Title: Renormalization and Effective Field Theory - Lecture 4

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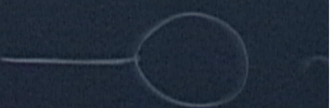
Abstract:

$$W(P, I) = \hbar \log e^{\hbar \partial_P} e^{I/\hbar} = \sum_{\substack{\text{diagrams } \gamma \\ \text{connected} \\ \text{w. external} \\ \text{lines}}} \hbar^{l(\gamma}$$



Example

ϕ^3 on $\mathbb{R}^n$

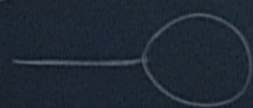
Expand $I^{CT}(\varepsilon)$ as a sum

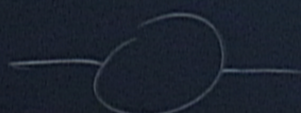
$$\sum \hbar^k I^{CT}_{i,k}(\varepsilon)$$

i = # of external lines

$$e^{I/\hbar} = \sum_{\substack{\text{diagrams } \gamma \\ \text{connected} \\ \text{w. external} \\ \text{lines}}} \hbar^{l(\gamma)} W_\gamma \left(\begin{array}{l} P \text{ on edges} \\ I \text{ on vertices} \end{array} \right)$$

a sum

 $\rightsquigarrow I^{CT}_{1,1}(\varepsilon) \sim \varepsilon^{-n/2} \int \phi$

 $\rightsquigarrow I^{CT}_{1,2}(\varepsilon) \sim \log \varepsilon \int \phi^2$

$(n=4)$

Last time

Given a Lagrangian $\int \phi \Delta \phi + I(\phi)$

we showed that $\exists$ counter terms

$I^{CT}(\varepsilon)$ such that

$$W(P(\varepsilon, L), I - I^{CT}(\varepsilon))$$

has an $\varepsilon \to 0$ limit.

Def^n

A scalar field theory on a manifold M is a collection $I[L](\phi)$ of fnls of ϕ (als[illegible] de[illegible]

$$I[L] = \sum \hbar^k I_{i,k}[L](\phi)$$

$I_{i,k}[L]$ is homogeneous of deg. i as a func[illegible] of ϕ

…eory on a manifold M

…$[L](\phi)$ of fnls of ϕ (also depend on t_i)

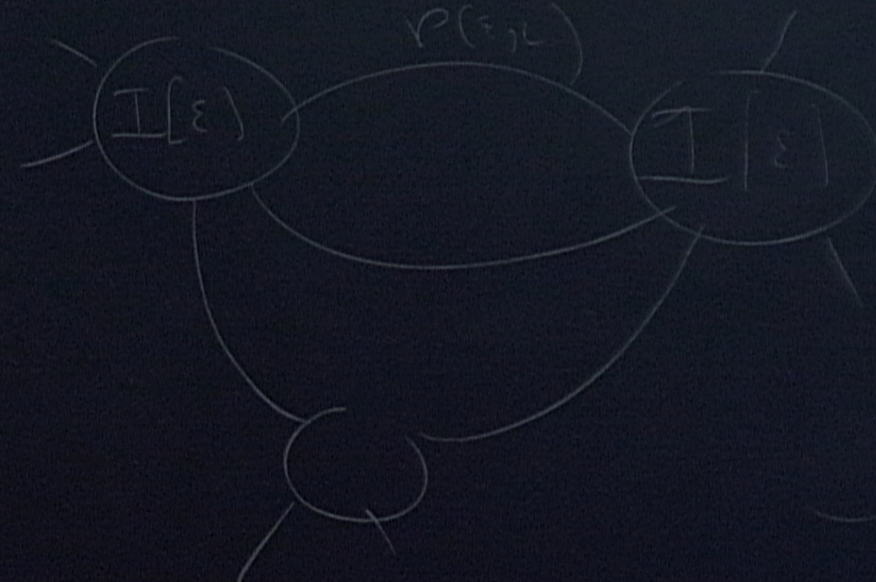
…$_{i,j,k}[L](\phi)$

homogeneous of deg. i as a function

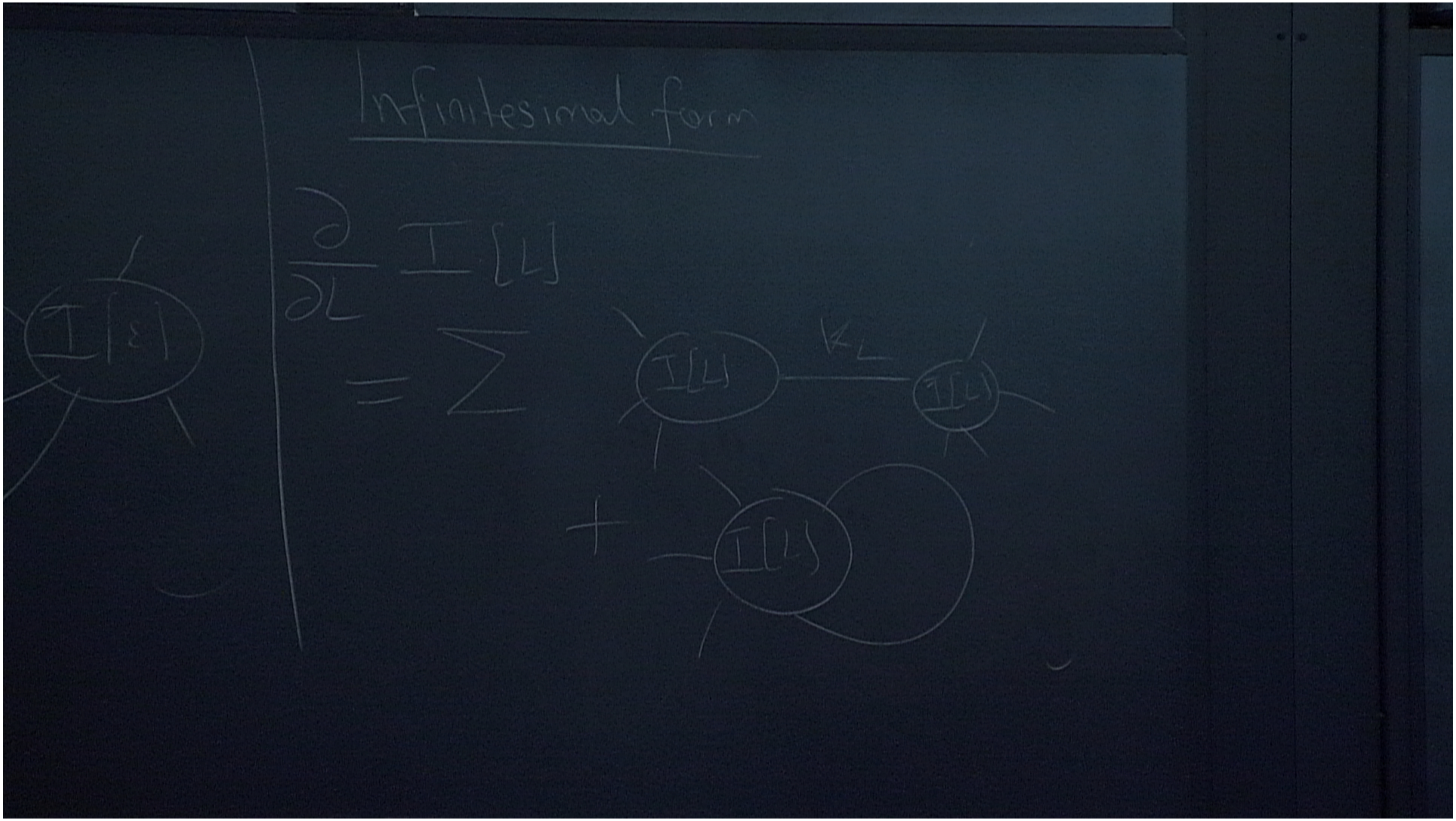
Related by:

$$W\left(\rho(\varepsilon, L), I[\varepsilon]\right) = I[L]$$

$$I[L] = \sum_{\gamma}$$



$(\varepsilon < L)$


Infinitesimal form
∂/∂L I[L]
= Σ
I[L]
K_L
I[L]
+
I[L]
I[ε]

ire

is close to local as $\varepsilon \to 0$

If, say, $\varphi_1 = 0$ outside $\|x\| < C$

$\varphi_2 = 0$ outside $\|x - x_0\| < C'$

φ_1 φ_2 disjoint regions

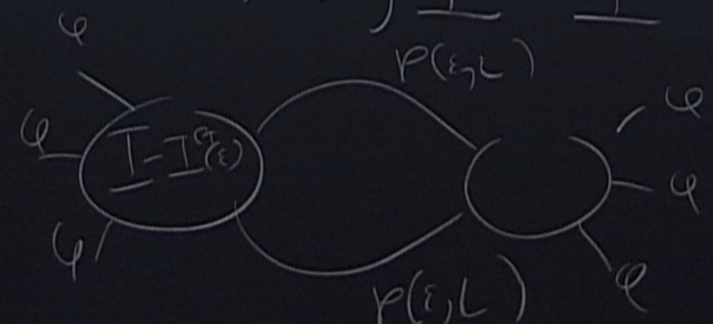
Then, $I_{1,k}[\varepsilon] \longrightarrow 0$ as $\varepsilon \to 0$

Proof.

Given the Lagrangian, call it I, we choose a scheme and let

$$I[L] = \lim_{\varepsilon \to 0} W\left(P(\varepsilon, L), I - I^{CT}(\varepsilon)\right)$$

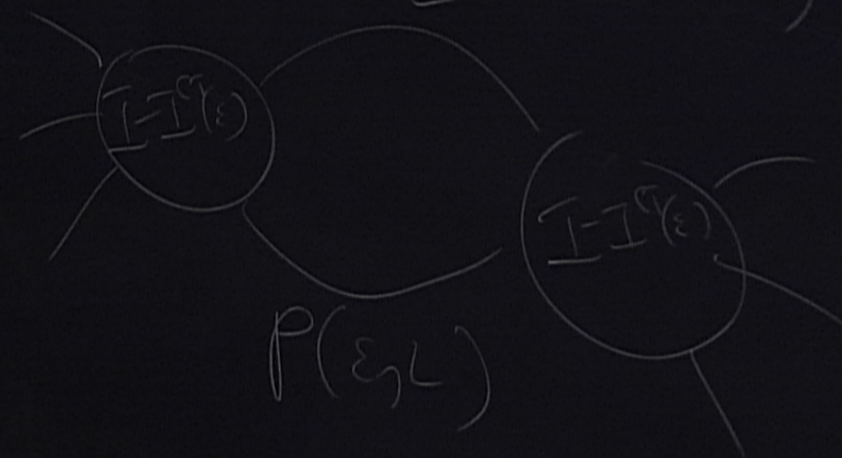
$$= \lim_{\varepsilon \to 0} \sum$$



$$\frac{d}{dL} I[L] = \lim_{\varepsilon \to 0} \frac{d}{dL} W\left(P(\varepsilon, L), I - I^{CT}(\varepsilon)\right)$$

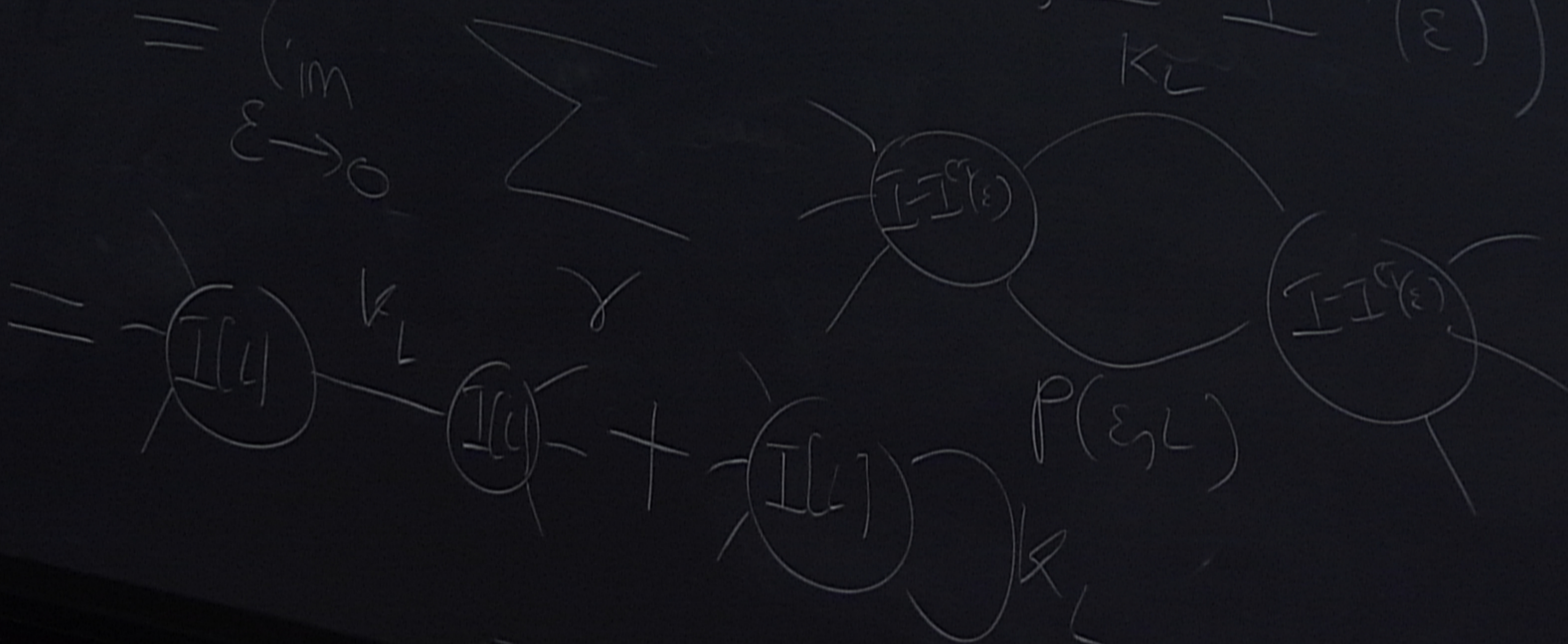
K_L

$$= \lim_{\varepsilon \to 0} \sum_{\gamma}$$



$$\frac{d}{dL} I[L] = \lim_{\varepsilon \to 0} \frac{d}{dL} W\left(P(\varepsilon, L), I - I^{CT}(\varepsilon)\right)$$

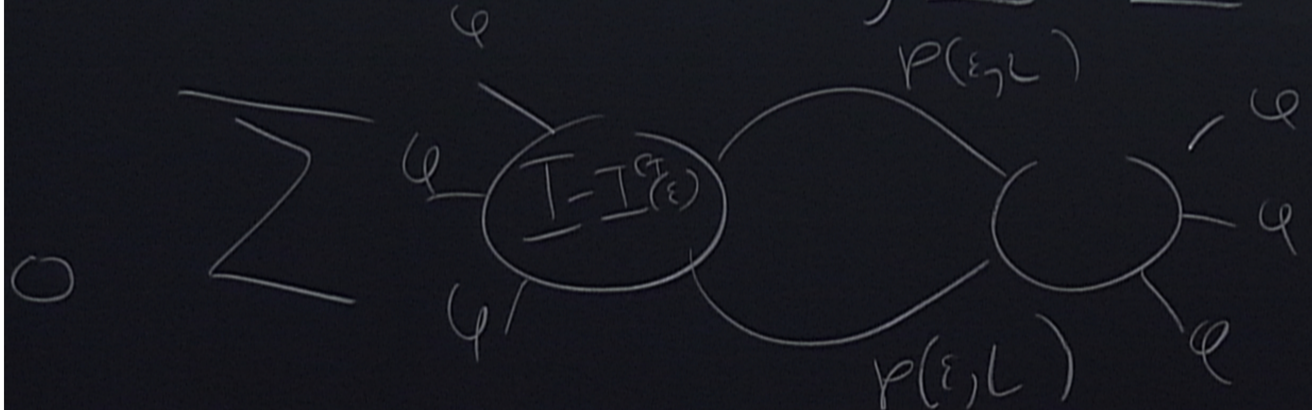
$$= \lim_{\varepsilon \to 0} \sum$$



Lagrangian, call it I,

a scheme and let

$$\lim_{\varepsilon \to 0} W\left(\varphi(\varepsilon, L), I - I^{CT}(\varepsilon)\right)$$



Converse:

If $I[L]$, $\tilde{I}[L]$ s.t

$$I_{r,s}[L] = \tilde{I}_{r,s}[L] \quad \text{for } s<k, \; s=k, r<i$$

then

$$I_{i,k}[L] - \tilde{I}_{i,k}[L] = \text{a local Lagrangian.}$$

As, $\frac{d}{dL}\left(I_{i,k}[L] - \tilde{I}_{i,k}[L]\right) = 0$ Since it's local as $L \to 0$

$\Longrightarrow$ it is a Lagrangian

... since it's local as $L \to 0$

PHILOSOPHY OF RENORMALIZATION

1) $\exists$ an ∞ dim^n manifold of QFT's
(in our set ups given by a collection $\{I[L]\}$)

2) Choosing a scheme gives coordinates on the manifold.

... ITY OF RENORMALIZAT...

1) $\exists$ an ∞ dim^n manifold of QFT...
(in our set ups given by a collection $\{I[L]\}$)

2) Choosing a scheme gives coordinates on the m...

3) Manifold of QFTs $\approx$ {Lagrangians}
Changes of schemes $\approx$ CRAZY coordinate changes

4) Field theories can only be specified by symmetry properties

5) Most Important Symmetry

Consider translation invariant field theories on $\mathbb{R}^n$ (kinetic term $\int \varphi \Delta \varphi$)

Scaling $\mathbb{R}^n$, sending $\varphi(x) \longrightarrow \lambda^{1-n/2} \varphi(\lambda x)$

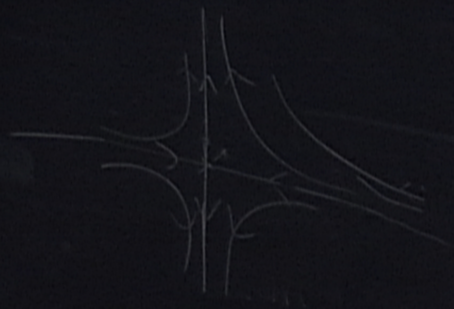
gives an action of $\mathbb{R}_{>0}$ on theories

Good = UV complete / renormalizable theories (live on a trajectory converging to a fixed point in the UV ✓

Typical behaviour

At the fixed point p given by free theory, T_p(Space of theories) has finitely many negative eigenspaces

∞ many +ve eigenspaces



...rangian

Classically, scalar f't on $\mathbb{R}^4$

Look at $SO(4) \ltimes$ translation

invariant Lagrangians

$\int \varphi \Delta \varphi \, d^4x$

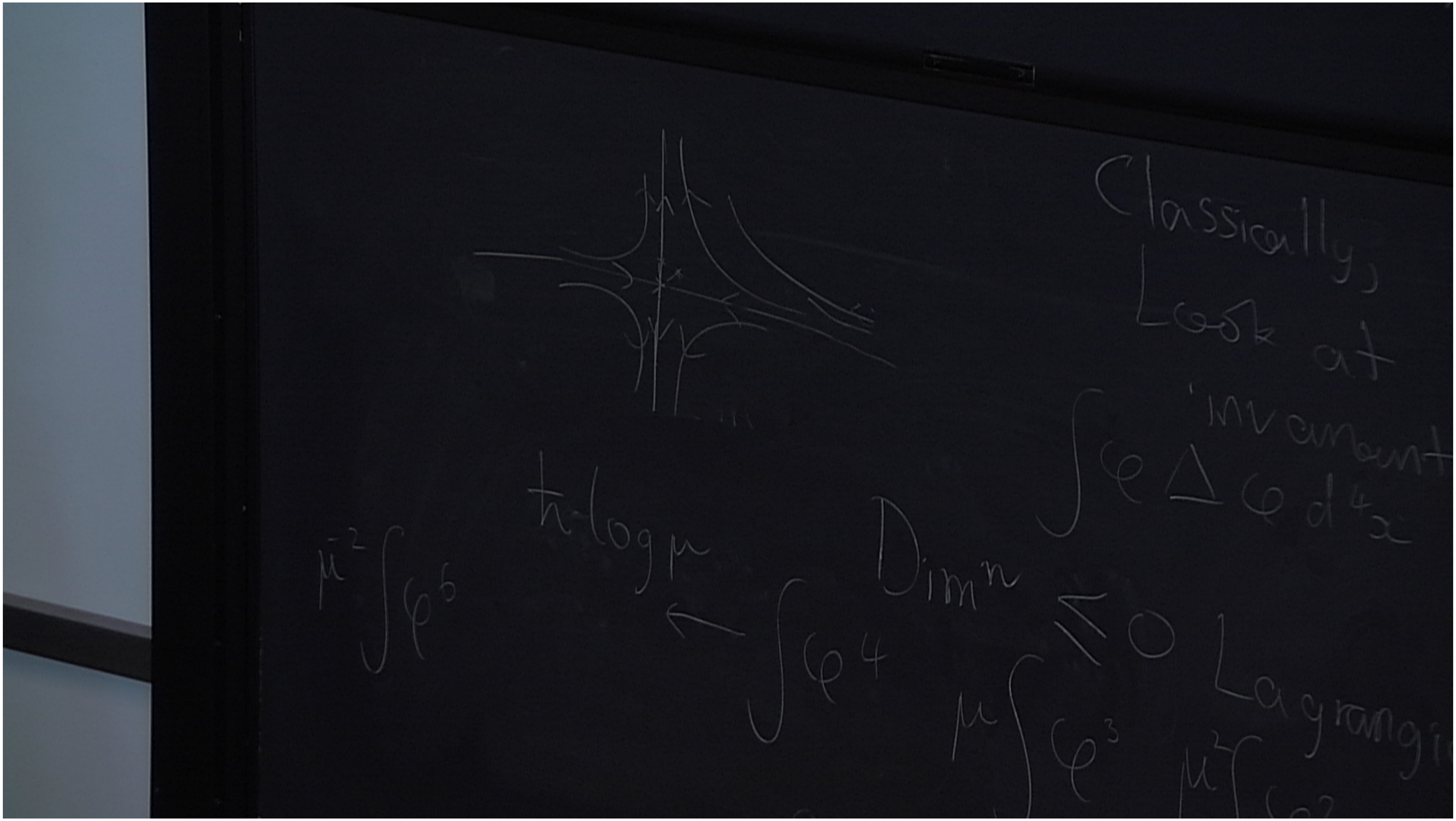
φ has dim^n 1

Dim^n ≤ 0 Lagrangians $\Rightarrow$ UV complete trajectories

$\int \varphi^4$ $\quad \mu \int \varphi^3$ $\quad \mu^2 \int \varphi^2$ $\leftarrow$ only possibilities

0 $\quad -1$ $\quad -2$

UV $\mu \to 0$


Classically,
Look at
'invariant'
∫φΔφ d⁴x
ħ log μ
μ⁻² ∫φ⁶
∫φ⁴
Dimⁿ ≤ 0
Lagrangi
μ ∫φ³
μ² ∫φ²