “islands”. Realizes MFL of itinerant electrons.

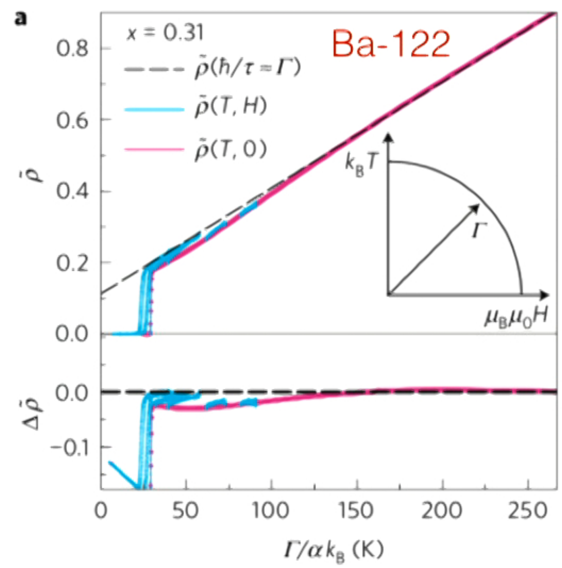
A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, **Phys. Rev. X 8, 021049 (2018)**

Strange metals just got stranger…

B-linear transverse magnetoresistance *and* scaling between B and T!?



I. M. Hayes et. al., Nat. Phys. 12, 916 (2016)



$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

I. M. Hayes et. al., Nat. Phys. 12, 916 (2016)

Magnetotransport: Marginal-Fermi liquid

- Thanks to large *N*,*M*, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields in the "idealized" MFL model…

$$(1-\partial_\omega \mathrm{Re}[\Sigma^c_R(\omega)])\partial_t \delta n(t,k,\omega) + v_F \hat{k}\cdot \mathbf{E}(t)\; n_f'(\omega) + v_F(\hat{k}\times \mathcal{B}\hat{z})\cdot \nabla_k \delta n(t,k,\omega) = 2\delta n(t,k,\omega)\mathrm{Im}[\Sigma^c_R(\omega)],$$

$(\mathcal{B} = eBa^2/\hbar)$ (i.e. flux per unit cell)

$$\sigma_L^{\mathrm{MFL}} = M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{-\mathrm{Im}[\Sigma^c_R(E_1)]}{\mathrm{Im}[\Sigma^c_R(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2},$$

$$\sigma_H^{\mathrm{MFL}} = -M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{(v_F/(2k_F))\mathcal{B}}{\mathrm{Im}[\Sigma^c_R(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2}.$$

$$\sigma_L^{\mathrm{MFL}} \sim T^{-1}s_L((v_F/k_F)(\mathcal{B}/T)),\quad \sigma_H^{\mathrm{MFL}} \sim -\mathcal{B}T^{-2}s_H((v_F/k_F)(\mathcal{B}/T)).$$

$$s_{L,H}(x\to\infty)\propto 1/x^2,\quad s_{L,H}(x\to 0)\propto x^0.$$

Scaling between magnetic field and temperature in **orbital** magnetotransport!

Macroscopic magnetotransport in the MFL

- Let us consider the MFL with additional **macroscopic** disorder (charge puddles etc.)

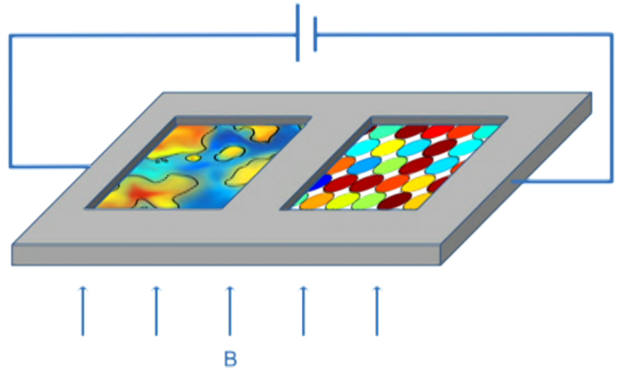


Figure: N. Ramakrishnan et. al., PRB 96, 224203 (2017)

- No macroscopic momentum, due to momentum relaxation at the microscopic level, so equations describing charge transport are just

$$\nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla\Phi(\mathbf{x}).$$

- Very weak thermoelectricity for large FS, so charge effectively decoupled from heat transport.

Random-resistor network: physical picture

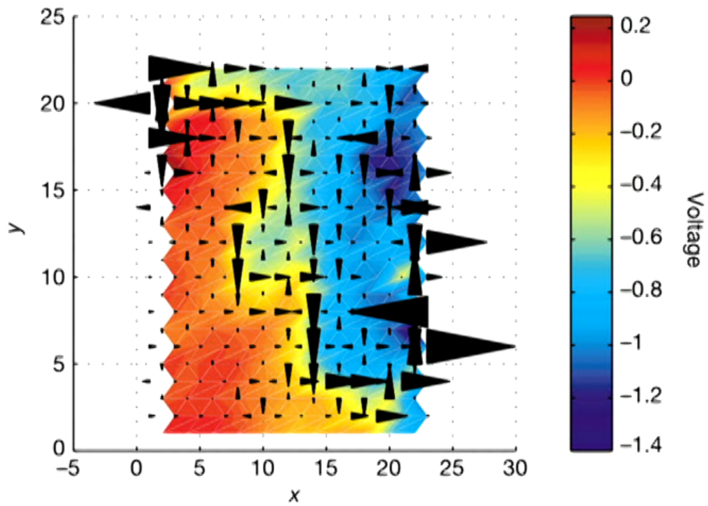
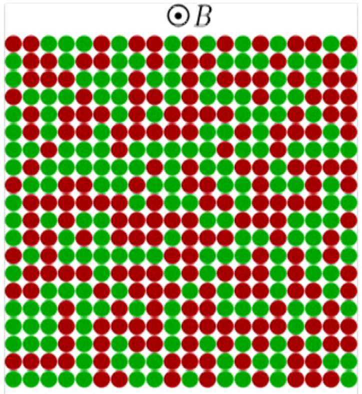


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in *H*.

- Due to disorder in the local conductivity tensor, the global Hall electric field does not cancel Hall current uniformly throughout the sample.
- Hence, fluctuations in the local Hall resistance lead to a distortion of the current path due to charge conservation, which contributes the local Hall resistance, which is linear in *B* to the global longitudinal resistance.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Solvable toy model: two types of resistors



- Two types of domains *a,b* with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly
(A. M. Dykhne, JETP 32, 348 (1971))
(V. Guttal and D. Stroud, PRB 71, 201304 (2005))

$$\left(\mathbb{I}+\frac{\sigma^a-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^a-\sigma^e)+\left(\mathbb{I}+\frac{\sigma^b-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^b-\sigma^e)=0.$$

$$\rho^e_L\equiv\frac{\sigma^e_L}{\sigma^{e2}_L+\sigma^{e2}_H}=\frac{\sqrt{(\mathcal{B}/m)^2\left(\gamma_a\sigma^{\mathrm{MFL}}_{0a}-\gamma_b\sigma^{\mathrm{MFL}}_{0b}\right)^2+\gamma_a^2\gamma_b^2\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)^2}}{\gamma_a\gamma_b(\sigma^{\mathrm{MFL}}_{0a}\sigma^{\mathrm{MFL}}_{0b})^{1/2}\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)},$$

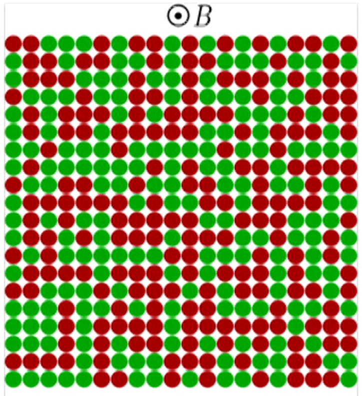
$$\rho^e_H\equiv-\frac{\sigma^e_H/\mathcal{B}}{\sigma^{e2}_L+\sigma^{e2}_H}=\frac{\gamma_a+\gamma_b}{m\gamma_a\gamma_b\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)}.\quad (m=k_F/v_F\sim 1/t)$$

$\gamma_{a,b}\sim T$ (i.e. effective transport scattering rates)

$$\rho^e_L\sim\sqrt{c_1T^2+c_2B^2}$$ (Hayes et. al.'s result!)

Scaling between *B* and *T* at microscopic orbital level has been transferred to global MR!

Solvable toy model: two types of resistors



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$$\rho_H^e \equiv -\frac{\sigma_H^e/\mathcal{B}}{\sigma_L^{e2}+\sigma_H^{e2}} = \frac{\gamma_a+\gamma_b}{m\gamma_a\gamma_b\left(\sigma_{0a}^{\mathrm{MFL}}+\sigma_{0b}^{\mathrm{MFL}}\right)}. \quad (m = k_F/v_F \sim 1/t)$$

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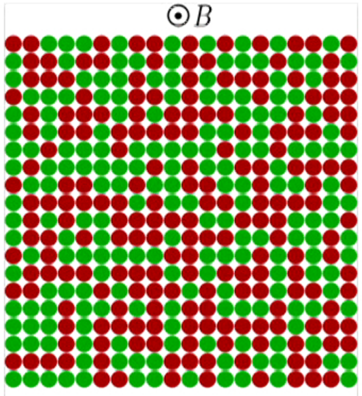

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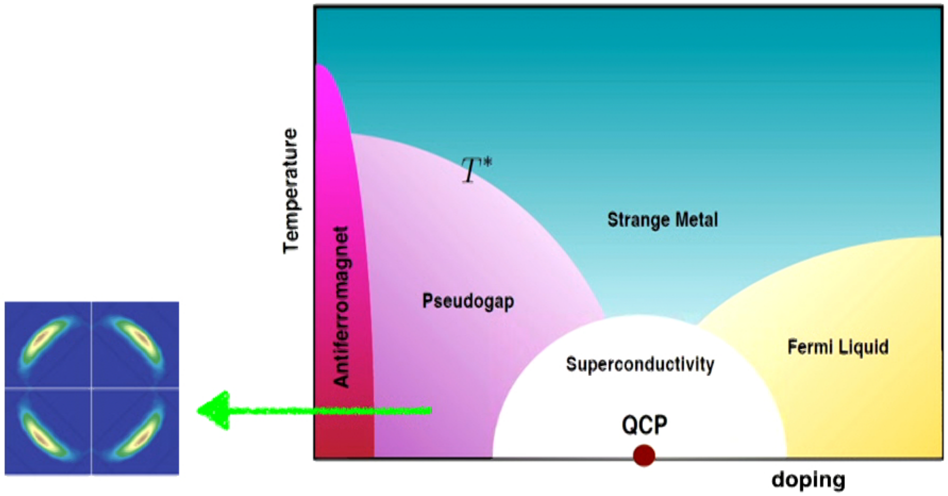
Strange metals with dynamic gauge fields



- Pseudogap Fermi surface looks like it's reconstructed by long-range antiferromagnetism (AFM) in ARPES, but no AFM is measured by neutron scattering.

- **S. Sachdev**: Electrons are fractionalized into gapped bosonic spinons and gapless fermionic chargons. An SU(2) gauge redundancy in this description, and the suppression of 2π vortices in the AFM landscape, allows the chargons to be subject to an effective long-range AFM order that shows up only as short-range fluctuating AFM for the electrons. However, the electron spectral function tracks that of the chargons, and they look like they have AFM, without having any actual AFM.

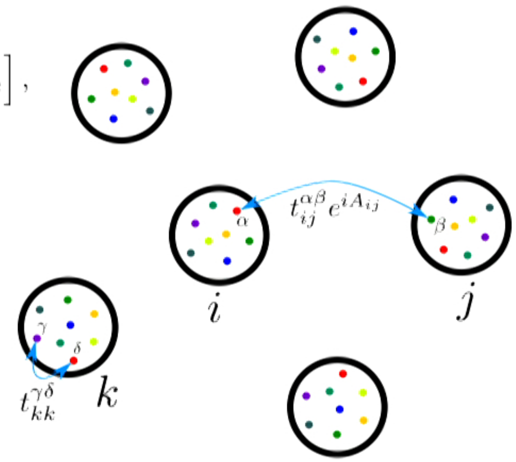
S. Sachdev, arXiv:1801.01125 (Review).

Strange metals with dynamic gauge fields: SYK-like toy model

$$\mathcal{H} = -\frac{1}{(MN)^{1/2}} \sum_{ij=1}^{N\to\infty} \sum_{\alpha\beta=1}^{M\to\infty} \left[t_{ij}^{\alpha\beta} e^{iA_{ij}} f_{i\alpha}^\dagger f_{j\beta} + (MN)^{1/2} \mu \delta_{ij}^{\alpha\beta} f_{i\alpha}^\dagger f_{i\alpha} \right],$$

$$\ll t_{ij}^{\alpha\beta} t_{ji}^{\beta\alpha} \gg = \ll |t_{ij}^{\alpha\beta}|^2 \gg = t^2,$$

$$A_{ji} = -A_{ij}.$$



A. A. Patel and S. Sachdev,
Phys. Rev. B 98, 125134 (2018)

- Clustered random all-to-all hopping model of fermionic chargons coupled to dynamic U(1) gauge fields. Has a large-*N* number of clusters, with a large-*M* number of orbitals per cluster. The ratio *M/N* is an *O(1)* number.

- Unlike the SYK models, this model combines both hopping and interaction effects.

Strange metals with dynamic gauge fields: SYK-like toy model

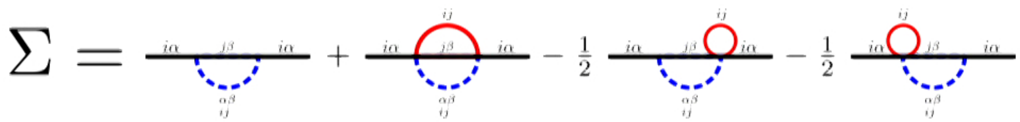
The model again self-averages in the large-*M,N* limits. Disorder averaged action after expanding $e^{iA_{ij}}$ to keep only IR-relevant terms:

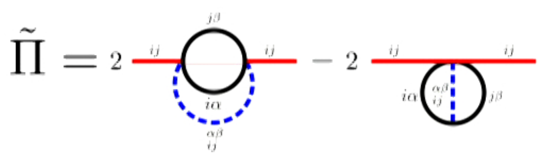
$$S = \int d\tau \sum_{i=1}^{N} \sum_{\alpha=1}^{M} f_{i\alpha}^{\dagger}(\tau)(\partial_\tau + iA_i^0(\tau)) f_{i\alpha}(\tau)$$

$$+ t^2 \frac{M}{N} \int d\tau d\tau' \sum_{ij=1, i\leq j}^{N} \left[1 + i(A_{ij}(\tau) - A_{ij}(\tau')) - \frac{1}{2}A_{ij}^2(\tau) - \frac{1}{2}A_{ij}^2(\tau') + A_{ij}(\tau)A_{ij}(\tau')\right] G_j(\tau - \tau')G_i(\tau' - \tau)$$

$$- M \int d\tau d\tau' \sum_{i=1}^{N} \Sigma_i(\tau - \tau') \left[G_i(\tau' - \tau) - \frac{1}{M}\sum_{\alpha=1}^{M} f_{i\alpha}(\tau') f_{i\alpha}^{\dagger}(\tau)\right]. \quad + \text{“Maxwell” terms.}$$

Varying w.r.t. *G* and Σ for each cluster *i* after integrating out *A* (which can be done easily in the large-*N* limit) and *f* yields SYK-like Dyson equations for the large-*M,N* saddle-point solution.

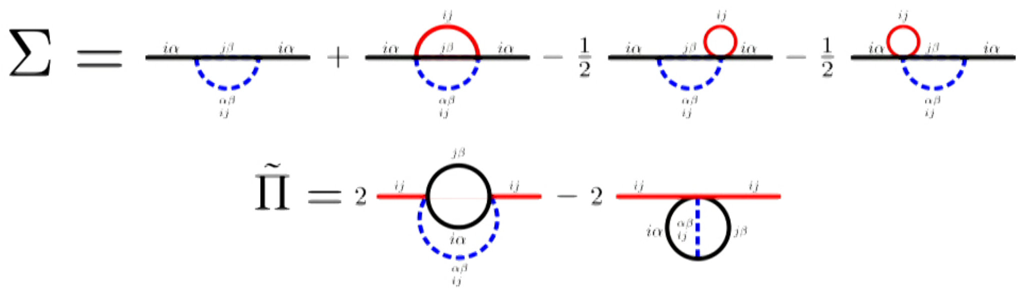




Strange metals with dynamic gauge fields: SYK-like toy model

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Omega_m^2/g^2 + \Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

$$\Pi(i\Omega_m) = t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n) G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)}.$$



No SL(2,R) invariance like SYK, but still possesses a *scale-invariant* solution in the IR.

Strange metals with dynamic gauge fields: SYK-like toy model

$$\Sigma(i\omega_n) = t^2 G(i\omega_n)\left[1 - T\sum_{\Omega_m \neq 0}\frac{1}{\Omega_m^2/g^2 + \Pi(i\Omega_m) - \Pi(i\Omega_m = 0)}\right]$$

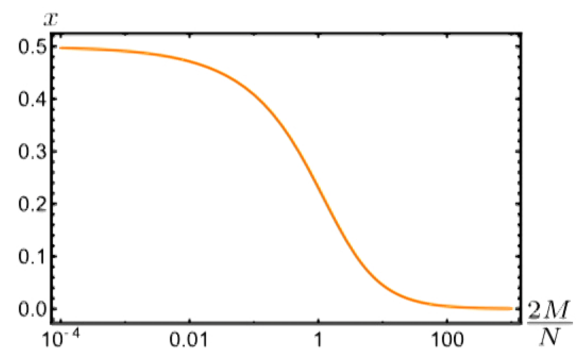
$$+ t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m)}{\Omega_m^2/g^2 + \Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

This term cancels at *T = 0*.

Power-law Green's function with tunable exponent at *T = 0*.

$$G(\tau) = -C\frac{\mathrm{sgn}(\tau)}{t^{1-x}|\tau|^{1-x}}, \quad G(i\omega_n) = -2iCt^{x-1}\sin\left(\frac{\pi x}{2}\right)\Gamma(x)\mathrm{sgn}(\omega_n)|\omega_n|^{-x}, \quad 0 < x < \frac{1}{2}, \quad C > 0.$$

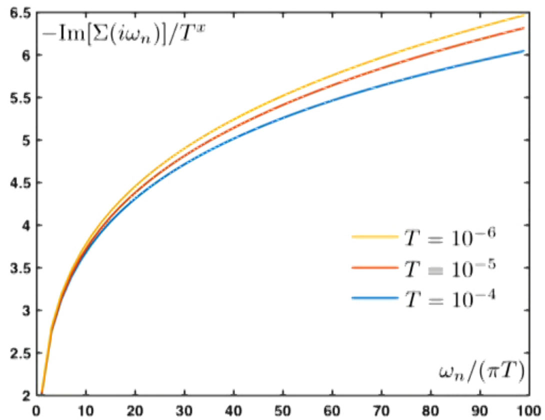
$$\frac{1/x - 2}{1 + \sec(\pi x)} = \frac{2M}{N}$$



Strange metals with dynamic gauge fields: SYK-like toy model

Scaling solution at finite T, but the scaling function is **not** the conformal SYK scaling function, and is determined numerically.

$$G(i\omega_n, T) = \frac{C}{t^{1-x}T^x} F_G\left(\frac{\omega_n}{T}\right), \quad F_G(y \to 0) \propto y^0, \quad F_G(y \to \infty) \propto \frac{1}{y^x}.$$



Strange metals with dynamic gauge fields: SYK-like toy model

- We can add spatial structure to the theory by defining each the clusters *i* to lie on the sites of a large-*N*-dimensional hypercubic lattice with nearest-neighbor hopping, without changing the saddle point.
- Then, splitting the chargons into two species $\pm$, coupling to the internal gauge fields *A* with opposite charges, we can derive transport properties in linear response to an external gauge field Ξ, to which the chargons couple with equal charges. This is the structure of a U(1) ACL (See S. Sachdev, arXiv:1801.01125).

$$\mathcal{H}' = -\frac{1}{(2MN)^{1/2}} \sum_{\langle ij \rangle} \sum_{\alpha\beta=1}^{M} \sum_{ss'=\pm} t_{ij}^{\alpha\beta} f_{i\alpha s}^{\dagger} e^{iA_{ij}\sigma^z_{ss'}} f_{j\beta s'}$$

- We get non-Fermi liquid conductivities from linear response to Ξ

$$\sigma^{\rm DC}(T) \sim \frac{M}{N}\left(\frac{t}{T}\right)^{2x}, \qquad \sigma(\Omega \gg T) = -2(M/N)C^2 \sin(\pi x)\Gamma(2x-1)\left(\frac{it}{\Omega}\right)^{2x}$$

Strange metals with dynamic gauge fields: SYK-like toy model

- We can also couple the chargons and the gauge fields to charge-2 complex scalar Higgs fields

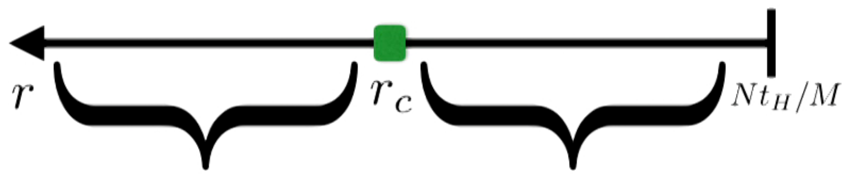
$$\mathcal{H}'' = -\frac{1}{(2MN)^{1/2}} \sum_{\langle ij \rangle} \sum_{\alpha\beta=1}^{M} \sum_{s=\pm} t_{ij}^{\alpha\beta} f_{i\alpha s}^{\dagger} e^{isA_{ij}} f_{j\beta s}$$
$$+ \sum_{i} \left[Mr|H_i|^2 + g_H \left(H_i \sum_{\alpha=1}^{M} f_{i\alpha+}^{\dagger} f_{i\alpha-} + \text{h.c.} \right) \right] - \frac{t_H}{2} \sum_{\langle ij \rangle} \left[H_i^* H_j e^{2iA_{ij}} + \text{h.c.} \right]$$

- The Dyson equations at the large-*M*,*N* saddle point are

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \int \frac{d\Omega_m}{2\pi} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Omega_m^2/g^2 + \tilde{\Pi}(i\Omega_m) + 4t_H|H|^2}, \quad G(i\omega_n) = \frac{i\omega_n - \Sigma(i\omega_n)}{(i\omega_n - \Sigma(i\omega_n))^2 - g_H^2|H|^2},$$
$$H\left[r - \frac{N}{M} t_H + \int \frac{d\omega_n}{2\pi} \frac{g_H^2}{(i\omega_n - \Sigma(i\omega_n))^2 - g_H^2|H|^2} + \frac{2N}{M} \int \frac{d\Omega_m}{2\pi} \frac{t_H}{\Omega_m^2/g^2 + \tilde{\Pi}(i\Omega_m) + 4t_H|H|^2} \right] = 0,$$
$$\tilde{\Pi}(i\Omega_m) = 2t^2 \frac{M}{N} \int \frac{d\omega_n}{2\pi} G(i\omega_n)(G(i\omega_n + i\Omega_m) - G(i\omega_n))$$

- The parameter *r* tunes the Higgs transition. Condensing the Higgs field breaks the U(1) gauge invariance down to Z_2, and gaps out its singular low-energy fluctuations.

Strange metals with dynamic gauge fields: SYK-like toy model



$(T = 0)$

Higgs fields uncondensed	Higgs fields condensed, A_{ij} gapped
U(1) ACL, non-Fermi liquid Green's function (no quasiparticles)	Z_2 ACL, random-matrix Green's function (has quasiparticles)
Low energy fermion density of states ~ $1/E^x$	Reduced low energy fermion density of states ~const. —> "pseudogap"

Transition exponent: $|H| \sim (r - r_c)^{1/2}, \;\; \nu = 1/2.$

Mean-field behavior at large-*M*,*N*.

Future directions

- SYK-like large-*N* limit for fermions coupled to quantum critical order parameter? Obtain quantum critical strange metal "fan" in the cuprates phase diagram.
- 2+1 dimensional lattice-SYK like model of chargons coupled to gauge fields, with Higgs transition from a strange metal with a "large" Fermi surface to a pseudogap with a "small" Fermi surface.
- Model that can give linear-in-*T* resistivity at both high and low *T*, keeping a fixed slope corresponding to a single-particle Drude lifetime of $\hbar/(k_B T)$ on both sides of the MIR limit.
- Model anomalous features in the optical conductivity of strange metals using crossovers between different SYK-based non-Fermi liquids.
- Try to understand if SYK-like local criticality can emerge in rare regions in disordered Hubbard models, and the effects of these local critical regions on the rest of the system.