

Title: Null conservation laws for gravity

Date: Oct 11, 2018 02:30 PM

URL: <http://pirsa.org/18100058>

Abstract: <p>I discuss the canonical degrees of freedom of metric Einstein gravity on a null surface. The constraints are interpreted as conservation equations of a boundary current. Gravitational fluxes are identified, and the Hamiltonians of diffeomorphism symmetry are discussed. Special attention is given to the role of a modification of the phase space at the boundary of the null surface. Based on 1802.06135 with Laurent Freidel.</p>

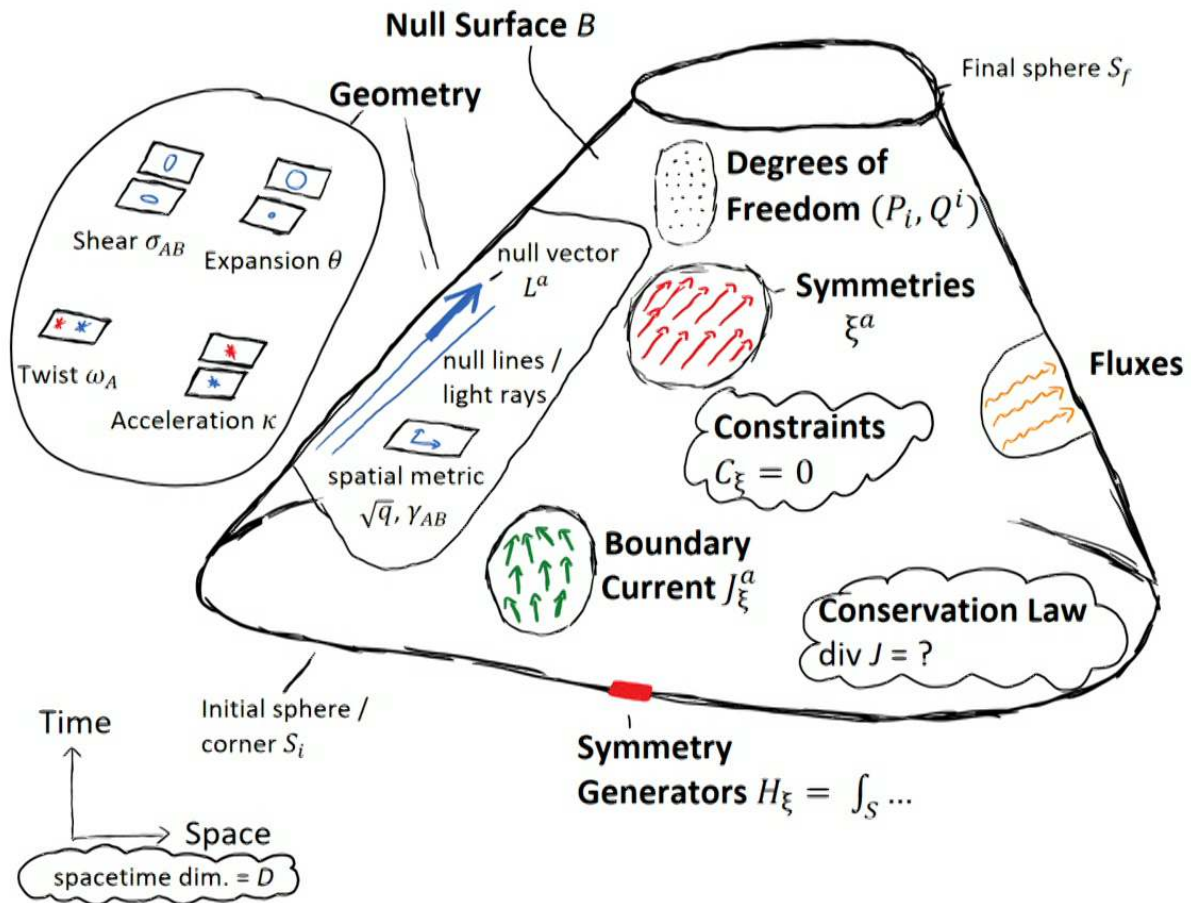
Null Conservation Laws for Gravity

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Oct 11 2018, PI QG Seminar

Based on 1802.06135, 1611.03096 with Laurent Freidel.

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Canonical GR on a null surface

- › **Why null?** Null=physically special. Examples
- › **Why not gauge fix?** Gauge vs. symmetry, edge modes.
- › **Why canonical conservation laws?** Import understanding from null infinity, canonical perspective on membrane paradigm
- › **Why worry about corner phase space?**
Different corner phase spaces lead to different Hamiltonians, what does it mean?

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Chapters

- › Motivation
- › Degrees of Freedom
- › Conservation Laws
- › Hamiltonians

Degrees of Freedom

- › Are contained in **presymplectic potential** θ defined by

$$\delta L = EoM + d\theta$$

- › Canonical data on Σ : $\int_{\Sigma} \theta = \text{“} \int_{\Sigma} \sum P_i \delta Q^i \text{”}$

- › Ambiguities: $\theta \rightarrow \theta + d\alpha + \delta\beta$

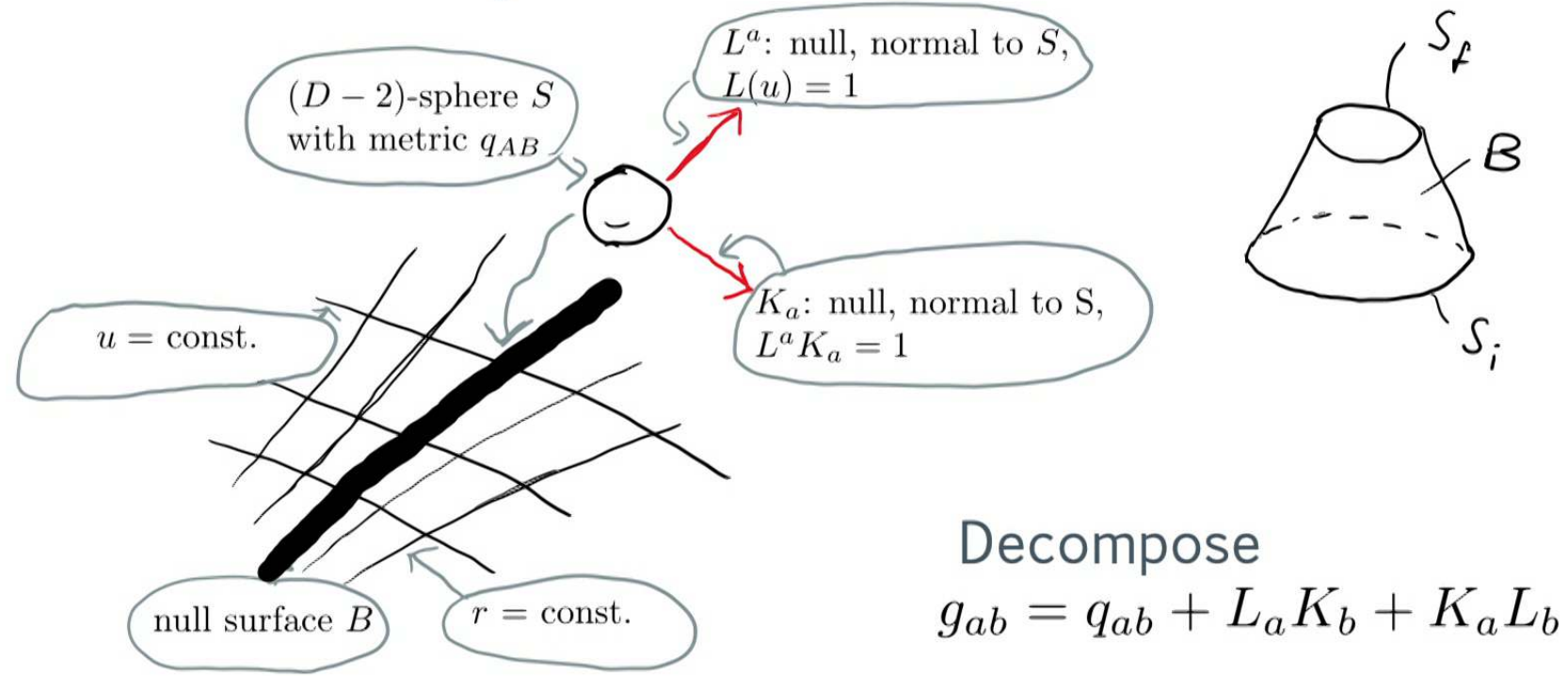
- › For Einstein-Hilbert action: Wald symplectic potential

$$\theta_W = (\nabla_b \delta g^{ab} - \nabla^a (g^{bc} \delta g_{bc})) \epsilon_a$$

- › ADM: $\Theta_{ADM} = \sqrt{h} (K^{ab} - K h^{ab}) \delta h_{ab}$ doesn't work for null

--> need to resolve degenerate geometry

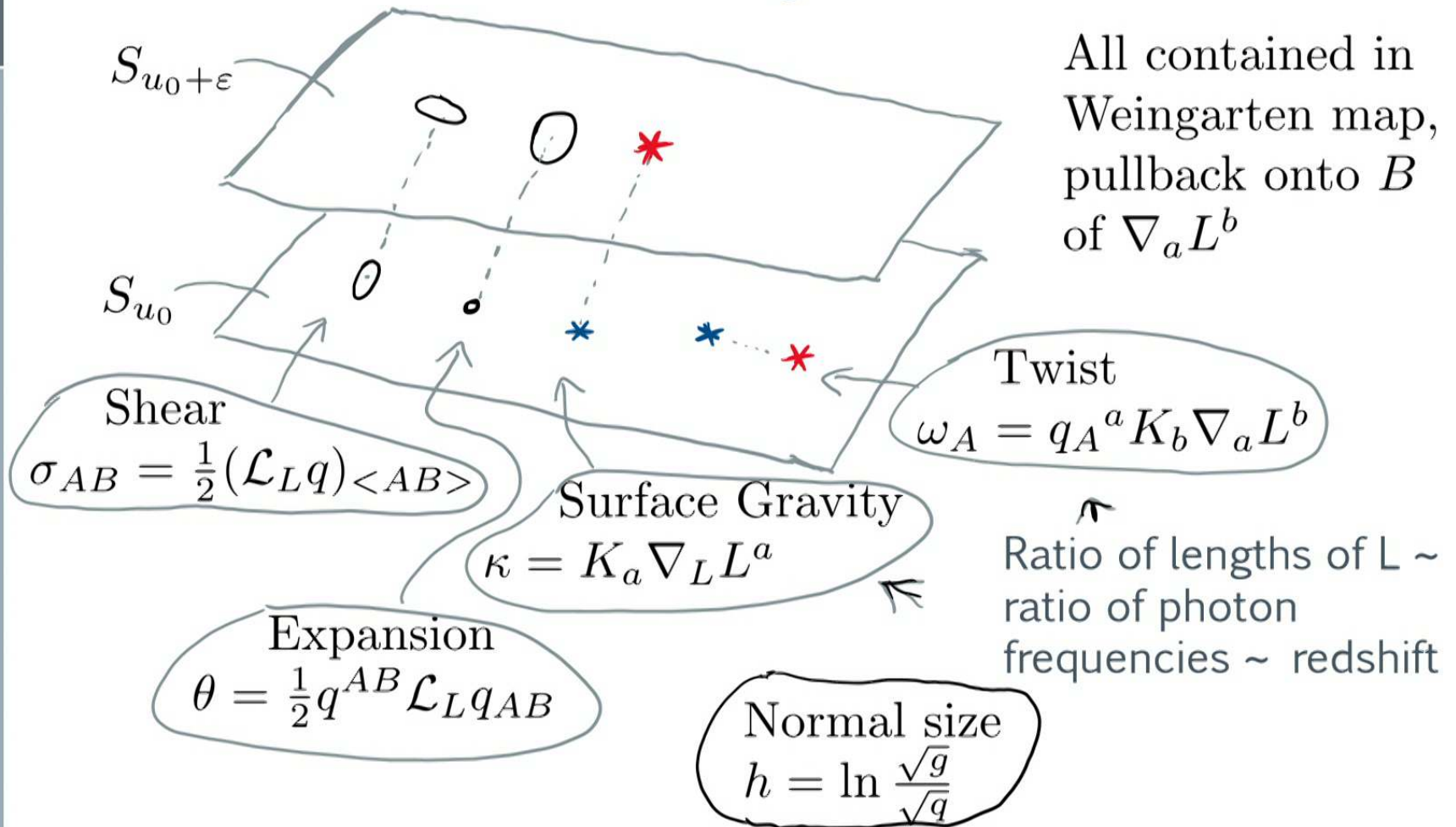
DoF - Geometry



Determines area $(D-2)$ -form ϵ_S ,
 “volume form” $dB = du \wedge \epsilon_S$

No gauge choice!

DoF – Extrinsic Geometry



DoF – Variational Data

- › Compare two similar geometries.
- › Restriction: variations leave B null
- › $\delta q_{AB} = \delta \gamma_{AB} + \frac{1}{D-2} q_{AB} \delta q$: variation of sphere metric
- › $\delta L^a \rightarrow \delta L^A$: small translation of adjacent cross-sections
- › Variation of size of normal geometry $\rightarrow \delta h$

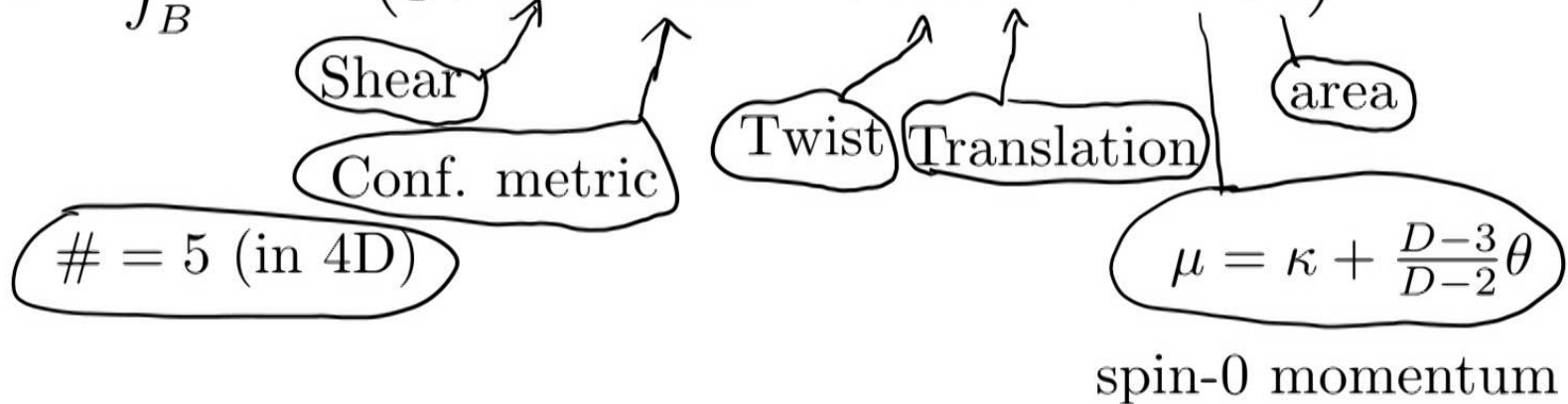
Degrees of Freedom

$$\triangleright \Theta_W = \int_B \theta_W = \int_B \sqrt{q} L_a (\nabla_b \delta g^{ab} - \nabla^a (g^{bc} \delta g_{bc}))$$

\triangleright sprinkle lots of $g_{ab} = q_{ab} + L_a K_b + K_a L_b$,
integrate by parts until nothing is left ...

\triangleright Result: $\Theta_W = \text{“} \int P \delta Q \text{”} = \Theta_B + \Theta_{\partial B}$

$$\Theta_B = \int_B dudS \left(\frac{1}{2} \sqrt{q} \sigma^{AB} \delta \gamma_{AB} - \sqrt{q} \omega_A \delta L^A + \delta \mu \sqrt{q} \right)$$




Degrees of Freedom

$$\Theta_B = \int_B dudS \left(\frac{1}{2} \sqrt{q} \sigma^{AB} \delta \gamma_{AB} - \sqrt{q} \omega_A \delta L^A + \delta \mu \sqrt{q} \right)$$

- › Conformal metric conjugate to its on “time” derivative, well-known
- › Translation conjugate to twist, or Hajicek field
twist is momentum in membrane paradigm,
gives b.h. angular momentum
(Price Thorne McDonald Znajek Damour)
- › spin-0 mom. $\mu = \kappa + \frac{D-3}{D-2} \theta$ involves surface gravity, expansion. For black holes, temperature. In membrane pic, contains pressure and bulk viscosity.
- › Connects to b.h. thermo

DoF – Corner Phase Space I

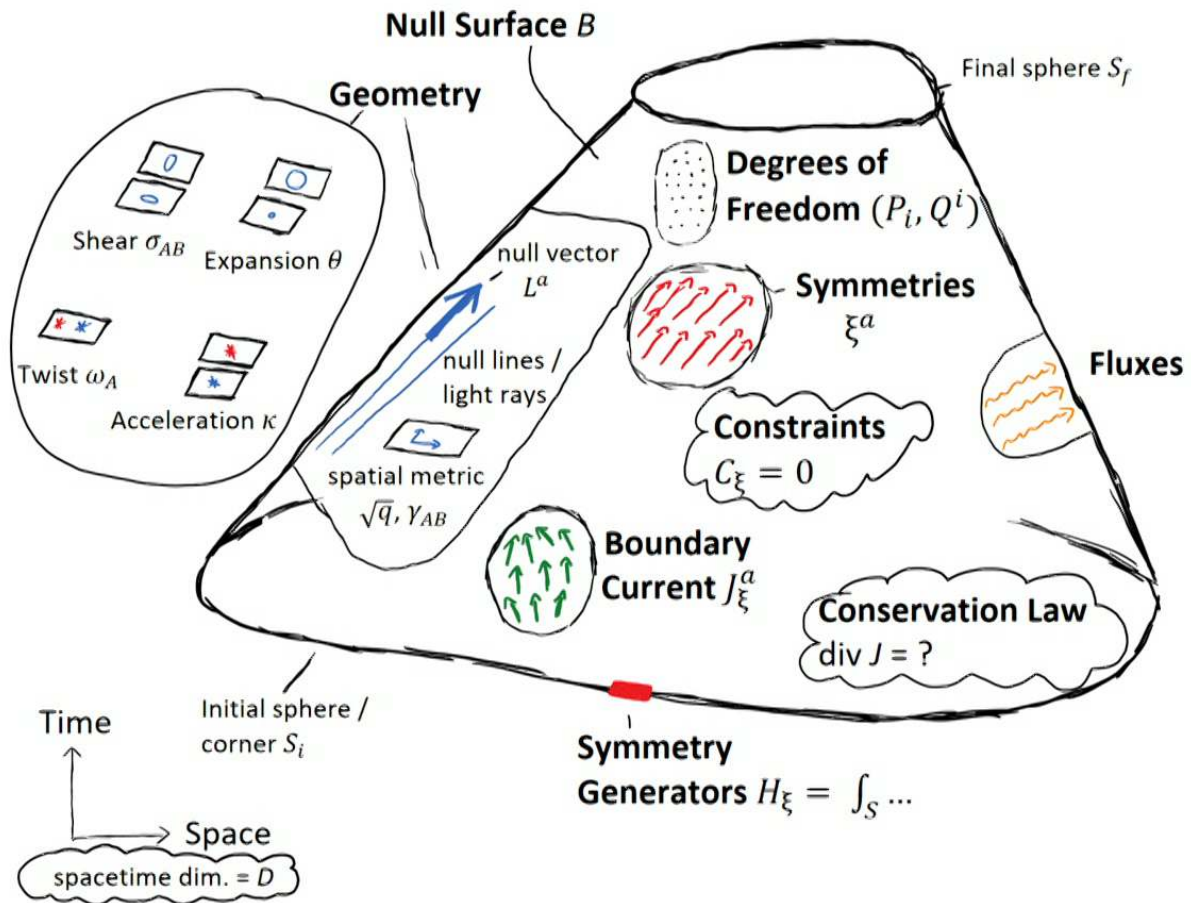
$$\Theta_{\partial B} = \int_{\partial B} dS \left(-\frac{1}{2} \delta h \sqrt{q} + \frac{1}{D-2} \delta(\sqrt{q}) \right)$$


normal size area

- › Remember: $\delta L = EoM + d\theta$, determines θ up to exact piece, Θ up to $\int_{\partial B}$
- › Most covariant choice: $\theta_W = (\nabla_b \delta g^{ab} - \nabla^a (g^{bc} \delta g_{bc})) \epsilon_a$
- › “Minimal” choice for null surface: Drop $\Theta_{\partial B}$.

DoF – The missing momentum

- › Aghapur, Jafari, Golshani 1808.07352
- › We have 5 pairs – where is the 6th?
- › What is the momentum conjugate to departure from nullness?
- › Answer: $\sim \kappa_K = L_a \nabla_K K^a$ transverse acceleration
- › Subtlety: if one wants brackets / Hamiltonians allowing departure from nullness:
Need to allow non-null configurations in the calculation of all momenta.



Conservation – Example

› Time translation symmetry for massless scalar field:

$$H = \frac{1}{2} \left(\int_{\Sigma} (\partial_t \phi)^2 + (\vec{\nabla} \phi)^2 \right), \text{ EoM } (-\partial_t^2 + \vec{\nabla}^2 \phi) = 0, \text{ then}$$
$$\frac{d}{dt} H = \int_{\partial \Sigma} \partial_n \phi \partial_t \phi$$

→ Energy conserved, up to flux $P\dot{Q}$ on the boundary.

Gravity: Consider spacetime region bounded by B.

› Energy → Energy-momentum, local expression on B

› Conservation $\frac{d}{dt}$ → covariant local conservation $\partial_a (\sqrt{q} J_{\xi}^a)$

› J_{ξ}^a is the **boundary current** on B, associated with arbitrary local symmetry $\xi^a \parallel B$

→ what is the boundary current? What are the fluxes?

Conservation – Symmetries

- › Diffeomorphism are local symmetries. How do they act?
- › We have introduced background structure (u coordinate)
→ partially breaks covariance

- › Define field space Lie derivative, on any tensor T :

$$\mathfrak{L}_\xi T = \frac{\delta T}{\delta g_{ab}} \mathcal{L}_\xi g_{ab}$$

- › Let $\xi^a = fL^a + v^a$, $v \parallel S$: the actions are

$$\mathfrak{L}_\xi \gamma_{AB} = f\sigma_{AB} + D_{\langle A} v_{B \rangle} \quad \mathfrak{L}_\xi \sqrt{q} = \sqrt{q}(f\theta + D_A v^A)$$

$$\mathfrak{L}_\xi L^a = \mathcal{L}_v L^a$$

$$\mathfrak{L}_\xi \mu = v^a \partial_a \mu + L^a \partial_a (f\mu + L^b \partial_b f)$$

$$\mathfrak{L}_\xi h = K_a \nabla_L \xi^a + L_a \nabla_K \xi^a \leftarrow \text{Odd one out!}$$

- transformations are not quite covariant, but under control → transfo of h involves transverse derivs of ξ

Constraints as Conservation Equations

Want: $\partial_a(\sqrt{q}J_\xi^a) = \text{flux} = P\mathfrak{L}_\xi Q$.

How: the constraints are $L_a G^a{}_b \xi^b = L_a T^a{}_b \xi^b$.

Use $L^a R_{ab} = (\nabla_a \nabla_b - \nabla_b \nabla_a)L^a$ and $g_{ab} = q_{ab} + L_a K_b + K_a L_b$ to find $G_{La}[q_{AB}, L^a, \sigma_{AB}, \theta, \kappa, \omega_A]$. Smear and i.b.p.

Result:

$$\partial_a(\sqrt{q}J_\xi^a) = \sqrt{q} \left(T_{L\xi} + \frac{1}{2} \sigma^{AB} (\mathfrak{L}_\xi \gamma_{AB}) - \omega_a (\mathfrak{L}_\xi L^a) + \mathfrak{L}_\xi \mu \right)$$

Recall: $\Theta_B = \int_B \text{dud}S \left(\frac{1}{2} \sqrt{q} \sigma^{AB} \delta \gamma_{AB} - \sqrt{q} \omega_A \delta L^A + \delta \mu \sqrt{q} \right)$

Flux = $P\mathfrak{L}_\xi Q$ ✓

So what is J_ξ^a ?

Conservation – The boundary current

So what is J_ξ^a ? Recall $\xi = fL + v$:

mom. for δL

$$J_v^a = L^a(\underbrace{v^b \omega_b}_{\substack{\text{momentum aspect} \\ \uparrow}}}) + \sigma^a_b v^b$$

momentum aspect

spatial
momentum current

mom. for \sqrt{q}

$$J_{fL}^a = L^a(\underbrace{f(\mu - \theta) - L^b \partial_b f}_{\substack{\text{energy aspect} \\ \downarrow}}})$$

energy aspect

no spatial energy current!

Conservation – Corner Phase Space II

› Recall: $\Theta_W = \Theta_B + \Theta_{\partial B}$

› We had: $\partial_a(\sqrt{q}J_\xi^a) = I_\xi\Theta_B + \int T_{L\xi}$, no $\Theta_{\partial B}$!

› $\Theta_{\partial B}$ contains h , transformation depends on $\nabla_K\xi$

› If we had kept $\Theta_{\partial B}$, we would have found instead of J_ξ :

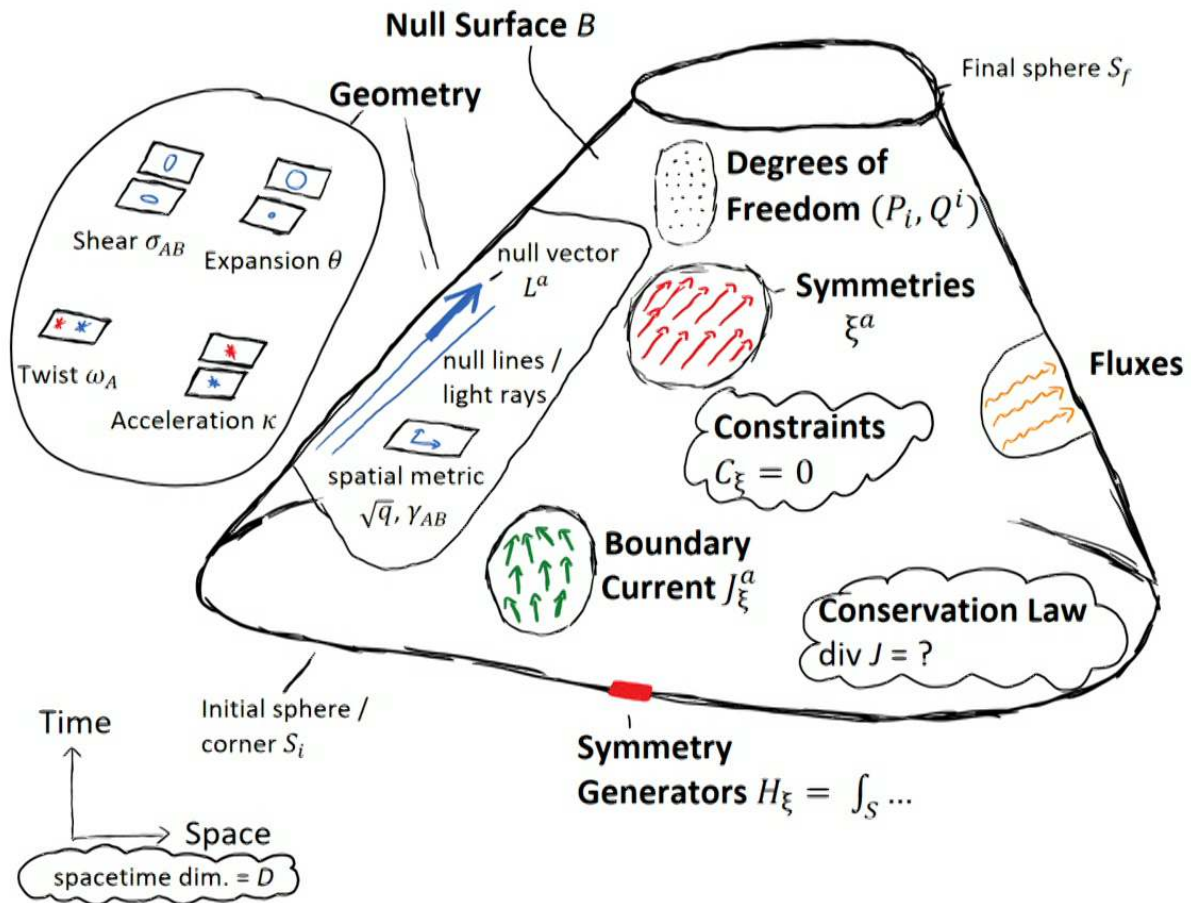
Komar charge $K_\xi \sim L_a\nabla_K\xi^a - K_a\nabla_L\xi^a$, depends on $\nabla_K\xi$

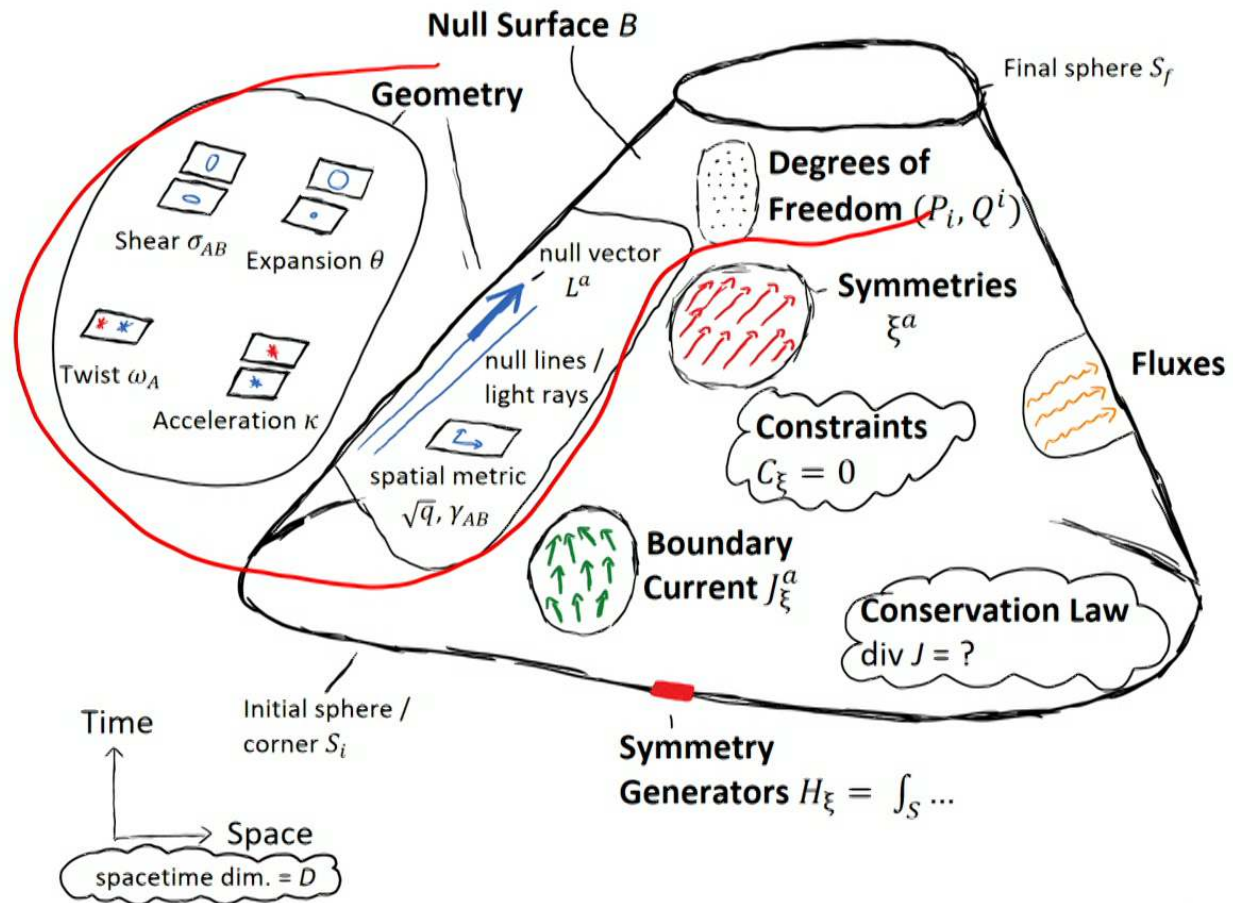
➔ Constraints suggest using Θ_B . By dropping $\Theta_{\partial B}$, we have removed the “extension dependence” from J_ξ

➔ Possible criterion for fixing corner phase space: “extension independence” of transformations of var’s

Call Θ_B the **intrinsic symplectic potential**.

(Almost) covariant under diffeos of B





Hamiltonians

- › But what are symmetries generated by?
- › Symplectic form $\Omega_B = \delta \wedge \Theta_B$.
 $h_\xi = \Omega(\delta g, \mathcal{L}_\xi g)$. ξ is pure gauge iff $h_\xi = 0$ on-shell,
Hamiltonian symmetry iff $h_\xi = \delta(H_\xi)$
- › Choose $\xi = v + fL$, $v \parallel S$.
Metric dependent – “tracks null lines”
- › Result: Hamiltonian lives on the corner, u-component of boundary current is the integrable bit of the Hamiltonian:

$$h_\xi = \delta \left(\int_{\partial B} \sqrt{q} J_\xi^u \right) - \int_{\partial B} dS \sqrt{q} \left(\frac{1}{2} f \sigma^{AB} \delta \gamma_{AB} + \delta(\mu + L(f)) \right)$$

$$J_\xi^u = f(\mu - \theta) + L^b \partial_b f + v^b \omega_b$$

- › Hamiltonian depends on corner phase space!

Hamiltonians: which symmetries are hamiltonian?

$$h_\xi = \delta \left(\int_{\partial B} \sqrt{q} J_\xi^u \right) - \int_{\partial B} dS \sqrt{q} \left(\frac{1}{2} f \sigma^{AB} \delta \gamma_{AB} + \delta(\mu + L(f)) \right)$$

$$J_\xi^u = f(\mu - \theta) + L^b \partial_b f + v^b \omega_b$$

No δL^a

→ take symmetries that don't move the corner.

$$\rightarrow \text{choose } f \stackrel{\partial B}{=} 0 \rightarrow H_\xi = \int_{\partial B} \sqrt{q} (v^b \omega_b - L^b \partial_b f) .$$

→ fix geometry of corner (\sqrt{q}, γ_{AB})

$$\rightarrow H_\xi = \int_{\partial B} dS \sqrt{q} (-f\theta + v^b \omega_b)$$

→ fix momenta

$$\rightarrow H_\xi = \int_{\partial B} dS \sqrt{q} (f(\mu - \theta) + L(f) + v^b \omega_b)$$

→ Something in between

Note: ~ 2 copies of “supertranslations” → 1807.11499

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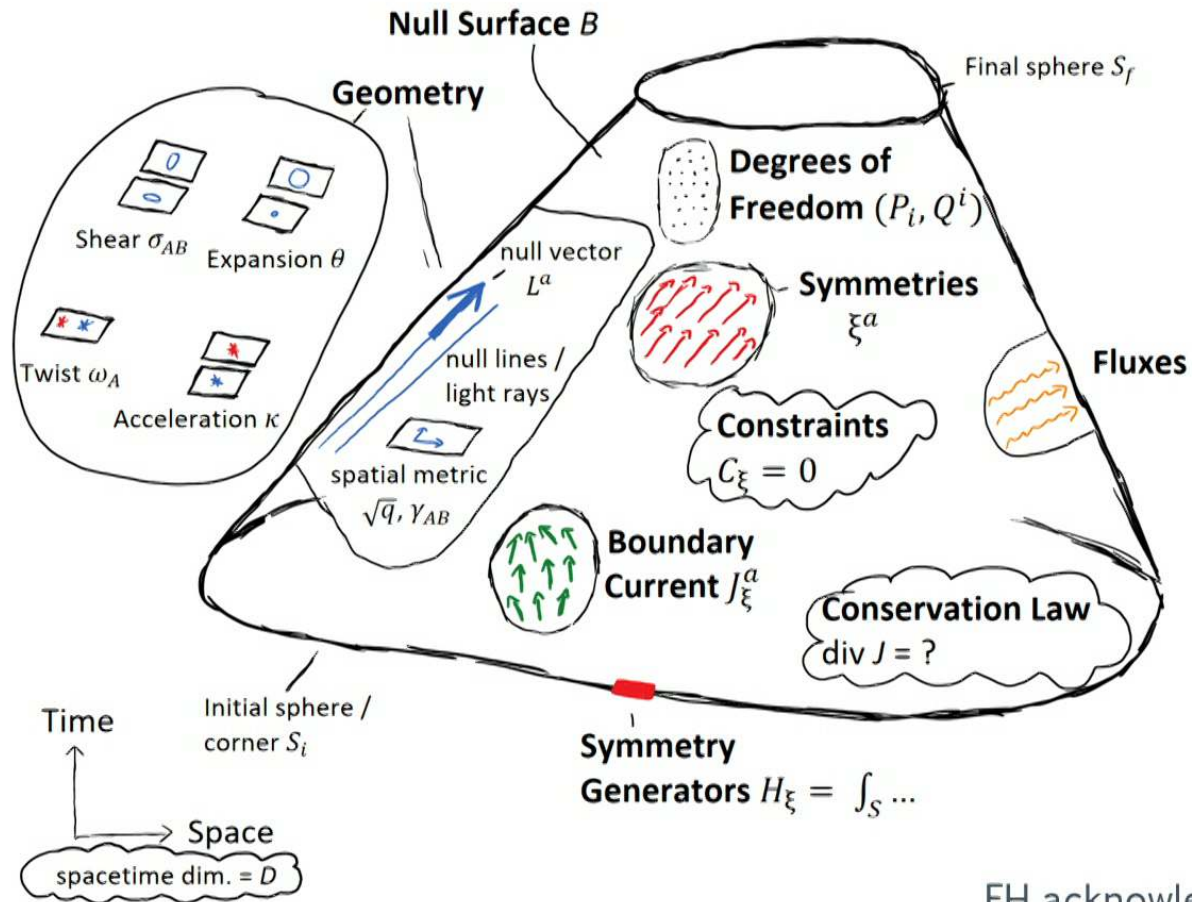
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Recap

- › Expressed DoF (including gauge DoF) of GR on a null surface in terms of null intrinsic & extrinsic geometry
- › Interpreted smeared tangential constraints as conservation law of a (induced) **boundary current**, with fluxes $P\mathcal{L}_\xi Q$
- › By a choice of corner phase space, removed dependence of boundary current and Hamiltonians on extension of symmetry parameter
- › Analyzed Hamiltonians & conditions for “Hamiltonicity” for tangential symmetries

Thanks!



FH acknowledges a Vanier CGS.