Title: Fault-tolerant magic state preparation with flag qubits

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Abstract: <p>Despite considerable effort, magic state distillation remains one of the leading candidates to achieve universal fault-tolerant quantum computation. However, when analyzing magic state distillation schemes, it is often assumed that gates belonging to the Clifford group can be implemented perfectly. In many current quantum technologies, two-qubit Cliffords gates are amongst the noisiest components of quantum computers. In this talk I will present a new scheme for preparing magic states with very low overhead that uses flag qubits. I will then compare our scheme to leading magic state distillation methods and show that the overhead can be reduced by orders of magnitude.</p>

Fault-tolerant magic state preparation with flag qubits

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Ingredients needed to build a universal quantum computer

- Qubits with long coherence times.
- Reliable and fast gates/measurements (also fast classical resources).
- Error correction (errors unavoidable, must be able to correct some of them with high probability).
- Fault-tolerant quantum error correction (must have a way to do error correction with noisy gates and measurements without errors spreading too badly).
- **Fault-tolerant quantum computation**: need the ability to perform logical gates reliably.

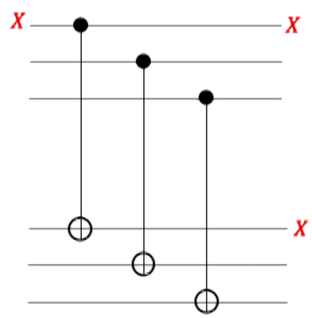
Focus of this talk

[1] B. Eastin and E. Knill, “Restrictions on transversal encoded quantum gate sets," Phys. Rev. Lett., vol. 102,p. 110502, 2009.

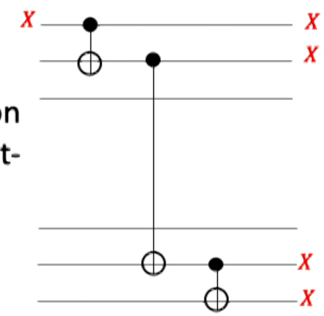
Fault-tolerance with transversal gates

- Transversal operation: *1) Only applies one qubit gates to the qubits in a code block. 2) It only interacts the i^{th} qubit in one code block with the i^{th} qubit in a different code block or ancilla block*

Example of a transversal CNOT gate on a 3-qubit code



Example of non-transversal CNOT gate on a 3-qubit code. Not fault-tolerant



- Eastin-Knill Theorem: The set of transversal gates for a given code generates a finite group. Cannot be universal for quantum computation. [1]

Several schemes to achieve fault-tolerant quantum computing on a universal gate set

- Magic state distillation (the focus of this talk)
- Universal concatenated quantum codes [2]
- Code switching [3,4]
- Transversal gates + error correction [5]
- Intermediate error correction rounds after non-transversal gates [6]

[2] T. Jochym-O'Connor and R. Laflamme, Phys. Rev. Lett (2014)
[3] J. T. Anderson, G. Duclos-Cianci and D. Poulin, Phys. Rev. Lett. (2014)
[4] H. Bombin N.J.P (2015)
[5] A. Paetznick and B. W. Reichardt Phys. Rev. Lett (2013)
[6] T. J. Yoder, R. Takagi and I. L. Chuang Phys. Rev. X (2016)

[7] S.Bravyi and A. Kitaev, Phys. Rev. A (2005)

Magic state distillation

- A state that can both be used to achieve universal quantum computation and be distilled **using only Clifford gates** (and |0> state prep + Z basis measurements) [7].

Clifford group: $$\mathcal{P}_n^{(2)} = \{U : UPU^\dagger \in \mathcal{P}_n^{(1)} \forall P \in \mathcal{P}_n^{(1)}\}$$

Generated by: $$\mathcal{P}_n^{(2)} = \langle H_i, S_i, \mathrm{CNOT}_{ij} \rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \text{and} \quad S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

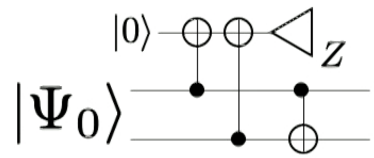
Universal quantum computing with Clifford's + resource states

Phase shift gate

- Suppose you have the resource state $|A_\theta\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle)$ and want to apply $\Lambda(e^{i\theta})|\psi\rangle$ where $\Lambda(e^{i\theta}) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ and $|\psi\rangle = a|0\rangle + b|1\rangle$.

Step 1): Prepare $|\Psi_0\rangle = |\psi\rangle \otimes |A_\theta\rangle$ and measure $S_1 = Z \otimes Z$. Apply CNOT from qubit 1 to qubit 2 and discard second qubit.

$$|\Psi_3^{\pm}\rangle = \Lambda(e^{\pm i\theta})|\psi\rangle$$



Step 2): Apply circuit repeatedly to get transformations $\Lambda(e^{ip_1\theta}), \Lambda(e^{ip_2\theta}), \cdots$ for some integers $p_1, p_2, \cdots$ obeying random walk statistics.

Eventually get $p_k = 1$. $P(n > N) = cN^{-1/2}$. If $\theta = \frac{p}{q}2\pi$ where $p, q \in \mathbb{Z}$ then probability of more than N steps decreases exponentially with N .

Distilling resource states

- Idea is to encode the resource state in an error correcting/detecting code.
- Measure the state to see if there is a logical fault. Also measure the error syndrome. If logical fault or non-trivial syndrome, reject the state.
- Accepted resource states are measured again.
- Above procedure repeated multiple times until resource states have desired failure rate (depends on the particular algorithm that one wants to implement).

Above can all be done using only Clifford gates, $|0\rangle$ states and Z-basis measurements.

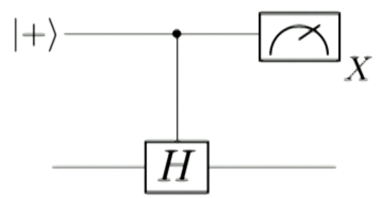
Example: Meier-Eastin-Knill (MEK) scheme (1)

Consider the following state: $|H\rangle = \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle = T|0\rangle$ with $T = e^{-i\frac{\pi Y}{8}} = \begin{pmatrix} \cos\frac{\pi}{8} & -\sin\frac{\pi}{8} \\ \sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{pmatrix}$

$$Y|H\rangle = -|H\rangle \qquad H|\pm H\rangle = \pm|H\rangle$$

Related to $|A_{\frac{\pi}{4}}\rangle$ by $|A_{\frac{\pi}{4}}\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle) = e^{i\frac{\pi}{8}}HS^{\dagger}|H\rangle$ So can use $|H\rangle$ as a resource state.

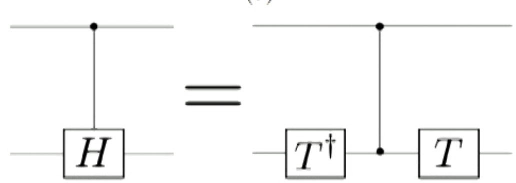
Can measure the Hadamard operator using

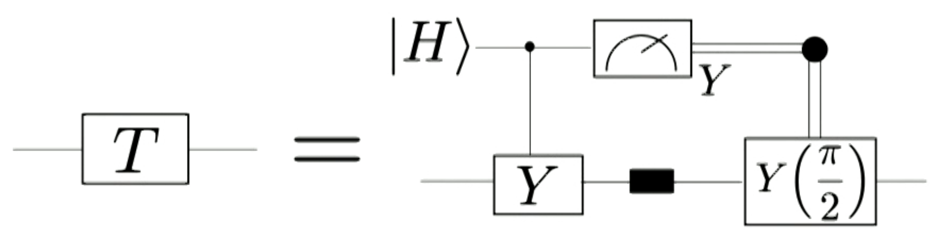


(a)

With additional resource states the circuits use only Clifford's.

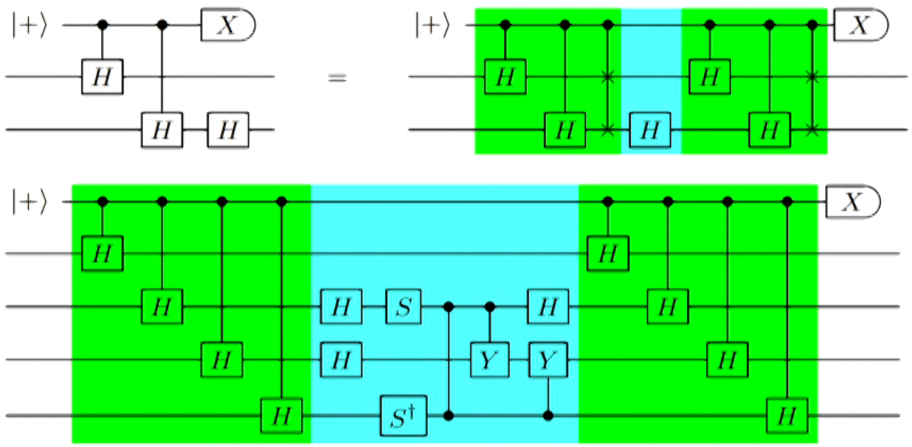
$$Y\left(\frac{\pi}{2}\right) = e^{-i\frac{\pi Y}{4}}$$





Example: Meier-Eastin-Knill (MEK) scheme (2)

Errors in the extra resource states could result in the wrong measurement outcome. Perform an encoded version.

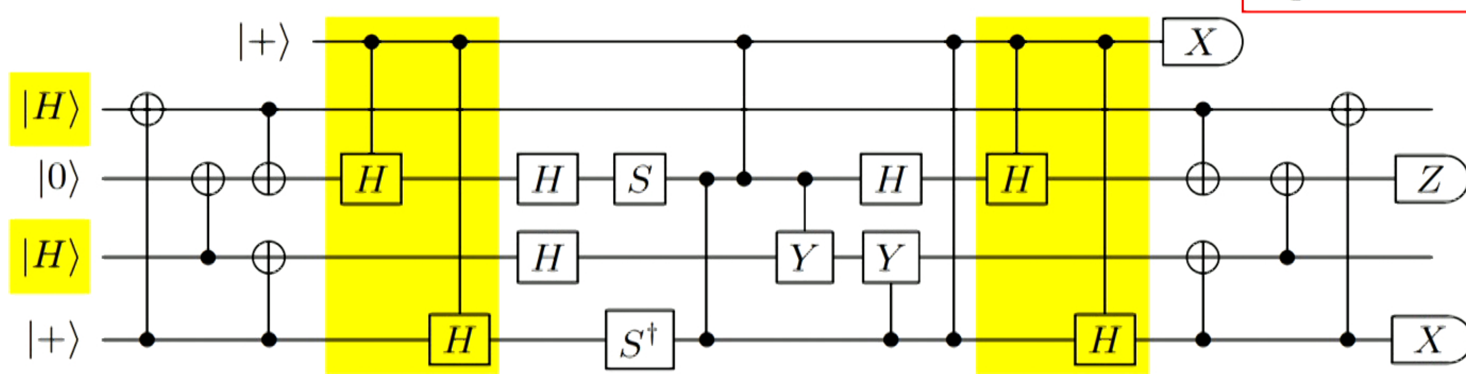


Encode in the [[4,2,2]] code.

Stabilizers $\begin{bmatrix} X \otimes X \otimes X \otimes X \\ Z \otimes Z \otimes Z \otimes Z \end{bmatrix}$

$$\bar{X}_1 = X \otimes X \otimes I \otimes I,$$
$$\bar{Z}_1 = Z \otimes I \otimes I \otimes Z,$$
$$\bar{X}_2 = X \otimes I \otimes I \otimes X,$$
$$\bar{Z}_2 = Z \otimes Z \otimes I \otimes I.$$

Full distillation routine



Problems when Clifford gates are noisy

- With current quantum devices, Clifford gates (in particular two qubit gates) can be very noisy.
- Must use **encoded** Clifford operations in Magic state distillation circuits.
- To achieve very low logical failure rates ($\leq 10^{-10}$ to 10^{-15}), will require encoded Clifford gates in very high distance codes.
- Consequently the qubit and gate overhead for magic state distillation routines can be very large.

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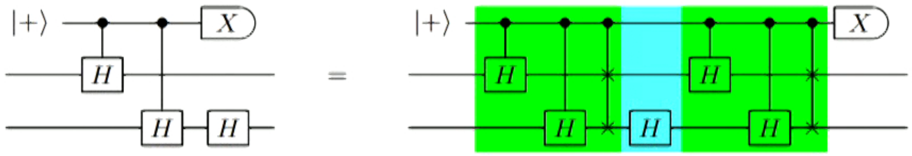

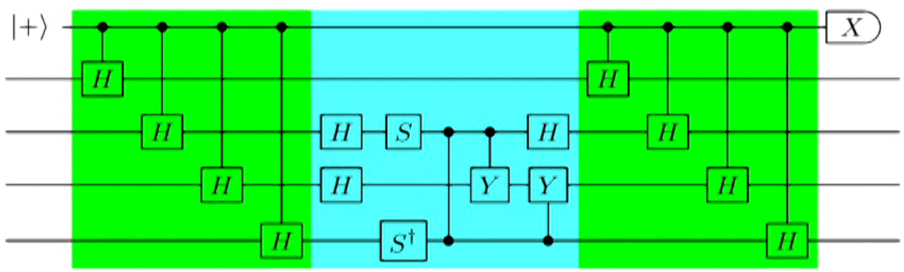

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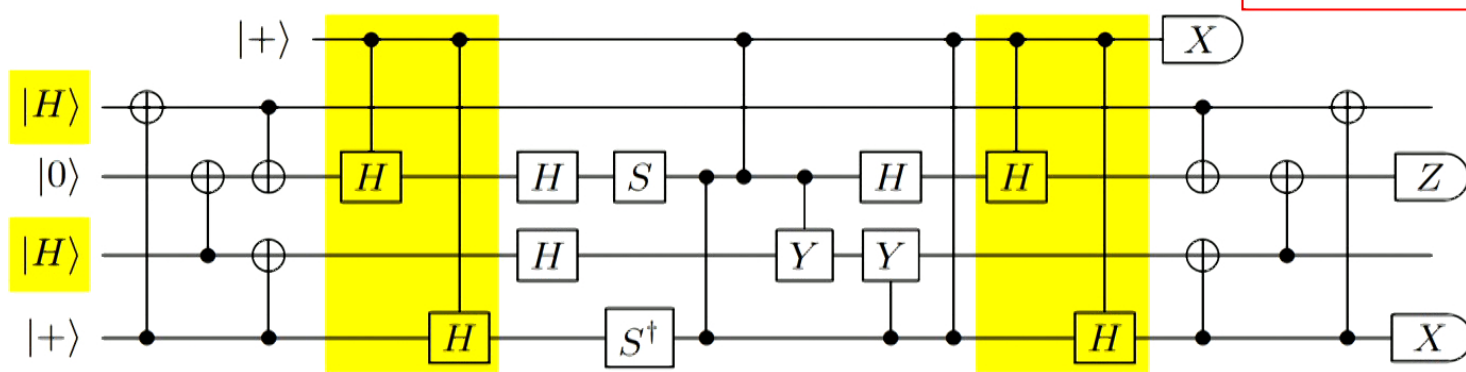

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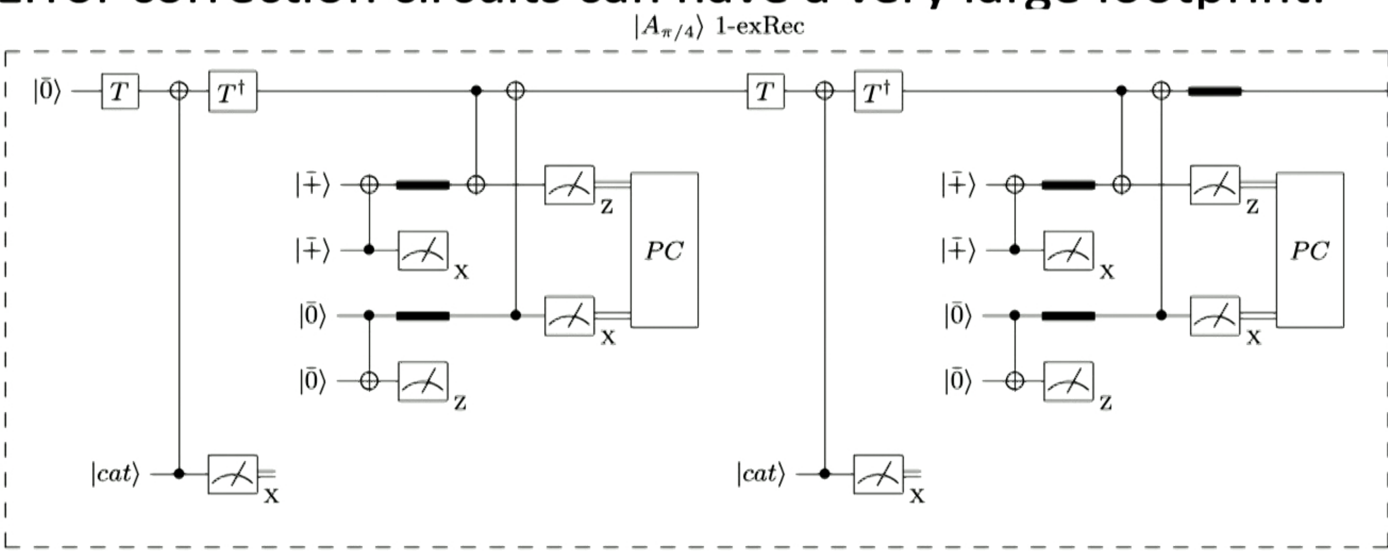
Magic state preparation with fault-tolerant circuits

- Instead of using magic state distillation routines, we can directly prepare magic states using fault-tolerant circuits.
- Previous methods require large ancilla states and have very low thresholds.
- Error correction circuits can have a very large footprint.

Example:[8]



- Requires large cat states that need to be verified for errors.
- Steane EC has a large footprint.

[8] P. Aliferis, D. Gottesman and J. Preskill, QIC (2006).

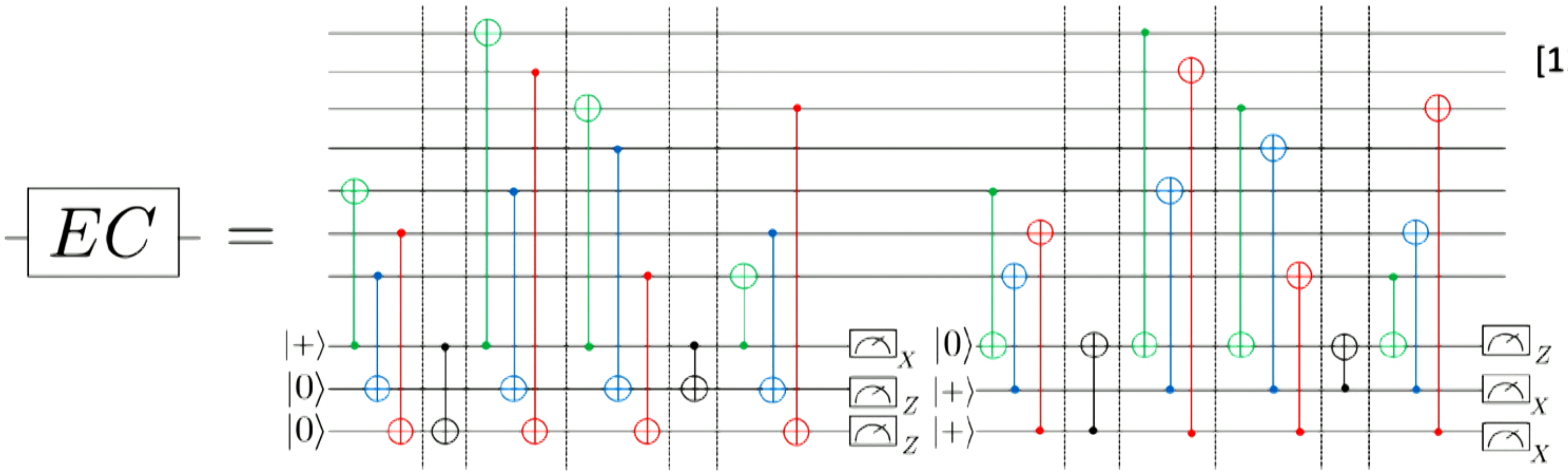
Use flag qubits to construct fault-tolerant circuits to prepare magic states

- Flag qubits were introduced in order to do fault-tolerant error correction using the minimum number of ancillas [9].
- Further generalizations were obtained in [10,11].
- Can use flag qubits to fault-tolerantly prepare magic states with very low qubit overhead.
- No need to prepare large ancilla states that need to be verified.
- Lower overhead compared to MEK by several orders of magnitude.

[9] R. Chao and B. Reichardt, Phys. Rev. Lett (2018)
[10] C. Chamberland and M. Beverland, Quantum (2017)
[11] T. Tansuwannont, C. Chamberland and D. Leung (2018)

Error correction and state-preparation circuits

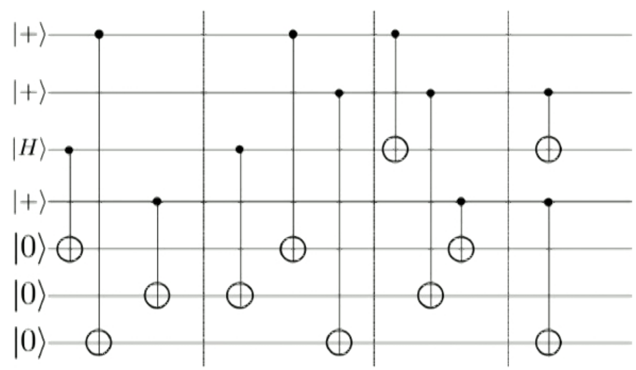
[12] B. Reichardt (2018)



$[\![7,1,3]\!]$ Steane code

$$g_1 = XIXIXIX$$
$$g_2 = IIIXXXX$$
$$g_3 = IXXIIXX$$
$$g_4 = ZIZIZIZ$$
$$g_5 = IIIZZZZ$$
$$g_6 = IZZIIZZ$$
$$\overline{X} = X^{\otimes 7}$$
$$\overline{Z} = Z^{\otimes 7}$$

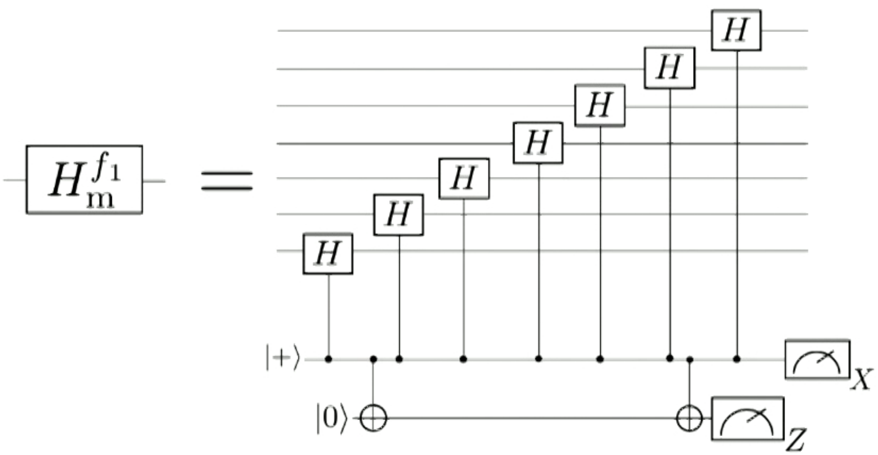
$|\overline{H}\rangle_{\mathrm{nf}} =$



Non fault-tolerant circuit for preparing the Hadamard eigenstate.

Magic state preparation protocol

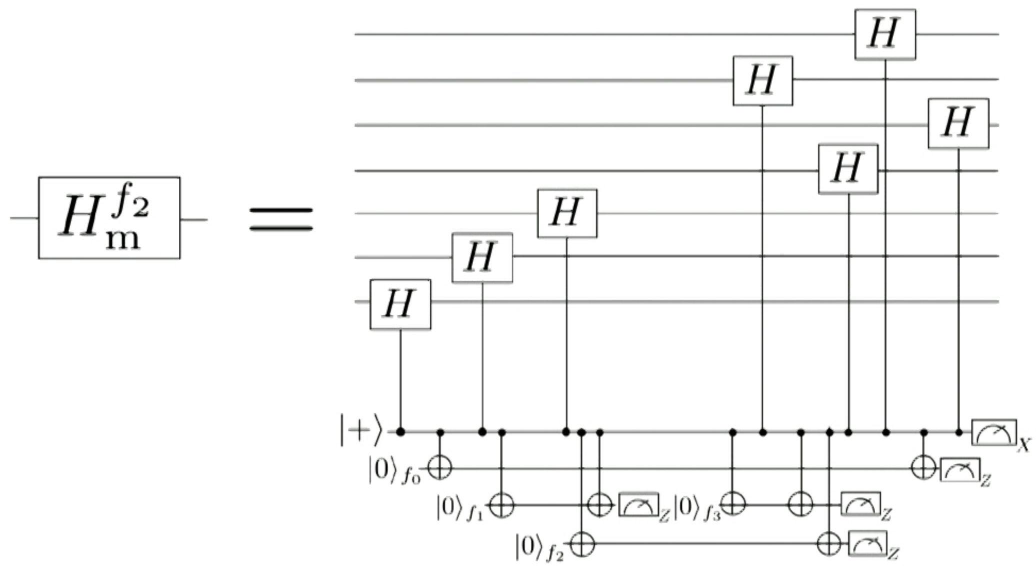
Error detection scheme



(a)

$|\overline{H}\rangle_{\mathrm{nf}}$ — $H_{\mathrm{m}}^{f_1}$ — EC —

Error correction scheme



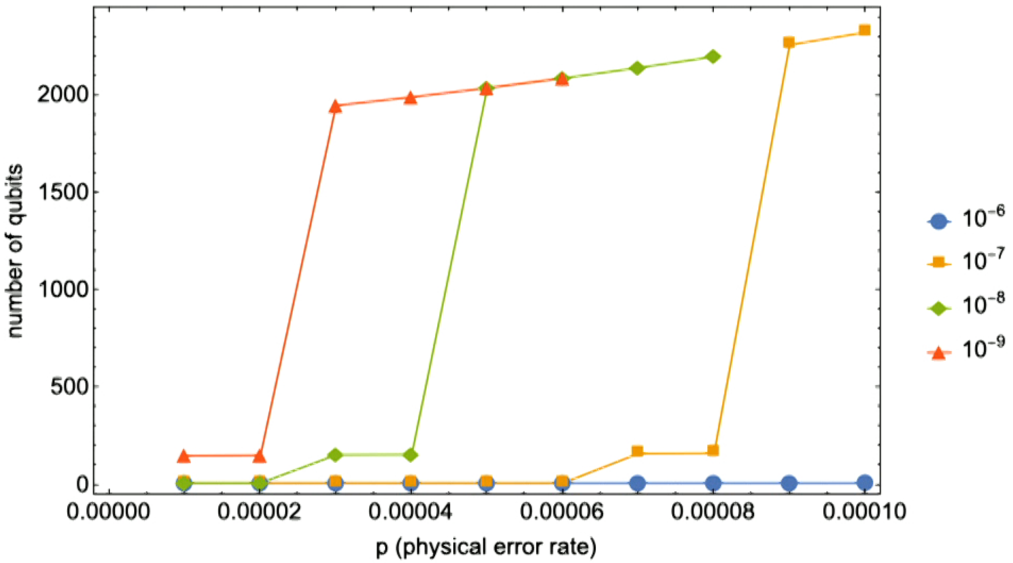
(a)

$|\overline{H}\rangle_{\mathrm{nf}}$ — $H_{\mathrm{m}}^{f_2}$ — EC — $H_{\mathrm{m}}^{f_2}$ — EC — $H_{\mathrm{m}}^{f_2}$ —

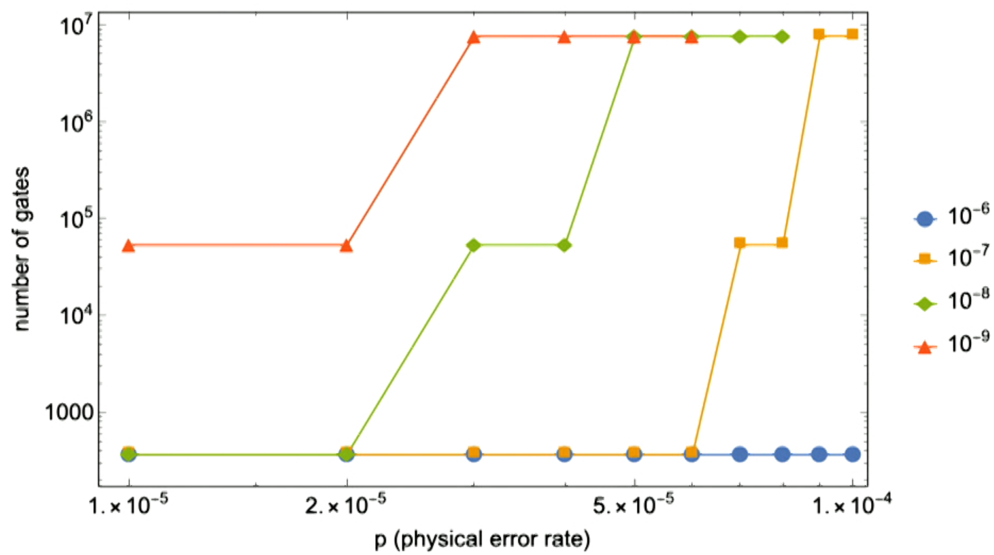
Overhead analysis of our scheme

Increase distance through code concatenation.

Qubit overhead



Gate overhead



Conclusions

- Difficult to measure high-weight logical Hadamard operators using flags (we have a circuit for the [[17,1,5]] color code).
- Our scheme requires code concatenation in order to scale to higher distances.
- Overhead results look very promising.
- Currently noisy Clifford gates seem nearly unavoidable.
- Our results could pave the way for further exploration of fault-tolerant magic state preparation.