Title: Computational Physics - Lecture 7

Speakers: Erik Schnetter

Collection: Computational Physics (Schnetter)

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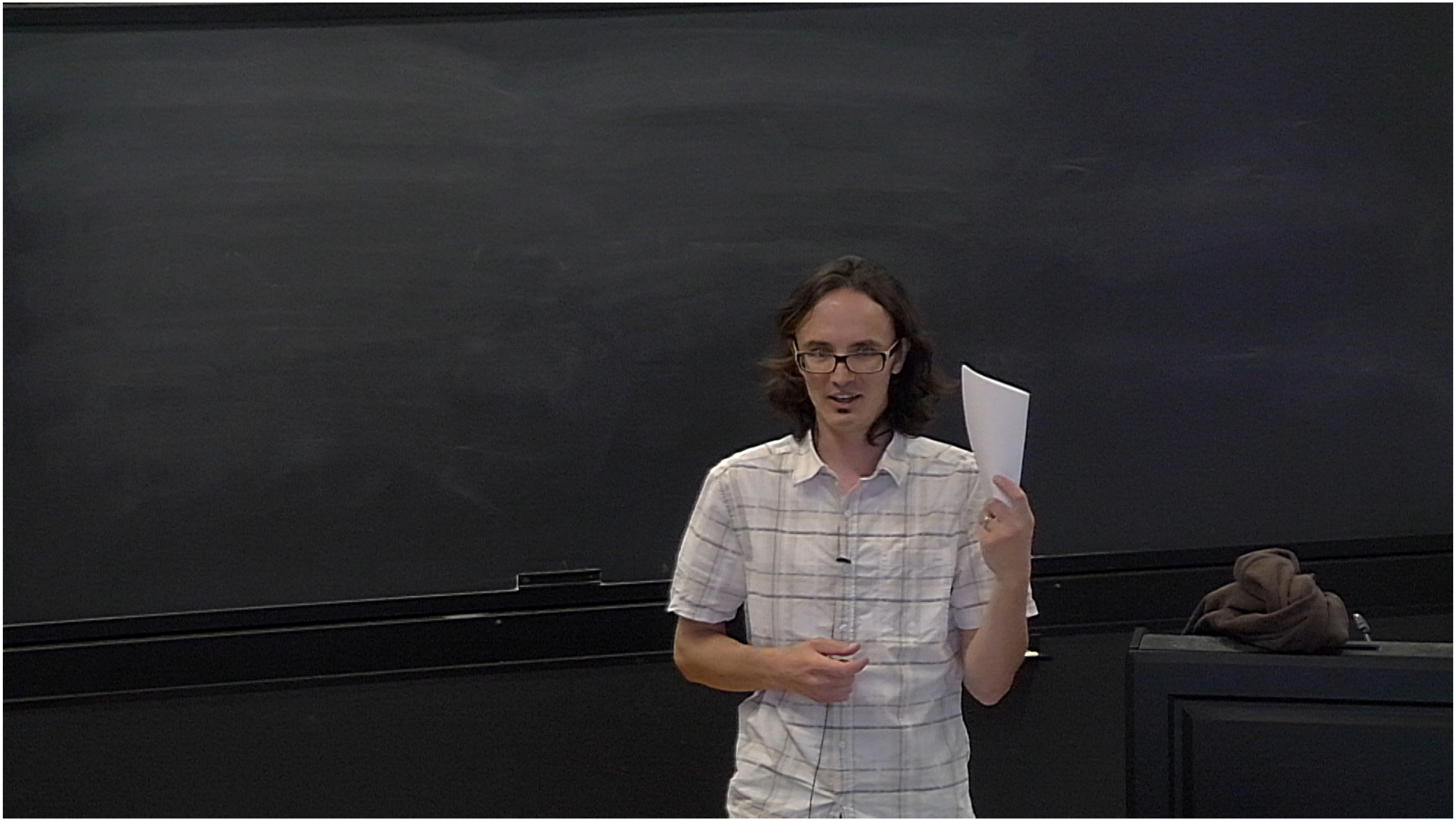
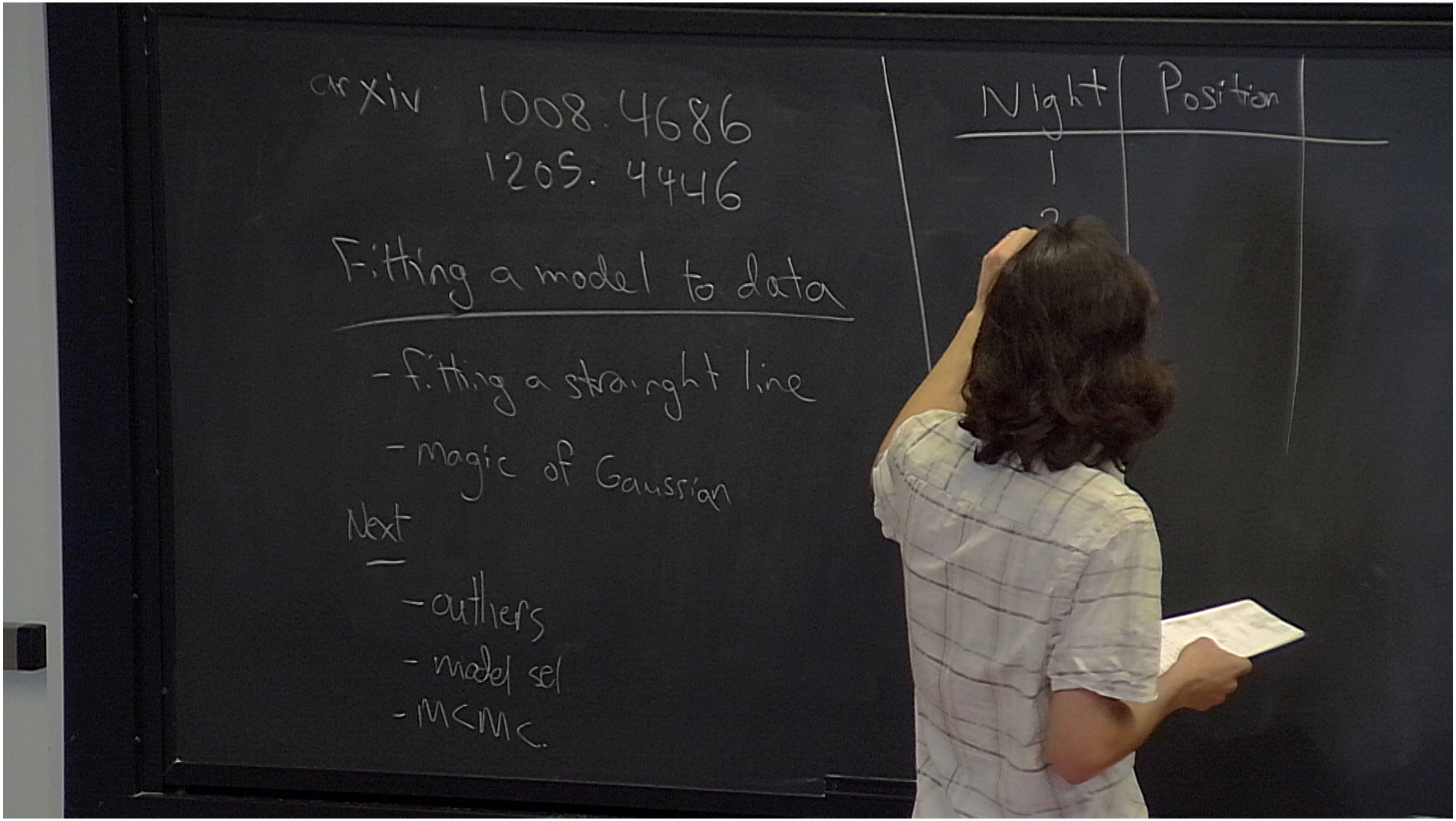
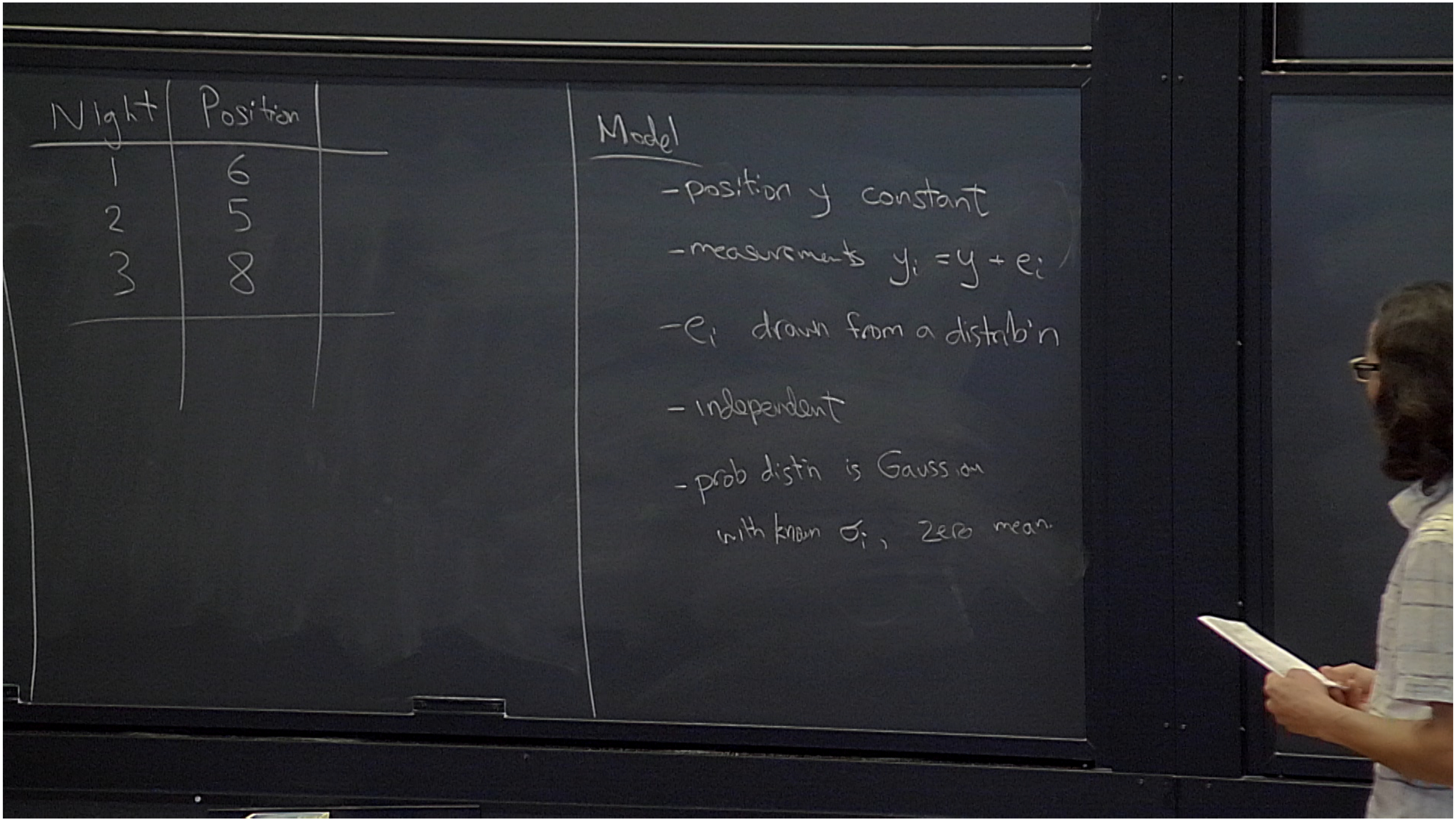



arxiv 1008.4686
1205.4446
Fitting a model to data
- fitting a straight line
- magic of Gaussian
Next
- outliers
- model sel
- MCMC
Night
Position
1
2

Night	Position
1	6
2	5
3	8

Model

- position y constant
- measurements $y_i = y + e_i$
- e_i drawn from a distrib'n
- independent


Night | Position
1 | 6
2 | 5
3 | 8
Model
- position y constant
- measurements $y_i = y + e_i$
- e_i drawn from a distrib'n
- independent
- prob dist'n is Gaussian
with known σ_i, zero mean

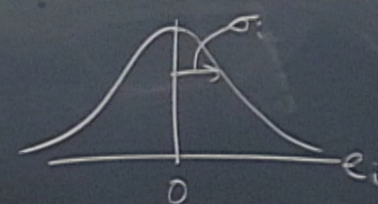
data

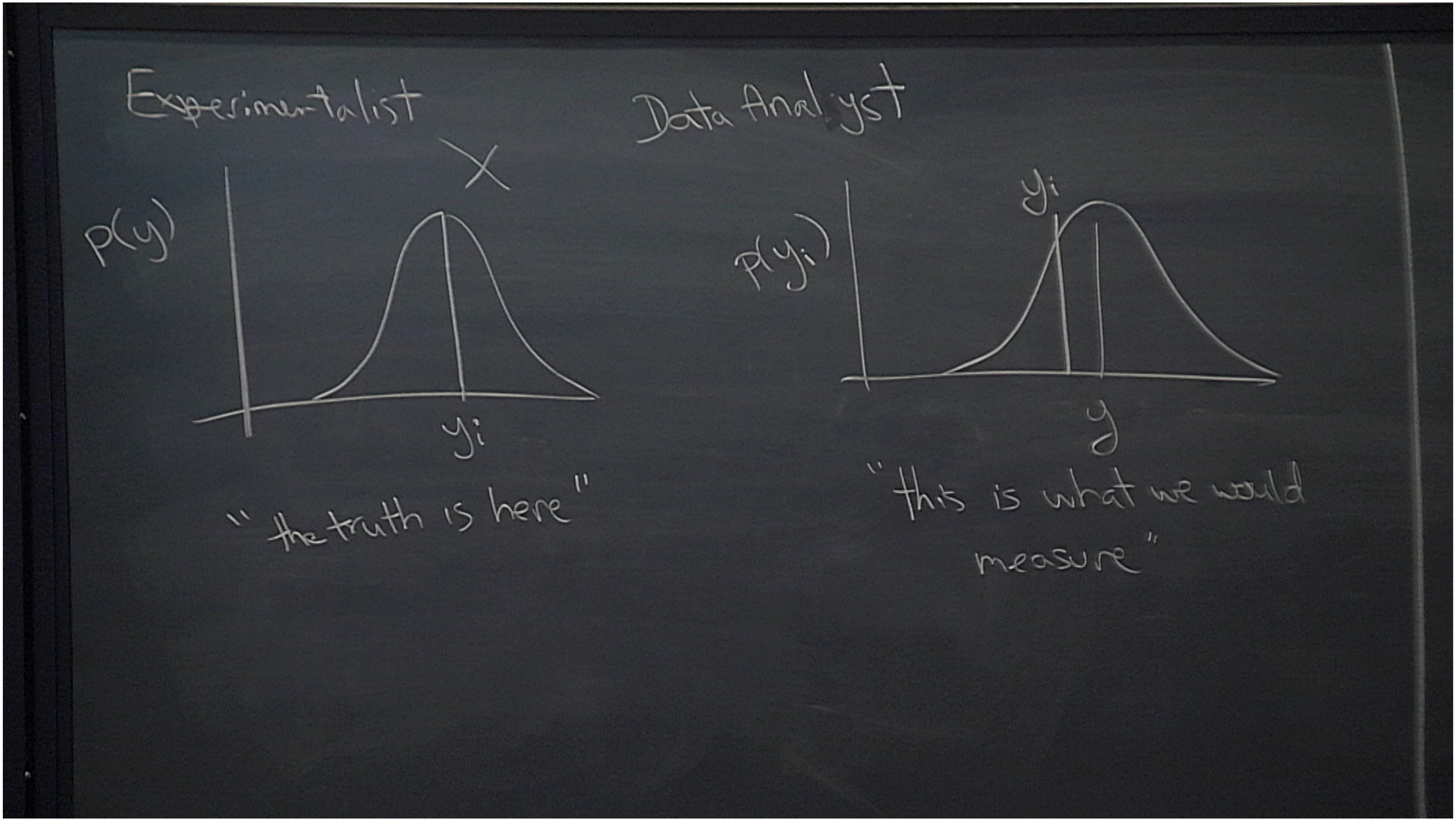
line

Night	Position	Uncertainty
1	6	1
2	5	1
3	8	1

Model

- position y constant
- measurements $y_i = y + e_i$
- e_i drawn from a distrib'n
- independent
- prob distn is Gaussian with known σ_i, zero mean.

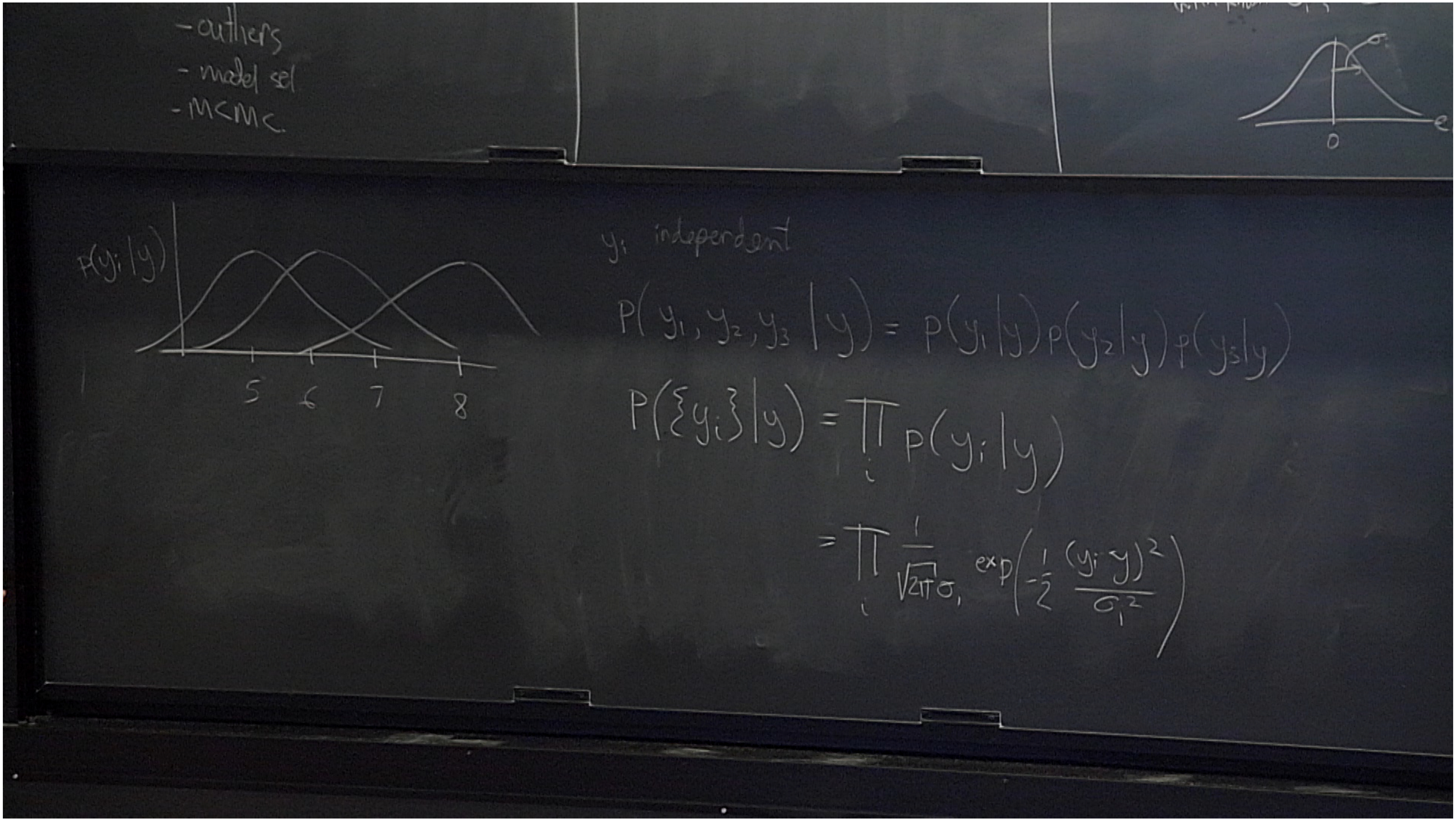



Experimentalist
Data Analyst
x
p(y)
yi
"the truth is here"
yi
p(yi)
y
"this is what we would measure"

$$p(y_i \mid y) = \frac{1}{\sqrt{2\pi}\,\sigma_i} \exp\left(-\frac{1}{2}\frac{(y_i - y)^2}{\sigma_i^2}\right)$$

$$N(\mu, \sigma^2)$$

$$e_i \sim N(0, \sigma_i^2)$$


- outliers
- model sel
- M<Mc.
$P(y_i|y)$
5 6 7 8
σ_i
0
e
y_i independent
$P(y_1, y_2, y_3 | y) = p(y_1|y)\,p(y_2|y)\,p(y_3|y)$
$P(\{y_i\}|y) = \prod_i p(y_i|y)$
$= \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{1}{2}\frac{(y_i-y)^2}{\sigma_i^2}\right)$

y_i independent

$$P(y_1, y_2, y_3 \mid y) = P(y_1 \mid y)\,P(y_2 \mid y)\,P(y_3 \mid y)$$

$$\underbrace{P(\{y_i\} \mid y)}_{\mathcal{L}} = \prod_i P(y_i \mid y)$$

$$= \prod_i \frac{1}{\sqrt{2\pi}\,\sigma_i} \exp\left(-\frac{1}{2}\frac{(y_i - y)^2}{\sigma_i^2}\right)$$

$$\frac{d}{dy}\log(\mathcal{L}) = -\sum_i \frac{y_i - y}{\sigma_i^2}$$

$$0 = \hat{y}\sum_i \frac{1}{\sigma_i^2} - \sum_i \frac{y_i}{\sigma_i^2}$$

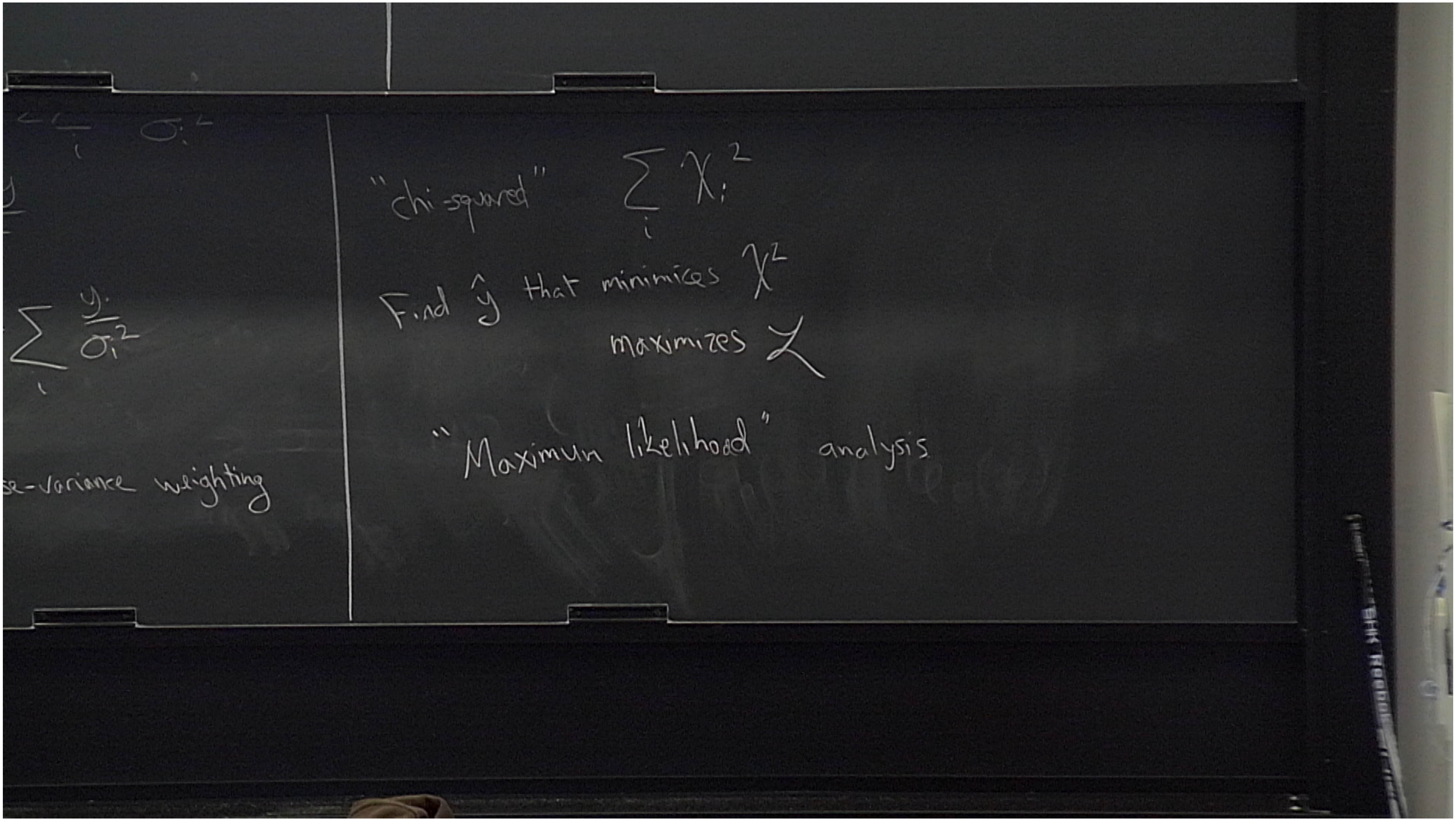
$$\boxed{\hat{y} = \frac{\sum_i \frac{y_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}}}$$

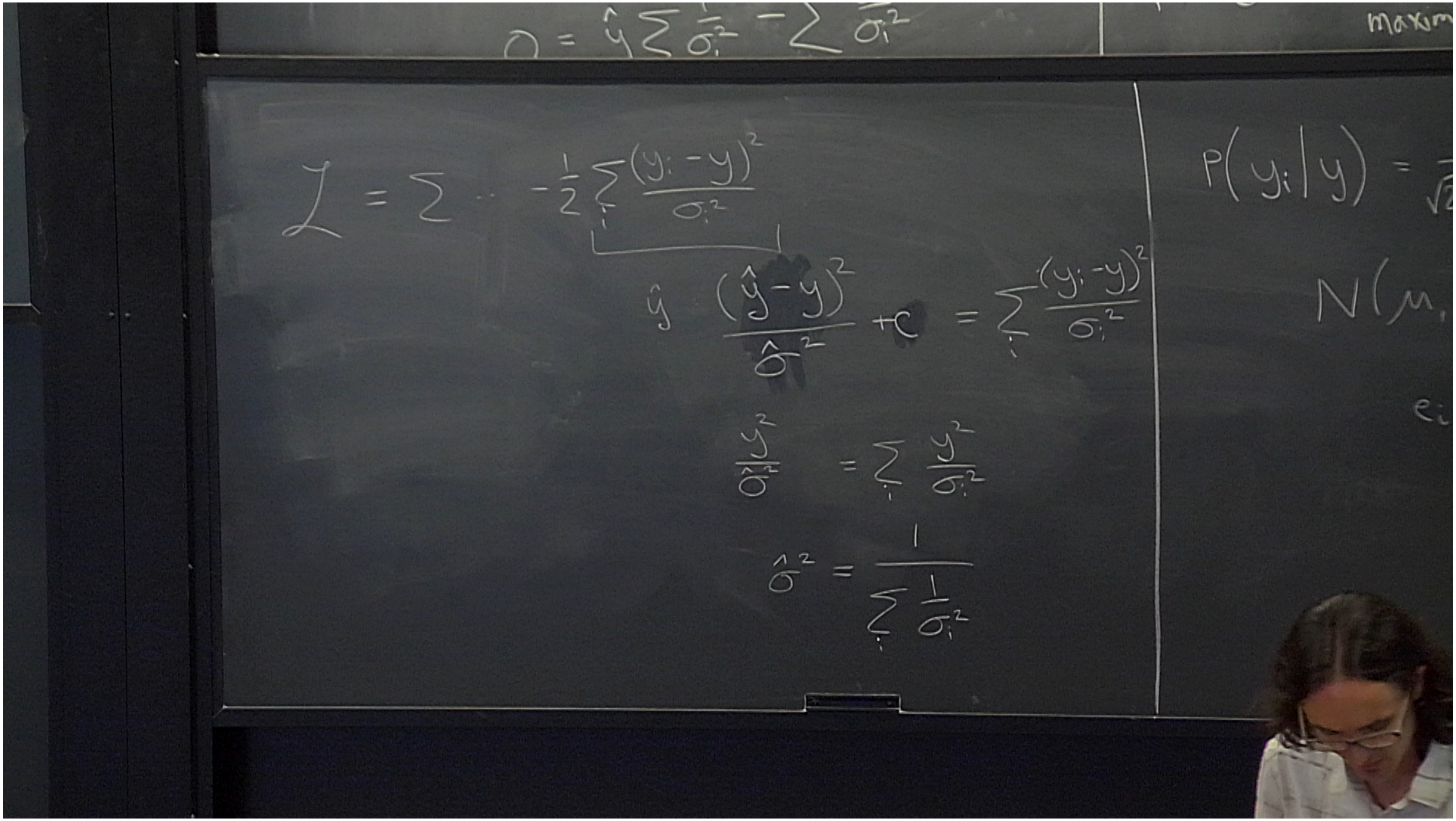
inverse-variance weighting

"chi-squared"

Find $\hat{y}$ tha

"Maxim


$\sum_i \frac{y_i}{\sigma_i^2}$
se-variance weighting
"chi-squared" $\sum_i \chi_i^2$
Find $\hat{y}$ that minimizes χ^2
maximizes $\mathcal{L}$
"Maximum likelihood" analysis



maxim
$\mathcal{L} = \sum \cdots -\frac{1}{2}\sum_i \frac{(y_i - y)^2}{\sigma_i^2}$
$\hat{y} \quad \frac{(\hat{y} - y)^2}{\hat{\sigma}^2} + c = \sum_i \frac{(y_i - y)^2}{\sigma_i^2}$
$\frac{y^2}{\hat{\sigma}^2} = \sum_i \frac{y^2}{\sigma_i^2}$
$\hat{\sigma}^2 = \frac{1}{\sum_i \frac{1}{\sigma_i^2}}$
$P(y_i | y) =$
$N(\mu,$
e_i

arxiv 1008.4686
1205.4446

Fitting a model to data

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Next

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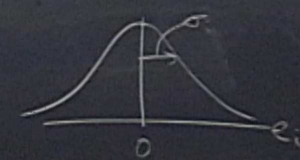
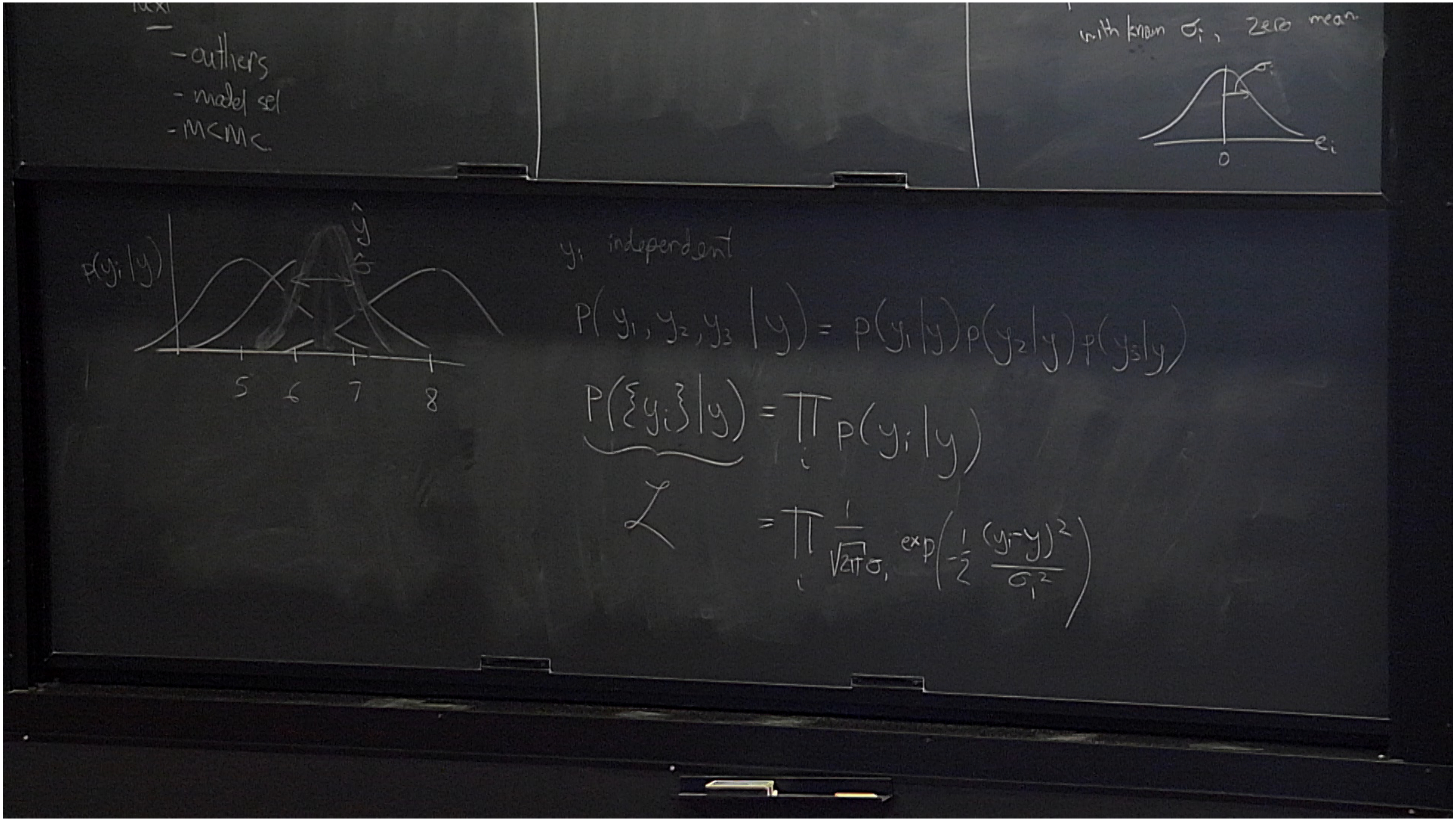
- MCMC

Night	Position	Uncertainty
1	6	1
2	5	1
3	8	1
ML	$6\frac{1}{3}$	$\frac{1}{\sqrt{3}} \simeq 0.6$

Model

- position y constant
- measurements $y_i = y + e_i$
- e_i drawn from a distrib'n
- independent
- prob distn is Gaussian with known σ_i, zero mean





- outliers
- model sel
- MCMC
with known σi, zero mean
σi
0
ei
p(yi|y)
ŷ
σ
5 6 7 8
yi independent
P(y1, y2, y3 | y) = p(y1|y) p(y2|y) p(y3|y)
P({yi}|y) = ∏i p(yi|y)
L
= ∏i 1/(√(2π) σi) exp(−1/2 (yi−y)²/σi²)

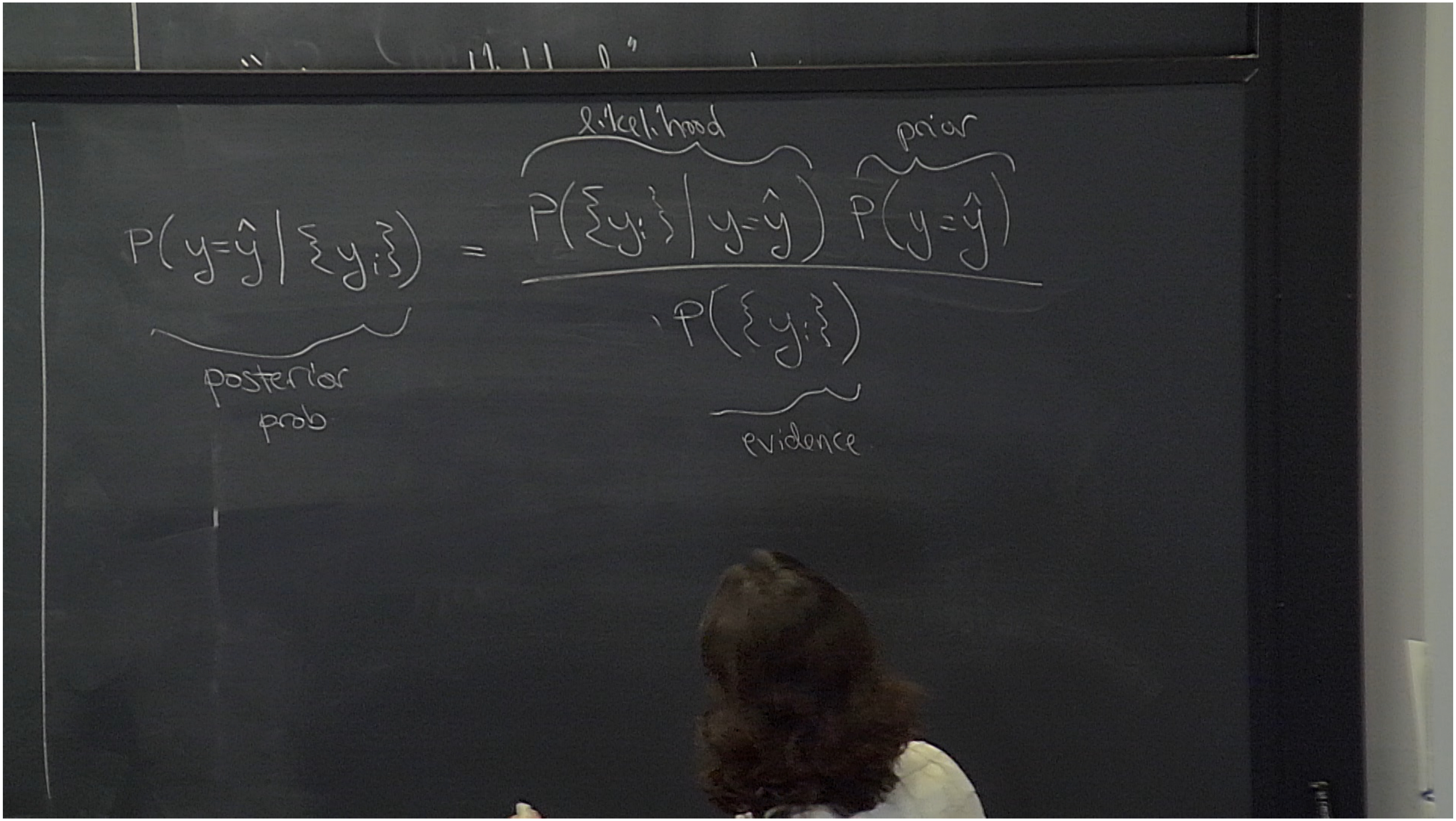
Frequentist | Bayesian

$$P(A,B) = P(A)\,P(B|A) = P(B)\,P(A|B)$$

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

B: y has the value $\hat{y}$

A: we measure $\{y_i\}$


likelihood
prior
$P(y=\hat{y} \mid \{y_i\}) = \frac{P(\{y_i\} \mid y=\hat{y})\, P(y=\hat{y})}{P(\{y_i\})}$
posterior prob
evidence

$$\chi_i = \frac{y_i - y}{\sigma_i}$$

"chi-squared" = $\sum_i \chi_i^2$

Find $\hat{y}$ that minimizes χ^2

maximizes $\mathcal{L}$

"Maximum likelihood" analysis

Evidence

$$p(\{y_i\}) = \underbrace{\int p(\{y_i\}|y)\, p(y)\, dy}_{\text{all the different ways you could get } \{y_i\}}$$