Title: Renormalization and Effective Field Theory - Lecture 6

Date: Oct 24, 2018 10:30 AM

URL: http://pirsa.org/18100074

Abstract:

RG flow for ϕ^4 theory

c coupling constant

- We saw

If we consider the space of theories $\{I[L]\}$ s.t. $\{I_\lambda[L]\}$ this has log-growth at $\lambda = 0$ <u>and</u> $\lambda = \infty$

This is a 1d manifold

At each order in $\hbar$, can add <u>only</u> $\hbar^k \int \phi^4$

General formul

$\{I[L]\}$ s.t. $\{I_\lambda[L]\}$ this has log-growth

at $\lambda = 0$ and $\lambda = \infty$

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At each order in $\hbar$, can add only $\hbar^k \int \phi^4$

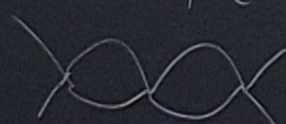
General formalism

1d manifold   Scheme $\to$ coordinate c

Change of scheme $c \longrightarrow c + f_2(\hbar)c^2 + f_3(\hbar)c^3 + \ldots$

RG flow is a v. field

$$\hbar c^2 \frac{\partial}{\partial c} + \hbar^2 c^3 \lambda_1 \partial_c + \ldots\ldots$$



Changing $c \longrightarrow c + O(c^2)$

fixes $c^2 \partial_c$ but changes the

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Higher order terms **entirely** scheme dependent.

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More abstractly

Manifold of nice theories

is a germ of a 1d manifold, with a v. field V

$\exists$ a critical point of v. field,

Higher order terms <u>entirely</u> scheme dependent.

<u>More abstractly</u>

Manifold of nice theories

is a germ of a 1d manifold, with a v. field V

+ a critical point of v. field,

+ a non-zero tangent vector at critical point.

Only coordinate-independent data = coeff. of $c^2\partial_c$.

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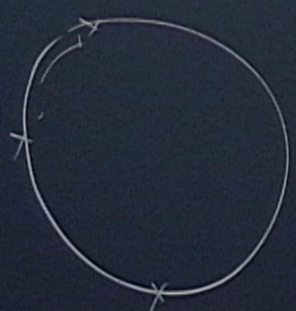

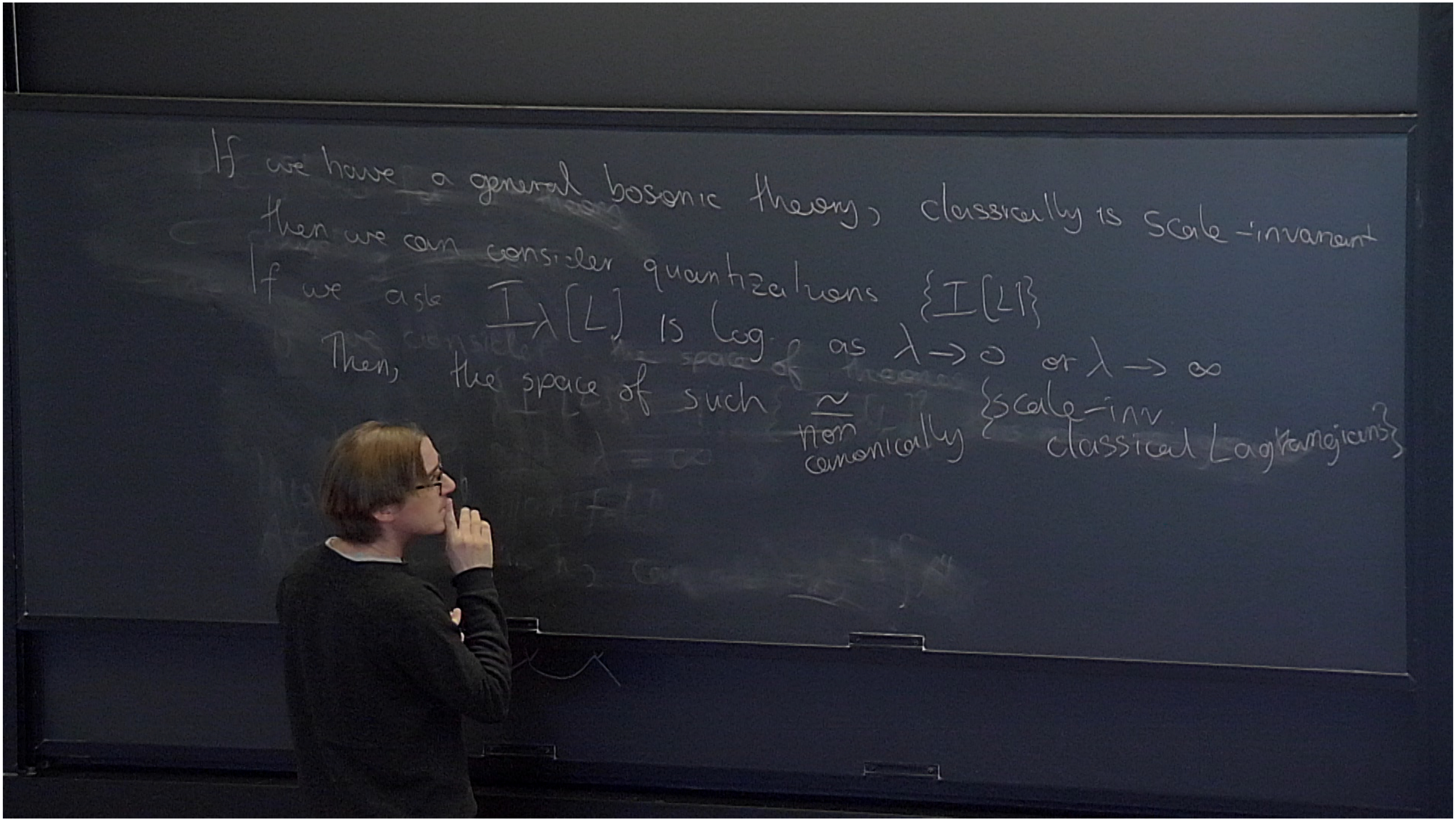
If we have a general bosonic theory, classically is scale-invariant
then we can consider quantizations {I[L]}
If we ask I λ[L] is log as λ → 0 or λ → ∞
Then, the space of such ≃ {scale-inv classical Lagrangians}
non canonically

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then we can consider quantizations $\{I[L]\}$

If we ask $I_\lambda[L]$ is log. as $\lambda \to 0$ or $\lambda \to \infty$

Then, the space of such $\underset{\text{non canonically}}{\simeq}$ $\{$scale-inv classical Lagrangians$\}$

Technical Fact

$I_\lambda[L]$ is as a fn of λ, is polynomial in $\lambda, \lambda^{-1}, \log\lambda$

non canonically { classical Lagrangians

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2d Scalar theories

$\varphi_1, \ldots, \varphi_n$ scalar fields

$\int \varphi_i \Delta \varphi_i = \int d\varphi_i \wedge * d\varphi_i$ is scale invariant even if φ_i are of dim^n 0

Most general $SO(2)$ and scale invariant Lagrangian

$\int g^{ij}(\vec{\phi})\, d\phi_i \wedge d\phi_j$

g^{ij} is some function of $\vec{\phi}$

View $\vec{\phi}$ as a map $\mathbb{R}^2 \longrightarrow \mathbb{R}^n$

is the σ-model for some metric g on $\mathbb{R}^n$

Changing $c \longrightarrow c + O(c^2)$

fixes $c^2 \partial_c$ but changes the

is the σ-model for some metric g on $\mathbb{R}^n$

We find

Scale-invariant classical Lagrangians

$\supseteq$ all metrics on $\mathbb{R}^n$!!

∞ d manifold of theories $\{I[L]\}$ where $I_\lambda[L]$ has log-growth at $\lambda = 0, \lambda = \infty$

Things are better if we take σ-model

$$\sigma : \mathbb{R}^2 \longrightarrow X$$

where X is a homogeneous space for some Lie group

Best: X = Riemannian symmetric space

e.g. $\mathbb{H}^n$, S^n, ...

there is 1 G-inv. metric up to scale.

non canonically ≤ classical Lagrangians

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1-loop β-fn = Ricci flow

X a Riemannian manifold

$\sigma : \mathbb{R}^2 \longrightarrow X$

$\int g_X(d\sigma, \star d\sigma)$

$= \int g'_{ij}(\sigma) d\sigma_i \wedge \star d\sigma_j$ and scale

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Choose Riemann normal coordinates near some point $x \in X$

$g^{ij} = \delta^{ij} + x_k x_l R^{klij} + \dots$

Riemann curvature tensor

non canonically ... classical Lagrangians

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Choose Riemann normal coordinates near some point $p \in X$

$$g^{ij} = \delta^{ij} + x_k x_l R^{klij}(p) + \dots$$

Riemann curvature tensor

$$R^{ij}(p) = \left(\sum \partial_{x_k} \partial_{x_k} g^{ij}\right)(p)$$

Compute one-loop β-fn:

$$\int d\varphi^i \wedge d\varphi^i + \varphi^k \varphi^l (d\varphi^i \wedge d\varphi^j) R_{ijkl} + \cdots$$

Propagator $= \int_{t=\varepsilon}^{L} t^{-1} e^{-\|y\|^2/t} \, dt$

$\varepsilon \to 0, L \to \infty, \rightsquigarrow \log y$

classical Lagrangians

metrics on $\mathbb{R}^n$ //

$$= \int_{t=\varepsilon} t \, e \cdots dt$$

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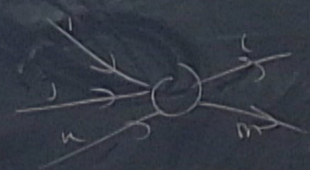
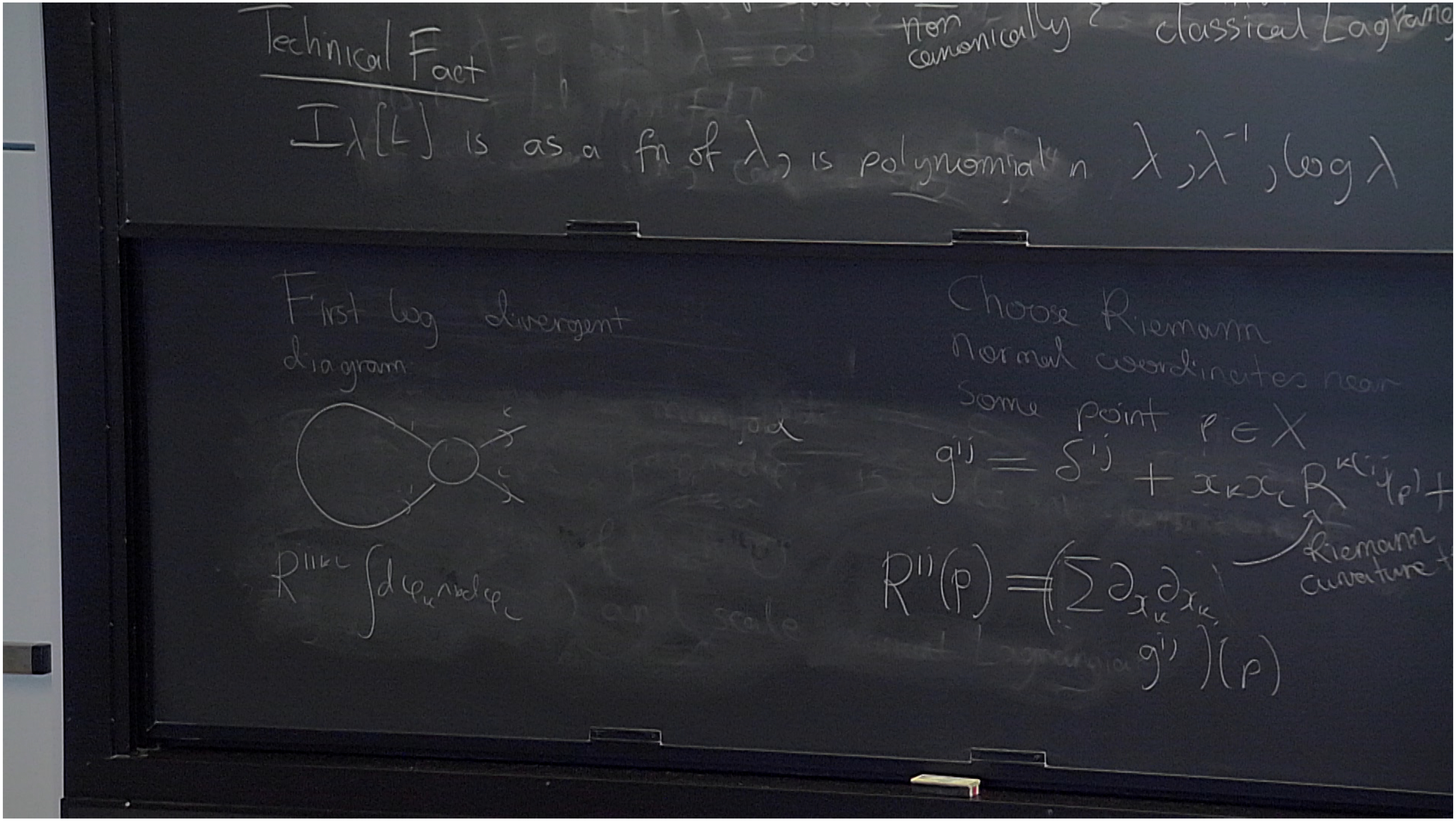


$\varphi_i \varphi_j \varphi_k (d\varphi_l \wedge \cdots \wedge d\varphi_m)$

One loop log divergent diagrams:

$$\sim \left(\int_{t=\varepsilon}^{L} t^{-1} \partial_y e^{-\|y\|^2/t} \right)_{y=0}$$


non canonically
classical Lagrange
Technical Fact
Iλ[L] is as a fn of λ, is polynomial in λ, λ⁻¹, log λ
First log divergent diagram
Choose Riemann normal coordinates near some point p ∈ X
gⁱʲ = δⁱʲ + x_k x_l R^{kl ij}(p) +
Riemann curvature
Rⁱʲ(p) = (Σ ∂_{x_k} ∂_{x_k} gⁱʲ)(p)
scale

non canonically ... classical Lagrang...

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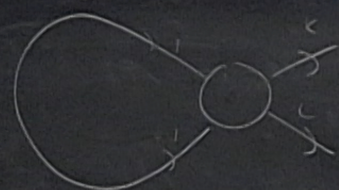
First log divergent diagram



$R^{ijkl} \int d\varphi_k \wedge d\varphi_l \int_{t=\varepsilon}^{L} t^{-1}\, dt$

Choose Riemann normal coordinates near some point $p \in X$

$g^{ij} = \delta^{ij} + x_k x_l R^{klij}(p) +$

Riemann curvature t...

$R^{ij}(p) = \left(\sum \partial_{x_k} \partial_{x_k} g^{ij}\right)(p)$

non canonically $\cong$ classical Lagrangians

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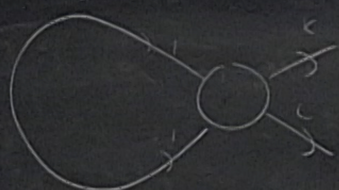

$R^{ijkl}\int d\varphi_k \wedge * d\varphi_l \int_{t=\varepsilon}^{L} t^{-1}\, dt$

1st one-loop log divergence

$= R^{kl}(\rho)\int d\varphi^k \wedge * d\varphi^c \log \varepsilon$

So, one-loop RG flow sends

$g^{ij} \to g^{ij} + R^{ij}(\rho) + \ldots$

$$= \int_{t=\varepsilon} t\, e \quad dt$$

$\varepsilon \to 0, L \to \infty, \rightsquigarrow \log y$

$$R^{ij} = c\, g^{ij}$$

E.g.

2d σ-model on S^n

This argument tells us

- 1d space of good theories, given by size of S^n
- 1 loop RG flow scales sphere

$$= \int_{t=\varepsilon} t \, e \quad dt$$

$\varepsilon \to 0, L \to \infty, \rightsquigarrow \log y$

Eg.

$R^{ij} = c \, g^{ij}$

2d σ-model on S^n

This argument tells us

- 1d space of good theories, given by size of S^n
- 1 loop RG flow scales sphere
- Free theory: S^n ∞ large

non canonically ... classical Lagrangians

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2d Fermions

ψ_i, ψ^i chiral fermions

$\psi_i, \psi^i \in \Omega^{1/2,0}(\mathbb{C})$ $\qquad \psi_i \bar\partial \psi^i$

$\sim \sqrt{dz}$

$\bar\psi_i, \bar\psi^i \in \Omega^{0,1/2}(\mathbb{C})$ $\qquad \bar\psi_i \partial \bar\psi^i$

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Interaction

$c_{ij}^{kl} \psi_k \psi^i \bar\psi_l \bar\psi^j$

for some c_{ij}^{kl}

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scale

Interaction

$C_{ij}^{kl} \psi_k \psi^i \bar{\psi}_l \bar{\psi}^j$

for some C_{ij}^{kl}

Finite number of scale-inv. Lagrangians! Good

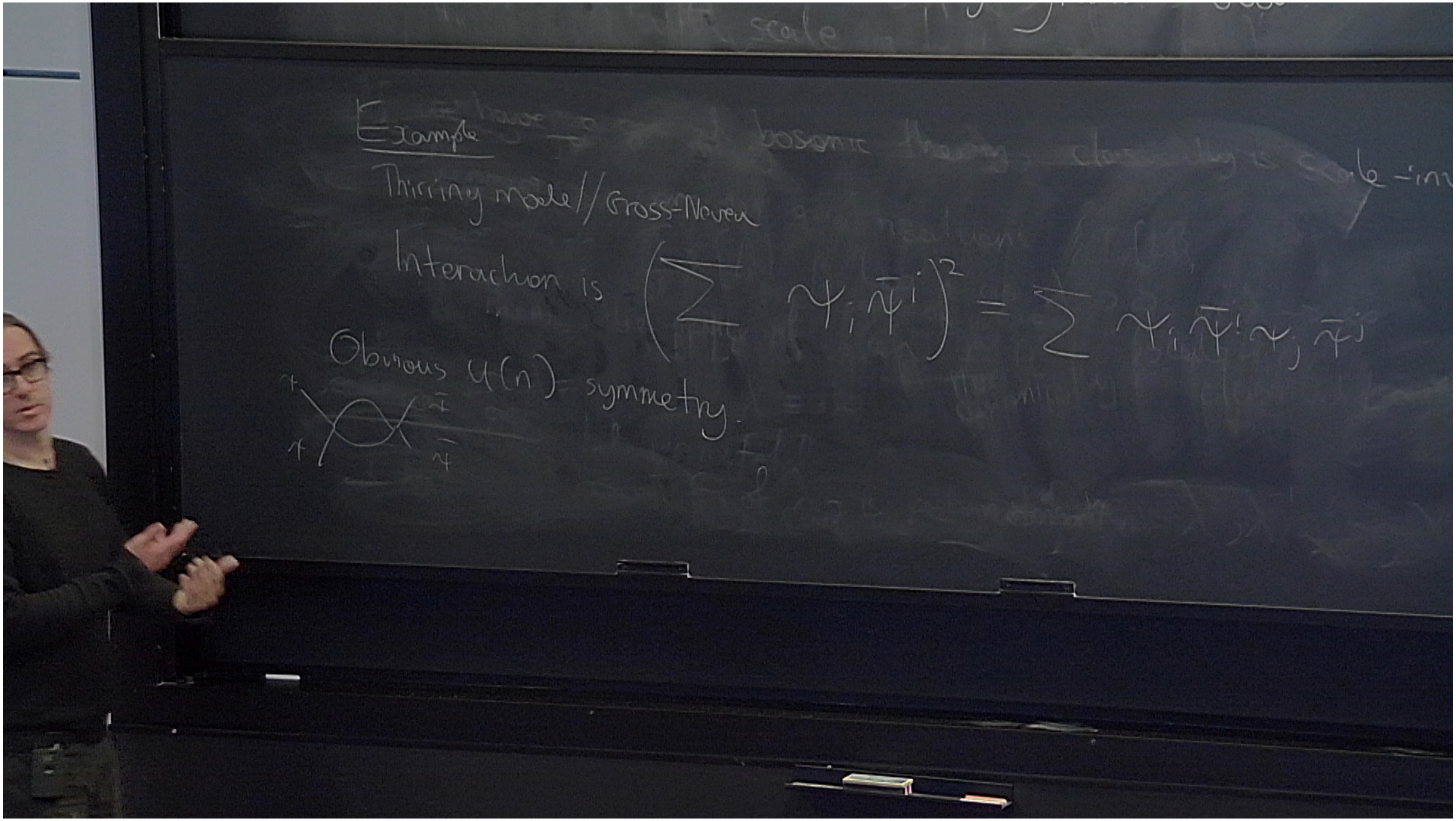
... scale-invariant

consider quantization

Example

Thirring model // Gross-Neveu

Interaction is $\left(\sum \psi_i \bar{\psi}^i\right)^2 = \sum \psi_i \bar{\psi}^i \psi_j \bar{\psi}^j$


Example
Thirring model // Gross-Neveu
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Obvious $U(n)$ symmetry.

scale-invariant

Example

Thirring model // Gross-Neveu

Interaction is $\left(\sum_i \psi_i \bar{\psi}^i\right)\left(\sum_j \bar{\psi}_j \psi^j\right) = \sum \psi_i \bar{\psi}^i \psi^j \bar{\psi}_j$

Obvious $U(n)$ symmetry.