

Title: A triangular-lattice spin-valley Hubbard model in the ABC trilayer graphene/h-BN moire system

Date: Oct 30, 2018 03:30 PM

URL: <http://pirsa.org/18100044>

Abstract: <p>This year there appear several amazing experiments in the graphene moire superlattices. In this talk I will focus on the ABC trilayer graphene/h-BN system. Mott-like insulators at $1/4$ and $1/2$ of the valence band have already been reported by Feng Wang's group at Berkeley. The sample is dual gated on top and bottom with voltage V_t and V_b . V_t+V_b controls the density of electrons. Interestingly we find that the displacement field $D=V_t-V_b$ can control both the topology and the bandwidth of the valence band. For one sign of D (for example $D>0$), there are two narrow Chern bands with opposite Chern numbers $C=3,-3$ for the two valleys. For $D<0$, the bands of the two valleys are trivial and have localized Wannier orbitals on a triangular lattice. As a result, the physics is governed by a spin-valley Hubbard model on a triangular lattice. This talk focuses on the $D<0$ side and consists of two parts: (1) I will provide the details of this spin-valley Hubbard model and discuss some subtleties special to the moire systems. (2) In the second part I want to show some of our theoretical attempts on this Hubbard model. First I will show that this system is a perfect platform for studying metal-insulating transition. I will provide a theory of continuous Mott transition between a Fermi liquid and a spinon Fermi surface Mott insulator. Second I will discuss some possible metallic phases upon doping away from the Mott insulator. Unlike the familiar spin $1/2$ case, the spin-valley Hubbard model may not be in a conventional Fermi liquid phase even in the over-doped region. I will provide some candidates of possible unconventional metals based on a six-flavor slave boson theory for the hole doped side.</p>

<p> </p>

<p>References:</p>

<p>Feng Wang et.al. arxiv: 1803.01985</p>

<p>Ya-Hui Zhang, Dan Mao, Yuan Cao, Pablo Jarillo-Herrero and T. Senthil. arXiv:1805.08232</p>

<p>Ya-Hui Zhang and T. Senthil, arxiv: 1809.05110</p>

<p>Ya-Hui Zhang and T. Senthil, ongoing work</p>



Bridging Hubbard Model Physics and Quantum Hall Physics in trilayer Graphene moire superlattice

**Ya-Hui Zhang
(MIT)**

References:

Y-H. Zhang, D. Mao, C. Yuan, P. Jarillo-Herrero, T. Senthil arxiv:1805.08232



Y-H. Zhang and T. Senthil, arxiv: 1809.05110

Collaborators at MIT

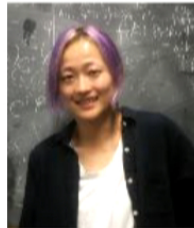


T. Senthil

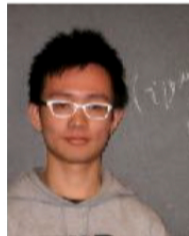


Cécile Repellin

Collaborators in Bilayer Graphene/ Bilayer Graphene System:



Dan Mao



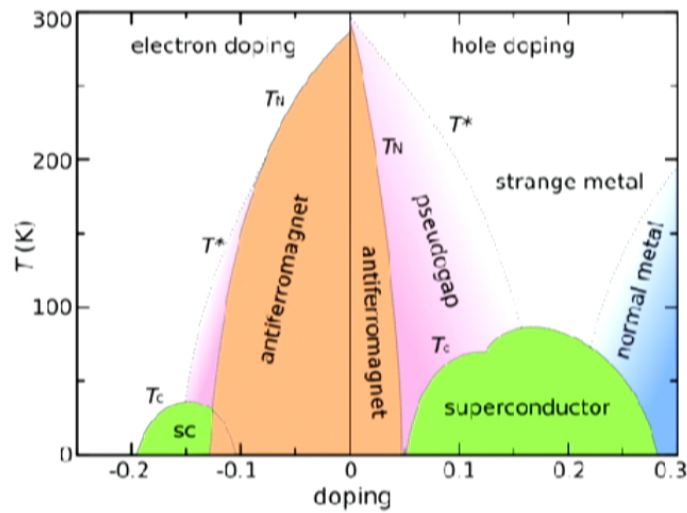
Yuan Cao



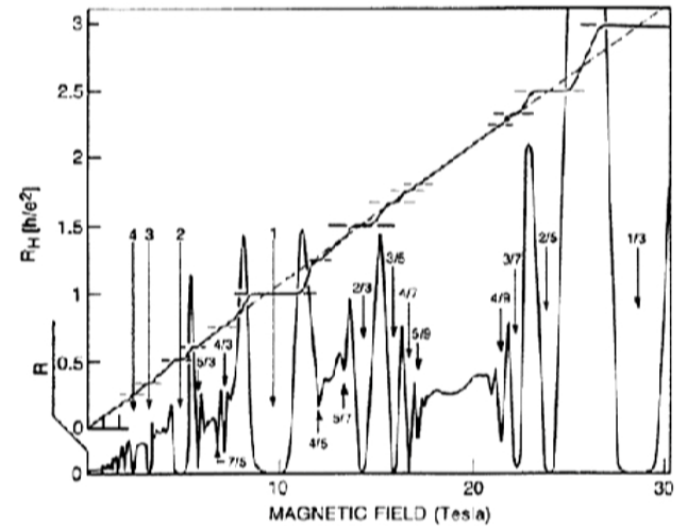
Pablo Jarillo-Herrero

Motivation: Cuprate and Quantum Hall System

30 years old problems:



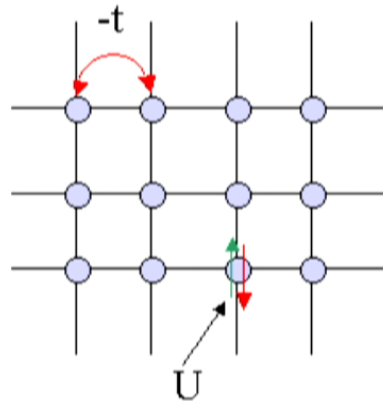
Cuprate



Quantum Hall

Cuprate

Square Lattice Hubbard Model



No theoretical agreement.

Is the physics governed by doped Mott Insulator ?

Are these phases universal to Hubbard model?

We need other realizations of doped Mott insulator.

Exotic Phases:

high-T_c superconductor

pseudogap metal

strange metal

Quantum Hall states

Theory is well established.



Need more experimental controls of anyons.

Can we trap anyon inside a vortex in a quantum Hall/ SC heterostructure?

Is FQHE at zero magnetic field possible?

Narrow Chern Band

Gate tunable density.



Need Strongly Correlated Systems with high Controllability

Hard in conventional solid-state material.

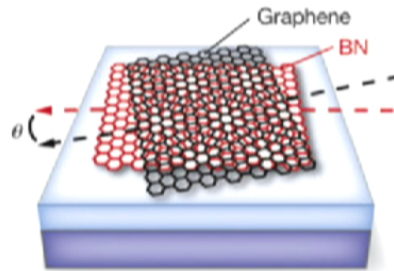
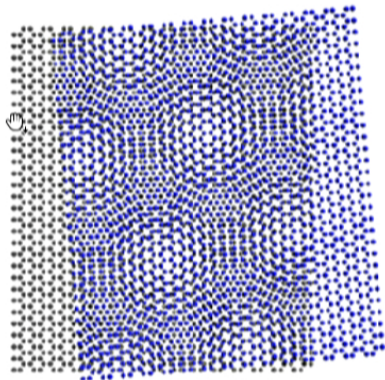
Cold atom: Temperature is too high (1000K now!)

Moire Superlattice is a perfect platform!

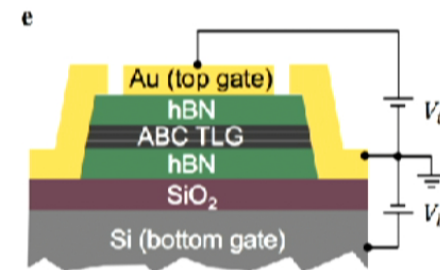
Bandwidth and Interaction: 100K.

Experiment regime: 100 mK; Low enough for superconductor.

ABC Trilayer Graphene on Hexagonal Boron Nitride



Feng Wang et.al. arxiv:1803.0198



D: a vertical electric field

Switch between two different physics:

Triangular Lattice Hubbard Model \longleftrightarrow **Nearly Flat** $C = \pm 3$ **Chern Bands**

D<0

D>0

“Cuprate” Physics

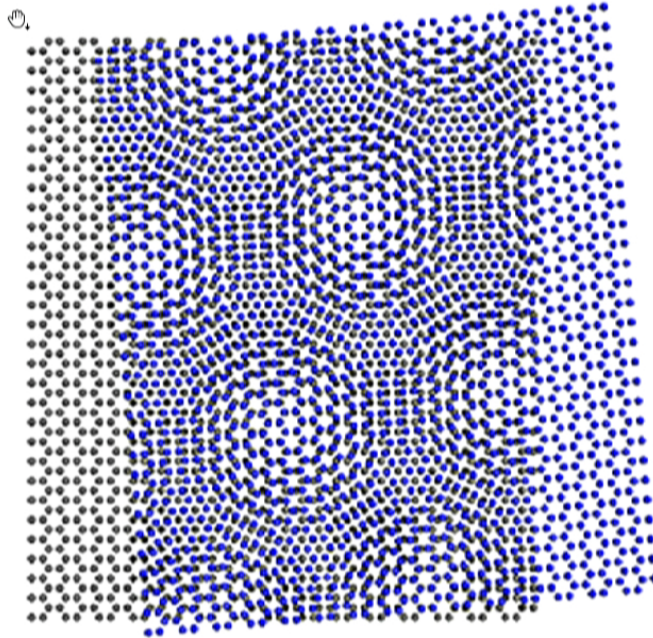
“Quantum Hall” Physics

Y-H. Zhang and T. Senthil, arxiv: 1809.05110

Y-H. Zhang, et.al and T. Senthil,
arxiv:1805.08232

Moire Superlattices

Explored for one decade



$$|e^{iG_1x} + e^{iG_2x}|^2 \sim e^{i(G_1-G_2)x} + \dots$$

Moire reciprocal vector: $\vec{G}_M = \vec{G}_1 - \vec{G}_2$

Almost identical lattice constant:

$$|G_1| \approx |G_2| \approx |G_0|$$

θ : twisted angle

$$\epsilon = \frac{||G_1| - |G_2||}{|G_0|} = \frac{|a_1 - a_2|}{a_0}$$

$$G_M \sim \sqrt{\epsilon^2 + \theta^2} G_0 \ll G_0$$

Typical value: $a_M \sim 60a_0 \sim 15 \text{ nm}$

Strongly correlated effects from moire superlattice: band folding!

Bandwidth decreases faster than interaction $U \sim 1/a_M$

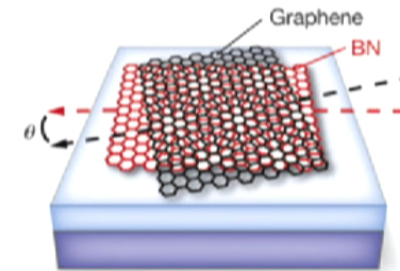
Graphene aligned with Hexagonal Boron Nitride

h-BN: band gap at order eV

$$G_M \sim \sqrt{\epsilon^2 + \theta^2} G_0$$

Zero twist angle:

$$\epsilon = \frac{a_1 - a_2}{a_1} = 1.7\% \quad a_M \approx 58a \text{ around } 15 \text{ nm.}$$

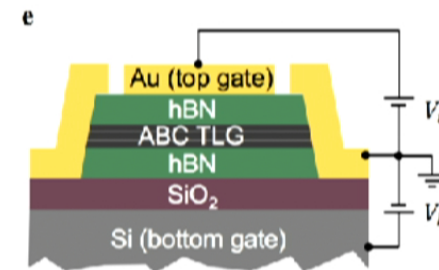


ABC Trilayer Graphene/h-BN (TG/h-BN):

$$\theta_1 = 0 \quad \theta_2 \text{ very large}$$

Effects of h-BN:

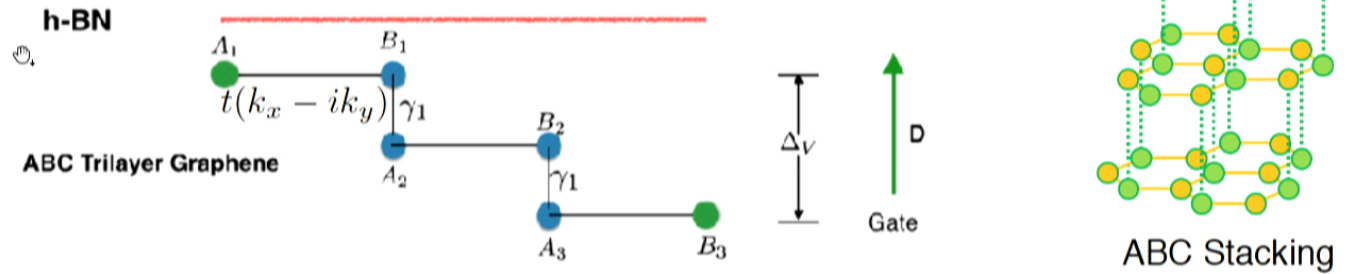
super-lattice potential for electrons in graphene.



Strongly correlated effects emerge!

Feng Wang et.al. arxiv:1803.0198

Model Single ABC Trilayer Graphene



$\gamma_1 \approx 400$ meV. Relevant Energy scale in this talk: 30 meV.

Two valleys at K and K'. Three Dirac cones in the decoupling limit.

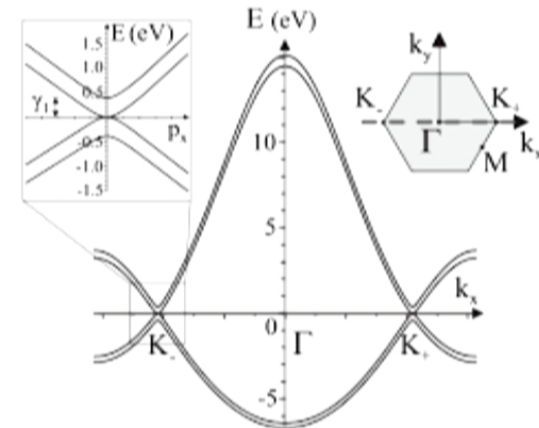
At neutrality, only two bands at low energy:

$$h_+(\mathbf{k}) = \begin{pmatrix} \frac{\Delta_V}{2} & \frac{t^3}{\gamma_1^2}(k_x - ik_y)^3 + 2\frac{t\gamma_3}{\gamma_1}|\mathbf{k}|^2 \\ \frac{t^3}{\gamma_1^2}(k_x + ik_y)^3 + 2\frac{t\gamma_3}{\gamma_1}|\mathbf{k}|^2 & -\frac{\Delta_V}{2} \end{pmatrix}$$

$\psi = (c_{A_1}, c_{B_n})$ Δ_V : Voltage between top and bottom

Time reversal: $h_-(\mathbf{k}) = h_+^*(-\mathbf{k})$

Mass term of the cubic touching is gate tunable!



A.H. MacDonald, arxiv: 0806. 2792

Moire Superlattice Potential

For one valley:

$$H_M = \sum_{\mathbf{G}_j} \sum_{\mathbf{k}} \psi^\dagger(\mathbf{k} + \mathbf{G}_j) \begin{pmatrix} V_0 e^{i\theta_j} & 0 \\ 0 & 0 \end{pmatrix} \psi(\mathbf{k}) + h.c.$$

DFT estimation: $V_0 \sim 10 meV$ Jeil Jung, et.al. , A.H. Macdonald, Phys. Rev . B 89, 205414 (2014)

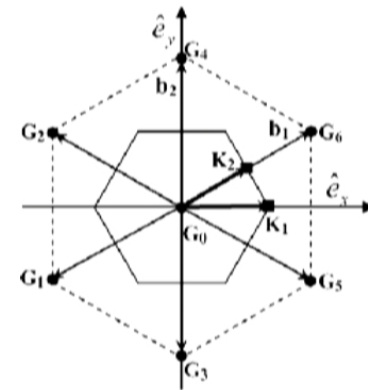
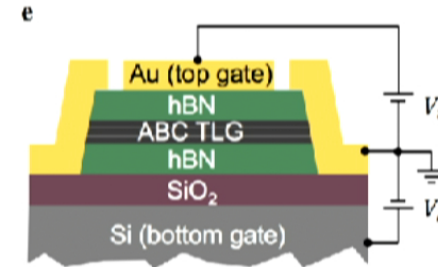
Free Electron Approximation.

G_j : Reciprocal vector of a Hexagon Moire Brillouin Zone (MBZ)

No valley scattering: $|G_j| = |G_M| \ll |K - K'|$

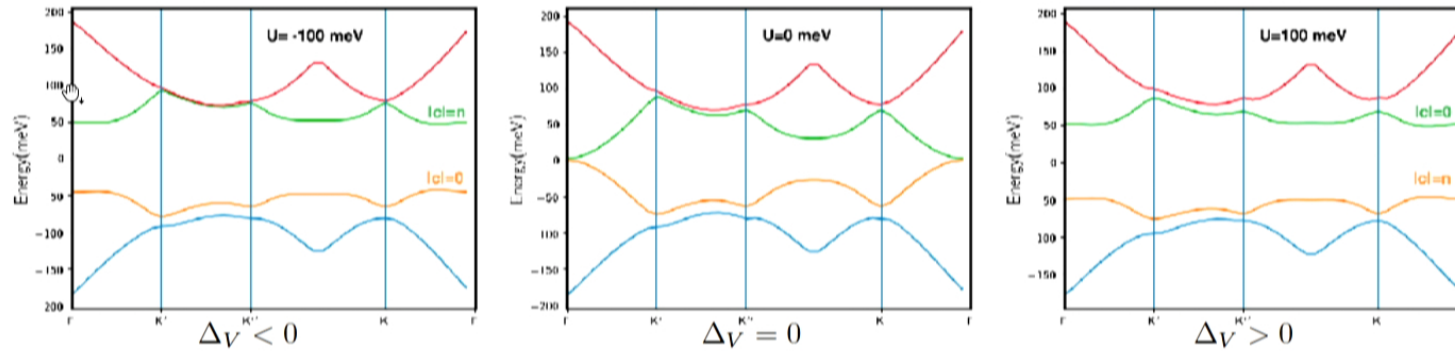
We move band crossing point of two valleys to Γ point of MBZ.

Valley can be viewed as a pseudo-spin.



$$|G_M| \sim \frac{1}{60} |G_{original}|$$

Bilayer Graphene/h-BN band dispersion for one valley:



Trilayer Graphene/h-BN is similar.

Y-H. Zhang, et.al and T. Senthil,
arxiv:1805.08232

Higher Angular-Momentum Band Inversion:

$$\text{For valley } +: \quad C(\Delta_V > 0) - C(\Delta_V < 0) = n$$

For TG/h-BN:

$\Delta_V > 0$ Narrow $C = \pm 3$ Chern Bands

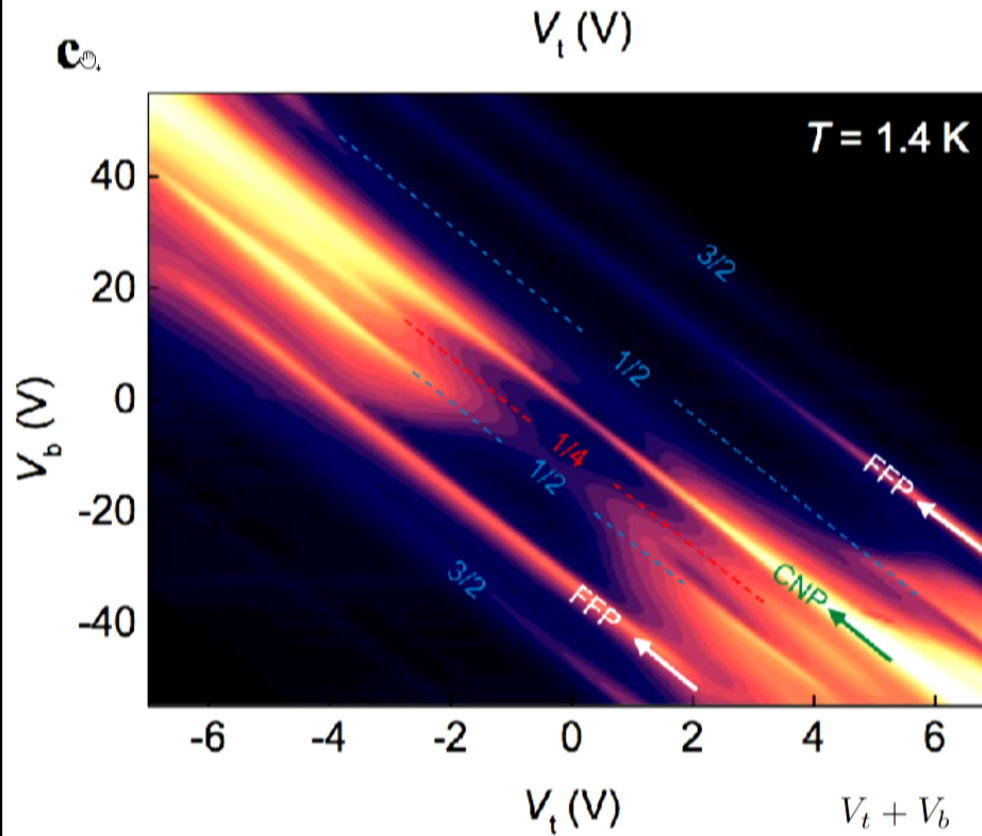
$\Delta_V < 0$ Narrow Trivial Bands

$$h_+(\mathbf{k}) = \begin{pmatrix} \frac{\Delta_V}{2} & \frac{t^3}{\gamma_1^2}(k_x - ik_y)^3 + 2\frac{t\gamma_3}{\gamma_1}|\mathbf{k}|^2 \\ \frac{t^3}{\gamma_1^2}(k_x + ik_y)^3 + 2\frac{t\gamma_3}{\gamma_1}|\mathbf{k}|^2 & -\frac{\Delta_V}{2} \end{pmatrix}$$

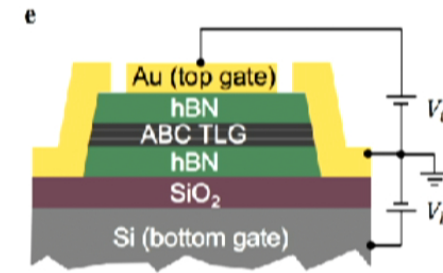
We focus on the valence band: four species + up, + down, - up, -down.

Experimental Results for TG/h-BN

Correlated Insulators has been found at $\nu_T = 1, 2$



Feng Wang et.al. arxiv:1803.0198



Valence band: four bands; spin, valley

number of holes per unit cell

$$\nu_T = 4\nu$$

Color map: Resistance

$V_t + V_b$ Controls the filling.

$\Delta_V = V_t - V_b$ Controls the bandwidth and topology.

The other system: twisted bilayer graphene

☞ Monolayer graphene on monolayer graphene
Correlated insulator and superconductor

Cao, Fatemi et al, Nature **556**, 80 (2018)

Cao, Fatemi et al, Nature **556**, 43 (2018)

However, no simple lattice model so far!

Yankowitz, Chen et.al. arxiv: 1808.07865

A simple Hubbard model seems not very likely.

Advantage of TG/h-BN

Gate Tunable Bandwidth and Topology

One band per spin valley

Simple model

The other system: twisted bilayer graphene

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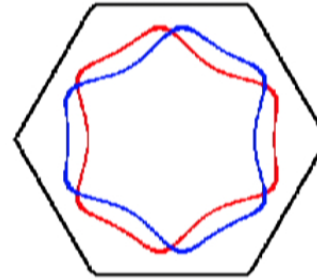
Simple model

Symmetry

Four Flavors of valence band: + up, + down, - up, - down.



Both kinetic term and interaction break U(4).



Density Operator:

$$\rho_{a,\sigma}(\mathbf{q}) = \sum_{\mathbf{k}} \lambda_a(\mathbf{k}, \mathbf{k} + \mathbf{q}) c_{a,\sigma}^\dagger(P(\mathbf{k} + \mathbf{q})) c_{a,\sigma}(\mathbf{k}) + \dots$$

Form factor: contains information of Berry connection.

Approximating Symmetry: $U(2)_+ \times U(2)_- \times T$

$$c_+^\dagger c_- V(2K) c_-^\dagger c_+ \quad \text{leads to:} \quad -J_H \mathbf{S}_+ \cdot \mathbf{S}_-$$

Good Symmetry: $U(1)_c \times U(1)_v \times SU(2)_s \times T$

Next Part



I. Trivial Side: Spin-Valley Hubbard Model

(1) Spin model for the strong Mott insulators.

(2) Metal-Insulator Transition

II. Chern Side: C=3,-3 Narrow Bands

(1) Flat band limit: Chern Insulator with $\sigma_{xy} = 3 \frac{e^2}{h}$

(2) Fractional Filling at Flat Band limit

Trivial Side

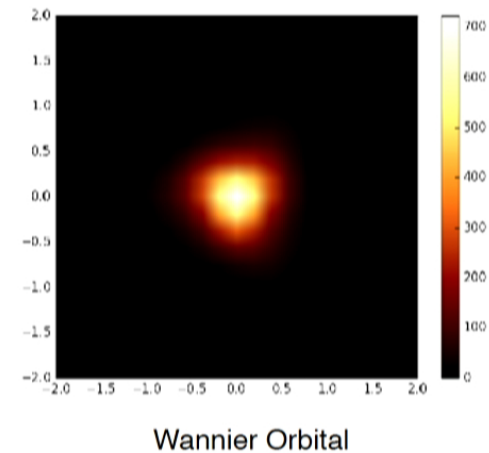
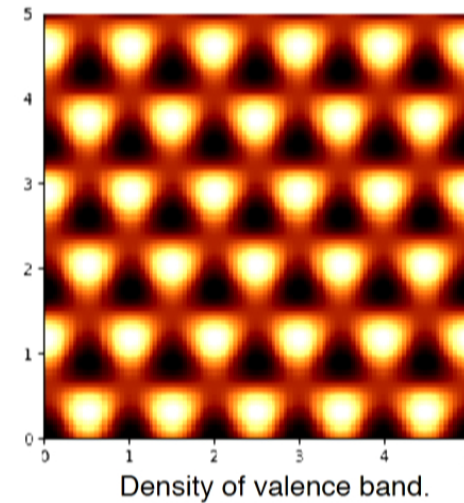
Wannier Orbital

☞ Bright points form Triangular Lattice.

Each valley: Wannier orbital at bright point.

$$|w(\mathbf{x}_0)\rangle = \frac{1}{\sqrt{N}} \sum_k e^{i\theta(\mathbf{k})} c_k^\dagger |0\rangle$$

Projection methods to choose $\theta(\mathbf{k})$



Spin-Valley Hubbard Model on Triangular Lattice:

$$H = H_t + H_U + \dots$$

Building block: $c_{i;a\sigma}$

H_U : on-site Hubbard U

$a=+,-$ valley index

$\sigma = \uparrow, \downarrow$ spin index

... terms:

on-site Inter-valley Hund's $-J_H \mathbf{S}_+ \cdot \mathbf{S}_-$

Nearest neighbor Hund's: Ferromagnetic spin-spin coupling

both at order 1% U

U is around 25 meV if dielectric constant is 8.

Trivial Side

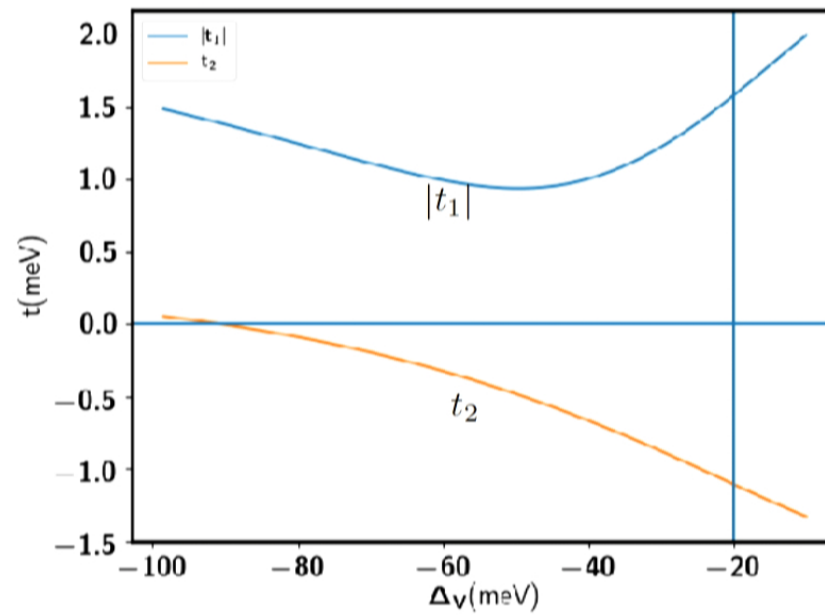
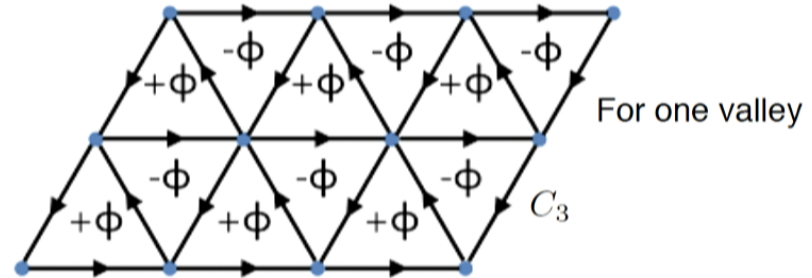
Valley-Contrasting Flux



$$\Phi \sim \frac{\pi}{2} - \pi$$

Time Reversal: $\Phi_+ = -\Phi_-$

Hopping Pattern



Vertical line: estimation for potential difference to make bandwidth= $U=25$ meV



Next Part

I. Trivial Side: Spin-Valley Hubbard Model

(1) Spin model for the strong Mott insulators.

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II. Chern Side: C=3,-3 Narrow Bands

(1) Flat band limit: Chern Insulator with $\sigma_{xy} = 3\frac{e^2}{h}$

(2) Fractional Filling at Flat Band limit

Trivial Side

Strong Mott Region: $U \gg t$

Number of particles at each site $\nu_T = 1, 2$

Feng Wang et.al. arxiv:1803.0198

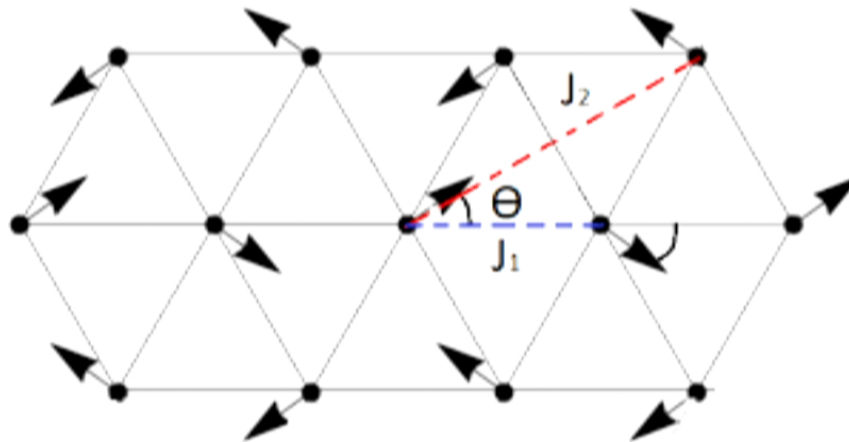


Charge is frozen at each site.

Low energy only has spin-valley neutral degree of freedom

t/U expansion: $J_1 - J_2$ spin-valley model

$SU(4)$ is strongly broken to $U(2)_+ \times U(2)_-$

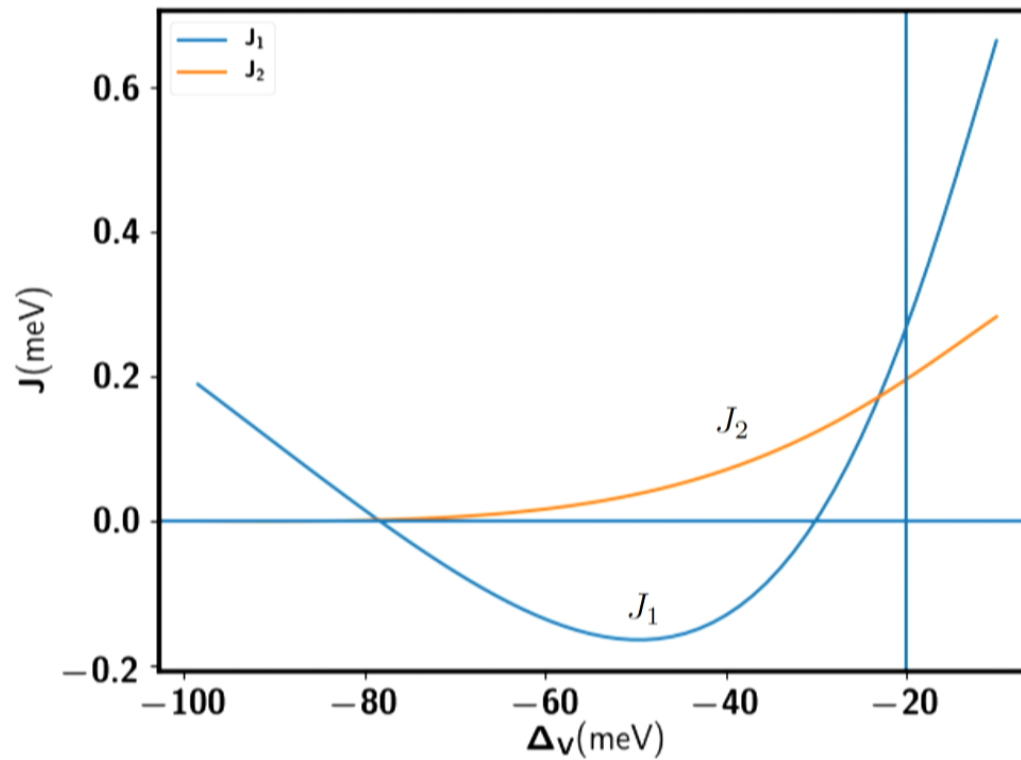


Trivial Side

Ferromagnetic vs Anti-Ferromagnetic

Two Contributions for : J_1

1. Inter-site Hund's: 1% of U 2. Super-exchange: $\frac{t^2}{U} \sim 0.01U$



Trivial Side

Strong Mott Insulator: Some Special Cases

DMRG is needed to map out the phase diagram.

👉 **Ferromagnetic Region:** $J_1 < 0, J_2 \sim 0$

$\nu_T = 2$: Spin polarized; Valley singlet

$\nu_T = 1$: Spin and valley polarized

Anti-Ferromagnetic Region:

Valley Zeeman coupling: $-g\mu_B B\tau_z$ $g \approx 45$

$B=0.2$ T: valley splitting 0.4 meV; at order of J

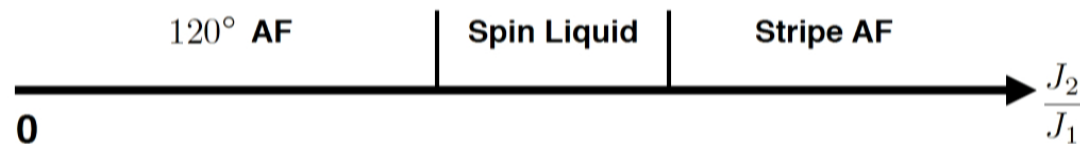
After Valley polarization: spin 1/2 model

Possible U(1) Dirac Spin Liquid!

Z. Zhu and S. R. White, Phys. Rev. B 92, 041105 (2015).

W.-J. Hu, et.al., Phys. Rev. B 92, 140403 (2015)

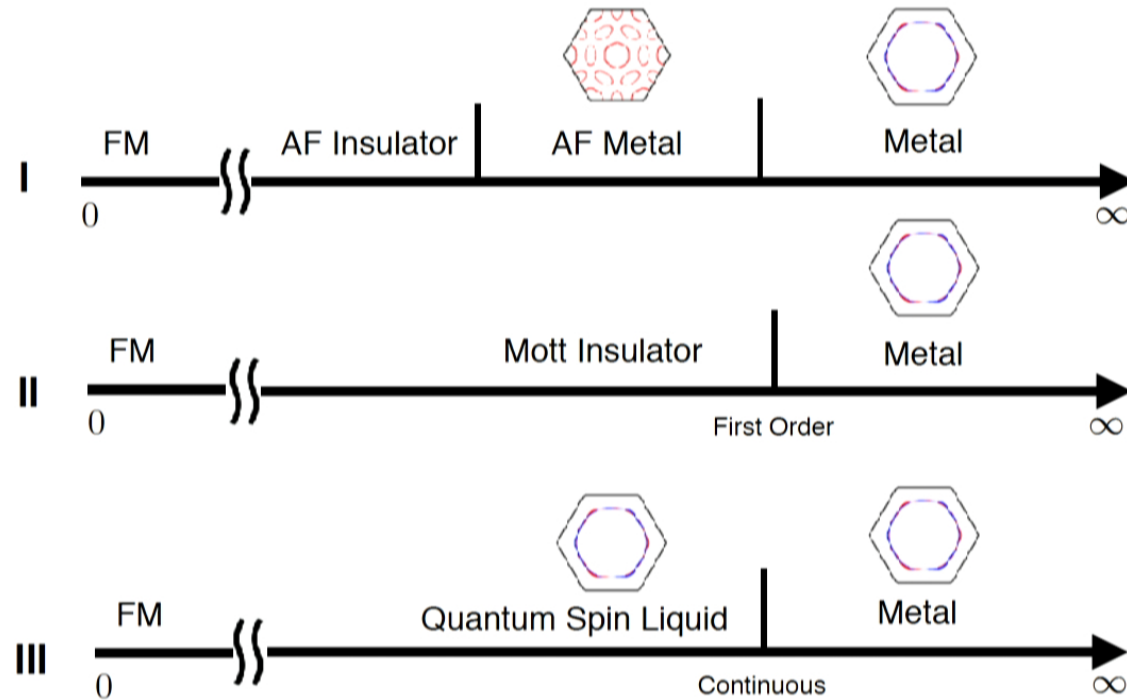
Yasir Iqbal, et.al., Phys. Rev. B 93, 144411 (2016)



Trivial Side

Weak Mott Insulator: Metal-Insulator Transition

Possible Scenarios:



$$\frac{t}{U}$$

Organics: pressure tuned transition

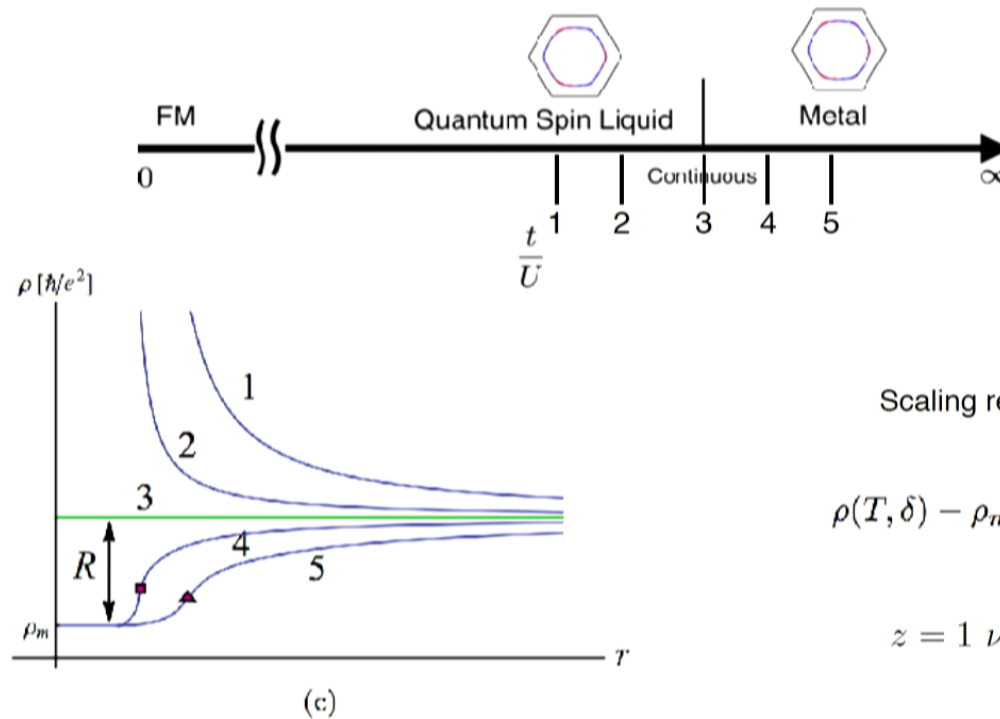
T. Furukawa, et. al. Nature Physics 11, 221 EP (2015)

Trivial Side

Continuous Mott Transition

T. Senthil Phys. Rev. B **78**, 045109 (2008)

Experimental Detection: Universal Residual Resistivity Jump



Scaling relation:

$$\rho(T, \delta) - \rho_m = \frac{\hbar}{e^2} G \left(\frac{\delta^{z\nu}}{T} \right)$$

$$z = 1 \quad \nu \approx 0.672$$

Witczak-Krempa et.al. , Phys. Rev. B 86, 245102 (2012)

Chern Side

General Remarks



Y-H. Zhang, D. Mao, C. Yuan, P. Jarillo-Herrero, T. Senthil arxiv:1805.08232

Other Materials: $C = \pm 1, \pm 2, \pm 3$

Twisted Bilayer/Bilayer, Trilayer/Bilayer, Trilayer/Trilayer

Two control variables: potential difference, twist angle: magic angle

Time Reversal Invariant: Two valleys have opposite Chern numbers.

Free fermion level: quantum Valley Hall effect

Strongly Correlated Effects:

Integer ν_T : correlated insulators

Fractional filling?

Chern Side

Quantum Hall Ferromagnetism: Chern Insulator

Focus on $\nu_T = 1$ flat band limit:

Flat band limit: similar to quantum Hall bilayers from opposite magnetic fields.

Two candidates:

Quantum Anomalous Hall Insulator $\sigma_{xy} = 3 \frac{e^2}{h}$

Inter-valley-coherent order $\langle c_+^\dagger c_- \rangle \neq 0$

Energy difference:

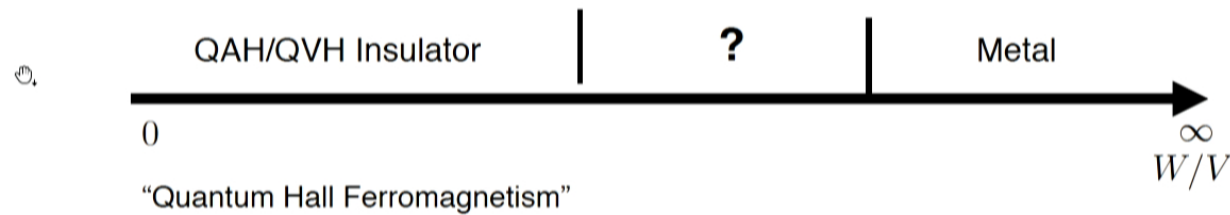
Hartree Fock

$$\begin{aligned} \Delta E_{IVC} &= H_{F,IVC} - H_{F,0} + H_{K,IVC} - H_{K,0} \\ &= \frac{1}{4} \sum_{\mathbf{k}, \mathbf{q}} V(\mathbf{q}) (|\lambda_+(\mathbf{k}, \mathbf{q})| - |\lambda_-(\mathbf{k}, \mathbf{q})|)^2 \\ &> 0 \end{aligned} \quad (26)$$

Chern insulator is favored at flat band limit!

Verified by ED calculation, Cécile Repellin, Y-H Zhang, T. Senthil, to appear

Beyond Mott Insulator



What happens at inter-mediate regime?

No simple Lattice model: Wannier Obstruction.

Charge can not be frozen; Non-trivial entanglement between charge-spin

Is a "Quantum Hall Spin Liquid" possible?

A completely new theoretical framework is needed!

A new representation, for example Landau Level?

Two Landau Levels with opposite chiralities.

Summary

We provide models for Trilayer Graphene moire superlattice:

Two sides:

spin-valley Hubbard Model

Nearly flat Chern bands

Correlated Insulators

Trivial Side:

J1-J2 spin-valley model

continuous metal-insulator transition

Chern Side:

Flat band limit: Chern insulator

Inter-mediate Region: need new theoretical framework

Ongoing and Future Work



I. Numerics at integer fillings for both sides.

II. Doped Mott insulator: t-J model

Superconductor

Pseudogap and Strange metal?

III. Fractional filling of Chern side:

Fractional Chern insulator (FCI) or Fractional topological insulator?

Transition between FCI and Fermi Liquid ?

IV. Quantum Hall Bilayers with Opposite Magnetic fields

Composite Fermion Insulator in Opposite-Fields Quantum Hall Bilayers,

Ya-Hui Zhang, arxiv:1810. 03600