Title: Numerical Simulation of the Axion Field through the QCD Phase Transition

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Abstract: <p>We perform a full (3+1)-dimensional numerical simulation of the axion field around the QCD epoch. Our aim is to fully resolve large dynamical non-linear effects in the inhomogenous axion field. These effects are important as they lead to large overdensities in the field at late times. Those overdensities will eventually evolve into axion minicluster, which have various phenomenological implications like microlensing events. It is therefore important to have a reliable estimate of the number of overdensities and their mass relation.</p>

Simulations of a Cosmological Axion through the QCD Phase Transition

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University of Michigan

10/23/2018
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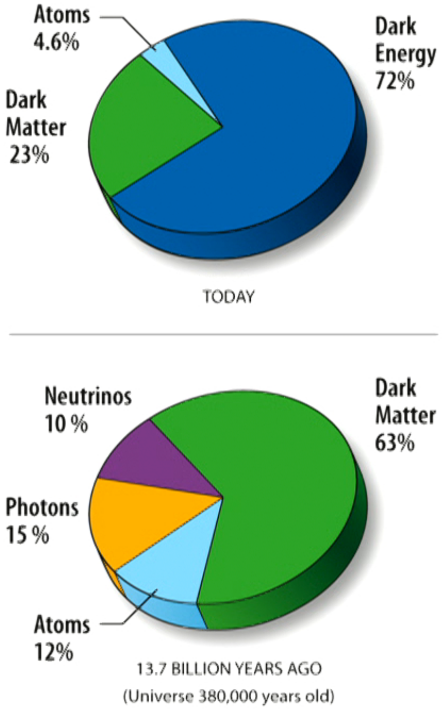
Dark Matter Key Facts

- ~23% of the total energy in the Universe is matter of an unknown kind
- Many viable explanations exist: One of the best motivated is **axion dark matter**
- Much of its properties today depend on the dynamics in the early Universe



NASA / WMAP Science Team

Axions

- Axions originally introduced to solve the **strong CP problem**:

$$\mathcal{L} = \theta \frac{1}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a} \longrightarrow \mathcal{L}_{axion} = (\partial_\mu a)^2 + \frac{(a/f_a + \theta)}{32\pi^2} F\tilde{F}$$

- U(1) PQ symmetry **spontaneously broken** at high scale, roughly $f_a \approx 10^{12}$ GeV

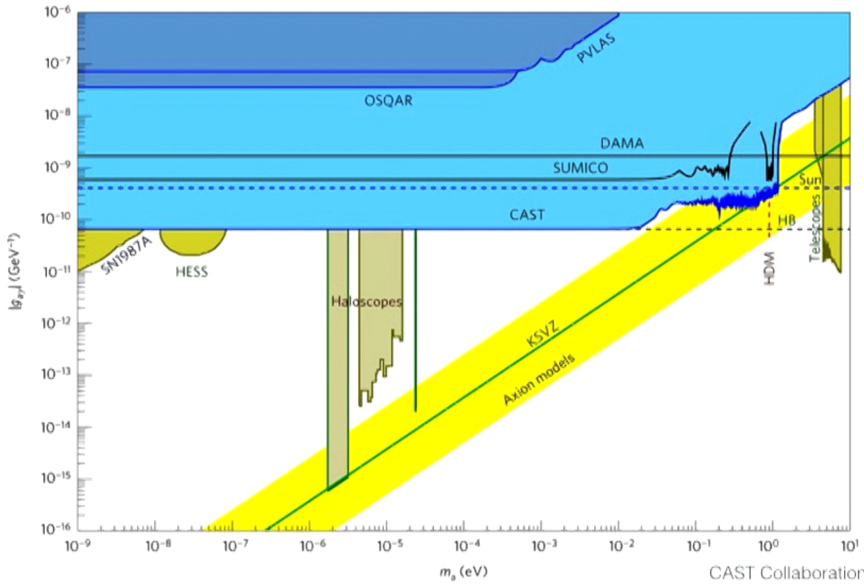
- **Axion mass** is small (QCD effects),

$$m_a^2 \approx \frac{m_\pi^2 f_{\pi^2}}{f_a^2}$$

as are its couplings

↓

good cold **DM candidate**

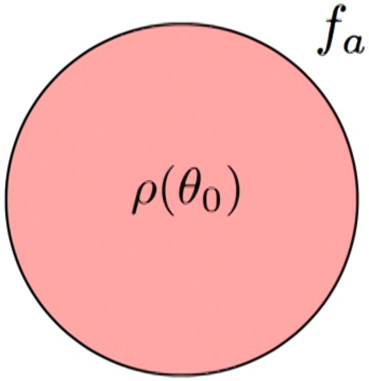


Post- vs Pre-inflationary scenario

Two different scenarios can be considered:
Breaking the PQ symmetry before or after inflation

before inflation:

$f_a > \frac{H_I}{2\pi}$

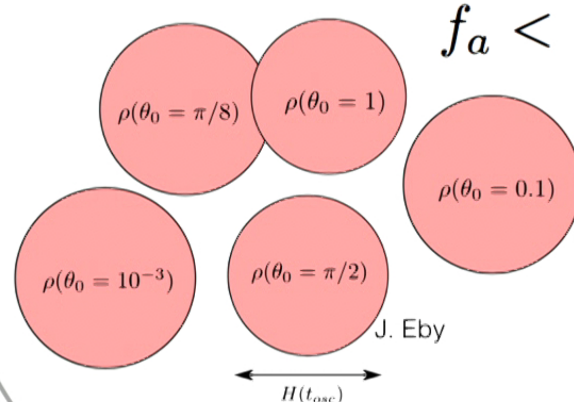


$$\Omega_{a,0} \sim {\theta_0}^2$$

two free parameter: θ_0 , f_a

after inflation:

$f_a < \frac{H_I}{2\pi}$



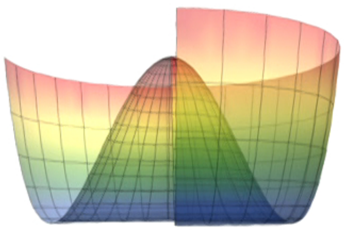
$$\Omega_{a,0} \sim \langle {\theta_0}^2 \rangle$$

one free parameter: f_a
inhomogeneous at small scales

PQ phase transition:

$$@\ T \approx f_a$$

$$V(\Phi, T) = \frac{\lambda}{4}\left(|\Phi|^2 - f_a^2\right)^2$$

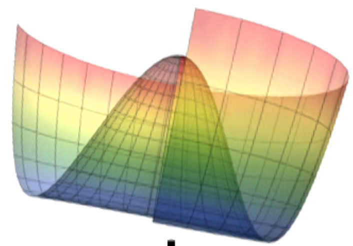


Formation of a string network

QCD phase transition:

$$@\ T \approx 1\ \mathrm{GeV}$$

$$V(\Phi, T) = \frac{\lambda}{4}\left(|\Phi|^2 - f_a^2\right)^2 + m_a(T)^2 f_a^2\left[1 - \cos \mathrm{Arg}(\Phi)\right]$$



Formation of domain walls

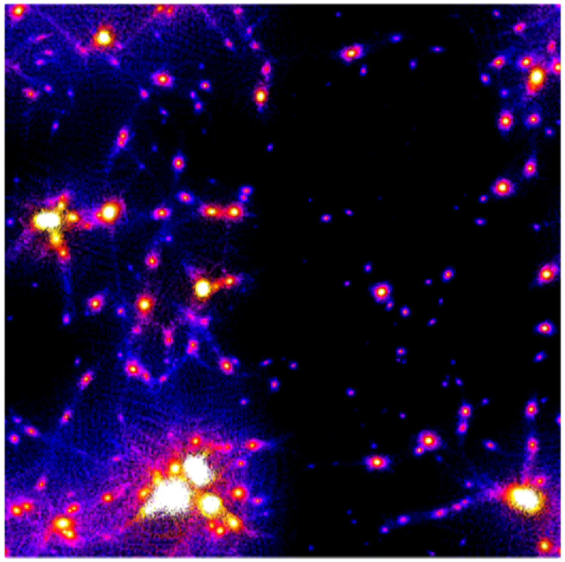
But what are the implications today?

Topological defects introduce additional energy and fluctuations

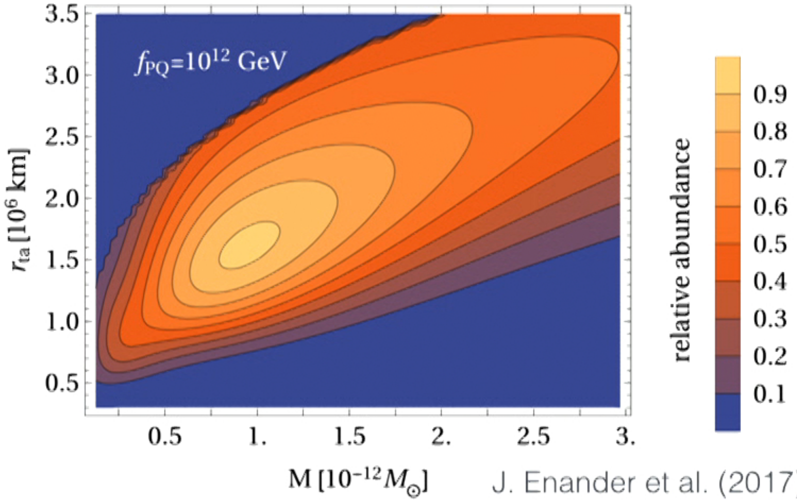
↓

Structure formation!

For example: **Axion Miniclusters**



K. Zurek et al. (2006)



J. Enander et al. (2017)

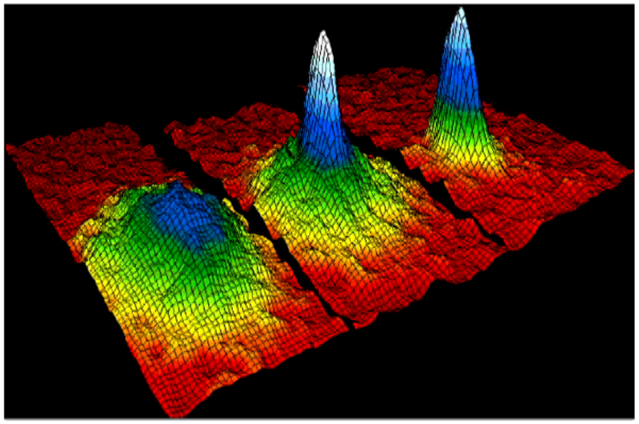
But what are the implications today?

Topological defects introduce additional energy and fluctuations

↓

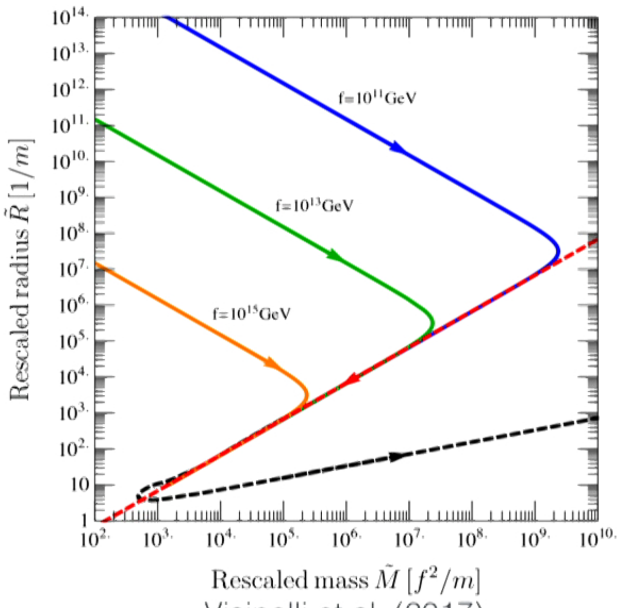
Structure formation!

For example: **Axion Stars**



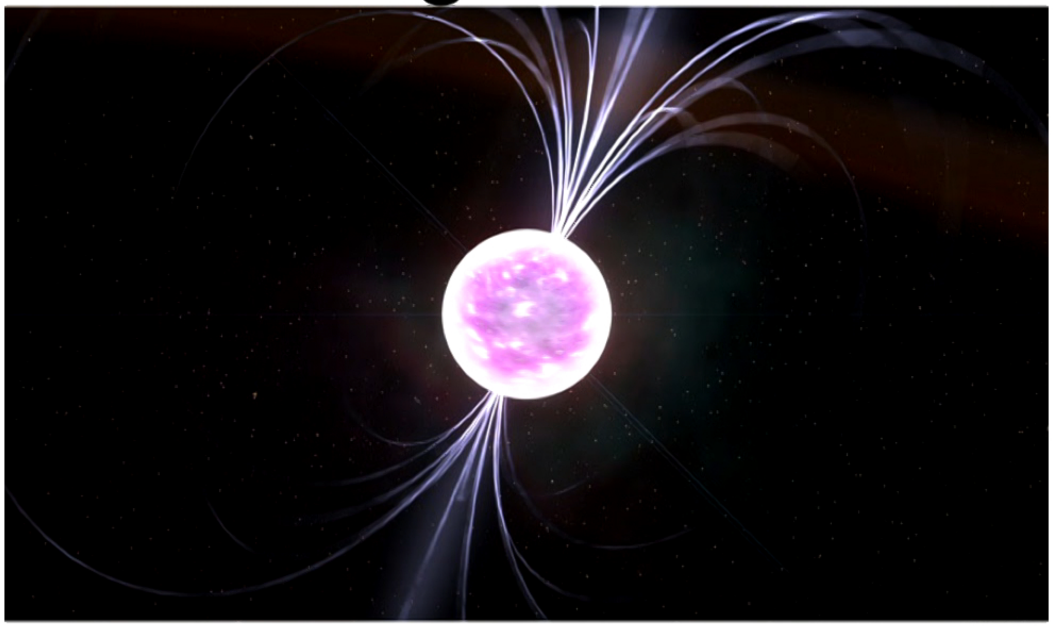
NIST/JILA/CU-Boulder

Axion star radius vs mass



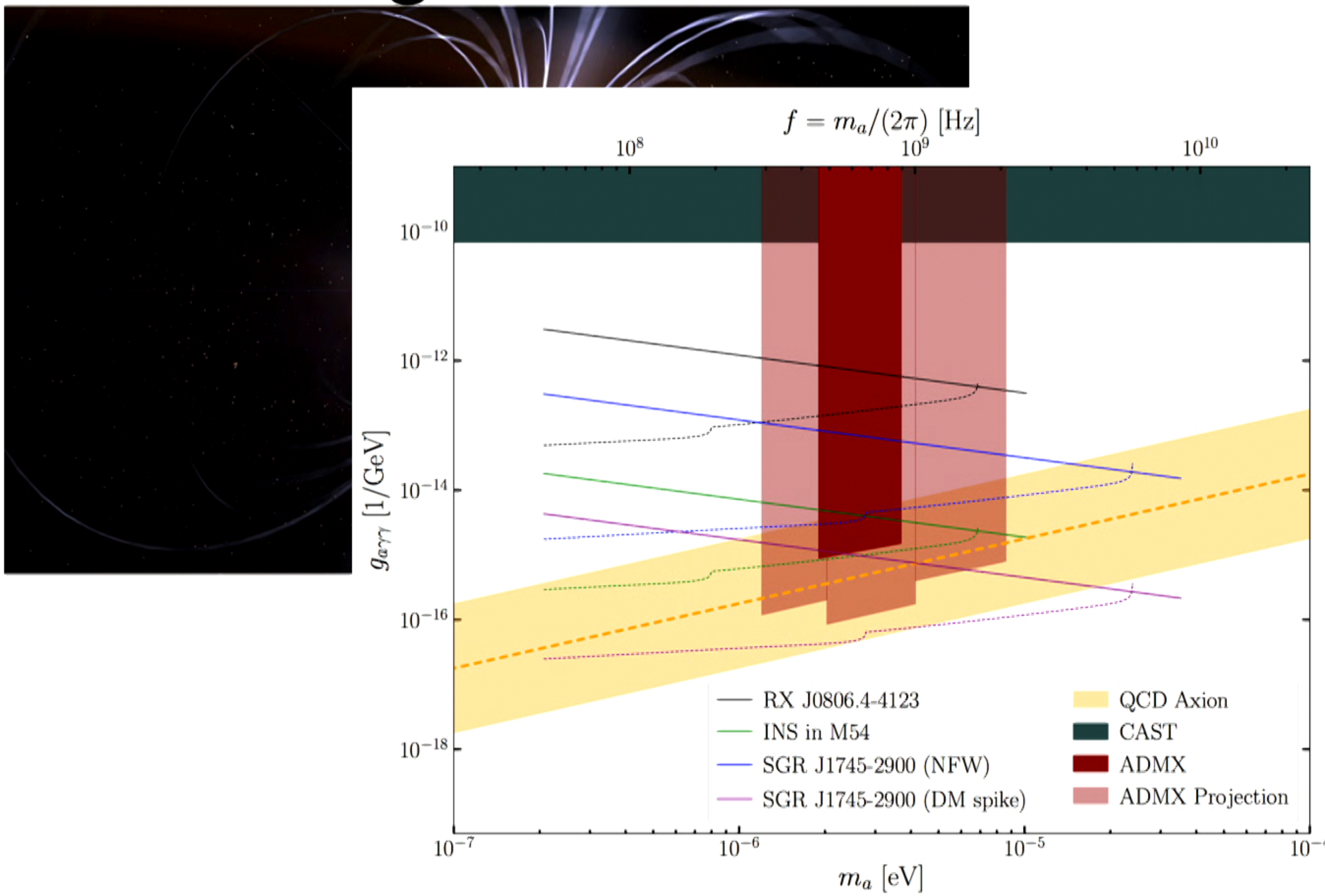
Visinelli et al. (2017)

Radio Signals from Neutron Stars



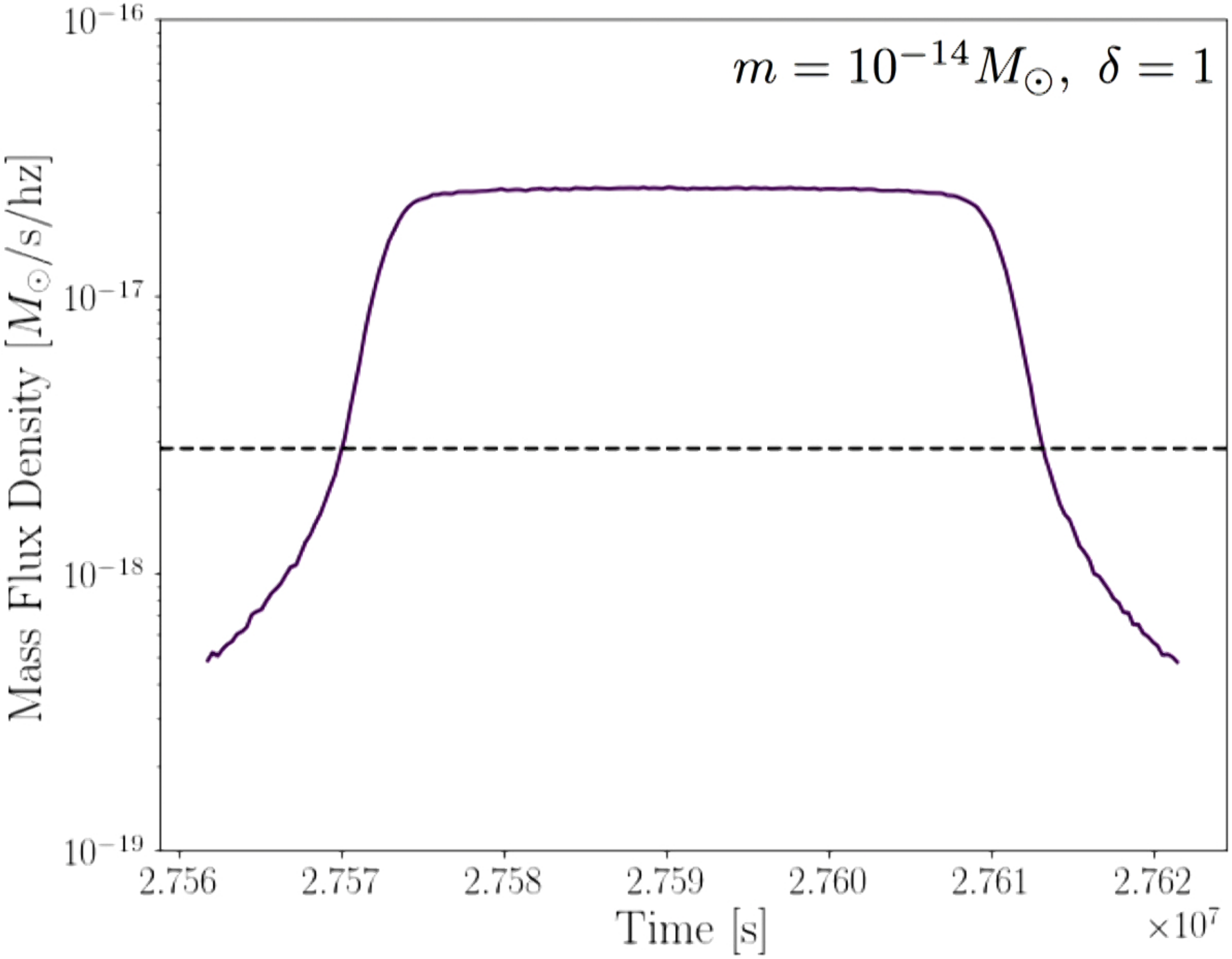
K. Gill

Radio Signals from Neutron Stars

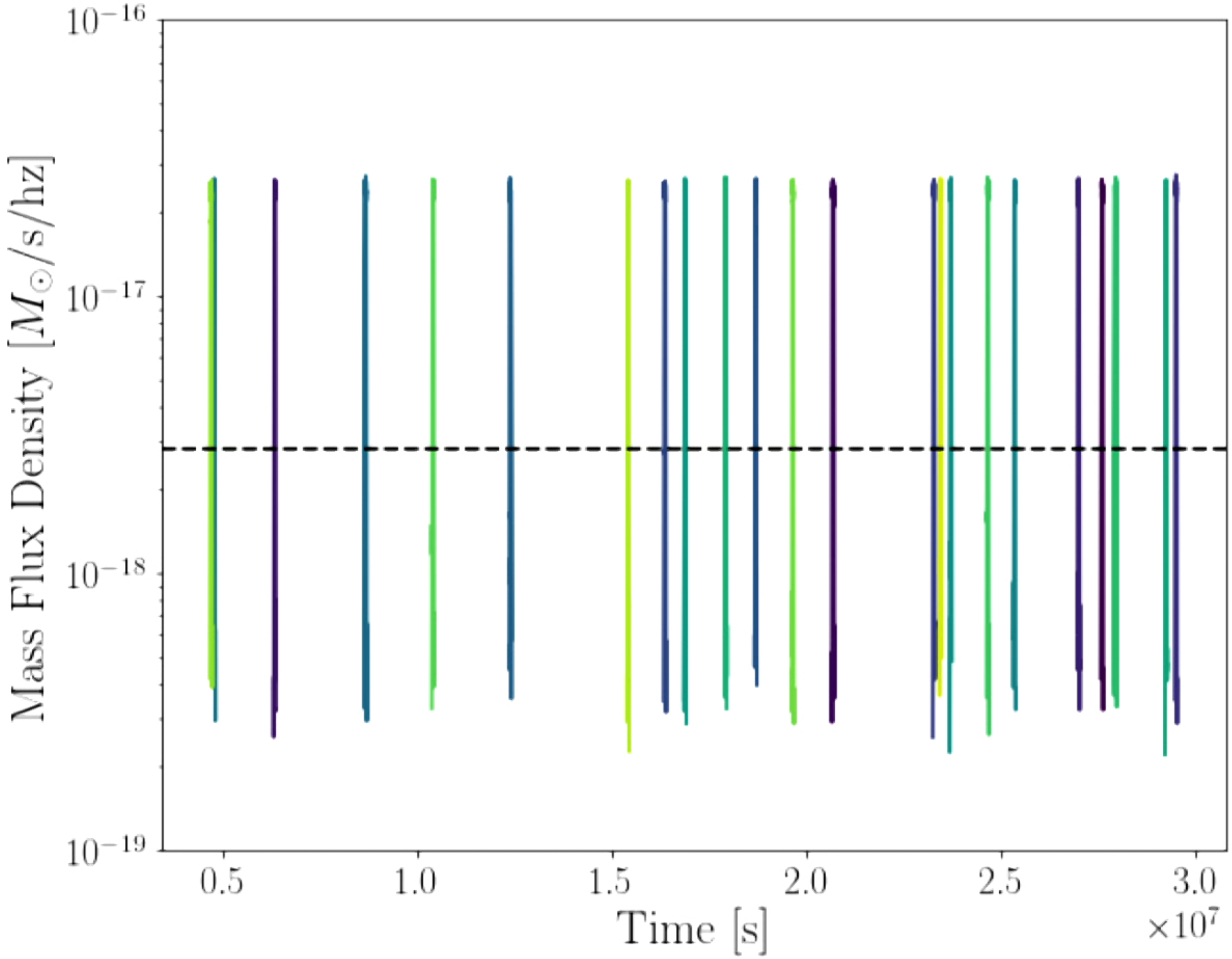


A. Hook et al. (1804.03145)

Radio Signals from Neutron Stars



Radio Signals from Neutron Stars



Questions that needs to be answered:

How much of the total mass is in clusters?

How heavy are they?

This ultimately boils down to:

What is the spectrum of overdensities after the QCD PT?

Simulations are needed!

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Nonlinear axion dynamics and the formation of cosmological pseudosolitons

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(Received 18 November 1993)

The (3+1)-dimensional evolution of an inhomogeneous axion field configuration around the QCD epoch is studied numerically, including important nonlinear effects due to the attractive self-interaction. It is found that axion perturbations on scales corresponding to causally disconnected regions at $T \sim 1$ GeV can lead to very dense pseudosoliton configurations we call axitons. These configurations evolve to axion miniclusters with a present density $\rho_a \gtrsim 10^{-8}\ \mathrm{g\,cm^{-3}}$. This is high enough for the collisional $2a \to 2a$ process to lead to Bose-Einstein relaxation in the gravitationally bound clumps of axions, forming Bose stars.

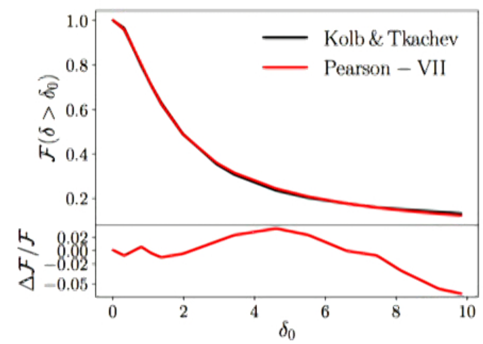


FIG. 2. **Minicluster Overdensity Distribution**: We show the cumulative mass fraction of miniclusters with overdensity parameter $\delta > \delta_0$. The black line shows the simulation results of Ref. [22], which we have fit using a Pearson-VII distribution. The overdensity distribution determines the halo concentration parameter (i.e. compactness) of miniclusters.

M. Fairbairn et al. (1707.03310)

Getting started

- State before the QCD PT: White noise with high modes that already entered the horizon redshifted accordingly

- Problem: Creating such a initial state properly difficult (Result often depends on the details -> not good)

- Instead: Evolve a white noise initial state through the PQ PT first!

Equation of Motion for PQ breaking

Lagrangian:

$$\mathcal{L} = \frac{1}{2}|\partial\Phi|^2 - V(\Phi, T)$$

with temperature dependent potential:

$$V(\Phi, T) = \frac{\lambda}{4}\left(|\Phi|^2 - f_a^2\right)^2 + \frac{\lambda T^2}{6}|\Phi|^2 + m_a(T)^2 f_a^2\left[1 - \cos \mathrm{Arg}(\Phi)\right]$$

Decompose and reparameterise:

$$\psi_1'' + \frac{2}{\eta}\psi_1' - \bar{\nabla}^2\psi_1 + \frac{1}{H_1^2}\left[\lambda\psi_1\left(\eta^2 f_a^2\left(\psi_1^2 + \psi_2^2 - 1\right) + \frac{1}{3}T_1^2\right) - m_a^2(T_1)\eta^{2+n}\left(\frac{\psi_2^2}{(\psi_1^2 + \psi_2^2)^{3/2}}\right)\right] = 0$$

$$\psi_2'' + \frac{2}{\eta}\psi_2' - \bar{\nabla}^2\psi_2 + \frac{1}{H_1^2}\left[\lambda\psi_2\left(\eta^2 f_a^2\left(\psi_1^2 + \psi_2^2 - 1\right) + \frac{1}{3}T_1^2\right) + m_a^2(T_1)\eta^{n+2}\left(\frac{\psi_1\psi_2}{(\psi_1^2 + \psi_2^2)^{3/2}}\right)\right] = 0$$

saxion: $\sqrt{\psi_1^2 + \psi_2^2}$

axion: $\arctan \frac{\psi_2}{\psi_1}$

Constructing an initial state

In the early Universe:
The two fields are in a **thermal equilibrium**

Relevant quantity: $\omega_k = \sqrt{m_{eff}^2 + k^2}$

We find: $$\frac{m_{eff}^2}{a_1^2 H_1^2} \approx \frac{\lambda}{3}\left(\frac{T}{T_0}\right)^2 \left(\frac{T_1}{f_a}\right)^2 \gg 1$$

$$\longrightarrow \omega_k \approx \sqrt{\frac{\lambda}{3}}T, \qquad n_k = \frac{1}{e^{\sqrt{\lambda/3}} - 1}$$

Only one relevant parameter (and this even changes only the overall amplitude $\longrightarrow$ irrelevant after PQ phase transition once radial mode reaches its vev)

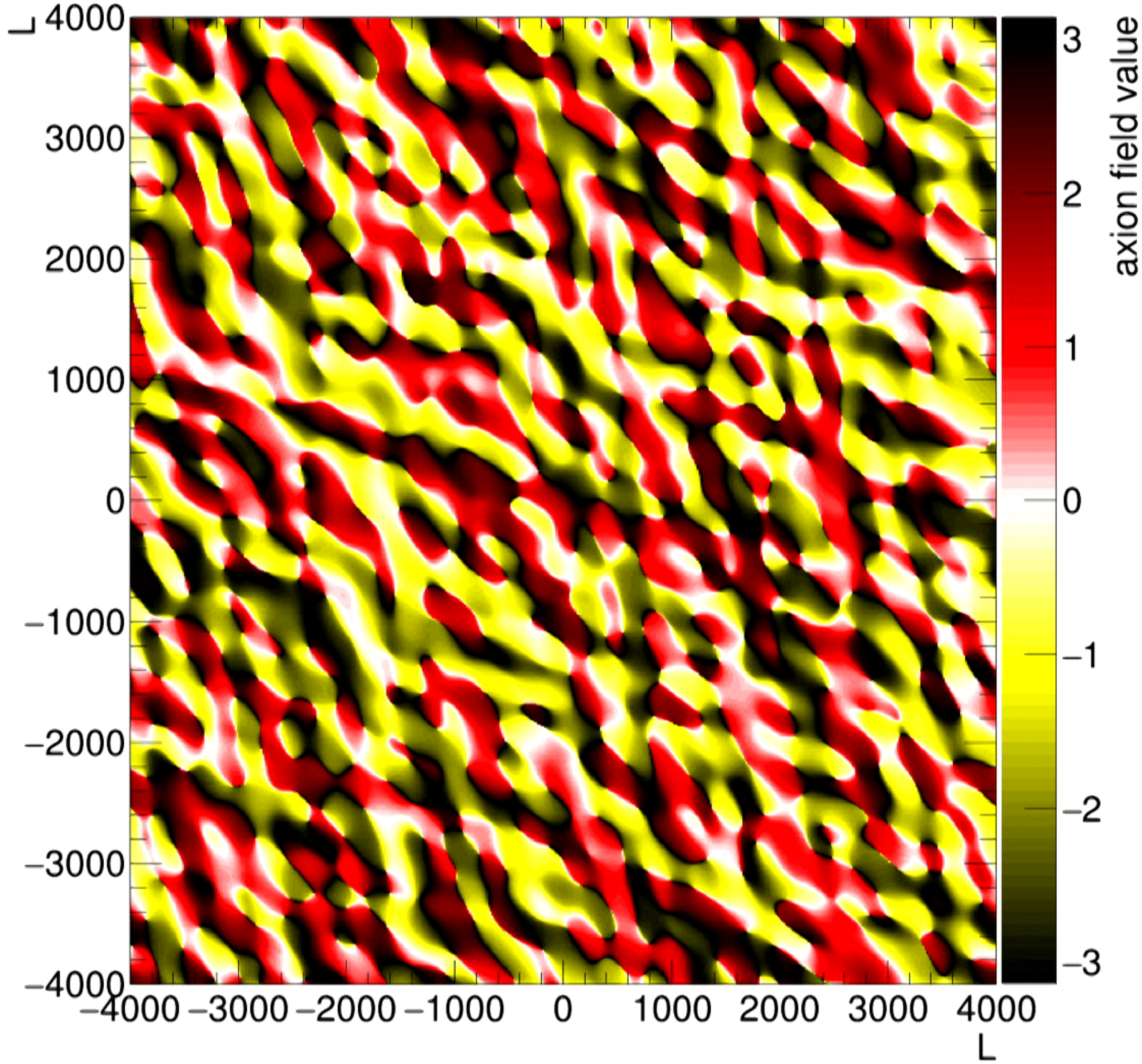
Unitless: $$\psi_1'' + \frac{2}{\eta}\psi_1' - \bar{\nabla}^2\psi_1 + \left[\lambda\psi_1\left(\eta^2\left(\psi_1^2+\psi_2^2-1\right)+\frac{1}{3}\left(\frac{T_1}{f_a}\right)^2\right)\right] = 0$$

$$\psi_2'' + \frac{2}{\eta}\psi_2' - \bar{\nabla}^2\psi_2 + \left[\lambda\psi_2\left(\eta^2\left(\psi_1^2+\psi_2^2-1\right)+\frac{1}{3}\left(\frac{T_1}{f_a}\right)^2\right)\right] = 0$$

with $\left(\frac{T_1}{f_a}\right)^2 = \sqrt{\frac{45}{4\pi^2 g_*}}\frac{M_{pl}}{f_a} = 237320. \times \frac{10^{12}\mathrm{GeV}}{f_a}$

Solve this numerically on a **3-dimensional grid**:

- box of 2040 x 2040 x 2040 = 8.5 billion grid sites
- periodic boundary conditions
- O(1 million) time steps
- O(10 quadrillion) function calls
- Computation time O(1 day) on O(1000) CPUs
- 374 GB of memory needed
- Output several TB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4000
3000
2000
1000
0
−1000
−2000
−3000
−4000
−4000 −3000 −2000 −1000 0 1000 2000 3000 4000
L
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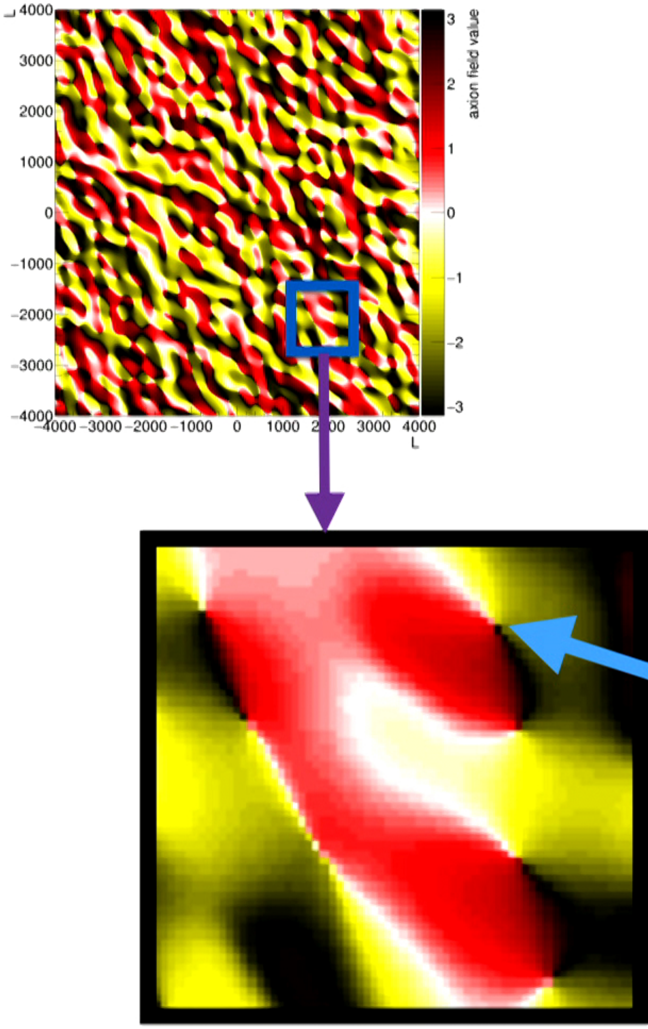
3
2
1
0
−1
−2
−3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2000
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0
−1000
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−4000 −3000 −2000 −1000 0 1000 2000 3000 4000
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3
2
1
0
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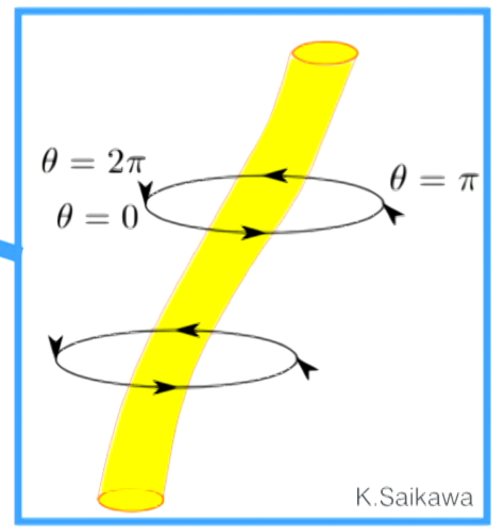
−3

Strings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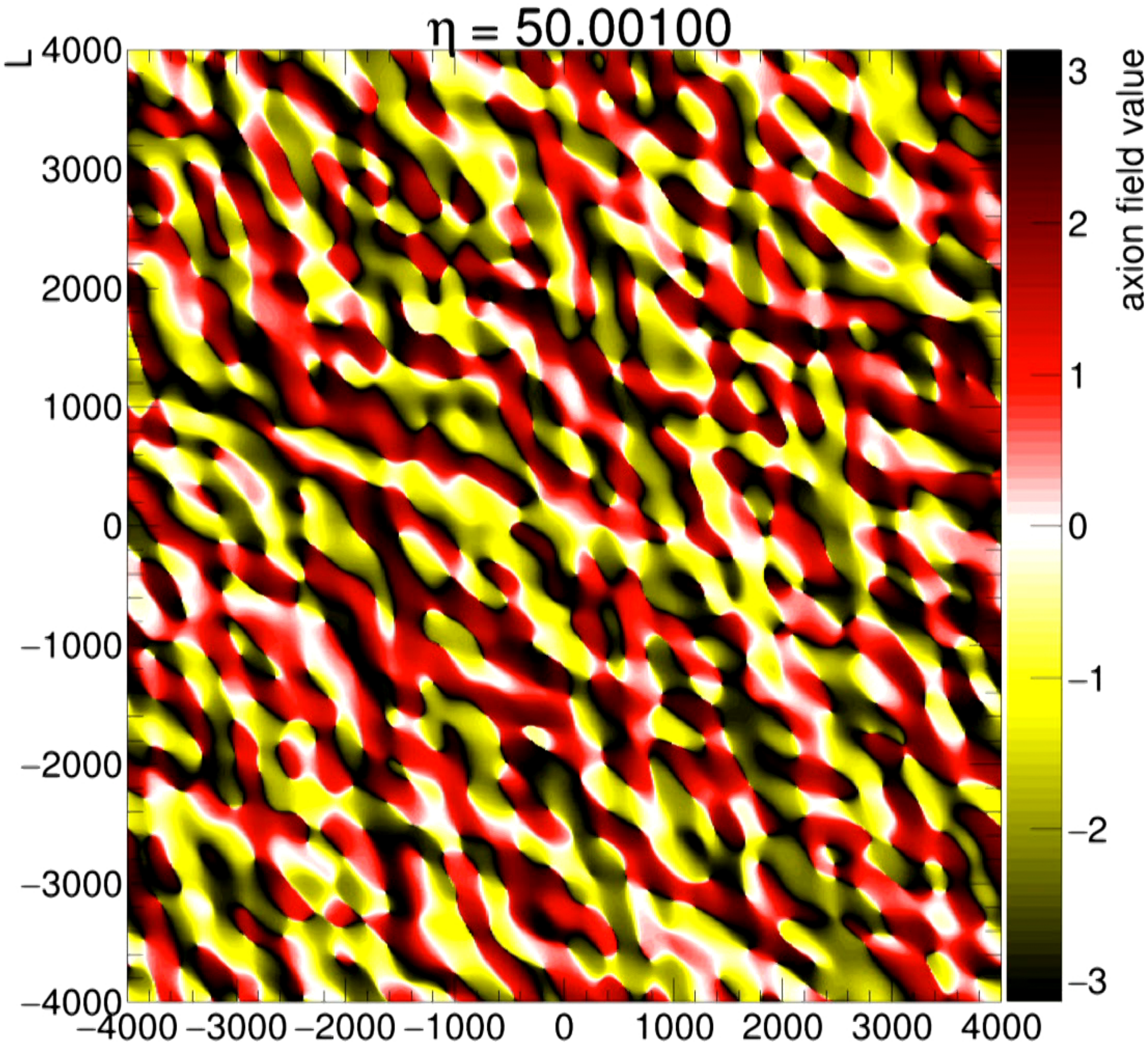
$\theta = \pi$
$\theta = 0$
K.Saikawa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2000
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−4000 −3000 −2000 −1000 0 1000 2000 3000 4000
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1
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−1
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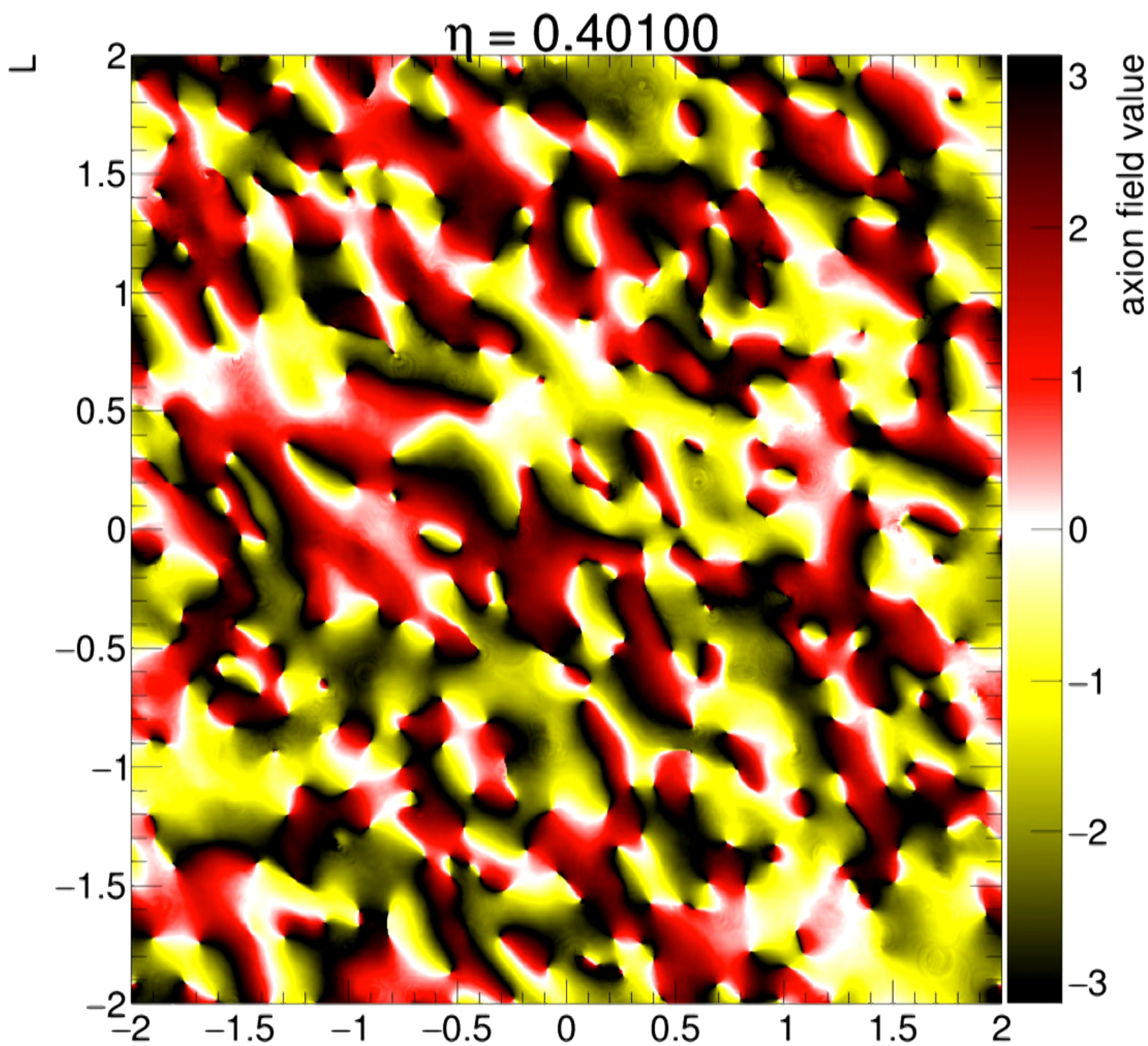
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0
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1
1.5
2
axion field value
3
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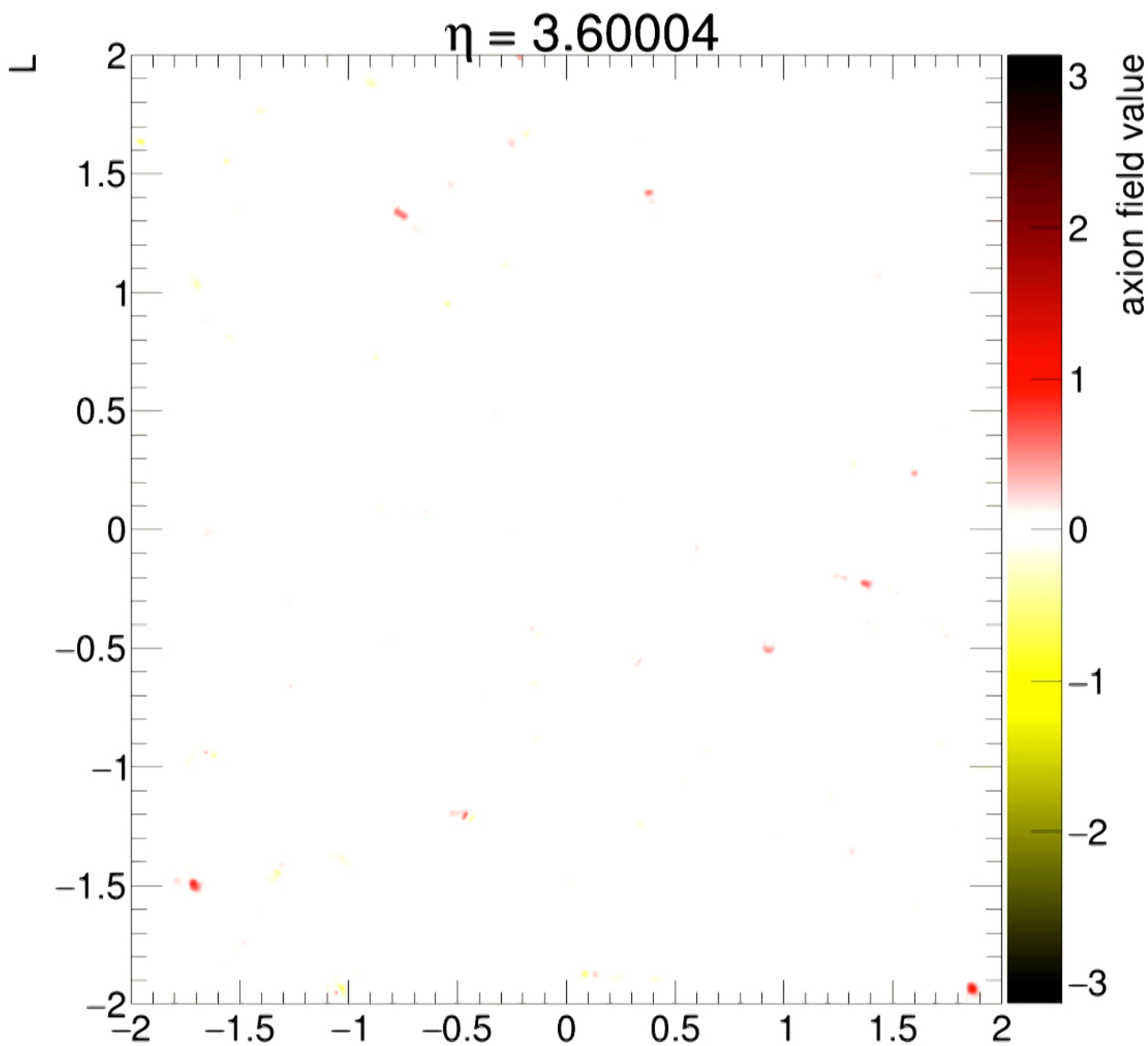
0
−1
−2
−3



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1
1.5
2
axion field value
3
2
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0
−1
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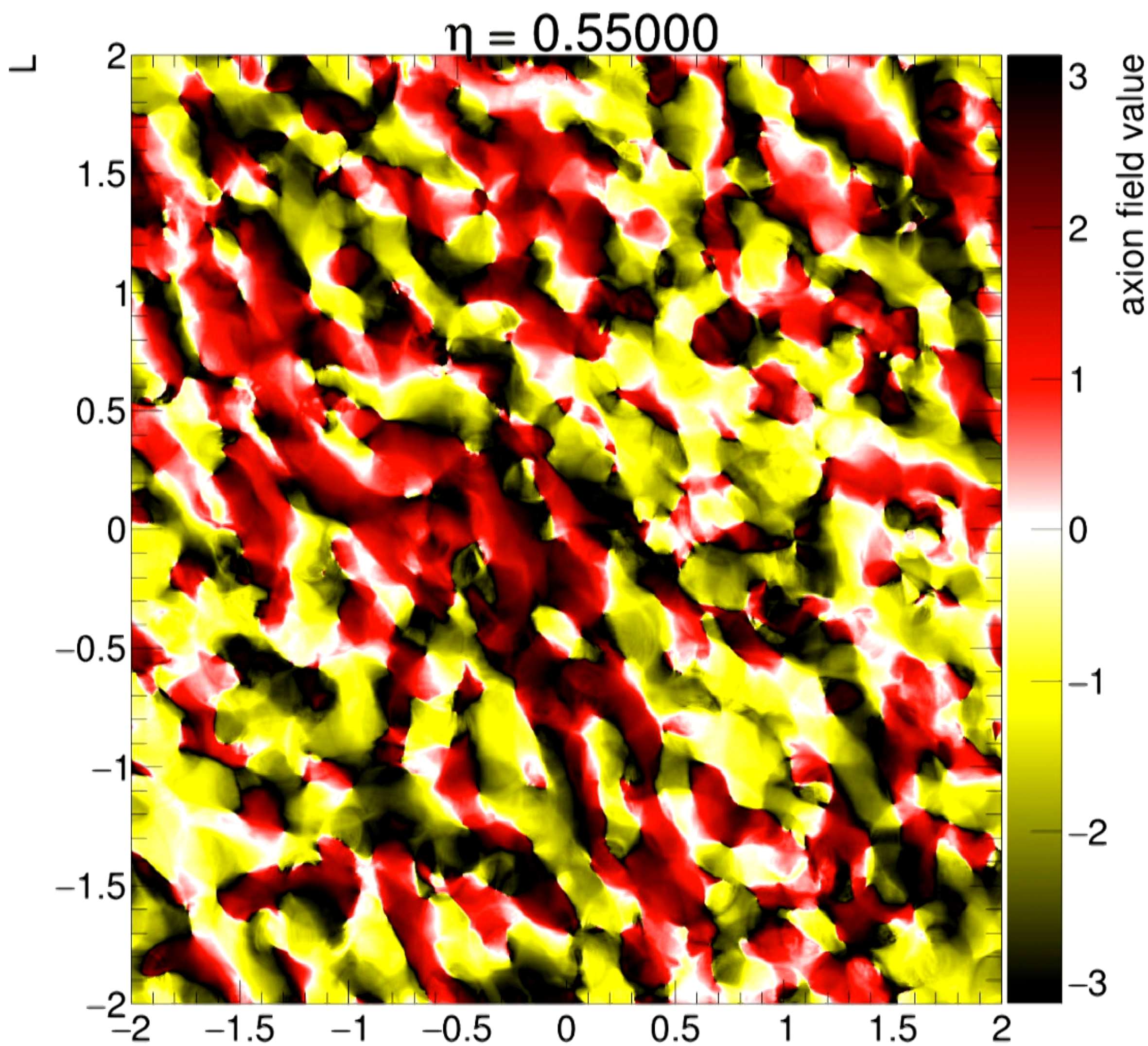
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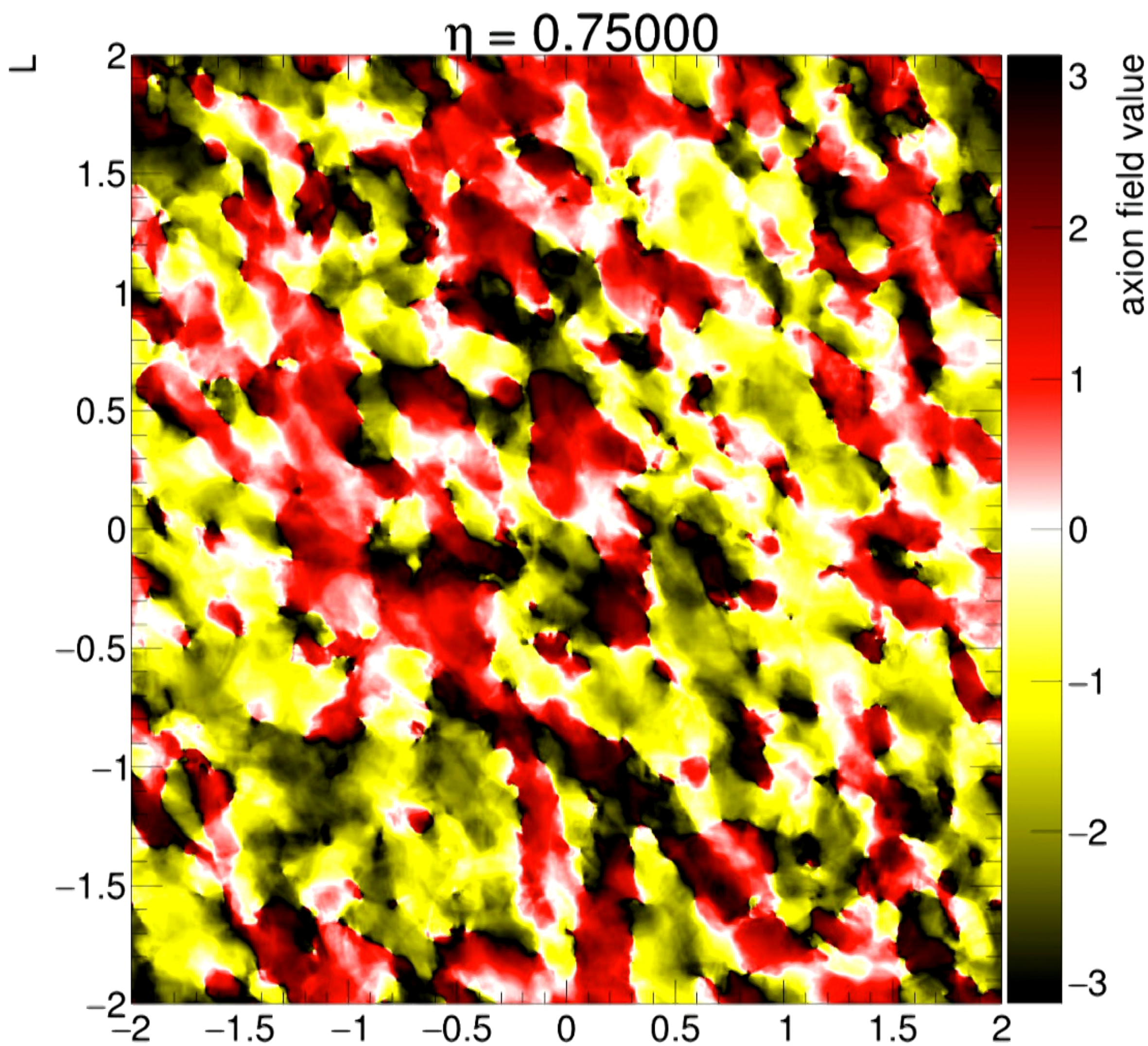
3 2 1 0 −1 −2 −3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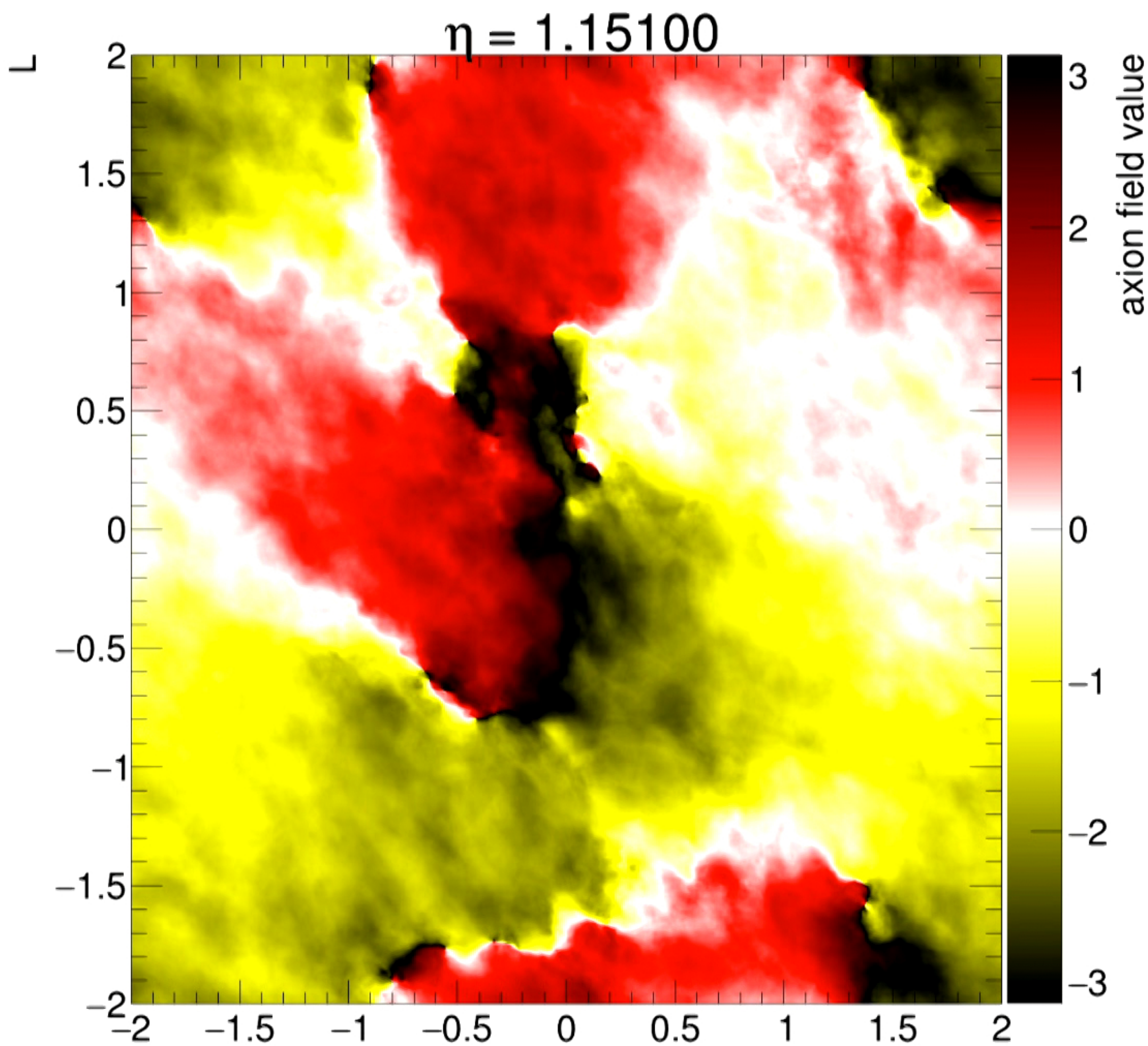
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1
0
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−2
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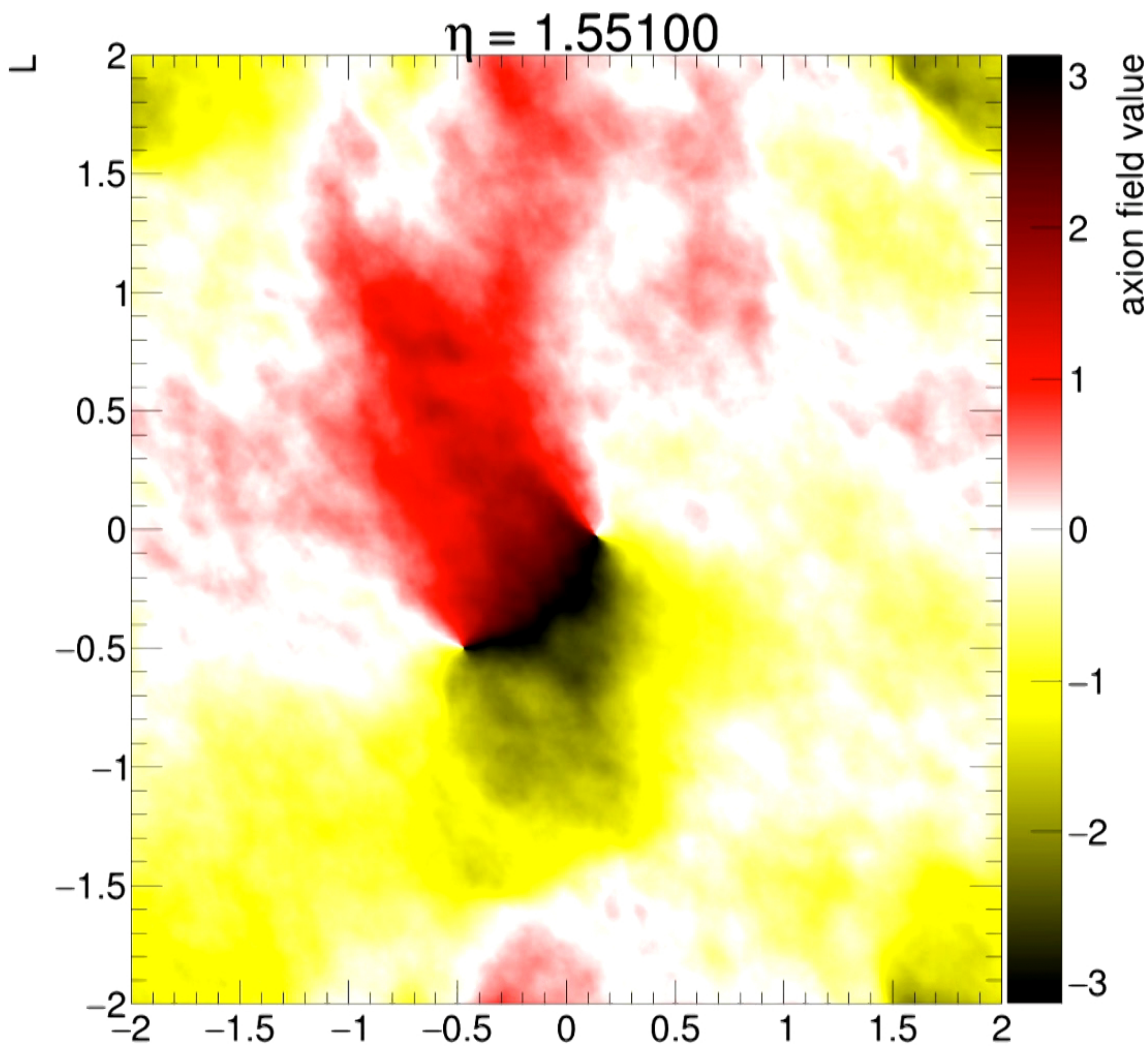
axion field value



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−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
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1.5
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axion field value
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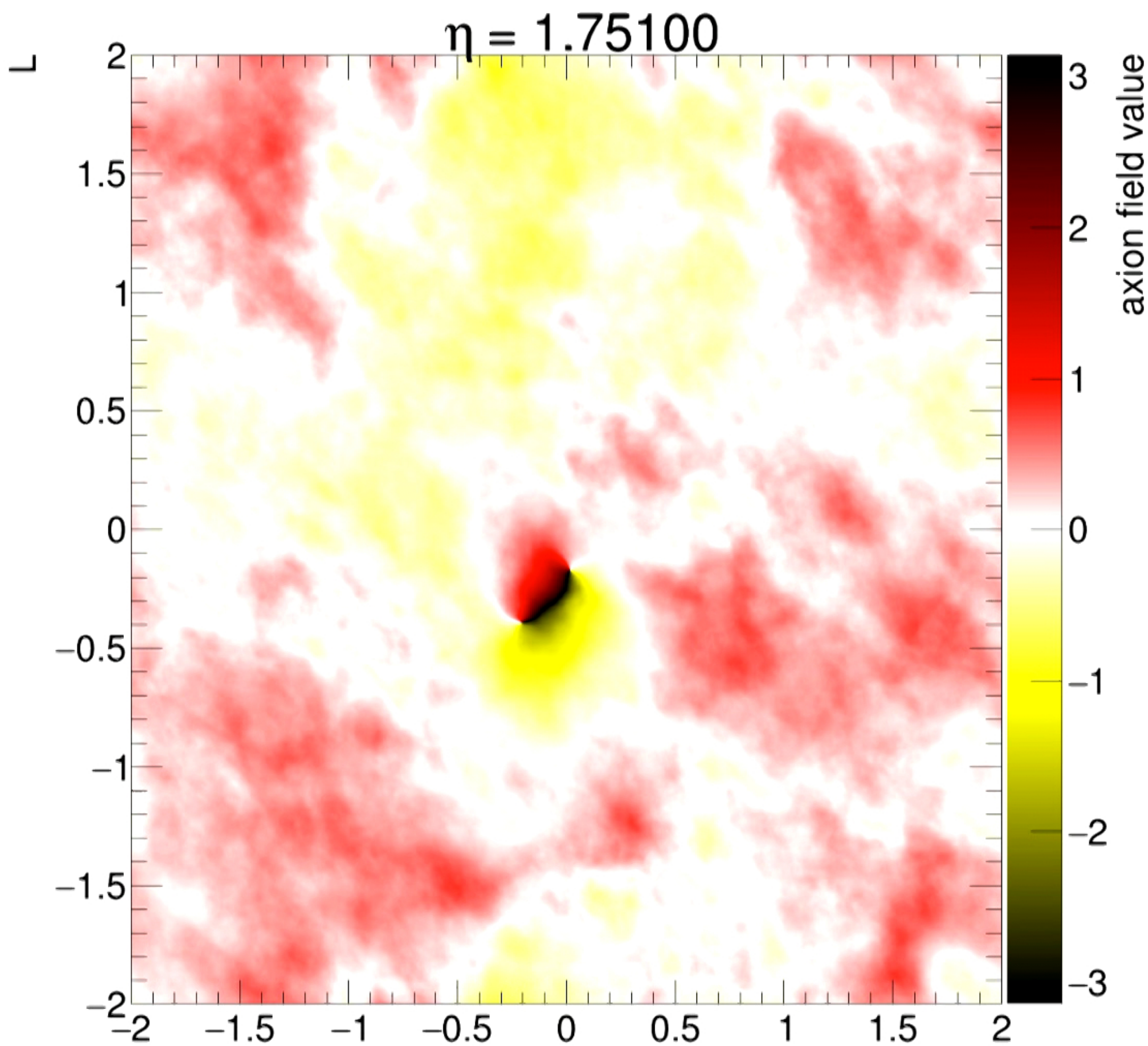
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axion field value
3
2
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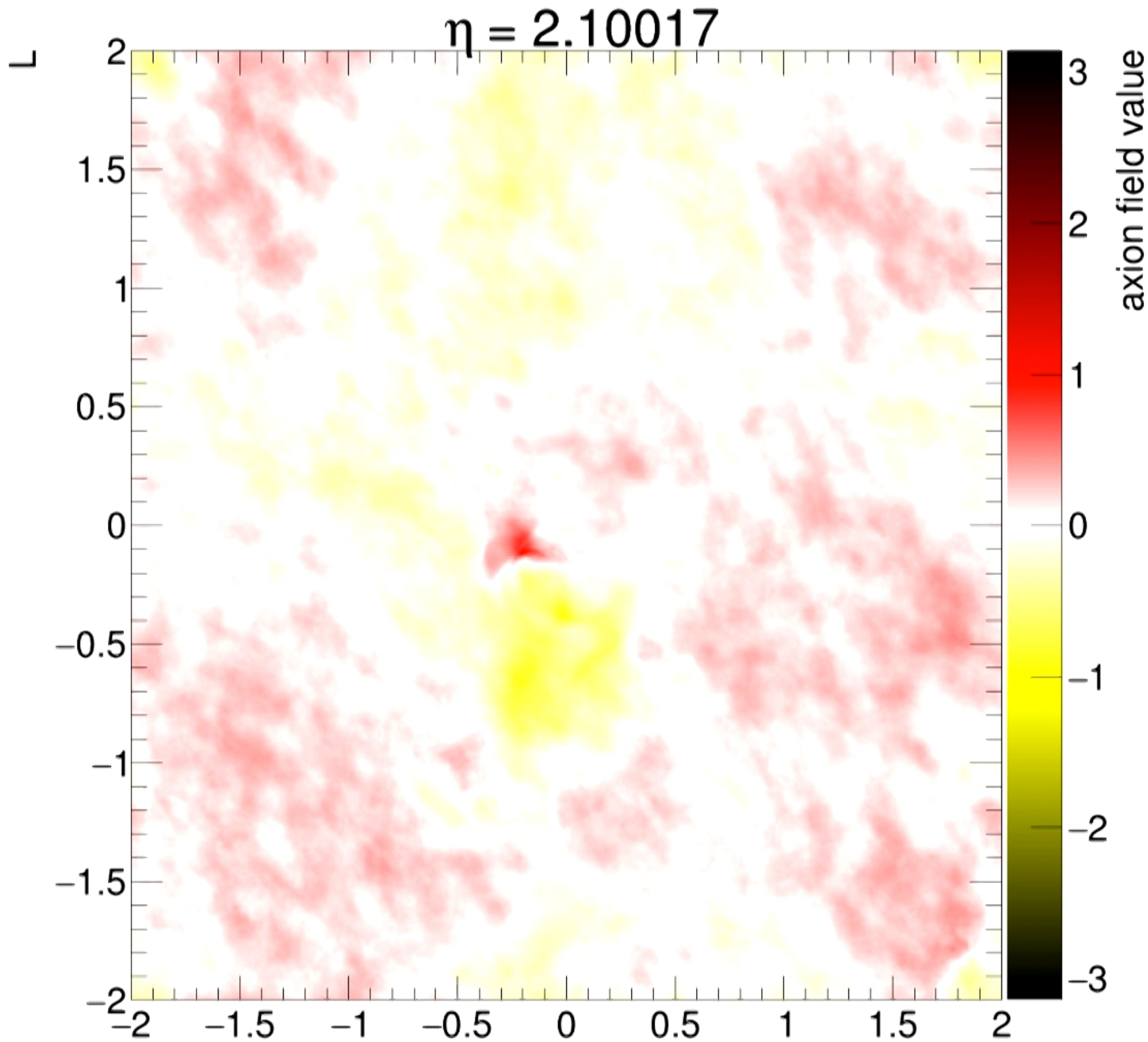

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axion field value
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−1
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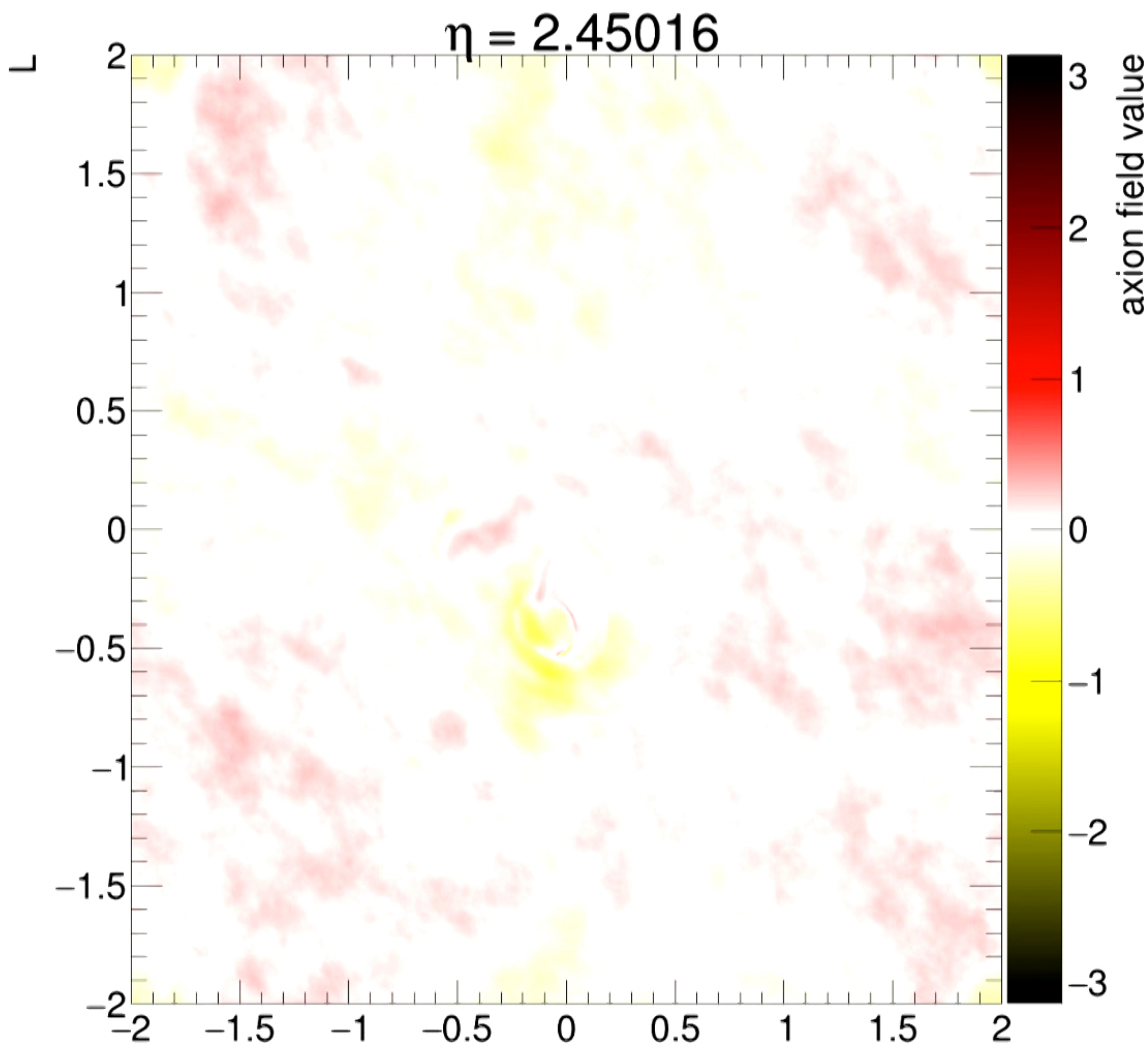

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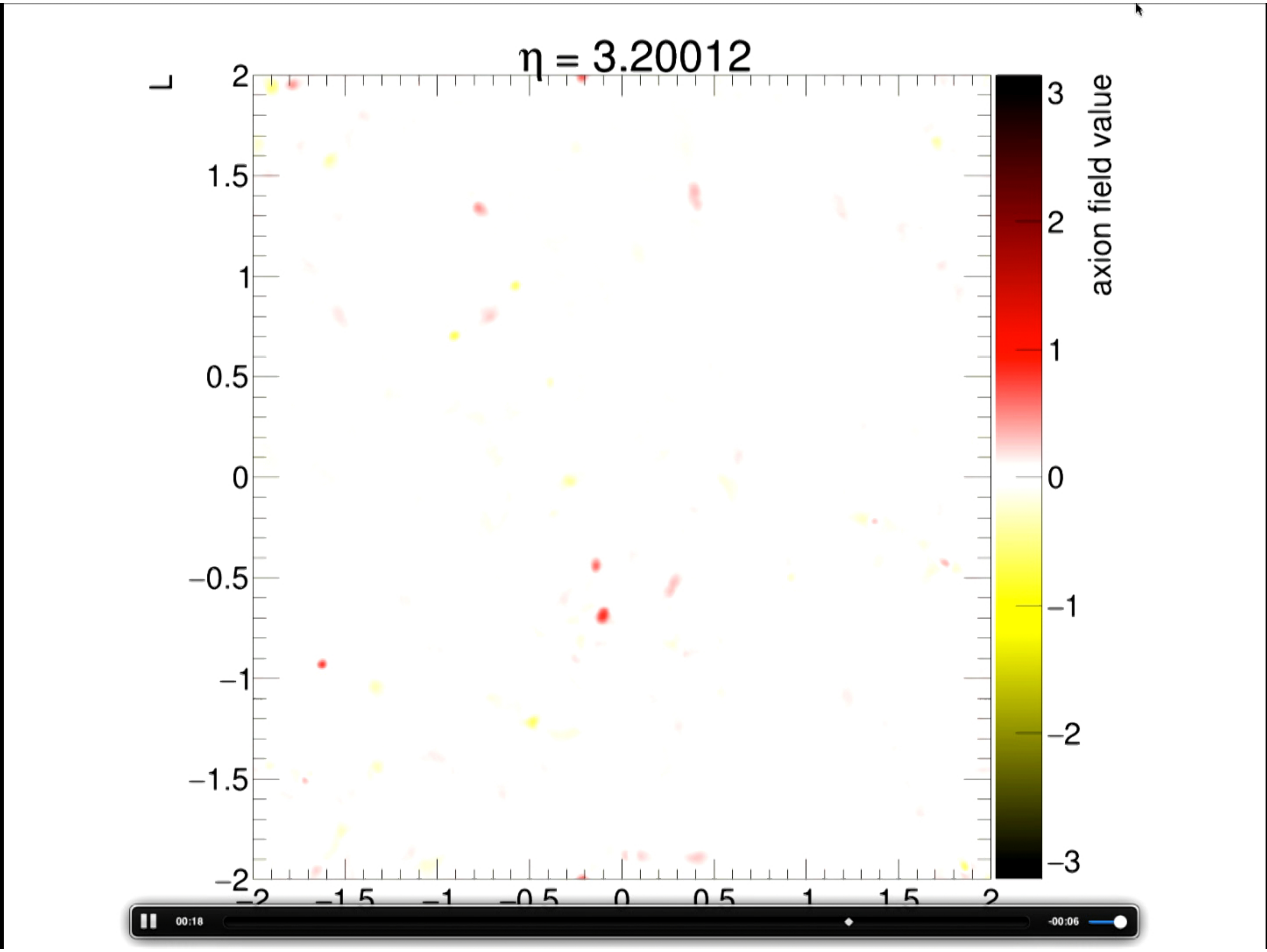
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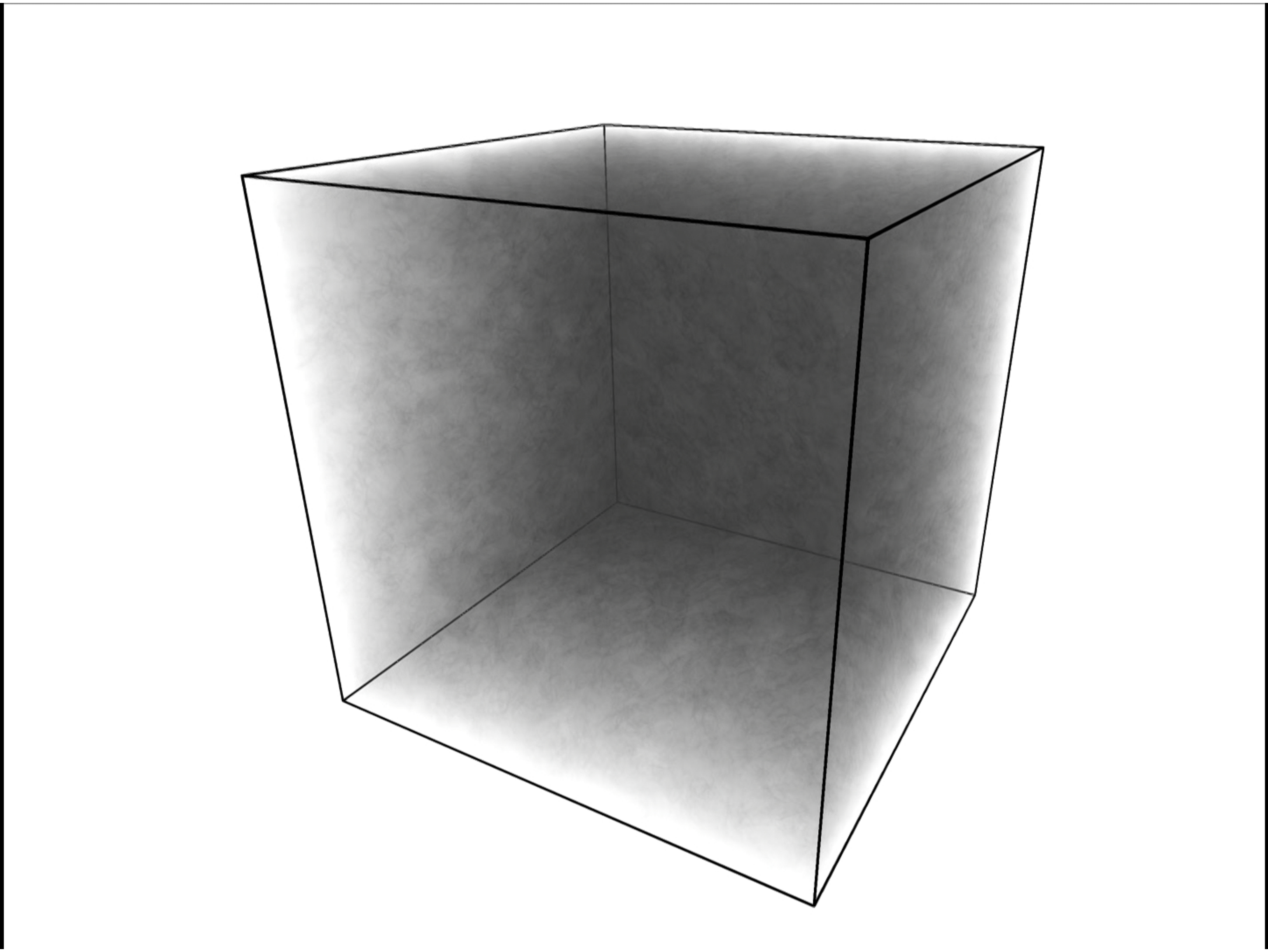

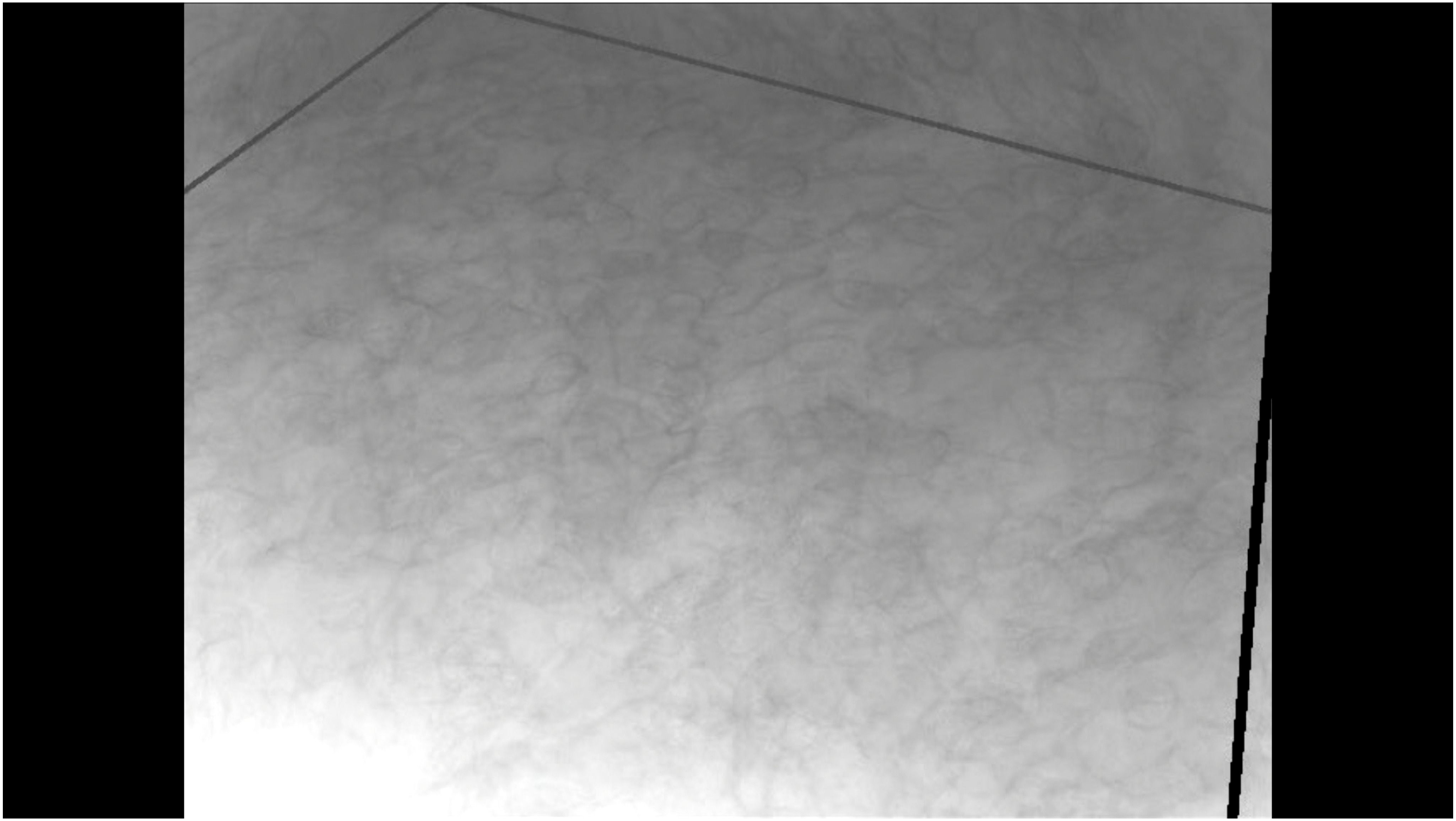

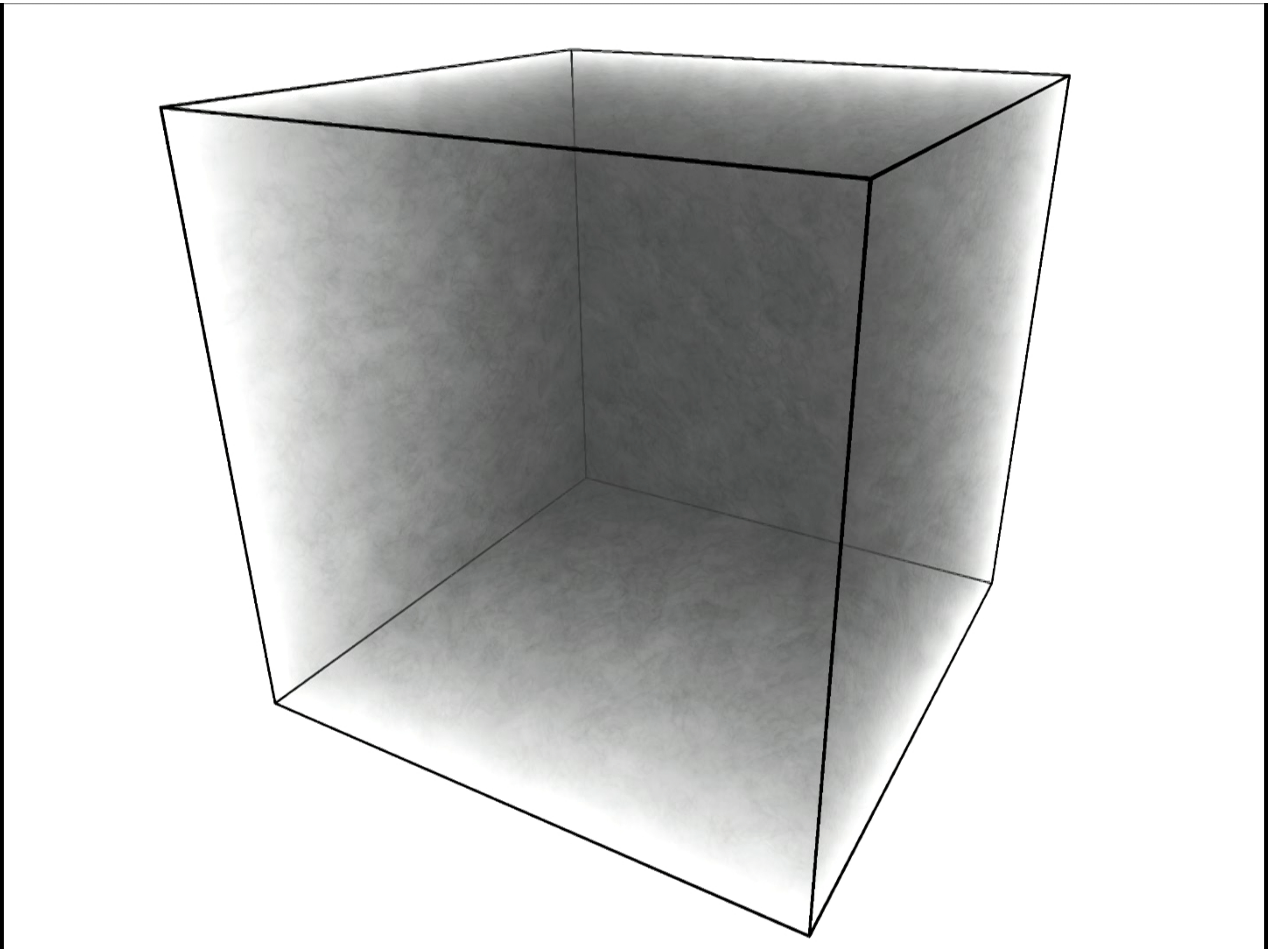

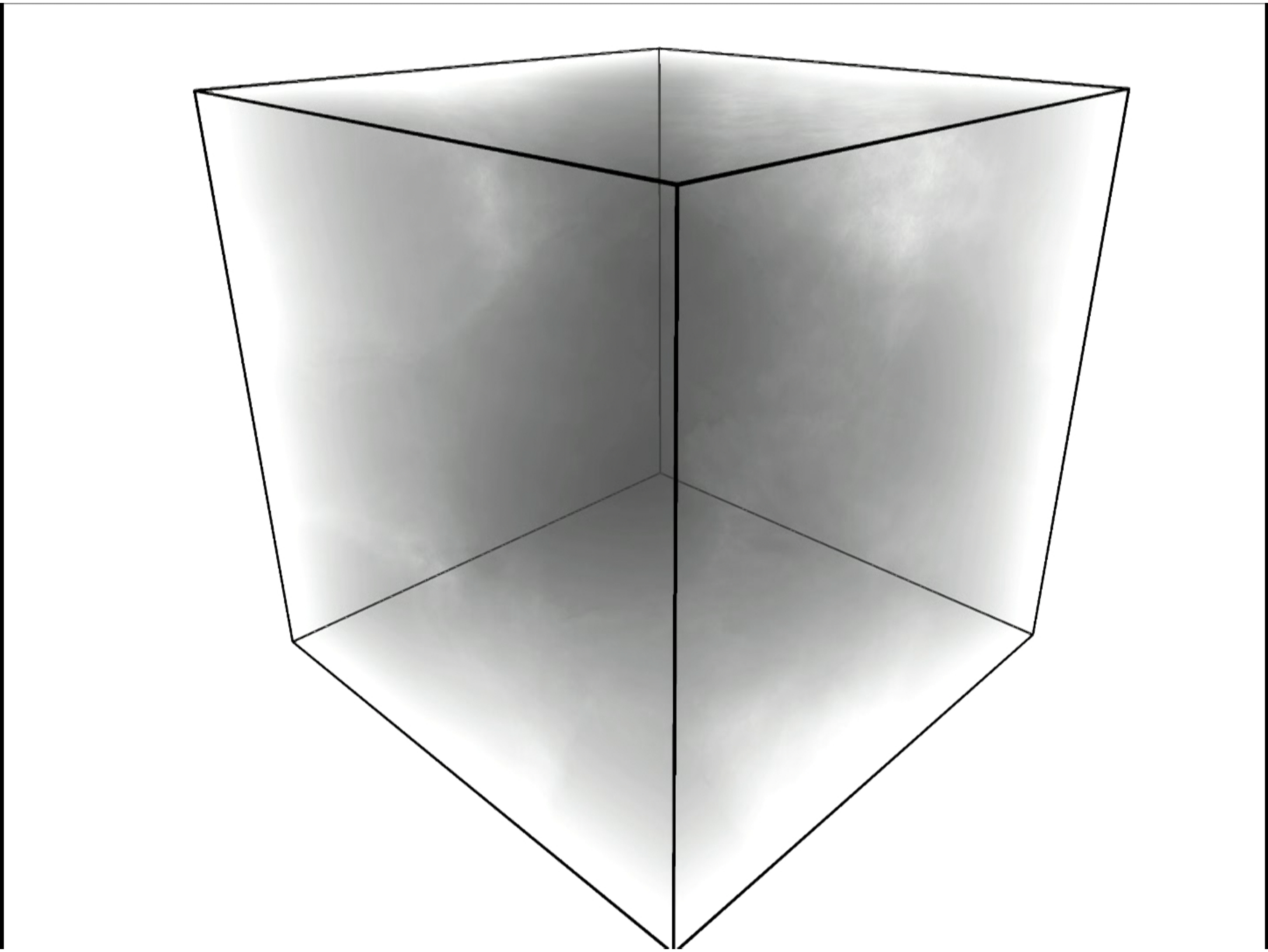

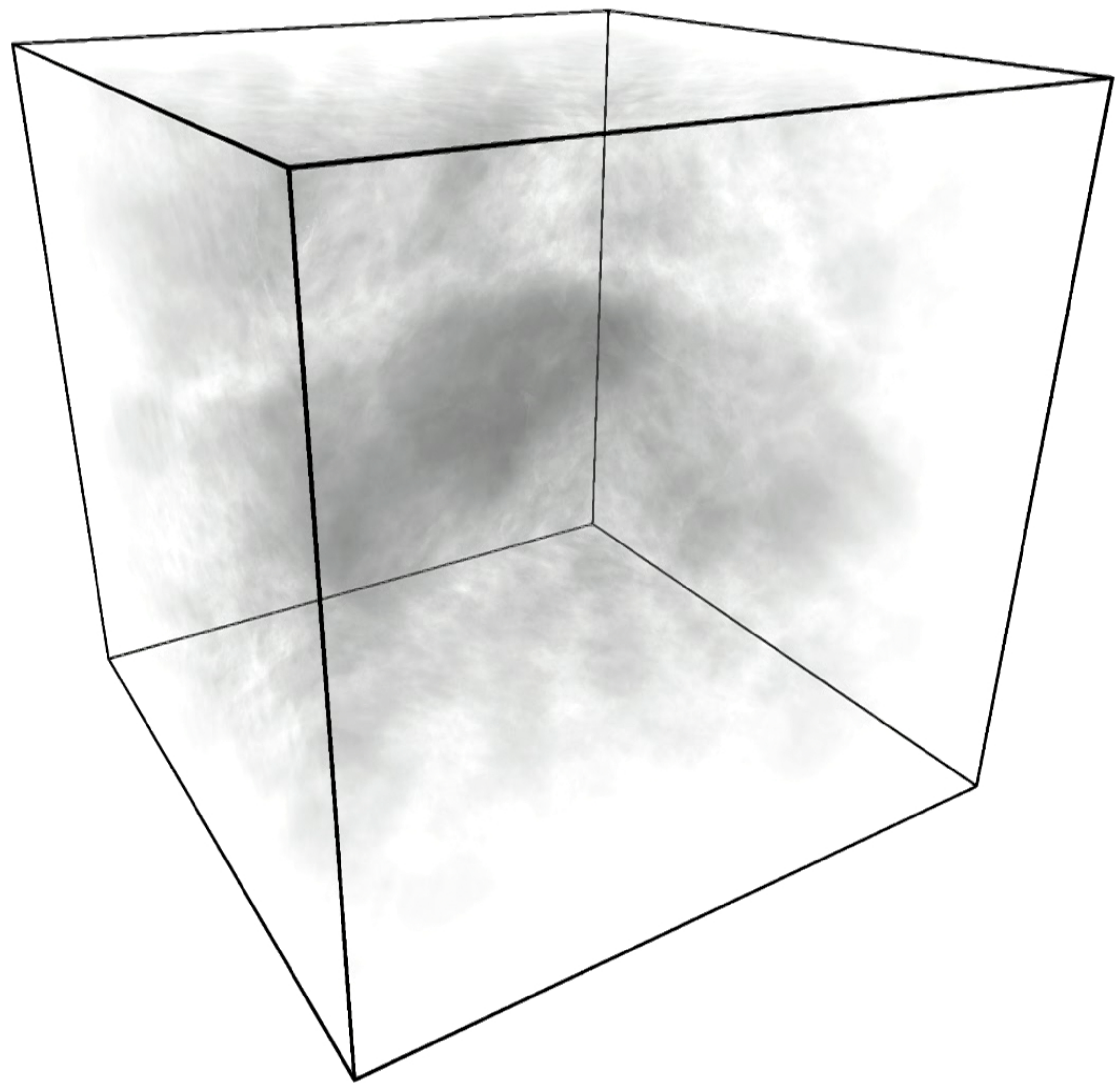

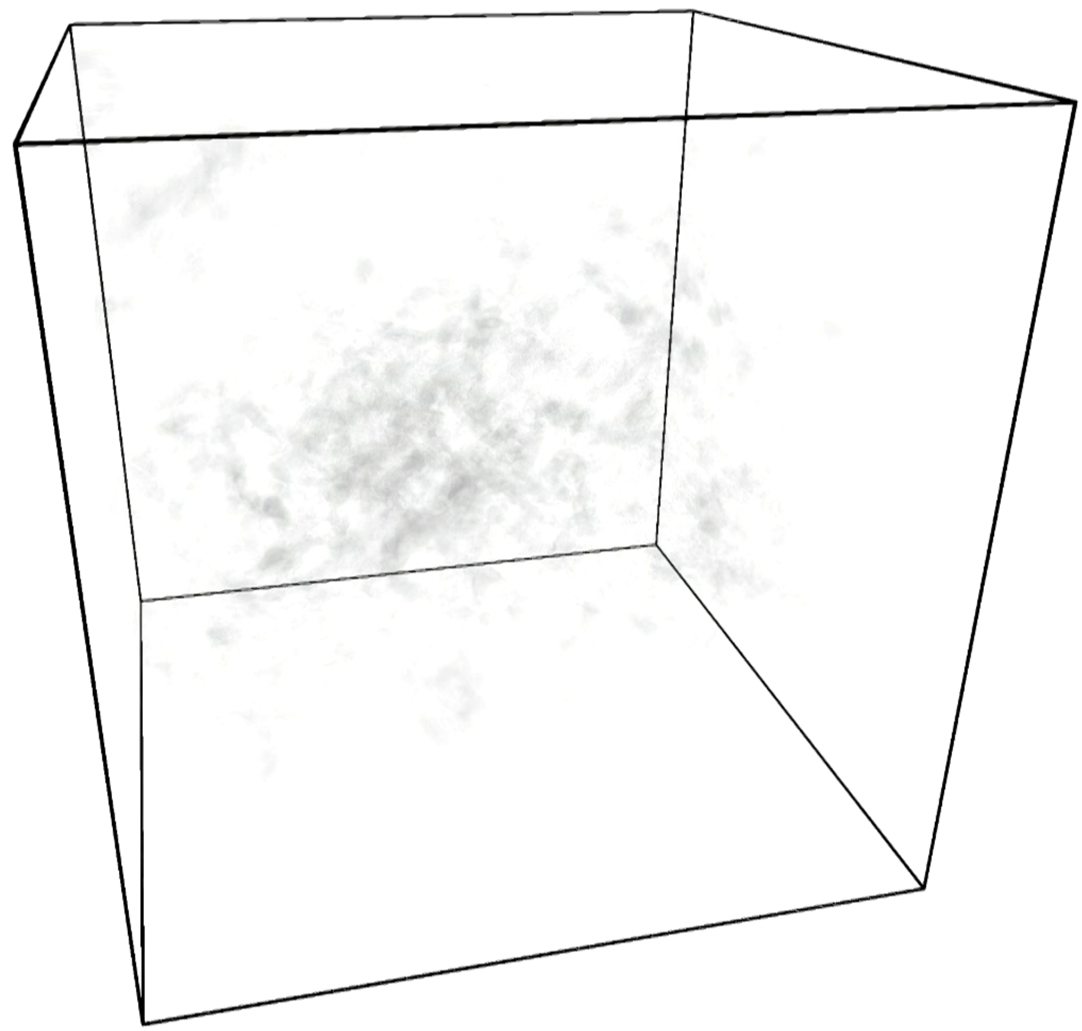

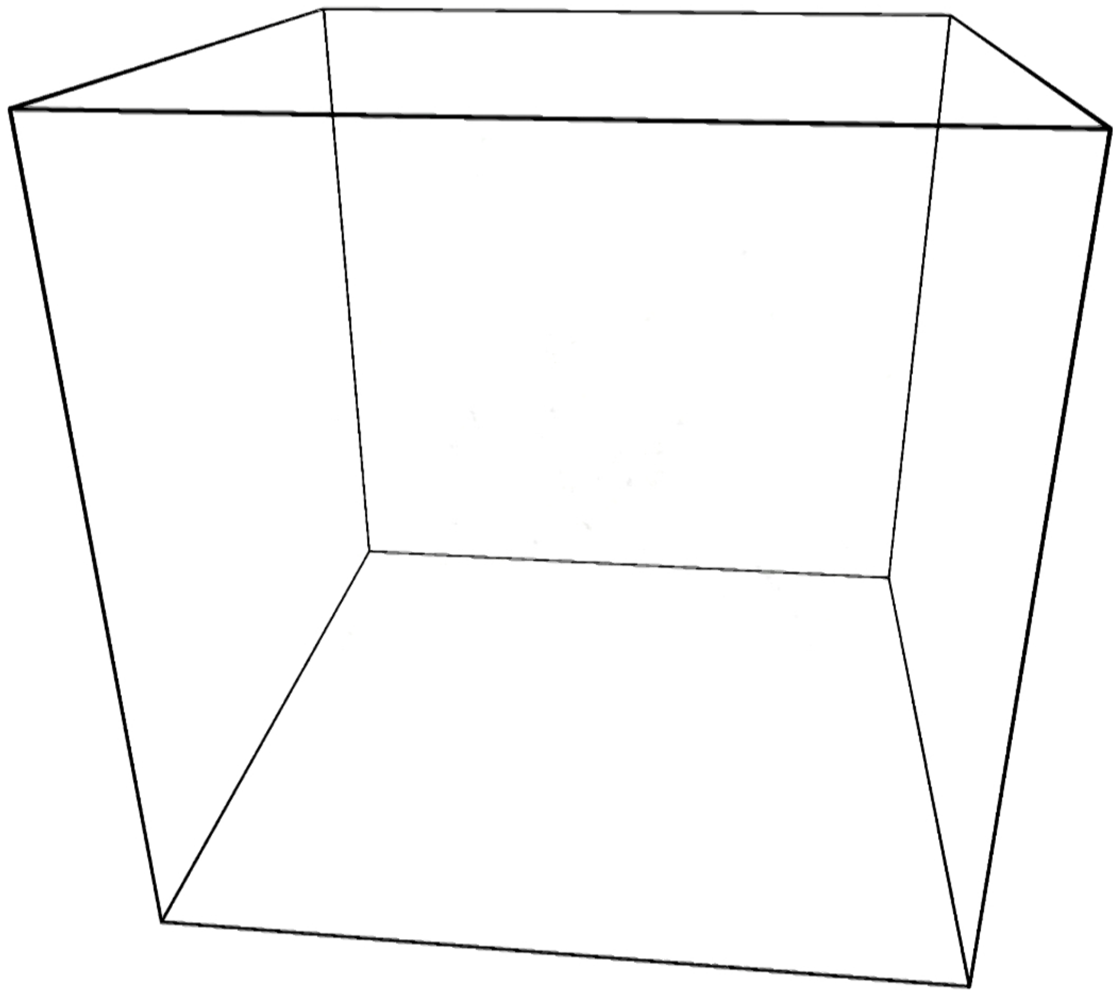

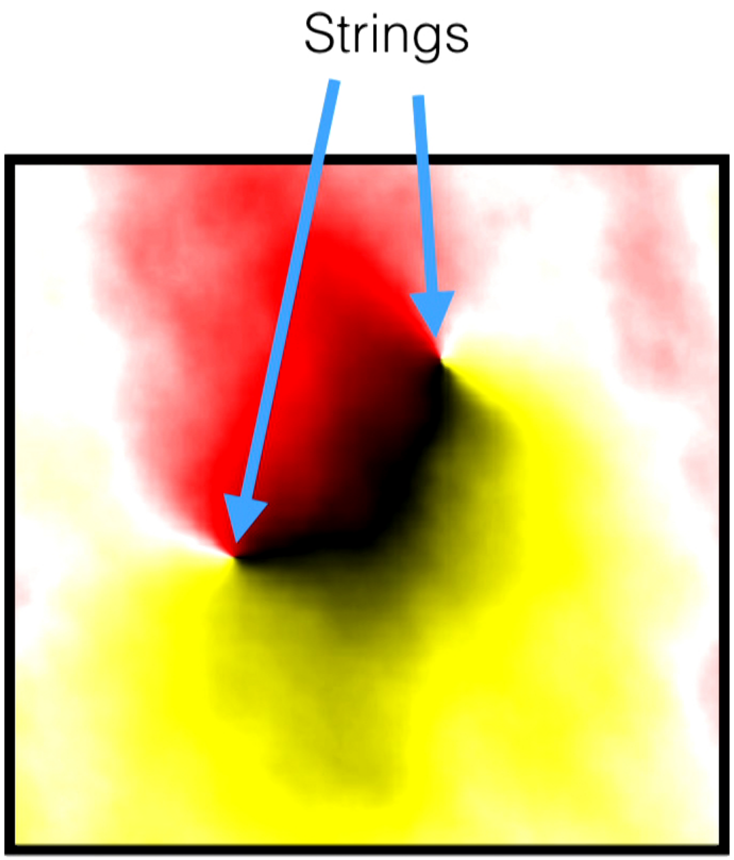
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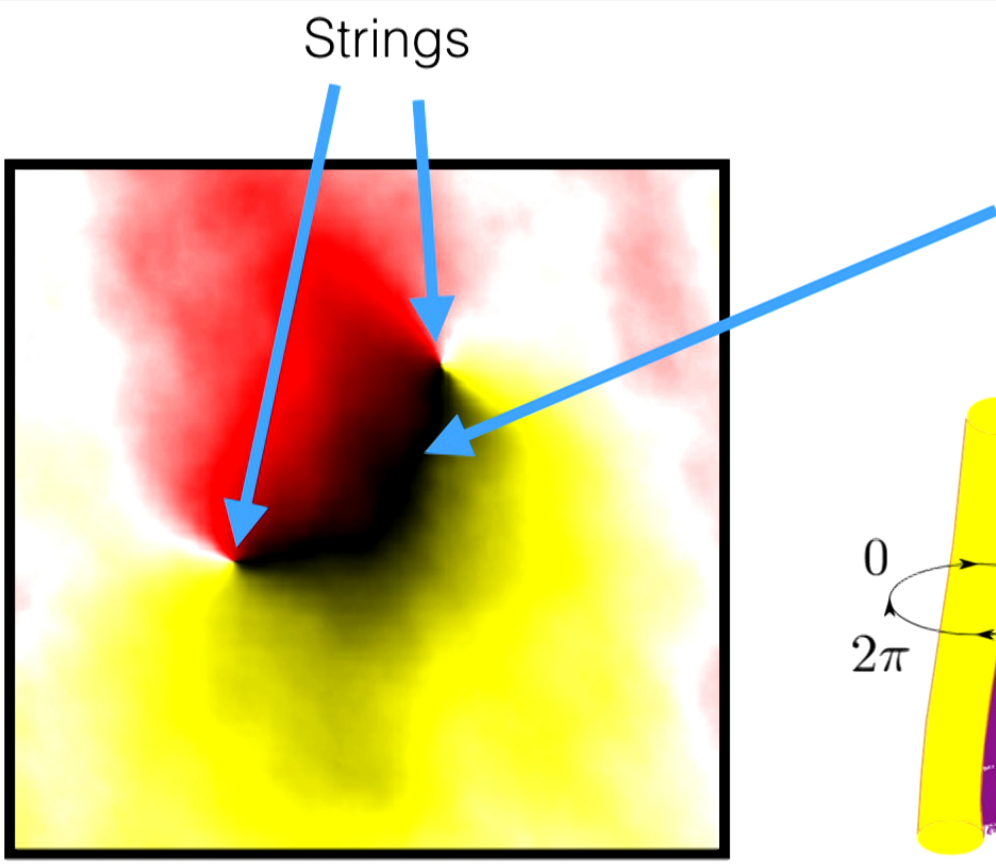
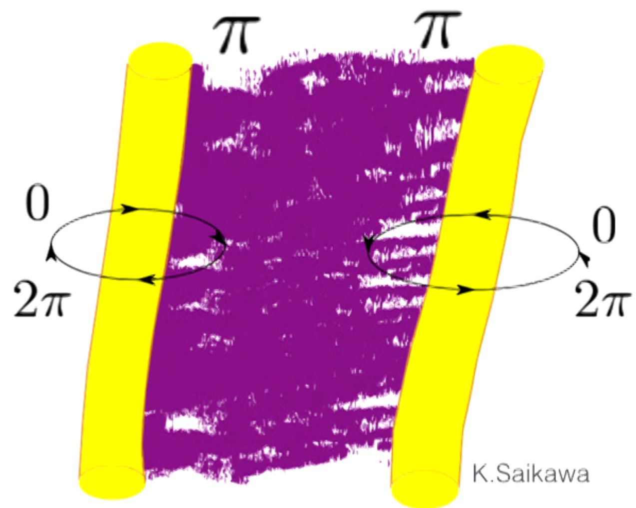
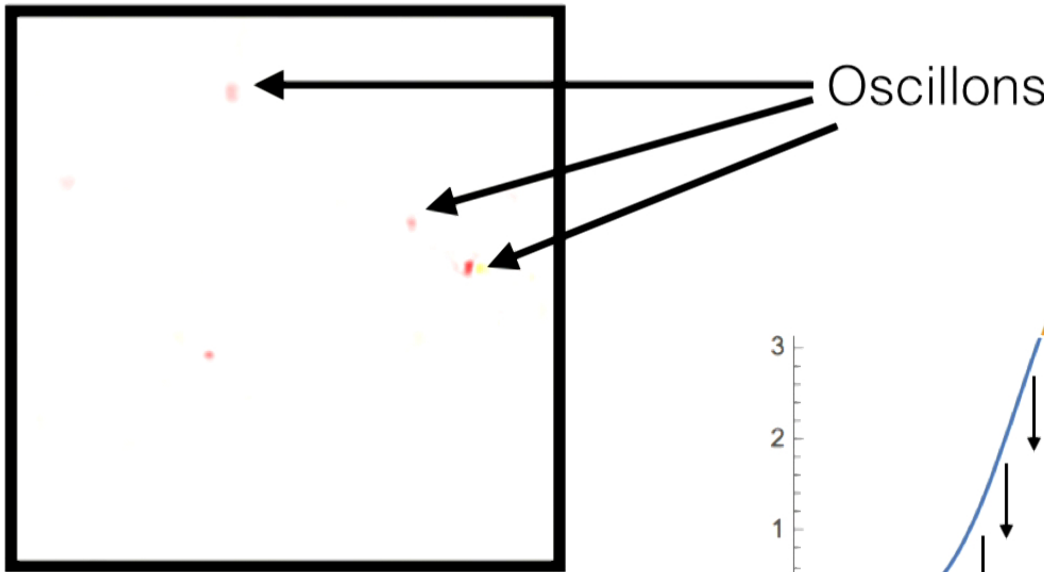
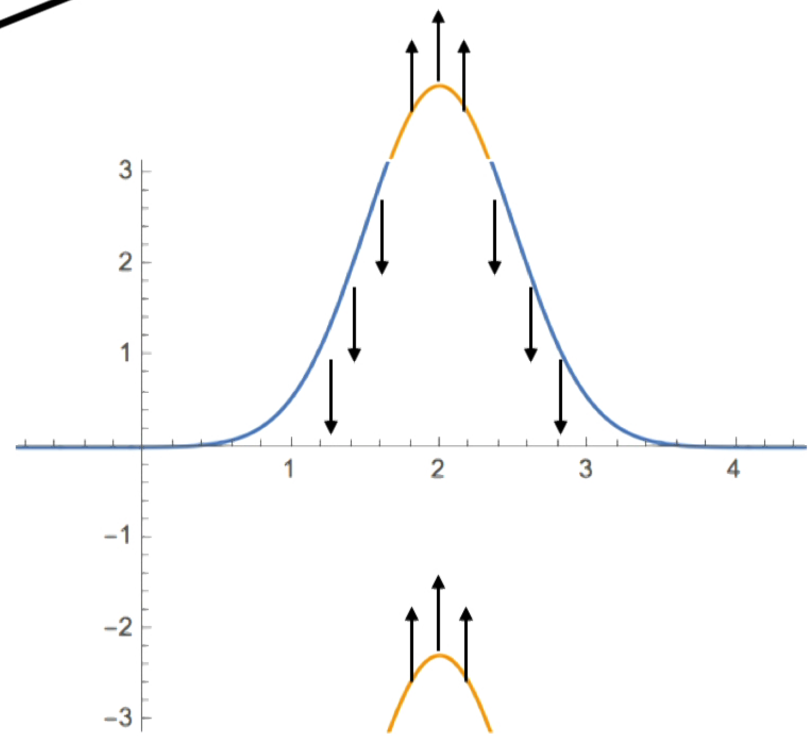




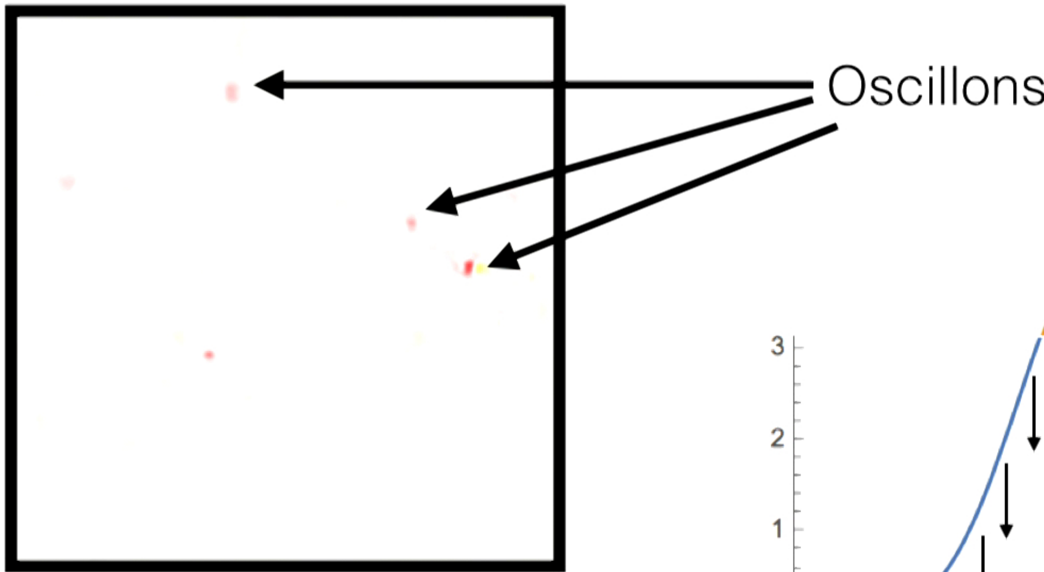
$$m_a^2(T_1)\eta^{2+n}\left(\frac{\psi_2^2}{(\psi_1^2+\psi_2^2)^{3/2}}\right)$$
$$m_a^2(T_1)\eta^{n+2}\left(\frac{\psi_1\psi_2}{(\psi_1^2+\psi_2^2)^{3/2}}\right) \longrightarrow \eta^{8.68}\sin\theta$$

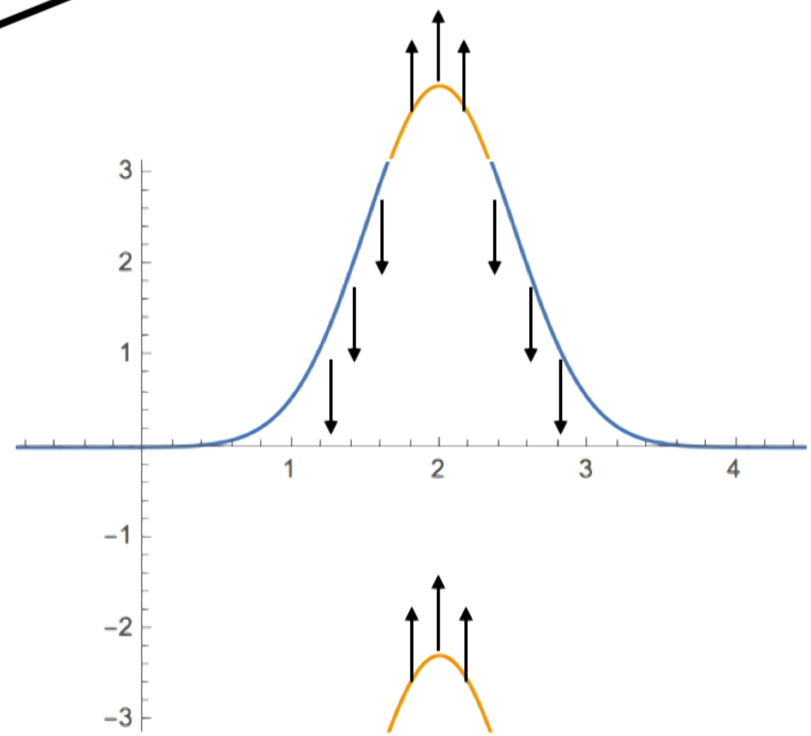




$$m_a^2(T_1)\eta^{2+n}\left(\frac{\psi_2^2}{(\psi_1^2+\psi_2^2)^{3/2}}\right)$$

$$m_a^2(T_1)\eta^{n+2}\left(\frac{\psi_1\psi_2}{(\psi_1^2+\psi_2^2)^{3/2}}\right) \longrightarrow \eta^{8.68}\sin\theta \text{ for increasing mass}$$

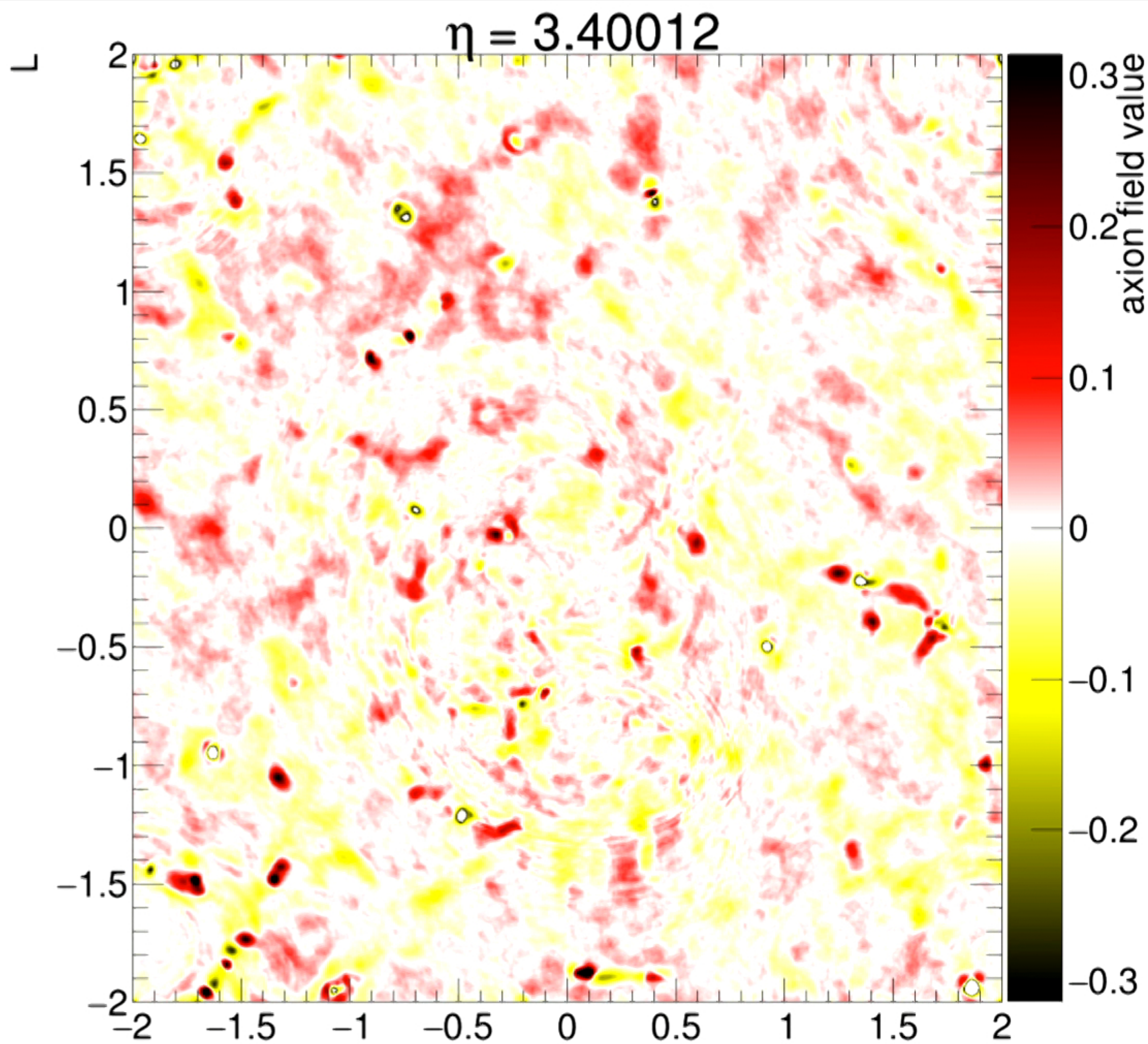


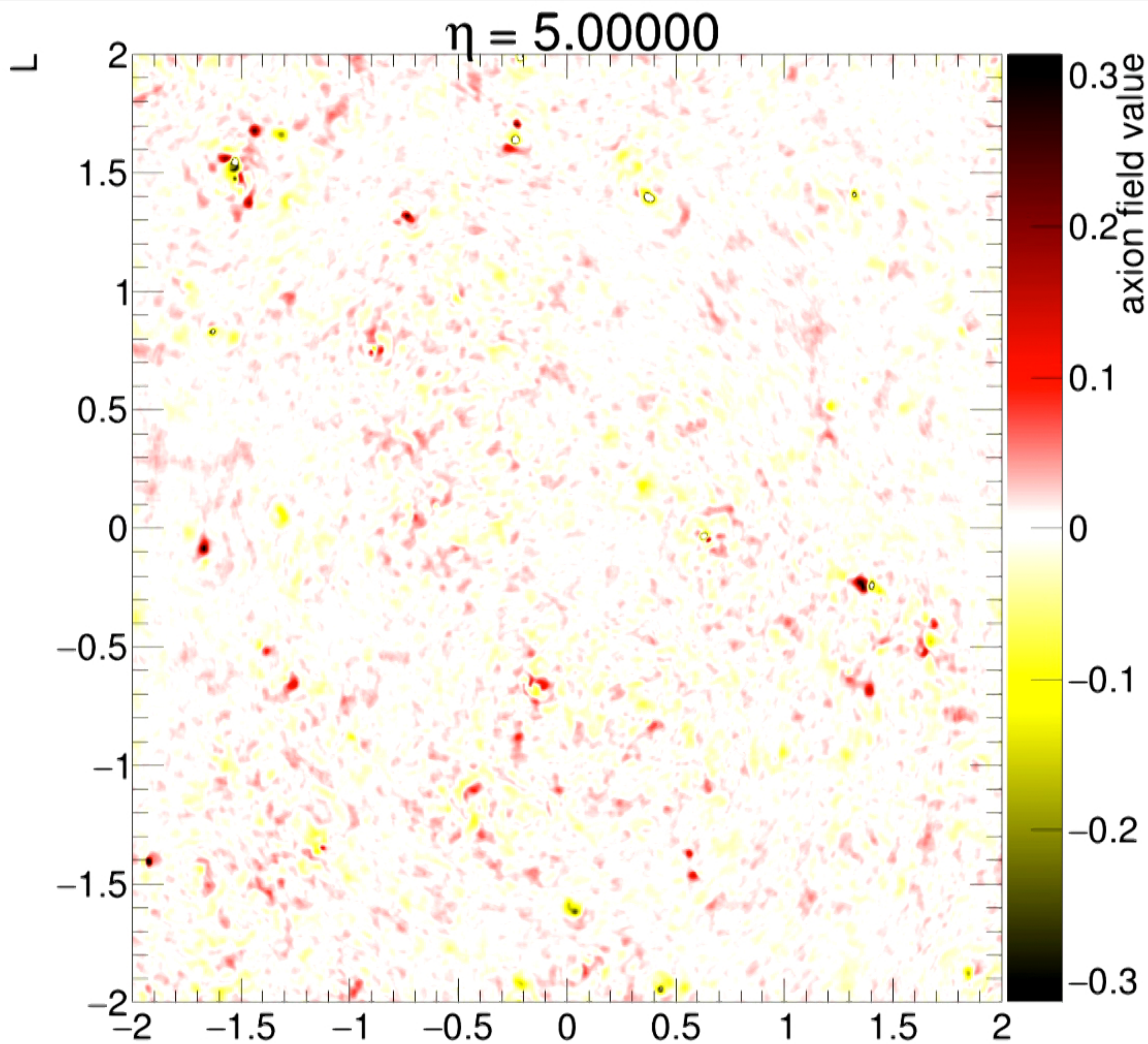


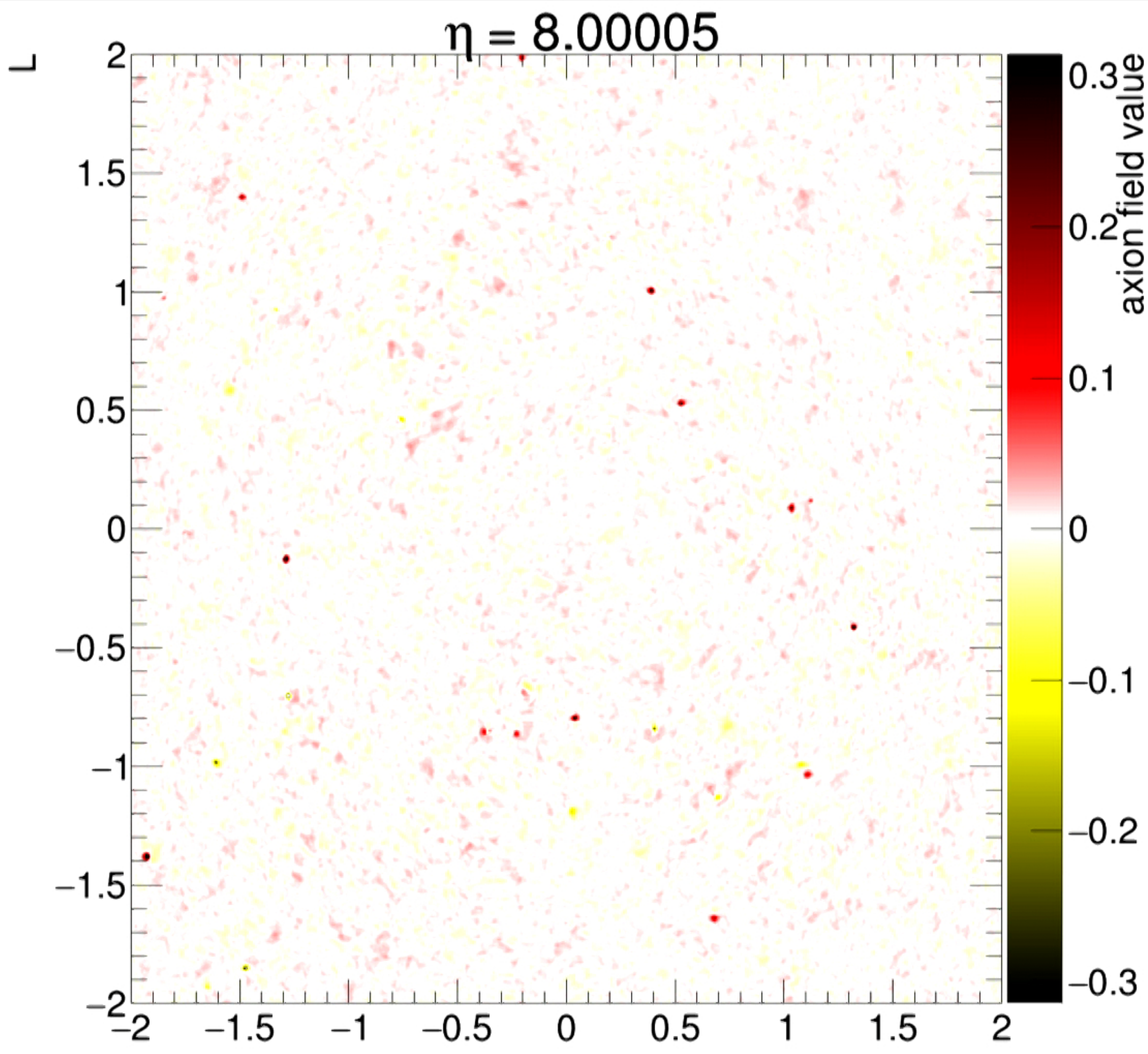
mass growth
truncated eventually:

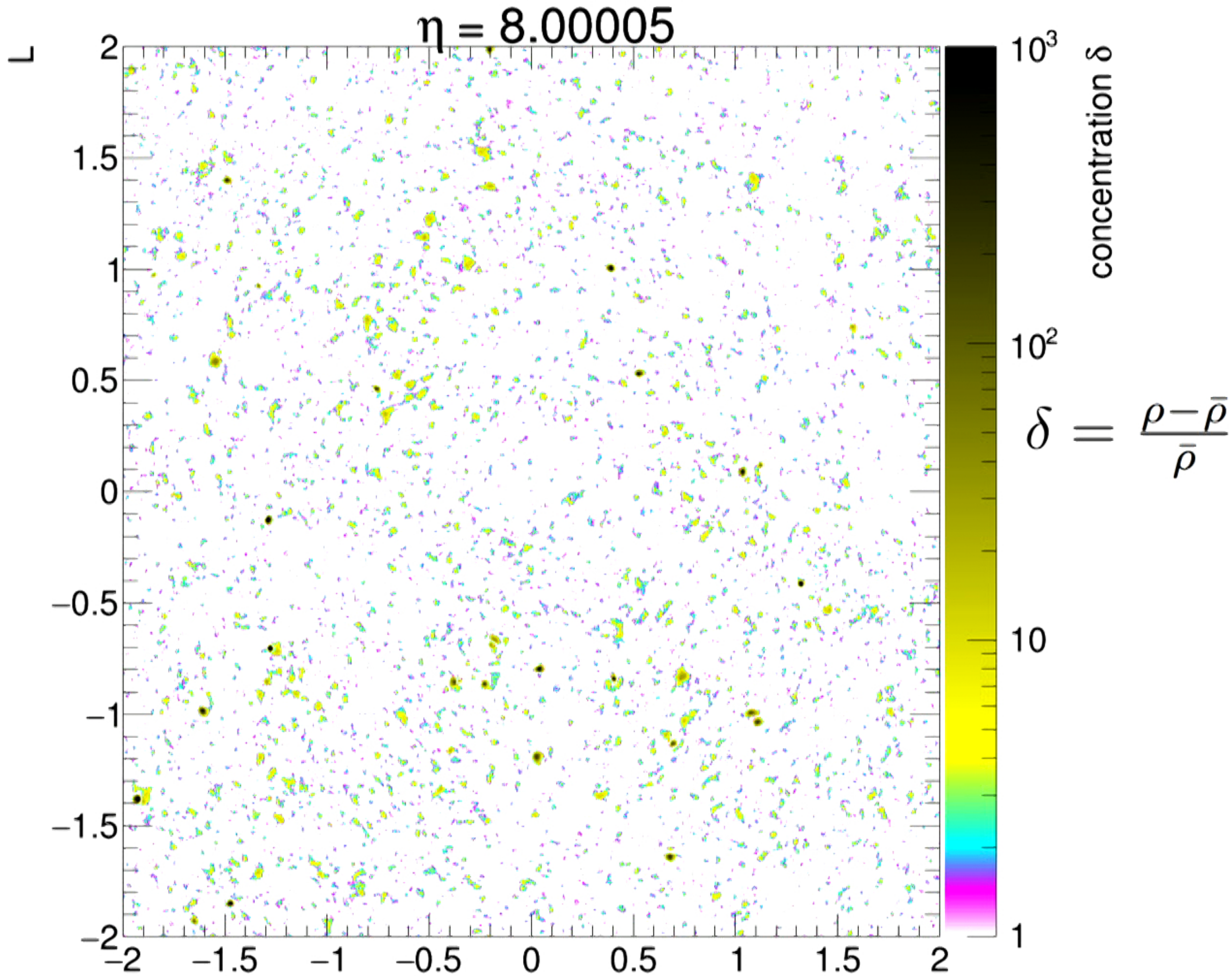
$$m_a^2(T_1)\eta^{2+n}\left(\frac{\psi_2^2}{(\psi_1^2+\psi_2^2)^{3/2}}\right)$$

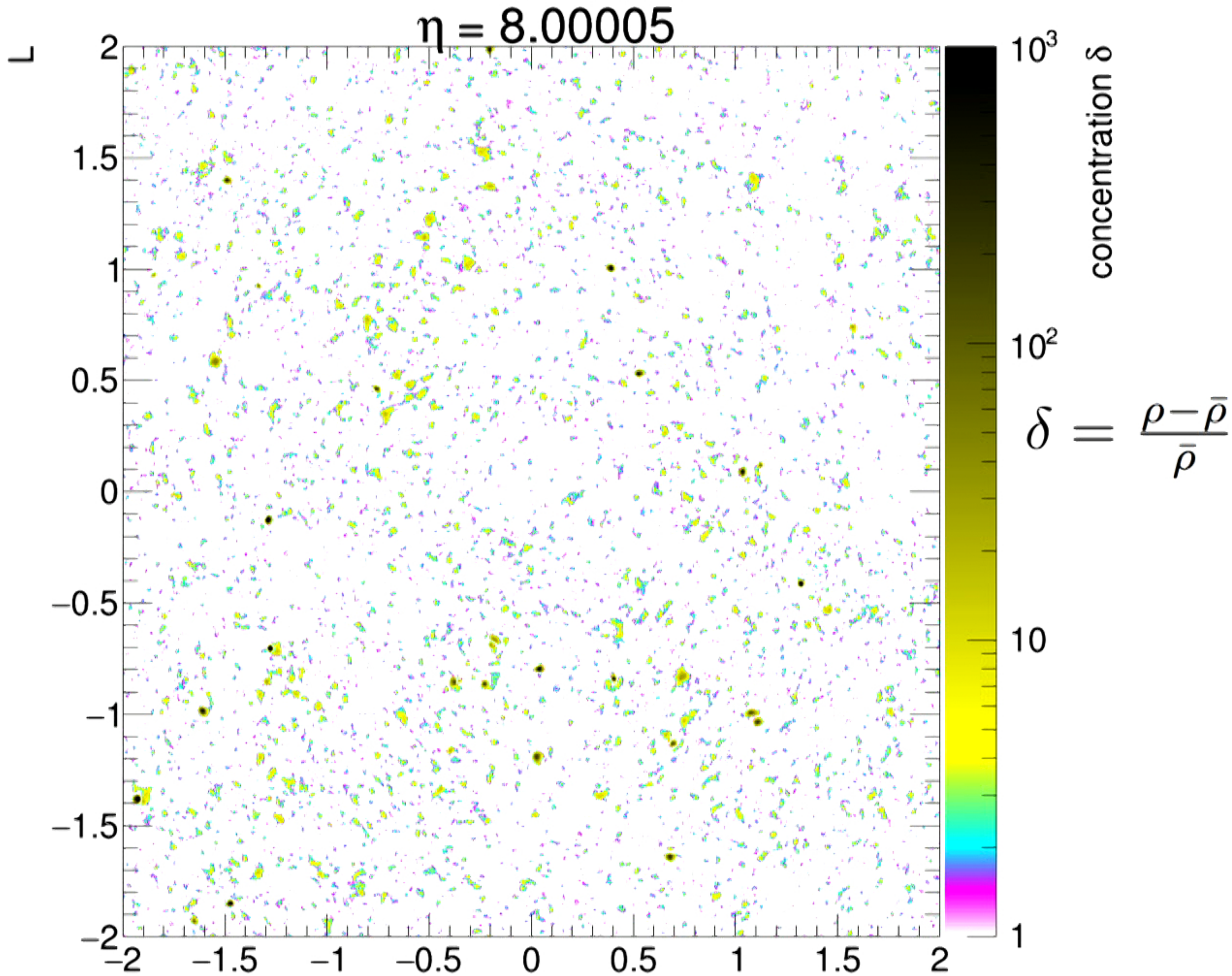
$$m_a^2(T_1)\eta^{n+2}\left(\frac{\psi_1\psi_2}{(\psi_1^2+\psi_2^2)^{3/2}}\right) \longrightarrow \eta^{8.68}\sin\theta \longrightarrow \eta^2\eta_s^{6.68}\sin\theta$$

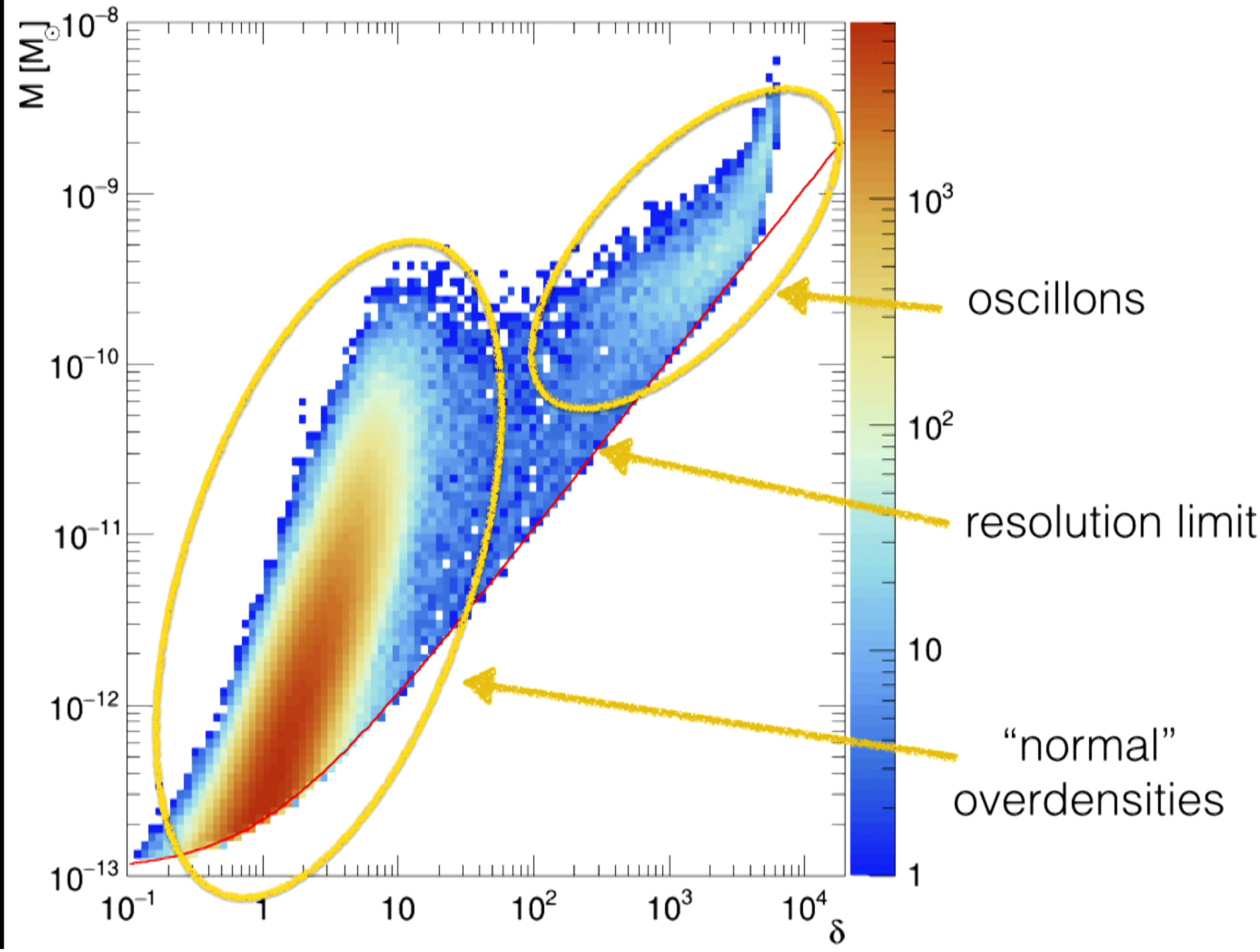

η = 3.40012
L
axion field value
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2

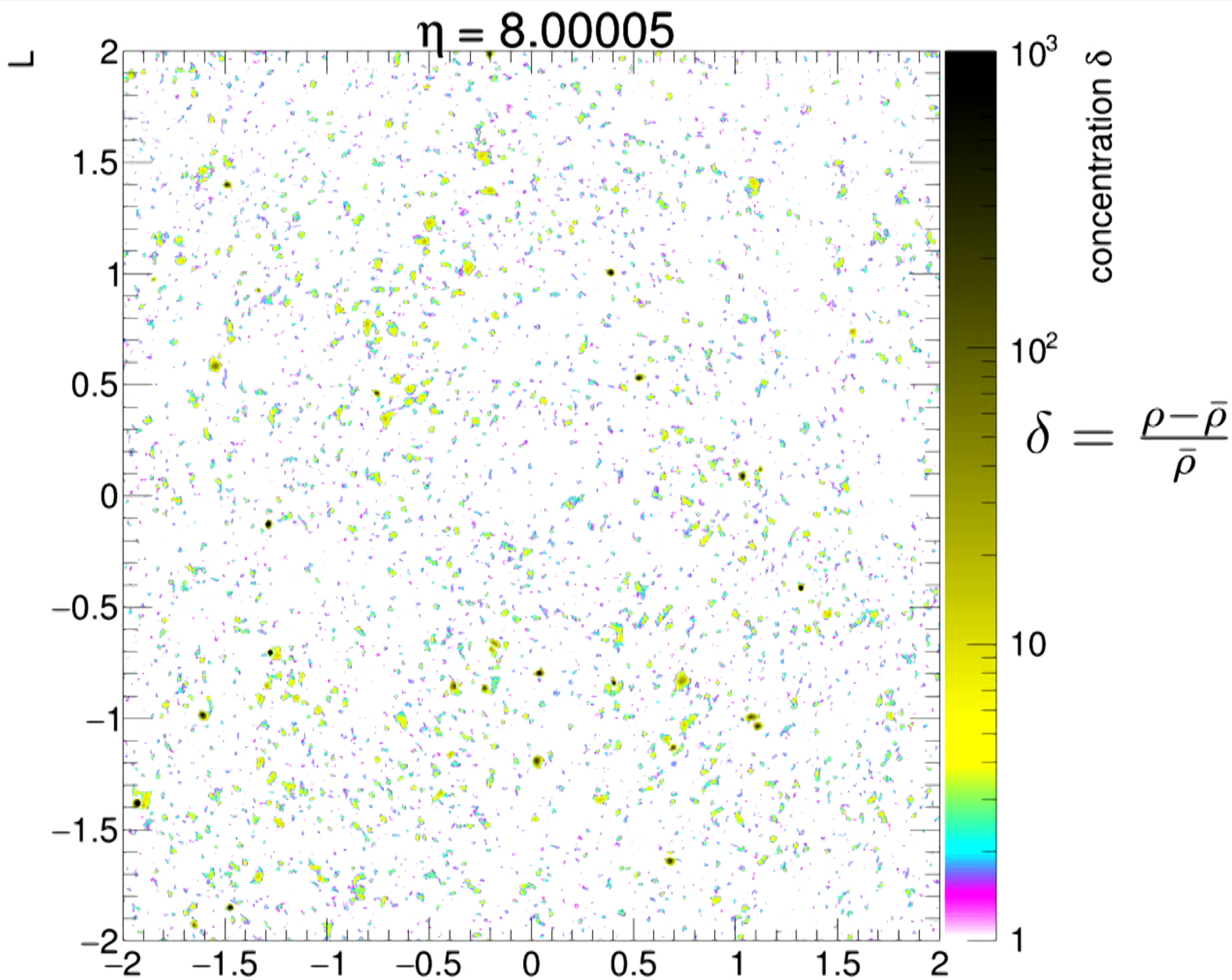

η = 5.00000
axion field value
L
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
0.3
0.2
0.1
0
−0.1
−0.2
−0.3


η = 8.00005
L
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
axion field value


η = 8.00005
L
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
10^3
10^2
10
1
concentration δ
$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

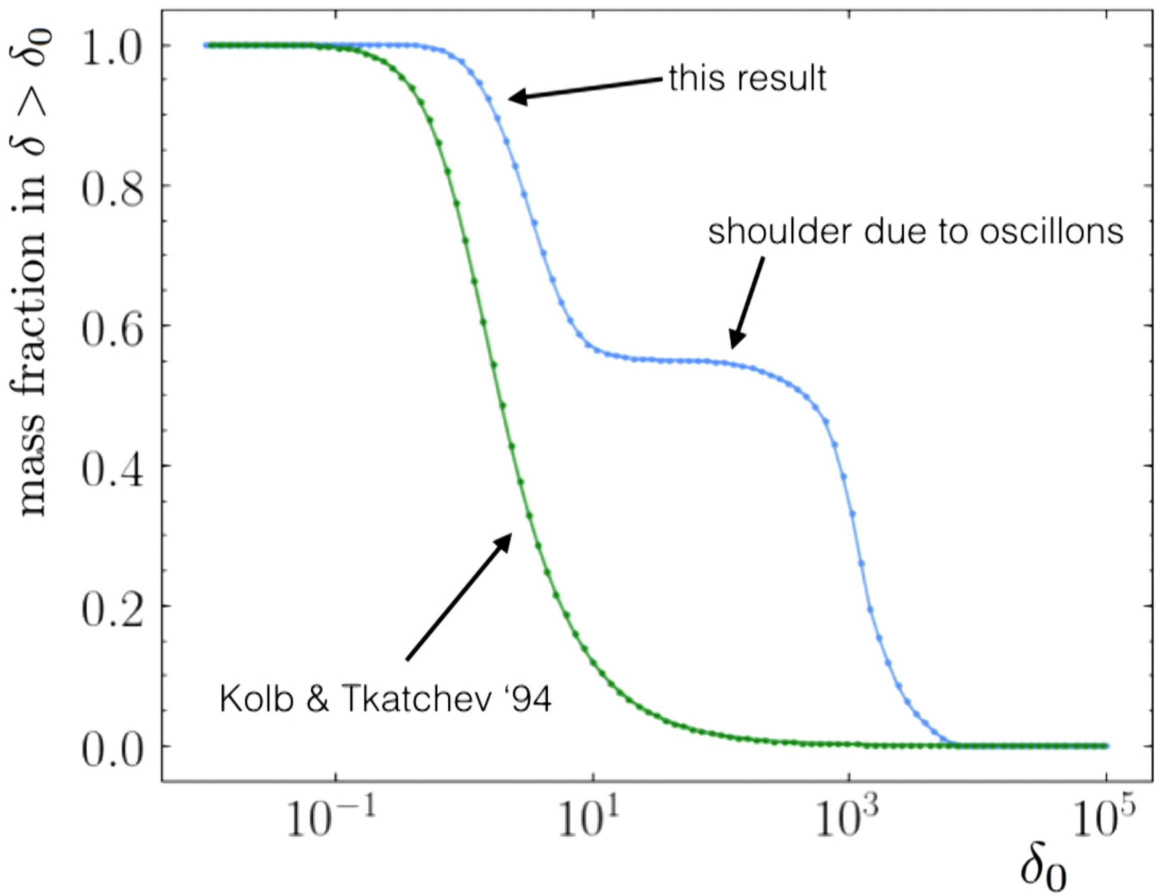

η = 8.00005
L
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
10^3
10^2
10
1
concentration δ
$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$


M [M$_\odot$]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-1}
1
10
10^2
10^3
10^4
δ
10^3
10^2
10
1
oscillons
resolution limit
"normal"
overdensities



η = 8.00005
L
concentration δ
$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

Oscillons carry a significant amount of the total mass:



These results are preliminary! There are three extrapolations to be made:

1. Simulating out to late times in η

Problem: Computation time + numerical noise starts dominating

However: We know how the axion field and oscillons scale with time once we are sufficiently away from the QCD PT

↓

Extrapolation can be done analytically

These results are preliminary! There are three extrapolations to be made:

2. Extrapolating the quartic coupling $\lambda \to 10^{57}$

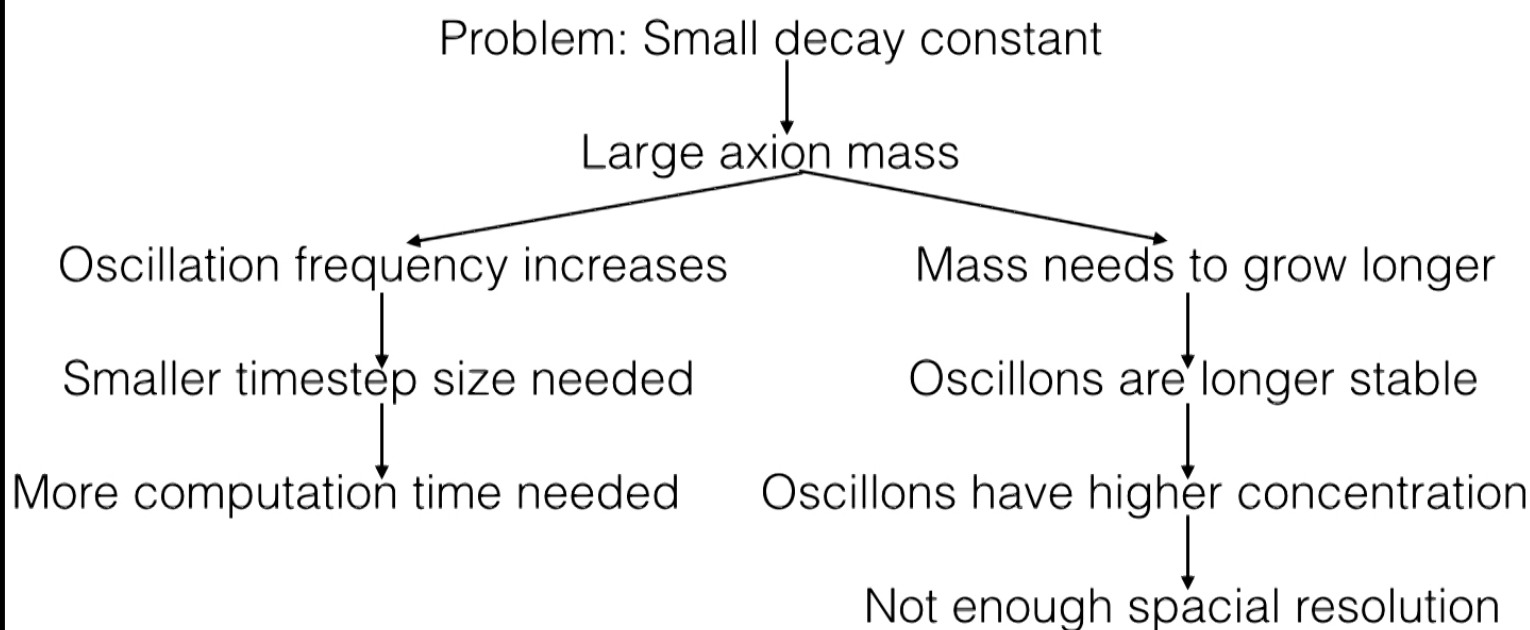
Problem: Strings/Domain Walls become too small to be resolved

However: Final result should only depend logarithmically on this coupling (and is thus often ignored).

Possible Solution: Runs at different couplings and subsequent fit to mass-delta data can be extrapolated

These results are preliminary! There are three extrapolations to be made:

3. Extrapolation to $f_a \approx 10^{12}$ GeV



Conclusions

- Post-inflationary scenario leads to small scale structure: Axion miniclusters, axion stars, …
- To predict the mass that is bound in such objects simulations through the QCD phase transition needs to be done
- Oscillons play an important role for the creation of compact objects
- Preliminary results show that a significant amount of energy is stored in oscillons and thus in clusters


Thank you!