

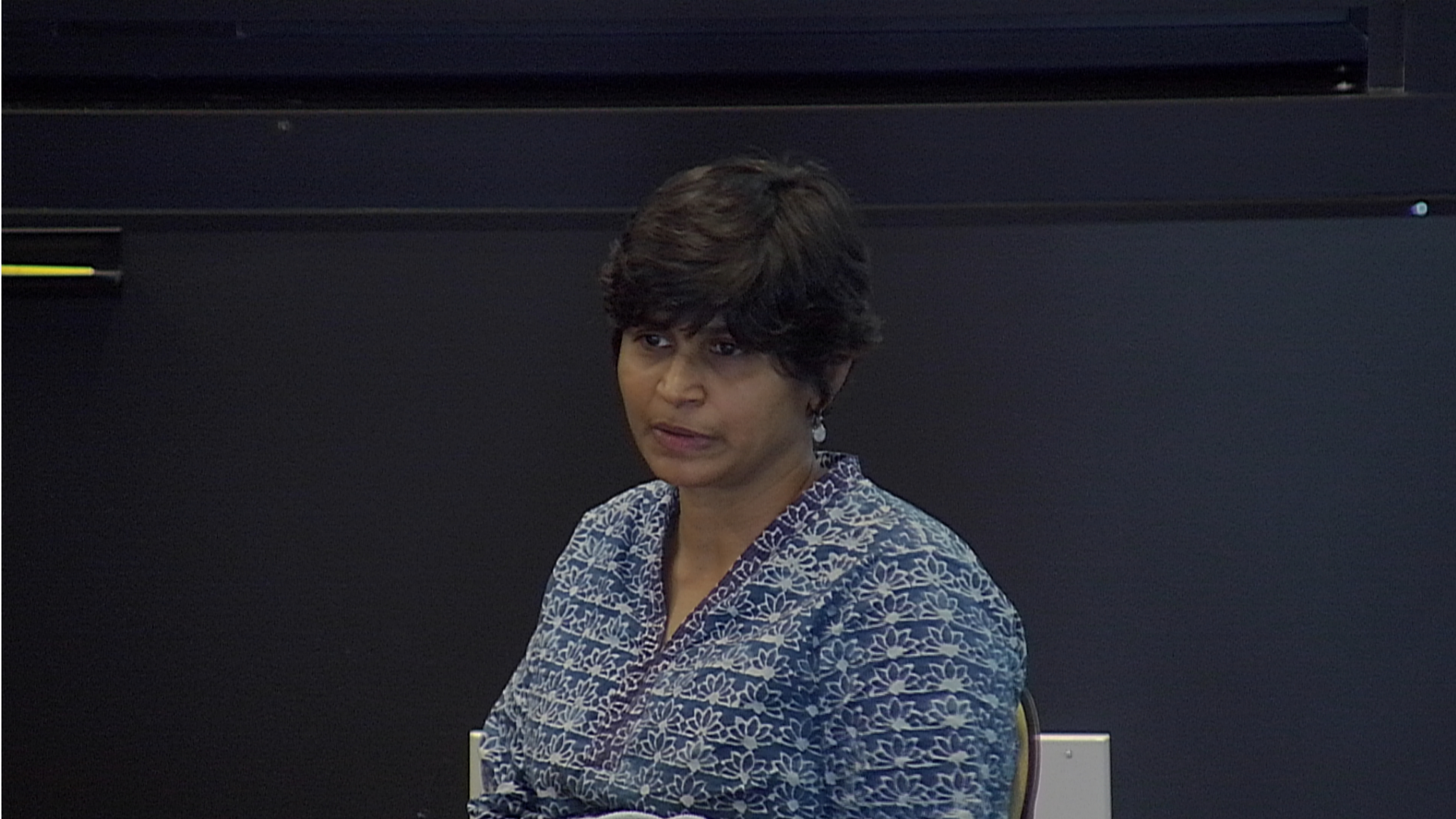
Title: Defining Spatial geometry via Spacetime Causal Structure

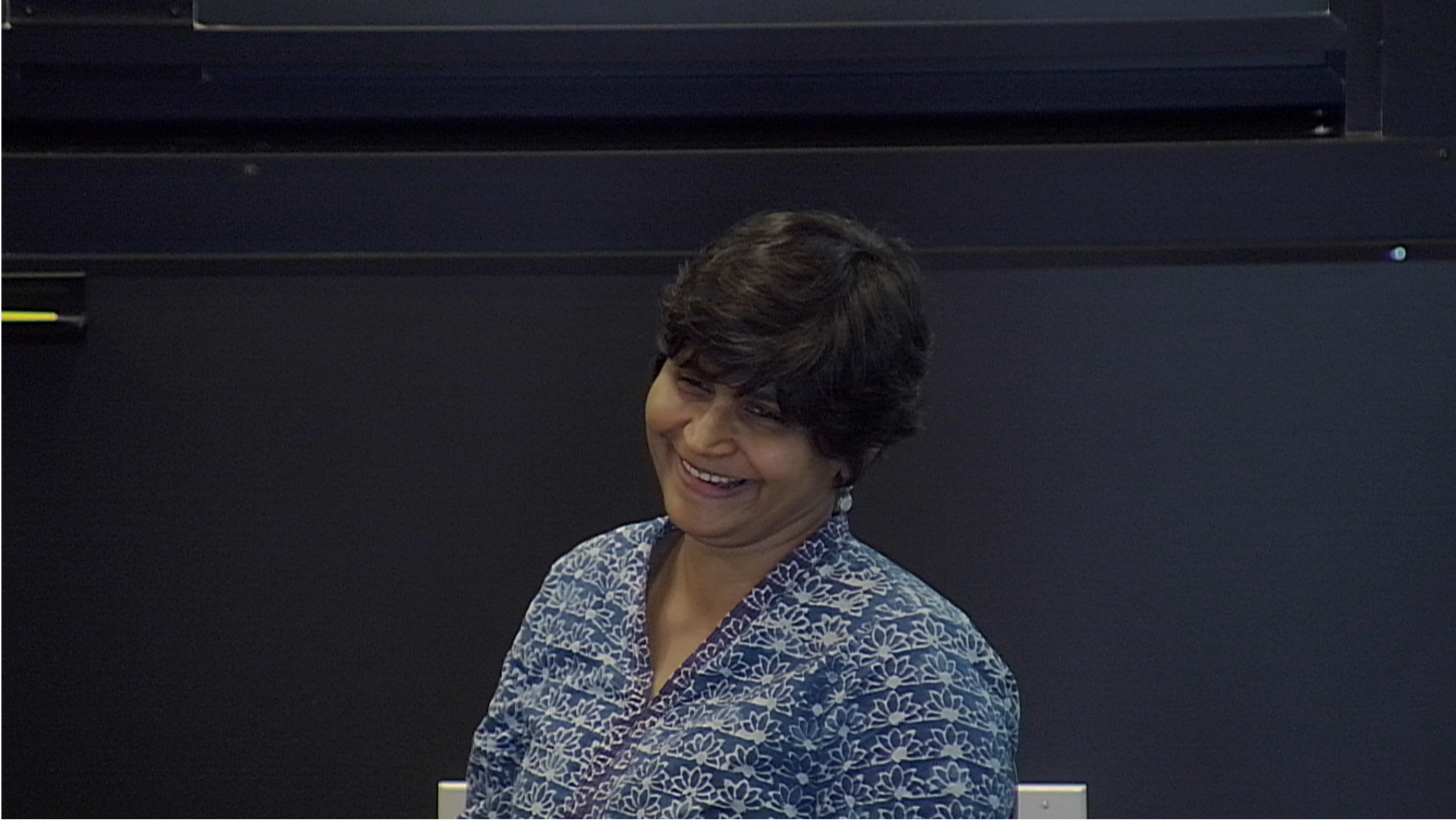
Date: Oct 25, 2018 02:30 PM

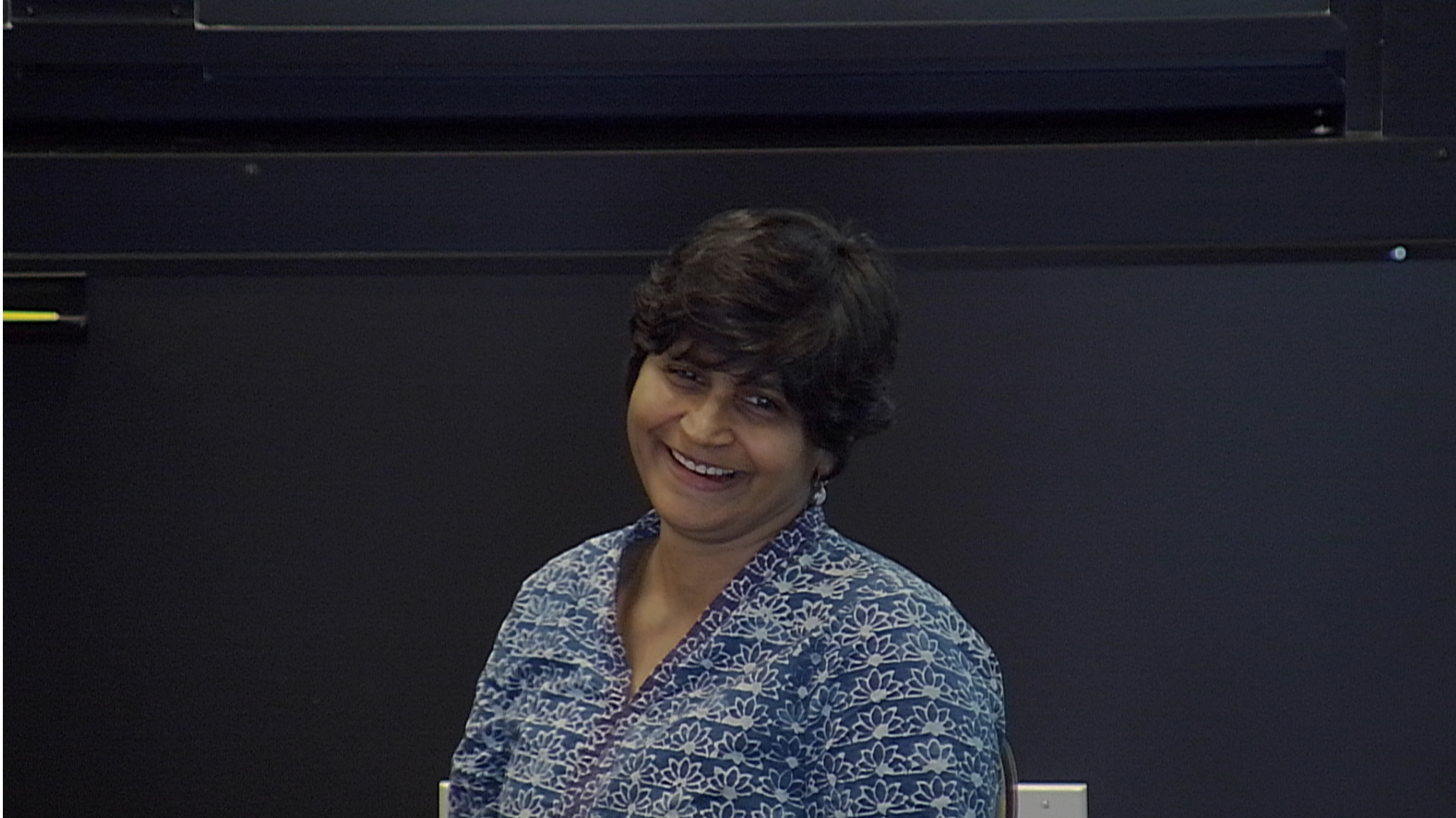
URL: <http://pirsa.org/18100059>

Abstract: <p>We find an approximation of the induced spatial distance on a Cauchy hypersurface using only the causal structure and local volume element. The approximation can be made arbitrarily precise for a globally hyperbolic spacetime with compact Cauchy hypersurfaces. This prescription has a discrete analog which we use to evaluate the induced spatial distance in a continuum-like causal set. As expected, because of discrete asymptotic silence, this gives a poor approximation to the continuum distance in the "UV". For larger distances, on the other hand, the approximation works very well, as long as the discreteness scale is significantly smaller than the extrinsic and intrinsic curvature scale of the spatial hypersurface. This paves the way for obtaining detailed spatial geometric information from the causal structure, which has implications for the continuum approximation of causal set theory.</p>









DEFINING SPATIAL GEOMETRY VIA SPACETIME CAUSAL STRUCTURE

w/ ASTRID EICHHORN & FLEUR VERSTEEGEN

arXiv:1809.06192

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OUTLINE:-

- GEOMETRIC RECONSTRUCTION: THE HKMM(L) THEOREM.
- SPACE VS SPACETIME

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- GEOMETRIC RECONSTRUCTION: THE HKMM(L) THEOREM.
- SPACE V/S SPACETIME
- CONTINUUM CONSTRUCTION OF "PREDISTANCE" & DISTANCE
- CST CONSTRUCTION.

HKMM(L) THEOREM

CONSTRUCTION OF PREDISTANCE & DISTANCE • CST CONSTRUCTION.

HKMM(L) THEOREM

- If a causal bijection $f \exists (M_1, g_1) \& (M_2, g_2)$ [future and past distinguishable] then they are conformally isometric.

$$\underbrace{(M_1, <_1)}_{+ \in} \quad \begin{array}{l} x <_1 y \\ y <_1 z \end{array} \Rightarrow x <_1 z \quad f: M_1 \rightarrow M_2.$$

$$x <_1 y \Leftrightarrow f(x) <_2 f(y)$$

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HKMM(L) THEOREM

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$$\underbrace{(M_1, <_1)}_{\substack{x <_1 y \\ y <_1 z}} \Rightarrow x <_2 z \quad f: M_1 \rightarrow M_2$$

SPACE V/S SPACETIME

$(M, g) \rightarrow C: \text{locally finite poset}$

$(M, g) \rightarrow \{C_i\}$



$$P_V(n) = \frac{(8V)^n e^{-8V}}{n!}$$

SPACE V/S SPACETIME

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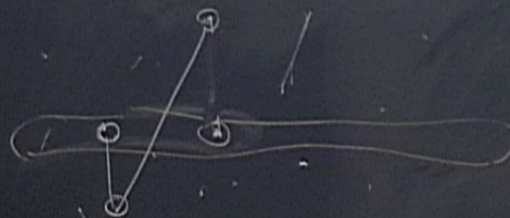
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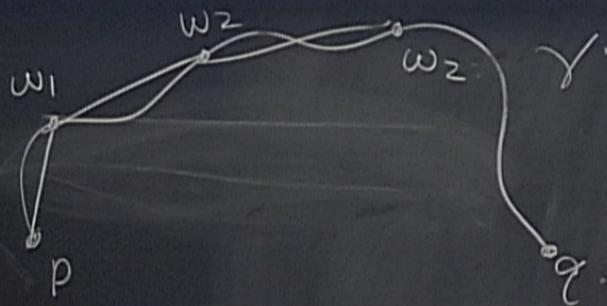


maximal.
antichain



$$P_V(n) = \frac{(8V)^n e^{-8V}}{n!}$$

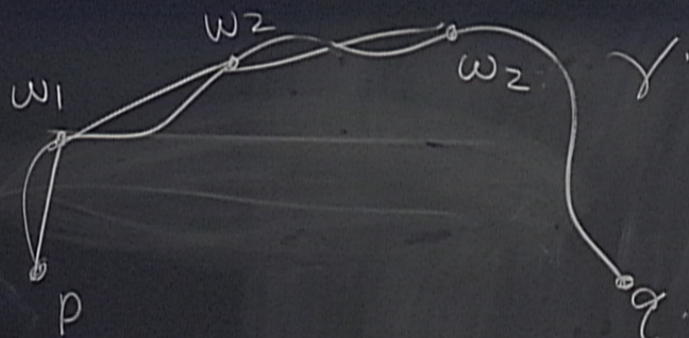
CONT: $(M, \angle) \in$



$$W_k = (w_0, \dots, w_k)$$

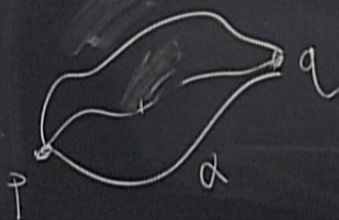
$$d_{W_k}(p, q) = \sum_{i=0}^{k-1} d_{\mathbb{R}^2}(w_i, w_{i+1})$$

CONT: $(M, \alpha) \in$



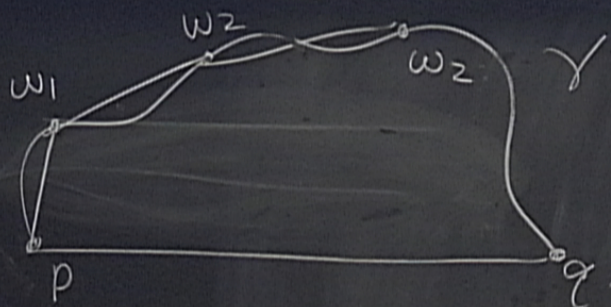
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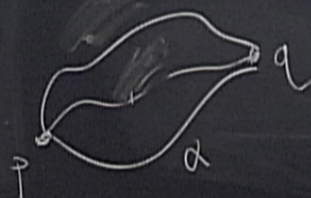
$$l_\alpha(p, q) = \int_\alpha ds$$

CONT: $(M, \langle \cdot, \cdot \rangle) \otimes \mathbb{R} \in$



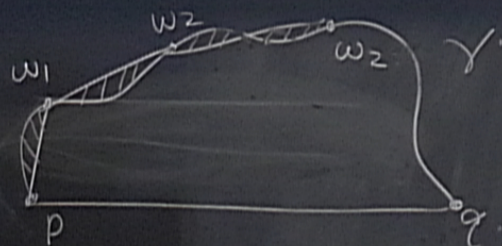
$$W_k = (w_0, \dots, w_k)$$

$$d_{W_k}(p, q) = \sum_{i=0}^{k-1} d_{\mathbb{R}^2}(w_i, w_{i+1})$$



$$l_\alpha(p, q) = \int_\alpha ds$$

CONT. $(M, \langle \cdot, \cdot \rangle) \in$



$$W_k = (\omega_0, \dots, \omega_k)$$

$$d_{W_k}(p, q) = \sum_{i=0}^{k-1} d_{\mathbb{R}^2}(\omega_i, \omega_{i+1})$$

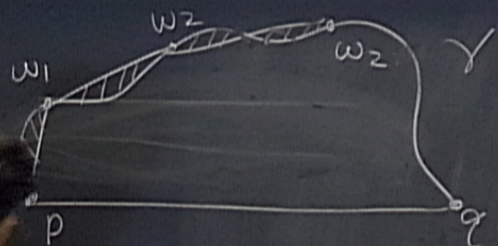


$$l_\alpha(p, q) = \int_\alpha ds$$

$$A < \epsilon^2$$

$$|d(\cdot, \cdot) - d_\gamma(\cdot, \cdot)| < \epsilon$$

CONT: $(M, d) \in$



$$d(p, q) = \inf_{W_k} d(p, q)$$

$$W_k = (w_0, \dots, w_k)$$

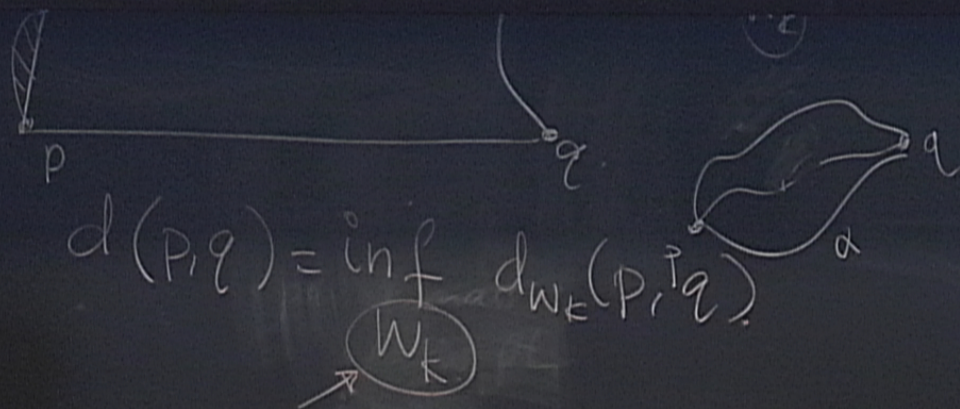
$$d_{W_k}(p, q) = \sum_{i=0}^{k-1} d_{\mathbb{R}^2}(w_i, w_{i+1})$$



$$l_\alpha(p, q) = \int_\alpha ds$$

$$A < \epsilon^2$$

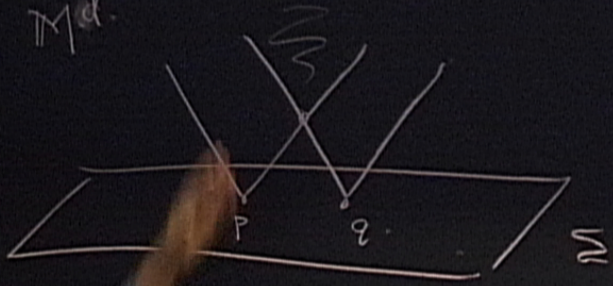
$$|d(\cdot, \cdot) - d_\gamma(\cdot, \cdot)| < \epsilon$$



$$l_\alpha(p, q) = \int_\alpha ds$$

• CST CONSTRUCTION.

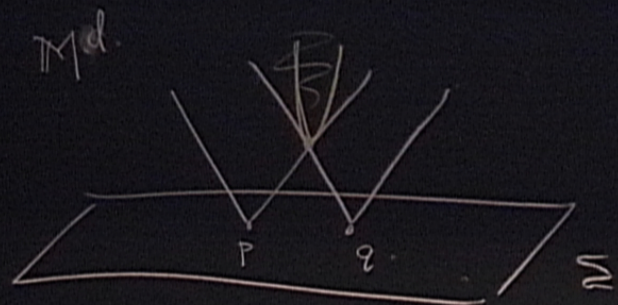
M^d



$$\mathcal{H} = \partial J^+(p) \cap \partial J^+(q)$$

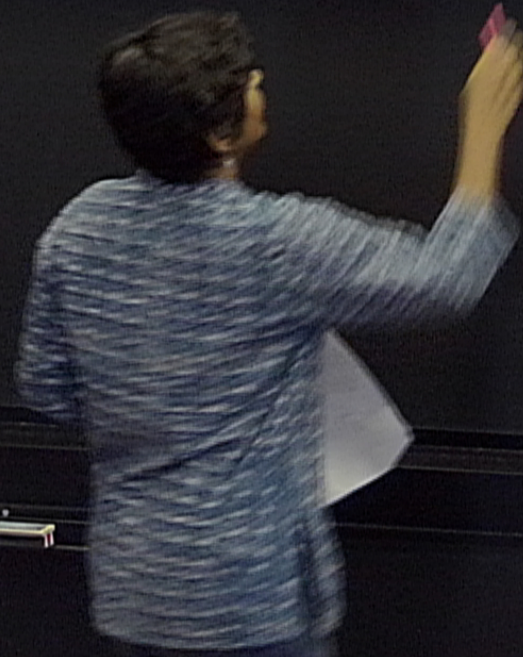
$$J^-(p) \cap J^+(q) = F(p, q)$$

• CST CONSTRUCTION.



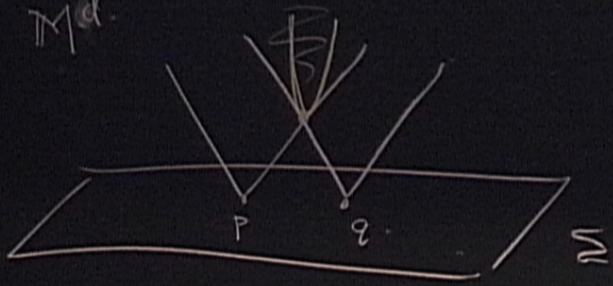
$$J^+(p) \cap J^+(q) = F(p, q)$$

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• CST CONSTRUCTION.

M^d

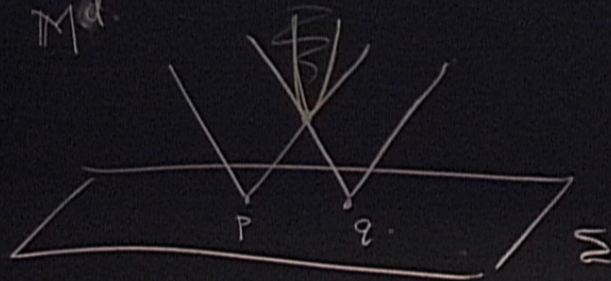


$$J^+(p) \cap J^+(q) = F(p, q)$$

$$\mathcal{C} = \partial J^+(p) \cap \partial J^+(q) \quad n-2$$

• CST CONSTRUCTION.

M^d

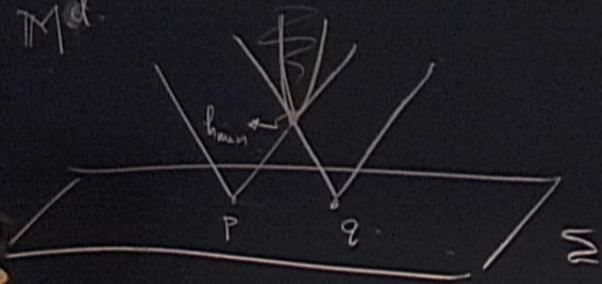


$$J^+(p) \cap J^+(q) = F(p, q)$$

$$\mathcal{L} = \partial J^+(p) \cap \partial J^+(q) \quad (n-2): \mathbb{H}^{n-2}$$

CST CONSTRUCTION.

M^d

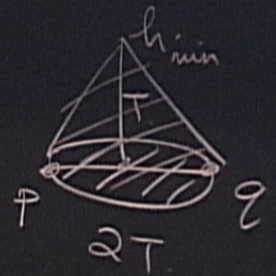
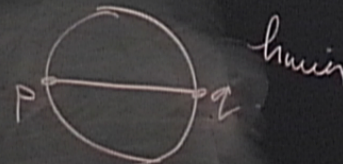


$$J^+(p) \cap J^+(q) = F(p, q)$$

$$\mathcal{H} = \partial J^+(p) \cap \partial J^+(q) \quad (n-2) \cdot \mathbb{H}^{n-2}$$

$$h \in \mathcal{H}: \quad J^-(h) \cap J^+(\varepsilon)$$

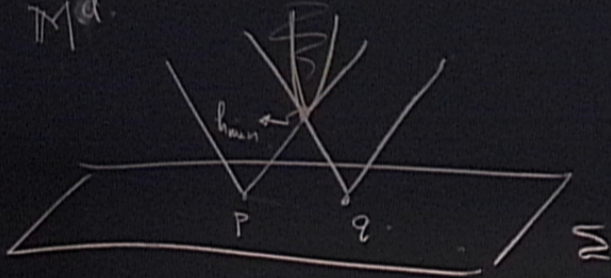
$$V(h) = \text{vol}(\quad)$$



$$V(h_{\min}) = \int_d T^d$$

CST CONSTRUCTION.

M^d

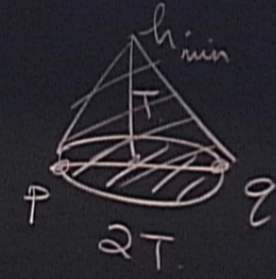
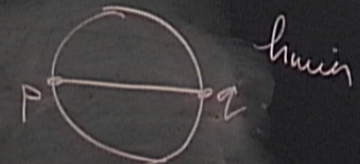


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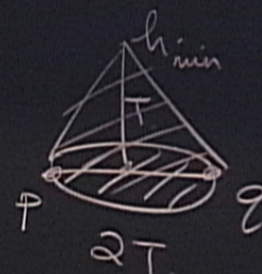
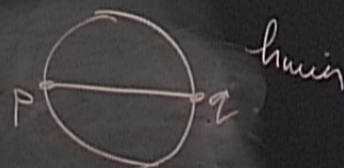
CST CONSTRUCTION.

$$\tilde{d}(p, q) =$$

$$\gamma = \partial J^+(p) \cap \partial J^+(q) \quad (n-2): \mathbb{H}^{n-2}$$

$$h \in \mathcal{H}: \quad J^-(h) \cap J^+(\varepsilon)$$

$$V(h) = \text{vol}(\quad)$$



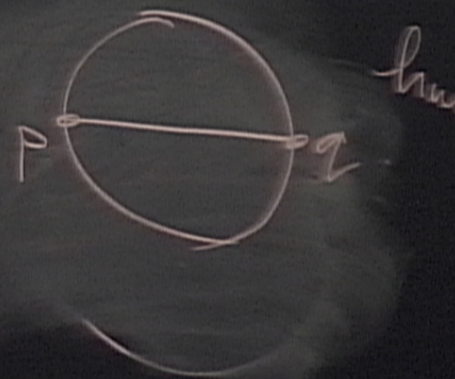
$$V(h_{\min}) = \int_{\mathcal{D}} T^d$$

$$\tilde{d}(p, q) = 2 \left[\frac{V(h_{\min})}{5d} \right]^{1/d}$$

$$\mathcal{H} = \partial J^+(p) \cap$$

$$h \in \mathcal{H} : \quad \overline{J^-(h)}$$

$$V(h) = \text{vol}(\overline{J^-(h)})$$



• CST CONSTRUCTION.

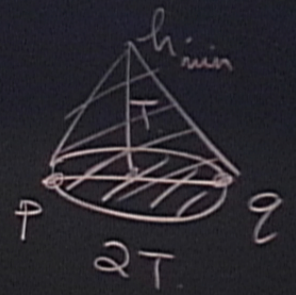
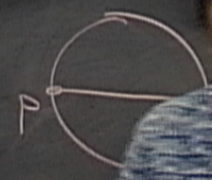
$$\tilde{d}(p, q) = 2 \left[\frac{V(h_{\min})}{\sum d} \right]^{1/d}$$



$$\mathcal{H} = \partial J^+(p) \cap \partial J^+(q) \quad (n-2) \cdot \mathbb{H}^{n-2}$$

$$h \in \mathcal{H}: \quad J^-(h) \cap J^+(\varepsilon)$$

$$V(h) = \text{vol}(\dots)$$



$$V(h_{\min}) = \sum d T^d$$

CST CONSTRUCTION.

$$\tilde{d}(p, q) = 2 \left[\frac{V(h_{\min})}{\tau_d} \right]^{1/d}$$

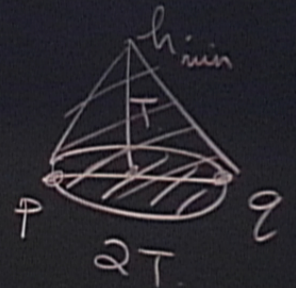
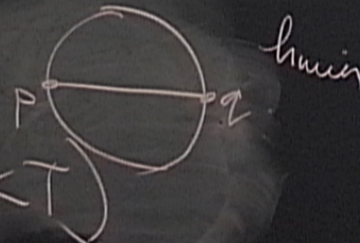
$$\gamma = \partial J^+(p) \cap \partial J^+(q) \quad (n-2) \cdot \mathbb{H}^{n-2}$$

$$h \in \mathcal{H}: \quad J^-(h) \cap J^+(\varepsilon)$$

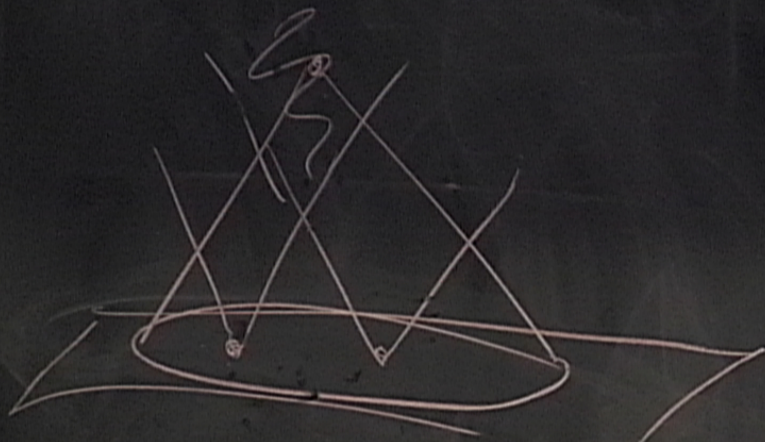
$$V(h) = \text{vol}(\quad)$$



$$\varepsilon \quad V = V_\gamma (1 + (\downarrow)KT)$$



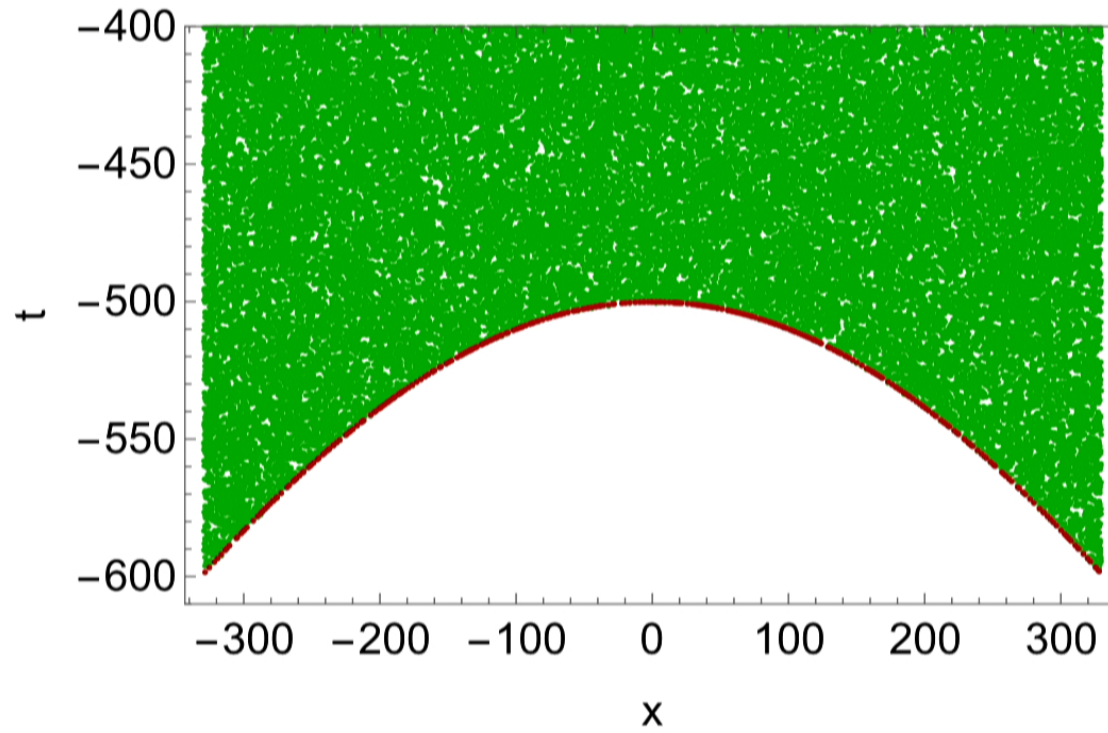
$$V(h_{\min}) = \tau_d T^d$$



l_p, l_{AS}, l_k

$\uparrow$
 l

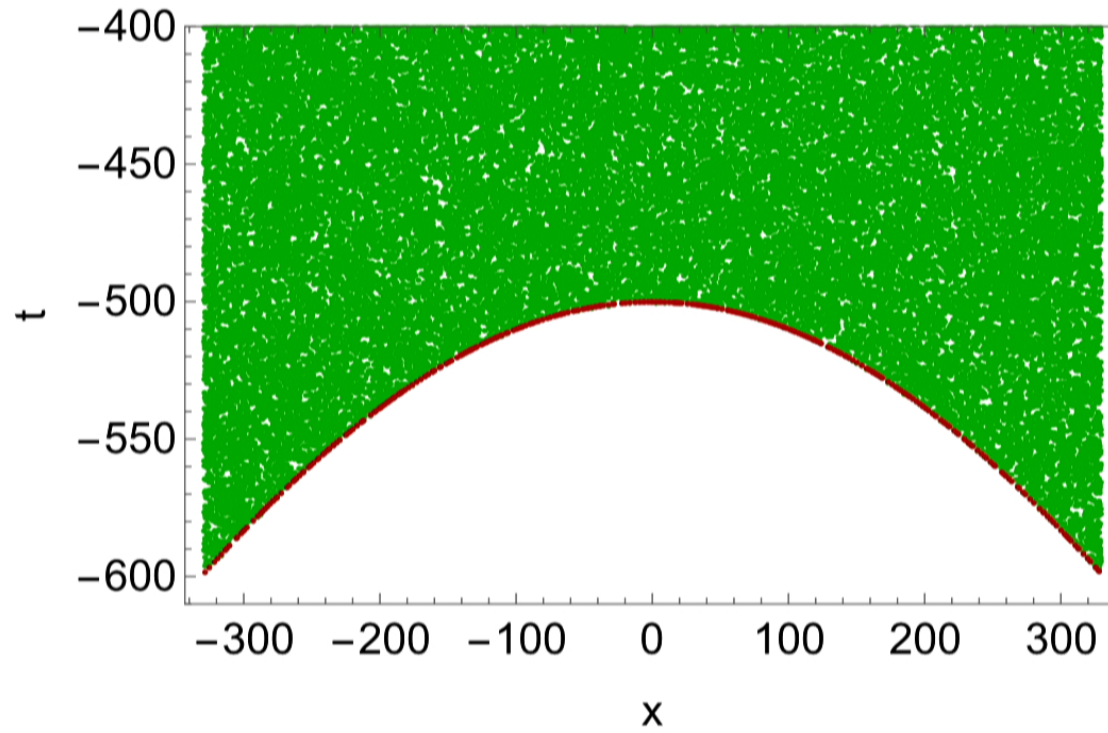
Hyperbola I



d=2: $N \sim 58\text{ K}$, $|A| \sim 200$

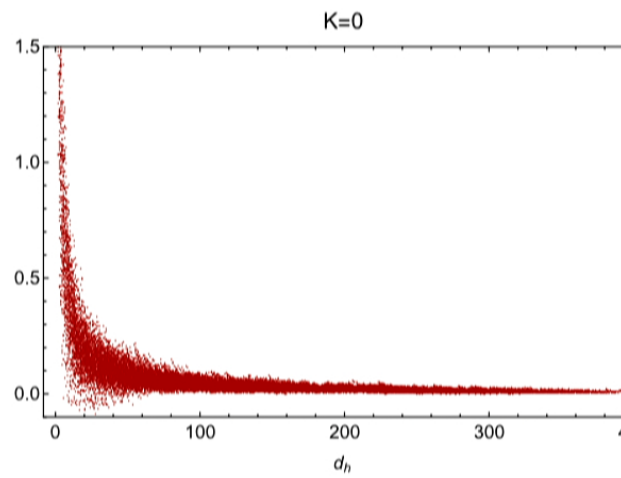
d=3: $N \sim 75\text{ K}$, $|A| \sim 1500$

Hyperbola I

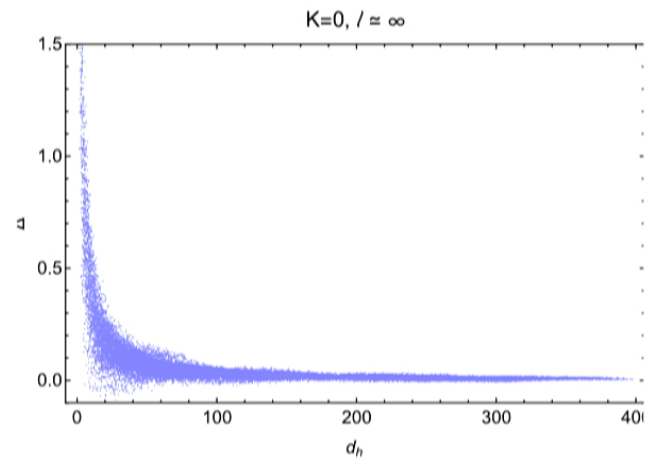


d=2: $N \sim 58\text{ K}$, $|A| \sim 200$

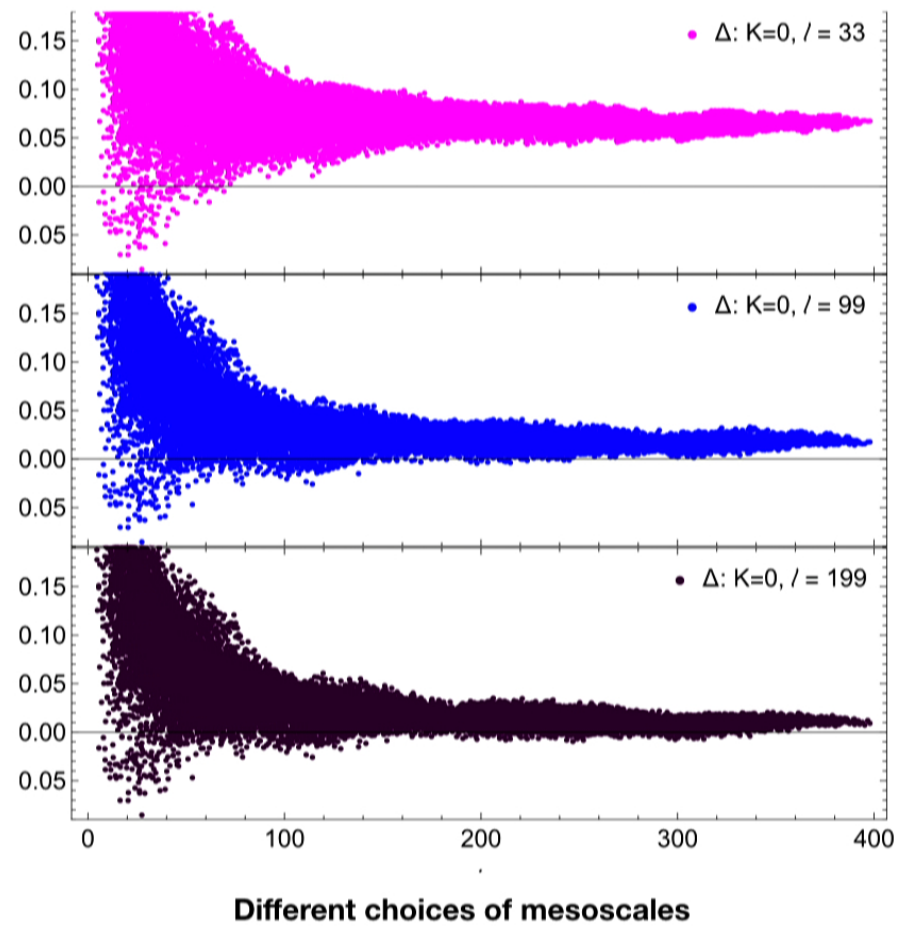
d=3: $N \sim 75\text{ K}$, $|A| \sim 1500$

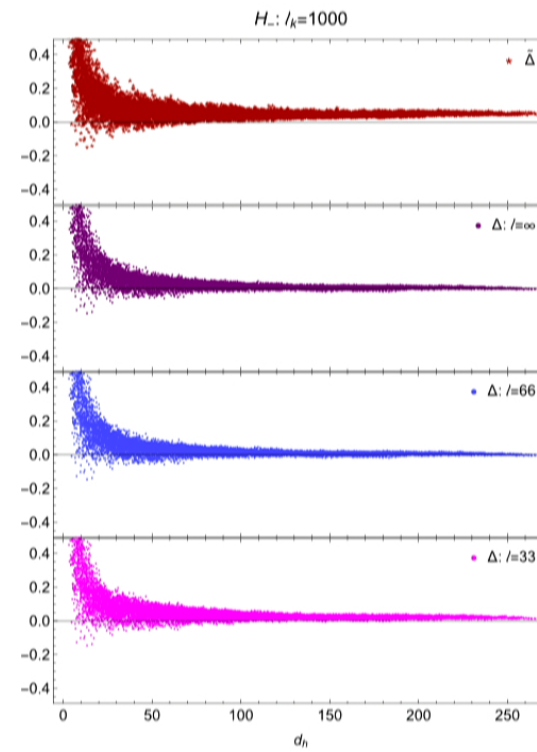
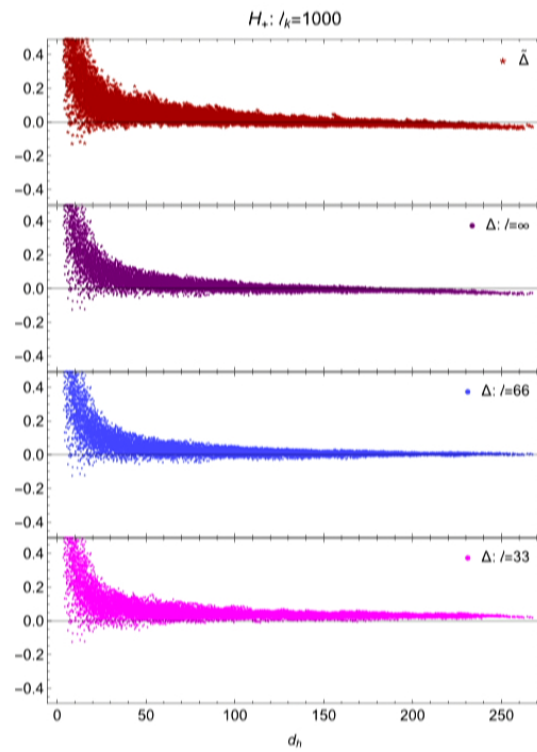


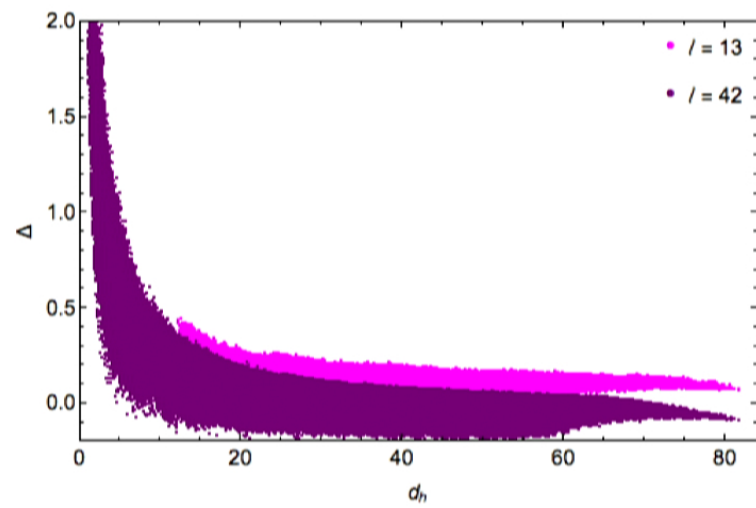
Predistance, d=2



Distance, d=2







d=3, K=0

