Title: Null conservation laws for gravity

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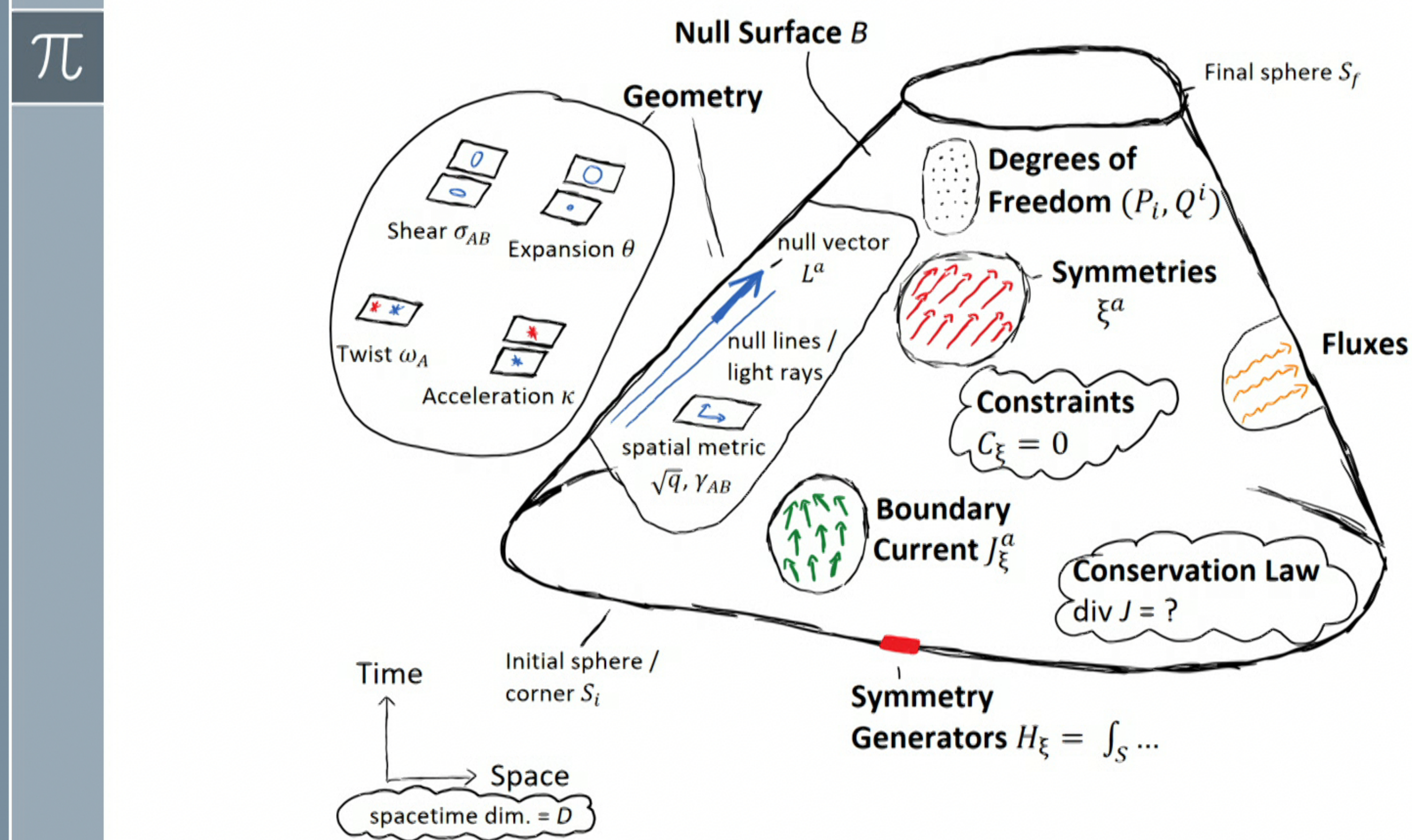
Abstract: <p>I discuss the canonical degrees of freedom of metric Einstein gravity on a null surface. The constraints are interpreted as conservation equations of a boundary current. Gravitational fluxes are identified, and the Hamiltonians of diffeomorphism symmetry are discussed. Special attention is given to the role of a modification of the phase space at the boundary of the null surface. Based on 1802.06135 with Laurent Freidel.</p>

Null Conservation Laws for Gravity

Florian Hopfmueller

Oct 11 2018, PI QG Seminar

Based on 1802.06135, 1611.03096 with Laurent Freidel.



Null Surface B
Final sphere S_f
Geometry
Shear σ_{AB}
Expansion θ
Twist ω_A
Acceleration κ
null vector L^a
null lines / light rays
spatial metric $\sqrt{q}, \gamma_{AB}$
Degrees of Freedom (P_i, Q^i)
Symmetries ξ^a
Fluxes
Constraints $C_\xi = 0$
Boundary Current J^a_ξ
Conservation Law div J = ?
Initial sphere / corner S_i
Symmetry Generators $H_\xi = \int_S \dots$
Time
Space
spacetime dim. = D

Canonical GR on a null surface

- **Why null**? Null=physically special. Examples

- **Why not gauge fix?** Gauge vs. symmetry, edge modes.

- **Why canonical conservation laws?** Import understanding from null infinity, canonical perspective on membrane paradigm

- **Why worry about corner phase space?** Different corner phase spaces lead to different Hamiltonians, what does it mean?

Chapters

- Motivation
- Degrees of Freedom
- Conservation Laws
- Hamiltonians

Degrees of Freedom

- Are contained in **presymplectic potential** θ defined by

$$\delta L = EoM + \mathrm{d}\theta$$

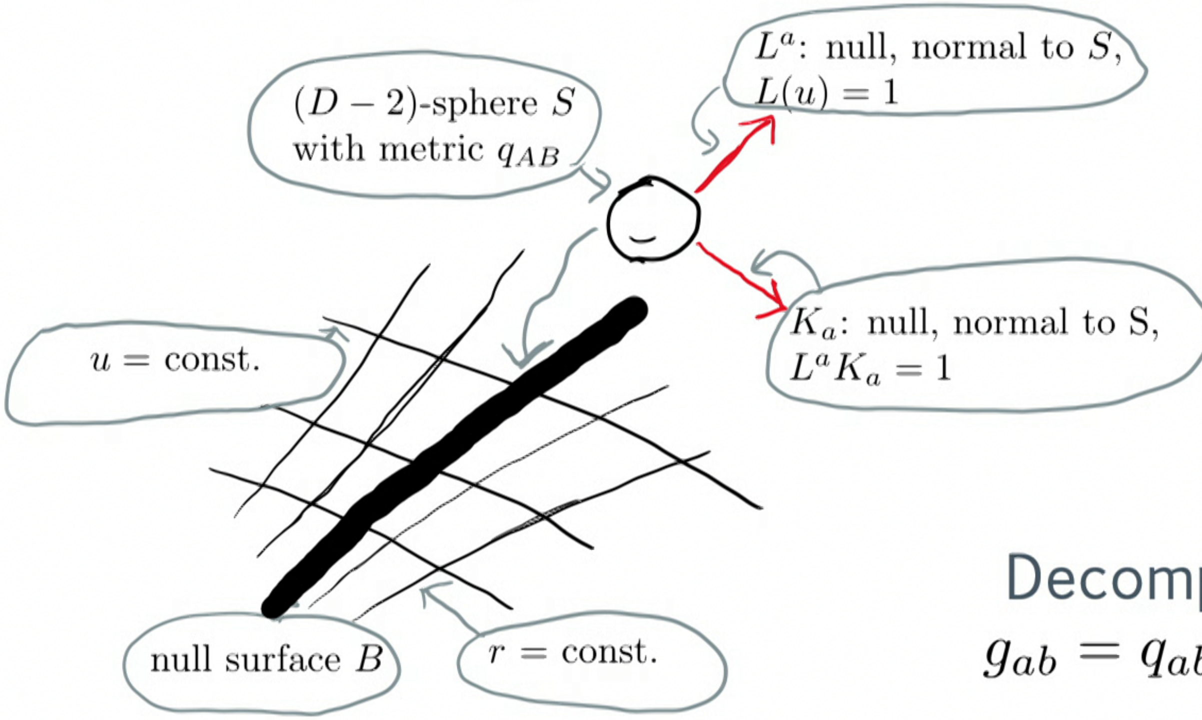
- Canonical data on Σ : $\int_\Sigma \theta =$ "$\int_\Sigma \sum P_i \delta Q^i$"
- Ambiguities: $\theta \to \theta + \mathrm{d}\alpha + \delta\beta$
- For Einstein-Hilbert action: Wald symplectic potential

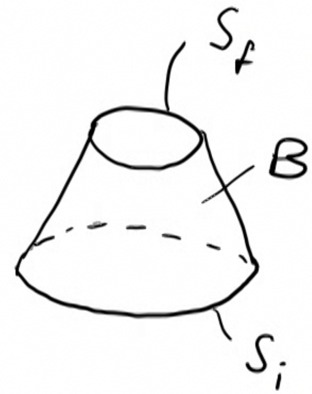
$$\theta_W = \big(\nabla_b \delta g^{ab} - \nabla^a (g^{bc}\delta g_{bc})\big)\epsilon_a$$

- ADM: $\Theta_{ADM} = \sqrt{h}(K^{ab} - K h^{ab})\delta h_{ab}$ doesn't work for null

--> need to resolve degenerate geometry

DoF - Geometry





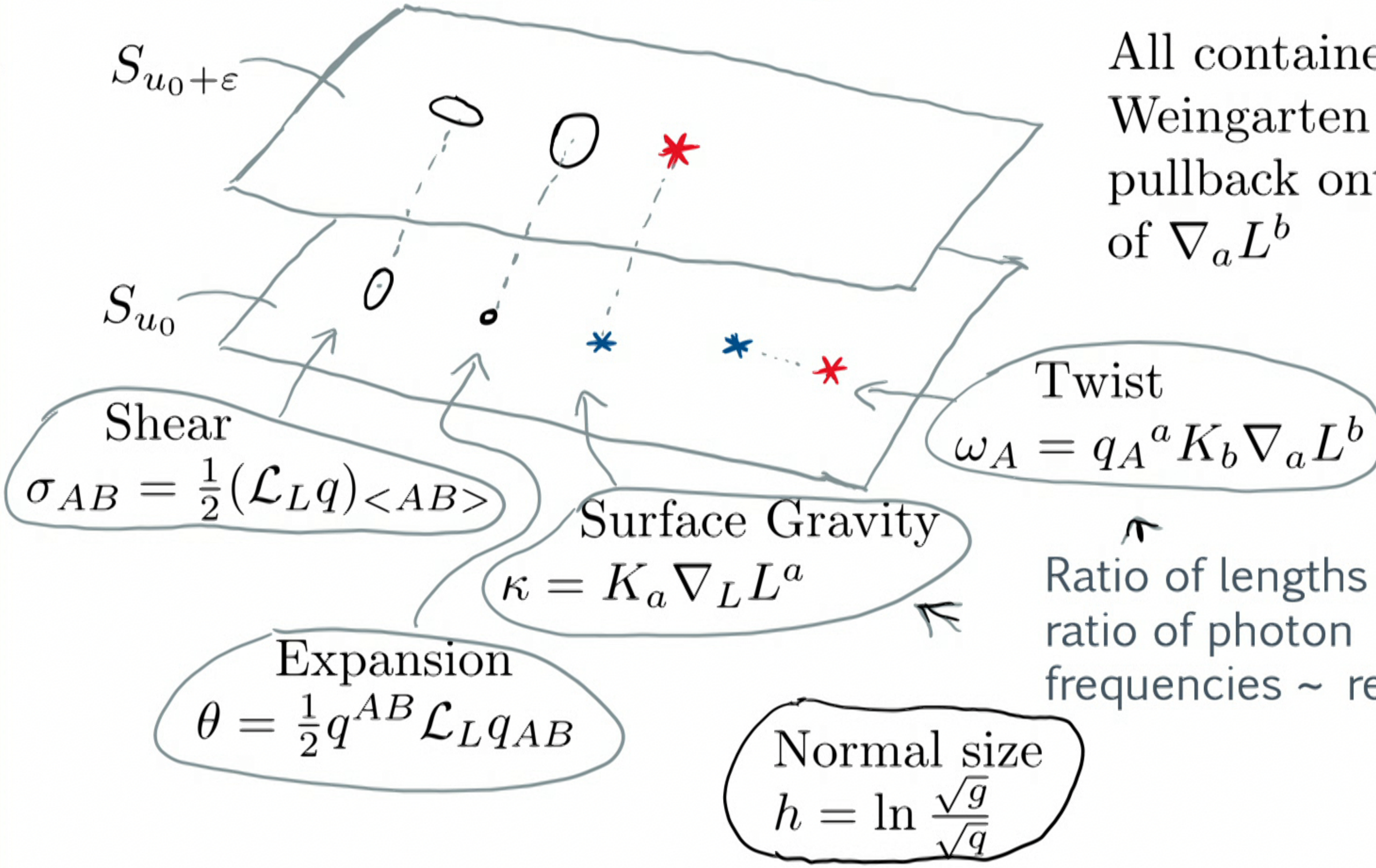
Decompose

$$g_{ab} = q_{ab} + L_a K_b + K_a L_b$$

Determines area (D-2)-form ϵ_S,
"volume form" $\mathrm{d}B = \mathrm{d}u \wedge \epsilon_S$

No gauge choice!

DoF – Extrinsic Geometry



All contained in Weingarten map, pullback onto B of $\nabla_a L^b$

Ratio of lengths of L ~ ratio of photon frequencies ~ redshift

DoF – Variational Data

- Compare two similar geometries.
- Restriction: variations leave B null
- $\delta q_{AB} = \delta\gamma_{AB} + \frac{1}{D-2} q_{AB}\delta q$: variation of sphere metric
- $\delta L^a \to \delta L^A$: small translation of adjacent cross-sections
- Variation of size of normal geometry $\to \delta h$

Degrees of Freedom

- $$\Theta_W = \int_B \theta_W = \int_B \sqrt{q} L_a \big(\nabla_b \delta g^{ab} - \nabla^a (g^{bc} \delta g_{bc})\big)$$
- sprinkle lots of $g_{ab} = q_{ab} + L_a K_b + K_a L_b$, integrate by parts until nothing is left ...
- Result: $\Theta_W = \text{“} \int P\delta Q \text{”} = \Theta_B + \Theta_{\partial B}$

$$\Theta_B = \int_B \mathrm{d}u \mathrm{d}S \left(\tfrac{1}{2} \sqrt{q} \sigma^{AB} \delta \gamma_{AB} - \sqrt{q} \omega_A \delta L^A + \delta \mu \sqrt{q} \right)$$

Shear

Conf. metric

Twist

Translation

area

$\# = 5$ (in 4D)

$\mu = \kappa + \frac{D-3}{D-2} \theta$

spin-0 momentum

Degrees of Freedom

$$\Theta_B = \int_B \mathrm{d}u \mathrm{d}S \left(\tfrac{1}{2}\sqrt{q}\sigma^{AB}\delta\gamma_{AB} - \sqrt{q}\omega_A \delta L^A + \delta\mu\sqrt{q} \right)$$

- Conformal metric conjugate to its on "time" derivative, well-known
- Translation conjugate to twist, or Hajicek field
  twist is momentum in membrane paradigm,
  gives b.h. angular momentum
  (Price Thorne McDonald Znajek Damour)
- spin-0 mom. $\mu = \kappa + \frac{D-3}{D-2}\theta$ involves surface gravity, expansion. For black holes, temperature. In membrane pic, contains pressure and bulk viscosity.
- Connects to b.h. thermo

DoF – Corner Phase Space I

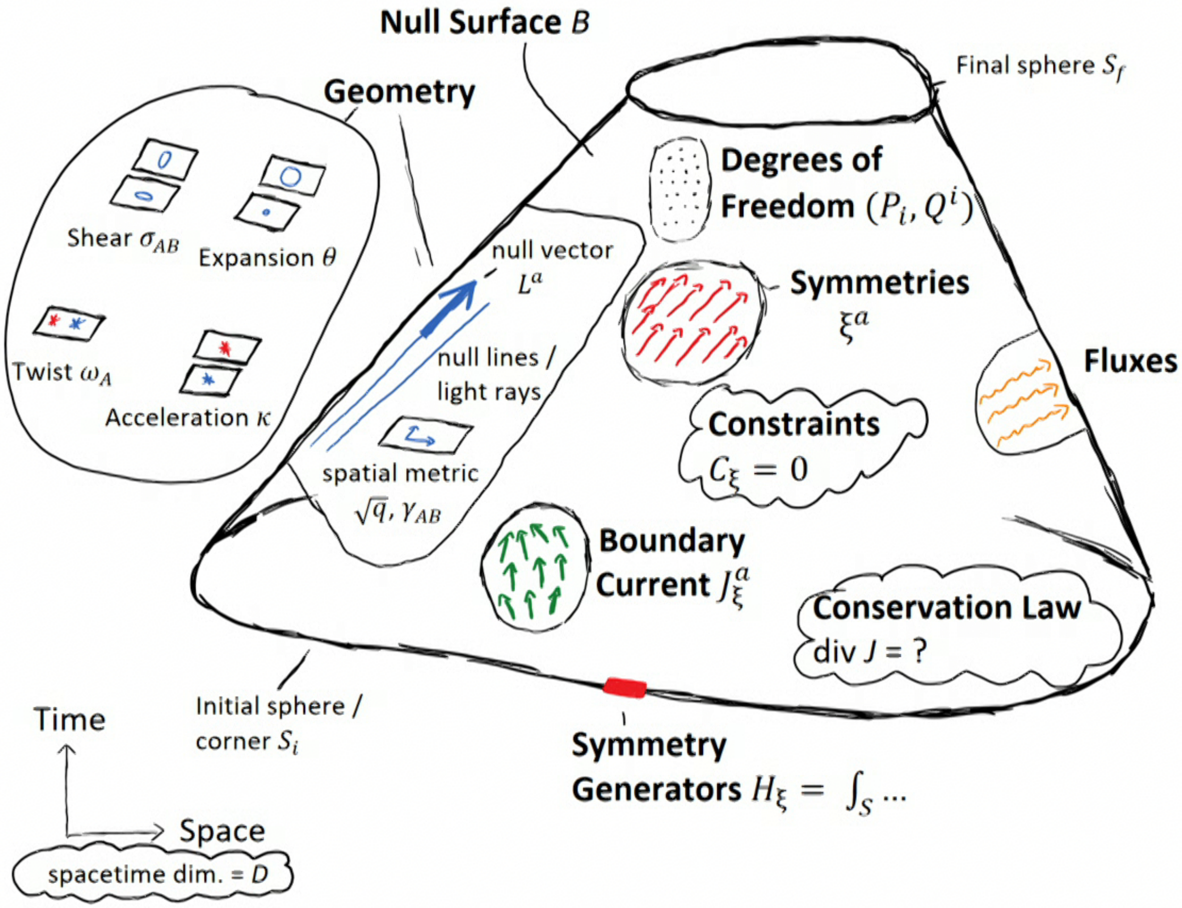
- $$\Theta_{\partial B} = \int_{\partial B} \mathrm{d}S\Big(-\tfrac{1}{2}\delta h\sqrt{q} + \tfrac{1}{D-2}\delta(\sqrt{q})\Big)$$

  normal size   area

- Remember: $\delta L = EoM + \mathrm{d}\theta$, determines θ up to exact piece, Θ up to $\int_{\partial B}$
- Most covariant choice: $\theta_W = \big(\nabla_b \delta g^{ab} - \nabla^a (g^{bc}\delta g_{bc})\big)\epsilon_a$
- "Minimal" choice for null surface: Drop $\Theta_{\partial B}$.

DoF – The missing momentum

- Aghapur, Jafari, Golshani 1808.07352
- We have 5 pairs – where is the 6th?
- What is the momentum conjugate to departure from nullness?
- Answer: $\sim \kappa_K = L_a \nabla_K K^a$ transverse acceleration
- Subtlety: if one wants brackets / Hamiltonians allowing departure from nullness:
  Need to allow non-null configurations in the calculation of all momenta.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Conservation – **Example**

› Time translation symmetry for massless scalar field:

$H = \frac{1}{2}\left(\int_\Sigma (\partial_t \phi)^2 + (\vec{\nabla}\phi)^2 \right)$, EoM $(-\partial_t^2 + \vec{\nabla}^2\phi) = 0$, then $\frac{\mathrm{d}}{\mathrm{d}t} H = \int_{\partial\Sigma} \partial_n \phi \partial_t \phi$

→ Energy conserved, up to flux $P\dot{Q}$ on the boundary.

Gravity: Consider spacetime region bounded by B.

› Energy → Energy-momentum, local expression on B

› Conservation $\frac{\mathrm{d}}{\mathrm{d}t} \to$ covariant local conservation $\partial_a(\sqrt{q} J_\xi^a)$

› J_ξ^a is the **boundary current** on B, associated with arbitrary local symmetry $\xi^a \parallel B$

→ what is the boundary current? What are the fluxes?

Conservation – Symmetries

- Diffeomorphism are local symmetries. How do they act?
- We have introduced background structure (u coordinate) →partially breaks covariance
- Define field space Lie derivative, on any tensor T:
  $\mathfrak{L}_\xi T = \frac{\delta T}{\delta g_{ab}} \mathcal{L}_\xi g_{ab}$
- Let $\xi^a = f L^a + v^a$, $v \parallel S$: the actions are
  $\mathfrak{L}_\xi \gamma_{AB} = f\sigma_{AB} + D_{<A} v_{B>}$ $\qquad \mathfrak{L}_\xi \sqrt{q} = \sqrt{q}(f\theta + D_A v^A)$
  $\mathfrak{L}_\xi L^a = \mathcal{L}_v L^a$ $\qquad \mathfrak{L}_\xi \mu = v^a \partial_a \mu + L^a \partial_a \big(f\mu + L^b \partial_b f\big)$
  $\mathfrak{L}_\xi h = K_a \nabla_L \xi^a + L_a \nabla_K \xi^a$ ← odd one out!

→ transformations are not quite covariant, but under control →transfo of h involves transverse derivs of ξ

Constraints as Conservation Equations

Want: $\partial_a(\sqrt{q}J^a_\xi) = \text{flux} = P\mathfrak{L}_\xi Q.$

How: the constraints are $L_a G^a{}_b \xi^b = L_a T^a{}_b \xi^b$.
Use $L^a R_{ab} = (\nabla_a \nabla_b - \nabla_b \nabla_a)L^a$ and $g_{ab} = q_{ab} + L_a K_b + K_a L_b$
to find $G_{La}[q_{AB}, L^a, \sigma_{AB}, \theta, \kappa, \omega_A]$. Smear and i.b.p.

Result:

$$\partial_a(\sqrt{q}J^a_\xi) = \sqrt{q}\Big(T_{L\xi} + \tfrac{1}{2}\sigma^{AB}(\mathfrak{L}_\xi \gamma_{AB}) - \omega_a(\mathfrak{L}_\xi L^a) + \mathfrak{L}_\xi \mu\Big)$$

Recall: $\Theta_B = \int_B \mathrm{d}u\mathrm{d}S \left(\tfrac{1}{2}\sqrt{q}\sigma^{AB}\delta\gamma_{AB} - \sqrt{q}\omega_A \delta L^A + \delta\mu\sqrt{q}\right)$

Flux $= P\mathfrak{L}_\xi Q$ ✓

So what is J^a_ξ?

Conservation – The boundary current

So what is J^a_ξ? Recall $\xi = fL + v$:

mom. for δL

$$J^a_v = L^a(v^b\omega_b) + \sigma^a{}_b v^b$$

momentum aspect

spatial momentum current

mom. for $\sqrt{q}$

$$J^a_{fL} = L^a\big(f(\mu - \theta) - L^b\partial_b f\big)$$

energy aspect

no spatial energy current!

Conservation – Corner Phase Space II

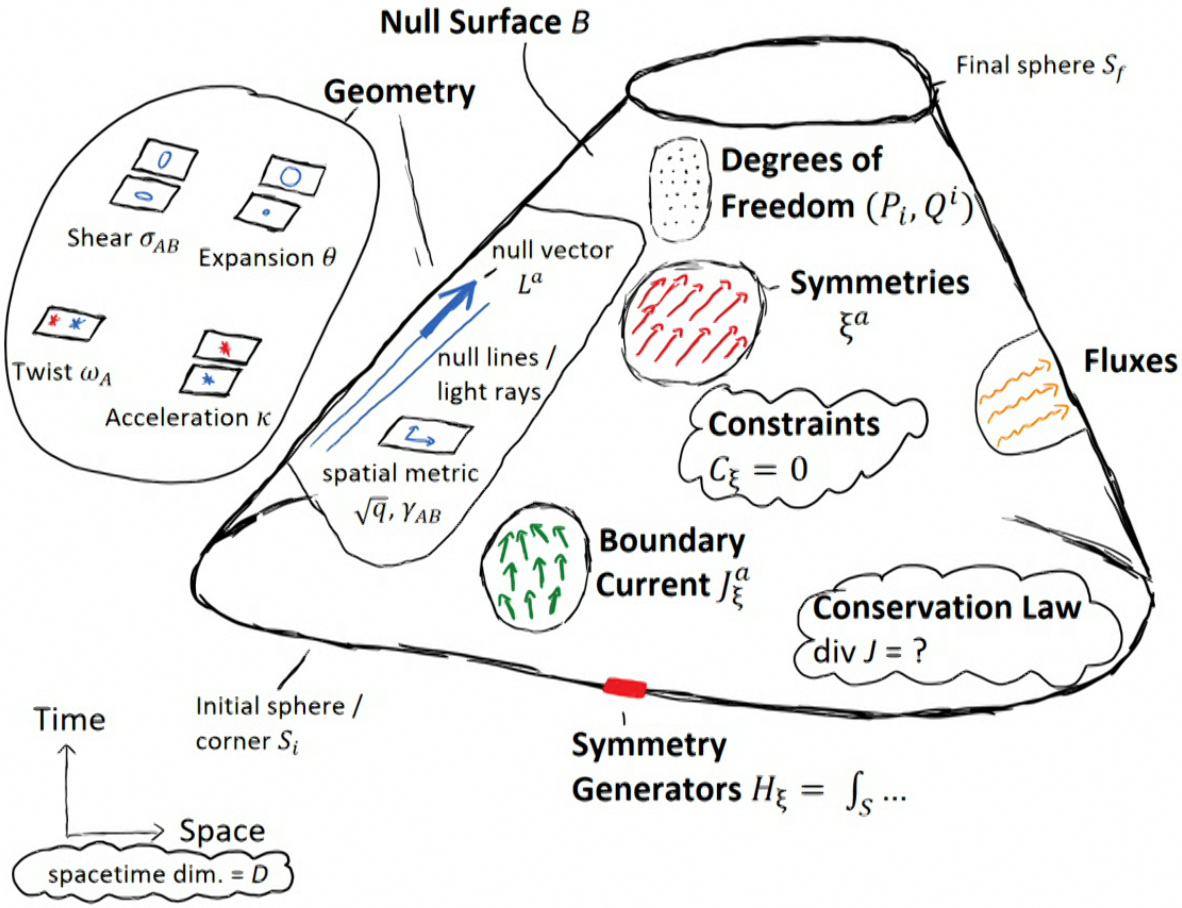
- Recall: $\Theta_W = \Theta_B + \Theta_{\partial B}$
- We had: $\partial_a(\sqrt{q}J^a_\xi) = I_\xi \Theta_B + \int T_{L\xi}$,no $\Theta_{\partial B}$!
- $\Theta_{\partial B}$ contains h, transformation depends on $\nabla_K \xi$
- If we had kept $\Theta_{\partial B}$, we would have found instead of J_ξ :

  Komar charge $K_\xi \sim L_a \nabla_K \xi^a - K_a \nabla_L \xi^a$, depends on $\nabla_K \xi$

➔ Constraints suggest using Θ_B. By dropping $\Theta_{\partial B}$, we have removed the “extension dependence” from J_ξ

➔ Possible criterion for fixing corner phase space: “extension independence” of transformations of var’s

Call Θ_B the **intrinsic symplectic potential.**
(Almost) covariant under diffeos of *B*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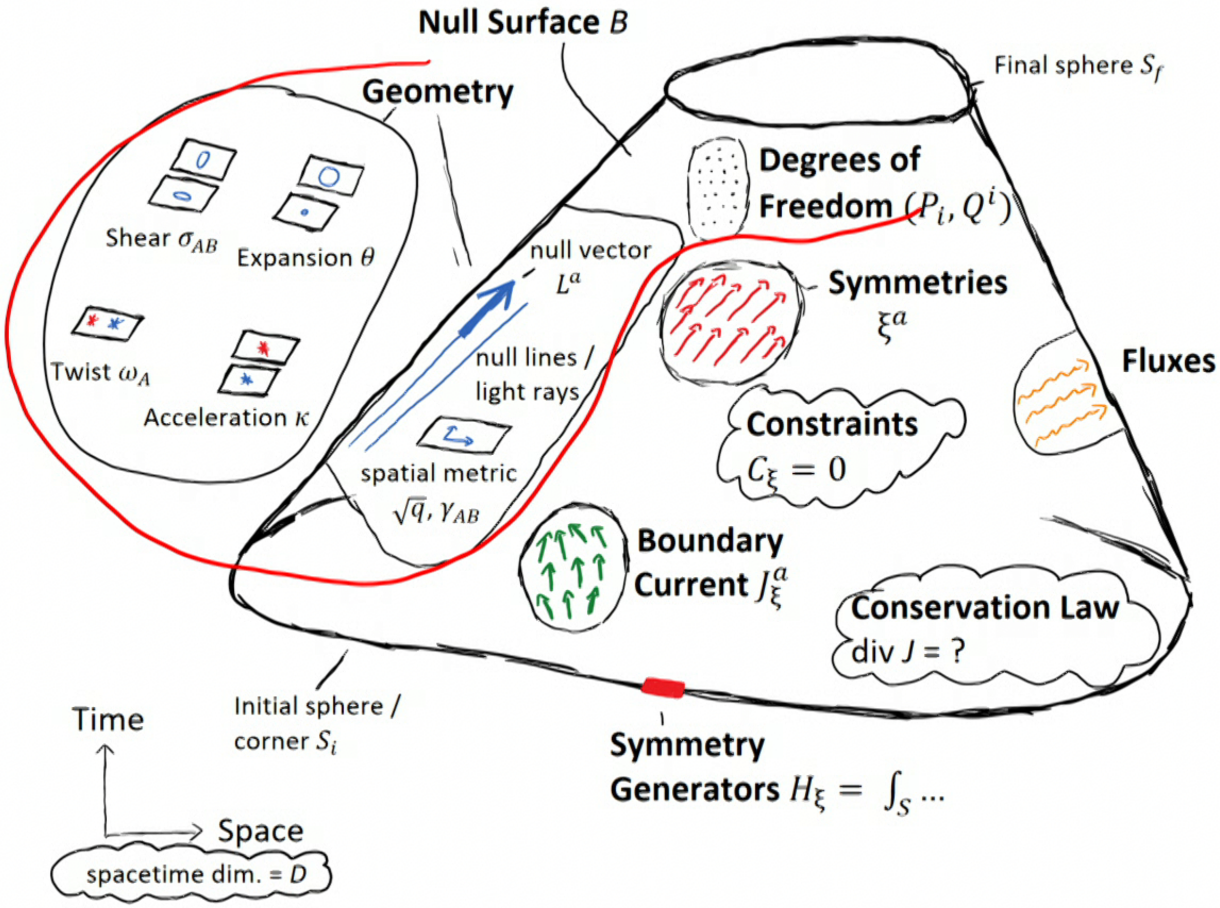

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Hamiltonians

- But what are symmetries generated by?
- Symplectic form $\Omega_B = \delta \wedge \Theta_B$.
  $h_\xi = \Omega(\delta g, \mathcal{L}_\xi g)$. ξ is pure gauge iff $h_\xi = 0$ on-shell,
  Hamiltonian symmetry iff $h_\xi = \delta(H_\xi)$
- Choose $\xi = v + fL$, $v \parallel S$.
  Metric dependent – "tracks null lines"
- Result: Hamiltonian lives on the corner, u-component of boundary current is the integrable bit of the Hamiltonian:

$$h_\xi = \delta\Big(\int_{\partial B} \sqrt{q} J^u_\xi\Big) - \int_{\partial B} \mathrm{d}S \sqrt{q}\Big(\frac{1}{2} f \sigma^{AB} \delta\gamma_{AB} + \delta\big(\mu + L(f)\big)\Big)$$

$$J^u_\xi = f(\mu - \theta) + L^b \partial_b f + v^b \omega_b$$

- Hamiltonian depends on corner phase space!

Hamiltonians: which symmetries are hamiltonian?

$$h_\xi = \delta\Big(\int_{\partial B} \sqrt{q} J^u_\xi \Big) - \int_{\partial B} \mathrm{d}S \sqrt{q} \Big(\frac{1}{2} f \sigma^{AB} \delta\gamma_{AB} + \delta(\mu + L(f)) \Big)$$

$$J^u_\xi = f(\mu - \theta) + L^b \partial_b f + v^b \omega_b$$

No δL^a

→ take symmetries that don't move the corner.

→ choose $f \overset{\partial B}{=} 0$ → $H_\xi = \int_{\partial B} \sqrt{q} \big(v^b \omega_b - L^b \partial_b f \big)$.

→ fix geometry of corner $(\sqrt{q}, \gamma_{AB})$

→ $H_\xi = \int_{\partial B} \mathrm{d}S \sqrt{q} (-f\theta + v^b \omega_b)$

→ fix momenta

→ $H_\xi = \int_{\partial B} \mathrm{d}S \sqrt{q} (f(\mu - \theta) + L(f) + v^b \omega_b)$

→ Something in between

Note: ~ 2 copies of "supertranslations" → 1807.11499

Hamiltonians: which symmetries are hamiltonian?

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→ Something in between

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Recap

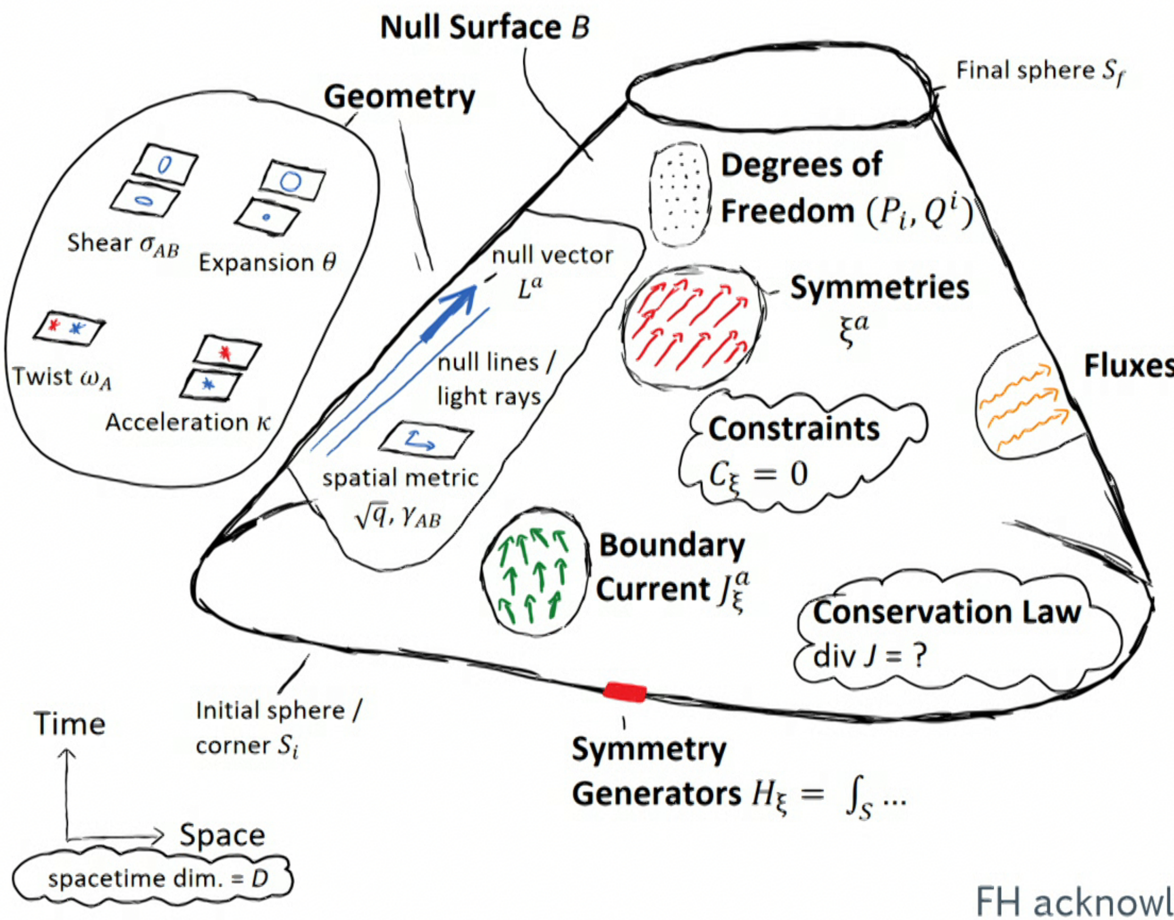
› Expressed DoF (including gauge DoF) of GR on a null surface in terms of null intrinsic & extrinsic geometry

› Interpreted smeared tangential constraints as conservation law of a (induced) **boundary current**, with fluxes $P\mathfrak{L}_\xi Q$

› By a choice of corner phase space, removed dependence of boundary current and Hamiltonians on extension of symmetry parameter

› Analyzed Hamiltonians & conditions for "Hamiltonicity" for tangential symmetries

Thanks!



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