

Title: Renormalization and Effective Field Theory - Lecture 4

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URL: <http://pirsa.org/18100072>

Abstract:

$$W(p, I) = \hbar \log e^{\hbar \mathcal{D}_p} e^{-I/\hbar} = \sum_{\text{diagrams } \gamma \text{ connected w. external lines}} \hbar^{L(\gamma)}$$

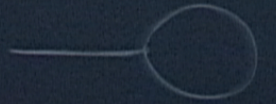
Example

ϕ^3 on \mathbb{R}^n

Expand $I^{\text{CT}}(\varepsilon)$ as a sum

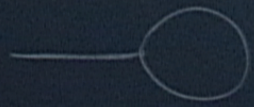
$$\sum \hbar^k I_{1;k}^{\text{CT}}(\varepsilon)$$

$k = \#$ of external lines



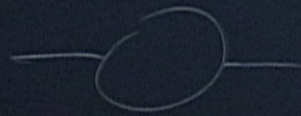
$$e^{-I/\hbar} = \sum_{\text{diagrams } \gamma} h^{L(\gamma)} W_{\gamma} \left(\begin{array}{l} P \text{ on edges} \\ I \text{ on vertices} \end{array} \right)$$

connected w. external lines



$$\rightsquigarrow I_{1,1}^{CT}(\epsilon) \sim \epsilon^{-n/2} \int \phi$$

a sum



$$\rightsquigarrow I_{1,2}^{CT}(\epsilon) \sim \log \epsilon \int \phi^2$$

(n=4)

Last time

Given a Lagrangian

we showed that \exists

$$I^{CT}(\varepsilon)$$

such that

$$W(\mathcal{P}(\varepsilon, L), \underline{I} - \underline{I}^{CT}(\varepsilon))$$

has an $\varepsilon \rightarrow 0$ limit.

$$\int \phi \Delta \phi + I(\phi)$$

counter term =

Defn

A scalar field theory on a manifold M is a collection $I[L](\phi)$ of fcts of ϕ (also den

$$I[L] = \sum h^k I_{i,k}[L](\phi)$$

$I_{i,k}[L]$ is homogeneous of deg. i as a func of ϕ

every on a manifold M
 $[L](\emptyset)$ of fcts of \emptyset (also depend on t)
 $H^k[L](\emptyset)$

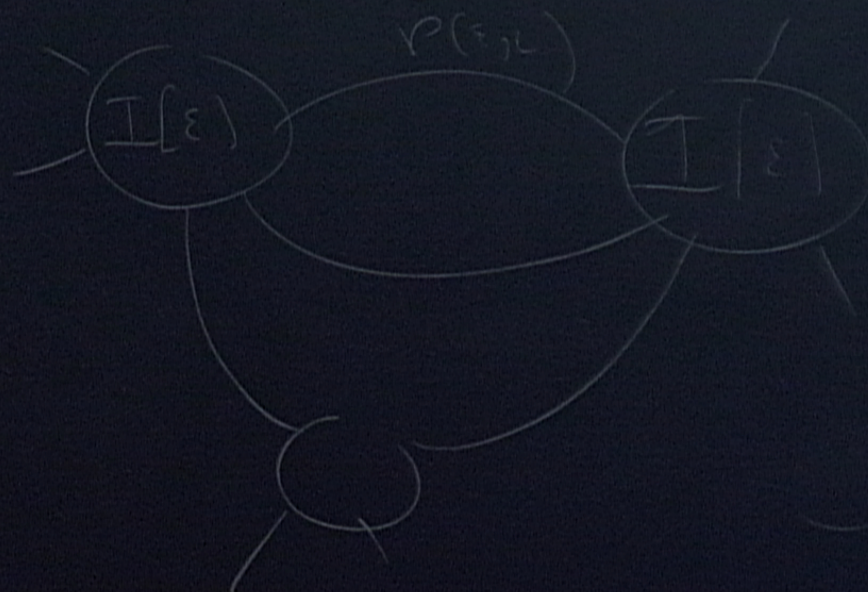
homogeneous of deg. i as a function

Related by

$$W(p(\varepsilon, L), I(\varepsilon)) = I[L]$$

$$I[L] = \sum_{\varepsilon} \dots$$

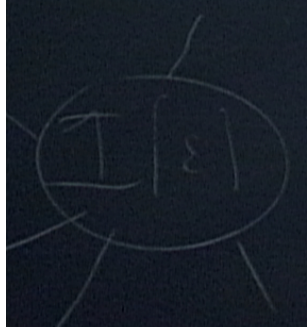
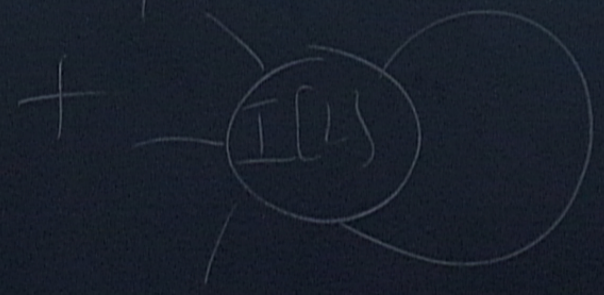
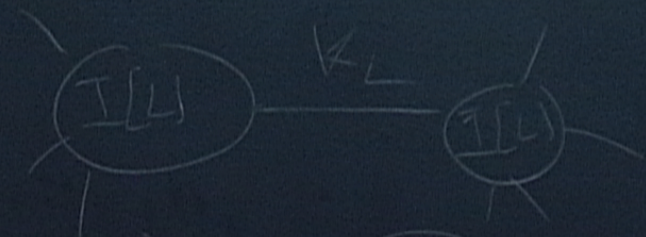
$(\varepsilon < L)$



Infinitesimal form

$$\frac{\partial}{\partial L} I[L]$$

$$= \sum$$



the

is close to local as $\varepsilon \rightarrow 0$

If, say, $\varphi_1 = 0$ outside $\|x\| < C$

$\varphi_2 = 0$ outside $\|x - x_0\| < C'$

Then, $\int_{I_{1,k}[\varepsilon]} (\varphi_1) (\varphi_2)$ disjoint regions $\rightarrow 0$ as $\varepsilon \rightarrow 0$

Proof.

Given the Lagrangian, call it I ,
we choose a scheme and let

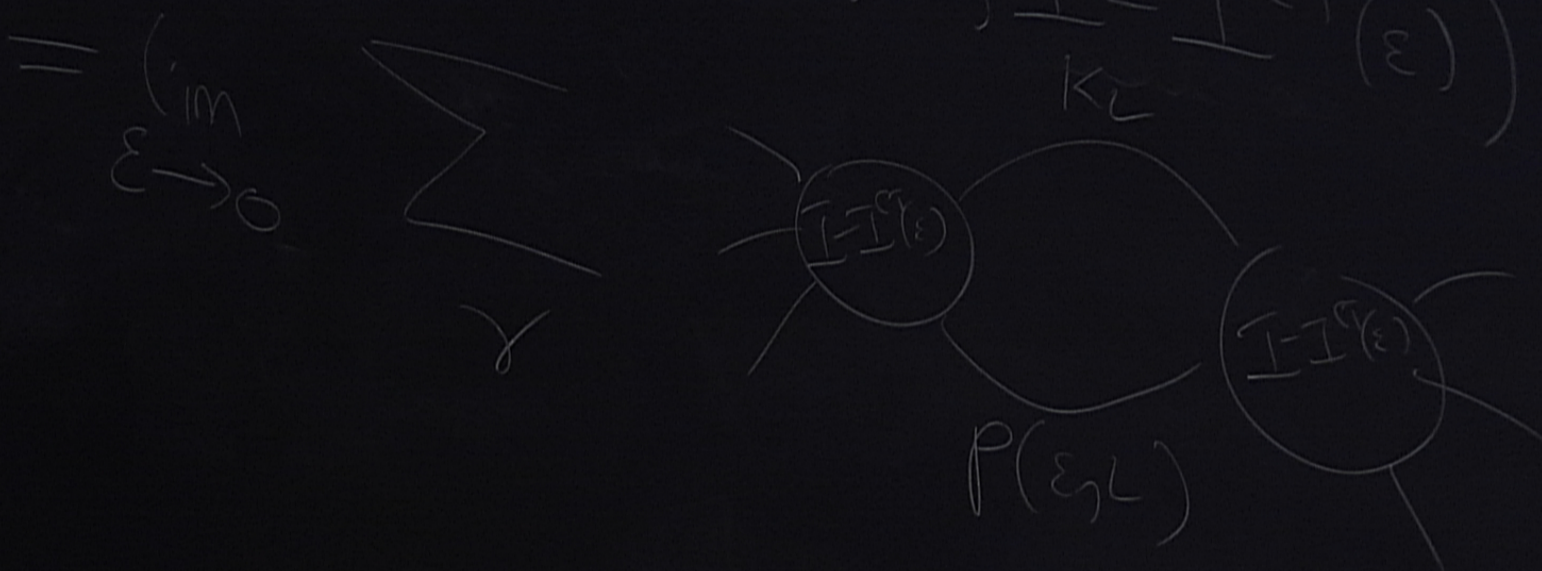
$$I[L] = \lim_{\epsilon \rightarrow 0} W(\rho(\epsilon, L), \overline{I - I^{\text{CT}}(\epsilon)})$$

$$= \lim_{\epsilon \rightarrow 0} \sum \int \text{[Diagram]}$$

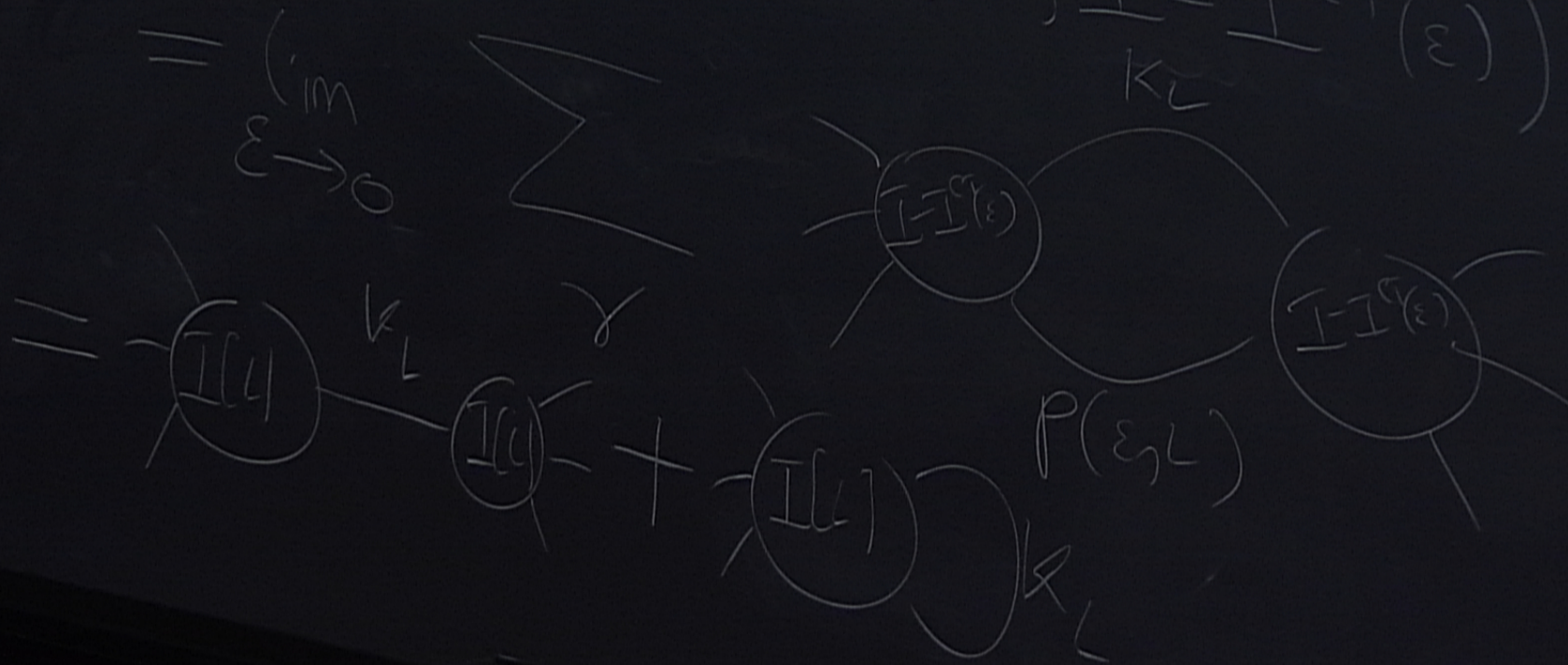
The diagram shows a sum over configurations of a system with two circular components. The left component is labeled $I - I^{\text{CT}}(\epsilon)$ and is surrounded by four external lines labeled φ . The right component is labeled $\rho(\epsilon, L)$ and is also surrounded by four external lines labeled φ . The two components are connected by two lines, one above and one below, both labeled $\rho(\epsilon, L)$.

$$\frac{d}{dL} I(L) = \lim_{\varepsilon \rightarrow 0} \frac{d}{dL} W(p(\varepsilon, L), I - I^{\text{GT}}(\varepsilon))$$

$$= \lim_{\varepsilon \rightarrow 0} \dots$$



$$\frac{d}{dL} I(L) = \lim_{\varepsilon \rightarrow 0} \frac{d}{dL} W(p(\varepsilon, L), I - I^{\text{CT}}(\varepsilon))$$



Lagrangian, call it I ,
 a scheme and let
 $\lim_{\varepsilon \rightarrow 0} W(P(\varepsilon, L), \underbrace{I - I^{GT}(\varepsilon)}_{P(\varepsilon, L)})$

The diagram consists of a large circle labeled $P(\varepsilon, L)$ at the bottom. Inside this circle, on the left side, is a smaller circle labeled $I - I^{GT}(\varepsilon)$. Several lines with the symbol φ at their ends connect the two circles. To the left of the large circle is a large summation symbol Σ . To the right of the large circle is another summation symbol Σ . The entire diagram is drawn in white on a dark background.

Converse:

If $I[L], \tilde{I}[L]$ s.t

$$I_{ss}[L] = \tilde{I}_{ss}[L] \text{ for}$$

then

$$I_{ijk}[L] - \tilde{I}_{ijk}[L] =$$

$s < k,$
 $s = k, r < 1$

a local Lagrangian.

$$\text{As } \frac{d}{dL} (I_{ijk}[L] - \tilde{I}_{ijk}[L]) = 0$$

Since it's local as $L \rightarrow 0$
 \Rightarrow it is a Lagrangian

since it's local as $L \rightarrow 0$

PHILOSOPHY OF RENORMALIZATION

- 1) \exists an ∞ dimⁿ manifold of QFT's
(in our set ups) given by a collection $\{I(L)\}$
gives coordinates on the manifold.
- 2) Choosing a scheme gives coordinates on the manifold.

THEORY OF RENORMALIZATION

- 1) Γ an ∞ dimⁿ manifold (in our set ups) given by a collection of QFTs $\{I_i\}$ on the manifold
- 2) Choosing a scheme gives coordinates on the manifold of QFTs \approx {Lagrangians}
- 3) Changes of schemes \approx CRAZY coordinate changes

4) Field theories can only be specified by symmetry properties

5) Most Important Symmetry
Consider translation invariant field theories on \mathbb{R}^n (kinetic term $\int \varphi \Delta \varphi$)
Scaling \mathbb{R}^n , sending $\varphi(x) \rightarrow \lambda^{1-n/2} \varphi(\lambda x)$
gives an action of $\mathbb{R}_{>0}$ on theories

Good = UV complete / renormalizable theories

(live on a trajectory converging to a fixed point in the UV)

Typical behaviour

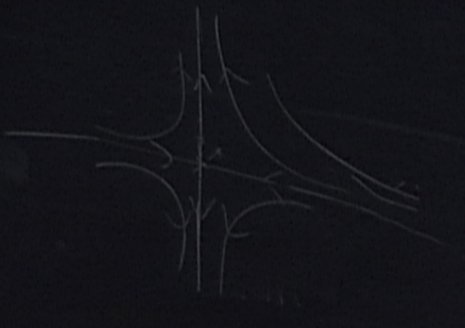
theory, T_p

(Space of theories)

At the fixed point^p given by free

has finitely many negative eigenspaces
 ∞ many +ve eigenspaces

Lagrangian



Classically,
Look at

Scalar f on \mathbb{R}^4

'invariant'
 $\int_{\mathcal{L}} \Delta \phi d^4x$

$SO(4) \times$ translations
Lagrangians

\mathcal{L} has $\dim n$

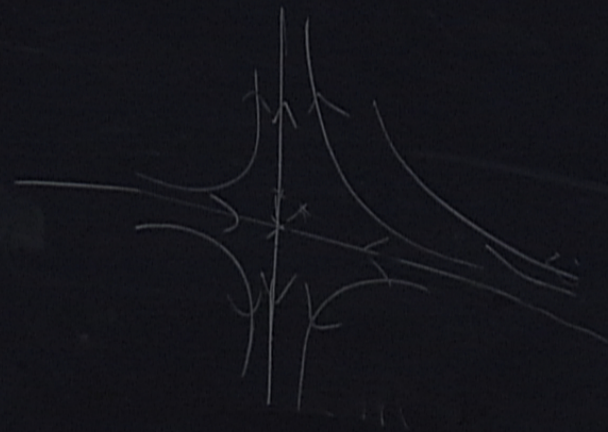
$\int_{\mathcal{L}^4} \dim n \leq 0$
 $\mu \int_{\mathcal{L}^3}$
0

Lagrangians

\Rightarrow UV complete trajectories
 \leftarrow only possibilities

-1
-2

UV $\mu \rightarrow 0$



Classically,
 Look at
 'invariant'
 $\int \mathcal{L} \Delta \phi d^4x$

$\mu^{-2} \int \phi^6$ $\ln \log \mu$ $\int \phi^4$ Dim n $\mu \int \phi^3$ $\mu^2 \int \phi^2$ $\int \phi \Delta \phi d^4x$ Lagrangian