

Title: Mechanisms of Fracton Phases

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URL: <http://pirsa.org/18100060>

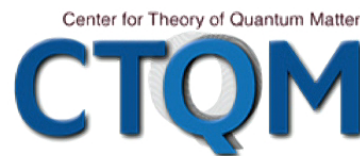
Abstract: <p>Three dimensional fracton phases are new type of phases featuring exotic excitations called fractons. They are gapped point-like excitations constrained to move in sub-dimensional space. In this talk, I will present the gapped fracton topological order discovered in exact solvable models and gapless fracton phase described by $U(1)$ symmetric tensor gauge theories. Their relation with ordinary topological ordered phase would be discussed in detail. Particularly, the fracton topological order exhibited in an exact solvable model called X-cube model can be constructed by coupling toric code layers. And it can also be obtained from a particular rank-2 symmetric tensor gauge theory called scalar charge theory by Higgs mechanism and partial confinement transition.</p>

Mechanisms of Fracton Phases

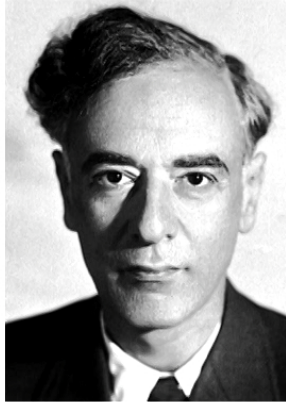
Han Ma
University of Colorado at Boulder

HM, Ethan Lake, Xie Chen, Mike Hermele, PRB 95, 245126, (2017)
HM, Mike Hermele, Xie Chen, PRB 98, 035111, (2018)

Perimeter
Oct 09, 2018



Quantum phases of matter

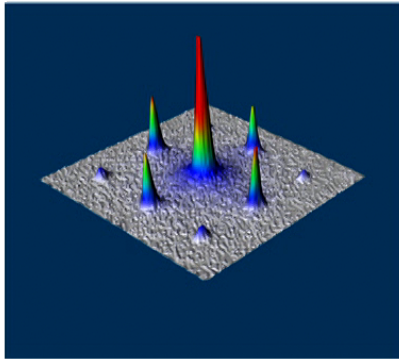


Landau's paradigm

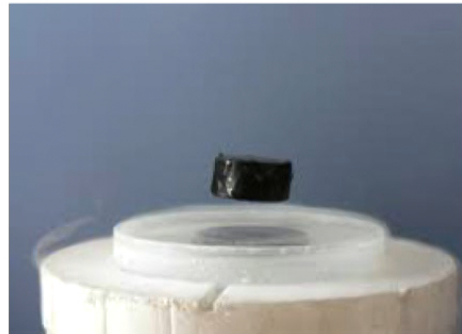
quantum states “are classified by symmetry breaking”

Ordered states:

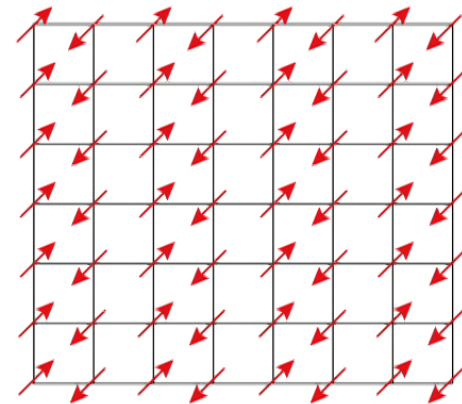
superfluid



superconductor



Neel state



Spin liquid: more than absence of order

- gapped spin liquid (topological order): Z_2 spin liquid (toric code), chiral spin liquid.....
- gapless spin liquid (critical phase): $U(1)$ spin liquid, algebraic spin liquid

Senthil, Fisher (1999, 2005),

Senthil, Vishwanath, Balents, Sachdev, Fisher(2004),

Wen (2002)

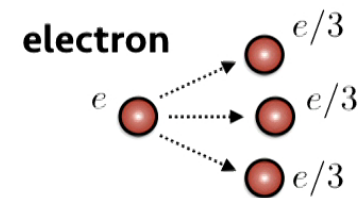
- deconfined criticality.....

Hermele, Ran, Lee, Wen (2008)

- Fractionalization in 2d/3d



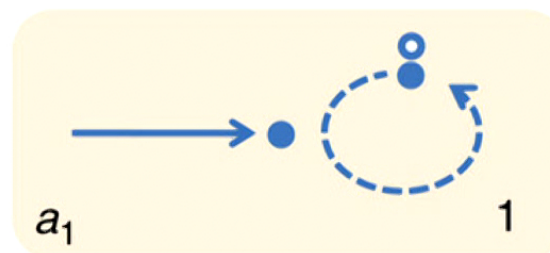
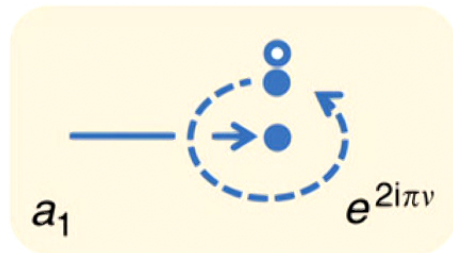
$1/3$ fractional quantum Hall



- Emergent gauge field: $U(1)$, Z_2

Gapped fractionalized excitations in 2d: anyons

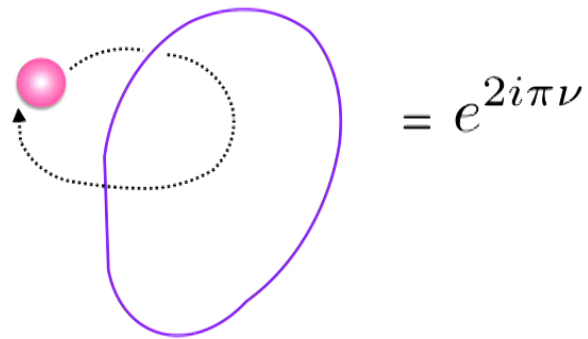
1. Point like excitations (e.g. e and m particle of toric code)
2. Braiding processes generates topological phase factors
3. Free to move



Gapped Fractionalization in 3d

1. No point-like anyons, only boson / fermion
2. Non-trivial braiding between loop excitations and point like excitations.

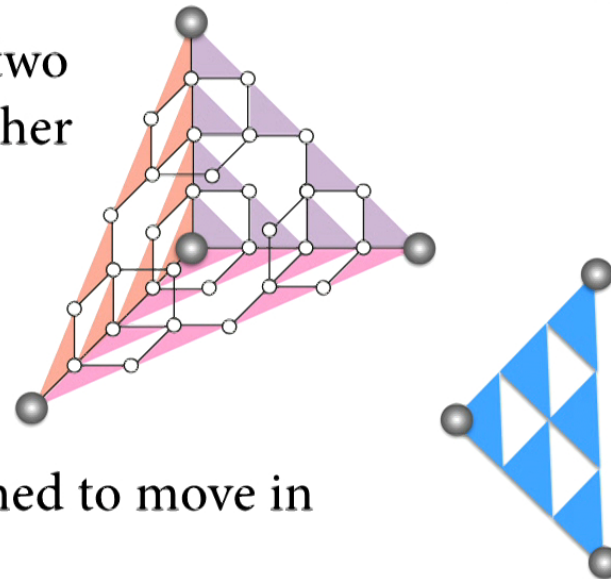
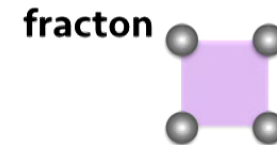
e.g.



Fractons

A new type of point-like excitations living in three spatial dimensions which:

1. cannot move without creating or moving other excitations.
2. cannot be created alone by local operators.
3. are always created in more than two and can move away from each other by acting membrane/fractal operators.



Sub-dimensional excitations

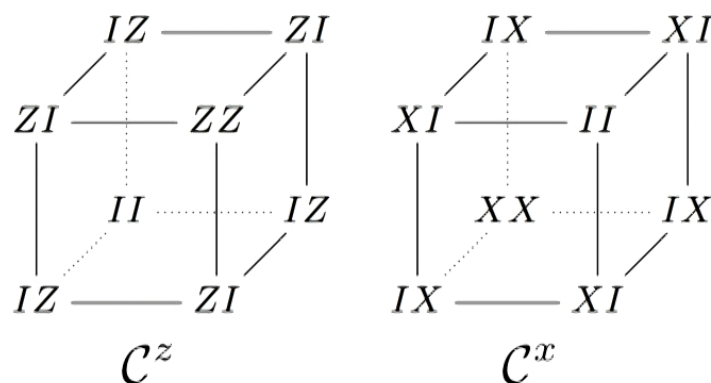
mobile excitations but only constrained to move in subdimensional space.

Example: Haah's cubic code

Haah (2011)

Haah (2013)

$$H_{Haah} = - \sum_C \mathcal{C}_C^z - \sum_C \mathcal{C}_C^x$$

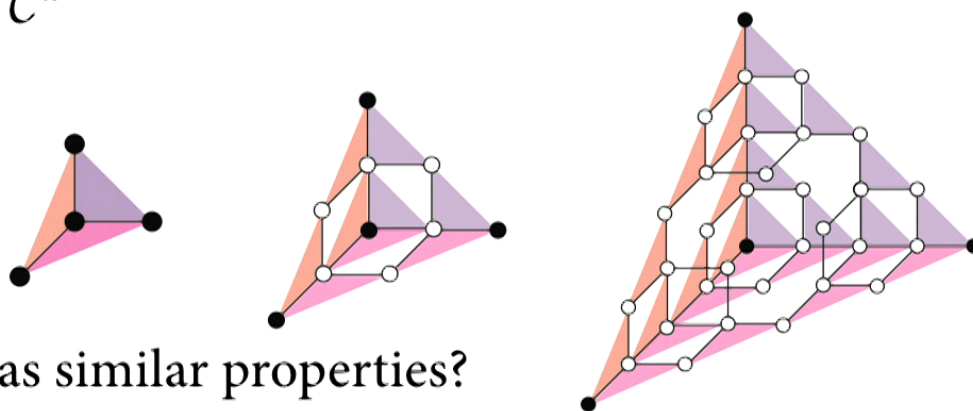


gsd: $2^{k(L)}$ on 3-torus

X,Z: Pauli matrices

I: Identity

Q: Any other model has similar properties?



3D Discrete Fracton models

- Chamon-Bravyi-Leemhuis-Terhal model
- Majorana (spin) Checkerboard model
- X-cube model

Chamon (2005)

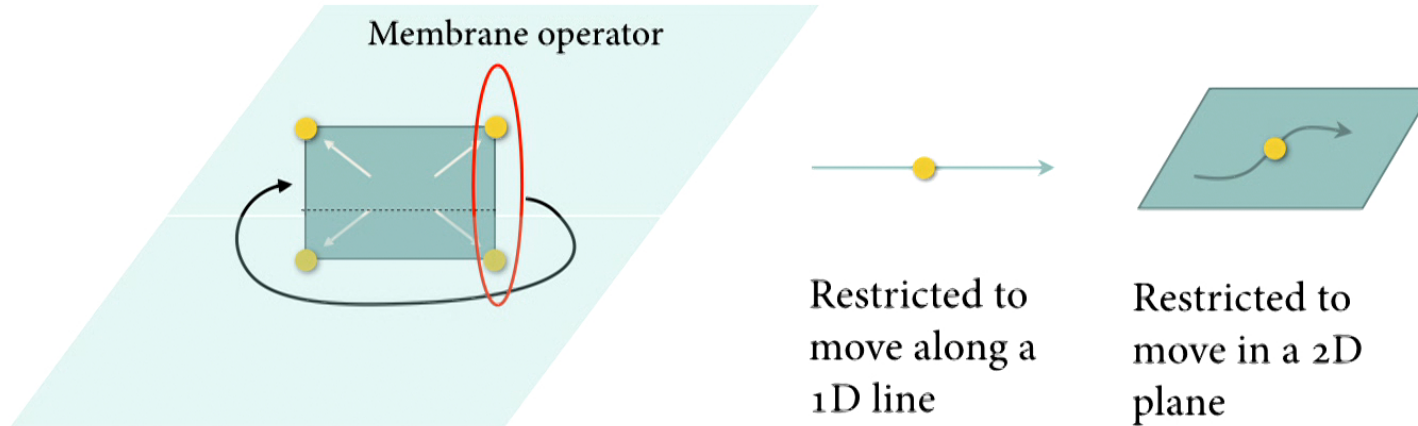
Yoshida (2013)

Vijay, Haah, Fu (2015), (2016)

HM, Lake, Chen, Hermele (2017)

Hsieh, Halasz (2017)

.....

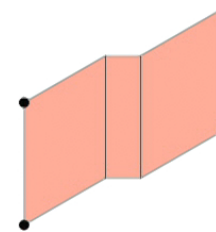


Mobile Subdimensional particles

3D Discrete Fracton models

- Type I:
Chamon-Bravyi-Leemhuis-Terhal model
Majorana (spin) Checkerboard model
X-cube model

Membrane operator
Subdimensional particles



- Type II: Haah's cubic code
Yoshida's classical fractal model

Fractal operator
Immobile particles



Fracton topological order

1. a gap to local excitations
2. fractionalized excitations that cannot be created alone by local operation.
3. Sub-extensive ground state degeneracy on 3-torus
4. fractionalized point-like excitations that are confined to move in zero (fractons), one or two-dimensional subspaces
5. beyond the description of topological quantum field theory (TQFT)

Gapless Fracton phase

Cenke Xu (2006)

Cenke Xu, Congjun Wu (2008)

Cenke Xu, Petr Horava (2010)

Alex Rasmussen, Yi-Zhuang You, Cenke Xu (2016)

Michael Pretko (2016)

U(1) symmetric tensor gauge theories:

Gauge variables are higher rank **symmetric** tensors instead of vectors.

Example: rank-2 gauge theory in 3d

$$3\text{d: } E_{\mu\nu} = \begin{bmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{bmatrix} \quad A_{\mu\nu} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

$A_{\mu\nu}, E_{\mu\nu}$: U(1) variables

$$H = U \sum_{\mathbf{r}, \mu \leq \nu} E_{\mu\nu}^2 - K \sum_{\mathbf{r}, \mu, \nu} \cos(B_{\mu\nu})$$

$$[A_{\mu\mu}, E_{\mu\mu}] = -i$$

$$[A_{\mu\nu}, E_{\mu\nu}] = -i$$

Unconventional Gauss laws $\rightarrow$ Gapped Fractons/subdimensional excitations

Gapless photon modes $\rightarrow$ Stable fixed point

Gapless Fracton phase

Michael Pretko (2016)

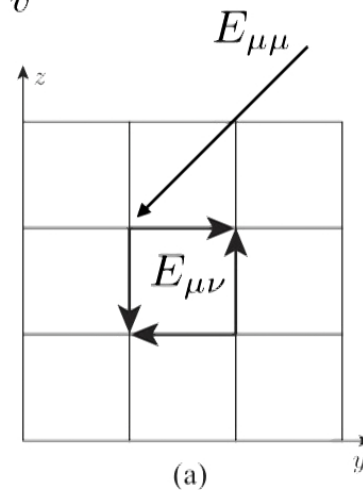
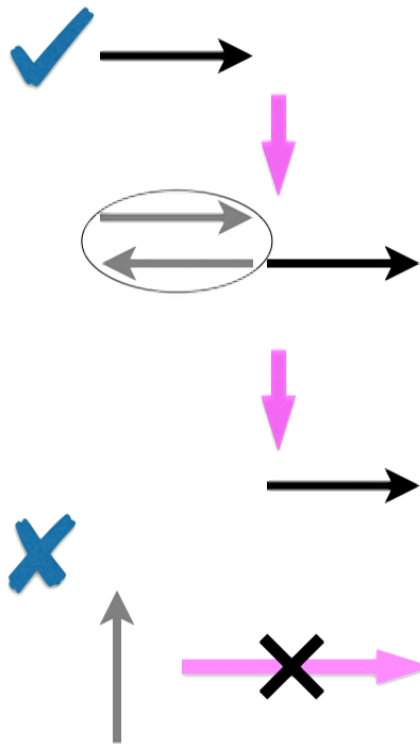
Example (1): Rank 2 vector charge theory

$$\partial_\mu E_{\mu\nu} = \rho_\nu \Rightarrow \int_v \vec{\rho} \times \vec{x} = 0$$

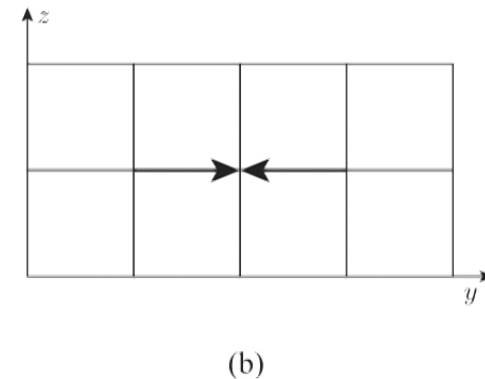
$$\int_v \rho = 0$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \alpha_\nu + \partial_\nu \alpha_\mu$$

$$B_{\mu\nu} = \epsilon^{\mu\lambda\eta} \epsilon^{\nu\delta\zeta} \partial_\lambda \partial_\delta A_{\eta\zeta}$$



$$E_{\mu\nu} \rightarrow E_{\mu\nu} + 1$$



$$E_{\mu\mu} \rightarrow E_{\mu\mu} + 1$$

Gapless Fracton phase

Michael Pretko (2016)

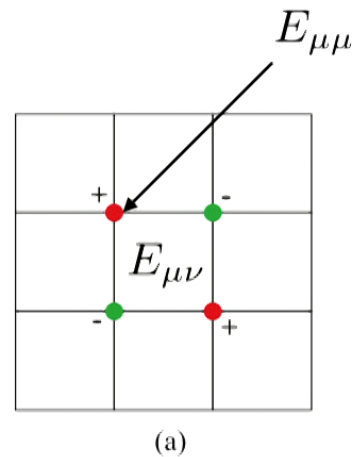
Example (2): Rank 2 scalar charge theory on the cubic lattice

$$\partial_\mu \partial_\nu E_{\mu\nu}(\mathbf{r}) = \rho_{\mathbf{r}}$$

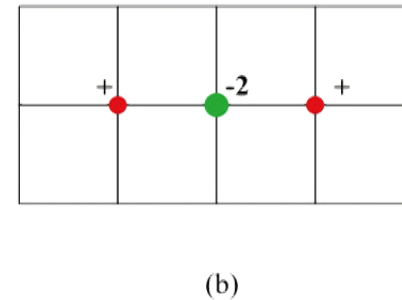
Charges:  

Conservation laws:

$$\int_v \rho = 0 \quad \int_v \vec{x} \rho = 0$$



$$E_{\mu\nu} \rightarrow E_{\mu\nu} + 1$$

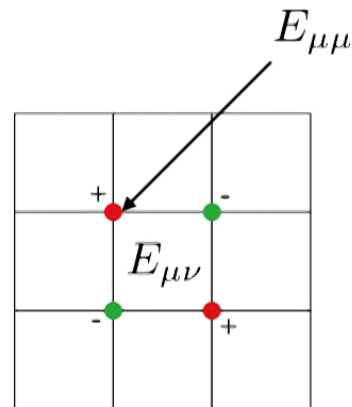
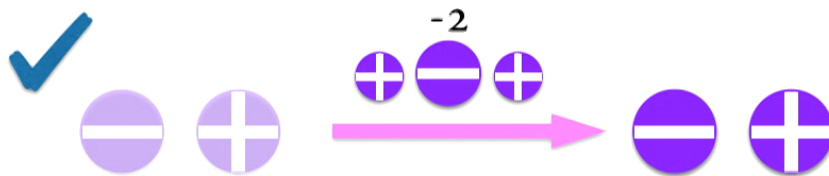
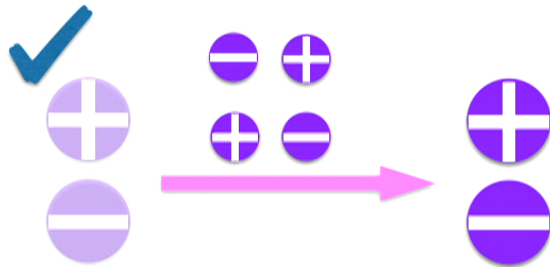
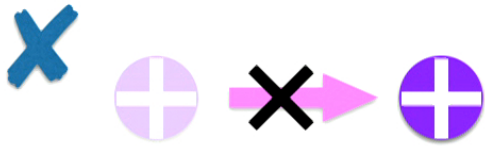


$$E_{\mu\mu} \rightarrow E_{\mu\mu} + 1$$

Gapless Fracton phase

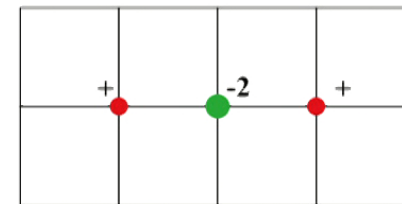
Michael Pretko (2016)

Example (2): Rank 2 scalar charge theory on the cubic lattice



(a)

$$E_{\mu\nu} \rightarrow E_{\mu\nu} + 1$$



(b)

$$E_{\mu\mu} \rightarrow E_{\mu\mu} + 1$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \partial_\nu f$$

$$B_{\mu\nu} = \epsilon_{\mu\lambda\sigma} \partial_\lambda A_{\sigma\nu}$$

$$\sum_\mu B_{\mu\mu} = 0$$

Fracton Phases

Questions:

- Relation to other topological ordered phases?
- Relation to the $U(1)$ fracton phases?
- Continuum field theory?
- Classification?
- Role of Lattice symmetry?
- How symmetry enrich this order?
-

Fracton Phases

HM, Lake, Chen, Hermele (2017)

HM, Hermele, Chen (2018)

Questions:

- Relation to other topological ordered phases?

√

- Relation to the $U(1)$ fracton phases?

√

- Continuum field theory?

- Classification?

- Role of Lattice symmetry?

Pretko (2016)

- How symmetry enrich this order?

Halasz, Hsieh, Balents (2017)

Gromov (2017)

Bulmash, Barkeshli (2018)

-

Shirley, Slagle, Wang, Chen (2018)

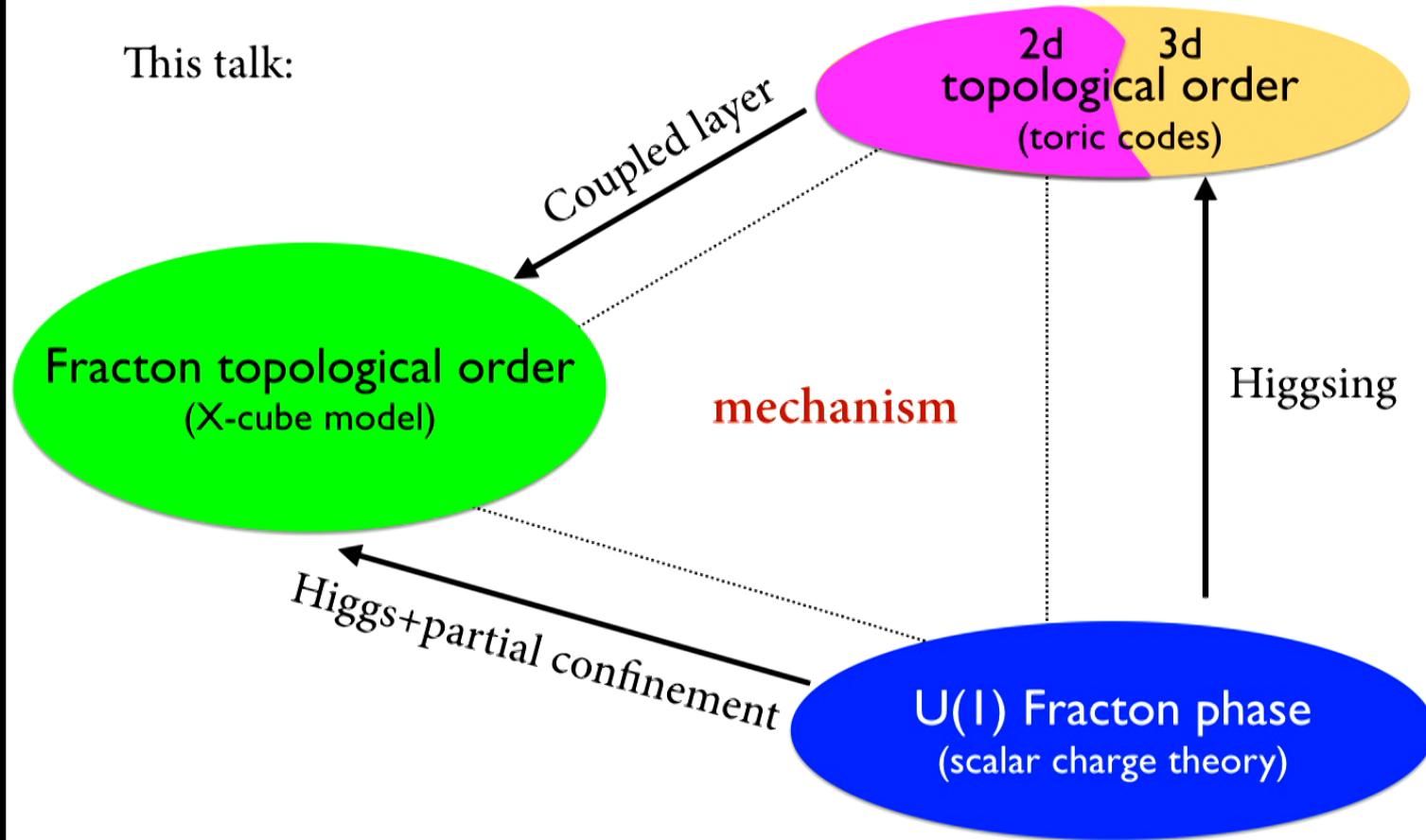
Song, Prem, Huang, Martin- Delgado (2018)

You, Devakul, Burnell, Sondhi (2018)

Williamson, Bi ,Cheng (2018)

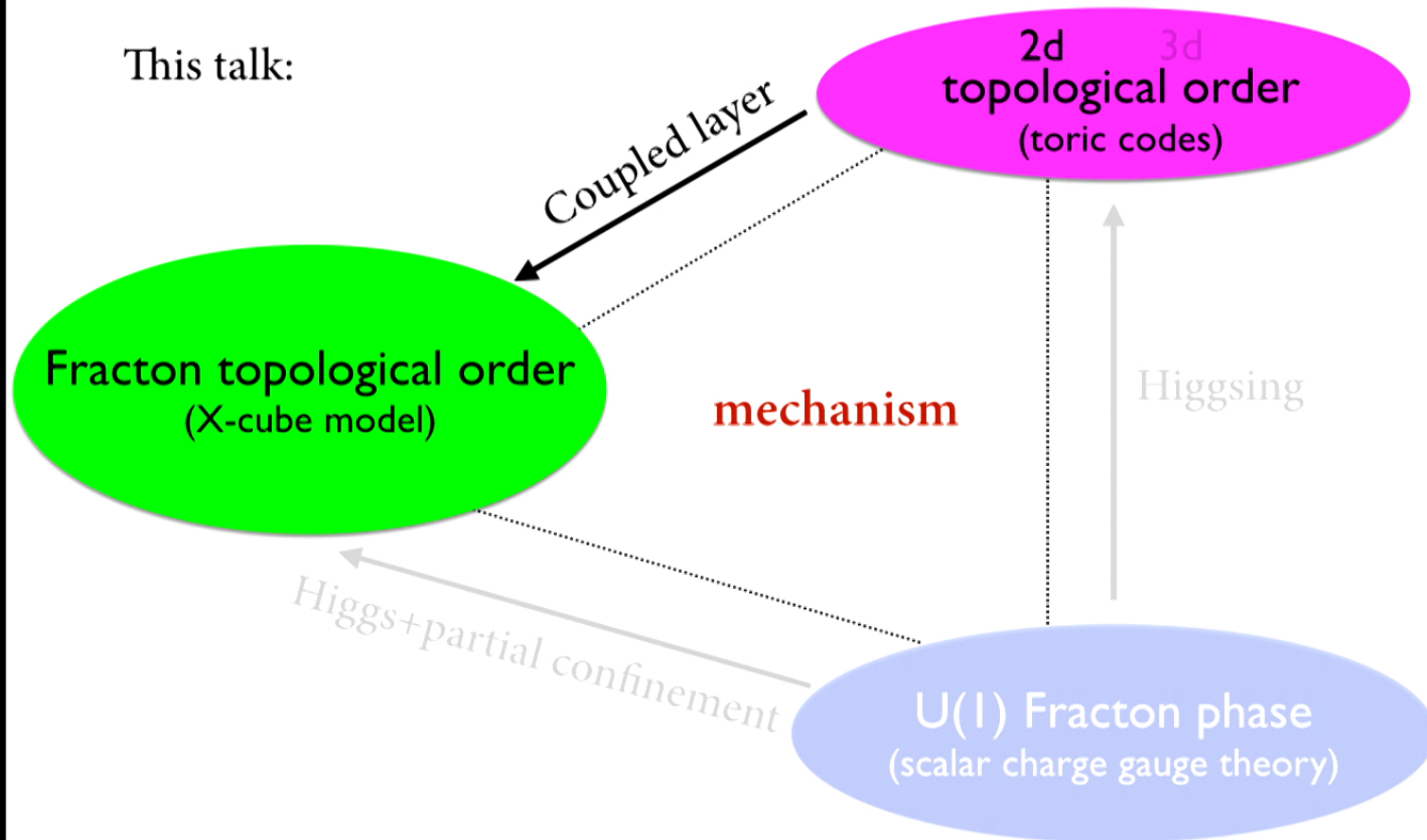
Fracton Phases — the phases have subdimensional excitations

This talk:



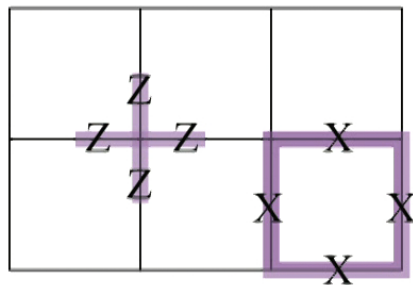
Fracton Phases — the phases have subdimensional excitations

This talk:



2d Toric code

$$H_{TC} = - \sum_v \prod_{l \in v} Z_l - \sum_p \prod_{l \in p} X_l$$



$$(\prod_{l \in v} Z_l) |GS\rangle = |GS\rangle$$

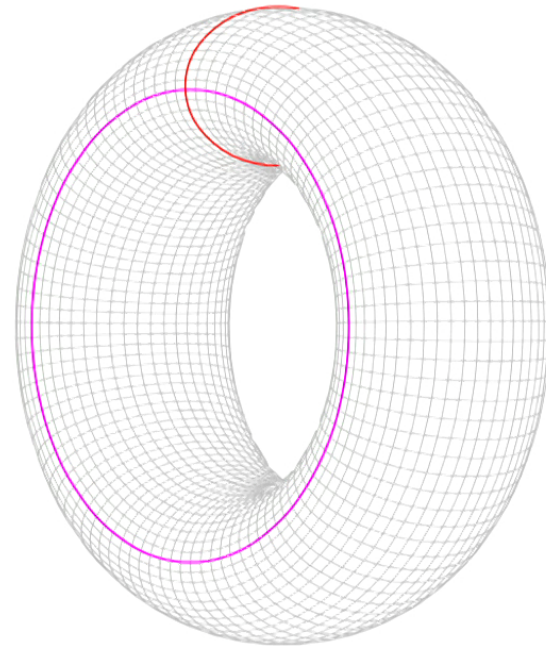
$$(\prod_{l \in p} X_l) |GS\rangle = |GS\rangle$$

$$\mathbf{m}: Z_l |GS\rangle = | \text{blue sphere} - \text{blue sphere} \rangle$$

$$\mathbf{e}: X_l |GS\rangle = | \text{pink sphere} - \text{pink sphere} \rangle$$

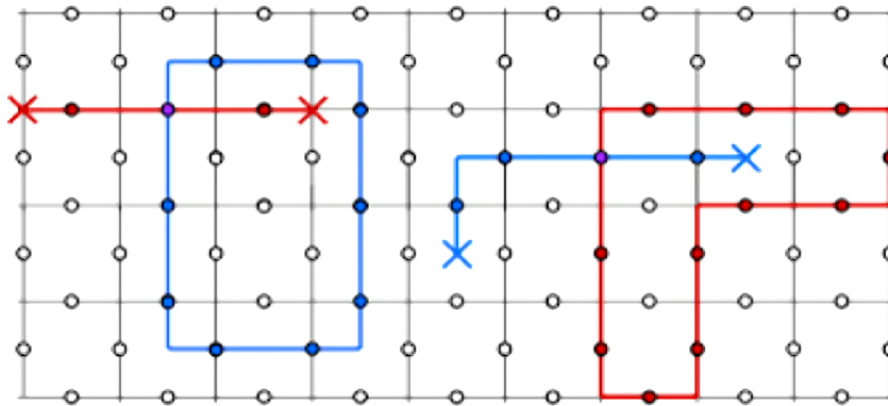
$$\text{gsd}=4$$

Kitaev, 1997

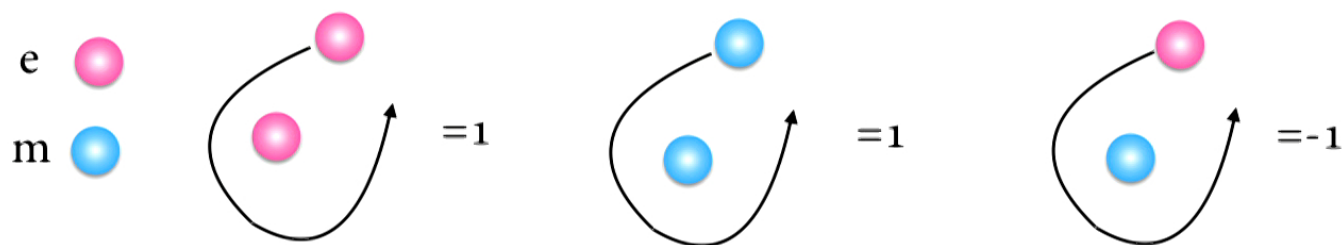


2d Toric code

Example: Toric code

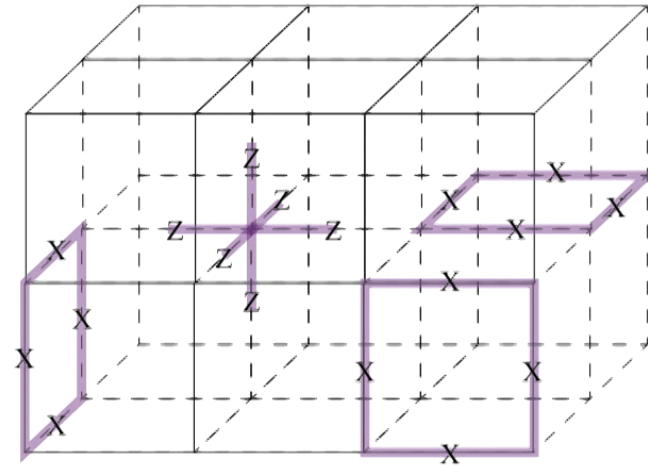


Semionic statistics



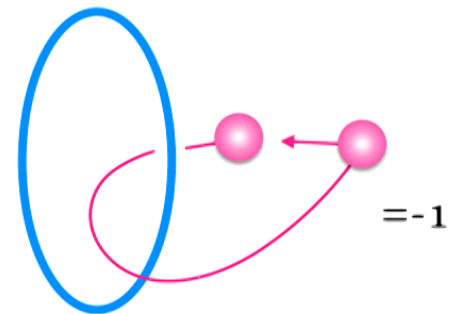
3d Toric code

$$H_{TC} = - \sum_v \prod_{l \in v} Z_l - \sum_p \prod_{l \in p} X_l$$



$$Z_l |GS\rangle = \left| \text{yellow star with blue circle} \right\rangle \quad X_l |GS\rangle = \left| \text{two pink spheres connected by a red line} \right\rangle$$

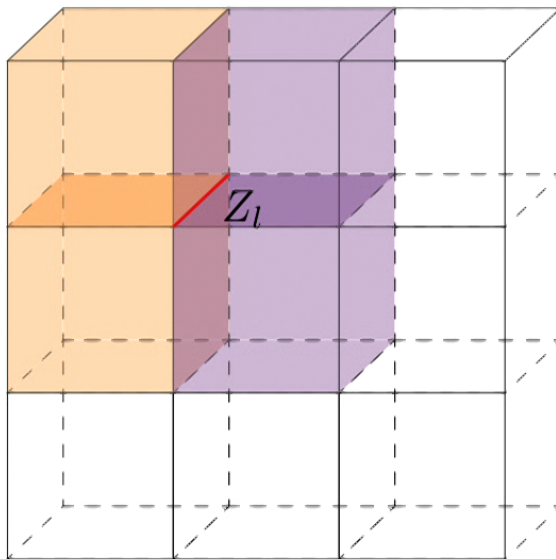
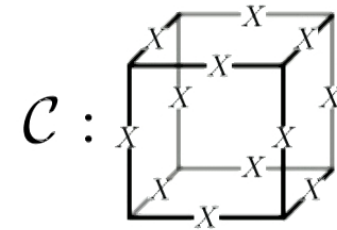
GSD = 2^3 on the 3-torus



X-cube model

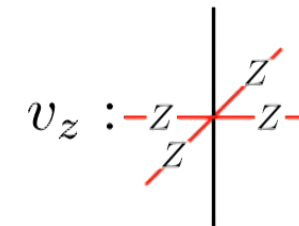
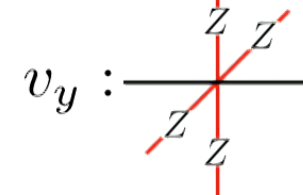
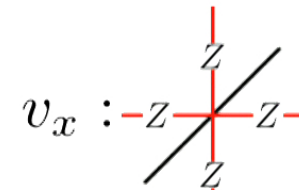
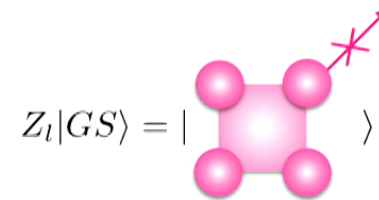
Vijay, Haah, Fu, 2015, 2016

$$H_{XC} = - \sum_{\mathcal{C}} \prod_{l \in \mathcal{C}} X_l - \sum_{\mu=x,y,z} \sum_{v_{\mu}} \prod_{l \in v_{\mu}} Z_l$$



$$\left(\prod_{l \in v_{\mu}} Z_l \right) |GS\rangle = |GS\rangle$$

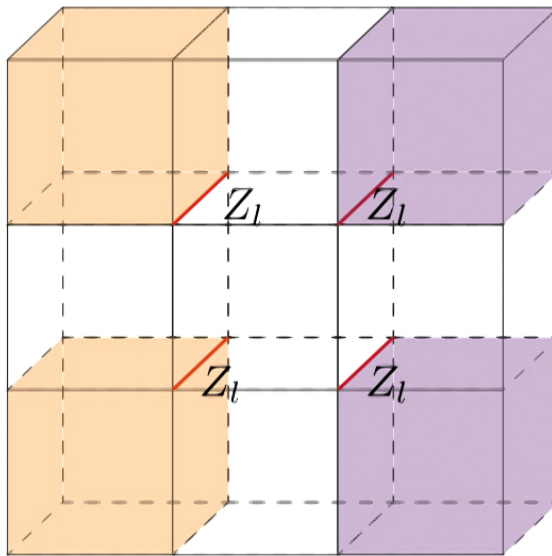
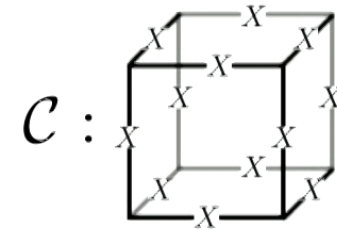
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X-cube model

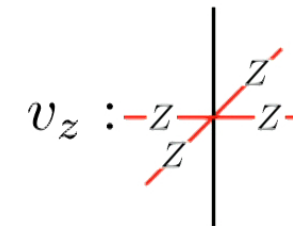
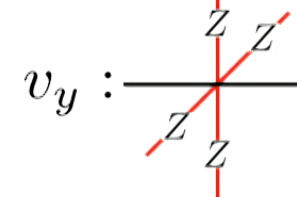
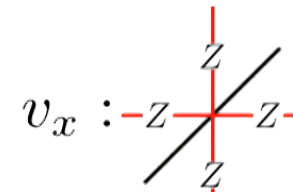
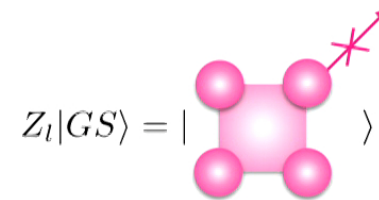
Vijay, Haah, Fu, 2015, 2016

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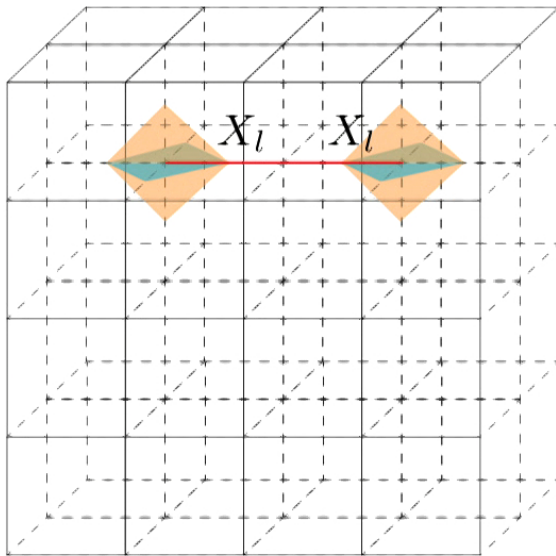
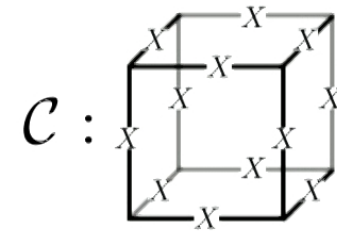
$$\left(\prod_{l \in \mathcal{C}} X_l \right) |GS\rangle = |GS\rangle$$



X-cube model

Vijay, Haah, Fu, 2015, 2016

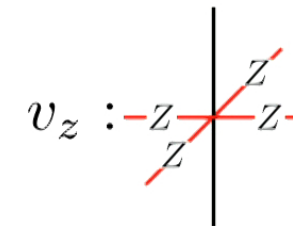
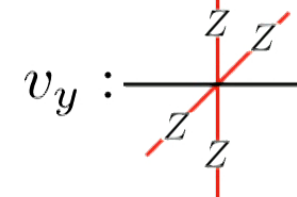
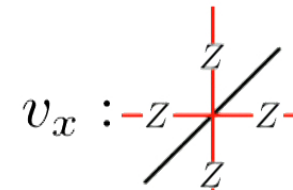
$$H_{XC} = - \sum_{\mathcal{C}} \prod_{l \in \mathcal{C}} X_l - \sum_{\mu=x,y,z} \sum_{v_{\mu}} \prod_{l \in v_{\mu}} Z_l$$



$$\left(\prod_{l \in v_{\mu}} Z_l \right) |GS\rangle = |GS\rangle$$

$$\left(\prod_{l \in \mathcal{C}} X_l \right) |GS\rangle = |GS\rangle$$

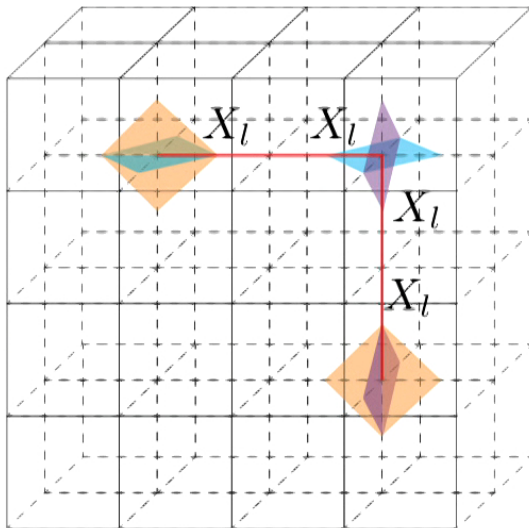
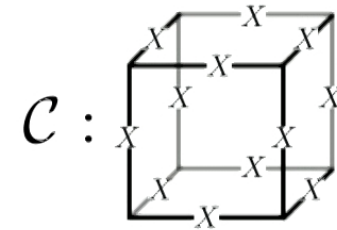
$$X_l |GS\rangle = | \text{blue sphere} \text{---} \text{blue arrow} \text{---} \text{blue sphere} \rangle$$



X-cube model

Vijay, Haah, Fu, 2015, 2016

$$H_{XC} = - \sum_{\mathcal{C}} \prod_{l \in \mathcal{C}} X_l - \sum_{\mu=x,y,z} \sum_{v_\mu} \prod_{l \in v_\mu} Z_l$$



$$\left(\prod_{l \in v_\mu} Z_l \right) |GS\rangle = |GS\rangle$$

$$\left(\prod_{l \in \mathcal{C}} X_l \right) |GS\rangle = |GS\rangle$$

$$X_l |GS\rangle = | \text{blue sphere} \text{---} \text{blue sphere} \rangle$$

The diagram shows two blue spheres connected by a horizontal blue line. A blue arrow points to the middle of this line, indicating the action of the X operator on the ground state.

$$v_x : \begin{array}{c} Z \\ | \\ -Z \text{---} Z \text{---} \\ | \\ Z \end{array}$$

A vertex labeled v_x. It has four edges: a vertical red line with 'Z' at both ends, a horizontal red line with 'Z' at both ends, and two diagonal black lines. The vertical and horizontal edges are labeled with 'Z' at the vertex.

$$v_y : \begin{array}{c} Z \\ | \\ Z \text{---} Z \\ | \\ Z \end{array}$$

A vertex labeled v_y. It has four edges: a vertical red line with 'Z' at both ends, a horizontal black line, and two diagonal red lines. The vertical and horizontal edges are labeled with 'Z' at the vertex.

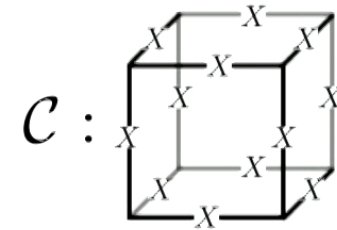
$$v_z : \begin{array}{c} | \\ -Z \text{---} Z \text{---} \\ | \\ Z \end{array}$$

A vertex labeled v_z. It has four edges: a vertical black line, a horizontal red line with 'Z' at both ends, and two diagonal black lines. The horizontal edge is labeled with 'Z' at the vertex.

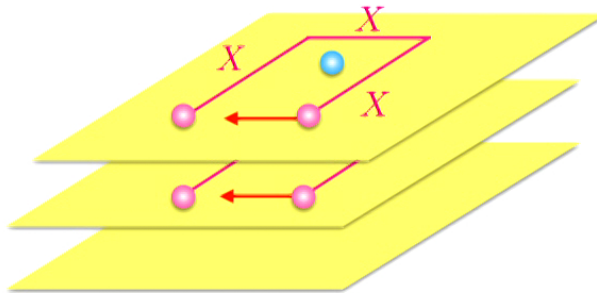
X-cube model

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Mutual statistics : π



$$\log_2(gsd) = 6L - 3$$

$$\left(\prod_{l \in v_{\mu}} Z_l \right) |GS\rangle = |GS\rangle$$

$$\left(\prod_{l \in \mathcal{C}} X_l \right) |GS\rangle = |GS\rangle$$

$$X_l |GS\rangle = | \text{blue sphere} \text{---} \text{blue sphere} \rangle$$

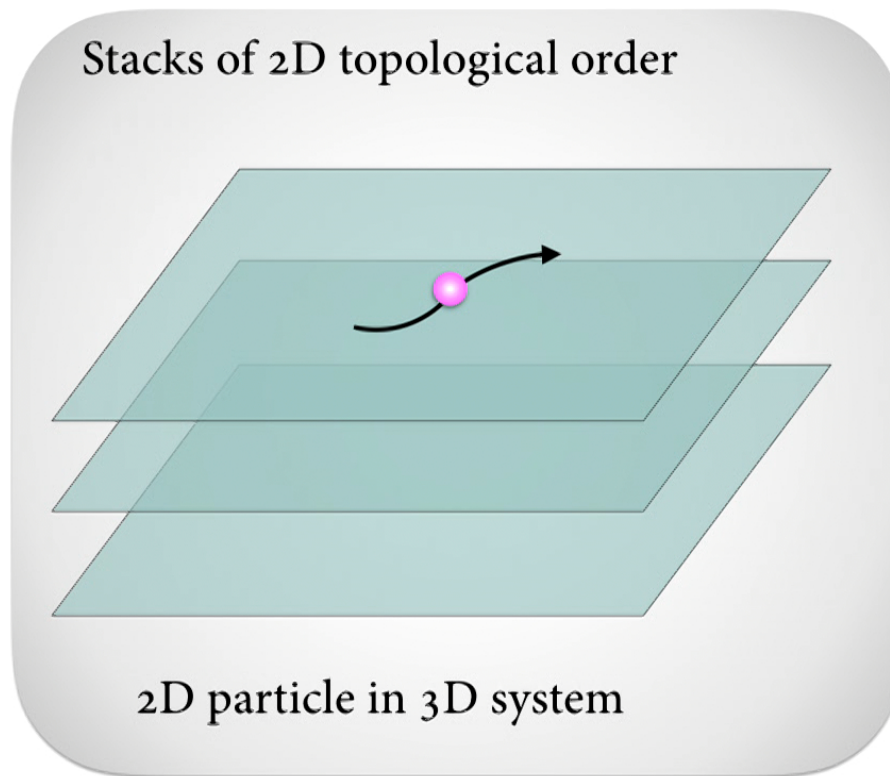
$$Z_l |GS\rangle = | \text{pink sphere} \text{---} \text{pink sphere} \rangle$$

$$v_x : \begin{array}{c} Z \\ \diagup \quad \diagdown \\ -Z \quad Z- \\ \diagdown \quad \diagup \\ Z \end{array}$$

$$v_y : \begin{array}{c} Z \\ \diagup \quad \diagdown \\ Z \quad Z \\ \diagdown \quad \diagup \\ Z \end{array}$$

$$v_z : \begin{array}{c} \diagup \quad \diagdown \\ -Z \quad Z- \\ \diagdown \quad \diagup \\ Z \end{array}$$

How to construct fracton topological order in the X-cube model?



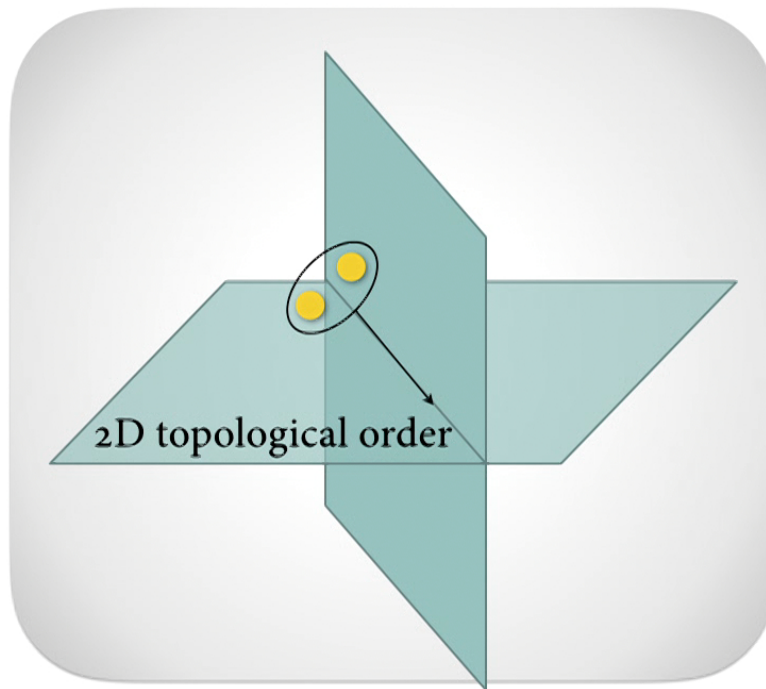
No fractons and 1d particles

Not pure stack of 2D layers!

How to construct fracton topological order in the X-cube model?

Possible construction: Intersecting layers

Composite particle move in lower dimensions



But things are not so simple

X-cube model :

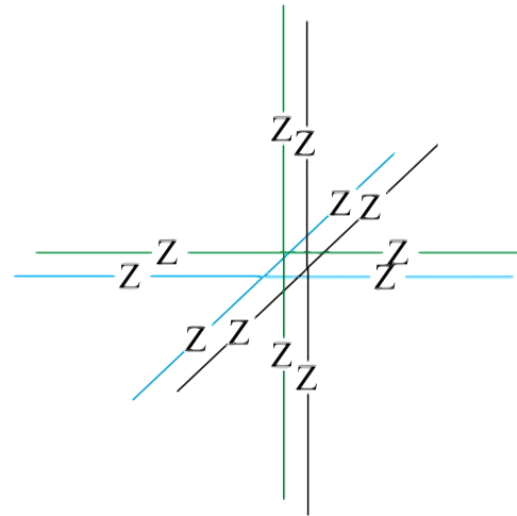
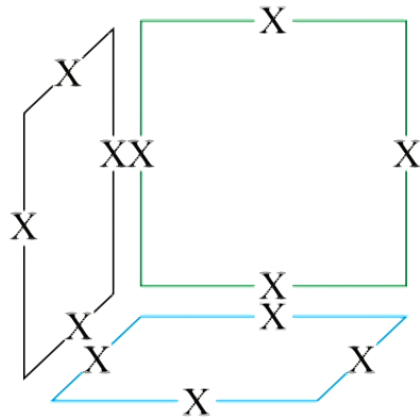
$$\log_2(gsd) = 6L - 3$$

Need coupling between layers!

Coupled layers?

X-cube model via toric code layers

HM, Lake, Chen, Hermele(2017)

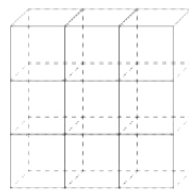


$$H_{cp} = -h \sum_l Z_l^\mu Z_l^\nu$$

Strong coupling limit: $h \rightarrow \infty$

$$\begin{aligned} Z_l^\mu &= Z_l^\nu = \mathcal{Z}_l \\ X_l^\mu X_l^\nu &= \mathcal{X}_l \end{aligned}$$

X-cube model via toric code layers

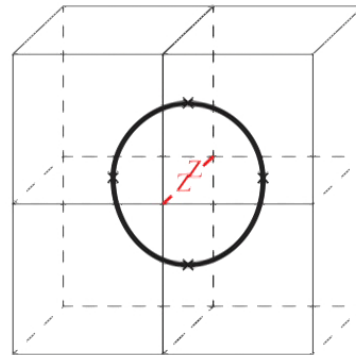
$$|\psi_0\rangle =$$


$$h = 0$$

$$|\psi\rangle = |gs\rangle_{X-cube}$$

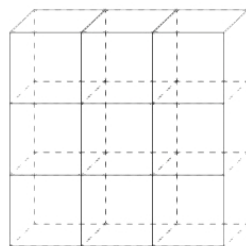
$$h = \infty$$

$$Z_l^\mu Z_l^\nu |\psi_0\rangle =$$

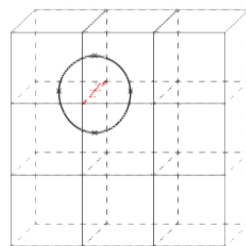


Particle-loop condensation

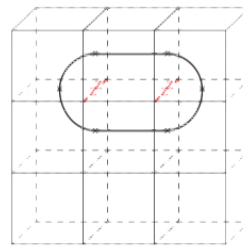
$$|\psi\rangle =$$



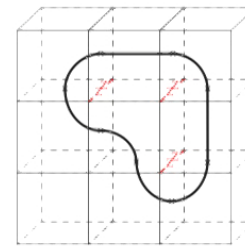
+



+

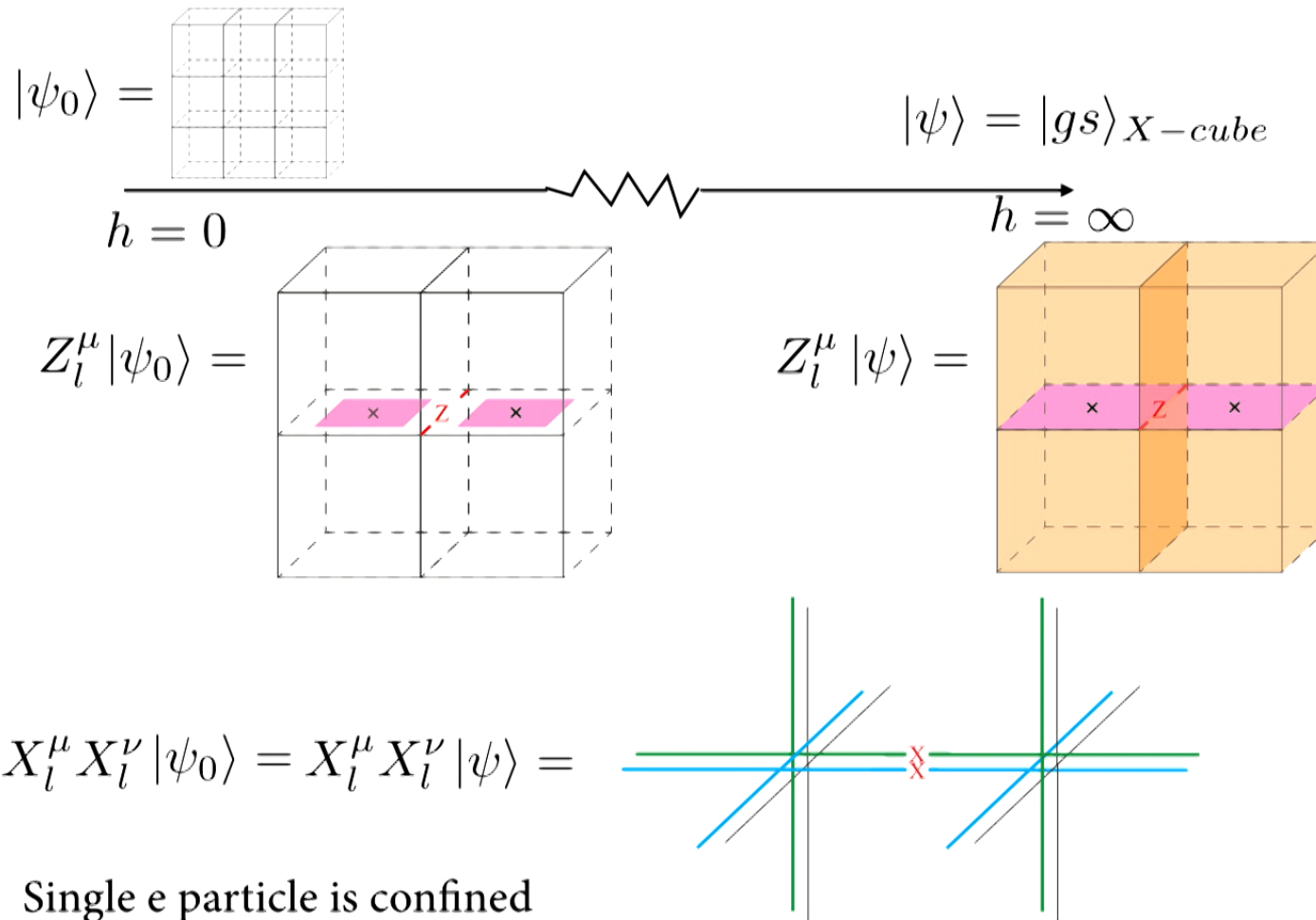


+

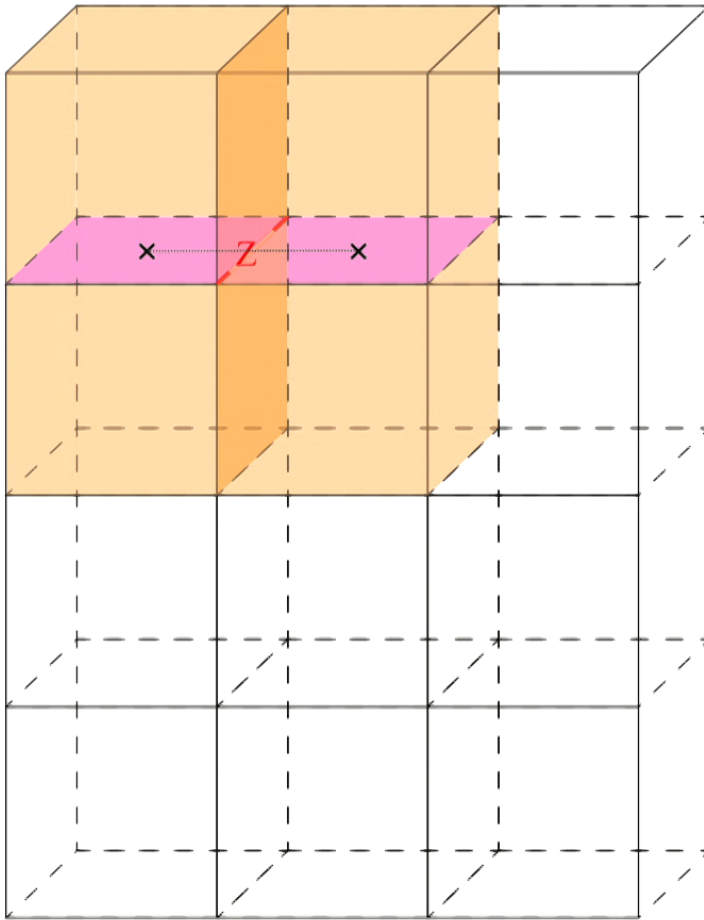


+ ...

X-cube model via toric code layers

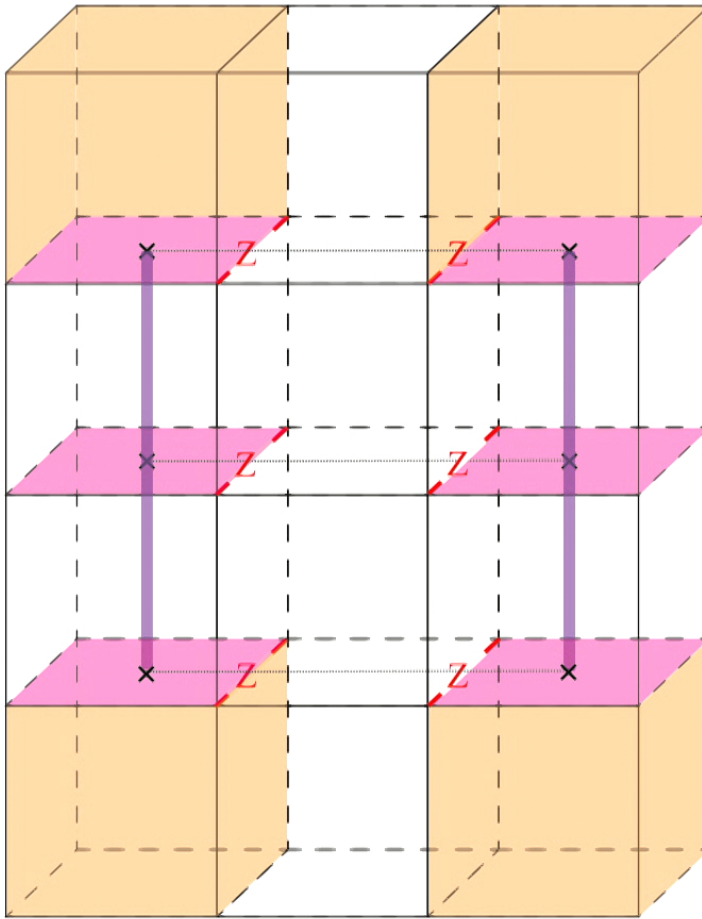


X-cube model via toric code layers



Identify the membrane operator as stacks of string operator from layers.

X-cube model via toric code layers



Identify the membrane operator as stacks of string operator from layers.

X-cube model via toric code layers

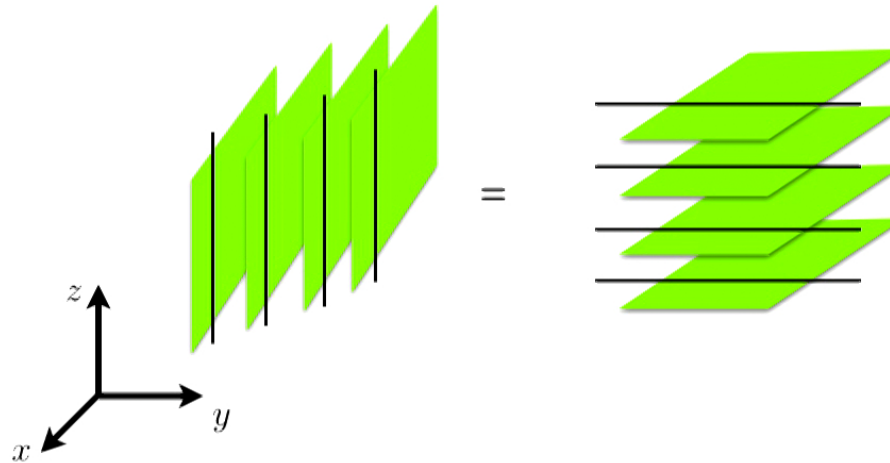
Decoupled toric code layers:

$6L$ Non-contractable Loop operators

2^{6L} degeneracy

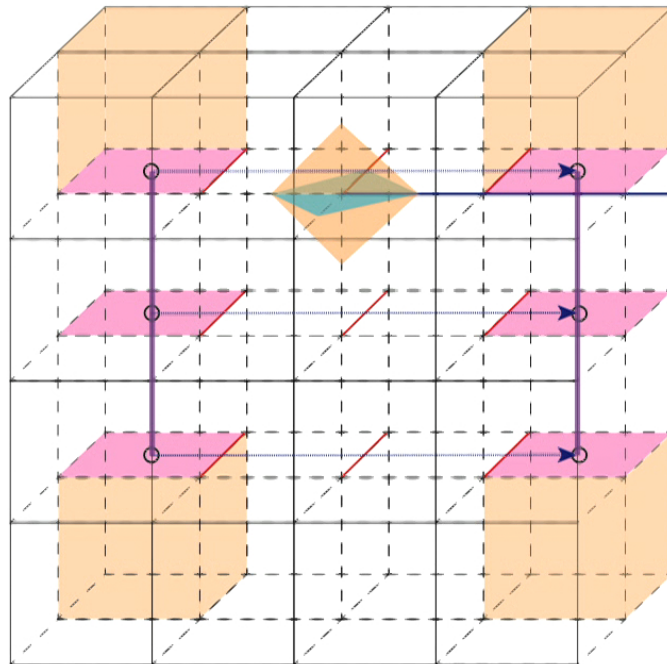
$$\log_2 D = 6L - 3$$

With coupling, there are 3 constraints:



X-cube model via toric code layers

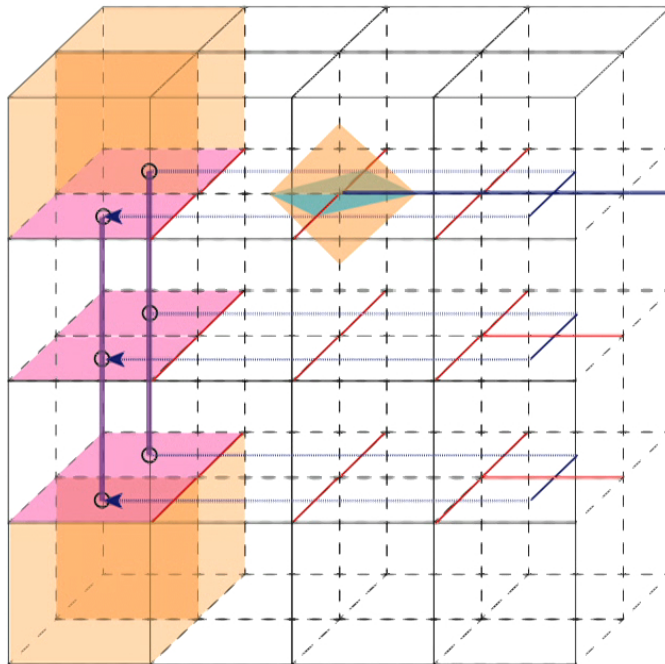
Semionic mutual statistics between fracton and 1d particle:



originates from the semionic statistics between e and m particle from the same layer.

X-cube model via toric code layers

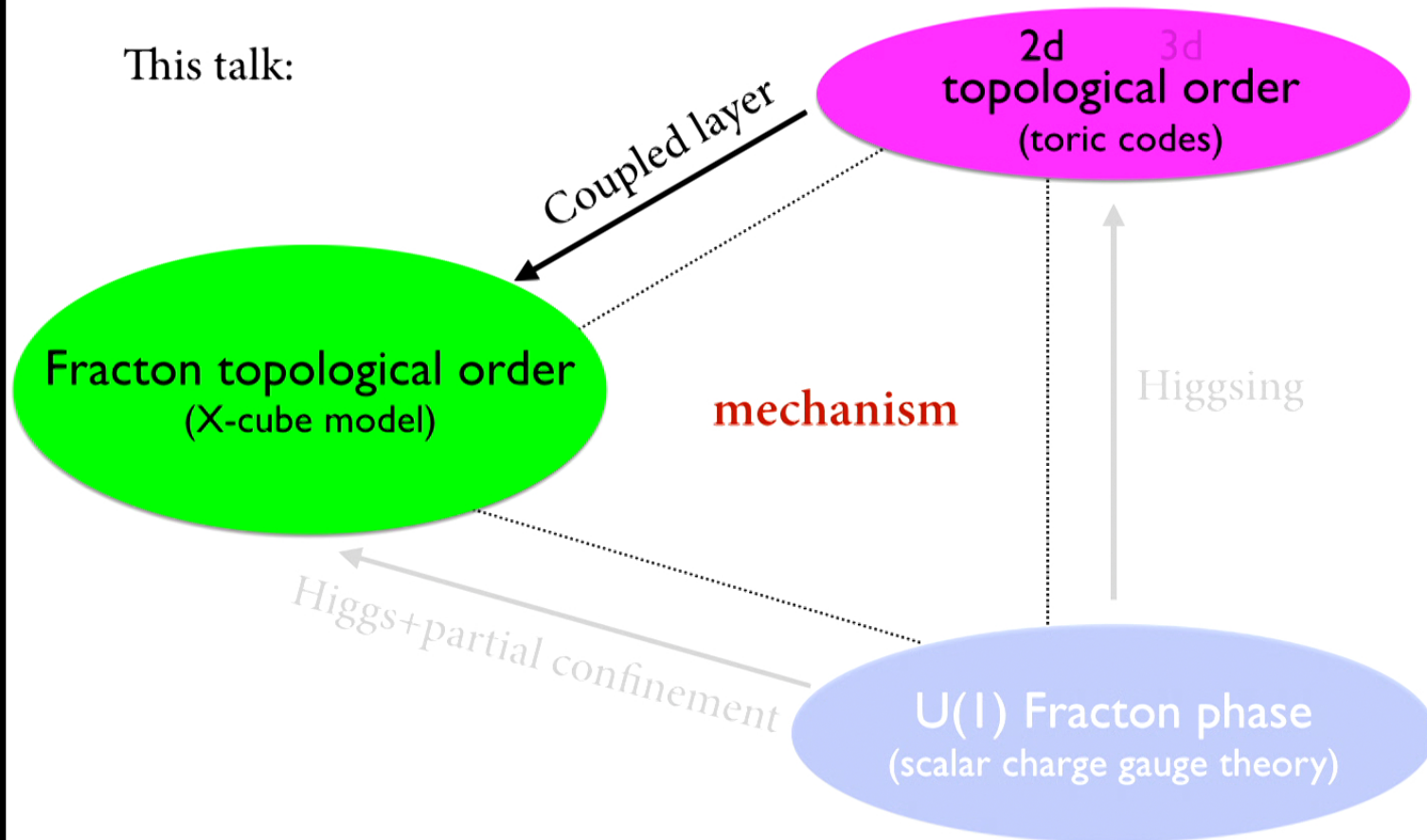
Semionic mutual statistics between fracton and 1d particle:



originates from the semionic statistics between e and m particle from the same layer.

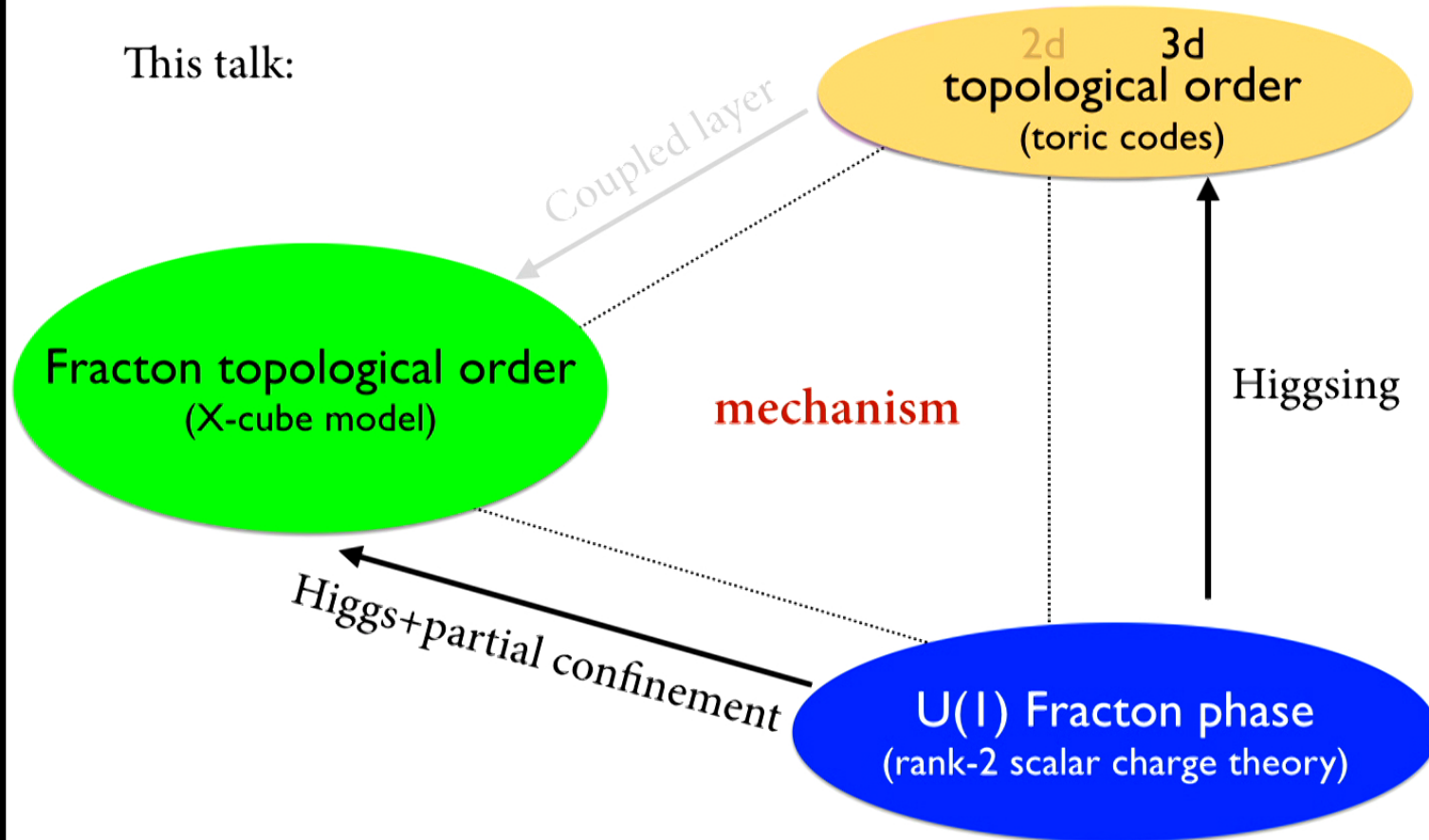
Fracton Phases — the phases have subdimensional excitations

This talk:



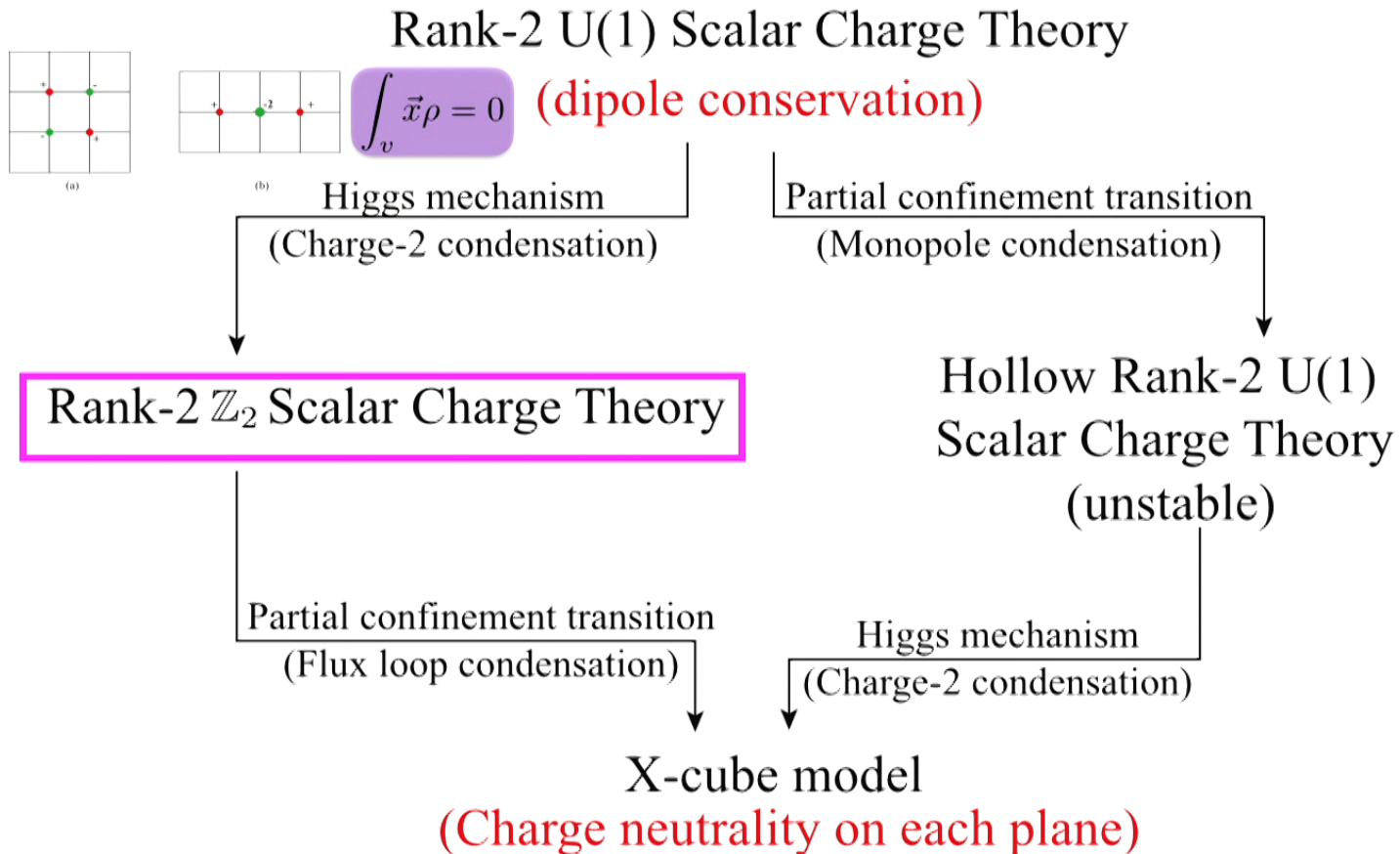
Fracton Phases — the phases have subdimensional excitations

This talk:



Higgsing rank-2 scalar charge theory

HM, Hermele, Chen (2018)



Rank-2 $\mathbb{Z}_2$ scalar charge theory

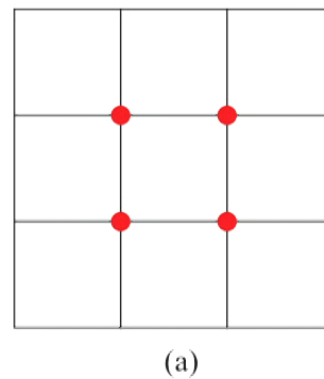
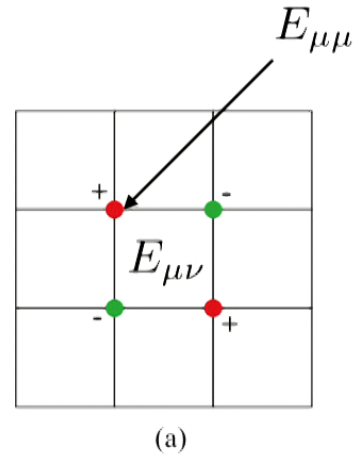
$$\Delta_\mu \Delta_\nu E_{\mu\nu}(\mathbf{r}) = \rho_{\mathbf{r}}$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \Delta_\mu \Delta_\nu f$$

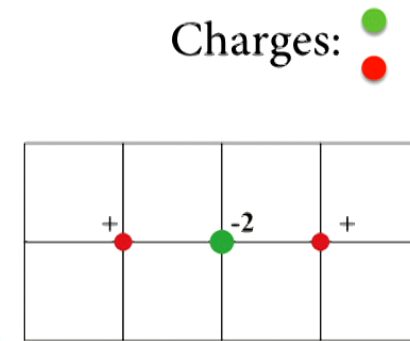
$$B_{\mu\nu} = \epsilon_{\mu\lambda\sigma} \Delta_\lambda A_{\sigma\nu}$$

Dipole conservation law:

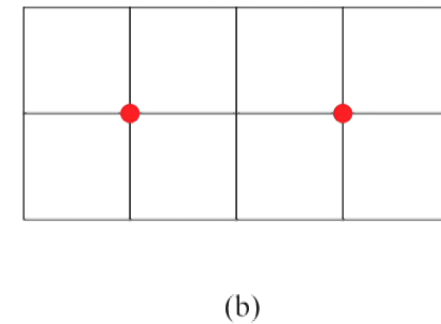
$$\sum_{\mathbf{r}} \mathbf{r} n_{\mathbf{r}} = 0 \text{ mod } 2$$



$$E_{\mu\nu} \rightarrow E_{\mu\nu} + 1$$



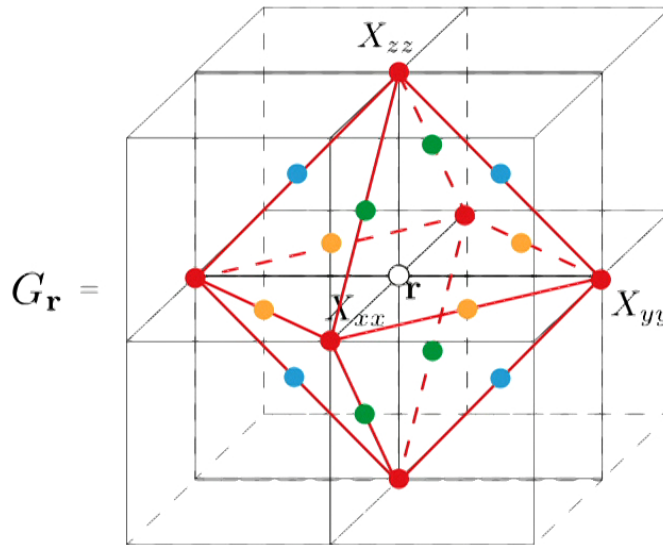
Higgsing



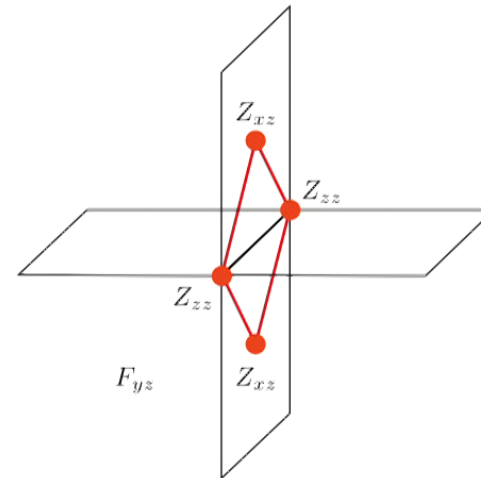
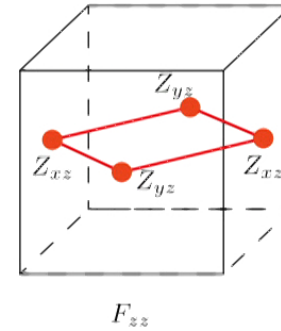
$$E_{\mu\mu} \rightarrow E_{\mu\mu} + 1$$

Rank-2 $\mathbb{Z}_2$ scalar charge theory

$$X_{\mu\nu} = e^{iE_{\mu\nu}} \quad Z_{\mu\nu} = e^{iA_{\mu\nu}}$$

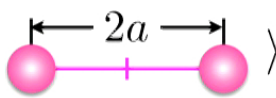


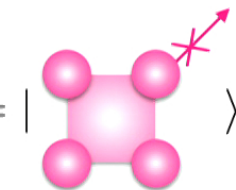
$$H_{4TC} = - \sum_{\mathbf{r}, \mu\nu} F_{\mu\nu} [Z_{\delta\lambda}(\mathbf{r})] - \sum_{\mathbf{r}} G_{\mathbf{r}} [X_{\mu\nu}(\mathbf{r})]$$

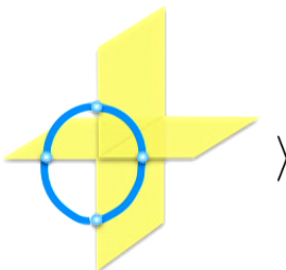


Rank-2 $\mathbb{Z}_2$ scalar charge theory

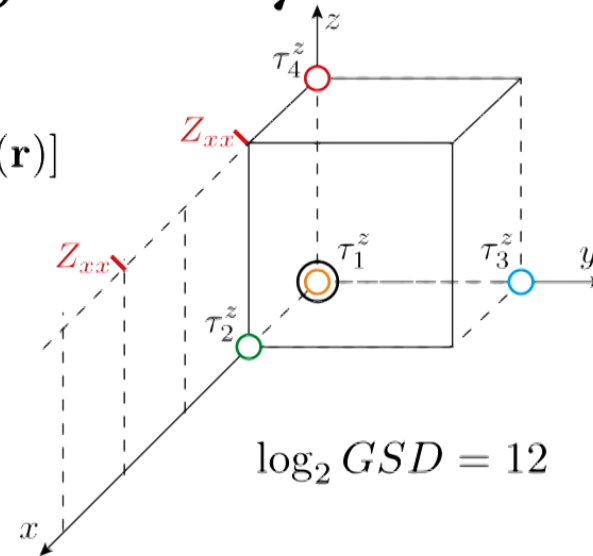
$$H_{4TC} = - \sum_{\mathbf{r}, \mu\nu} F_{\mu\nu} [Z_{\delta\lambda}(\mathbf{r})] - \sum_{\mathbf{r}} G_{\mathbf{r}} [X_{\mu\nu}(\mathbf{r})]$$

$$Z_{\mu\mu}|GS\rangle = | \text{---} 2a \text{---} \rangle$$


$$Z_{\mu\nu}|GS\rangle = | \text{---} \rangle$$


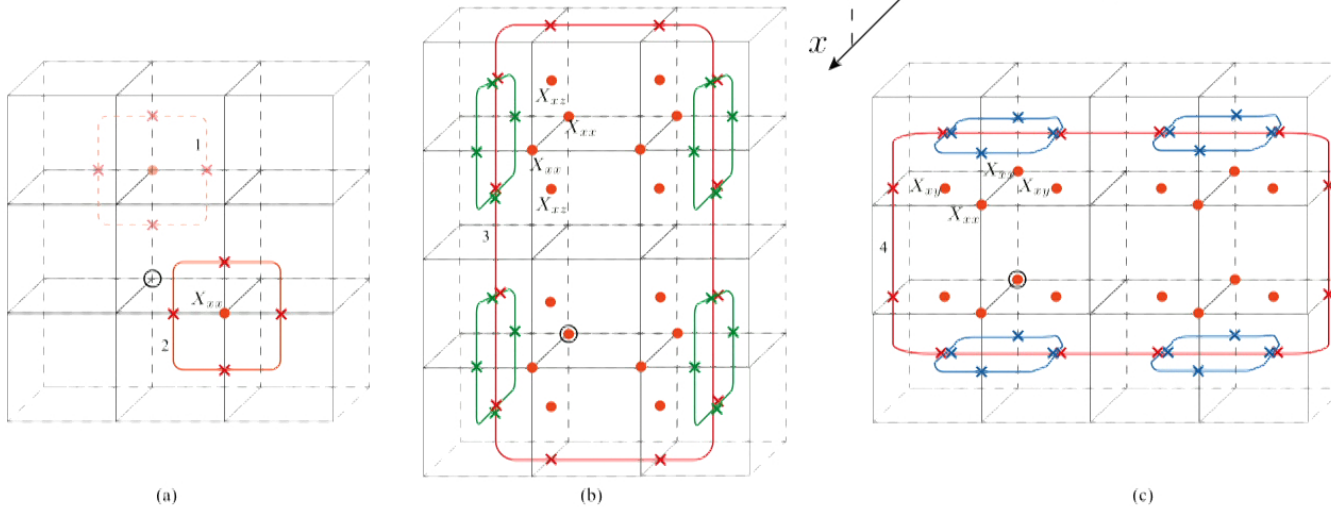
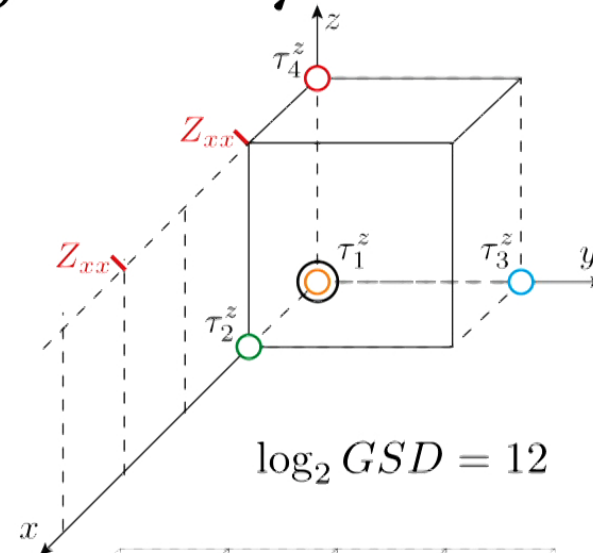
$$X_{\mu\mu}|GS\rangle = | \text{---} \rangle$$


$$X_{\mu\nu}|GS\rangle = | \text{---} \rangle$$

Rank-2 $\mathbb{Z}_2$ scalar charge theory

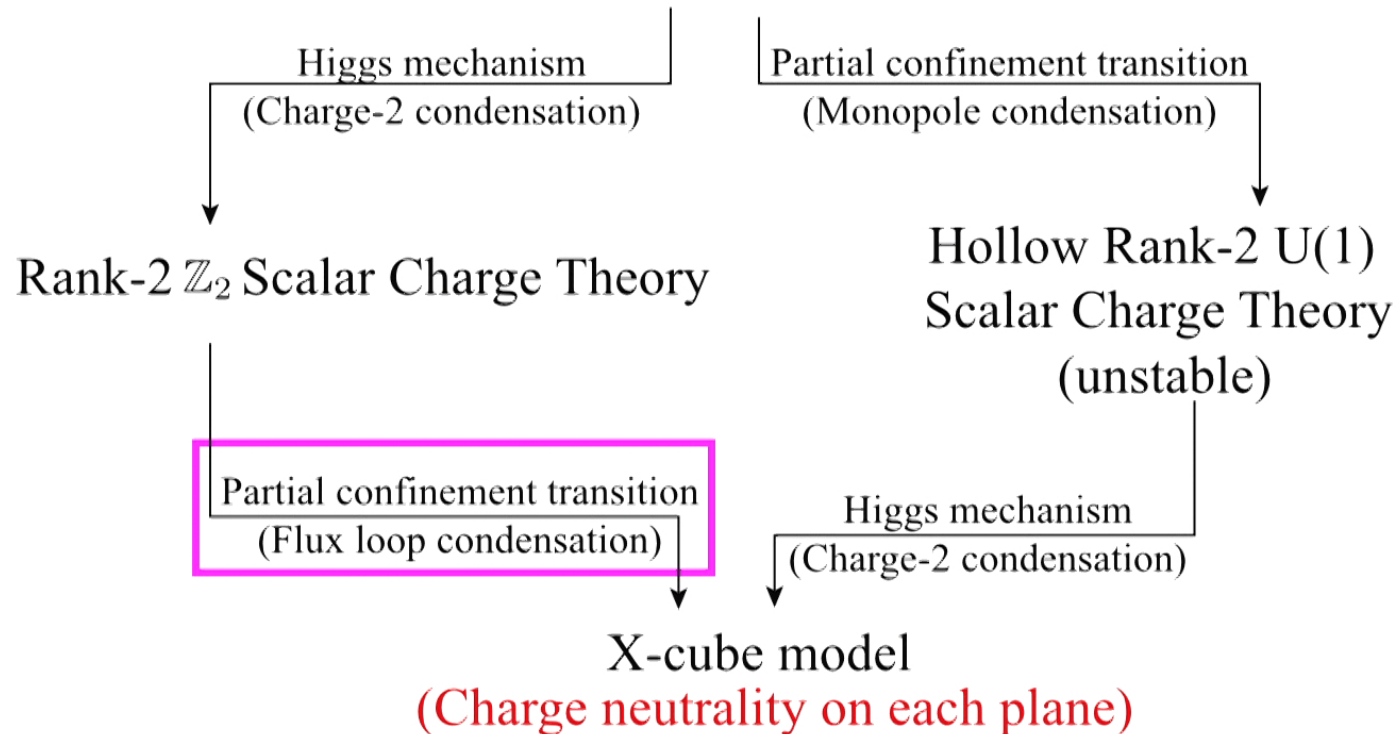
Loop type	Charge type
1	2
2	1,3,4
3	1,2,3
4	1,2,4



Higgsing rank-2 scalar charge theory

HM, Hermele, Chen (2018)

Rank-2 U(1) Scalar Charge Theory
(dipole conservation)

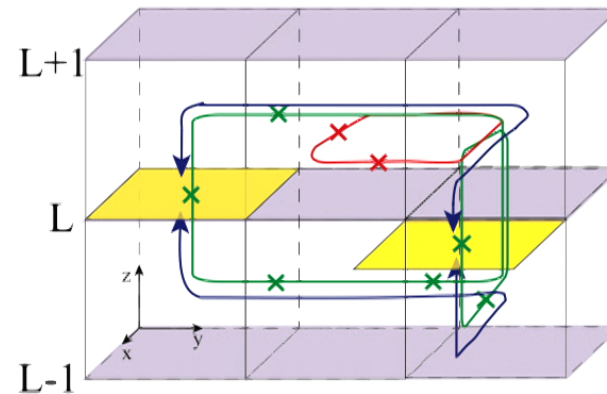


Rank-2 $\mathbb{Z}_2$ scalar charge theory $\longrightarrow$ X-cube model

3d Toric code $\longrightarrow$ 2d Toric code

$$H'_{3dTC} = -h \sum_{\ell \parallel z} X_{\ell}$$

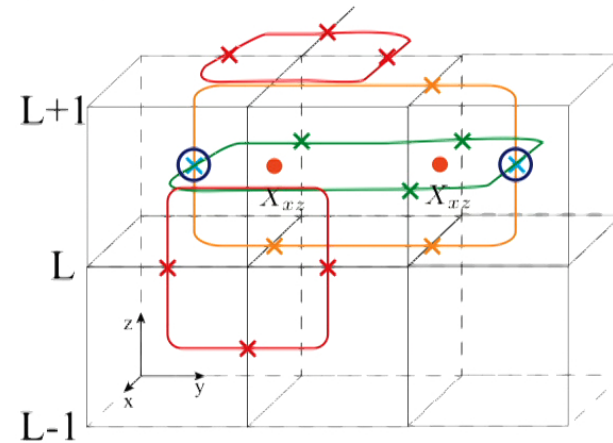
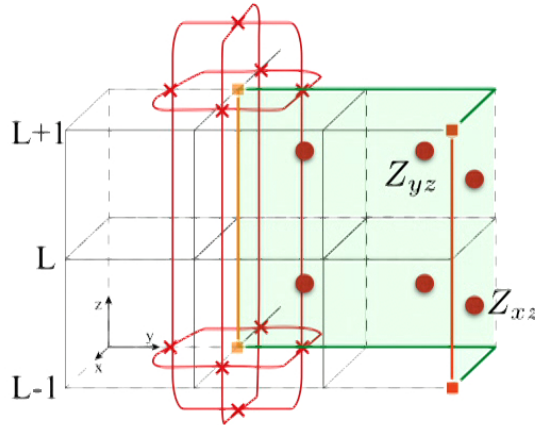
$$h \rightarrow \infty$$



1. e particle becomes 2d excitations, it can no longer move in z-direction
2. m loop reduces to 2d excitations where the loop intersects with xy-plane

Rank-2 $\mathbb{Z}_2$ scalar charge theory $\longrightarrow$ X-cube model

Condense loops: $H'_{\text{X-Cube}} = -h \sum_{\mu=x,y,z} X_{\mu\mu}$



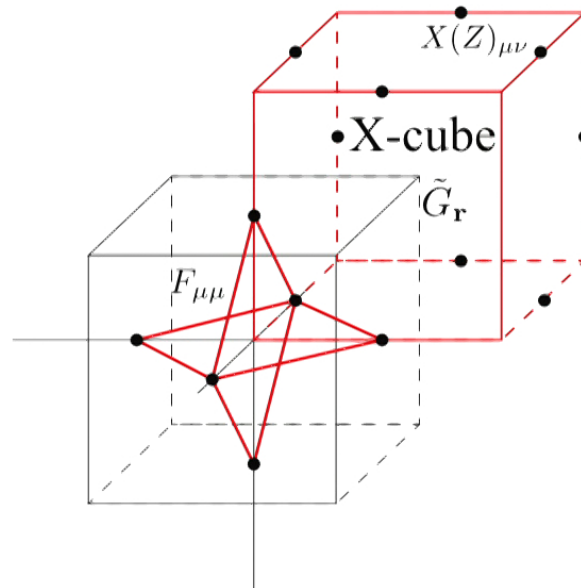
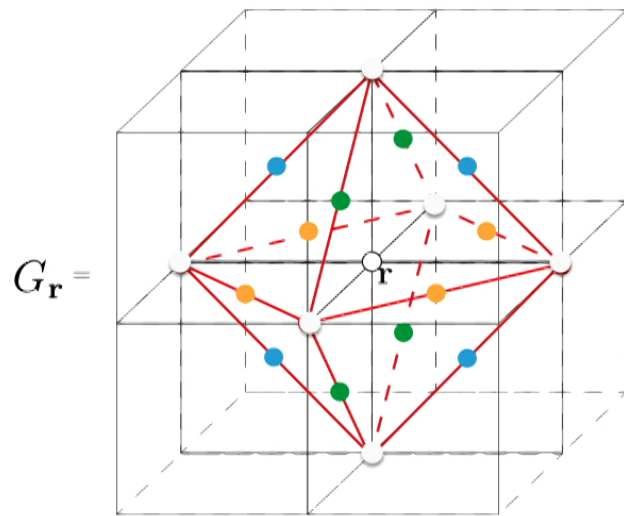
1. The point charge excitations are immobile due to non-trivial statistics with loops.
2. The bound states of two charges can propagate freely in 2d planes.
3. Four of the eight excitations created by acting $X_{\mu\nu}$ on a plaquette remain gapped.

Hollow Rank-2 $\mathbb{Z}_2$ gauge theory for X-cube model

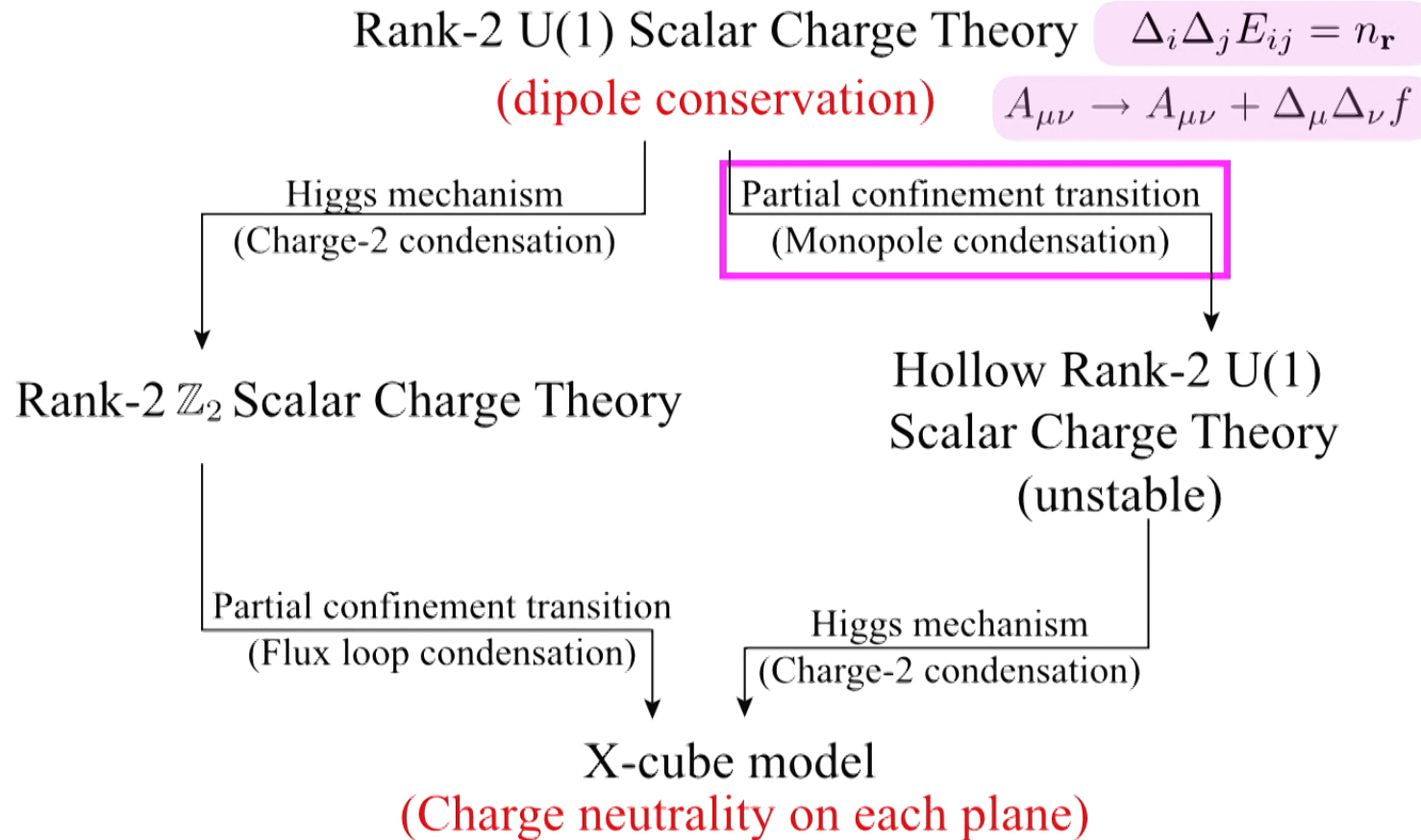
$$E_{ij} = \begin{bmatrix} 0 & E_{xy} & E_{xz} \\ E_{yx} & 0 & E_{yz} \\ E_{zx} & E_{zy} & 0 \end{bmatrix} \quad A_{ij} = \begin{bmatrix} 0 & A_{xy} & A_{xz} \\ A_{yx} & 0 & A_{yz} \\ A_{zx} & A_{zy} & 0 \end{bmatrix}$$

$$X_{\mu\nu} = e^{iE_{\mu\nu}}$$

$$Z_{\mu\nu} = e^{iA_{\mu\nu}}$$



Higgsing rank-2 scalar charge theory



Rank-2 scalar charge theory $\longrightarrow$ Hollow theory

$$S = i \sum_{\tau, \mathbf{r}} \sum_{\mu \leq \nu} E_{\mu\nu} \Delta_\tau A_{\mu\nu} - i \sum_{\tau, \mathbf{r}} \phi(\tau, \mathbf{r}) \sum_{\mu \leq \nu} \Delta_\mu \Delta_\nu E_{\mu\nu} + \sum_{\tau, \mathbf{r}} \left[u_d \sum_{\mu} E_{\mu\mu}^2 + u_{od} \sum_{\mu < \nu} E_{\mu\nu}^2 \right] - k \sum_{\tau, \mathbf{r}} \sum_{\mu, \nu} \cos(B_{\mu\nu})$$

$$S_{\text{dual}} = \frac{1}{4\pi^2} \sum_{\tau, r} \left[u_d \sum_{\mu} (\epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu})^2 + u_{od} \sum_{\mu < \nu} (\epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\nu} + \epsilon_{\nu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu})^2 \right]$$

$$+ 2\pi^2 k \beta_{\mu\nu}^2 - i \beta_{\mu\nu} (\Delta_\tau \alpha_{\mu\nu} - \Delta_\mu \psi_\nu - \delta_{\mu\nu} \theta) - t_m \sum_{\tau, r} \sum_{\mu \neq \nu} \cos(\Delta_\mu \phi_\nu - \alpha_{\mu\nu})$$

$$- t'_m \sum_{\tau, r} \sum_{\mu \neq \nu} \cos(\Delta_\mu \phi_\mu - \Delta_\nu \phi_\nu - \alpha_{\mu\mu} + \alpha_{\nu\nu}) \quad \begin{aligned} \psi_\mu &\rightarrow \psi_\mu + \Delta_\tau \lambda_\mu \\ \theta &\rightarrow \theta + \Delta_\tau f \\ \alpha_{\mu\nu} &\rightarrow \alpha_{\mu\nu} + \Delta_\mu \lambda_\nu + \delta_{\mu\nu} f \end{aligned}$$

$$- \frac{1}{t''_m} n_\mu^2 - i n_\mu (\Delta_\tau \phi_\mu - \psi_\mu)$$

$$E_{\mu\mu} = \frac{1}{2\pi} \epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu}$$

$$E_{\mu\nu} = \frac{1}{2\pi} (\epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\nu} + \epsilon_{\nu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu})$$

Monopole:

$$\begin{aligned} [\hat{\phi}_\mu(r), \hat{n}_\nu(r')] &= i \delta_{\mu, \nu} \delta_{r, r'} \\ [\hat{\beta}_{\mu\nu}(r), \hat{\alpha}_{\mu\nu}(r')] &= i \delta_{r, r'} \\ [\hat{\beta}_{\mu\mu}(r), \hat{\alpha}_{\mu\mu}(r')] &= i \delta_{r, r'} \\ \Delta_\mu \beta_{\mu\nu} &= n_\nu \quad \sum_{\mu} \beta_{\mu\mu} = 0 \end{aligned}$$

Rank-2 scalar charge theory $\longrightarrow$ Hollow theory

$$S = i \sum_{\tau, \mathbf{r}} \sum_{\mu \leq \nu} E_{\mu\nu} \Delta_\tau A_{\mu\nu} - i \sum_{\tau, \mathbf{r}} \phi(\tau, \mathbf{r}) \sum_{\mu \leq \nu} \Delta_\mu \Delta_\nu E_{\mu\nu} + \sum_{\tau, \mathbf{r}} \left[u_d \sum_{\mu} E_{\mu\mu}^2 + u_{od} \sum_{\mu < \nu} E_{\mu\nu}^2 \right] - k \sum_{\tau, \mathbf{r}} \sum_{\mu, \nu} \cos(B_{\mu\nu})$$

$$S_{\text{dual}} = \dots \quad \boxed{-t_m \sum_{\tau, \mathbf{r}} \sum_{\mu \neq \nu} \cos(\Delta_\mu \phi_\nu - \alpha_{\mu\nu})}$$

$$-t'_m \sum_{\tau, \mathbf{r}} \sum_{\mu \neq \nu} \cos(\Delta_\mu \phi_\mu - \Delta_\nu \phi_\nu - \alpha_{\mu\mu} + \alpha_{\nu\nu}) \dots$$

$$\boxed{t_m \rightarrow \infty} \xrightarrow{\text{monopole condensation}} \boxed{\begin{aligned} \alpha_{\mu\nu} &= 0 \\ E_{\mu\mu} &= 0 \end{aligned}}$$

$$E_{\mu\mu} = \frac{1}{2\pi} \epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu}$$

$$E_{\mu\nu} = \frac{1}{2\pi} (\epsilon_{\mu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\nu} + \epsilon_{\nu\lambda\sigma} \Delta_\lambda \alpha_{\sigma\mu})$$

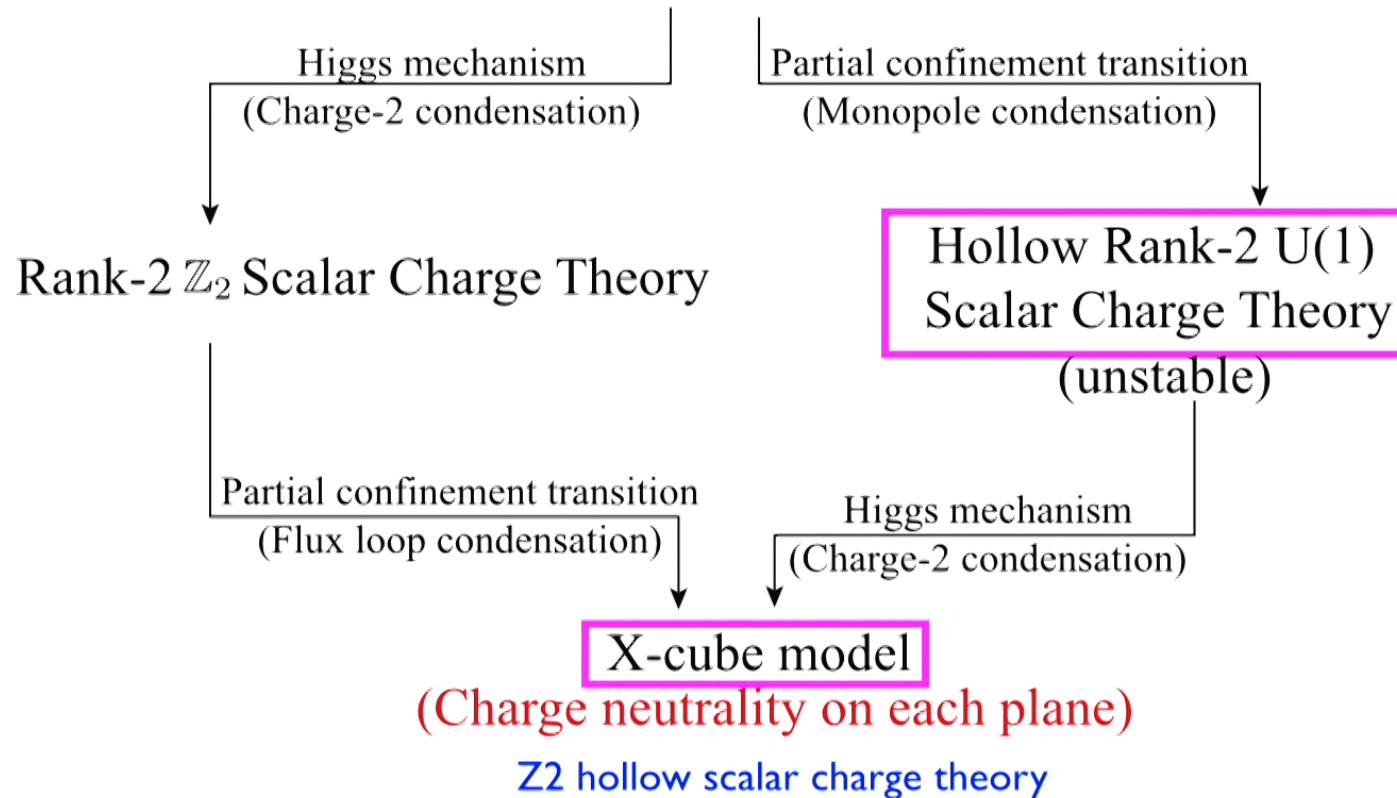
Monopole:

$$\begin{aligned} [\phi_\mu(r), n_\nu(r')] &= i\delta_{\mu,\nu} \delta_{r,r'} \\ [\beta_{\mu\mu}(r), \alpha_{\mu\mu}(r')] &= i\delta_{r,r'} \\ [\beta_{\mu\nu}(r), \alpha_{\mu\nu}(r')] &= i\delta_{r,r'} \\ \Delta_\mu \beta_{\mu\nu} &= n_\nu \quad \sum_{\mu} \beta_{\mu\mu} = 0 \end{aligned}$$

Higgsing rank-2 scalar charge theory

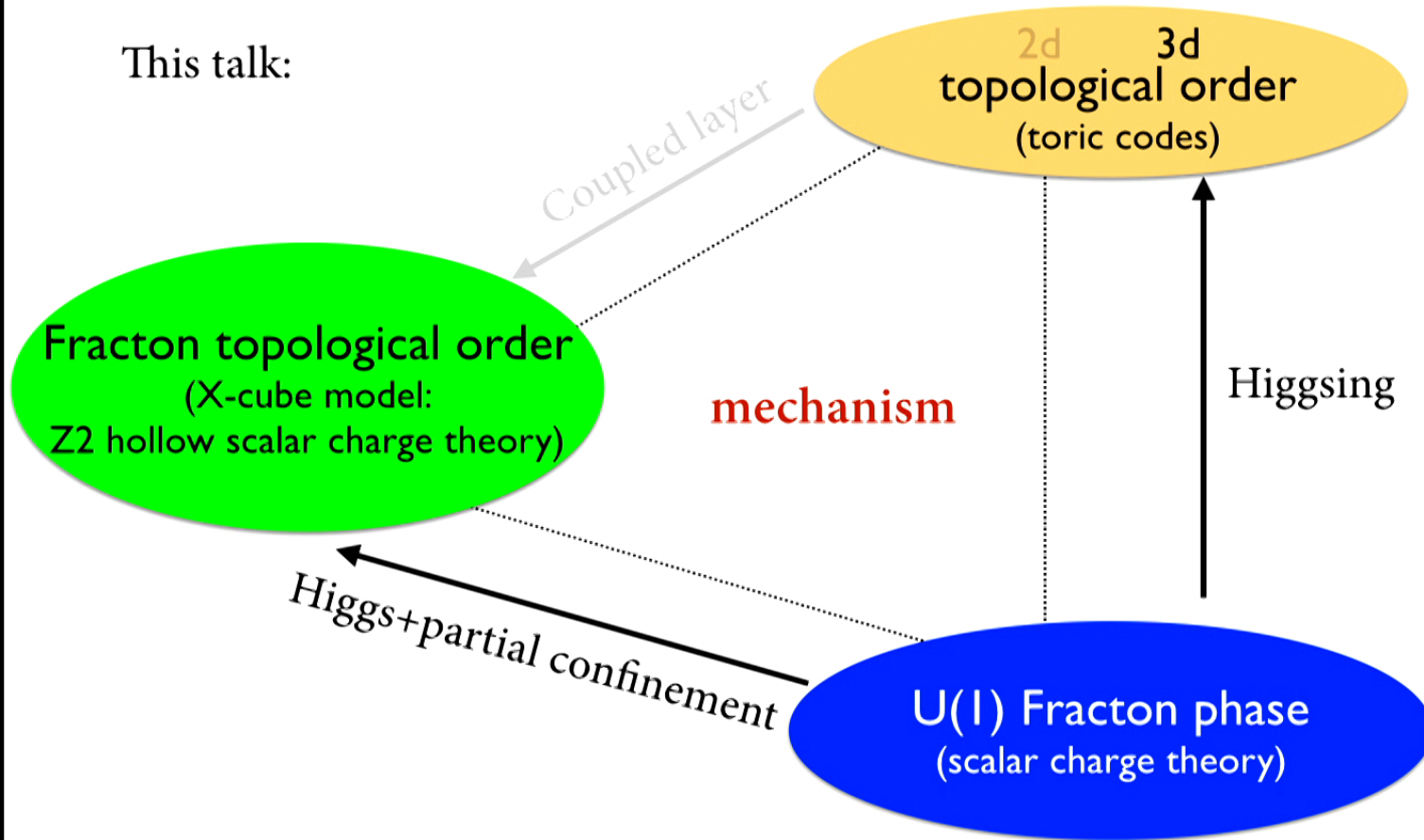
Cenke Xu, Congjun Wu (2008)

Rank-2 U(1) Scalar Charge Theory
(dipole conservation)



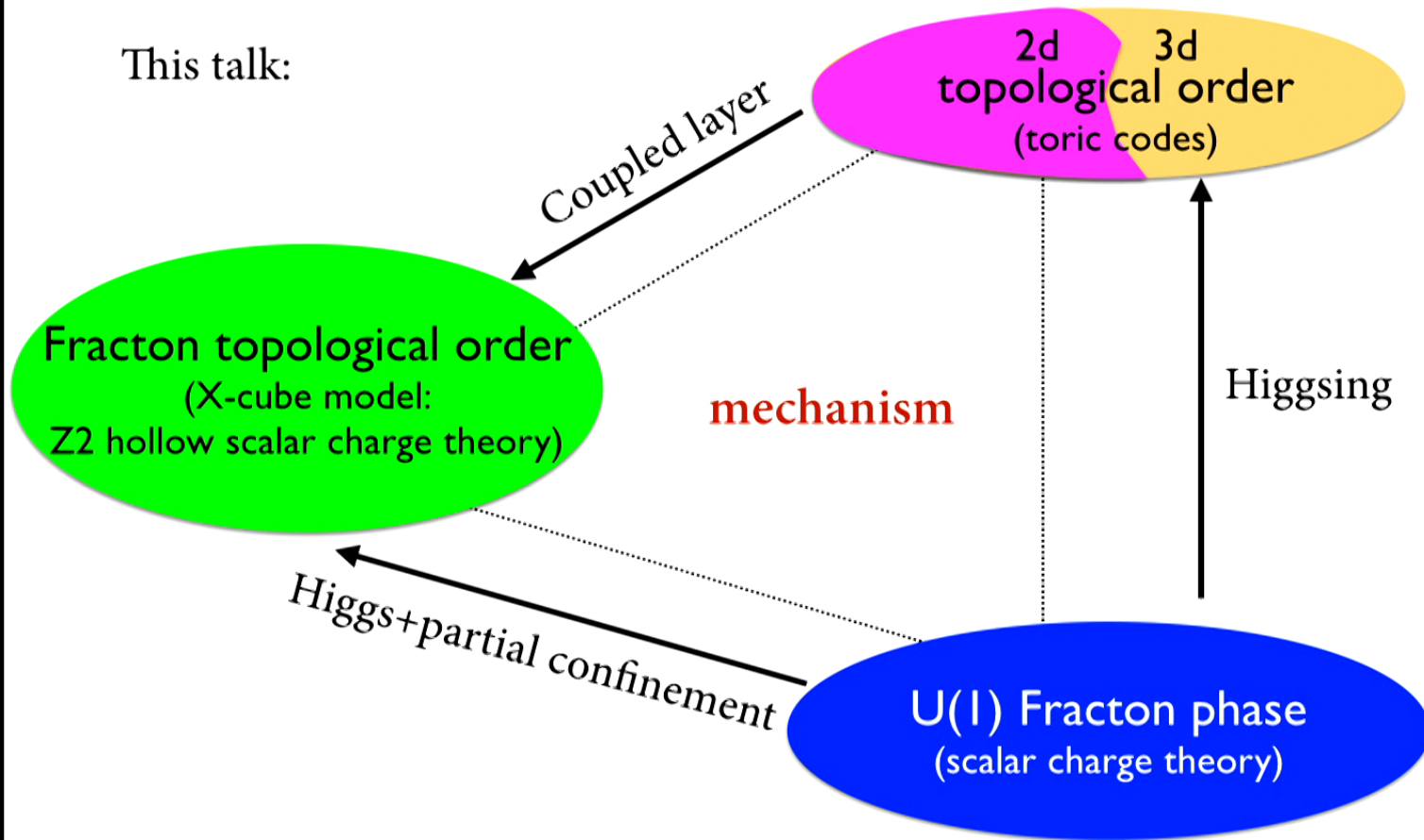
Fracton Phases — the phases have subdimensional excitations

This talk:



Fracton Phases — the phases have subdimensional excitations

This talk:



Other works

- Entanglement entanglement entropy of fracton stabilizer codes

HM, Schmitz, Parameswaran, Hermele, Nandkishore, (2018)

- Subdimensional excitations at Lifshitz transition and the Exciton Bose Liquid

HM, Pretko (2018)

- Possible phases of the spin-1/2 $J_1 - J_2$ XXZ model by boson-vortex duality

Zhu, Huse, White (2013)

HM (2018)

- Magnetic translation symmetry enriched topological order

- Pseudocriticality and complex CFTs

Thank you for your attention!