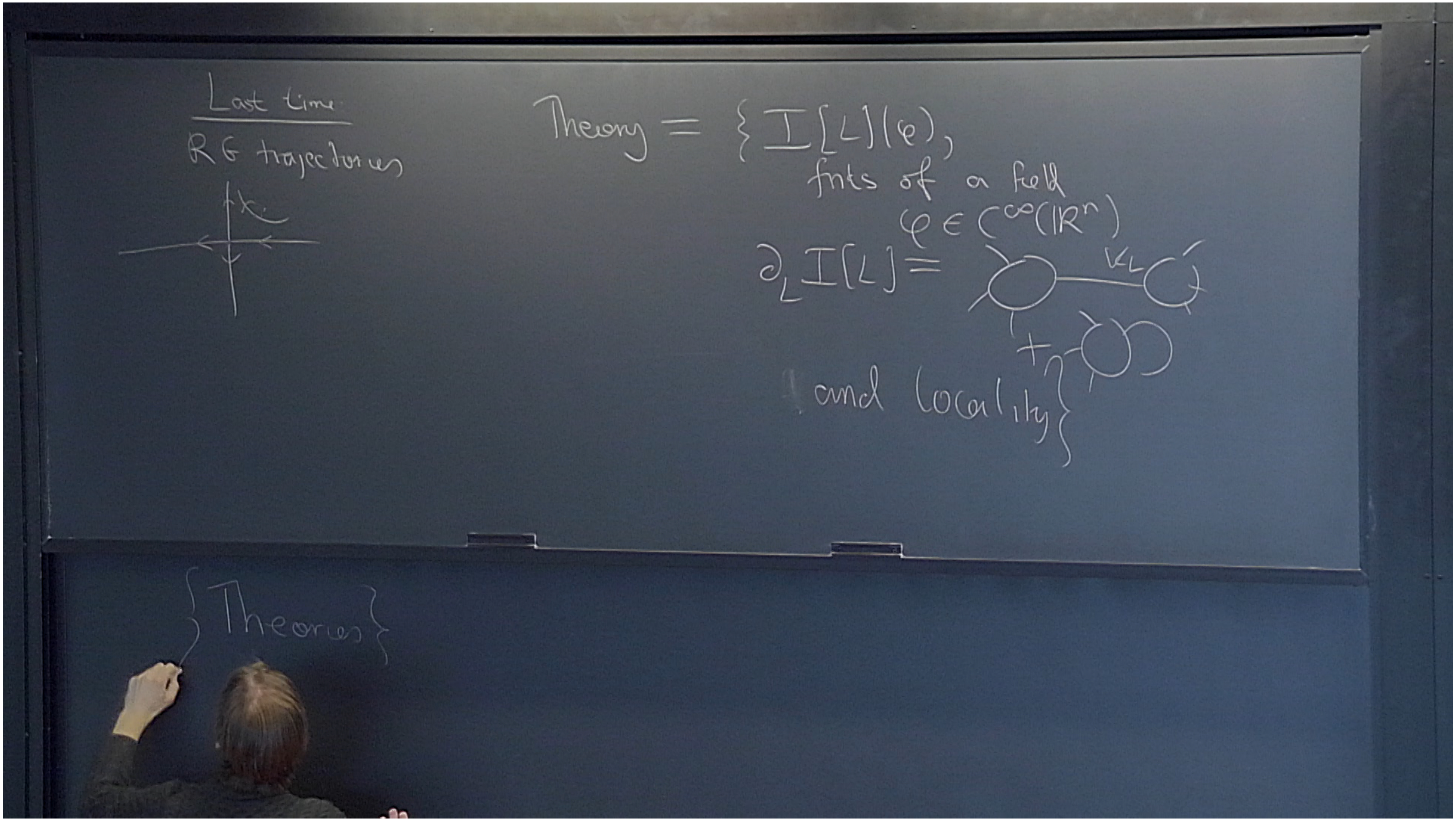
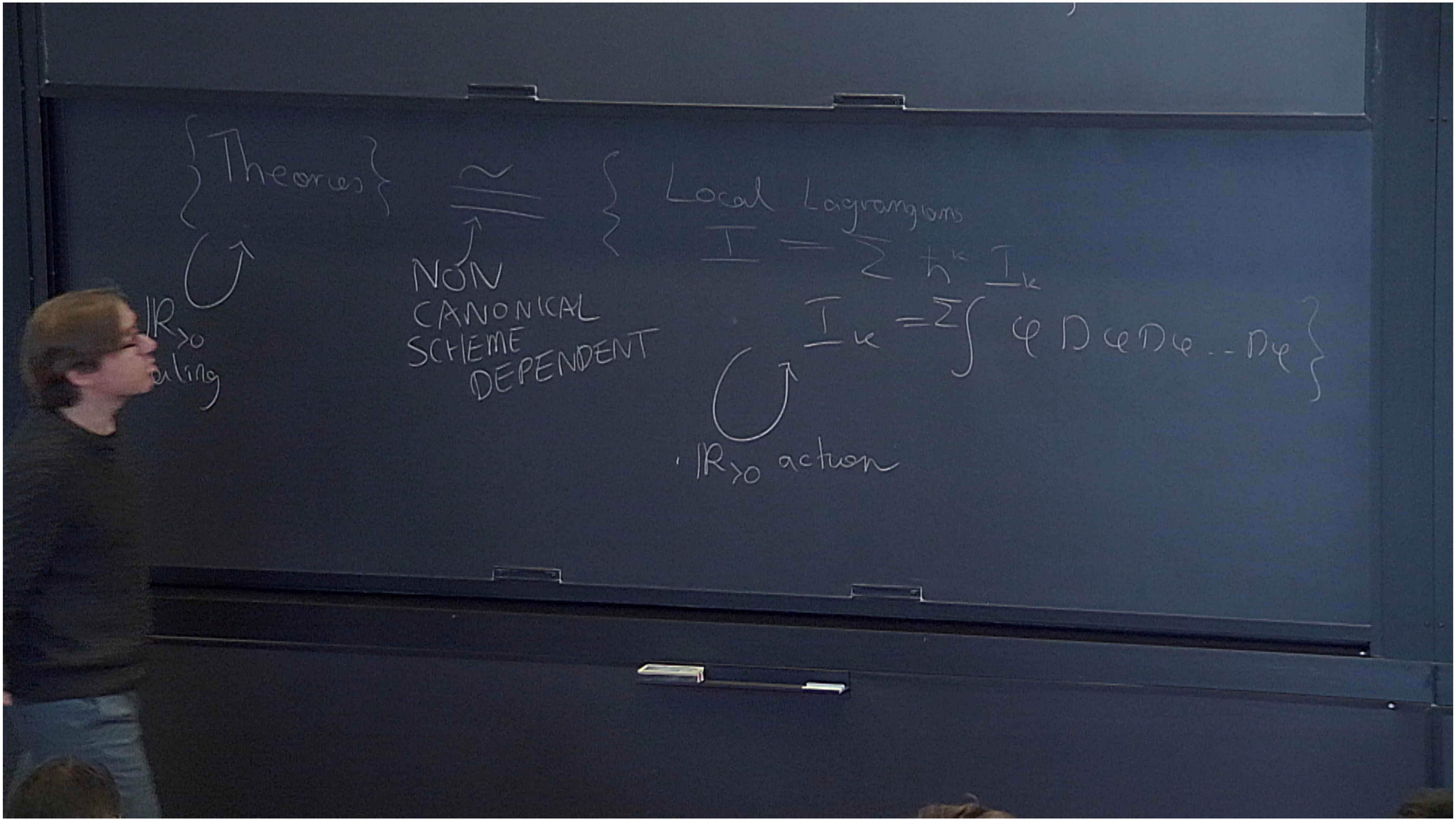
Title: Renormalization and Effective Field Theory - Lecture 5

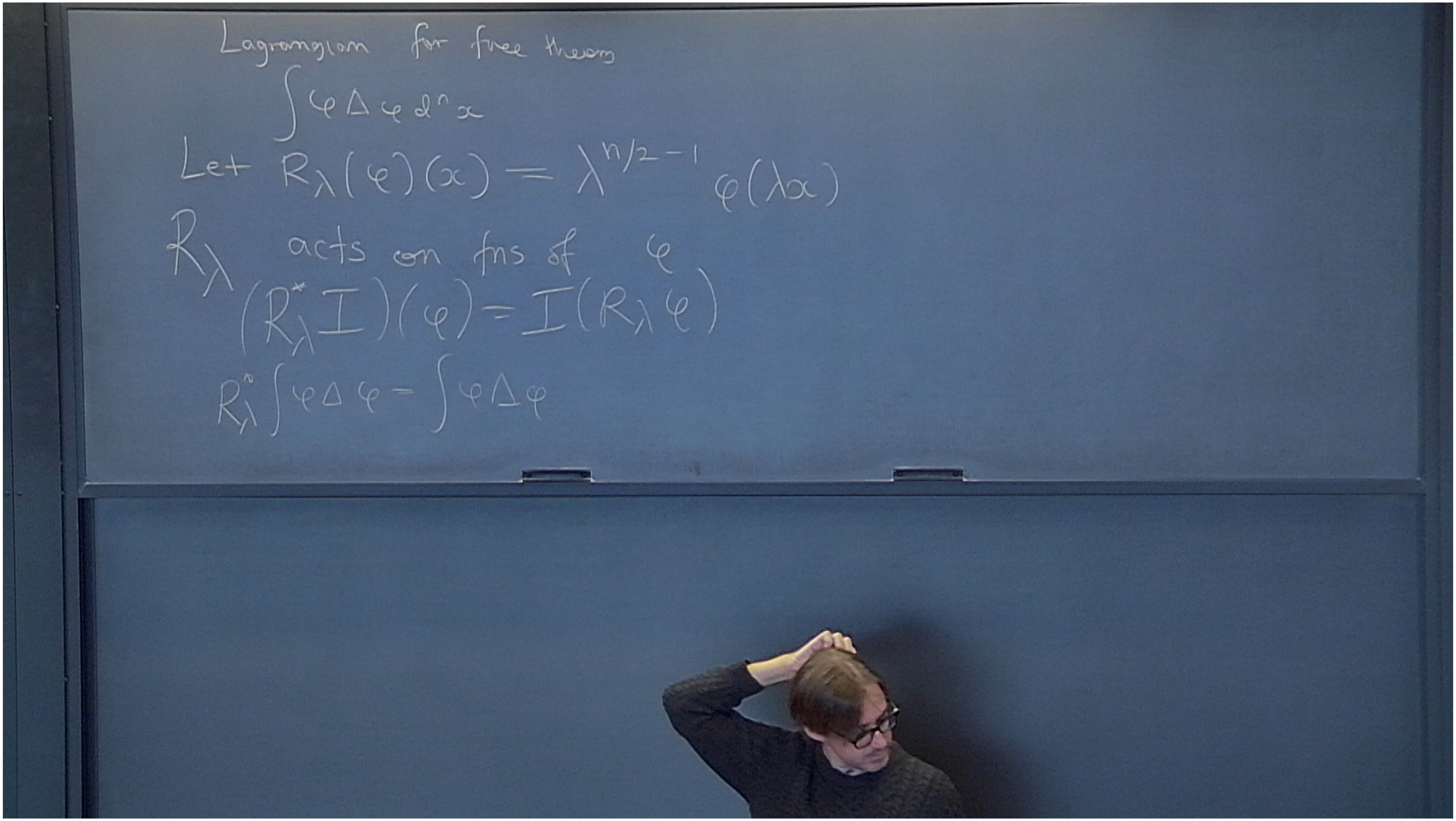
Date: Oct 17, 2018 10:30 AM

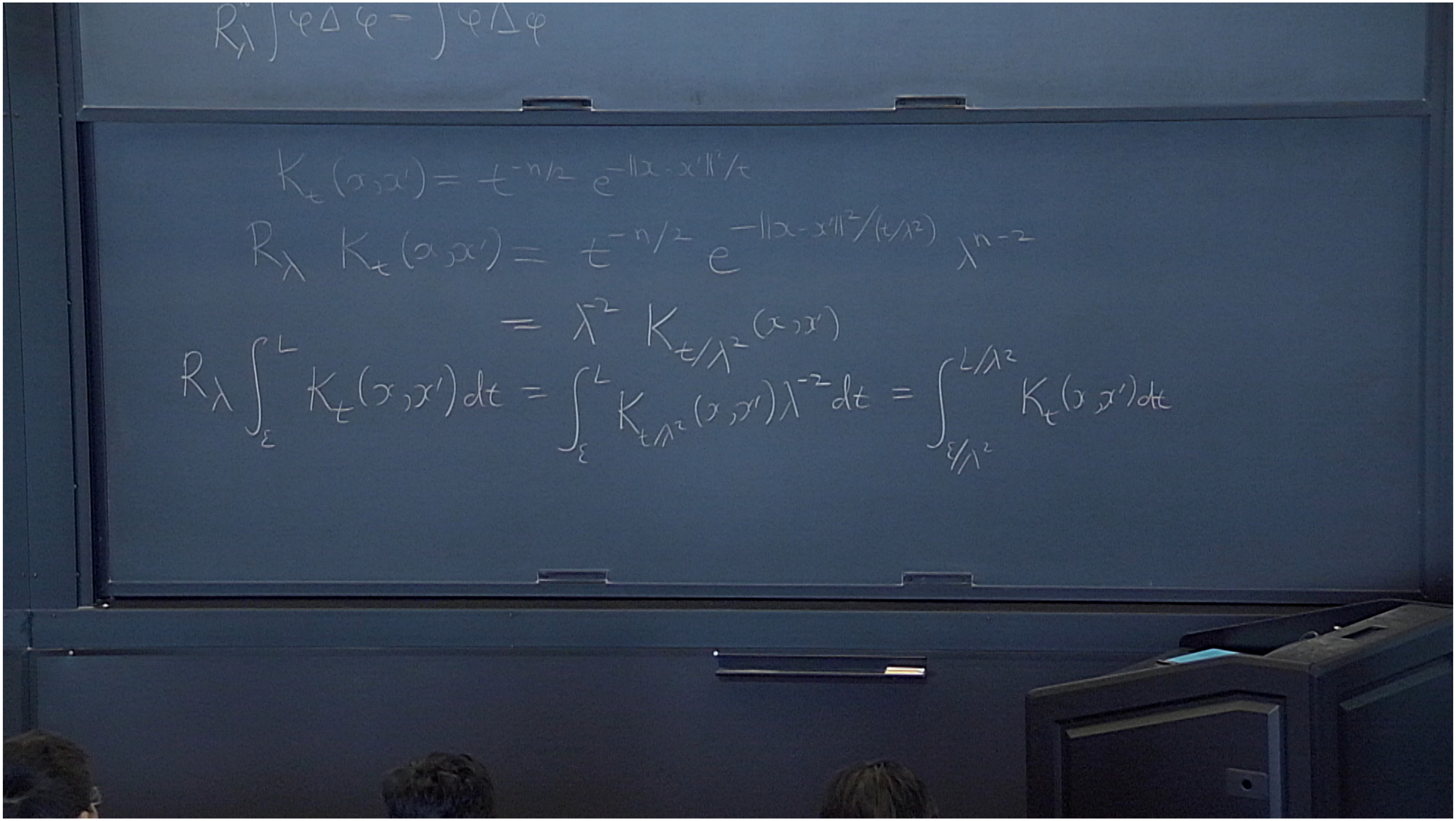
URL: http://pirsa.org/18100073

Abstract:


Last time
RG trajectories
Theory = { $I[L](\varphi)$,
fnts of a field
$\varphi \in C^\infty(\mathbb{R}^n)$
$\partial_L I[L] =$
K_L
and locality }
{ Theories }


{Theories}
≃
{ Local Lagrangians
I = Σ ħ^k I_k
I_k = Σ ∫ φ Dφ Dφ ... Dφ }
ℝ>0
NON
CANONICAL
SCHEME
DEPENDENT
ℝ>0 action


Lagrangian for free theory
$\int \varphi \Delta \varphi \, d^n x$
Let $R_\lambda(\varphi)(x) = \lambda^{n/2-1} \varphi(\lambda x)$
R_λ acts on fns of φ
$(R_\lambda^* I)(\varphi) = I(R_\lambda \varphi)$
$R_\lambda^* \int \varphi \Delta \varphi = \int \varphi \Delta \varphi$


$R_\lambda \int \varphi \Delta \varphi = \int \varphi \Delta \varphi$
$K_t(x,x') = t^{-n/2} e^{-\|x-x'\|^2/t}$
$R_\lambda K_t(x,x') = t^{-n/2} e^{-\|x-x'\|^2/(t/\lambda^2)} \lambda^{n-2}$
$= \lambda^{-2} K_{t/\lambda^2}(x,x')$
$R_\lambda \int_\varepsilon^L K_t(x,x')\,dt = \int_\varepsilon^L K_{t/\lambda^2}(x,x')\lambda^{-2}\,dt = \int_{\varepsilon/\lambda^2}^{L/\lambda^2} K_t(x,x')\,dt$

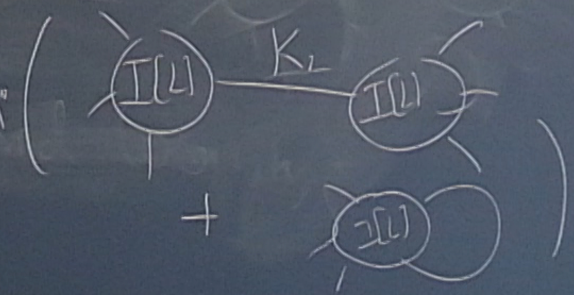
Claim

If $\{I[L]\}$ defines a theory then so does

$$I_\lambda[L](\varphi) = R_\lambda^* I[\lambda^2 L](\varphi)$$

Proof

$$\frac{d}{dL} I_\lambda[L/\lambda^2] = R_\lambda^* \frac{d}{dL} I[L](\varphi) = R_\lambda^* \Big(\quad \Big)$$



$$R_\lambda^\vee \int \varphi \Delta \varphi = \int \varphi \Delta \varphi$$

Define the $\mathbb{R}_{>0}$ action on the space of theories

by $\{I[L]\} \longrightarrow \{I_\lambda[L] = R_\lambda^\vee I[\lambda^2 L]\}$

... have a scheme, we can write

$$I[L] = \lim_{\varepsilon \to 0} W\left(P(\varepsilon, L), I - I^{CT}(\varepsilon)\right)$$

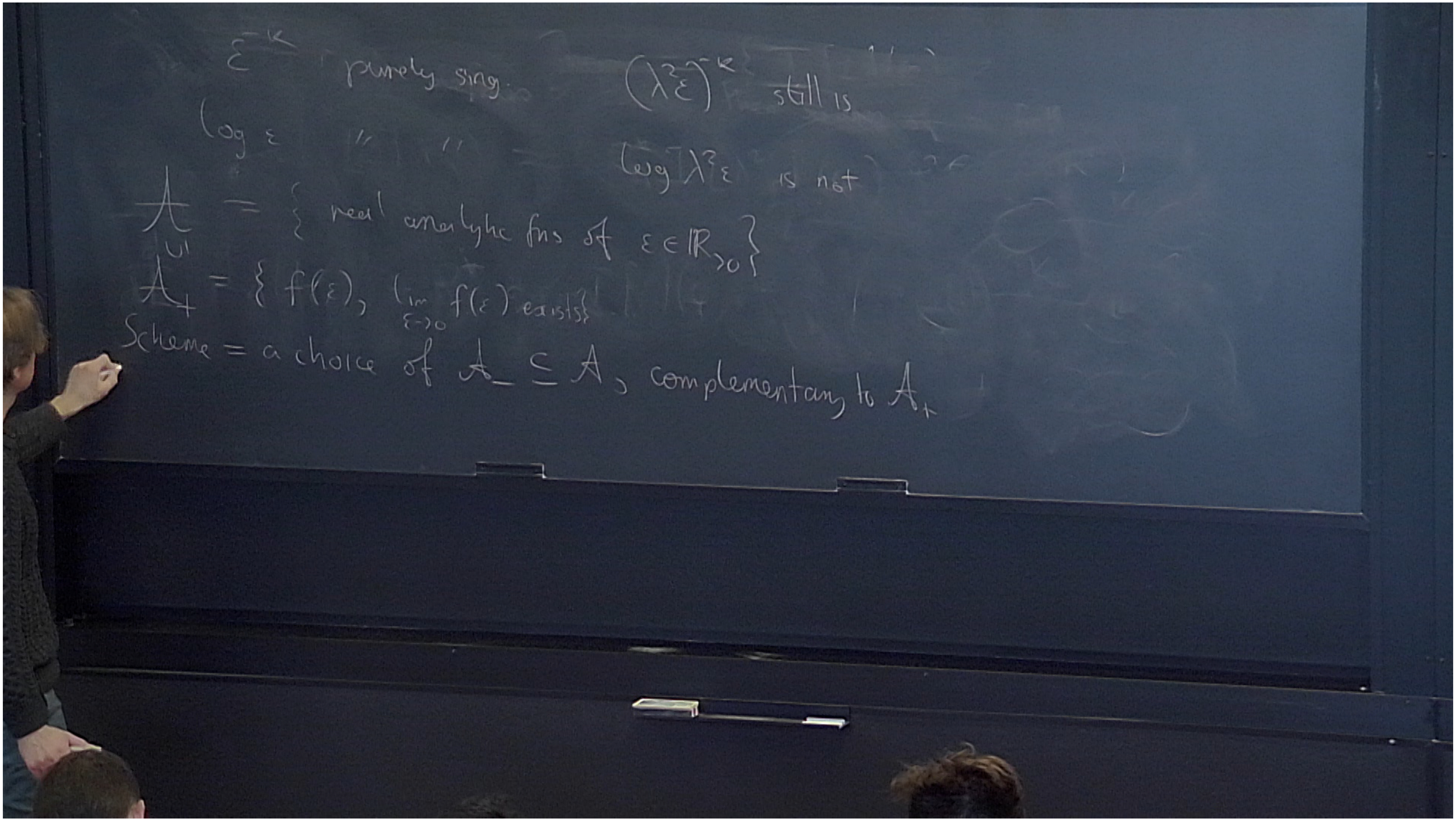
I $\hbar$ dependent Lagrangian

$I^{CT}(\varepsilon)$: Counter-terms – uniquely determined if we ask they are purely singular as fns of ε

$$R_\lambda^* I[\lambda^2 L] = \lim_{\varepsilon \to 0} R_\lambda^* W\left(P(\varepsilon, \lambda^2 L), I - I^{CT}(\varepsilon)\right)$$

$$= \lim_{\varepsilon \to 0} W\left(R_\lambda P(\varepsilon, \lambda^2 L), R_\lambda^* I - R_\lambda^* I^{CT}(\varepsilon)\right)$$

$$= \lim_{\varepsilon \to 0} W\left(P(\lambda^{-2}\varepsilon, L), R_\lambda^* I - R_\lambda^* I^{CT}(\varepsilon)\right)$$


ε^{-k} purely sing. $(\lambda^2 \varepsilon)^{-k}$ still is
$\log \varepsilon$ " " $\log \lambda^2 \varepsilon$ is not
$\mathcal{A} = \{$ real analytic fns of $\varepsilon \in \mathbb{R}_{>0} \}$
$\cup|$
$\mathcal{A}_+ = \{ f(\varepsilon), \lim_{\varepsilon \to 0} f(\varepsilon)$ exists$\}$
Scheme = a choice of $\mathcal{A}_- \subseteq \mathcal{A}$, complementary to $\mathcal{A}_+$

was in old scheme

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$\mathbb{R}_{>0}$ acts on $\mathcal{A}$, $f \to f(\lambda^2 \varepsilon)$, preserves $\mathcal{A}_+$

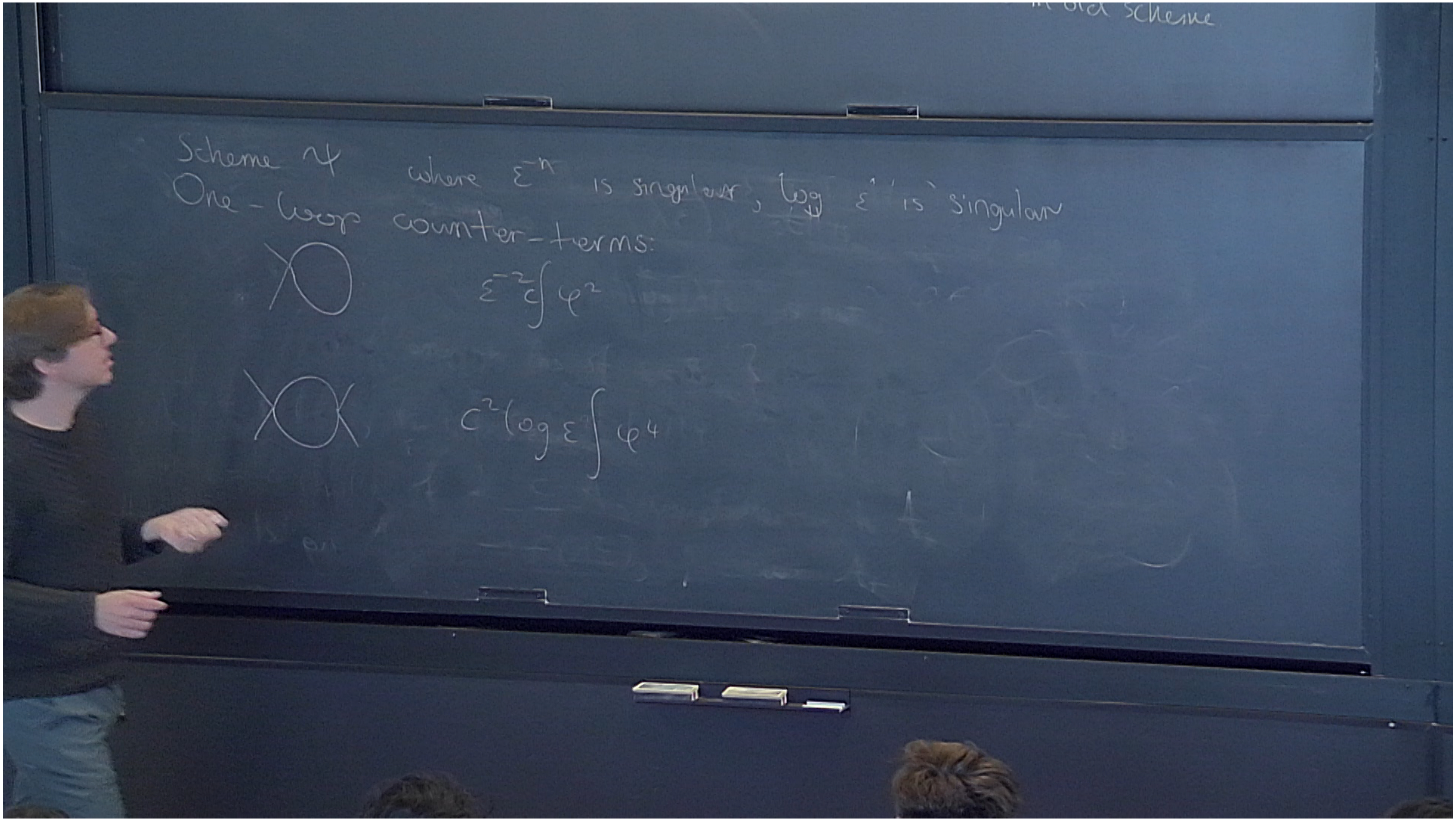
$\mathbb{R}_{>0}$ action on $\{\text{Lagrangians}\}$ sends

$$I \longrightarrow S^{-1}_{R_\lambda \psi} S_\psi R^*_\lambda I$$

φ^4 theory on $\mathbb{R}^4$

$R_\lambda(\varphi) = \lambda\varphi(\lambda x)$, so $\int \varphi^4$ is fixed by R_λ

Classically, $\int \varphi\Delta\varphi + c\int\varphi^4$ is fixed

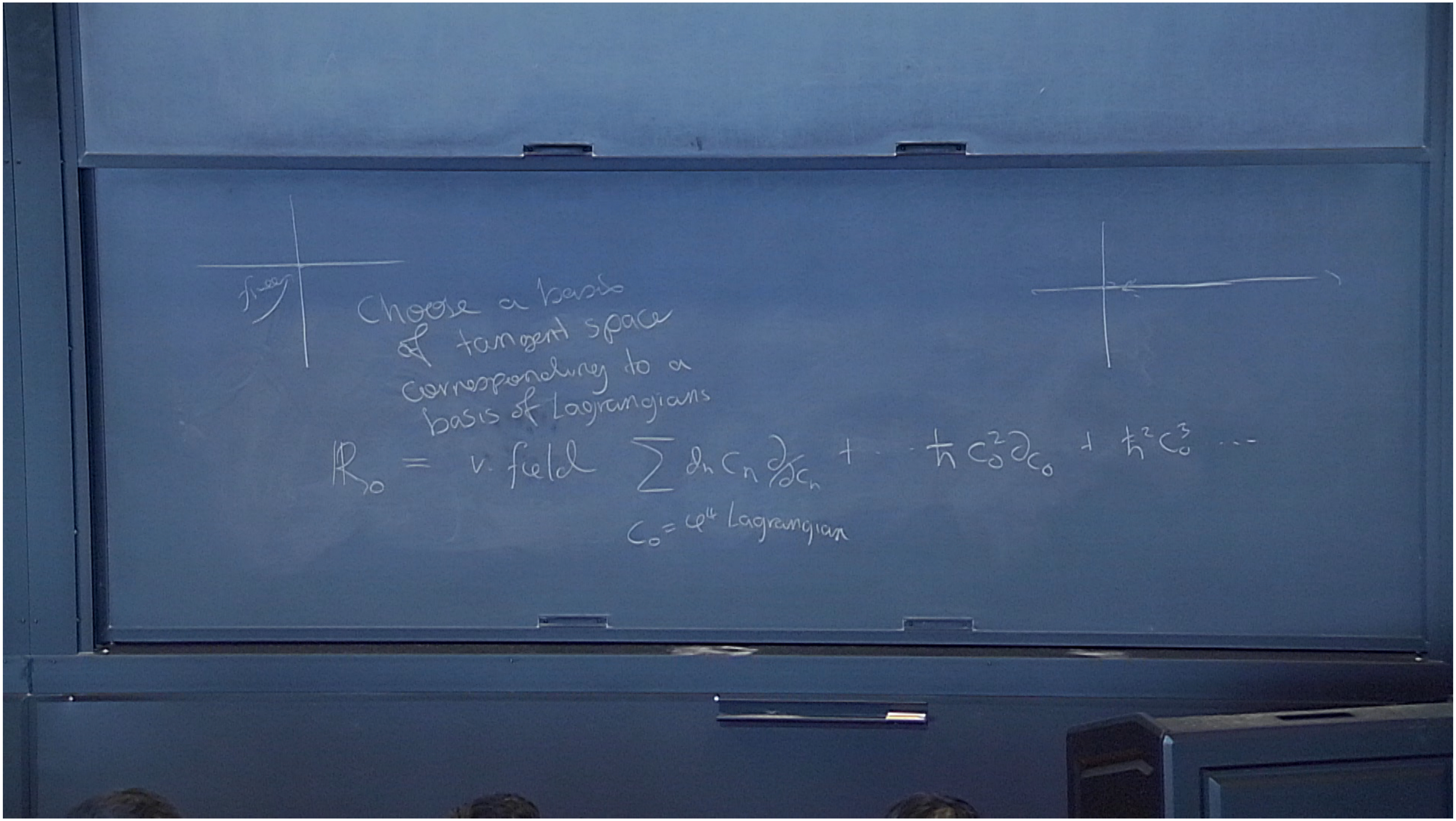


scheme
Scheme ~ψ where ε^{-n} is singular, log ε^{-1} is singular
One-loop counter-terms:
ε^{-2} c ∫ φ^2
c^2 log ε^{-1} ∫ φ^4

If we send $\varepsilon \to \lambda^2\varepsilon$ and apply $R_\lambda^{\cong}$ then

$$\varepsilon^{-2}\int \varphi^2 \longrightarrow \varepsilon^{-2}\int\varphi^2 \quad \text{unchanged}$$

$$\log\varepsilon\, C^2\int\varphi^4 \longrightarrow \left(\log\varepsilon\, C^2\right)\int\varphi^4 + 2C^2\log\lambda\int\varphi^4$$


free
Choose a basis
of tangent space
corresponding to a
basis of Lagrangians
$\mathbb{R}_{>0}$ = v. field $\sum d_n c_n \partial/\partial c_n + \cdots \hbar c_0^2 \partial_{c_0} + \hbar^2 c_0^3 \cdots$
$c_0 = \varphi^4$ Lagrangian

Say a theory is <u>perturbatively renormalizable</u>
if $I_\lambda[L]$ has log. growth as $\lambda \longrightarrow 0$

<u>For φ^4 theory</u>

One can show that the set of perturbatively renorm. theories
is $\left\{ \sum \hbar^k \left(c_k \int \varphi^4 + d_k \int \varphi^2 \right) \right\}$ (compatible w all symmetries)